

Radiation-reaction effects at 3PN order in scalar-tensor theories

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Motivation: toward precision waveforms beyond GR

As detectors become more sensitive, theoretical uncertainties are becoming more and more prominent : it is crucial to improve waveform predictions.

There is currently an important gap for GW predictions beyond-GR

- scattered analytical/numerical results in different theories
- one IMR template in EsGB theories: SEOBNRv5PHM_ESGB

The aim is to improve analytical (PN) predictions for the inspiral phase in scalar-tensor theories toward GR-like accuracy and implement these results in IMR waveform templates.

- Valid for isolated, compact, **slowly moving** and **weakly-self gravitating** systems
- Can be used to describe the inspiral phase of binary systems
- Provides an **approximate** but **analytical** solution to the Einstein field equations
- Small parameter $\varepsilon = \frac{v_{12}^2}{c^2} \sim \frac{Gm_{\text{tot}}}{r_{12}c^2} < 1$
- PN orders : $n\text{PN} = \mathcal{O}\left(\left(\frac{v}{c}\right)^{2n}\right)$

Field equations

We consider the following action $S = S_{\text{EH}} + S_m$, where S_m is the skeletonized action for point-particles :

$$S_m = - \sum_{A=1,2} \int dt m_A c^2 \sqrt{-(g_{\mu\nu})_A \frac{V_A^\mu V_A^\nu}{c^2}} .$$

We define

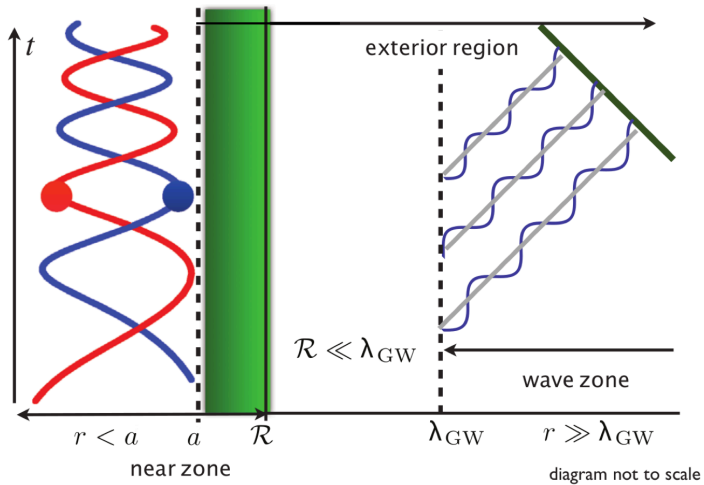
$$\sqrt{-g} g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu} .$$

Varying the action yields the following non-linear field equations :

$$\begin{cases} \square_\eta h^{\mu\nu} = \frac{16\pi G}{c^4} \tau^{\mu\nu} = \frac{16\pi G}{c^4} |g| T^{\mu\nu} + \Lambda^{\mu\nu}[h] \\ \partial_\mu h^{\mu\nu} = 0 \end{cases}$$

where $\Lambda^{\mu\nu}$ is at least quadratic in h , ∂h and $\partial^2 h$.

Gravitational-wave (GW) generation formalism (1/2)



Gravitational-wave (GW) generation formalism (2/2)

Near zone : post-Newtonian expansion

$$\bar{h}^{\mu\nu} = \sum_{m=2}^{\infty} \frac{1}{c^m} \bar{h}_m^{\mu\nu} \quad \text{with} \quad \square_{\eta} \bar{h}_m^{\mu\nu} = 16\pi G \bar{\tau}_{m-4}^{\mu\nu} \quad (m \geq 2)$$
$$\bar{\tau}^{\mu\nu} = \sum_{m=-2}^{\infty} \frac{1}{c^m} \bar{\tau}_m^{\mu\nu}$$

Exterior zone : multipolar post-Minkowskian expansion

$$\mathcal{M}(h^{\mu\nu}) = \sum_{n=1}^{\infty} G^n h_{(n)}^{\mu\nu} \quad \text{with} \quad \square_{\eta} h_{(n)}^{\mu\nu} = \Lambda_{(n)}^{\mu\nu}[h_{(1)}, \dots, h_{(n-1)}]$$

Buffer region (in green) : matching of the asymptotic expansions

$$\mathcal{M}(\bar{h}) = \overline{\mathcal{M}(h)}$$

Dynamical sector

- Compute the Lagrangian $L = L[y_A, v_A, \dots]$ with $A = 1, 2$
- Using the generalized Euler Lagrange equations, deduce the conservative equations of motion (EOM) $m_A \mathbf{a}_{A,\text{cons}} = \mathbf{F}_A(t)$
- Compute the 10 Noetherian conserved quantities $Q_{\text{cons}} \in \{E_{\text{cons}}, \mathbf{P}_{\text{cons}}, \mathbf{J}_{\text{cons}}, \mathbf{K}_{\text{cons}}\}$ such that $\left. \frac{dQ_{\text{cons}}}{dt} \right|_{\mathbf{a}_{A,\text{cons}}} = 0$
- Results expanded in **even** PN orders :

$$\mathbf{a}_{A,\text{cons}} = \mathbf{a}_A^{\text{N}} + \frac{1}{c^2} \mathbf{a}_A^{1\text{PN}} + \frac{1}{c^4} \mathbf{a}_A^{2\text{PN}} + \dots$$
$$Q_{\text{cons}} = Q^{\text{N}} + \frac{1}{c^2} Q^{1\text{PN}} + \frac{1}{c^4} Q^{2\text{PN}} + \dots$$

Computational steps (2/2)

Radiative sector

- Compute the radiation field $h^{\mu\nu}$ using the GW generation formalism
- Deduce the flux of energy $\mathcal{F}$
- **Assuming** an energy balance equation, compute the orbital phase

$$\frac{dE}{dt} = -\mathcal{F} \Rightarrow \phi(t) = \int \omega dt = - \int \frac{\omega dE/d\omega}{\mathcal{F}(\omega)} d\omega$$

where ω denotes the orbital frequency

- Deduce the GW phase $\phi_{\text{GW}}(t) = 2 \phi(t)$
- State-of-the-art in GR : 4.5PN phase for circular orbits

[Blanchet et al., 2023a]

Radiation-reaction effects at 4.5PN
order in general relativity

4PN conserved energy for circular orbits

Introducing $y = \left(\frac{Gm_{\text{tot}}\omega}{c^3}\right)^{2/3}$, the 4PN conserved energy reads [Damour et al., 2015]
[Bernard et al., 2018] :

$$\begin{aligned} E_{\text{cons}}(y) = & -\frac{m\nu c^2 y}{2} \left\{ 1 + y \left(-\frac{3}{4} - \frac{\nu}{12} \right) + y^2 \left(-\frac{27}{8} + \frac{19}{8}\nu - \frac{\nu^2}{24} \right) \right. \\ & + y^3 \left[-\frac{675}{64} + \nu \left(\frac{34445}{576} - \frac{205}{96}\pi^2 \right) - \frac{155}{96}\nu^2 - \frac{35}{5184}\nu^3 \right] \\ & + y^4 \left[-\frac{3969}{128} + \nu \left(-\frac{123671}{5760} + \frac{9037}{1536}\pi^2 + \frac{896}{15}\gamma_E + \frac{448}{15}\ln(16y) \right) \right. \\ & \left. \left. + \nu^2 \left(-\frac{498449}{3456} + \frac{3157}{576}\pi^2 \right) + \frac{301}{1728}\nu^3 + \frac{77}{31104}\nu^4 \right] + \mathcal{O}(y^5) \right\} \end{aligned}$$

4.5PN flux of energy for circular orbits

The 4.5PN flux of energy at $\mathcal{I}^+$ in terms of y reads [Blanchet et al., 2023b] :

$$\begin{aligned}
 \mathcal{F}(y; b_0) = & \frac{32c^5}{5G} \nu^2 y^5 \left\{ 1 + y \left(-\frac{1247}{336} - \frac{35}{12} \nu \right) + 4\pi y^{3/2} + y^2 \left(-\frac{44711}{9072} + \frac{9271}{504} \nu + \frac{65}{18} \nu^2 \right) + \pi y^{5/2} \left(-\frac{8191}{672} - \frac{583}{24} \nu \right) \right. \\
 & + y^3 \left[\frac{6643739519}{69854400} + \frac{16}{3} \pi^2 - \frac{1712}{105} \gamma_E - \frac{856}{105} \ln(16y) + \nu \left(-\frac{134543}{7776} + \frac{41}{48} \pi^2 \right) - \frac{94403}{3024} \nu^2 - \frac{775}{324} \nu^3 \right] \\
 & + \pi y^{7/2} \left(-\frac{16285}{504} + \frac{214745}{1728} \nu + \frac{193385}{3024} \nu^2 \right) \\
 & + y^4 \left[-\frac{323105549467}{3178375200} + \frac{232597}{4410} \gamma_E - \frac{1369}{126} \pi^2 + \frac{39931}{294} \ln 2 - \frac{47385}{1568} \ln 3 + \frac{232597}{8820} \ln y \right. \\
 & + \nu \left(-\frac{1467849789229}{1466942400} + \frac{10118}{245} \gamma_E - \frac{267127}{4608} \pi^2 - \frac{85418}{2205} \ln 2 + \frac{47385}{392} \ln 3 - \frac{26301}{245} \ln y + 128 \ln \left(\frac{Gm}{c^2 b_0} \right) \right) \\
 & + \nu^2 \left(\frac{1607125}{6804} - \frac{3157}{384} \pi^2 \right) + \frac{6875}{504} \nu^3 + \frac{5}{6} \nu^4 \left. \right] \\
 & + \pi y^{9/2} \left[\frac{265978667519}{745113600} - \frac{6848}{105} \gamma_E - \frac{3424}{105} \ln(16y) + \nu \left(\frac{2062241}{22176} + \frac{41}{12} \pi^2 \right) \right. \\
 & \left. \left. - \frac{133112905}{290304} \nu^2 - \frac{3719141}{38016} \nu^3 \right] + \mathcal{O}(y^5) \right\}
 \end{aligned}$$

Energy balance equation in terms of the orbital frequency y

If we use the energy balance equation

$$\frac{dE_{\text{cons}}(y)}{dt} = -\mathcal{F}(y; b_0)$$

the phasing will depend on b_0 , hence will not be a gauge invariant quantity.

Energy balance equation in terms of the waveform frequency x

We introduce Ω_{22} the half-frequency of the (2,2) mode, define $x = \left(\frac{Gm_{\text{tot}}\Omega_{22}}{c^3} \right)^{2/3}$

and recall that $y = \left(\frac{Gm_{\text{tot}}\omega}{c^3} \right)^{2/3}$.

The orbital and waveform frequencies start to differ at the 4PN order :

$$x = y \left\{ 1 + y^4 \nu \left[\frac{128}{5} \ln \left(\frac{Gm}{c^2 b_0} \right) - \frac{192}{5} \ln(y) - \frac{128}{5} \gamma_E - \frac{32}{15} - \frac{256}{5} \ln(2) \right] \right\}$$

The energy flux in terms of x does not depend on b_0 anymore but the energy balance equation becomes

$$\frac{dE_{\text{cons}}(x; b_0)}{dt} = -\mathcal{F}(x)$$

so the phase will still depend on b_0 .

Conserved vs binding energy

The quantity entering the energy balance equation is the binding energy, defined by

$$\left. \frac{dE_{\text{bind}}}{dt} \right|_{\mathbf{a}_{\text{cons}} + \mathbf{a}_{\text{diss}}} = -\mathcal{F}$$

The binding energy differs from the conserved energy by dissipative terms, starting from 2.5PN order :

$$E_{\text{bind}} = E_{\text{cons}} + \frac{1}{c^5} E_{\text{diss}}^{2.5\text{PN}} + \frac{1}{c^7} E_{\text{diss}}^{3.5\text{PN}} + \frac{1}{c^8} E_{\text{diss}}^{4\text{PN}} + \frac{1}{c^9} E_{\text{diss}}^{4.5\text{PN}}$$

- $\{E_{\text{diss}}^{2.5\text{PN}}, E_{\text{diss}}^{3.5\text{PN}}, E_{\text{diss}}^{4.5\text{PN}}\}$ are the Schott (instantaneous) terms $\rightarrow$ they vanish on circular orbits at the orders we consider
- $E_{\text{diss}}^{4\text{PN}}$ is a pseudo-Schott (tail) term which does contribute to the energy balance equation and reads [Trestini, 2025] :

$$E_{\text{diss}}^{4\text{PN}}(y; b_0) = -\frac{c^2 m \nu^2 y^5}{2} \left[\frac{128}{5} \ln \left(\frac{Gm}{c^2 b_0} \right) - \frac{192}{5} \ln(y) - \frac{128}{5} \gamma_{\text{E}} - \frac{256}{5} \ln(2) - \frac{32}{15} \right]$$

Coming back to the energy balance equation (1/2)

Considering the energy balance equation in the **orbital frequency** y

$$\frac{dE_{\text{bind}}(y; b_0)}{dt} = -\mathcal{F}(y; b_0),$$

one finds that the **orbital phase** is now given by

$$\phi(y) = -\frac{c^3}{Gm} \int y^{3/2} \frac{dE_{\text{bind}}(y; b_0)/dy}{\mathcal{F}(y; b_0)} dy,$$

and that the b_0 -scale drops out at 4.5PN order.

Coming back to the energy balance equation (2/2)

Alternatively, one can also use the energy balance equation in the **waveform frequency** x

$$\frac{dE_{\text{bind}}(x)}{dt} = -\mathcal{F}(x)$$

to derive the **half phase of the (2,2) mode**

$$\psi(x) = -\frac{c^3}{Gm} \int x^{3/2} \frac{dE_{\text{bind}}(x)/dx}{\mathcal{F}(x)} dx,$$

which is directly measurable by the GW detectors.

The b_0 -dependence comes from various part of the computation, namely

- the dissipative part of the acceleration
- the pseudo-Schott term in the binding energy
- the link between the orbital frequency y and the waveform frequency x
- the tail term in the energy flux at 1.5PN order.

The fact that all b_0 drop from the final phases is a strong consistency check !

Radiation-reaction effects at 3PN order in scalar-tensor theories

Generalized Brans-Dicke theories : extra massless scalar field

Action in the Jordan frame :

$$S_{\text{ST}}^{\text{J}} = \frac{c^3}{16\pi G} \int d^4x \sqrt{-g} \left[\phi R - \frac{\omega(\phi)}{\phi} g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi \right] + S_{\text{m}}(\mathbf{m}, g_{\alpha\beta}) ,$$

Conformal transformation (to decouple scalar and tensor degrees of freedom) :

$$\tilde{g}_{\mu\nu} = \frac{\phi}{\phi_0} g_{\mu\nu} = \varphi g_{\mu\nu} , \quad \text{with} \quad \phi_0 = \phi(\infty) = \text{const}$$

Action in the Einstein frame :

$$S_{\text{ST}}^{\text{E}} = \frac{c^3 \phi_0}{16\pi G} \int d^4x \sqrt{-\tilde{g}} \left[\tilde{R} - \frac{3 + 2\omega(\phi)}{2\varphi^2} \tilde{g}^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi \right] + S_{\text{m}}(\mathbf{m}, \varphi^{-1} \tilde{g}_{\alpha\beta}) ,$$

For S_{m} , we use the skeletonized matter action for point-particles

In the dynamical sector :

- EOM, including dissipative contributions, to 3PN order
[Mirshekari and Will, 2013] [Bernard, 2018] :

$$\mathbf{a}_A = \mathbf{a}_A^{0\text{PN}} + \frac{1}{c^2} \mathbf{a}_A^{1\text{PN}} + \frac{1}{c^3} \mathbf{a}_A^{1.5\text{PN}} + \frac{1}{c^4} \mathbf{a}_A^{2\text{PN}} + \frac{1}{c^5} \mathbf{a}_A^{2.5\text{PN}} + \frac{1}{c^6} \mathbf{a}_A^{3\text{PN}} + \frac{1}{c^6} \mathbf{a}_A^{3\text{PN},\text{tail}} \quad A = 1, 2$$

dissipative & instantaneous, dissipative & non-local in time

- conserved energy (and other Noetherian quantities) to 3PN order [Bernard, 2019] :

$$E_{\text{cons}} = E^{\text{N}} + \frac{1}{c^2} E^{1\text{PN}} + \frac{1}{c^4} E^{2\text{PN}} + \frac{1}{c^6} E^{3\text{PN}}$$

In the radiative sector :

- total flux of energy $\mathcal{F} = \mathcal{F}_g + \mathcal{F}_s$ to 1.5PN order [Lang, 2015] [Bernard et al., 2022]:

$$\mathcal{F}_g = \frac{1}{c^5} \left(\mathcal{F}_g^{0\text{PN}} + \frac{1}{c^2} \mathcal{F}_g^{1\text{PN}} + \frac{1}{c^3} \mathcal{F}_g^{1.5\text{PN},\text{tail}} \right)$$

$$\mathcal{F}_s = \frac{1}{c^5} \left(c^2 \mathcal{F}_s^{-1\text{PN}} + \mathcal{F}_s^{0\text{PN}} + \frac{1}{c} \mathcal{F}_s^{0.5\text{PN},\text{tail}} + \frac{1}{c^2} \mathcal{F}_s^{1\text{PN}} + \frac{1}{c^3} \mathcal{F}_s^{1.5\text{PN},\text{tail}} \right)$$

instantaneous, non-local in time

Towards GW to 3PN order in ST ($\sim$ to the 4PN accuracy in GR) :

- dissipative terms in Noetherian quantities to 3PN order
- energy flux to 2PN order
- GW phasing to 3PN order

Instantaneous Schott terms (1/2)

We define the binding quantities $Q = Q_{\text{cons}} + Q_{\text{diss}}^{\text{inst}} (+Q_{\text{diss}}^{\text{tail}})$, with $Q \in \{E, \mathbf{P}, \mathbf{J}, \mathbf{G}\}$.
They satisfy :

$$\left\{ \begin{array}{l} \frac{dQ_{\text{cons}}}{dt} \Big|_{\mathbf{a}_{\text{cons}} + \mathbf{a}_{\text{diss}}} + \mathcal{F}_Q = -\delta\mathcal{F}_Q \quad \text{for } Q = E, \mathbf{P}, \mathbf{J} \\ \frac{d\mathbf{G}_{\text{cons}}}{dt} \Big|_{\mathbf{a}_{\text{cons}} + \mathbf{a}_{\text{diss}}} + \mathcal{F}_{\mathbf{G}} - \mathbf{P} = -\delta\mathcal{F}_{\mathbf{G}} \end{array} \right.$$

where $\mathcal{F}_E = \mathcal{O}(1/c^3)$, $\mathcal{F}_{\mathbf{J}} = \mathcal{O}(1/c^3)$, $\mathcal{F}_{\mathbf{G}} = \mathcal{O}(1/c^5)$, and $\mathcal{F}_{\mathbf{P}} = \mathcal{O}(1/c^5)$.

The terms on the RHS can be written as total time derivatives

$$\begin{aligned} \delta\mathcal{F}_Q &= \frac{dQ_{\text{diss}}^{\text{inst}}}{dt} \quad \text{for } Q = E, \mathbf{P}, \mathbf{J} \\ \delta\mathcal{F}_{\mathbf{G}} &= \frac{d\mathbf{G}_{\text{diss}}^{\text{inst}}}{dt} \end{aligned}$$

Instantaneous Schott terms (2/2)

In the center-of-mass frame, we find [ED and Trestini, 2026 (in prep)] :

$$\begin{aligned} E_{\text{diss}}^{1.5\text{PN}} &= \frac{4\alpha^2 \tilde{G}^2 m^3 \zeta \mathcal{S}_-^2 \nu^2}{3c^3 r^2} \dot{r} \\ E_{\text{diss}}^{2.5\text{PN}} &= \frac{1}{c^5} \left[\frac{8\alpha^3 \tilde{G}^3 m^4 \zeta \mathcal{S}_- \nu^2}{15r^3} (-13\mathcal{S}_- - 5\beta^- \mathcal{S}_- - 5\bar{\gamma} \mathcal{S}_- + 5\beta^- \mathcal{S}_- \delta - 2\mathcal{S}_+ \delta + 6\mathcal{S}_- \nu) \right. \\ &\quad + \frac{\alpha^2 \tilde{G}^2 m^3 \nu^2}{r^2} \left((-4\beta^- + 2\bar{\gamma} - \zeta \mathcal{S}_-^2 - \zeta \mathcal{S}_- \mathcal{S}_+ \delta + 2\zeta \mathcal{S}_-^2 \nu) \dot{r}^2 \right. \\ &\quad \left. \left. - \frac{1}{15} (-24 - 60\beta^- + 20\bar{\gamma} + 5\zeta \mathcal{S}_-^2 + 20\zeta \bar{\gamma} \mathcal{S}_-^2 - 15\zeta \mathcal{S}_- \mathcal{S}_+ \delta + 80\zeta \mathcal{S}_-^2 \nu) \nu^2 \right) \right] \dot{r} \\ E_{\text{diss}}^{3\text{PN,inst}} &= \frac{2\alpha^4 \zeta \tilde{G}^4 m^5 \mathcal{S}_-^2 \nu^2}{9r^4} \left(\bar{\gamma} + 4\zeta \mathcal{S}_- (-\mathcal{S}_+ \delta + \mathcal{S}_- (-1 + 2\nu)) \right) \end{aligned}$$

Due to radiation-reaction, orbits are not perfectly circular, so $\dot{r} = (n \cdot v) = \mathcal{O}(1/c^3)$.

The Schott terms will contribute to GW already at 3PN order !

To compute the pseudo-Schott terms, we need to consider

$$\left. \frac{dQ_{\text{cons}}^N}{dt} \right|_{\mathbf{a}_{\text{diss}}^{\text{tail}}} + \mathcal{F}_Q^{\text{tail}} = -\delta\mathcal{F}_Q^{\text{tail}}$$

for $Q = E, \mathbf{P}, \mathbf{J}$ and $\mathbf{G}$. The associated pseudo-Schott terms then will be

$$Q_{\text{diss}}^{\text{3PN,tail}} = \int_{-\infty}^t dt' \delta\mathcal{F}_Q^{\text{tail}}(t').$$

Non-local-in-time pseudo-Schott terms : Energy

For the energy we have

$$E_N = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - \frac{\tilde{G}\alpha m_1m_2}{r_{12}}$$

$$\left. \frac{dE_N}{dt} \right|_{a_1^{\text{tail diss}}} = \frac{2G^2m(3+2\omega_0)}{3c^6} \mathcal{I}_i^{(1)} \int_0^{+\infty} d\tau \ln\left(\frac{c\tau}{2b_0}\right) \left[\mathcal{I}_i^{(5)}(t-\tau) + \mathcal{I}_i^{(5)}(t+\tau) \right] + \mathcal{O}\left(\frac{1}{c^8}\right)$$

$$\mathcal{F}_{s,E \text{ tail}}^i = \frac{4G^2M(3+2\omega_0)}{3c^6} \mathcal{I}_i^{(2)}(u) \int_0^{+\infty} d\tau \left[\ln\left(\frac{c\tau}{2b_0}\right) + 1 \right] \mathcal{I}_i^{(4)}(u-\tau) + \mathcal{O}\left(\frac{1}{c^8}\right),$$

After combination and integration, we find [ED and Trestini, 2026 (in prep)]

$$E_{\text{diss}}^{3\text{PN, tail}} = \frac{2G^2m(3+2\omega_0)}{3c^6} \left\{ \int_0^{+\infty} d\rho \mathcal{I}_i^{(3)}(t-\rho) \int_0^{+\infty} d\tau \ln\left(\frac{c\tau}{2b_0}\right) \left[\mathcal{I}_i^{(3)}(t-\rho-\tau) - \mathcal{I}_i^{(3)}(t-\rho+\tau) \right] \right.$$

$$- \mathcal{I}_i^{(1)}(t) \int_0^{+\infty} d\tau \ln\left(\frac{c\tau}{2b_0}\right) \left[\mathcal{I}_i^{(4)}(t-\tau) + \mathcal{I}_i^{(4)}(t+\tau) \right]$$

$$\left. - \mathcal{I}_i^{(2)}(t) \int_0^{+\infty} d\tau \ln\left(\frac{c\tau}{2b_0}\right) \left[\mathcal{I}_i^{(3)}(t-\tau) - \mathcal{I}_i^{(3)}(t+\tau) \right] - \mathcal{I}_i^{(2)}(t) \mathcal{I}_i^{(2)}(t) \right\}$$

- (Pseudo-) Schott terms are essential to ensure that PN predictions for the phasing are gauge-invariant
- In this work, we are extending the derivation of radiation-reaction effects to scalar-tensor theories.
- This will be crucial for computing the GW phase and amplitude at 3PN order, to achieve a level of accuracy equivalent to that of GR.

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