

Radial perturbations in modified theories of gravity

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Dixième Assemblée Générale du GdR Ondes Gravitationnelles

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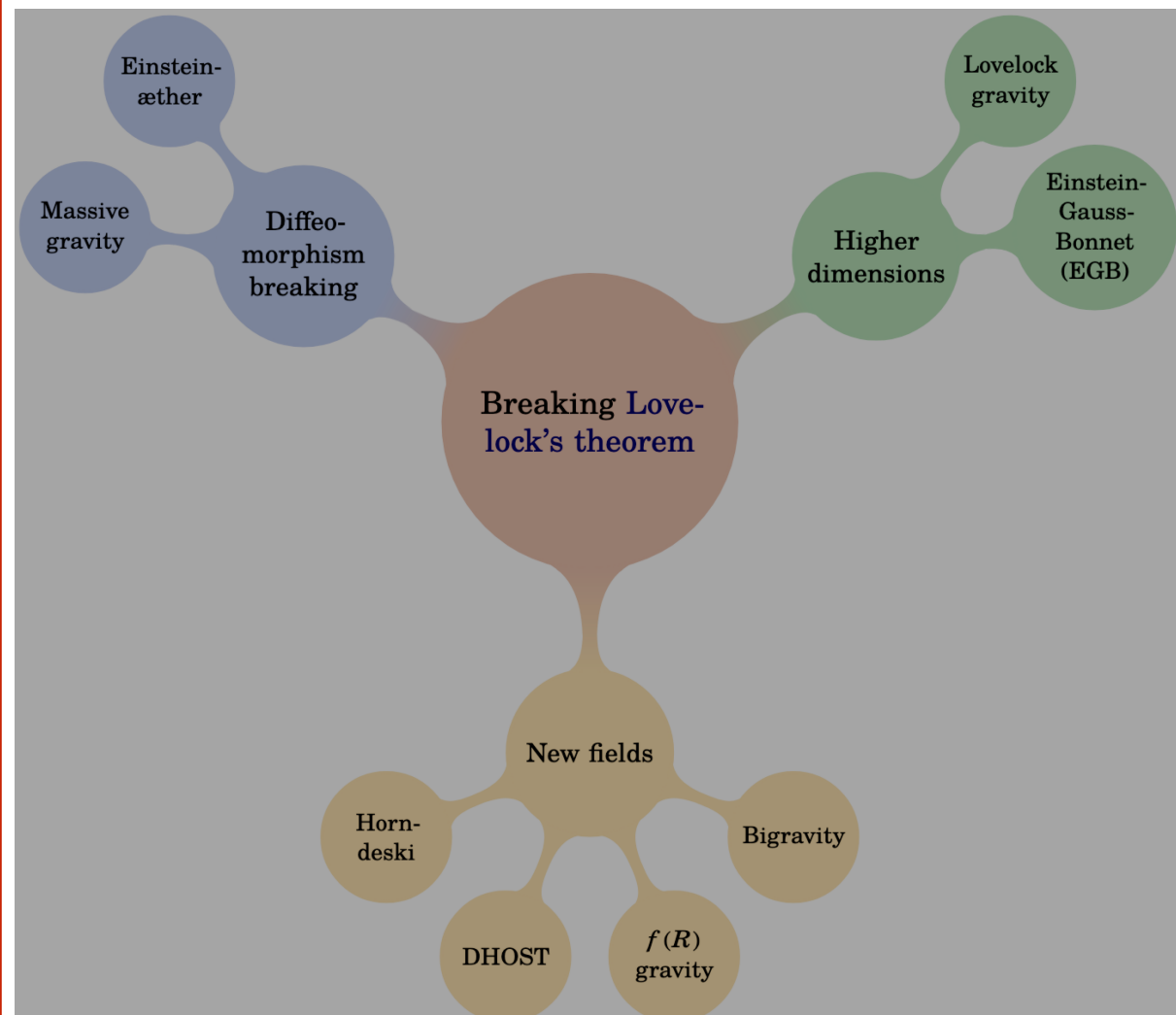


- GW astronomy provides a new window to test GR, in particular in the strong field regime.
- Why Modify GR ? $\rightarrow$ GR seems compatible with most observations
 - $\rightarrow$ Understand cosmological acceleration
 - $\rightarrow$ Explore alternative gravitational theories $\rightarrow$ what singles out GR as a theory ?
 - $\rightarrow$ Step toward quantum gravity
- In this talk: scalar-tensor theories (allowing for 2nd order derivatives in their Lagrangian)
 - $\rightarrow$ Extract quantitative deviation of DHOST theories

Modified Theories of gravity And DHOST theories

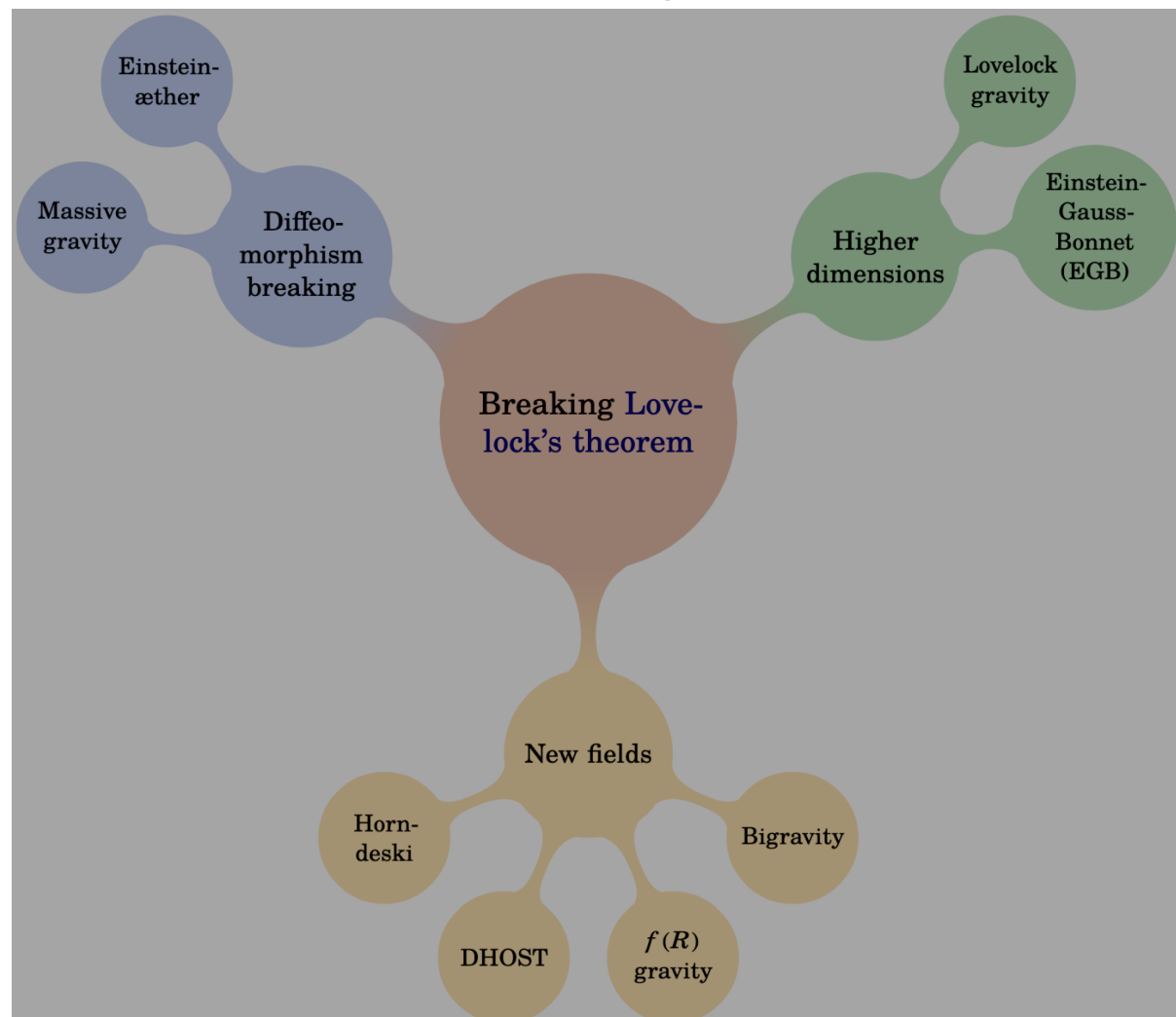
Lovelock Theorem

Lovelock's theorem: In 4d, the only rank-2 tensor constructed solely from the metric $g_{\mu\nu}$ and its derivatives up to second differential order, preserving diffeomorphism invariance and arising from a least action principle is the Einstein tensor plus a cosmological term.



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Scalar Tensor Theories

- Traditional scalar-tensor theories :
$$\mathcal{L}(\nabla_\lambda \phi, \phi)$$
- Generalized theories with second order derivatives
$$\mathcal{L}(\nabla_\mu \nabla_\nu \phi, \nabla_\lambda \phi, \phi)$$
- In general, they contain an extra degree of freedom, expected to lead to instabilities
$$L(\ddot{q}, \dot{q}, q)$$
- But this can be avoided if the theory is degenerate (2nd-order equations of motion not necessary !) → DHOST

Compact objects In DHOST theories

Non-Rotating BH

Consider a static spherically symmetric BH with a nontrivial scalar field

$$ds^2 = -\mathcal{A}(r)dt^2 + \frac{dr^2}{\mathcal{B}(r)} + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2)$$

- Scalar field: $\phi(t, r) = qt + \psi(r)$ [Babichev, Charmousis '13]

The scalar charge q can constitute an additional charge.

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Phenomenology

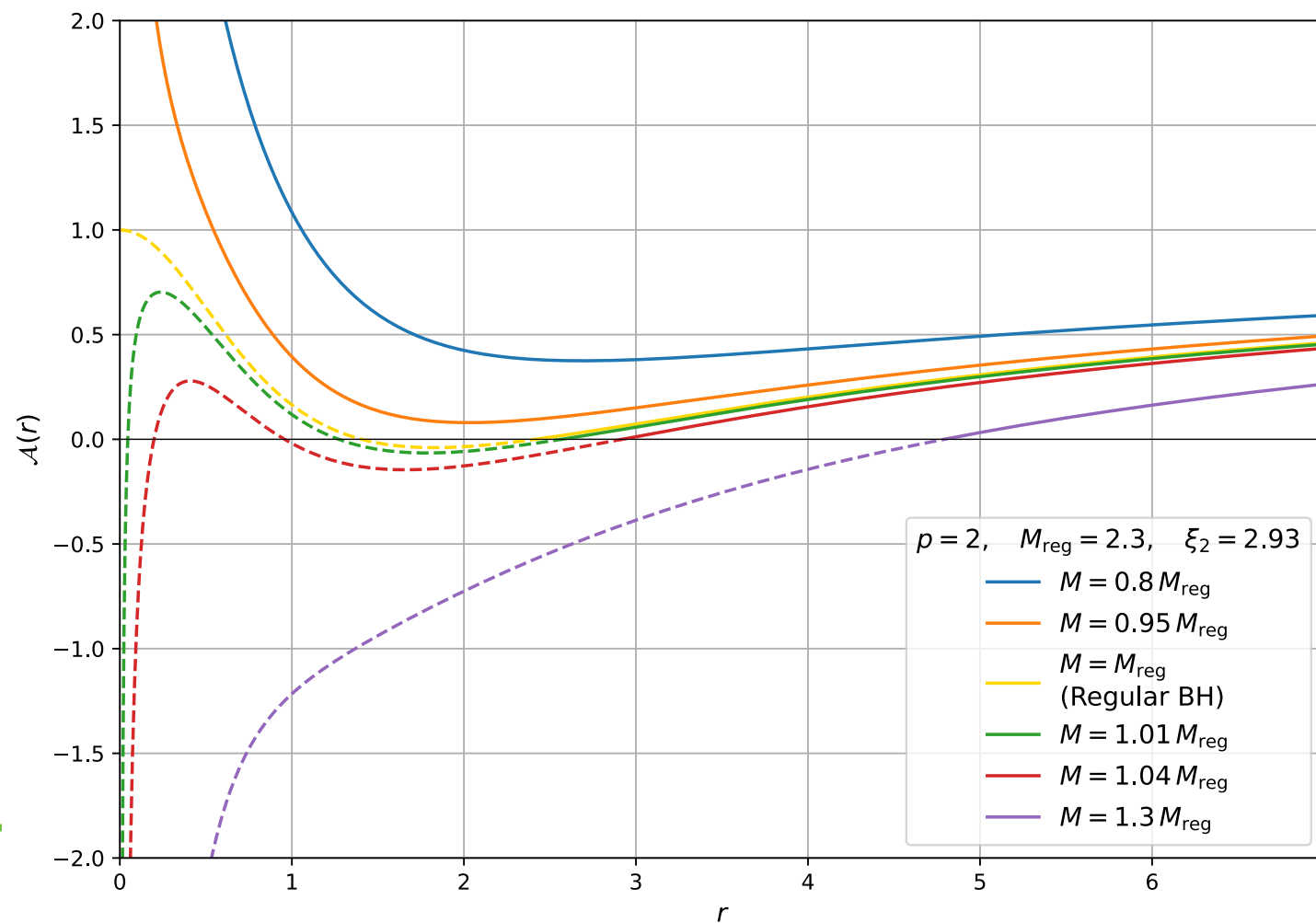
- ϕ lives close to the BH and modify the geometry
- On the background : induces parametrized modification in $\mathcal{A}(r), \mathcal{B}(r)$
- On the GW signal : new polarizations, new modes...

General Hairy BH

$$\mathcal{A}(r) = \mathcal{B}(r) = 1 - \frac{2M}{r} - \frac{2\xi_p}{r} \int_0^r \frac{u^2 du}{(1+u^2)^p}$$

$$\mathcal{A}(r) = 1 - \frac{2M}{r} + \xi_2 \left(\frac{\pi/2 - \arctan r}{r} + \frac{1}{1+r^2} \right)$$

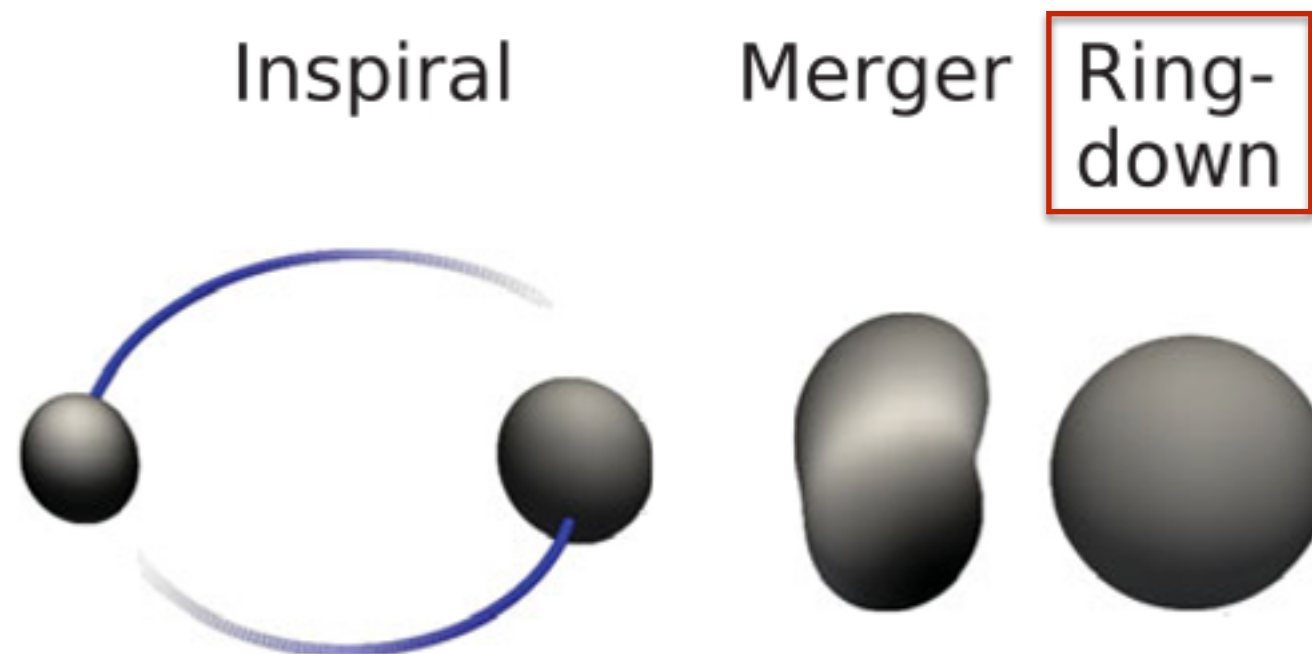
No singularity at $r = 0$,
 $M = M_{\text{reg}} = \frac{\pi}{4} \xi_2$



Could we distinguish
such objects from
GR BH in the
GW signal ?

BH Perturbation Theory

- Ringdown phase of a BH merger is interesting for (modified) gravity models, as it can be described by BH perturbations.



- Deviations to GR in the context of DHOST theories ?

In General Relativity

$$g_{\mu\nu} = g_{\mu\nu}^{\text{bgd}} + h_{\mu\nu}$$

- Perturbation parametrized by the 10 components of $h_{\mu\nu}$
- Expand on Fourier modes $f(t, r) = f(r)e^{-i\omega t}$ and on spherical harmonics $Y_{\ell m}(\theta, \varphi)$
- Axial Sector [Regge-Wheeler 1953] and Polar sector [Zerilli 1970]

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Modified Gravity

- Reproduce a similar analysis in scalar tensor theory
- Identify generic features that distinguish them from GR

GR

- In GR, radial perturbations are absent as a consequence of Birkhoff's theorem.

DHOST

- In DHOST: radial perturbations describes a single degree of freedom which combines both scalar and metric perturbations.
- Its propagation can be described by a simple Schrödinger-like equation in some appropriate coordinates (t_*, r_*) .

$$\left[\frac{\partial^2}{\partial t_*^2} - \frac{\partial^2}{\partial r_*^2} \right] \Psi + V_{\text{eff}} \Psi = 0$$

GR for $\ell \geq 2$: Tortoise Coordinates

- The domain of r_* extends from the horizon at $-\infty$ to spatial infinity at $+\infty$
- Vanishing Potential at both ends
- Boundary conditions imposed at both end.
- QNMs appear (**complex** frequencies)

GR for $\ell \geq 2$: Tortoise Coordinates

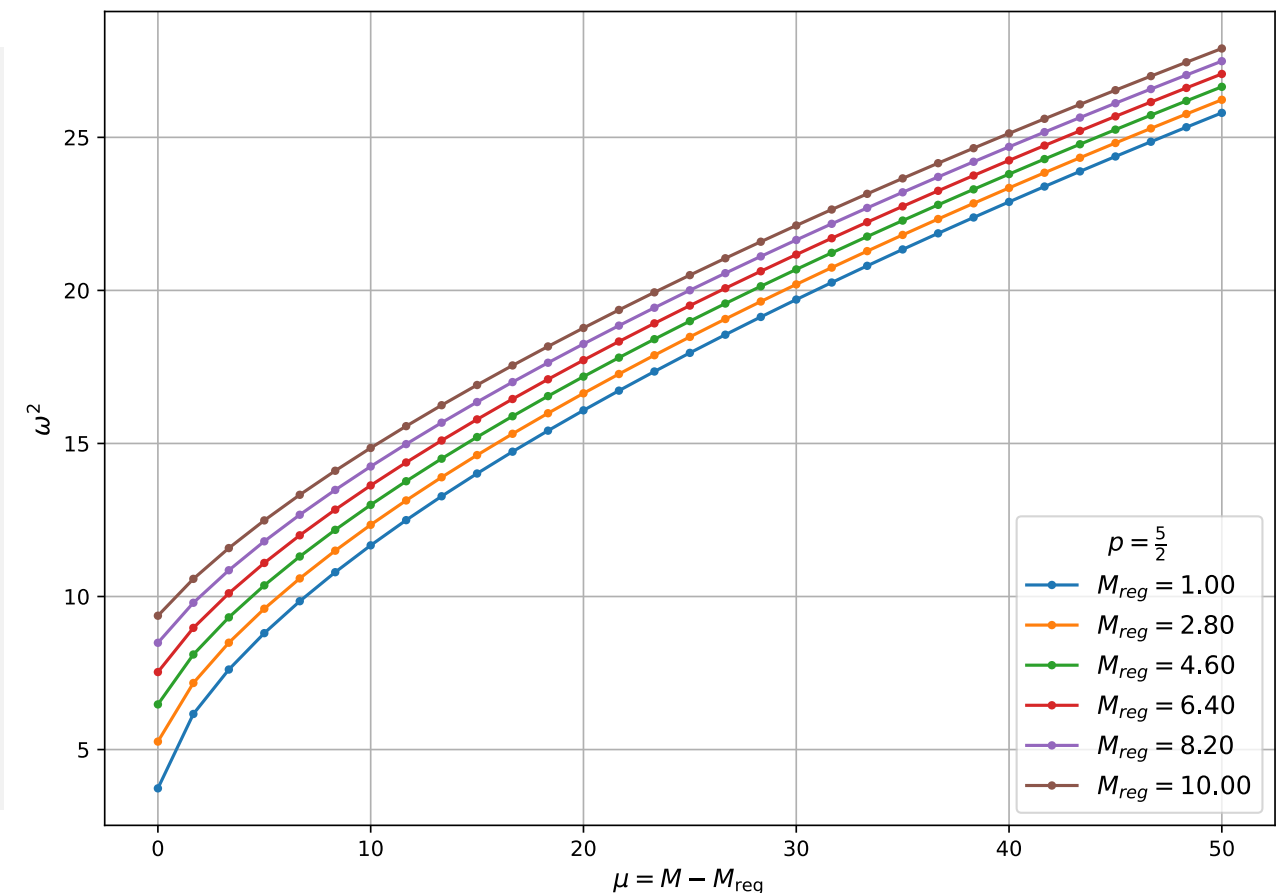
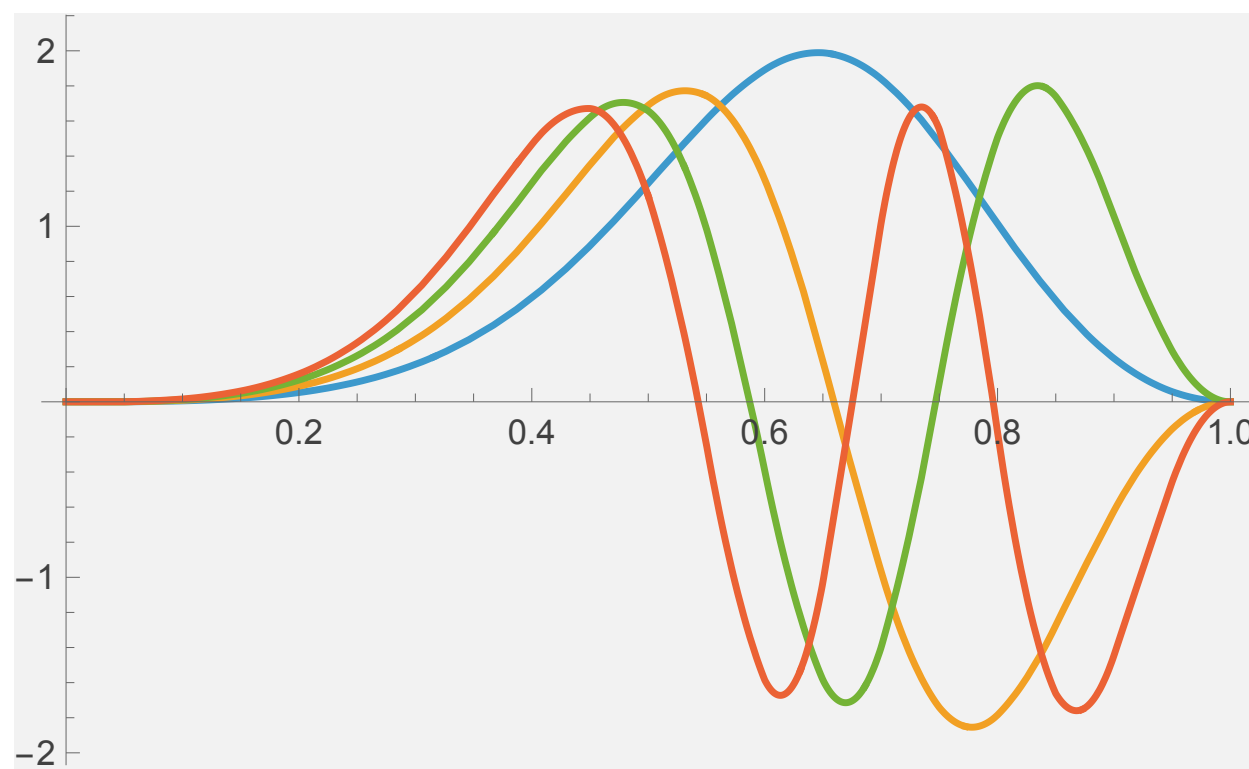
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Hairy BH Monopole

- Because ϕ is regular up to the singularity at $r=0$
$$r_* \in [0, r_*^{\max}]$$
- Different potential behaviors
- Result in different boundary conditions
- Now appearance of **normal** modes (**real** frequencies)

Hairy BH Monopole

- Positive Schrödinger operator $\rightarrow$ time evolution of Ψ remains bounded at all times.
- Real values of ω



Conclusions

- DHOST theories generically give rise to a monopole
- For an interesting subclass of theories, this monopole is described by real frequencies.

But how do we measure them in practice?

- Future perspective: higher ℓ with the dipole and the multipoles

Thank you for your attention!

Additional Slides

Quadratic DHOST

$$S = \int d^4x \sqrt{-g} \left[P(X, \phi) + Q(X, \phi) \square \phi + F(X, \phi) R + \sum_{i=1}^5 A_i(X, \phi) L_i^{(2)} \right]$$

Where $X \equiv -\frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi$ $\phi_\mu \equiv \nabla_\mu \phi$ $\phi_{\mu\nu} \equiv \nabla_\nu \nabla_\mu \phi$

$$L_1^{(2)} = \phi_{\mu\nu} \phi^{\mu\nu}, \quad L_2^{(2)} = (\square \phi)^2, \quad L_3^{(2)} = (\square \phi) \phi^\mu \phi_{\mu\nu} \phi^\nu$$

$$L_4^{(2)} = \phi^\mu \phi_{\mu\rho} \phi^{\rho\nu} \phi_\nu, \quad L_5^{(2)} = \left(\phi^\mu \phi_{\mu\nu} \phi^\nu \right)^2$$

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Possible extensions

- Possible extension to cubic order (in $\phi_{\mu\nu}$) [Ben Achour, Crisostomi, Koyama, Langlois, Noui & Tasinato '16]

$$L^{(3)} = F_3(X, \phi) G_{\mu\nu} \phi^{\mu\nu} + \sum_{i=1}^{10} B_i(X, \phi) L_i^{(3)}$$

- DHOST includes Horndeski, Beyond Horndeski, Einstein-scalar-Gauss-Bonnet
e.g. (quadratic) Horndeski: $A_2 = -A_1 = F, X$, $A_3 = A_4 = A_5 = 0$

Non-Rotating BH

Consider a static spherically symmetric BH with a nontrivial scalar field

$$ds^2 = -\mathcal{A}(r)dt^2 + \frac{dr^2}{\mathcal{B}(r)} + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2)$$

- Scalar field: $\phi(t, r) = qt + \psi(r)$ [Babichev, Charmousis '13]

The scalar charge $q \neq 0$ possible in shift-symmetric theories

Quadratic DHOST theories considered

$$P(X) = -\frac{2\alpha}{\lambda^2}X^p, \quad F(X) = 1 - 2XA_1, \quad A_1(X) = \frac{\alpha}{2}X^{p-1}, \quad A_3(X) = \frac{\alpha}{2}(2p-1)X^{p-2}$$

Then, one is able to find explicit solutions:

$$\mathcal{A}(r) = \mathcal{B}(r) = 1 - \frac{2\mu}{r} - \frac{2\xi_p}{r} \int_0^r \frac{u^2 du}{(1+u^2)^p}$$

The scalar Hair $\xi_p \equiv \alpha(2p-1) \left(\frac{q^2}{2} \right)^p$

$$\text{and } X = \frac{q^2}{2(1+r^2)}, \quad \psi'(r) = \frac{q}{\mathcal{A}(r)} \sqrt{1 - \frac{\mathcal{A}(r)}{1+r^2}}$$

Asymptotic Behavior

Schematically, $V(r_*) \propto \frac{\alpha}{r_*^2}$

Depending on the value of α we need to impose boundary conditions in order for the Schrödinger operator to be self-adjoint.

$$\psi(r_{max}) = \psi(0) = 0$$

Positivity

$$\int_0^{r_*^{\max}} dr_* \bar{\Psi} \mathcal{L}_E \Psi = \mathcal{B}_0 - \mathcal{B}_{\max} + \int_0^{r_*^{\max}} dr_* |\mathcal{D}\Psi|^2,$$
$$\mathcal{D}\Psi \equiv \partial_* \Psi + S\Psi,$$

$$\mathcal{B}_0 = \lim_{r_* \rightarrow 0} \bar{\Psi} \mathcal{D}\Psi, \quad \mathcal{B}_{\max} = \lim_{r_* \rightarrow r_*^{\max}} \bar{\Psi} \mathcal{D}\Psi.$$

Hairy BH Monopole

- Pass to Fourier space $\Psi(t_*, r_*) = e^{-i\omega t_*} \Psi(r_*)$ and define the Schrödinger operator $\mathcal{L}$:

$$\mathcal{L}\Psi = -\frac{d^2\Psi}{dr_*^2} + V_{\text{eff}}\Psi = \omega^2\Psi,$$

- Impose Boundary conditions at the zeros of $r^2 + 1 - \mathcal{A}(r)$ and at spatial infinity to define a self-adjoint-operator.
- Prove positivity of the spectrum using the S-deformation:

$$V_{\text{eff}} = S^2 - \frac{dS}{dr_*}$$

Disformal Transformation

Transformation [Bekenstein '92]

$$g_{\mu\nu} \longrightarrow \tilde{g}_{\mu\nu} = C(X, \phi)g_{\mu\nu} + D(X, \phi)\partial_\mu\phi\partial_\nu\phi$$

Generating new theories

From $\tilde{S}[\phi, \tilde{g}_{\mu\nu}]$, one gets the new action

$$S[\phi, g_{\mu\nu}] \equiv \tilde{S}[\phi, \tilde{g}_{\mu\nu} = Cg_{\mu\nu} + D\phi_\mu\phi_\nu]$$

Physically relevant
DHOST families are
closed under these
transformations

Generating new theories

When standard fields are minimally coupled, two disformally related theories are physically inequivalent!

$$S \left[g_{\mu\nu}, \phi \right] + S_m \left[\Psi_m, g_{\mu\nu} \right] \neq \tilde{S} \left[\tilde{g}_{\mu\nu}, \phi \right] + S_m \left[\Psi_m, \tilde{g}_{\mu\nu} \right]$$

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Some Examples

- «stealth» Schwarzschild: $\mathcal{A} = \mathcal{B} = 1 - \frac{\mu}{r}$
- «BCL» [Babichev, Charmousis, Lehébel '17]. $\mathcal{A} = \mathcal{B} = 1 - \frac{\mu}{r} - \xi \frac{\mu^2}{2r^2}$
- 4d Gauss-Bonnet [Glavan Lin '19, Lu Pang '20]. $\mathcal{A} = \mathcal{B} = 1 - \frac{2\mu/r}{1 + \sqrt{1 + 4\alpha\mu/r^3}}$
- Scalar-Gauss-Bonnet [Julié Berti '19] $\mathcal{A} = \mathcal{B} = 1 - \frac{\mu}{r} + a_2(r)\varepsilon^2 + \dots$

In General Relativity

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GR for $\ell \geq 2$: Tortoise Coordinates

$$t_* = t$$

$$r_*(r) \equiv \int^r \frac{dx}{\mathcal{A}(x)}$$

- Boundary conditions imposed at the zero of $\mathcal{A}(x)$
- The domain of r_* extends from $-\infty$ to $+\infty$

Hairy BH Monopole

$$t_* = \phi \quad r_*(r) \equiv \int^r \frac{dx}{n(x)},$$

$$n(r) = \frac{(r^2 + 1) \sqrt{r^2 + 1 - \mathcal{A}(r)}}{\sqrt{2p - 1} r}.$$

- Boundary conditions : zeros of $r^2 + 1 - \mathcal{A}(r)$
- $r_* \in [0, r_*^{\max}]$

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Axial Sector

$h_0(r), h_1(r)$ [Regge-Wheeler gauge]

$$h_{\mu\nu} = \sum_{\ell, m} \begin{pmatrix} 0 & 0 & \frac{1}{\sin\theta} h_0^{\ell m} \partial_\varphi & -\sin\theta h_0^{\ell m} \partial_\theta \\ 0 & 0 & \frac{1}{\sin\theta} h_1^{\ell m} \partial_\varphi & -\sin\theta h_1^{\ell m} \partial_\theta \\ \text{sym} & \text{sym} & 0 & 0 \\ \text{sym} & \text{sym} & 0 & 0 \end{pmatrix} Y_{\ell m}(\theta, \varphi)$$

Polar Sector

H_0, H_1, H_2, K [Zerilli gauge]

$$h_{\mu\nu} = \sum_{\ell, m} \begin{pmatrix} A(r)H_0^{\ell m}(r) & H_1^{\ell m}(r) & 0 & 0 \\ H_1^{\ell m}(r) & A^{-1}(r)H_2^{\ell m}(r) & 0 & 0 \\ 0 & 0 & K^{\ell m}(r)r^2 & 0 \\ 0 & 0 & 0 & K^{\ell m}(r)r^2 \sin^2\theta \end{pmatrix} Y_{\ell m}(\theta, \varphi)$$

(And $\delta\phi$ in DHOST)