

10e AG du GdR Ondes gravitationnelles · Lyon 2026

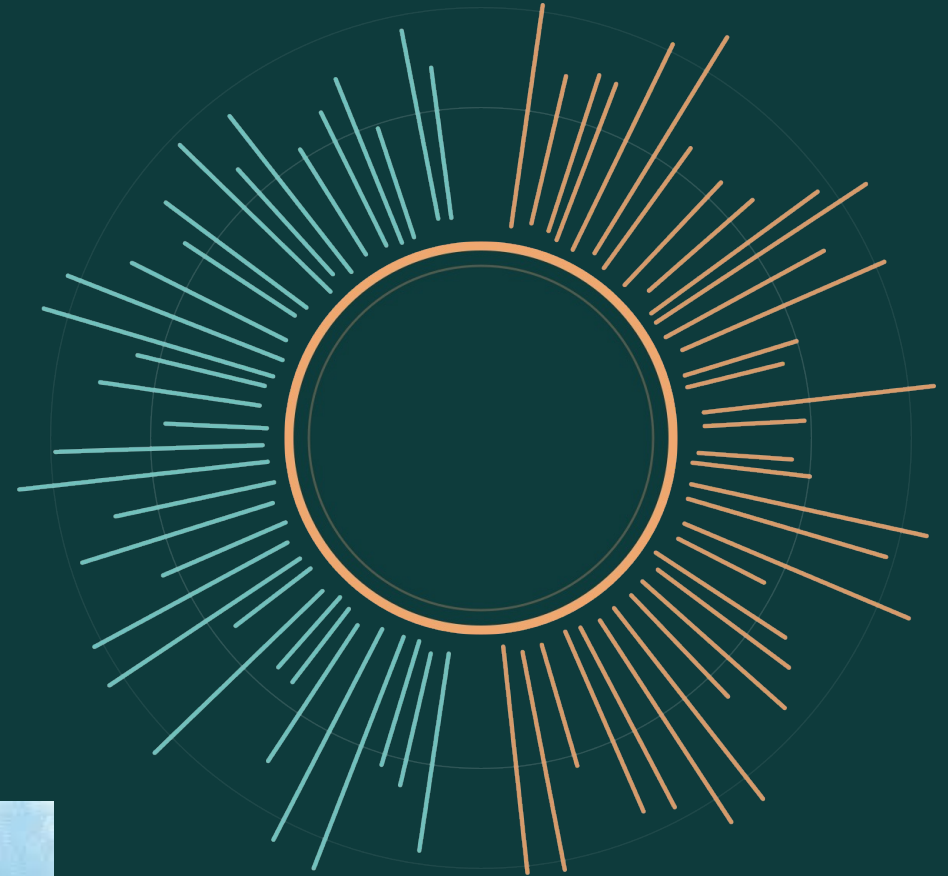
New aspects of spontaneous scalarization of black holes beyond General Relativity

Zakaria Belkhadria

IP2I Lyon · Université Claude Bernard Lyon 1

Work carried out at Université de Genève (Cosmology Group, GWSC) and Università di Cagliari (DMI, INFN)

Belkhadria & Mignemi · Phys. Rev. D 112, 044015 (2025) · arXiv:2506.12137



Outline

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GR and no-hair

theorems · how to grow hair

3

Scalarization

mechanism · landscape

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NEW: curvature + matter

motivation · model · results

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Beyond GR

why · new degrees of freedom

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Curvature-induced (EsGB)

threshold · instabilities · cures

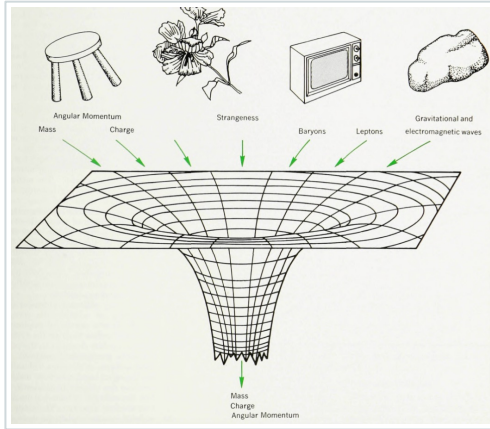
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Conclusions and outlook

summary · gravitational-wave roadmap

Black holes in GR have no hair, and there are two ways to grow some

Israel '67 · Carter '71 · Wheeler & Ruffini '71 · Hawking '72 · Bekenstein '72



Credit: Ruffini & Wheeler 1971

Stationary black holes in GR are fixed by **M, Q, J** only. Every other charge is radiated away in the collapse.

No general proof, only case-by-case theorems

- Bekenstein 1971–72: massive or minimal scalar
- Hawking 1972: Brans–Dicke theory
- Bekenstein 1995: novel no-hair theorem
- Sotiriou & Faraoni 2011: $f(R)$ gravity
- Hui & Nicolis 2012: shift-symmetric Horndeski

Modify gravity

generalise the gravitational action

- Higher-curvature terms: Gauss–Bonnet, R^2 , $f(R)$
- New propagating degrees of freedom: Horndeski, scalar-tensor theories
- Drop assumptions: locality, four dimensions, ...

Add matter fields

extend the matter sector

- Couple a scalar: Bekenstein '71, Damour & Esposito-Farèse '93
- Couple a Maxwell or Yang–Mills field
- Allow non-minimal couplings between matter and the scalar

Our route: one scalar field with non-minimal couplings, sourced by curvature and by matter at the same time.

GR passes every test, and that is exactly why we push further

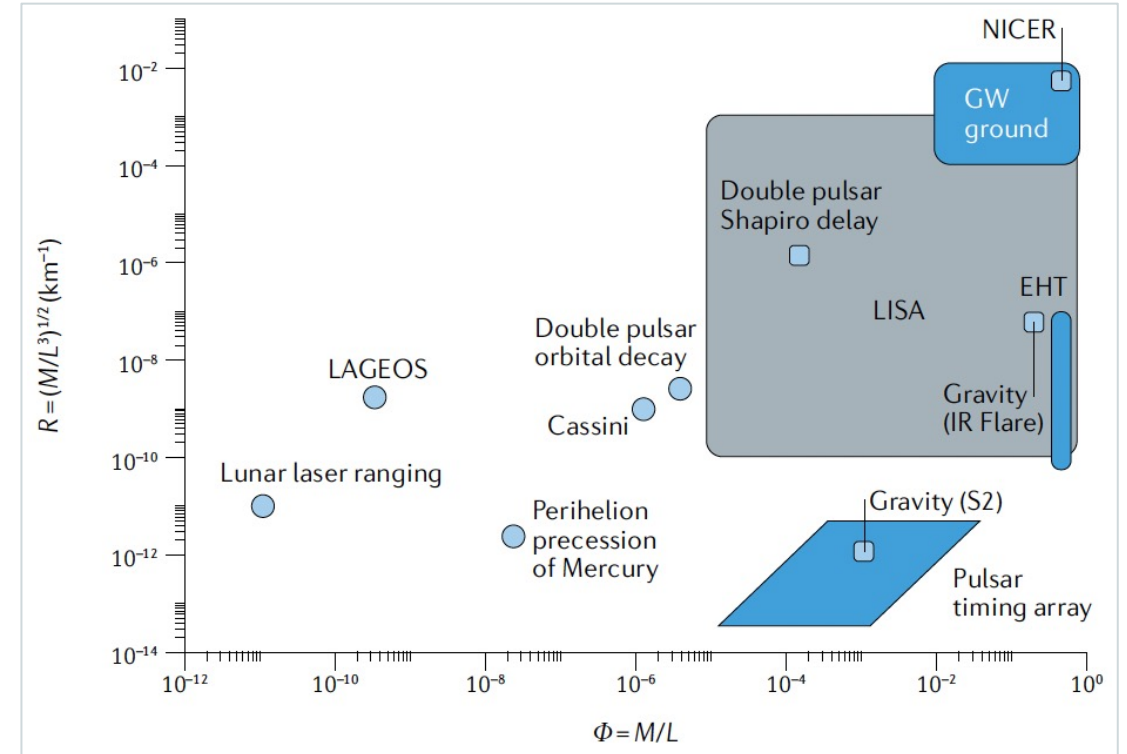
A golden age for strong-field gravity, and good reasons to look beyond it

What we have seen

- LIGO 2015: GW150914, the first black-hole merger
- EHT 2019 and 2022: the shadows of M87* and Sgr A*
- GR passes the solar system, pulsars, gravitational waves and cosmology

Why look beyond it

- Theoretical gaps: renormalizability, singularities, quantum gravity
- Dark sector: dark matter, cosmic acceleration, the Λ problem
- New regimes: strong-field gravity is barely tested; ET, LISA and Cosmic Explorer will reach it

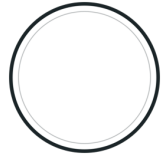


Curvature and potential probed by current and future experiments. Bailes et al., Nat. Rev. Phys. (2021)

A scalar field ϕ is the simplest new degree of freedom (low-energy strings, Kaluza–Klein, dark-energy candidates). Does it hair the black hole? Is there a mechanism that triggers it dynamically? What is the signature?

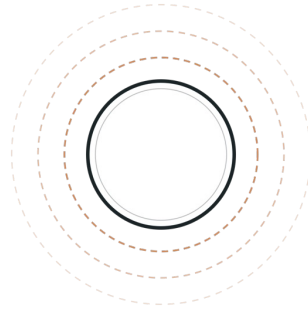
Spontaneous scalarization: a tachyonic instability of the GR solution

Damour & Esposito-Farèse (1993) for neutron stars · black holes since 2017



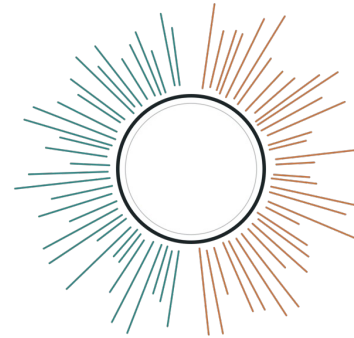
Scalar-free black hole

$$\phi = 0$$



Tachyonic mode

$$\mu_{\text{eff}}^2 < 0$$



Scalarized branch

$$\phi \neq 0$$

The idea: GR is recovered in the weak field, but in the strong field a tachyonic instability sources a non-trivial scalar profile: hairy compact objects.

1 Linearise the KG equation

$$(\square - \mu_{\text{eff}}^2) \delta\phi = 0$$

GR remains a solution ($\phi = 0$), but it is tested against perturbations on its own background.

2 Trigger: $\mu_{\text{eff}}^2 < 0$

Sourced by a curvature invariant or a matter source through a coupling $f(\phi)$. The negative region must be strong enough: a negative local mass is not yet the instability.

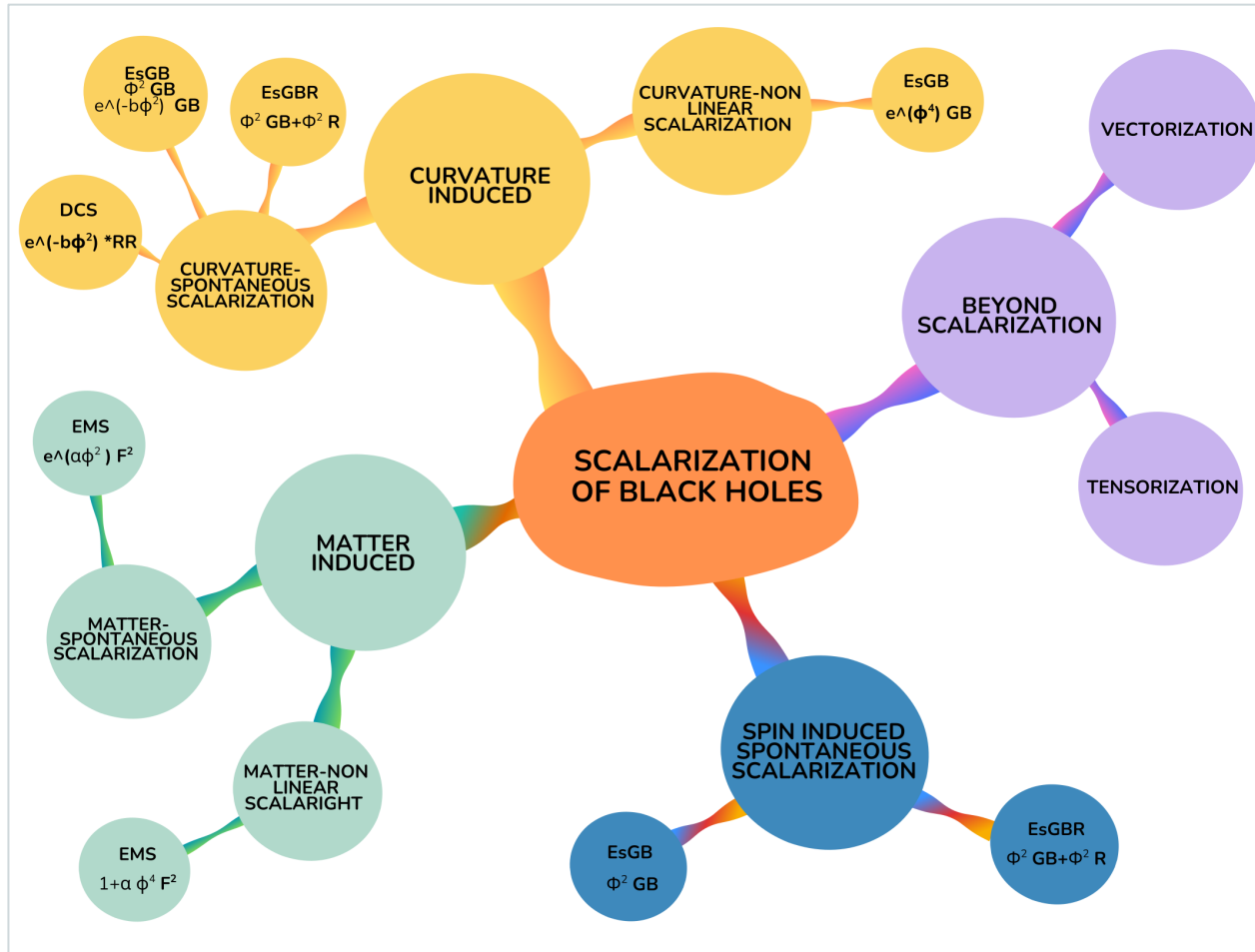
3 A new hairy branch

Bifurcates from the GR branch at a critical mass or charge. Finding it does not yet prove it is the stable dynamical end point.

Weak field: identical to GR. Strong field: a scalar profile appears spontaneously, so the mechanism survives every weak-field test.

Scalarization models: the landscape

Active research · many sources, many mechanisms



Curvature-induced

EsGB · EsGB-Ricci · ϕ^4 self-interaction

Matter-induced

Einstein-Maxwell-scalar (EMS)

Spin-induced

rotating black holes, negative Gauss–Bonnet coupling

Beyond scalarization

vectorization · tensorization

Our focus: curvature and matter, in one action

Einstein-scalar-Gauss–Bonnet: the scalarized family bifurcates from GR

Hairy black holes in EsGB sorted by the choice of $f(\phi)$

Dilatonic GB

$$f(\phi) = \eta e^{\phi}$$

GR is NOT a solution

Mignemi & Stewart 1993 · Kanti et al. 1996

Shift-symmetric GB

$$f(\phi) = \eta \phi$$

GR is NOT a solution

Sotiriou & Zhou 2014

Scalarized BH

$$f(\phi) = \frac{\eta}{2} \phi^2, \quad \eta e^{-\phi^2}$$

GR IS a solution

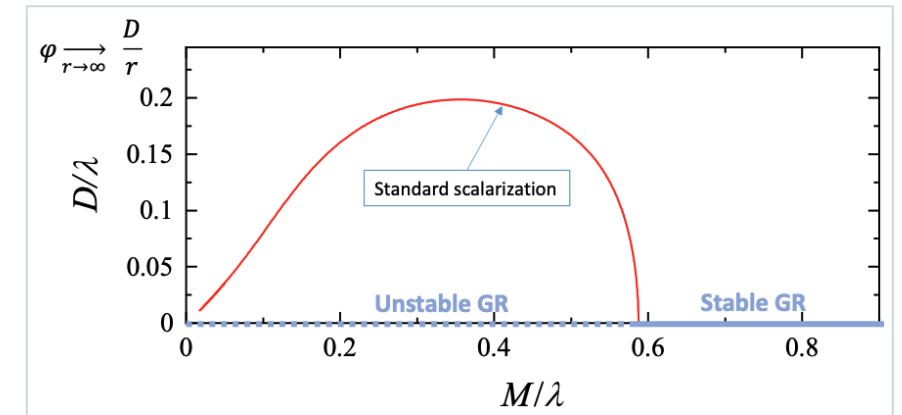
Silva et al. 2017 · Doneva & Yazadjiev 2017

Scalar–Gauss–Bonnet action, linearised around $\phi = 0$

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[R - 2\nabla_\mu \phi \nabla^\mu \phi + f(\phi) \mathcal{G} \right]$$

$$(\square - \mu_{\text{eff}}^2) \delta\phi = 0, \quad \mu_{\text{eff}}^2 = -\frac{1}{4} f_{,\phi\phi}(0) \mathcal{G} = -\frac{12 \eta M^2}{r^6} \quad (\text{Schwarzschild})$$

$f(0) = 0$ and $f'(0) = 0 \rightarrow$ GR ($\phi = 0$) is a solution. $f''(0) \neq 0 \rightarrow$ tachyonic instability where $\mu_{\text{eff}}^2 < 0$.



Scalar charge D against mass M/λ : the hairy branch (red) bifurcates from Schwarzschild at $\eta/M^2 \approx 2.9$ and exists where GR is unstable. Doneva & Yazadjiev, PRL (2018)

The scalarized family is the one we build on: GR is a solution, and it loses stability at a threshold.

Scalarized EsGB black holes are unstable, and the known cures change the theory

Motivation for a richer framework

$\ell = 0$ Radial instability

EsGB with quadratic $f(\phi) = \eta\phi^2/2$: every scalarized solution is radially unstable.

Blázquez-Salcedo et al. 2018 · arXiv:1805.05755

$\ell = 2$ Quadrupolar instability

EsGB-Ricci (and EsGB): quadrupolar perturbation modes can go unstable.

Kleihaus, Kunz, Utermöhlen & Berti 2023 · arXiv:2303.04107

$\ell \gg 1$ Gradient instability

EsGB, any coupling: the sound speed of angular even-parity modes turns imaginary at large ℓ .

Minamitsuji & Mukohyama 2023 · arXiv:2305.05185

PDE Loss of hyperbolicity

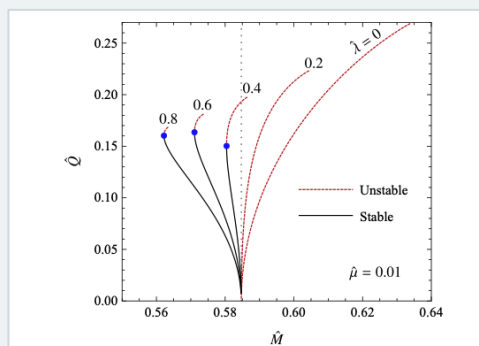
Dynamical evolution breaks down in several regimes: an obstruction for numerical relativity.

arXiv:1805.05755, 2105.08571, 2404.15526

Known cures: extend the gravitational sector

Scalar self-interaction

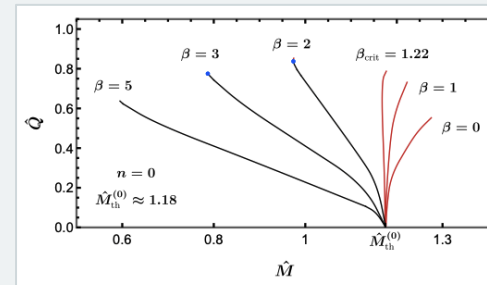
$$f(\phi) = \frac{1}{8} \eta \phi^2 + \frac{1}{16} \zeta \phi^4$$



Stabilises the radial mode. Macedo et al. 2019

Ricci coupling (EsGB-Ricci)

add $\frac{1}{2} \beta R \phi^2$ to the EsGB action: new parameter β



Restores stability, but an $\ell = 2$ instability appears at large β . Antoniou et al. 2021

Both cures act on the gravitational sector. **Can a MATTER coupling do the same job?**

A new framework: scalarization from curvature + matter

Goal

Couple the scalar field non-minimally to BOTH a curvature invariant (Gauss–Bonnet $\mathcal{G}$) AND a U(1) gauge field (Maxwell F^2).

In a nutshell

- unifies curvature- and matter-induced scalarization in one action
- two coupling constants: a richer phase space, multiple thresholds
- a subclass of Maxwell–Horndeski with controlled linear stability



Two sources, one scalar field

Why this combination is physically motivated

Belkhadria & Mignemi · Phys. Rev. D 112, 044015 (2025) · arXiv:2506.12137

Heterotic strings and Kaluza–Klein

Non-minimal couplings to G AND F^2 emerge naturally at low energy; reduction of higher-dimensional Einstein–Maxwell generates exactly these terms. First black holes: Mignemi & Stewart 1993.

Gross & Sloan 1987 · hep-th/9212146 · classical KK literature

Maxwell–Horndeski subset

The linear stability of this class has already been studied: a controlled testing ground for the new solutions.

Kase & Tsujikawa 2023 · arXiv:2301.10362

Scalar–photon coupling

May modify electromagnetic propagation in the same way as gravitational waves, reopening modified-gravity parameter space (with a grain of salt).

Babichev et al. 2024 · arXiv:2407.20339

Does it cure the instabilities?

Many scalarization models are unstable or lose hyperbolicity dynamically. A Maxwell coupling may relax this; the evidence so far is the branch geometry shown next.

arXiv:1805.05755 · 2105.08571 · 2404.15526

Hold on, black holes aren't neutral? We stay agnostic about the $U(1)$: a hidden-sector dark photon (arXiv:1604.07845), magnetic charge (Maldacena 2020), or simply a controlled model with an extremal limit. Q is a global $U(1)$ charge, whatever its origin. Details in backup.

The theory setup

Belkhadria & Mignemi, Phys. Rev. D 112, 044015 (2025) · conventions $4\pi G = 1 = 4\pi\epsilon_0$, signature $(-,+,+,+)$

4D action

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[R - 2\nabla_\mu \phi \nabla^\mu \phi + f(\phi) \mathcal{G} - g(\phi) F_{\mu\nu} F^{\mu\nu} \right]$$

$$\mathcal{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

CURVATURE (Gauss-Bonnet)

$$f(\phi) \mathcal{G}$$

higher-curvature term · classical EFT correction ·
topological in 4D unless coupled to ϕ

MATTER (Maxwell)

$$g(\phi) F_{\mu\nu} F^{\mu\nu}$$

the scalar couples to a U(1) gauge field ·
electric, magnetic or dyonic charge

Field equations

$$G_{\mu\nu} = T_{\mu\nu}^{(\phi)} + T_{\mu\nu}^{(\text{GB})} + T_{\mu\nu}^{(\text{EM})}$$

$$\square\phi + \frac{1}{4} f_{,\phi} \mathcal{G} - \frac{1}{4} g_{,\phi} F_{\mu\nu} F^{\mu\nu} = 0$$

$$\nabla_\nu (g(\phi) F^{\mu\nu}) = 0$$

Novelty: previous models switch on EITHER curvature (the EsGB family) OR matter (EMS). Here both are on at once, opening a two-parameter (η, α) family. Ansatz and boundary conditions in backup.

The scalarization threshold

Linearise the KG equation around the Reissner–Nordström background

Z_2 symmetry ($\phi \rightarrow -\phi$) keeps the scalar-free branch

$$f(0) = 0, \quad g(0) = 1, \quad f'(0) = g'(0) = 0 \quad \Rightarrow \quad \phi = 0 \text{ (Reissner–Nordström) is a solution}$$

Effective mass squared of the perturbation, $(\square - \mu_{\text{eff}}^2) \delta\phi = 0$

$$\mu_{\text{eff}}^2 = -\frac{1}{4} f_{,\phi\phi}(0) \mathcal{G} + \frac{1}{4} g_{,\phi\phi}(0) F_{\mu\nu} F^{\mu\nu}$$

Our concrete choice

$$f(\phi) = \frac{\eta}{2} \phi^2, \quad g(\phi) = 1 - \alpha \phi^2$$

On the Reissner–Nordström background

$$\mathcal{G}_{\text{RN}} = \frac{8}{r^8} (6M^2 r^2 - 12MQ^2 r + 5Q^4), \quad F_{\mu\nu} F^{\mu\nu} = -\frac{2Q^2}{r^4}$$

$$\mu_{\text{eff}}^2 = \underbrace{-\frac{2\eta}{r^8} (6M^2 r^2 - 12MQ^2 r + 5Q^4)}_{\text{curvature } (\eta > 0 \Rightarrow < 0)} + \underbrace{\frac{\alpha Q^2}{r^4}}_{\text{matter } (\alpha < 0 \Rightarrow < 0)}$$

How we locate the onset

We do not just inspect the sign of μ_{eff}^2 . We solve for a static scalar cloud ($\ell = 0$) that is regular at the horizon and decays at infinity:

$$[r^2 N_{\text{RN}} U']' = [\ell(\ell + 1) + r^2 \mu_{\text{eff}}^2] U, \quad \delta\phi = U(r) Y_{\ell m}$$

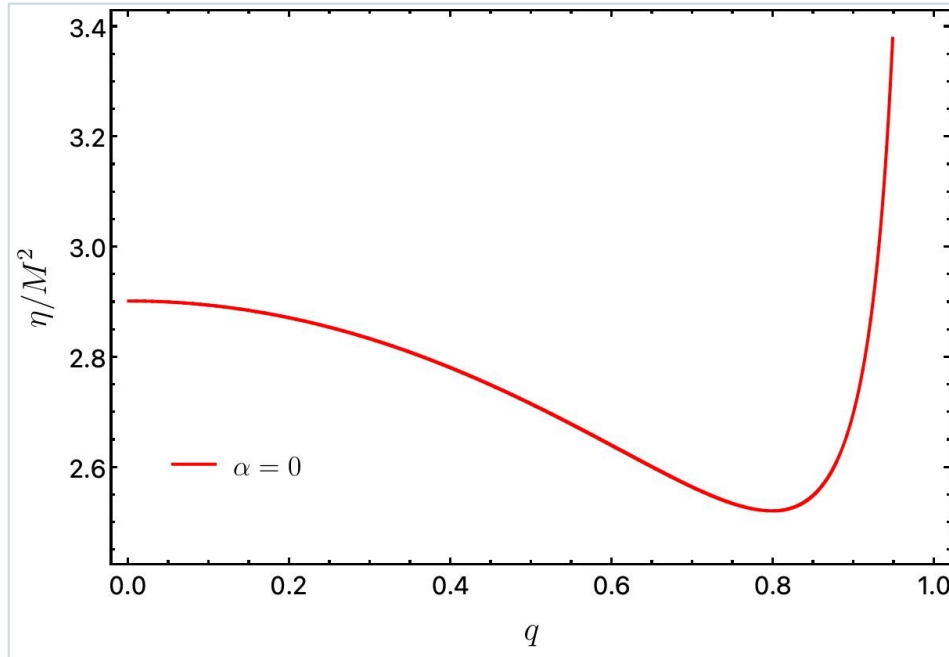
The plots that follow show the lowest $\gamma = \eta/M^2$ that supports a cloud, against $q = Q/M$, at fixed α :

$$q \equiv Q/M, \quad \gamma \equiv \eta/M^2$$

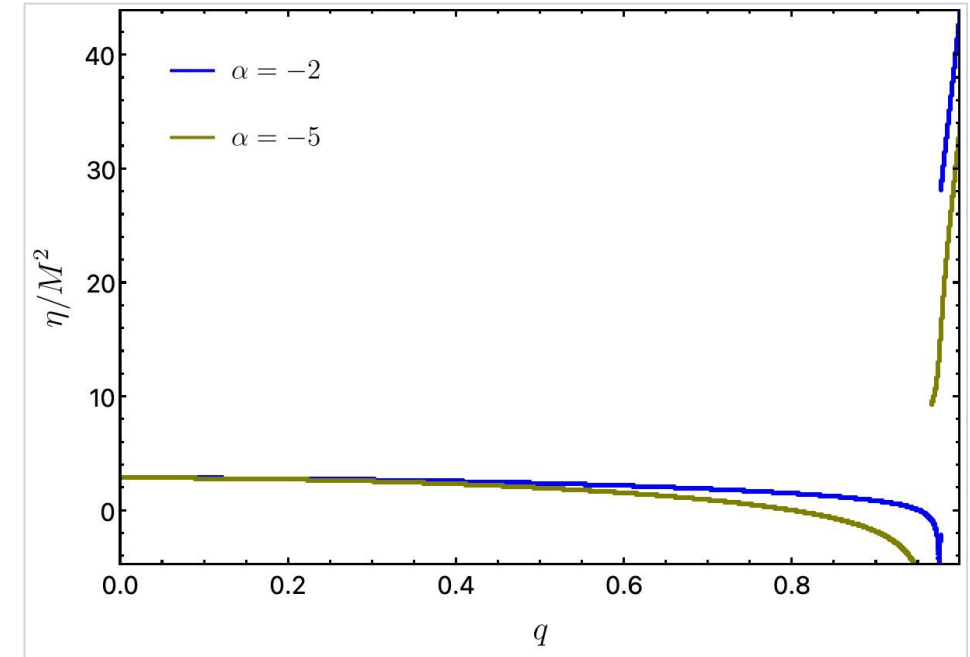
Tachyonic instability, strong enough $\Rightarrow$ a scalarized branch bifurcates from RN.

Threshold of scalarization · moderate Maxwell coupling

Lowest η/M^2 supporting a scalar cloud on Reissner–Nordström, against the reduced charge $q = Q/M$



$\alpha = 0$ (curvature only) · recovers Doneva '18 and Herdeiro '21

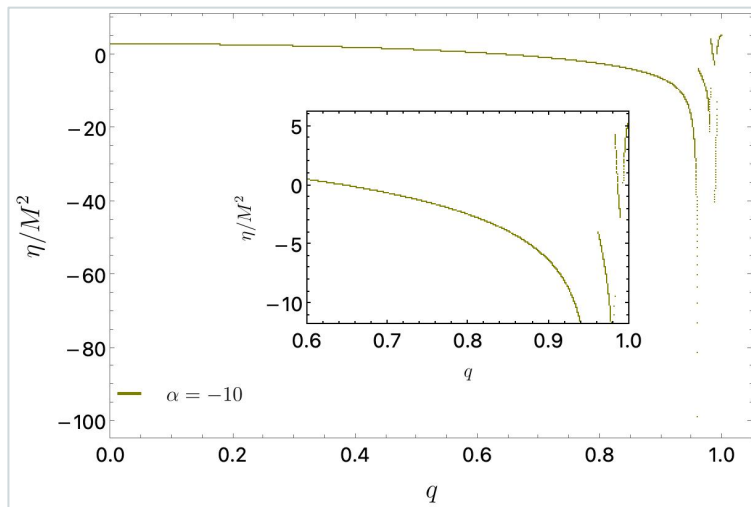


$\alpha = -2$ and $\alpha = -5$

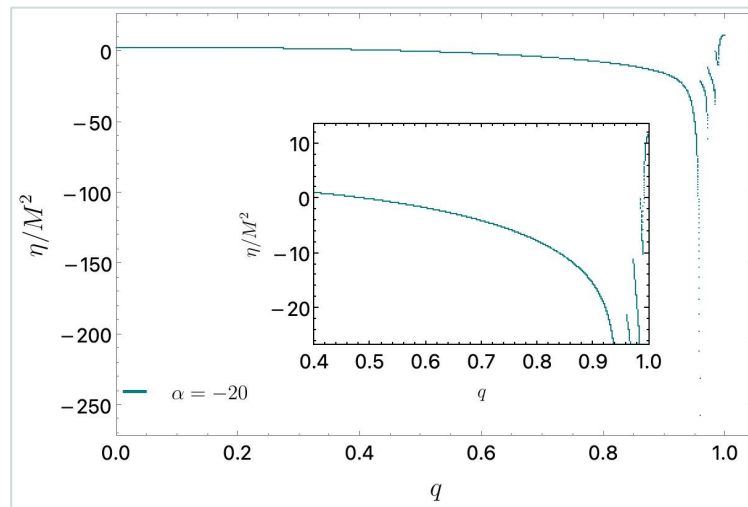
- For $\alpha = 0$ we reproduce exactly the EsGB result of Doneva 2018 and Herdeiro 2021: $\eta/M^2 \approx 2.9$ at $q = 0$, rising steeply near extremality
- Switching on a moderate $\alpha < 0$ deforms the bifurcation curve: the threshold η/M^2 drops at high q
- The $q = 0$ limit is unchanged: the gauge contribution to μ^2_{eff} vanishes without charge

Strong Maxwell coupling unlocks $\eta < 0$ scalarization

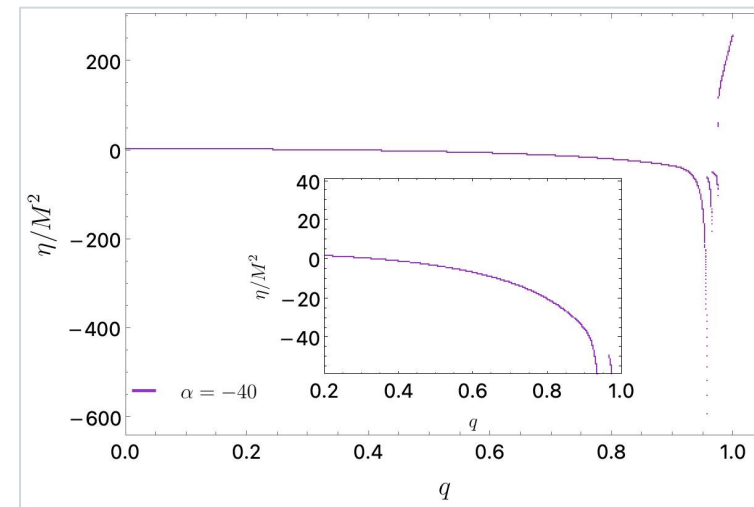
$\alpha = -10, -20, -40$ · qualitatively new bifurcation branches · note the different vertical scales



$\alpha = -10$



$\alpha = -20$



$\alpha = -40$

1 $\eta < 0$ at moderate charge

Earlier studies (Brihaye & Hartmann '19, Herdeiro '21) found RN clouds with $\eta < 0$ only near extremality, $q \approx 0.96$. Here the negative- η window opens down to $q \approx 0.2$ for large $|\alpha|$.

2 The critical charge migrates

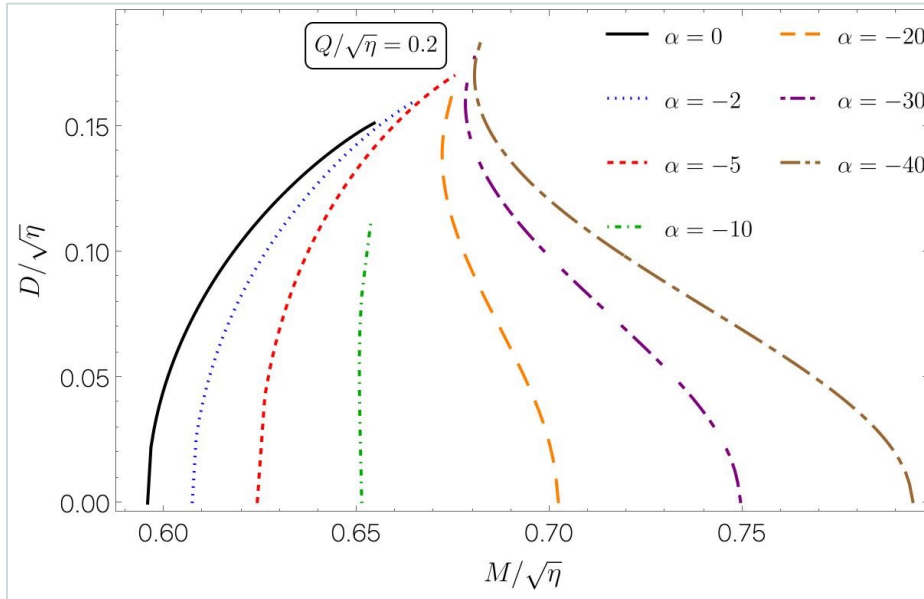
The q at which η flips sign decreases monotonically with $|\alpha|$: negative- η scalarization becomes accessible to a much broader class of RN black holes.

3 Extra bifurcation branches

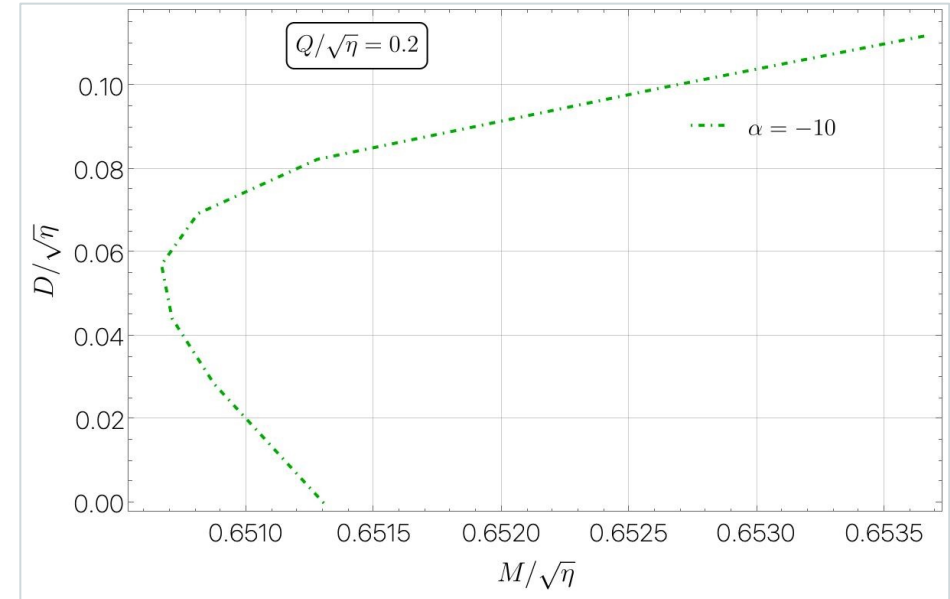
Activating $g(\phi)$ generates additional curves once $\alpha \leq -2$; from $\alpha = -10$ onward they multiply (higher-node scalar clouds). The near-extremal fine structure should not be over-read.

Black-hole solutions · low electric charge

$Q/\sqrt{\eta} = 0.2$ · scalar charge D ($\phi \approx D/r$ at infinity) against mass M · $\eta > 0, \alpha \leq 0$



All α values · full picture



$\alpha = -10$ · turning-point branch (zoom)

1 Distinct thresholds

Each α gives its own onset mass, unlike EsGB or EsGB-Ricci where all branches share one bifurcation point.

2 Splitting at $\alpha = -10$

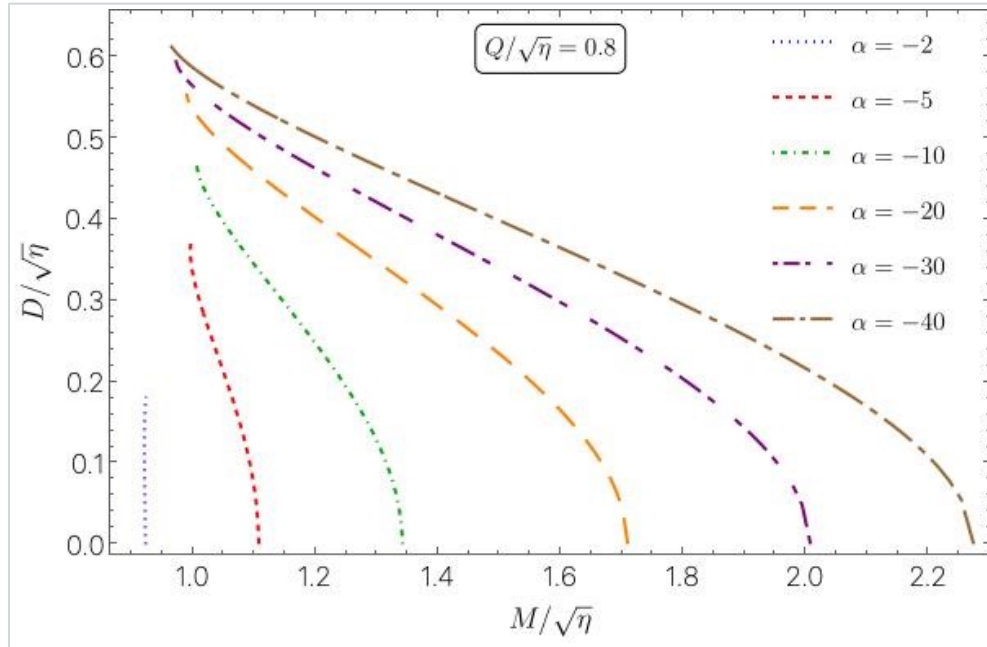
Solutions move to LOWER masses for large $|\alpha|$ (higher masses for $\alpha \geq -5$): a qualitative change of regime. The intermediate case has a turning point.

3 Hint of an attractor

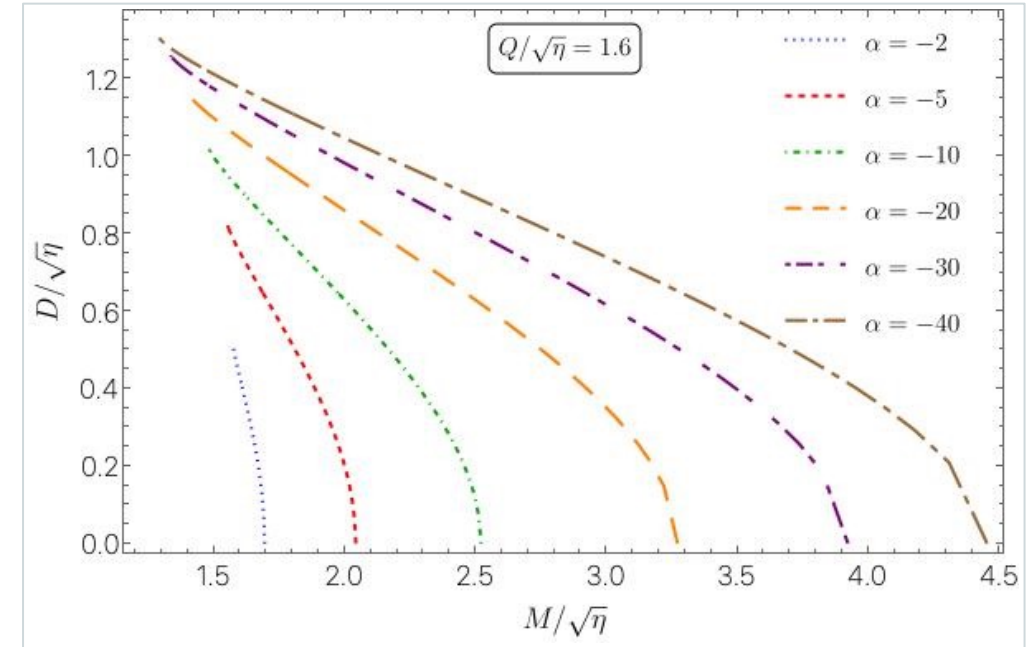
Despite different onsets, many branches converge to similar end states. Branch direction is a clue about stability, not a mode calculation.

Black-hole solutions · higher electric charge

$Q/\sqrt{\eta} = 0.8$ and $Q/\sqrt{\eta} = 1.6$



$Q/\sqrt{\eta} = 0.8$



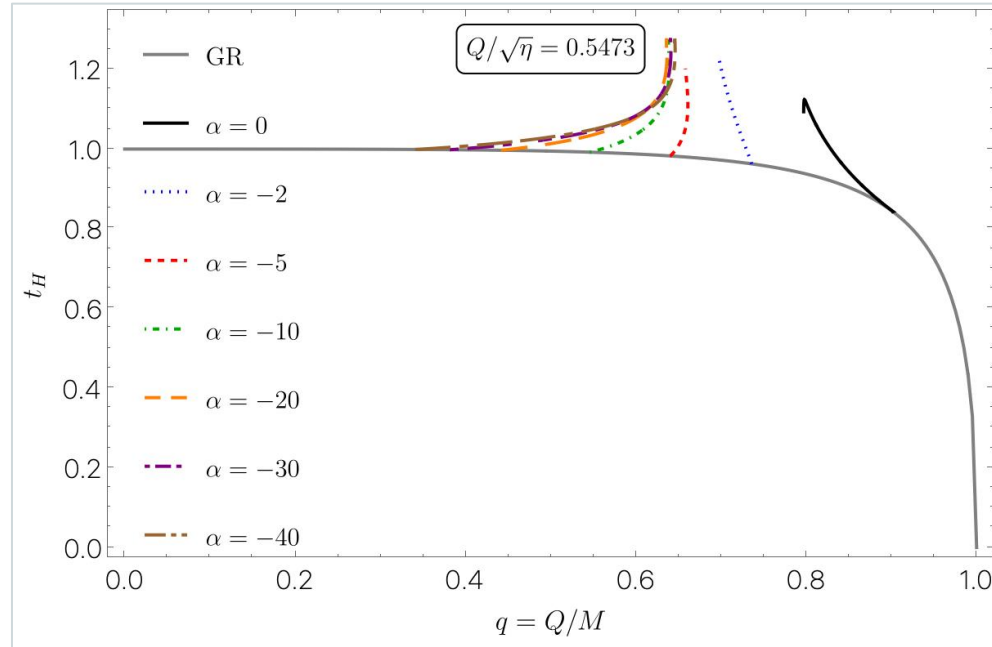
$Q/\sqrt{\eta} = 1.6$ · over-charged solutions appear ($q > 1$)

- All branches now evolve toward lower masses: a likely stability boost from the Maxwell coupling (cf. Macedo et al. 2019, Antoniou et al. 2021)
- Over-charged scalarized black holes ($q > 1$) appear for the first time with positive η ; previously they were seen only in the $\eta < 0$ regime
- Across α the onset masses span almost a factor of three

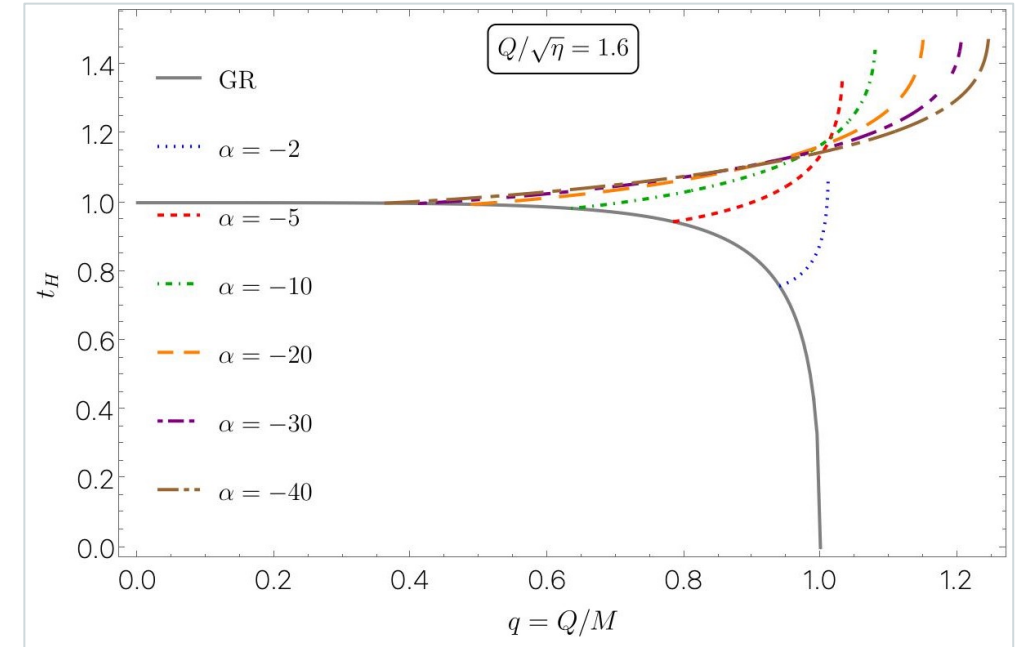
Thermodynamics · temperature

Dimensionless horizon temperature against $q = Q/M$

$$t_H \equiv 8\pi M T_H, \quad T_H = \frac{\sigma(r_H) N'(r_H)}{4\pi}$$



$Q/\sqrt{\eta} = 0.5473$



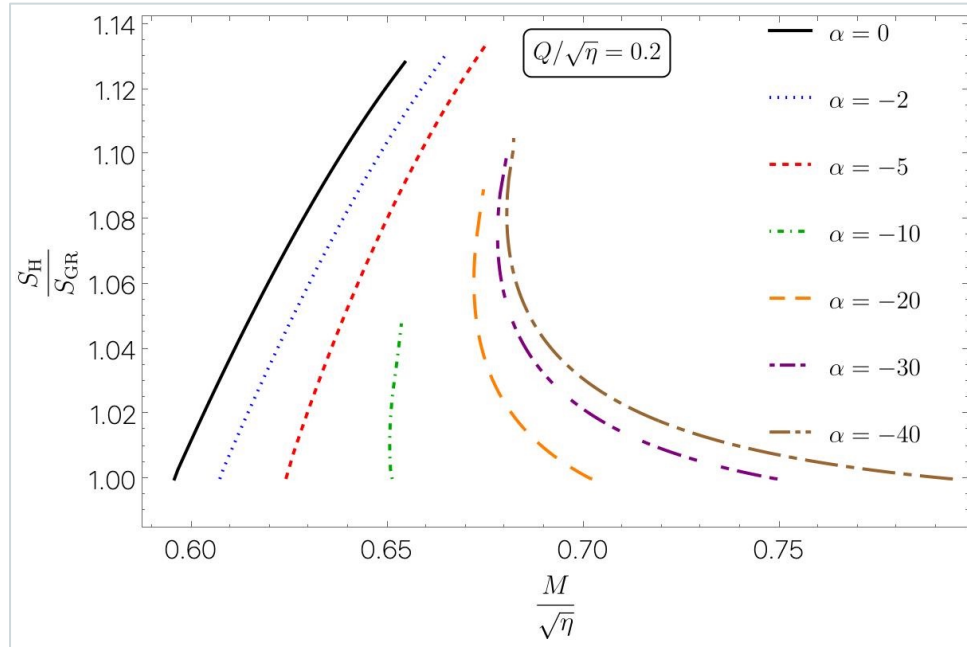
$Q/\sqrt{\eta} = 1.6$

- **Scalarized black holes are hotter than their RN counterparts and never reach extremality ($t_H \neq 0$)**
- At $Q/\sqrt{\eta} = 1.6$ the branches become over-charged ($q > 1$), a regime never observed for $\eta > 0$ scalarization before
- $q > 1$ relative to RN is not by itself a cosmic-censorship violation: that bound belongs to the RN geometry, and these solutions stay regular and non-extremal over the displayed range

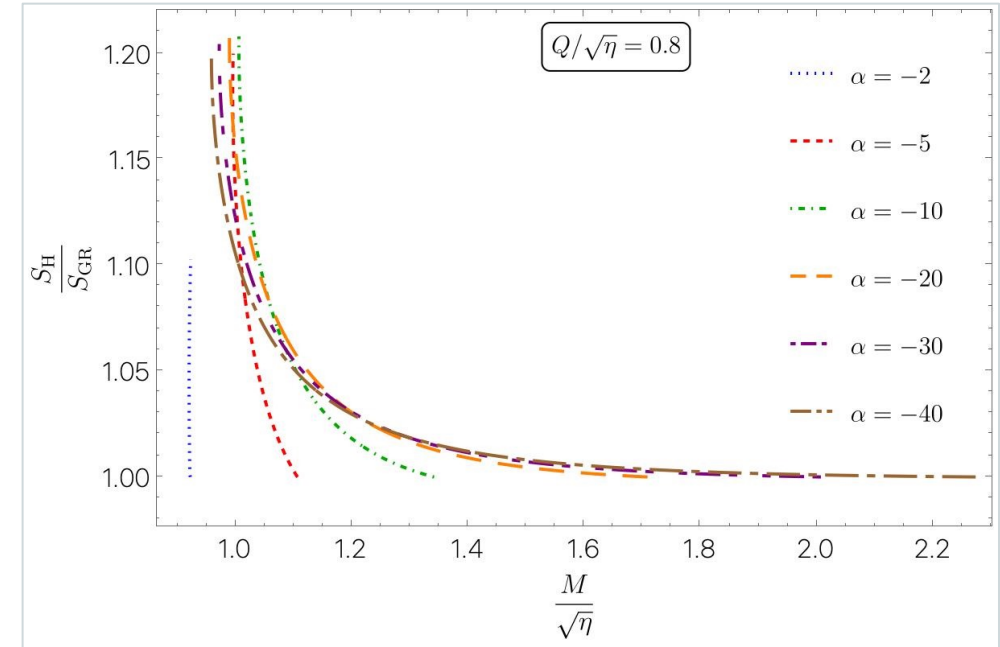
Thermodynamics · entropy

Scalarized black holes are entropically favoured at the same (M, Q)

$$S_H = \frac{A_H}{4} + 4\pi f(\phi_0)$$



$Q/\sqrt{\eta} = 0.2$



$Q/\sqrt{\eta} = 0.8$

$$\frac{S_H}{S_{RN}} > 1 \text{ at the same } (M, Q)$$

- **Every scalarized solution carries more entropy than the RN black hole with the same mass and charge: thermodynamically preferred**
- The Gauss–Bonnet coupling adds $4\pi f(\phi_0)$ to the area law, so this is more than an area comparison
- Entropy preference and branch geometry motivate the stability tests; they do not replace them

Conclusions

01 A new framework

A scalar field coupled non-minimally to both Gauss–Bonnet curvature AND a U(1) gauge field: curvature- and matter-induced scalarization in one action.

02 Richer parameter space

Two coupling constants: multiple mass thresholds, $\eta < 0$ scalarization accessible at moderate charge, additional bifurcation branches.

03 Distinctive observables

Hotter, never-extremal scalarized black holes and OVER-CHARGED solutions ($q > 1$) with positive η : a laboratory for cosmic-censorship tests.

04 Thermodynamically favoured

Scalarized black holes carry more entropy than RN at the same (M, Q) : energetically preferred end states, stability still to be tested.

Scalarization remains one of the most compelling mechanisms for large deviations from GR that survive every weak-field test.

From these backgrounds to gravitational-wave predictions

Many directions, many possible collaborations

Established here

I

Scalar-cloud thresholds on RN, stationary scalarized black holes in the $\eta > 0$, $\alpha < 0$ sector, and their temperature and entropy.

Next calculation

II

Full linear stability (radial, with J. L. Blázquez-Salcedo and C. Macedo) and the quasinormal-mode spectra, the ringdown signature (with J. L. Blázquez-Salcedo and J. Kunz).

Longer-term programme

III

Numerical-relativity evolutions and control of hyperbolicity (with M. Elley); rotating solutions; shadows; exponential couplings and the $\eta < 0$ branches; probing a dark U(1) sector with gravitational waves.

Status

Candidate non-GR backgrounds with a mechanism for their appearance and a tunable scalarization threshold.

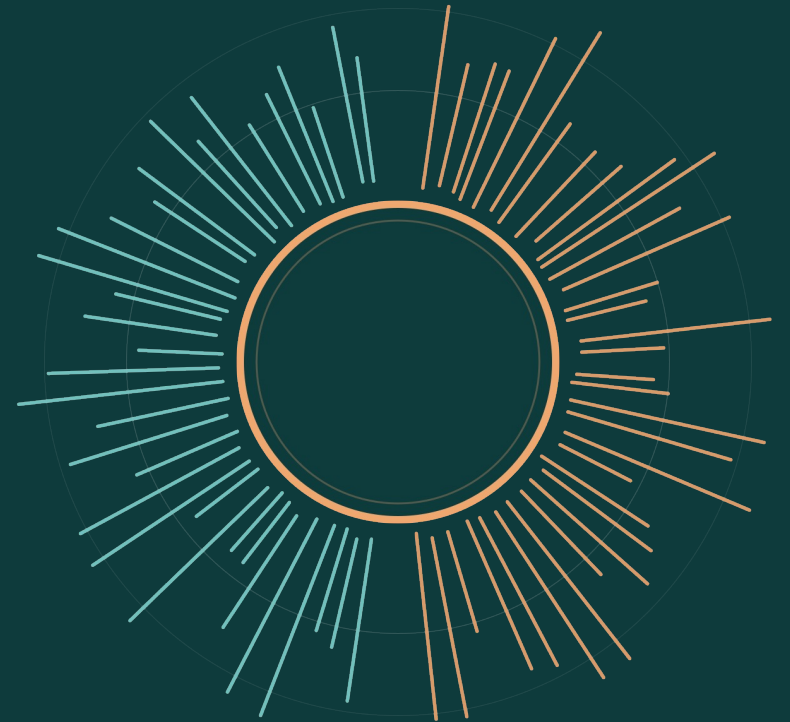
Not yet a ringdown-frequency shift, a waveform template or a detectability forecast. The gauge coupling adds structure that makes those calculations worth doing.

[arXiv:2506.12137](https://arxiv.org/abs/2506.12137)

Thank you for your attention · questions, comments and collaborations welcome

Backup slides

B1 ansatz, field equations and boundary conditions · B2 numerical procedure · B3 the U(1) charge · B4 neutron stars (DEF 1993) · B5 modified-gravity landscape · B6 what is, and is not, established



Ansatz, field equations and boundary conditions

Static, spherically symmetric, electrically charged (magnetic and dyonic cases work the same way) · unknowns N, σ, V, ϕ · global charges M, Q, D

$$ds^2 = -N(r) \sigma(r)^2 dt^2 + \frac{dr^2}{N(r)} + r^2 (d\theta^2 + \sin^2\theta d\varphi^2), \quad N(r) = 1 - \frac{2m(r)}{r}$$

$$A_\mu dx^\mu = V(r) dt, \quad \phi = \phi(r), \quad \left(\frac{r^2 g(\phi) V'}{\sigma} \right)' = 0$$

The Maxwell equation integrates once; the rest reduces to three coupled non-linear ODEs (N', σ', ϕ''), solved by shooting

At the horizon $r = r_H$

$$N = N_1(r - r_H) + \dots, \quad \sigma = \sigma_0 + \sigma_1(r - r_H) + \dots, \quad \phi = \phi_0 + \phi_1(r - r_H) + \dots$$

$$\phi_1 = \frac{B \pm \sqrt{\Delta}}{A}, \quad N_1 = \frac{r_H^2 g(\phi_0) - Q^2}{r_H^2 g(\phi_0) (2\phi_1 f_{,\phi}(\phi_0) + r_H)}, \quad V_1 = -\frac{Q \sigma_0}{r_H^2 g(\phi_0)}$$

At spatial infinity (RN tail, asymptotic flatness, scalar charge D , Coulomb tail)

$$N \simeq 1 - \frac{2M}{r}, \quad \sigma \simeq 1 - \frac{D^2}{2r^2}, \quad \phi \simeq \frac{D}{r}, \quad V \simeq \psi_e + \frac{Q}{r}$$

Linear threshold and the shooting method

Linear radial problem on the RN background ($\ell = 0$ for the threshold), then continuation of the non-linear branches

$$\delta\phi = U(r) Y_{\ell m}(\theta, \varphi)$$

$$[r^2 N U']' = U \left[\ell(\ell + 1) - \frac{\eta}{r^2} \left(\frac{12M^2}{r^2} - \frac{24MQ^2}{r^3} + \frac{10Q^4}{r^4} \right) + \frac{\alpha Q^2}{r^2} \right]$$

$$\rho^6 \left[(q^2 - 2\rho + \rho^2) u'' + 2(\rho - 1) u' \right] = \left[\alpha q^2 \rho^4 - 2\gamma (5q^4 - 12q^2 \rho + 6\rho^2) \right] u$$

$$\rho = r/M, \quad q = Q/M, \quad \gamma = \eta/M^2$$

Non-linear solutions

$$r_H = 1, \quad r_{\text{start}} = r_H + 10^{-4}, \quad r_{\text{max}} \sim 10^3, \quad \text{continuation at fixed } \alpha \text{ and } Q/\sqrt{\eta}$$

- Impose $N(r_H) = 0$, regularity of ϕ (discriminant $\Delta \geq 0$) at the horizon; require $\phi \rightarrow 0$ and $\sigma \rightarrow 1$ at large r
- Start from the bifurcation point given by the linear analysis, then step in $Q/\sqrt{\eta}$ at fixed α
- Adjust (ϕ_0, σ_0) until the field vanishes and the metric becomes RN at large r ; rescale σ to normalise
- If the solver fails, return to the previous solution and reduce the step; check the signs of N' , M and Q and monitor the local integration error

Black holes are not neutral? What the U(1) charge represents

We stay agnostic about the U(1) field and about how the charge was acquired

1 Dark photon, hidden U(1)

The U(1) may live in a hidden sector decoupled from the Standard Model. Black holes then carry a dark charge that evades the standard Schwinger discharge argument.

e.g. arXiv:1604.07845

2 Magnetically charged black holes

Motivated by hypothetical magnetic monopoles; renewed interest since Maldacena's 2020 paper. The model covers magnetic and dyonic cases.

Maldacena 2020 · arXiv:2004.06084

3 Toy model, stepping stone

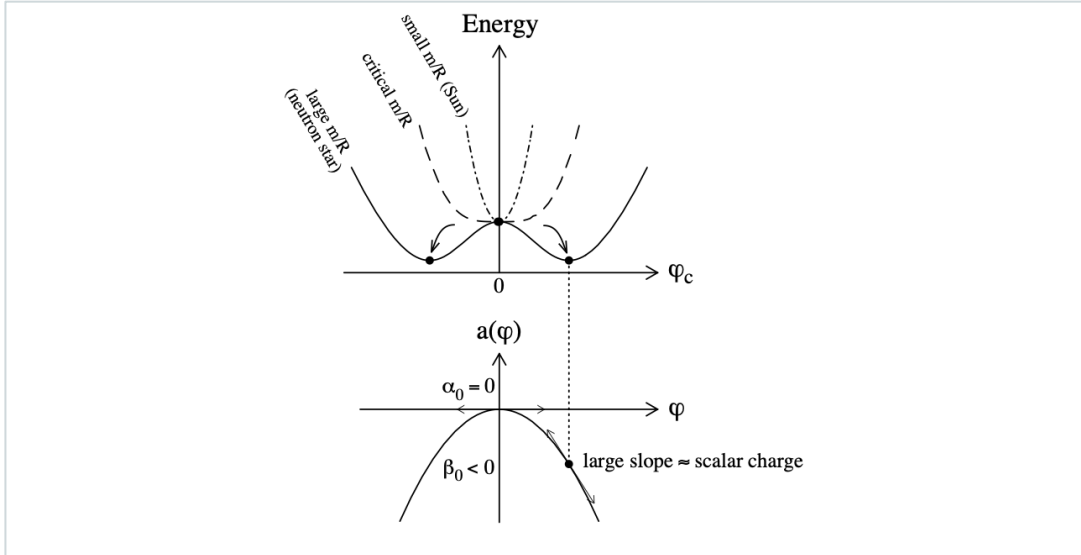
Reissner–Nordström admits an extremal limit ($q \rightarrow 1$): a useful spherically symmetric laboratory before rotating solutions, where spin-induced scalarization appears.

standard textbook setting

Scope of the results: static, spherical, electrostatic. We treat Q as a global U(1) charge, independent of its microscopic origin, and derive no formation mechanism or astrophysical charge distribution.

Spontaneous scalarization of neutron stars

Damour & Esposito-Farèse 1993 · the original heuristic argument



Heuristic energy argument. Damour & Esposito-Farèse, arXiv:gr-qc/0402007

Heuristic argument

Matter couples to the conformally related metric $A(\phi)^2 g_{\mu\nu}$; the effective potential for ϕ depends on the compactness $2Gm/Rc^2$:

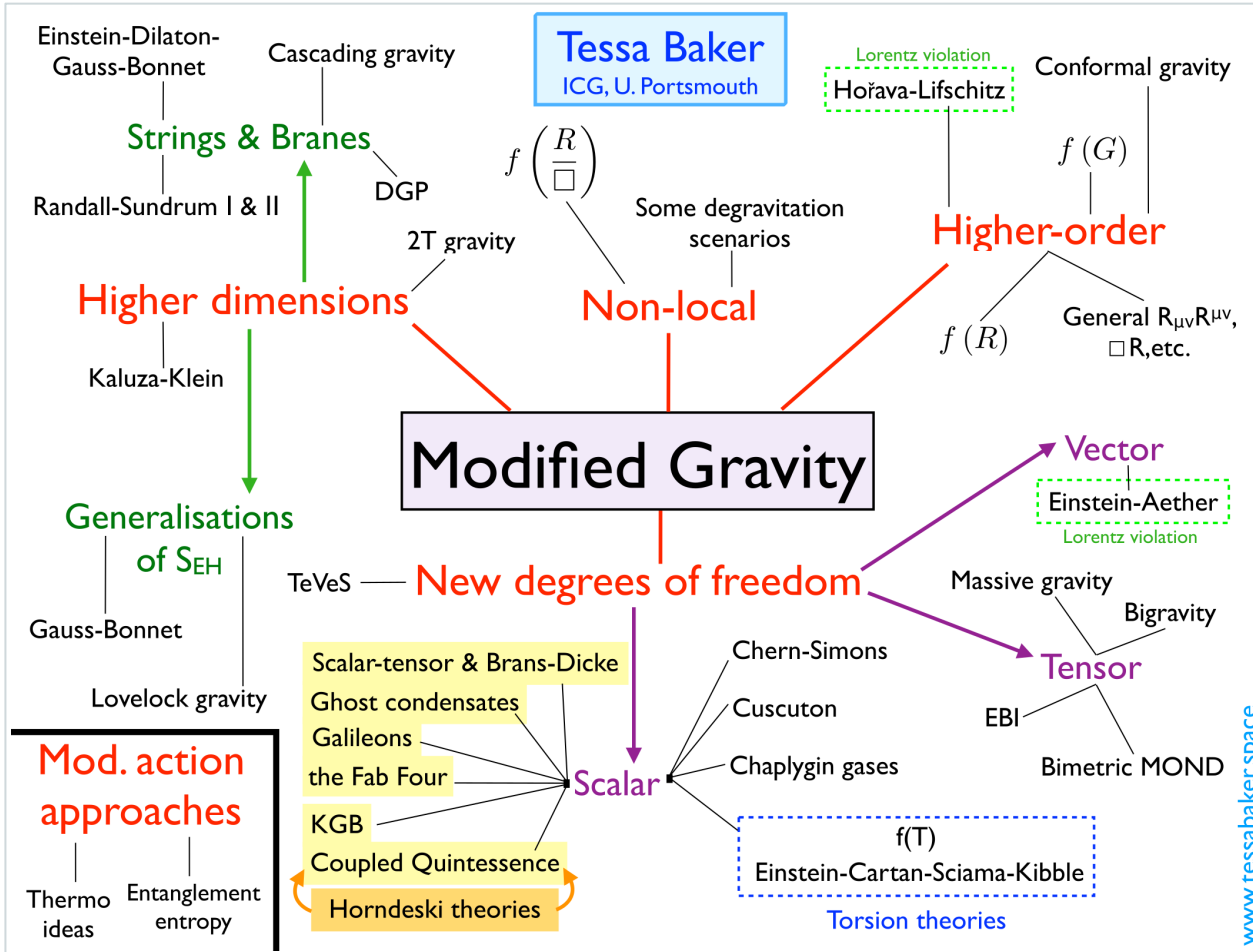
- Small m/R (the Sun): a single well at $\phi = 0$
- Critical m/R : the well flattens
- Large m/R (a neutron star): the well inverts, two new minima appear, the star rolls into one of them: scalarized

$$\square\phi = -4\pi G_* \alpha(\phi) T, \quad \mu_{\text{eff}}^2 = -4\pi G_* \alpha'(\phi_0) T, \quad \alpha(\phi) = \frac{d \ln A(\phi)}{d\phi}$$

Black holes need a vacuum source instead: $T = 0$ outside the star, so the invariant must come from curvature or from a gauge field.

Modified gravity: the landscape

Many paths, common roots · we work in the scalar-tensor sector



Credit: Tessa Baker (Portsmouth)

Extra fields

scalar · vector · tensor

Higher derivatives

R^2 · Gauss-Bonnet · $f(R)$

Extra dimensions

Kaluza-Klein · DGP

Massive gravity

bimetric · MOND-like

Non-local theories

$f(R/\square)$ · degravitation

What is, and is not, established

Separating completed results from open questions

ESTABLISHED IN THIS WORK

- Scalar-cloud thresholds on Reissner–Nordström, for α from 0 to -40
- Stationary scalarized black holes in the sampled $\eta > 0$, $\alpha < 0$ sector, with distinct onset masses
- Over-charged ($q > 1$) regular, non-extremal solutions with $\eta > 0$
- Temperature and entropy comparisons with RN at the same (M, Q)

STILL TO BE CALCULATED

- Full perturbative stability (radial and non-radial) and quasinormal-mode spectra
- Non-linear formation and evolution; hyperbolicity across the parameter space
- Non-linear construction of the $\eta < 0$ branches; rotating generalisation
- Waveforms, ringdown frequency shifts and detectability forecasts

Entropy preference and branch geometry are motivations for the stability tests, not their outcome.