From 3×2pt to 4×3pt

Calibrating galaxy bias and intrinsic alignments with galaxy clustering and weak lensing bispectra

Raphaël Kou, with Antony Lewis

IRAP, Toulouse · University of Sussex

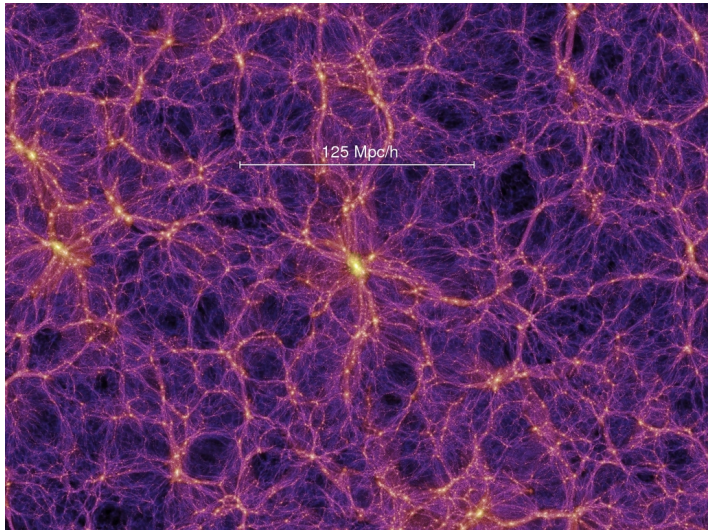
Colloque national du WG Dark Energy 2026 · 4th November 2026

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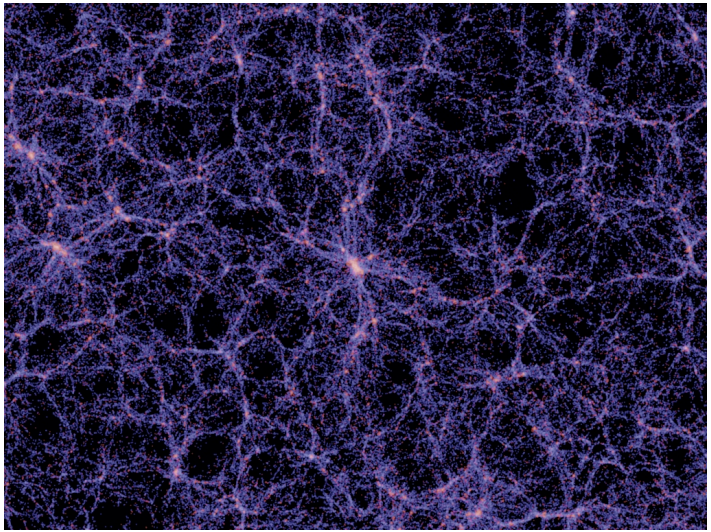
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Motivation

Credit: Springel et al. 2005 (Millennium Simulation)



Dark matter field



Galaxy field

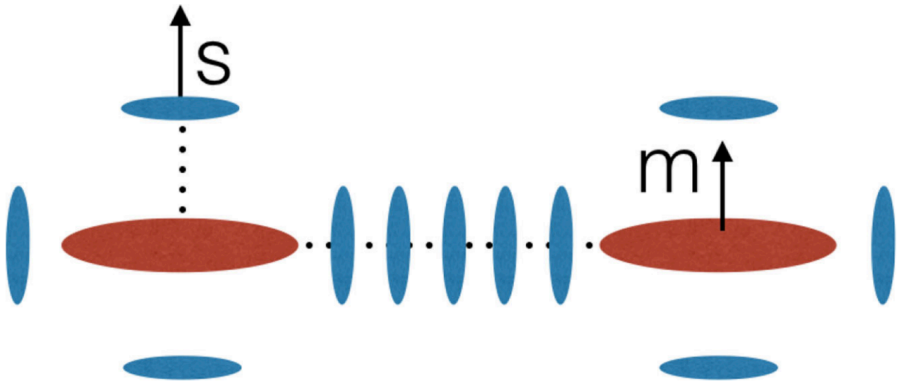
- Galaxies are biased tracers of matter.

$$\delta_g = b_1\delta + \frac{b_2}{2}\delta^2 + \frac{b_s}{2}s^2 + \frac{b_{3\mathrm{nl}}}{2}\psi_{\mathrm{nl}} + \frac{b_{k^2}}{2}\nabla^2\delta$$

Motivation

$$\gamma_{ij}^{I} = C_1 s_{ij} + C_{1\delta}\,\delta\, s_{ij} + C_2 \left(s_{ik} s_{kj} - \frac{1}{3}\delta_{ij}^{\mathrm{K}} s^2 \right)$$

Credit: Chisari et al. 2015



$$C_1(z) = -A_1 \frac{\bar{C}_1 \rho_{\mathrm{crit}} \Omega_{\mathrm{m}}}{D(z)} \left(\frac{1+z}{1+z_{\mathrm{piv}}} \right)^{\eta_1}$$

$$C_{1\delta}(z) = A_\delta\, C_1(z)$$

$$C_2(z) = 5A_2 \frac{\bar{C}_1 \rho_{\mathrm{crit}} \Omega_{\mathrm{m}}}{D^2(z)} \left(\frac{1+z}{1+z_{\mathrm{piv}}} \right)^{\eta_2}$$

- Intrinsic alignment effect(s) limit weak lensing constraining power.

Second-order parameters enter power spectra only through loops, but bispectra already at tree level

$$P^{XY}(k) = P^{XY}_{\rm L}(k) + \underbrace{P^{XY}_{22}(k) + P^{XY}_{13}(k)}$$

1-loop contribution:
where higher-order parameters enter

$$B^{XYZ}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = 2K^X_1(\mathbf{k}_1)K^Y_1(\mathbf{k}_2)\underbrace{K^Z_2(\mathbf{k}_1, \mathbf{k}_2)}P_{\rm L}(k_1)P_{\rm L}(k_2) + 2 \text{ cyclic perms.}$$

Quadratic kernel:
Where second-order parameters enter

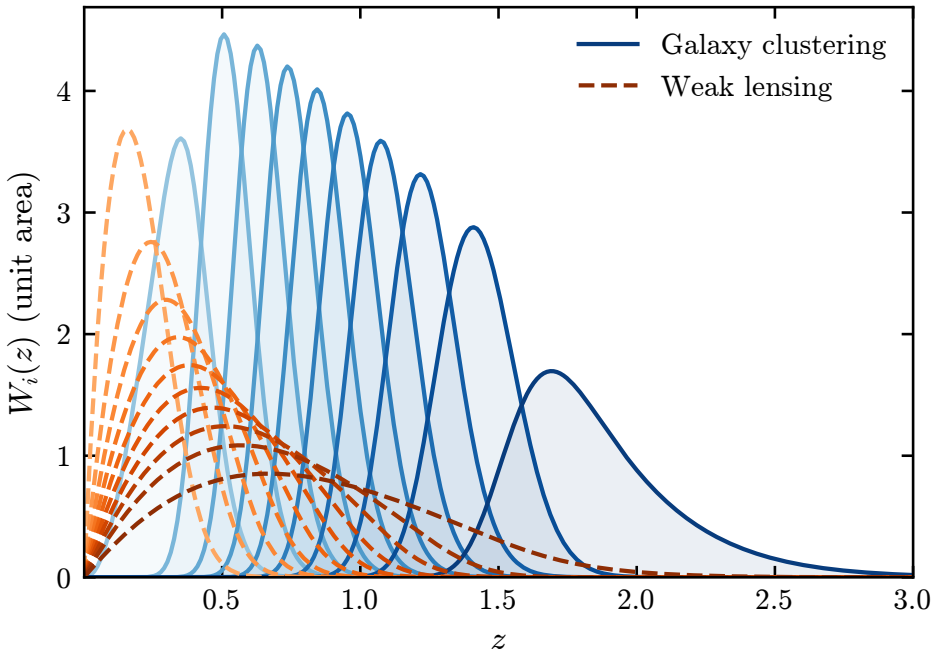
4×3pt: all auto- and cross-bispectra of clustering and lensing

3×2pt	GG	GL	LL	
4×3pt	GGG	GGL	GLL	LLL

G: galaxy clustering L: weak lensing (shear + intrinsic alignments)

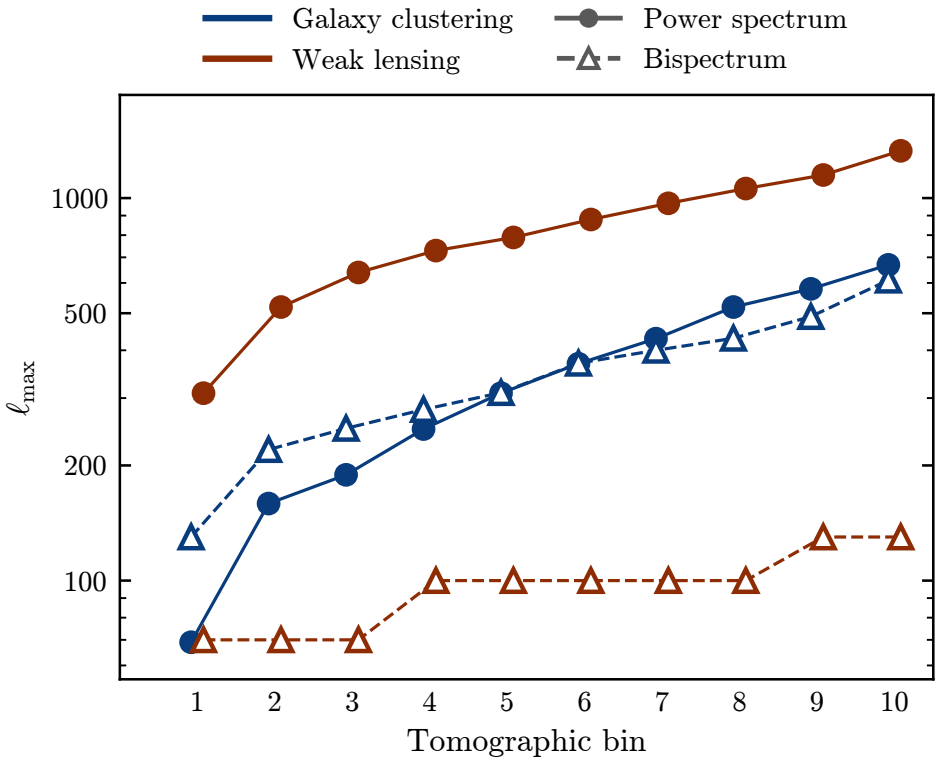
- $$C^{XY}_{\ell,ij} = \int_0^{z_{\max}} \mathrm{d}z \, \frac{H(z)}{c\,\chi^2(z)} \, W^i_X(z) \, W^j_Y(z) \, P_{XY}\left(k = \frac{\ell + 1/2}{\chi(z)}, z\right)$$
- $$b^{XYZ}_{\ell_1\ell_2\ell_3} = \int_0^{z_{\max}} \mathrm{d}z \, \frac{H^2(z)}{c^2\,\chi^4(z)} \, W_X(z) \, W_Y(z) \, W_Z(z) \, B_{XYZ}\left(k_1, k_2, k_3, z\right)$$

Fisher forecast for a Euclid-like survey, combined with the CMB



- 15 000 deg², 10 equipopulated bins, z < 2.5
- Cosmology: w_0w_aCDM
- Nuisance: 5 bias parameters per bin, 5 TATT parameters, baryonic feedback $\log(T_{AGN}/K)$

Conservative scale cuts, set by the survey's precision



Scale cuts criteria:

- Bispectra and GC power spectra: Bias on the amplitude below 0.3σ
- WL power spectra: 95% of the signal coming from $k < 3h\text{Mpc}^{-1}$

Results using the three lowest redshift bins

The gain comes from calibrating second-order galaxy bias

Gain from bispectra	All marginalised
H_0	23%
w_0	18%
w_a	13%
$\log(T_{AGN}/K)$	14%
Figure of merit	**×1.33**

Gain from bispectra	All marginalised
A_2	48 %
A_δ	39 %
η_2	34 %
b_2	92-98%
b_s	89-97%

The gain comes from calibrating second-order galaxy bias

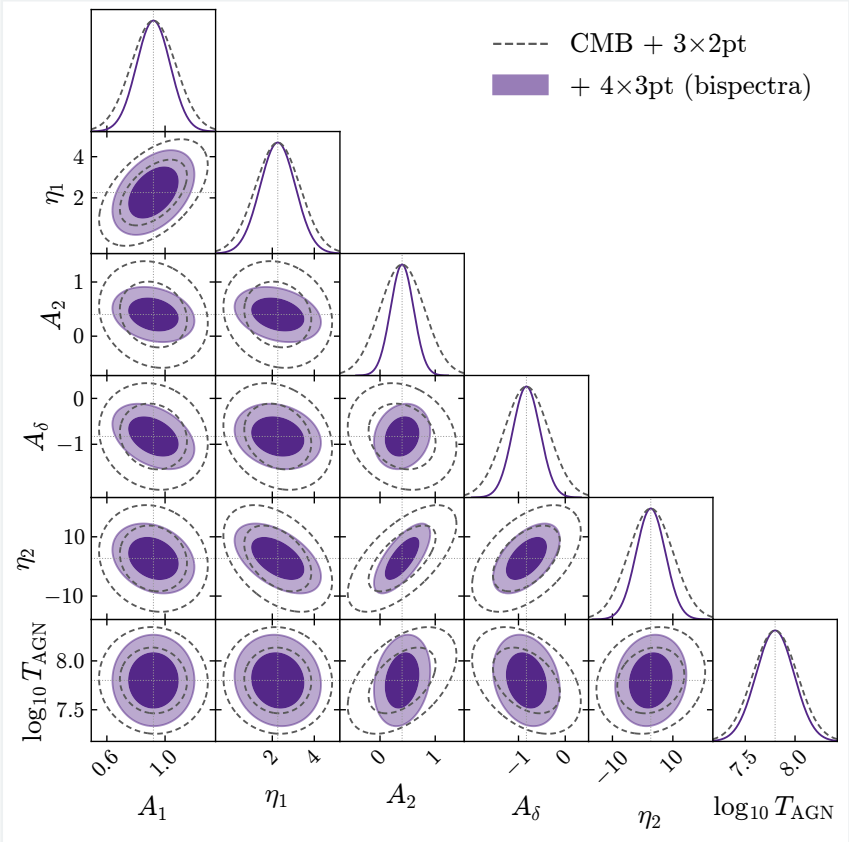
Gain from bispectra	All marginalised	Fix bias + IA
H_0	23%	4%
w_0	18%	3%
w_a	13%	3%
$\log(T_{AGN}/K)$	14%	1%
Figure of merit	**×1.33**	×1.07

The gain comes from calibrating second-order galaxy bias

Gain from bispectra	All marginalised	Fix bias + IA	Fix bias	Fix IA
H_0	23%	4%	**6%**	23%
w_0	18%	3%	**3%**	18%
w_a	13%	3%	**2%**	13%
$\log(T_{AGN}/K)$	14%	1%	14%	1%
Figure of merit	**×1.33**	×1.07	**×1.09**	×1.31

Fixing the second-order bias removes the dark energy gain; fixing the second-order IA does not. Baryons behave the other way round.

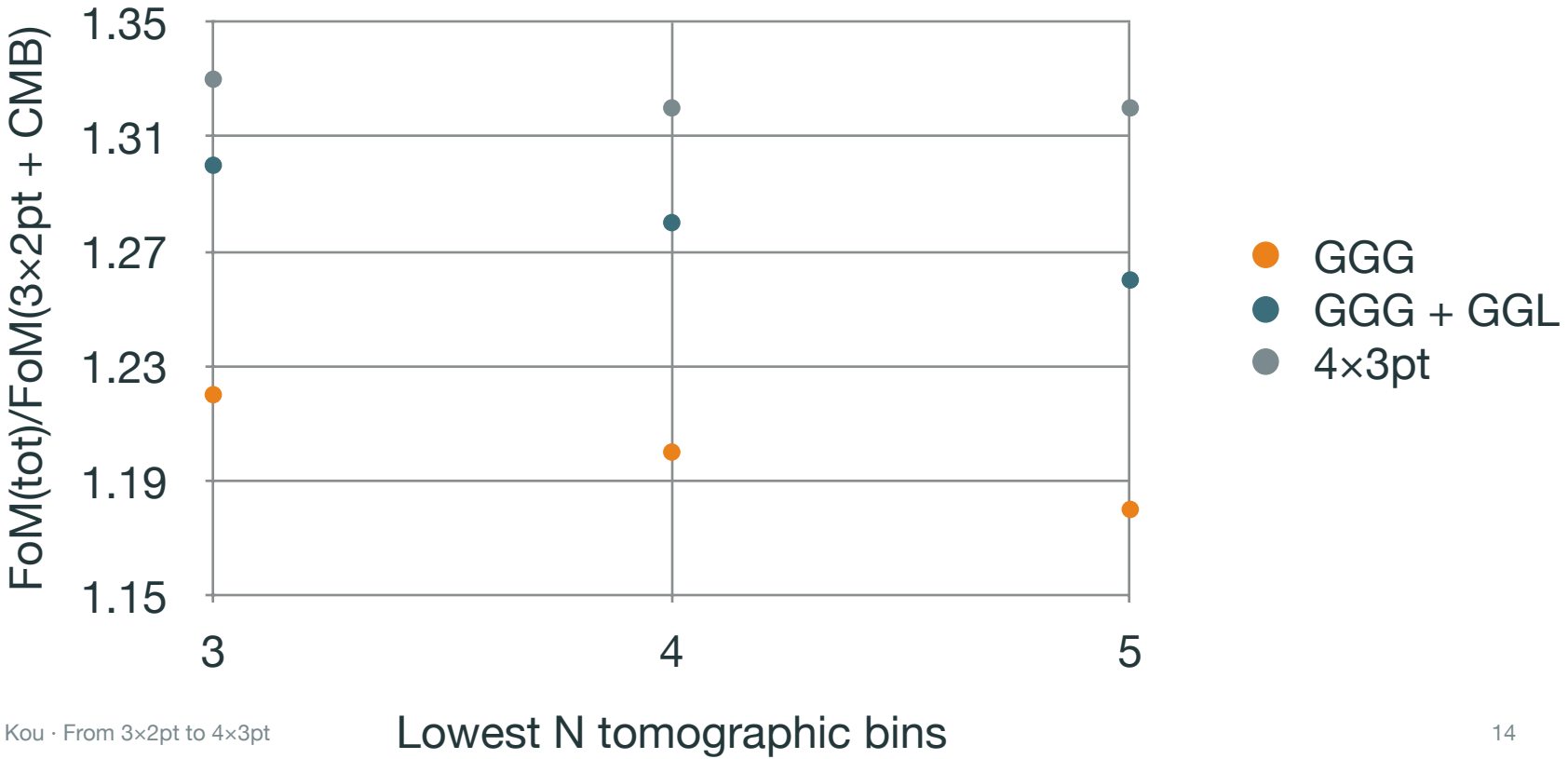
The 4×3pt reduces baryonic feedback — TATT correlations



The 4×3pt reduces IA/baryonic feedback correlations

Results using up to five redshift bins

Extension to four and five redshift bins



Data compression and extension to 10 bins

Skew-spectra: recipe



- Take maps X, Y, Z
- Filter X and Y with Gaussian filters with angular width θ_X and θ_Y
- Take the product of filtered maps X and Y
- Cross-correlate the resulting map with map Z

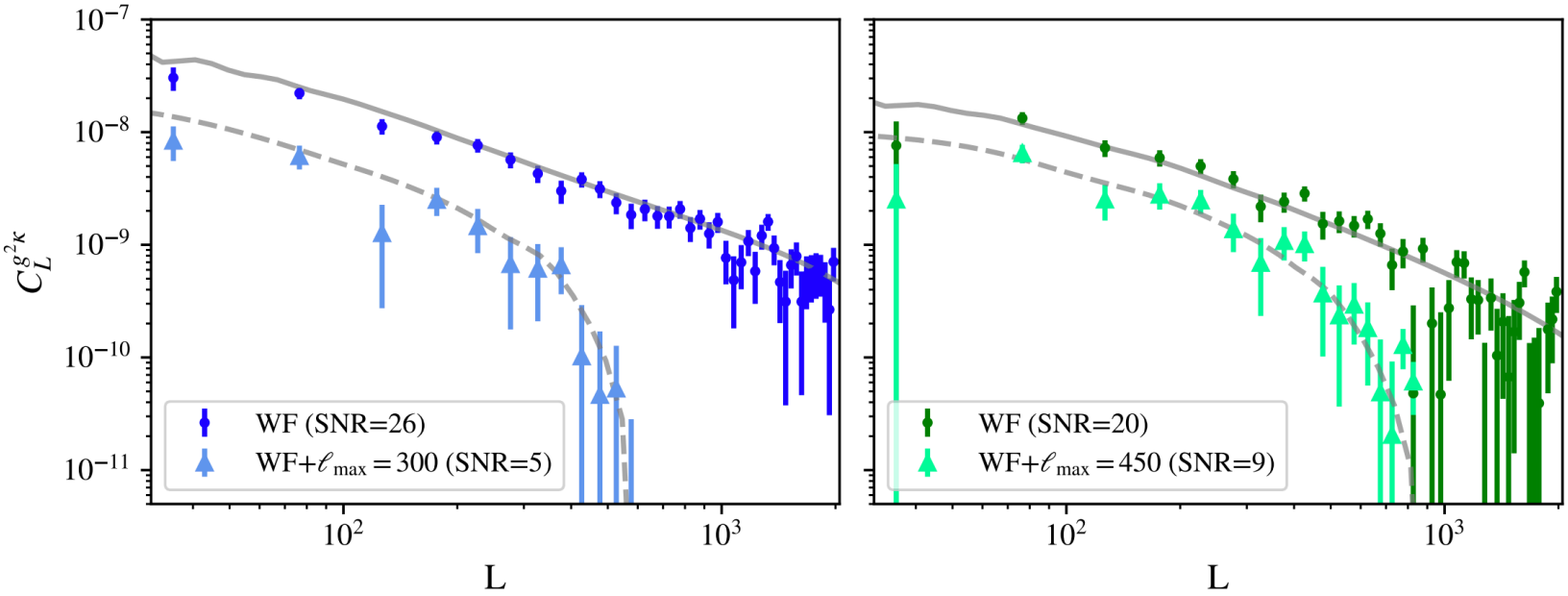
$$S_\ell^{XYZ}(\theta_X,\theta_Y)=\sum_{\ell_1,\ell_2}\sqrt{\frac{(2\ell_1+1)(2\ell_2+1)}{4\pi(2\ell+1)}}\begin{pmatrix}\ell_1 & \ell_2 & \ell\\ 0 & 0 & 0\end{pmatrix}\mathcal{F}_{\ell_1}(\theta_X)\,\mathcal{F}_{\ell_2}(\theta_Y)\,B^{XYZ}_{\ell_1\ell_2\ell}$$

We choose $\theta_X=\dfrac{\lambda_X}{\ell^{XYZ}_{\max}}$, where $\lambda_X\in\Lambda$, a set of allowed filters.

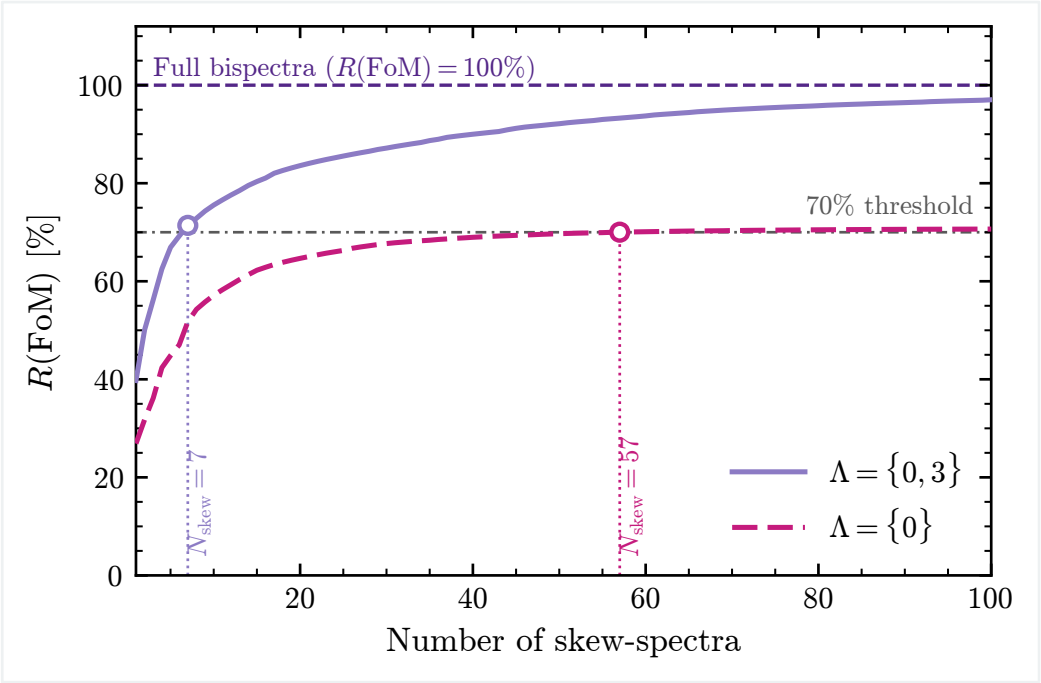
Skew-spectra: recipe

- CMB lensing — galaxy skew-spectrum measured with high significance with Planck/unWISE data

Farren et al (2023)



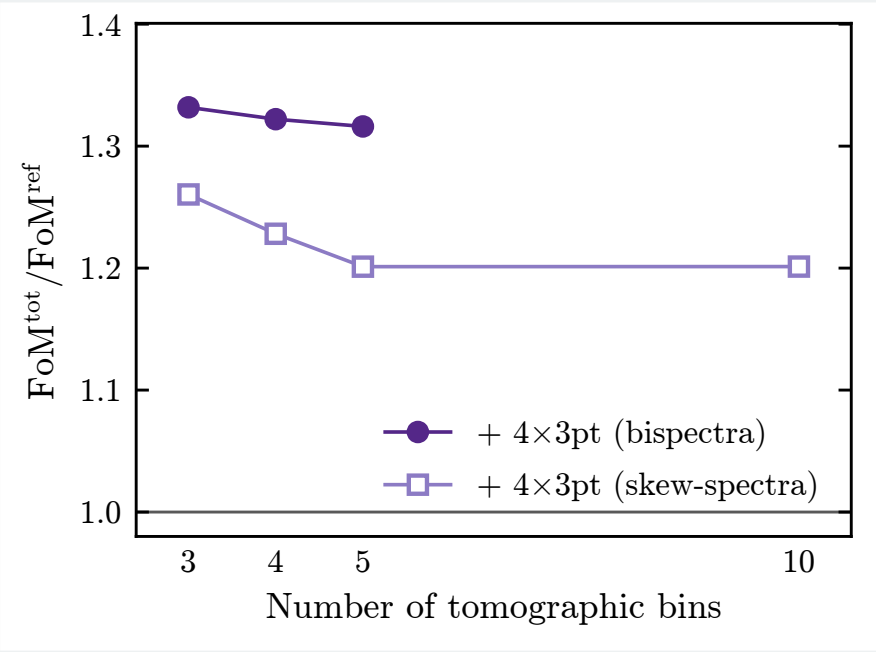
Seven skew-spectra recover about 70% of the gain



- Greedy selection on 3 bins: 7 skew-spectra recover 71%
- Without any filter ($\Lambda = \{0\}$), more than 50 skew-spectra required to reach the same R(FoM).
- All 7 selected skew-spectra are of type GGG or GGL.

(X, Y, Z)	λ_X	λ_Y	R(FoM)
(G_2, G_2, G_2)	3	3	39.8%
(G_2, G_3, G_2)	0	0	50.1%
(G_1, G_1, G_1)	0	0	56.2%
(G_1, G_1, L_3)	0	0	62.5%
(G_2, L_3, G_2)	3	0	66.9%
(G_2, G_3, G_3)	0	0	69.2%
(G_3, G_3, L_3)	3	3	71.4%

10 bins with skew-spectra



- Gain of 20% for 10 bins when using the compression with skew-spectra

Conclusions

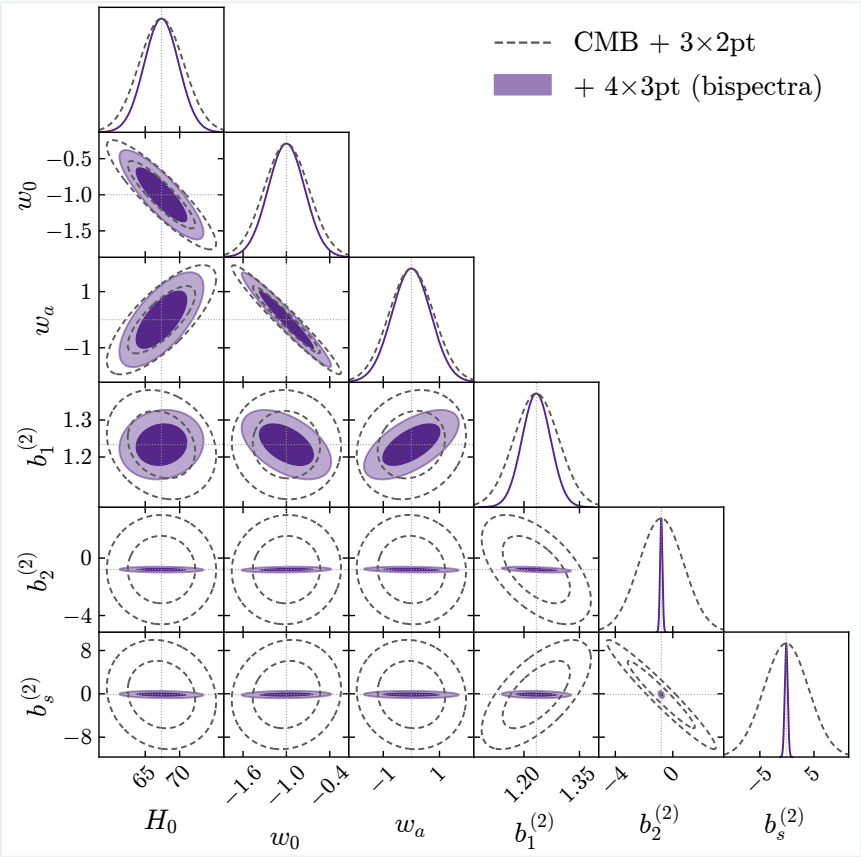
- 4×3pt bispectra improve the dark energy figure of merit by about 30% (CMB + 3×2pt, conservative cuts)
- The gain comes mostly from calibrating second-order galaxy bias with GGG and GGL bispectra
- Skew-spectra make it possible to extend the analysis to 10 tomographic bins with reduced gain (~20%)
- 4×3pt bispectra can also be useful to reduce correlations between IA and baryonic feedback

Questions?

Backup

Backup slides

The 4×3pt breaks degeneracies with second-order nuisance parameters



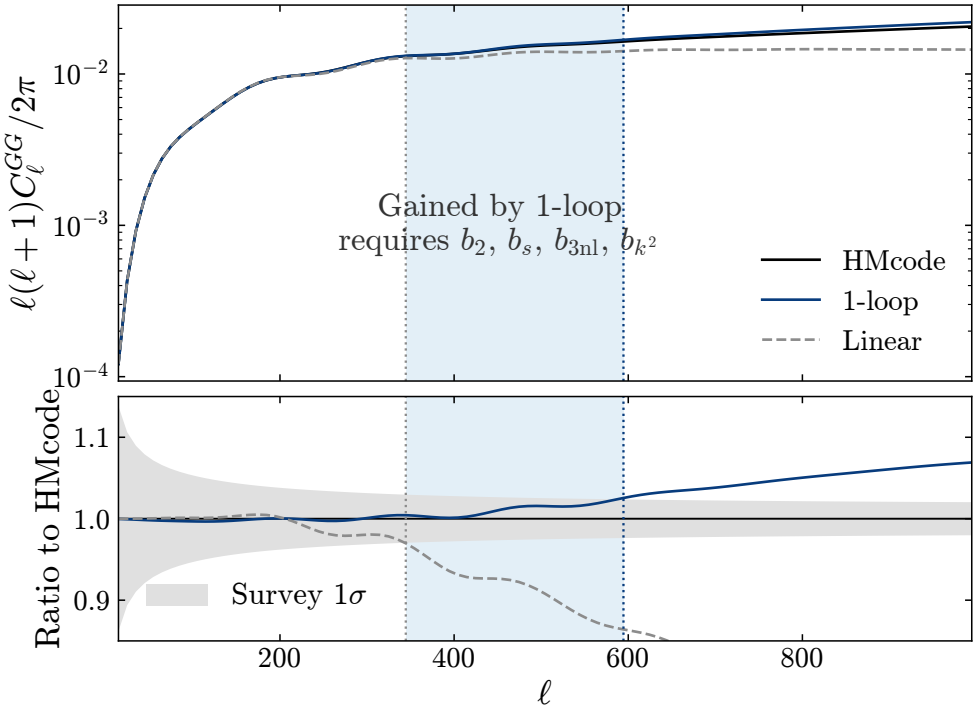
The 4×3pt breaks the b_2-b_s degeneracy

Skew-spectrum classes for N bins

Fields	Bins	Filters λ	Range
G G G	i i i	3, 3	$1 \le i \le N$
G G G	i j i	0, 0	$i < j$
G G G	i i i	0, 0	$1 \le i \le N$
G G L	i i j	0, 0	$i < j$
G L G	i j i	3, 0	$i < j$
G G G	i j j	0, 0	$i < j$
G G L	i i i	3, 3	$1 \le i \le N$

$3N + 2N(N-1)$ skew-spectra: 21, 36, 55 and 210 for 3, 4, 5 and 10 bins

Smaller scales need 1-loop models, and 1-loop models need nuisance parameters



- Much of the information sits on mildly non-linear scales
- 1-loop galaxy clustering adds higher-order bias:

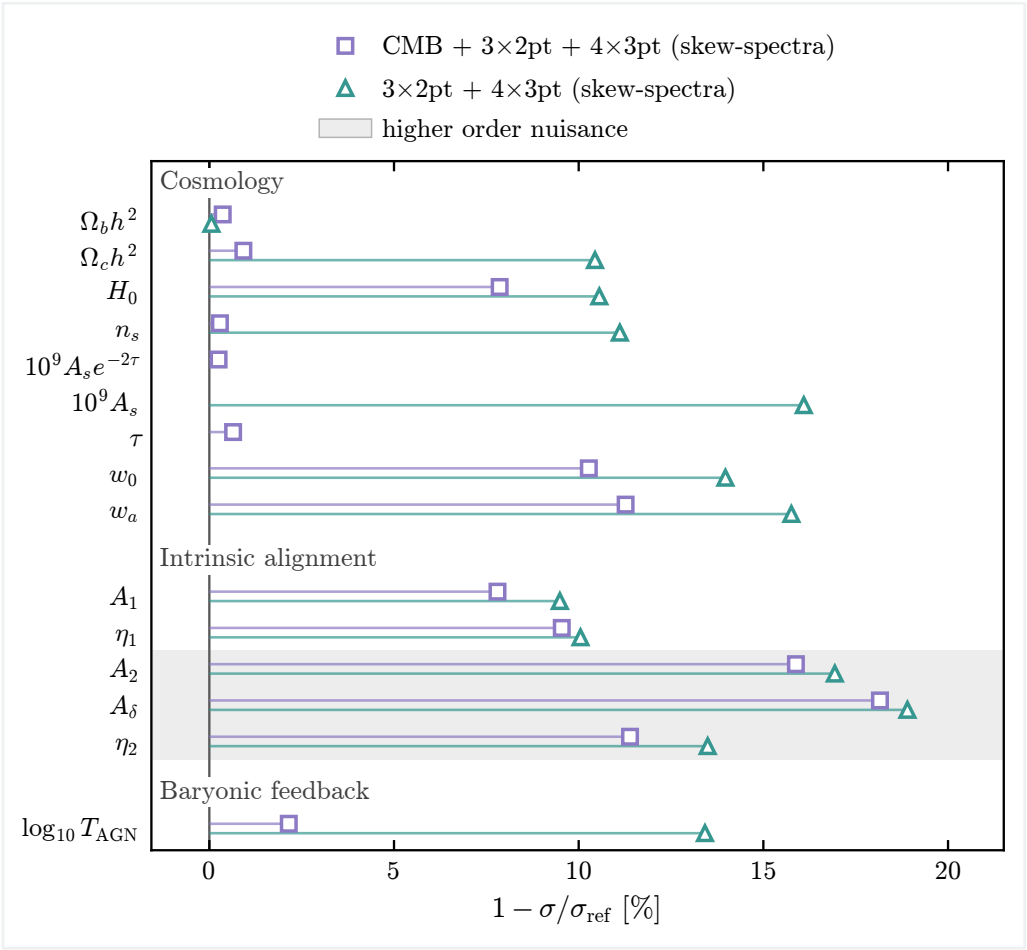
$$\delta_g = b_1\delta + \frac{b_2}{2}\delta^2 + \frac{b_s}{2}s^2 + \frac{b_{3\mathrm{nl}}}{2}\psi_{\mathrm{nl}} + \frac{b_{k^2}}{2}\nabla^2\delta$$

- Non-linear IA (TATT):

$$\gamma_{ij}^I = C_1 s_{ij} + C_{1\delta}\,\delta\, s_{ij} + C_2\left(s_{ik}s_{kj} - \frac{1}{3}\delta_{ij}^{\mathrm{K}} s^2\right)$$

- Marginalising over them costs cosmological information

10 bins with skew-spectra: FoM ×1.20 with the CMB, ×1.37 without



Intrinsic alignment — baryonic feedback correlation

