

Charges aspects of BMS symmetries (as opposed to a stress energy tensor)

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General context

- Bondi-Sachs formalism
→ Bondi et al. 1962, Sachs 1962
- Global quantities as Noether charges of asymptotic BMS symmetries at null infinity
→ Observables such as energy and angular momentum
- Can one extract local quantities such as a stress-energy tensor ?

Specific context

- Recent analysis of the covariance of BMS charges and currents
→ Rignon-Bret, Speziale 2024
- Takes into account arbitrary cross-sections of null infinity

Results presented here

- Identification of a tensorial formulation for the integrands
- Comparison with a stress-energy tensor

Bondi-Sachs Formalism

- Coordinate system (u, r, θ^A) adapted to null geodesics
 - Foliation by null hypersurfaces $u = \text{cst}$
 - Set conditions $g^{uu} = 0$, $g^{uA} = 0$ and $\det(g_{AB}) = r^4 \det(q_{AB})$
- Used to describe future null infinity $\mathcal{I}^+$ as $r \rightarrow \infty$
 - Asymptotic flatness defined as approaching Minkowski
 - Metric expansion in $\frac{1}{r}$

$$\begin{aligned}
 g_{\mu\nu} = & \begin{pmatrix} 0 & -1 & 0 \\ & 0 & 0 \\ & & r^2 q_{AB} \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ & 0 & 0 \\ & & r C_{AB} \end{pmatrix} \\
 & + \begin{pmatrix} -1 & * & * \\ & 0 & 0 \\ & & * \end{pmatrix} + \begin{pmatrix} \frac{2}{r} GM & * & \frac{2}{3r} G J_A + * \\ & 0 & 0 \\ & & * \end{pmatrix} + \dots
 \end{aligned}$$

↓ Sphere metric
↓ Shear
↘ Mass aspect
↘ Angular momentum aspect

Remark

There is some overlap with the term aspect

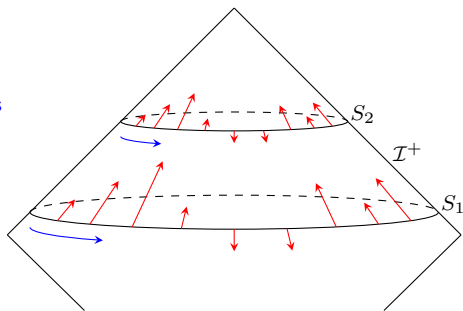
BMS symmetry group

- Asymptotic symmetry group:
coordinates transformations that preserve the asymptotic behaviour of the metric
- Associated algebra:

$$\xi = T\partial_u + \frac{1}{2}u(\mathcal{L}_A Y^A)\partial_u + Y^A\partial_A + \mathcal{O}(\Omega) \quad \text{with} \quad \Omega \stackrel{\mathcal{I}^+}{=} 0 \quad (1)$$

$T(\{\theta^A\})$ supertranslations
Infinite dimensions !

$Y^A(\{\theta^A\})$ Lorentz transformations
Conformal Killing Vectors
6 dimensions



Associated charges

On a section S of $\mathcal{I}^+$ at $u = \text{cst}$ that is a round sphere :

→ Supermomentum charge:

$$Q_T^{\text{BMS}} = \frac{1}{4\pi} \oint_S TM \epsilon_S \quad (2)$$

→ Lorentz charge:

$$Q_Y^{\text{BMS}} = \frac{1}{8\pi} \oint_S \left(Y^A J_A + u (\mathcal{D}_A Y^A) M \right) \epsilon_S \quad (3)$$

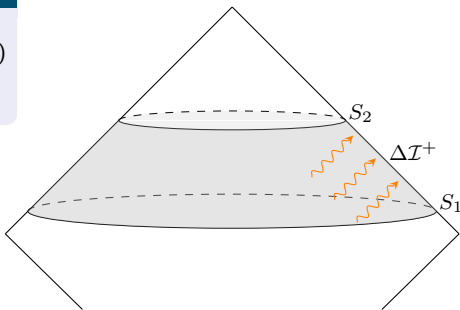
Flux-balance laws

$$Q_\xi[S_2] - Q_\xi[S_1] \approx \int_{\Delta\mathcal{I}^+} j_\xi \quad (4)$$

→ Conserved in the absence of radiation

Local formula

$$dq_\xi \approx j_\xi \quad (5)$$



Conformal transformation and non round spheres

→ Change of the conformal factor Ω linked to the unphysical metric $\hat{g}'_{\mu\nu} = \Omega'^2 g_{\mu\nu}$ with :

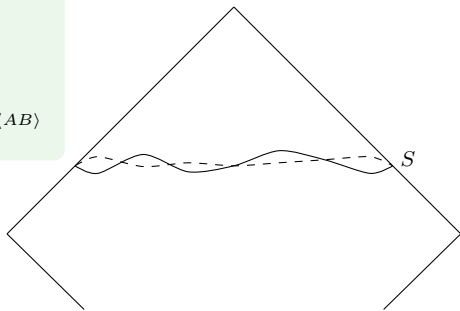
$$\Omega' = \omega \Omega \quad \text{with } \omega(\theta^A) \text{ arbitrary} \quad (6)$$

→ Higher order terms to keep adapted Bondi coordinates:

$$\Omega' = \omega \Omega + \frac{1}{2} \Omega^2 \alpha \quad u' = \omega u - \frac{\Omega}{2\omega} (\partial_C \omega) (\partial^C \omega) u^2 + \frac{1}{2} \Omega^2 \beta \quad \theta'^A = \theta^A - \frac{\Omega}{\omega} (\partial^A \omega) u + \frac{1}{2} \Omega^2 \gamma^A \quad (7)$$

Geroch tensor

- Tensor ρ_{AB}
- Conformal transformation ρ'_{AB} is known
- Is simply Ricci curvature $\rho_{AB} = \frac{1}{2} \mathcal{R} q_{AB}$ for round spheres
- Used to make the news $N_{AB} = \dot{C}_{AB} + \rho_{\langle AB \rangle}$ conformally invariant



Covariance of the charges and the currents

- Currents and charges for two BMS transforms verified before improvement:

$$\{\bar{J}_\xi, \bar{J}_\chi\}^* \approx \bar{J}_{[\xi, \chi]} \quad \{\bar{Q}_\xi, \bar{Q}_\chi\}^* \approx \bar{Q}_{[\xi, \chi]} + \bar{K}_{(\xi, \chi)} \quad \text{with} \quad \partial_u \bar{K}_{(\xi, \chi)} = 0 \quad (8)$$

- Cocycle $\bar{K}$ removed by adding a new term:

$$\theta^{\text{BMS}} = \theta^{\text{BT}} + \delta l^G \quad \text{with} \quad l^G = -\frac{1}{32\pi} \rho_{AB} C^{AB} \epsilon_{\mathcal{I}} \quad (9)$$

→ Note that $l^G = 0$ for round spheres

- Change of the charges:

$$Q_\xi^{\text{BMS}} = \frac{1}{8\pi} \oint_S (2T \underbrace{M_\rho}_{\text{red circle}} + u(\mathcal{D}_A Y^A) \underbrace{M_\rho}_{\text{red circle}} + Y^A J_A) \epsilon_{\mathcal{I}} \quad (10)$$

$M_\rho = M - \frac{1}{8} \rho_{AB} C^{AB}$ instead of M

Note that $M_\rho = M$ for round spheres

Determination of the charge aspects

→ Start from the Noether current:

$$j_{\xi}^{\text{BMS}} = \frac{1}{32\pi} N^{AB} (\delta_{\xi} C_{AB}) \epsilon_{\mathcal{I}} \quad (11)$$

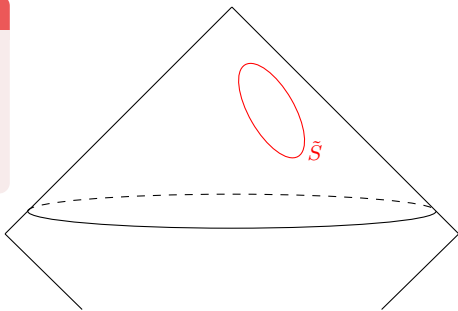
→ Identify q_{ξ}^A writing it as a total divergence:

$$8\pi j_{\xi}^{\text{BMS}} \approx (D_a q_{\xi}^a) \epsilon_{\mathcal{I}} \approx \left((\partial_u q_{\xi}^u) + (\mathcal{D}_A q_{\xi}^A) \right) \epsilon_{\mathcal{I}} \quad \text{with } q_{\xi}^u \text{ known} \quad (12)$$

→ We obtain q_{ξ} as a 3d vector (Hodge dual of a 2-form) on $\mathcal{I}^+$

Integration over an arbitrary surface

$$\begin{aligned} Q_{\xi}[\tilde{S}] &= \int_{\tilde{S}} \frac{1}{2} q_{\xi}^u \epsilon_{\mathcal{I} u A B} d\theta^A \wedge d\theta^B \\ &+ \int_{\tilde{S}} q_{\xi}^A \epsilon_{\mathcal{I} A u B} du \wedge d\theta^B \end{aligned} \quad (13)$$



Result for the BMS charge aspect q_ξ

$$\begin{aligned}
 q_\xi^u &= 2\xi^u M_\rho + Y^A J_A \\
 q_\xi^A &= \frac{1}{2}\xi^u (\mathcal{D}_B N^{AB}) + \frac{1}{2}Y^B N^{AC} C_{BC} - \frac{1}{8}Y^A N_{BC} C^{BC} \\
 &\quad + \frac{1}{4}Y^B (\mathcal{D}^A \mathcal{D}^C C_{BC}) - \frac{1}{4}Y^B (\mathcal{D}_B \mathcal{D}_C C^{AC}) - \frac{1}{2}Y^B \rho_{BC} C^{AC} - Y^A M_\rho \\
 &\quad - \frac{1}{2}(\mathcal{D}_B \xi^u) N^{AB} + \frac{1}{4}(\mathcal{D}^B Y^A)(\mathcal{D}^C C_{BC}) - \frac{1}{4}(\mathcal{D}^A Y^B)(\mathcal{D}^C C_{BC}) \\
 &\quad - \frac{1}{4}(\mathcal{D}_B \mathcal{D}_C Y^C) C^{AC}
 \end{aligned} \tag{14}$$

Comparison with stress-energy tensor

→ Standard Noether formulation:

$$Q_\xi = \int_S T^\mu{}_\nu \xi^\nu dS_\mu \tag{15}$$

→ Here presence of derivatives of ξ :

$$Q_\xi = \oint_S \left(T_{(0)\nu}^\mu \xi^\nu + T_{(1)\nu}^{\mu\rho} (D_\rho \xi^\nu) + T_{(2)}^{\mu\rho\sigma}{}_\nu (D_\rho D_\sigma \xi^\nu) \right) dS_\mu \tag{16}$$

Conclusion

- Identification of a charge aspect as a tensor at null infinity
- Corresponds to a notion of charge density
- Presence of 2d derivatives of the symmetry vector
 - Not a stress-energy tensor per se
- Residual freedom of the charge aspects as 3d total derivatives
 - Doesn't change this last characteristic

Thank you