



CSI - 1st year  
PCal Calibration of Virgo  
&  
Earth's rotation impact on ET





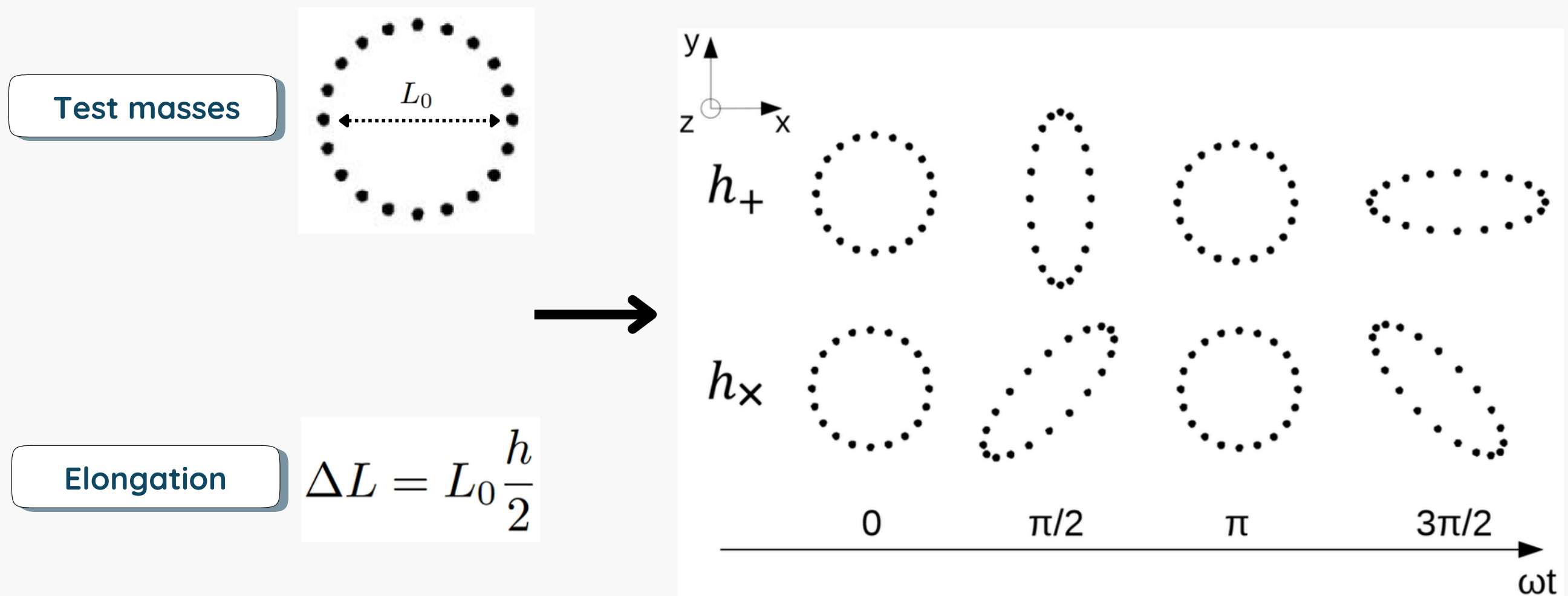
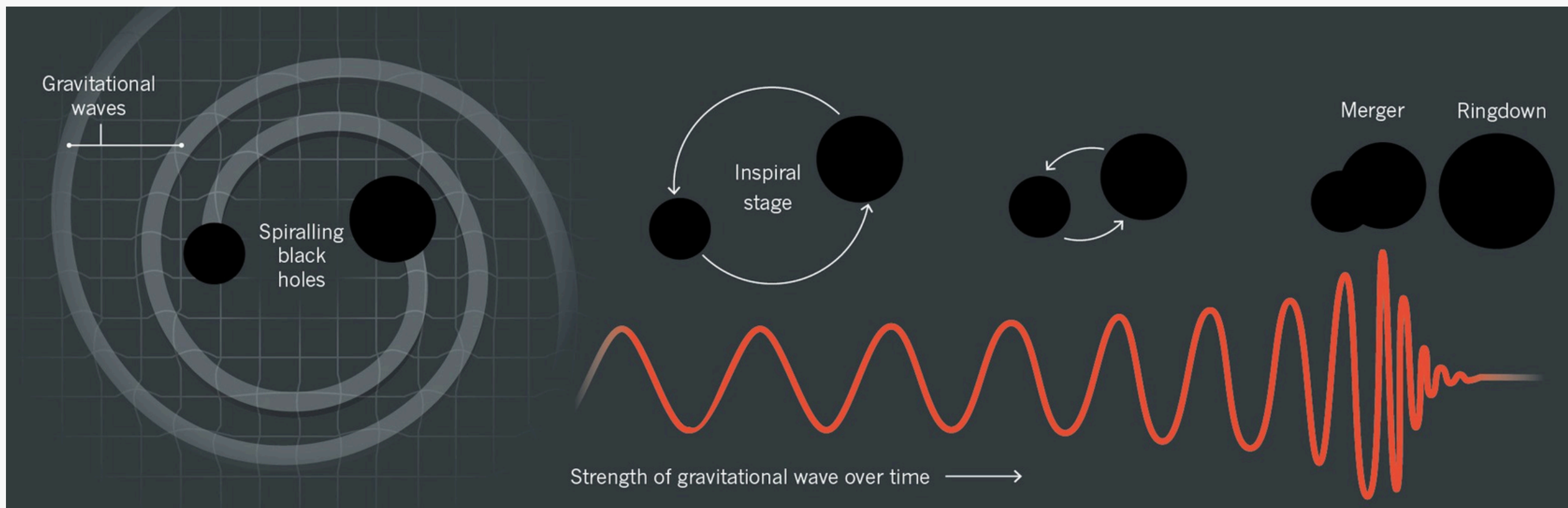
- 
- *Introduction*
  - *Virgo calibration (Loïc)*
  - *Earth's rotation impact on ET (Damir)*
  - *Conclusion - Miscellaneous*
- 



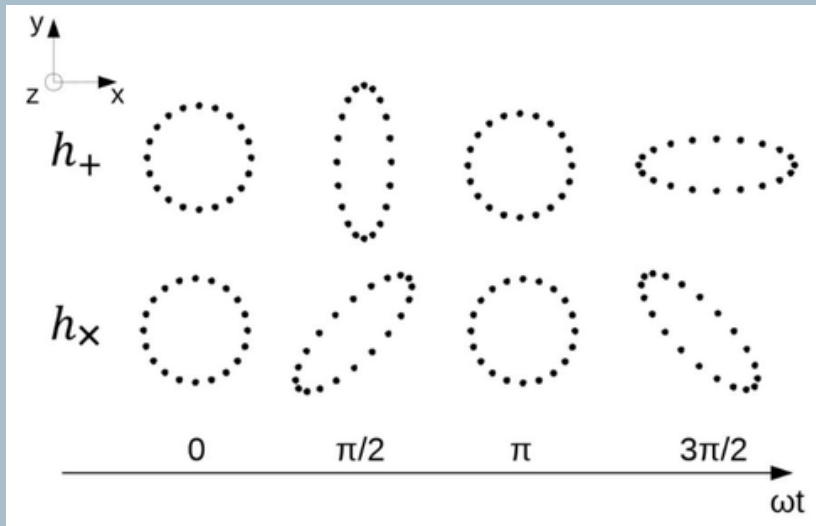


# *Introduction*

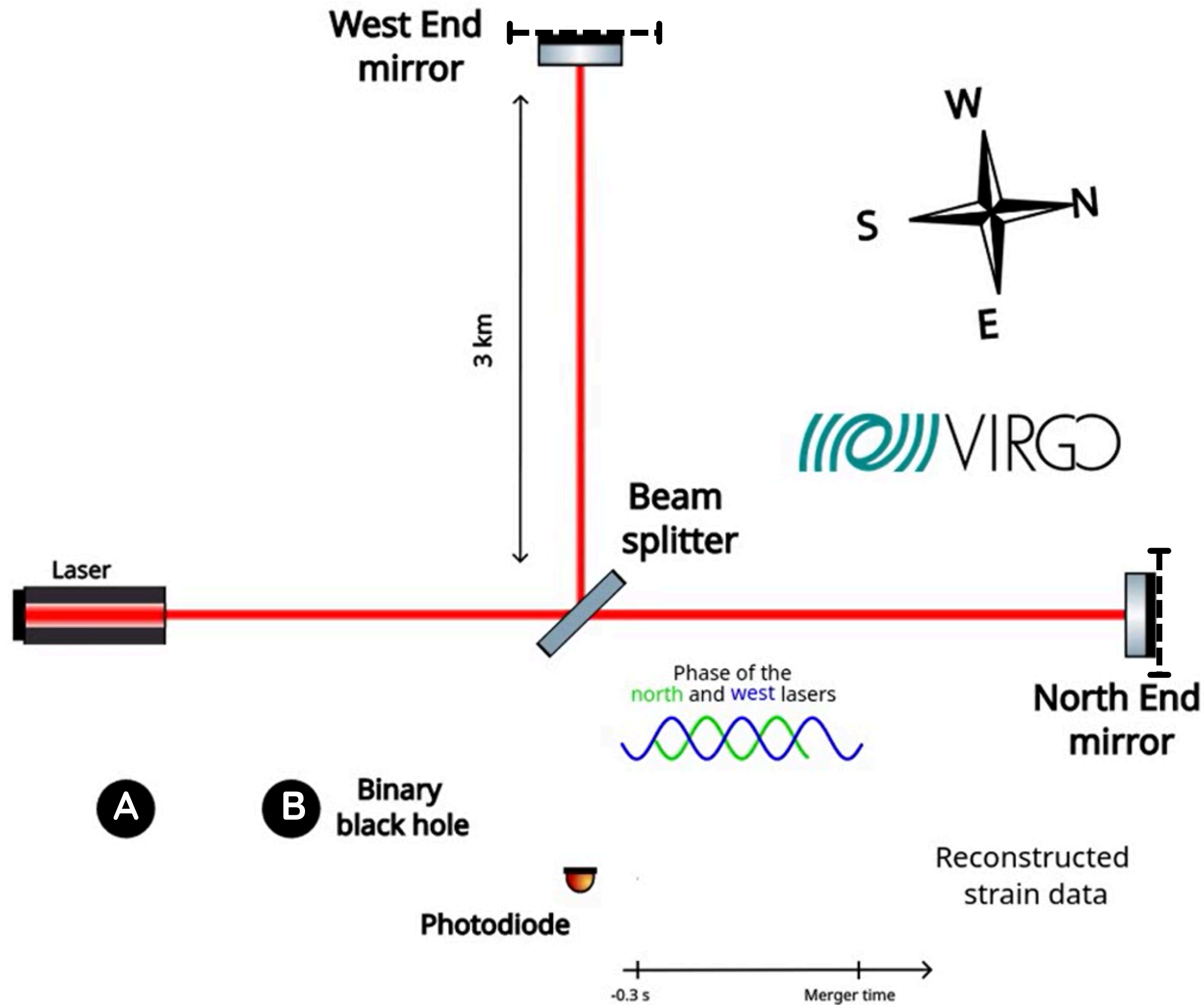




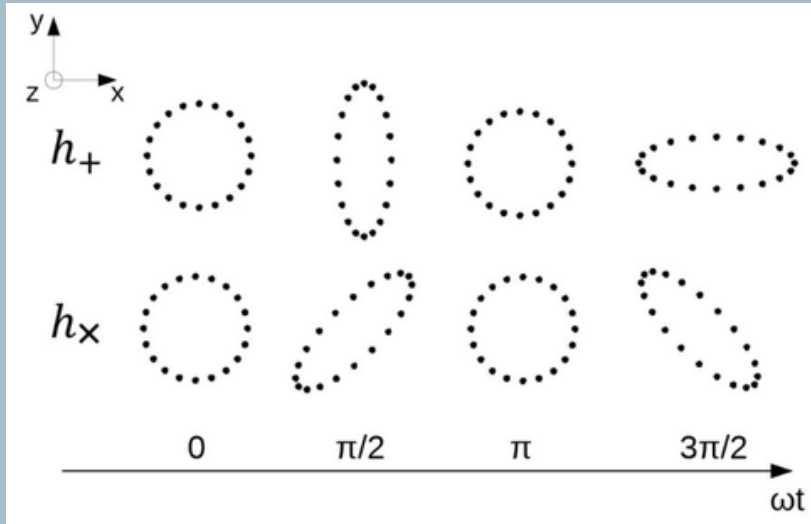
## (Reminders)



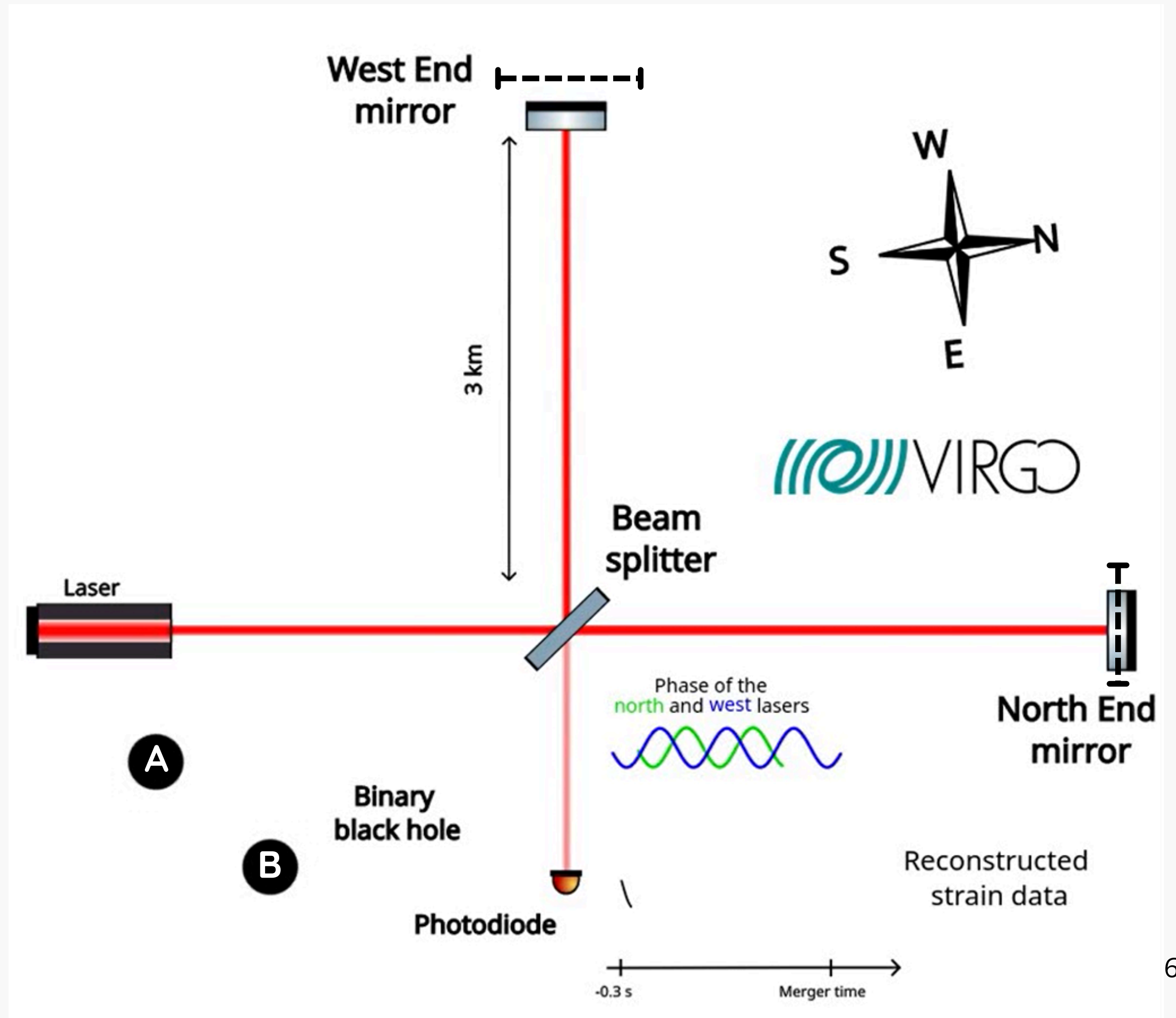
$$\Delta L = L_0 \frac{h}{2}$$



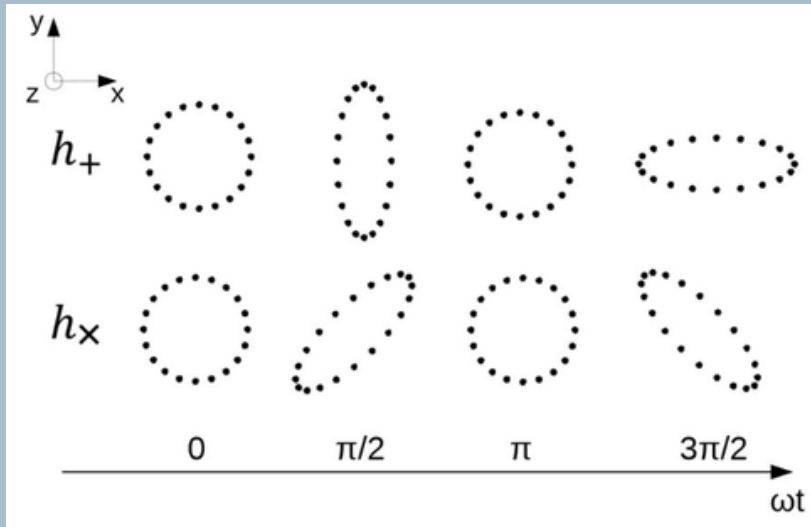
## (Reminders)



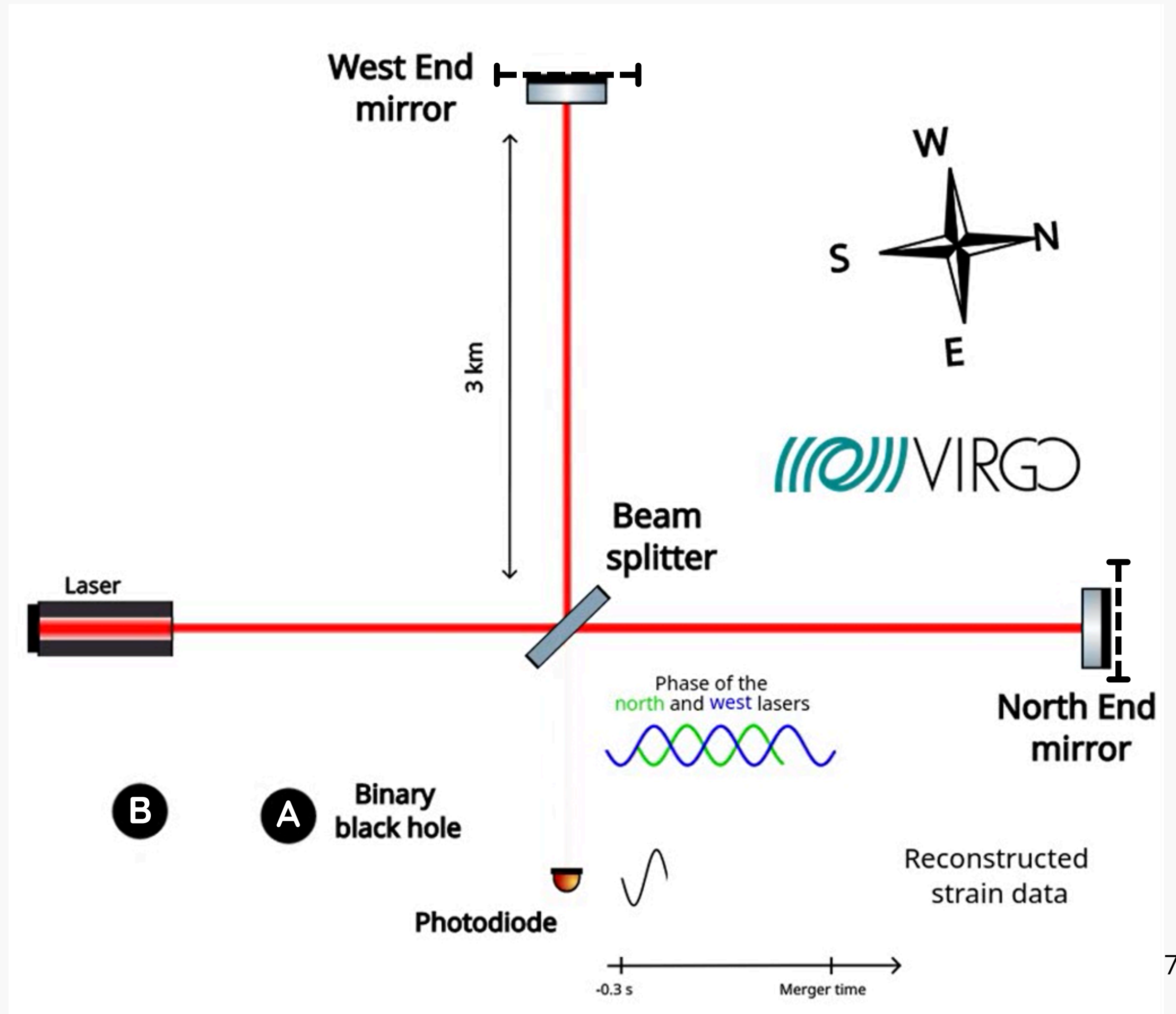
$$\Delta L = L_0 \frac{h}{2}$$



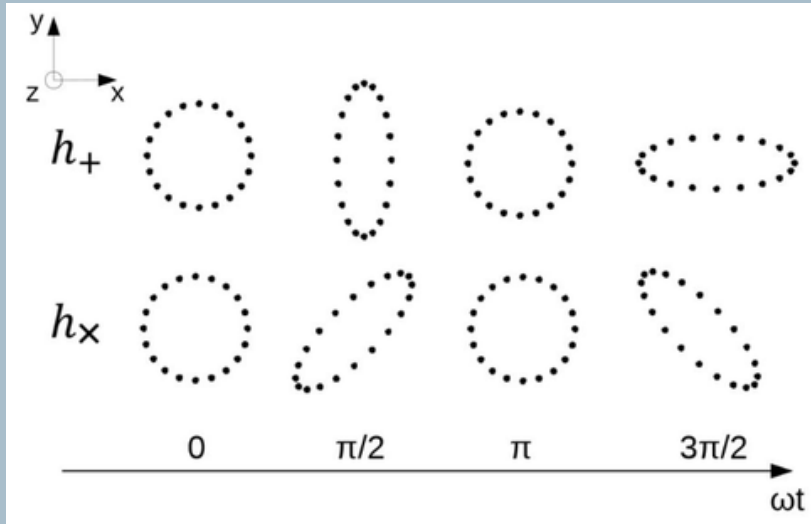
## (Rappels)



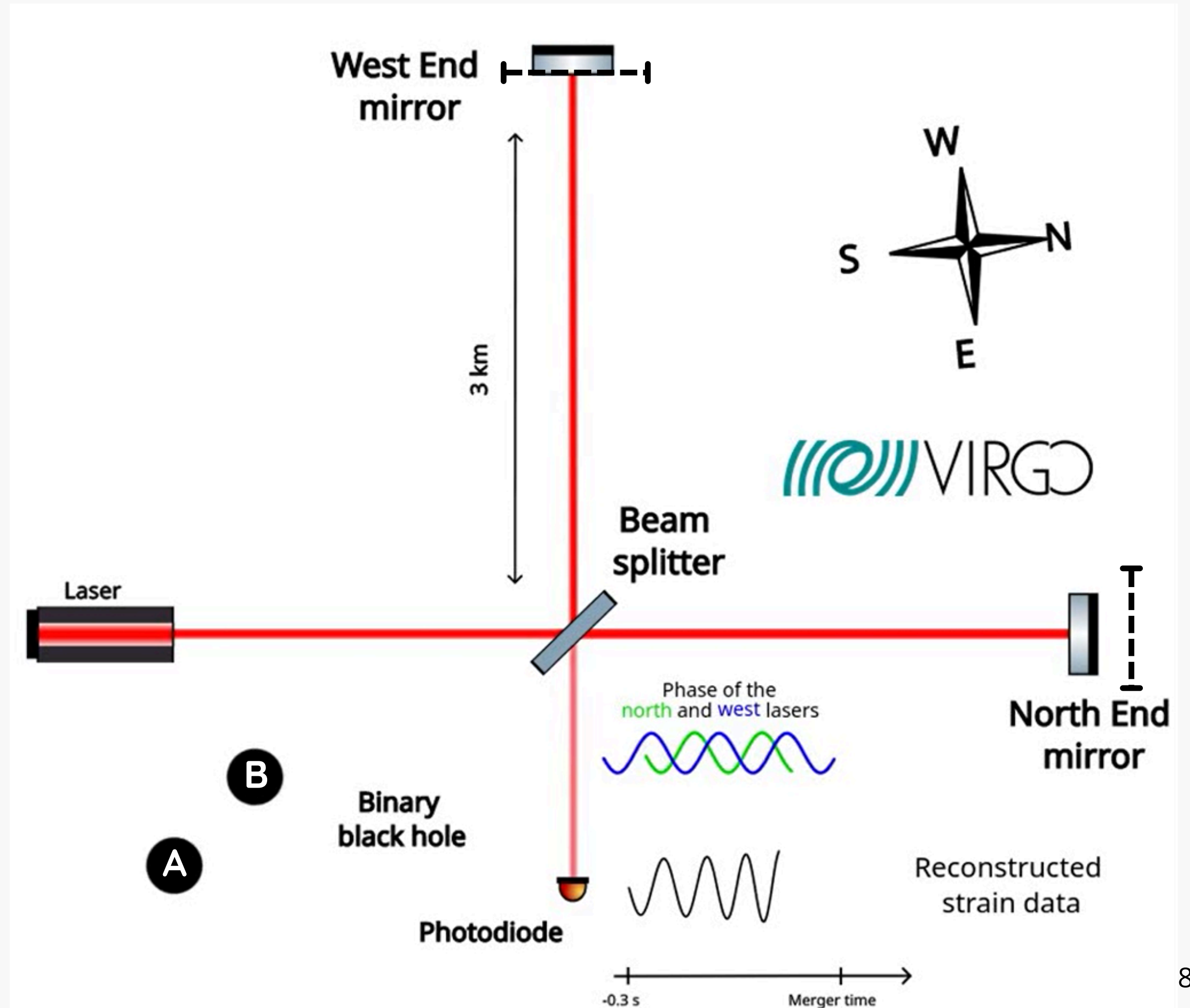
$$\Delta L = L_0 \frac{h}{2}$$



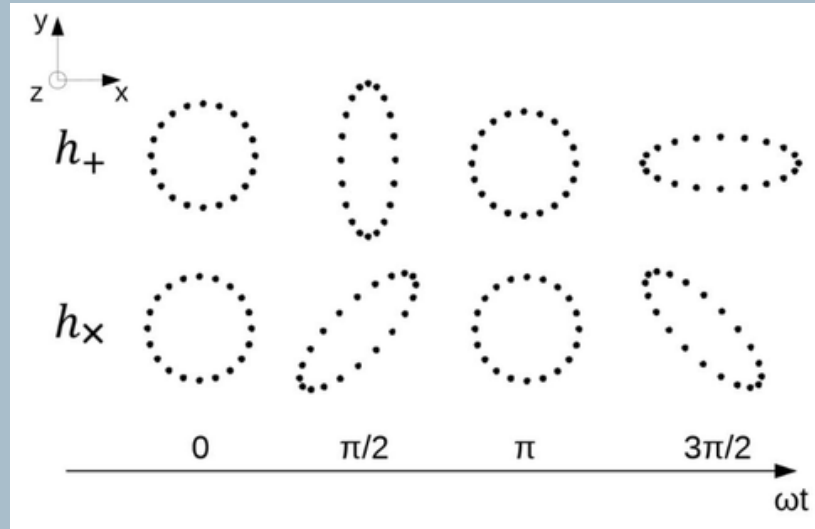
## (Reminders)



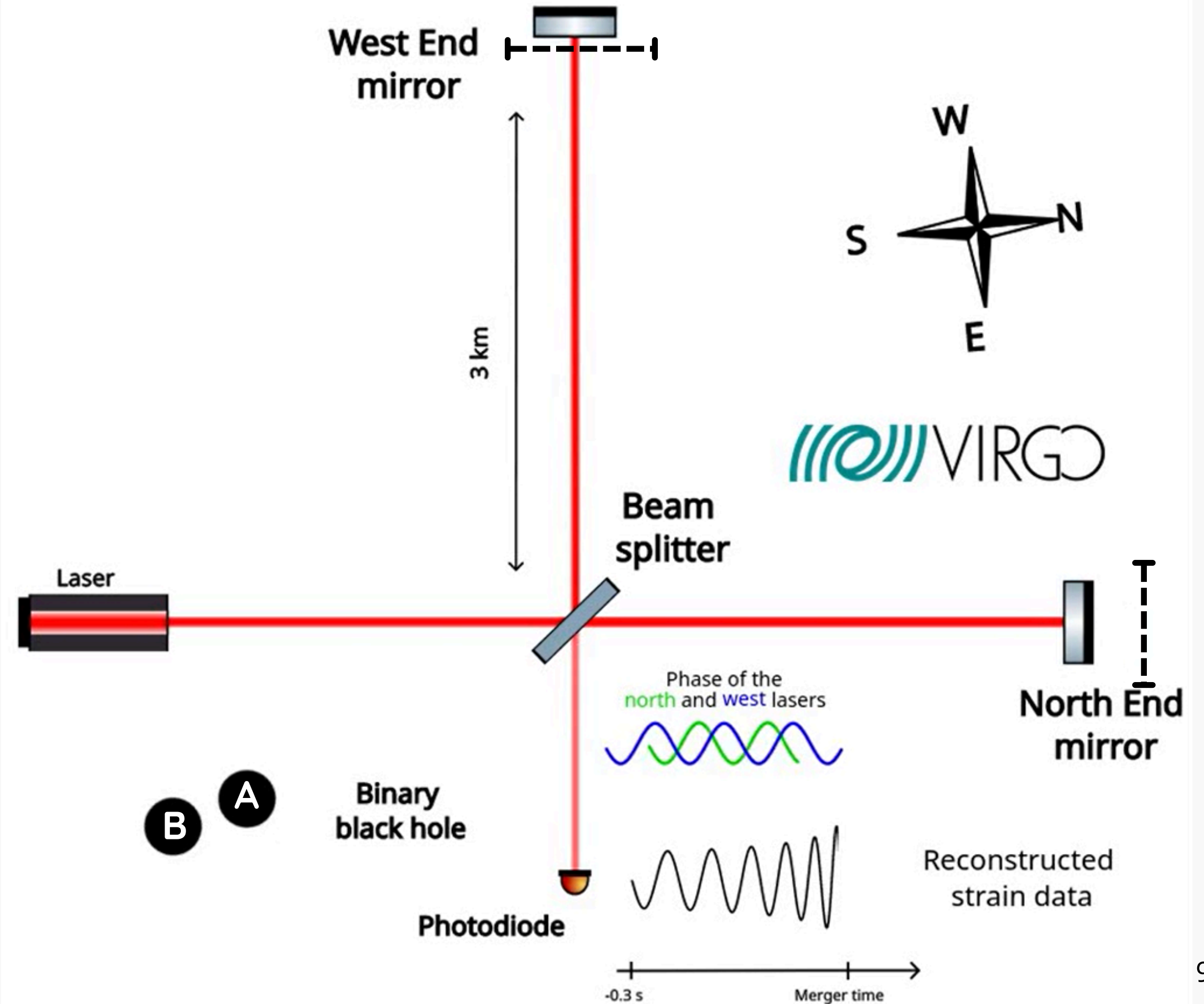
$$\Delta L = L_0 \frac{h}{2}$$



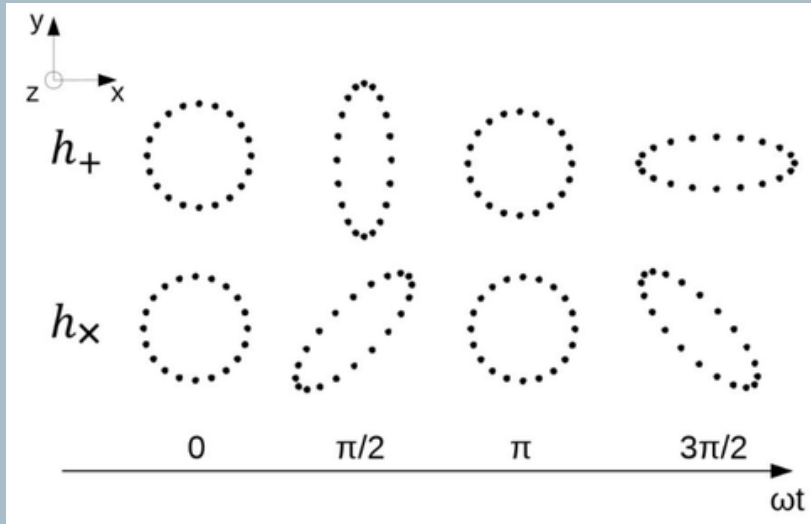
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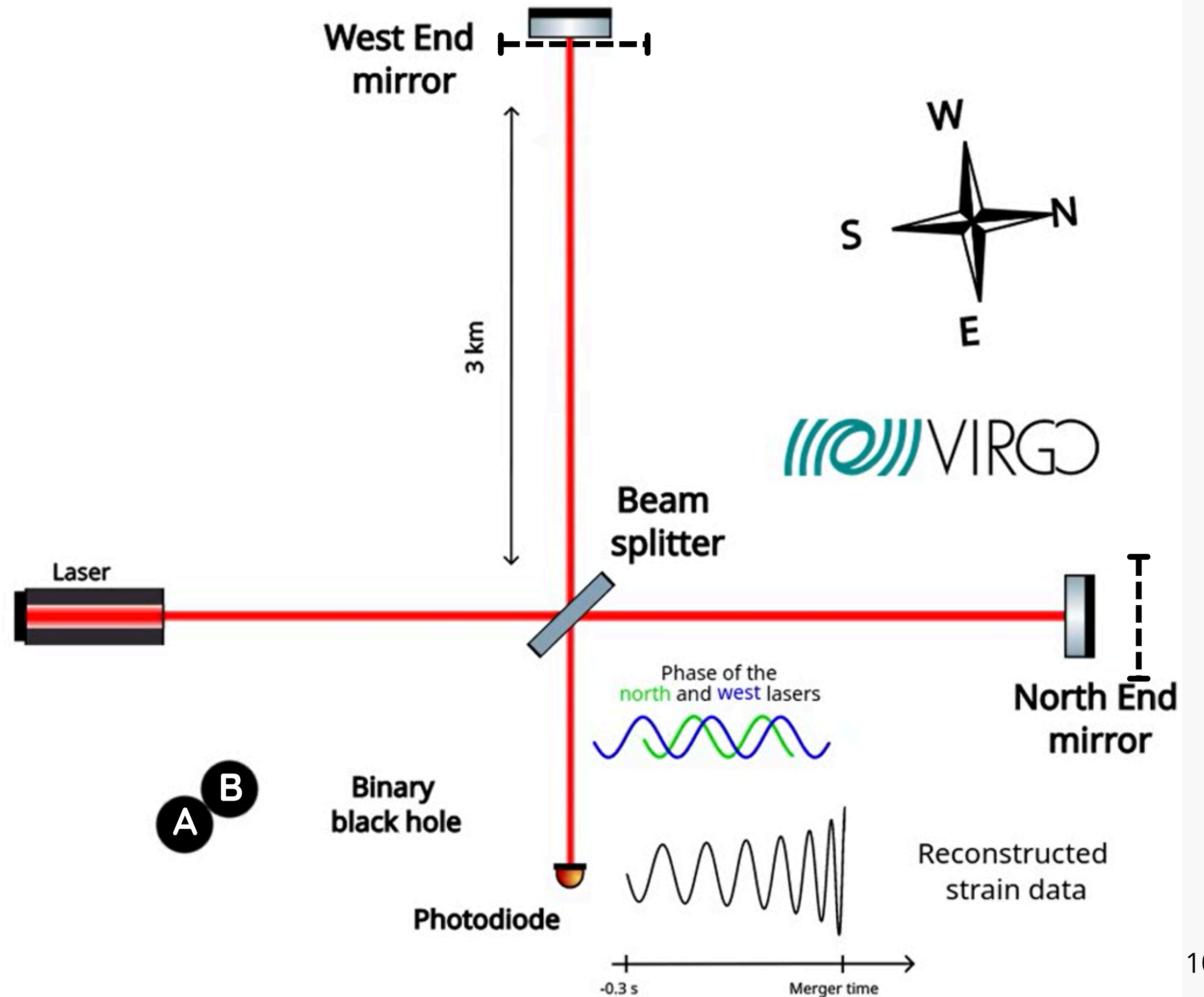
$$\Delta L = L_0 \frac{h}{2}$$



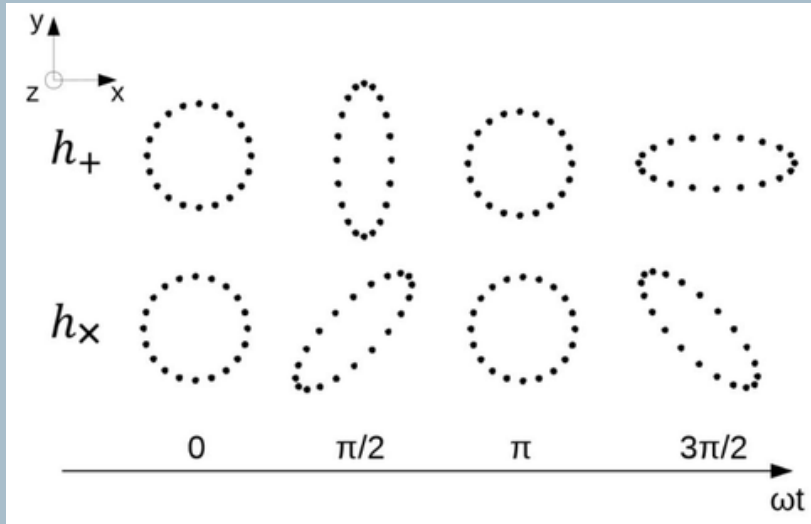
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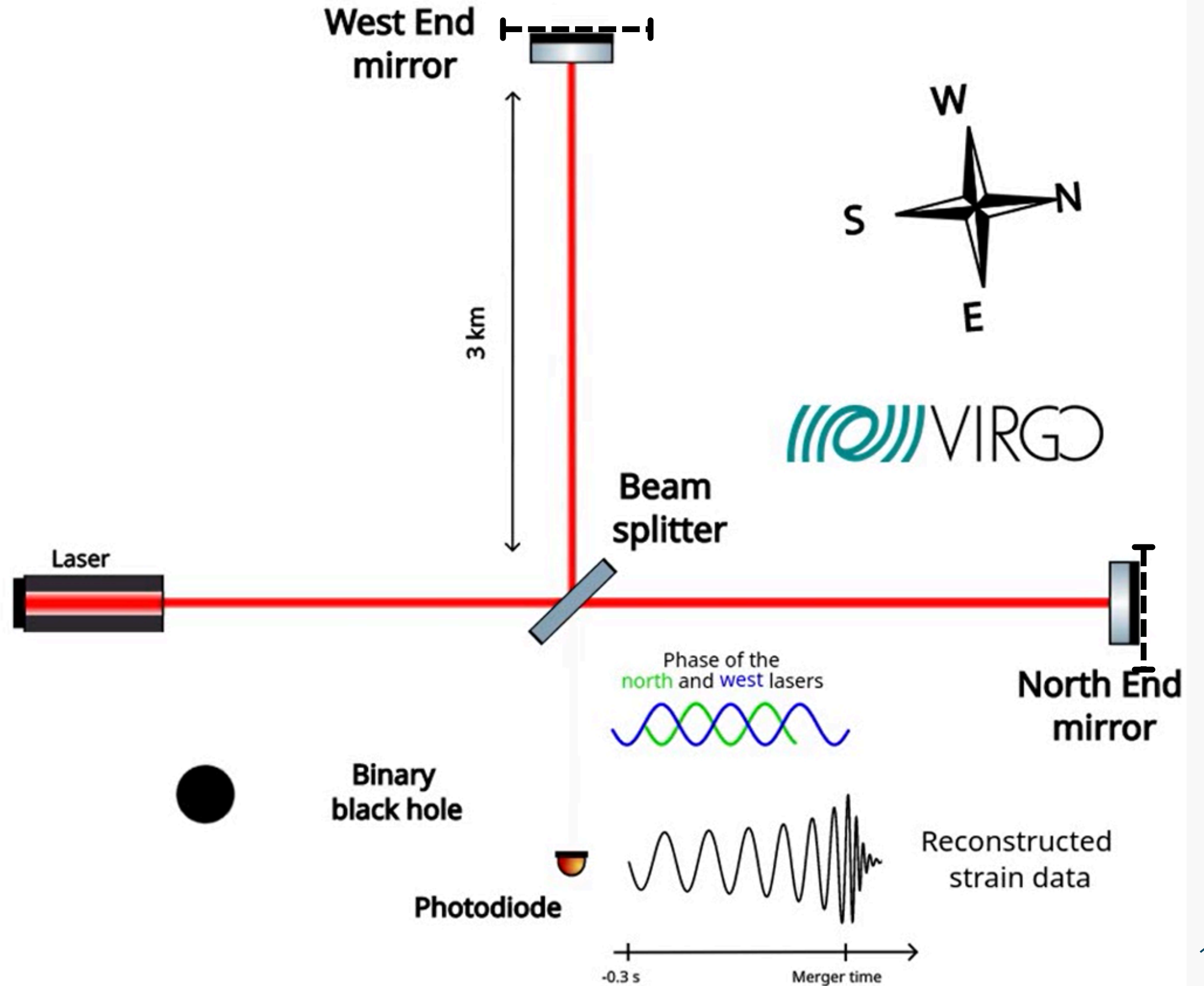
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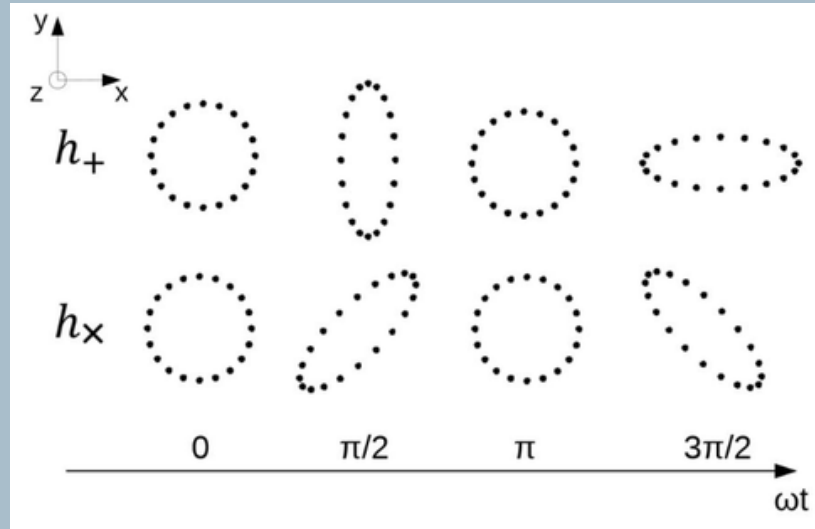
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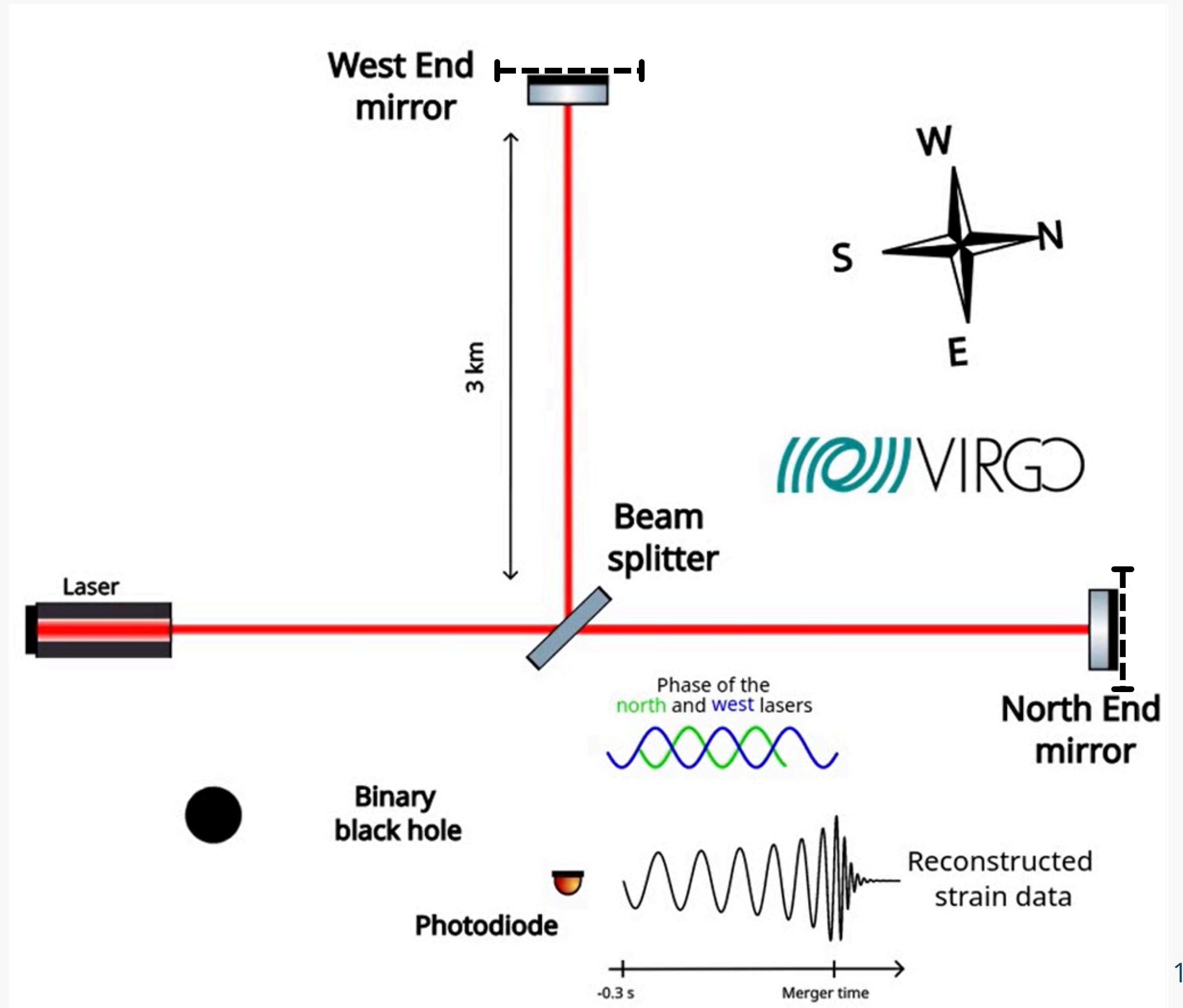
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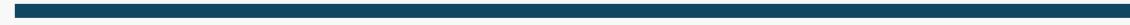


## (Reminders)



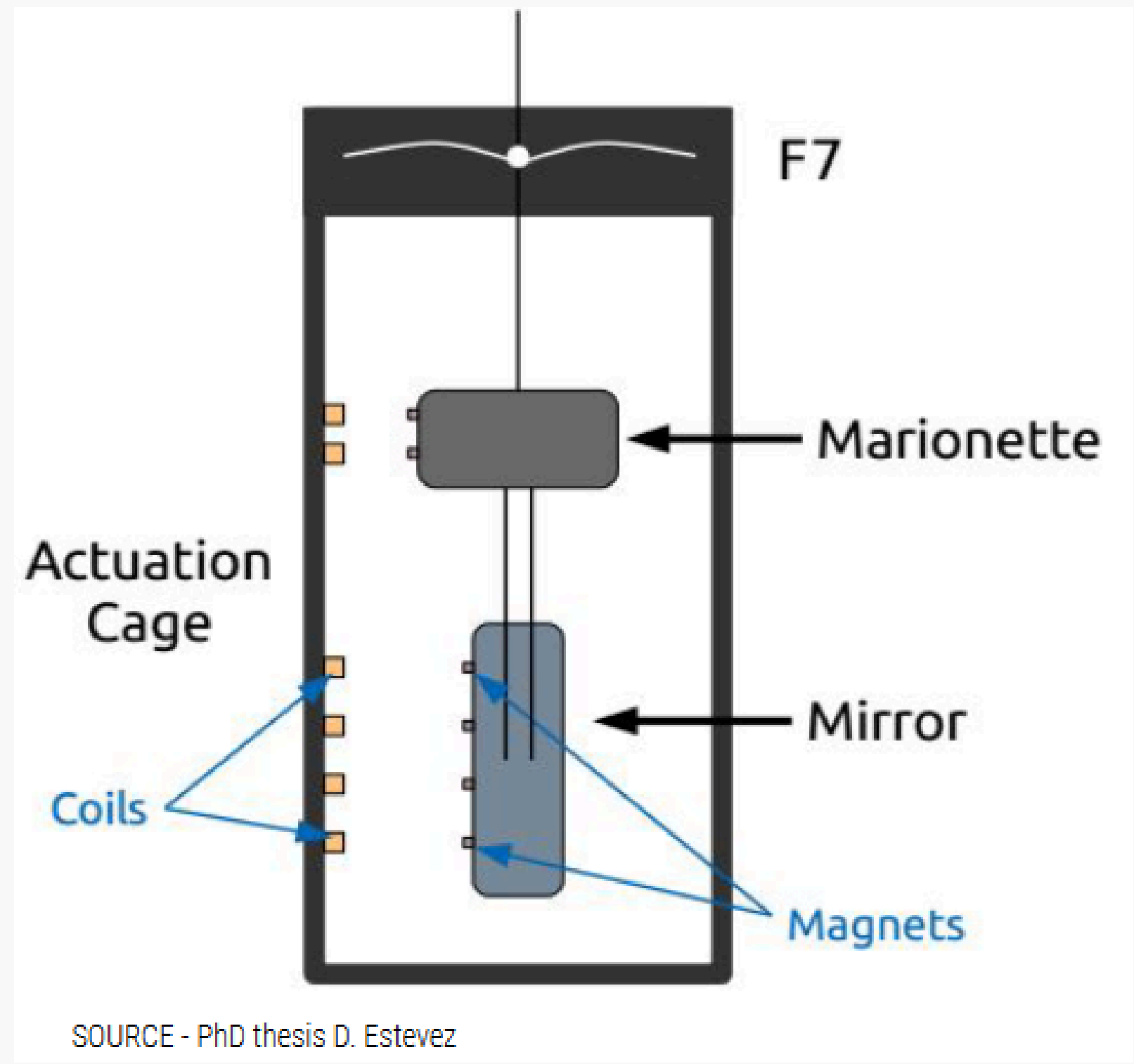
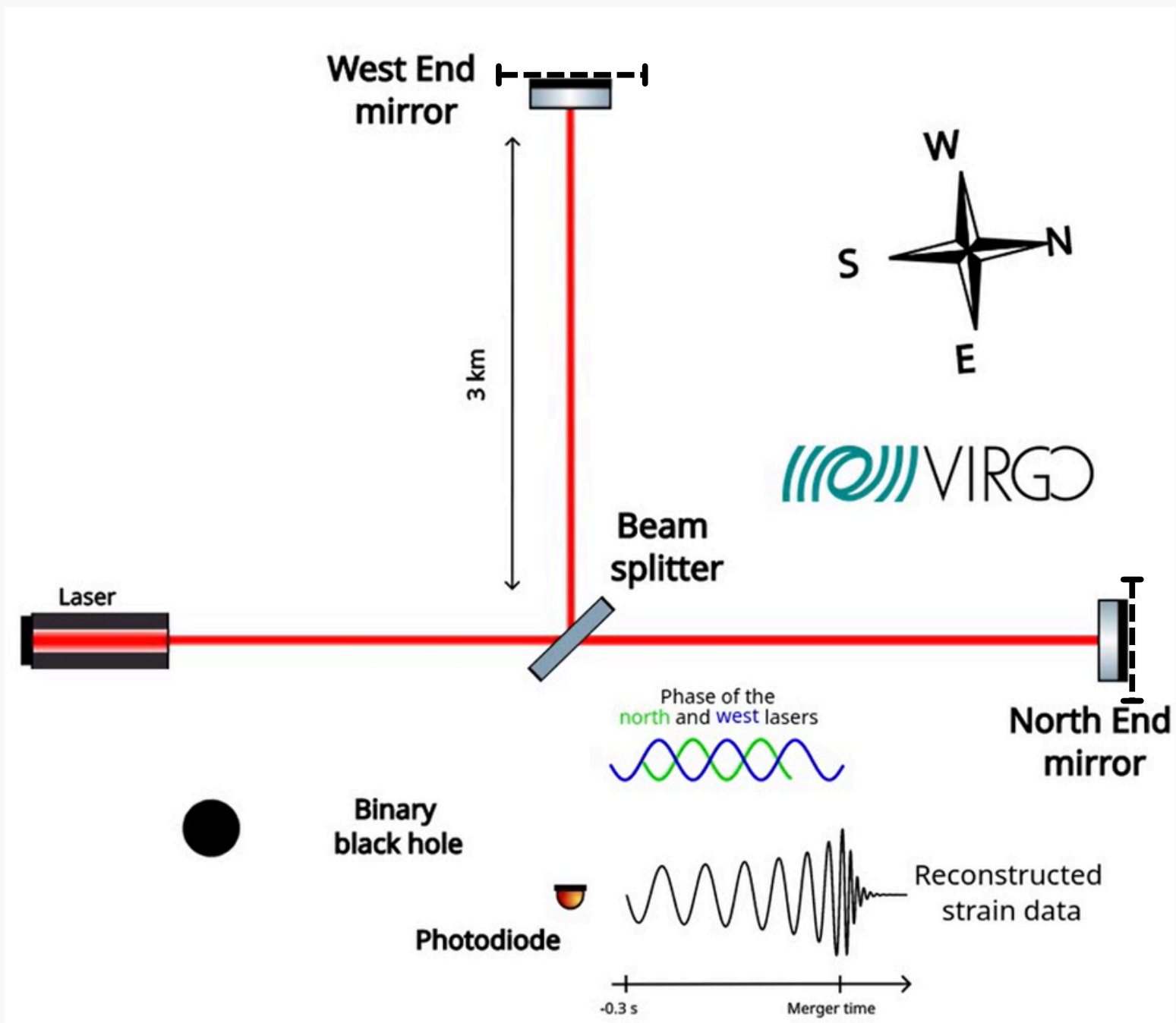
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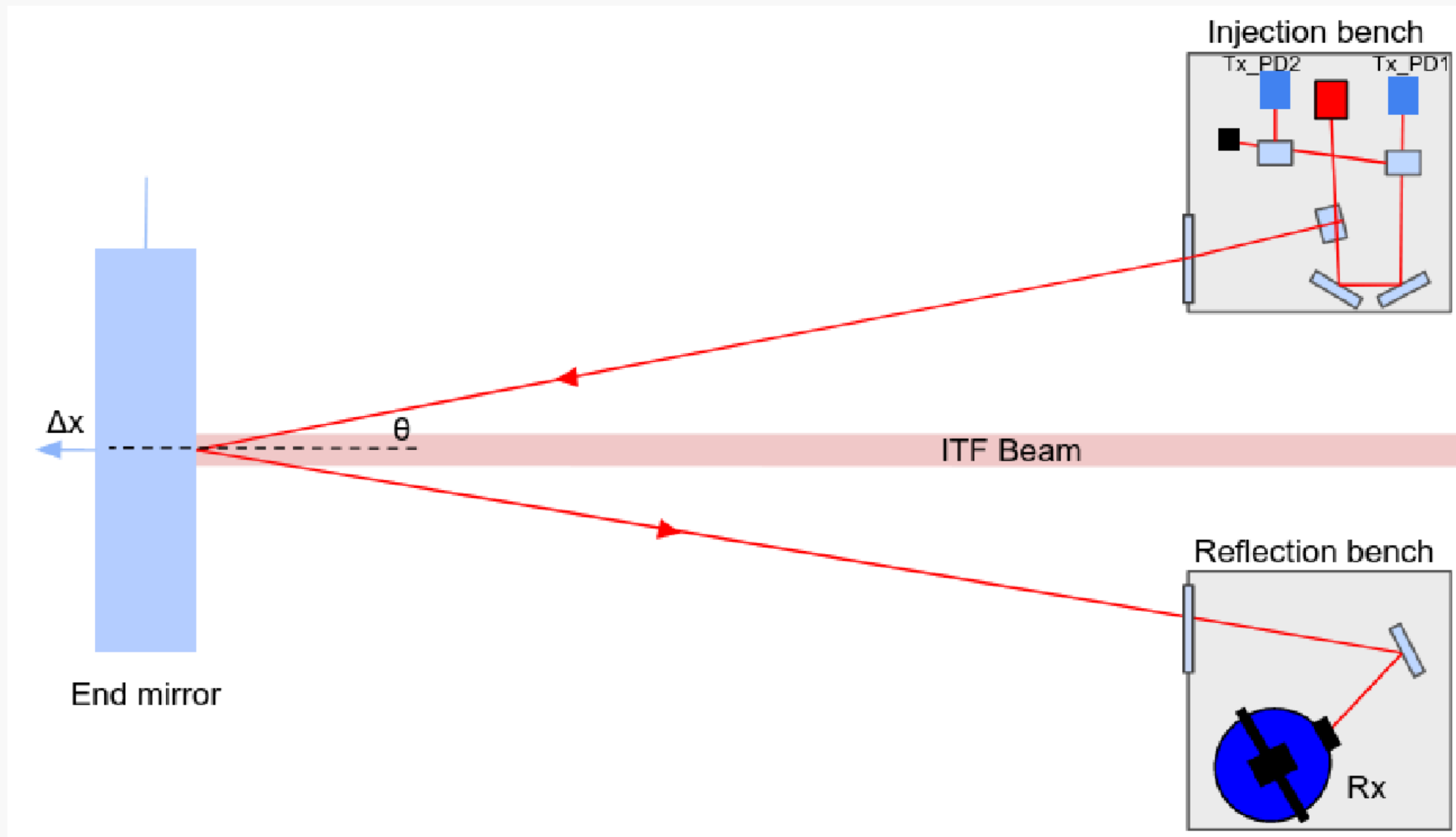


# *PCal calibration of Virgo*

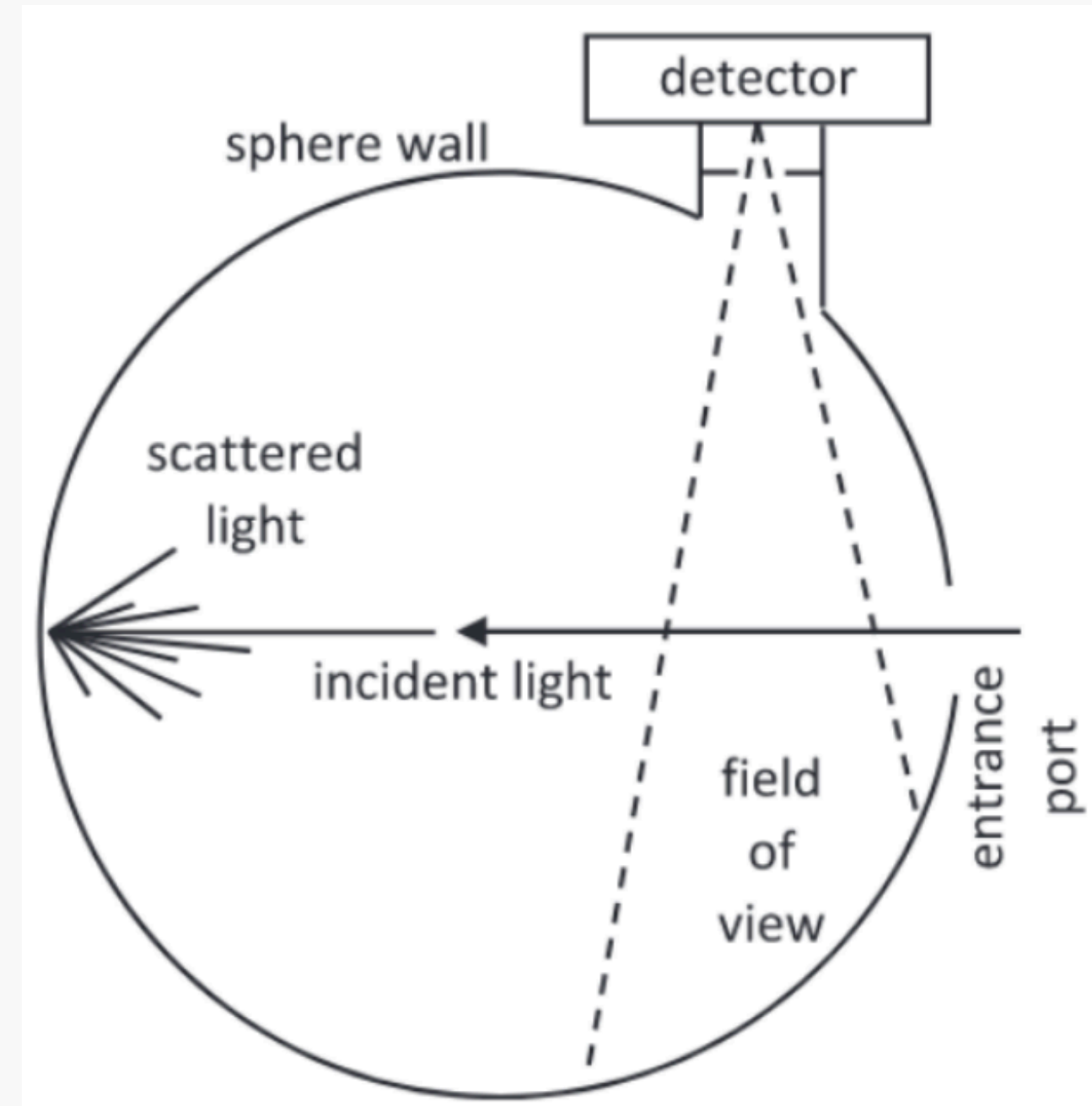




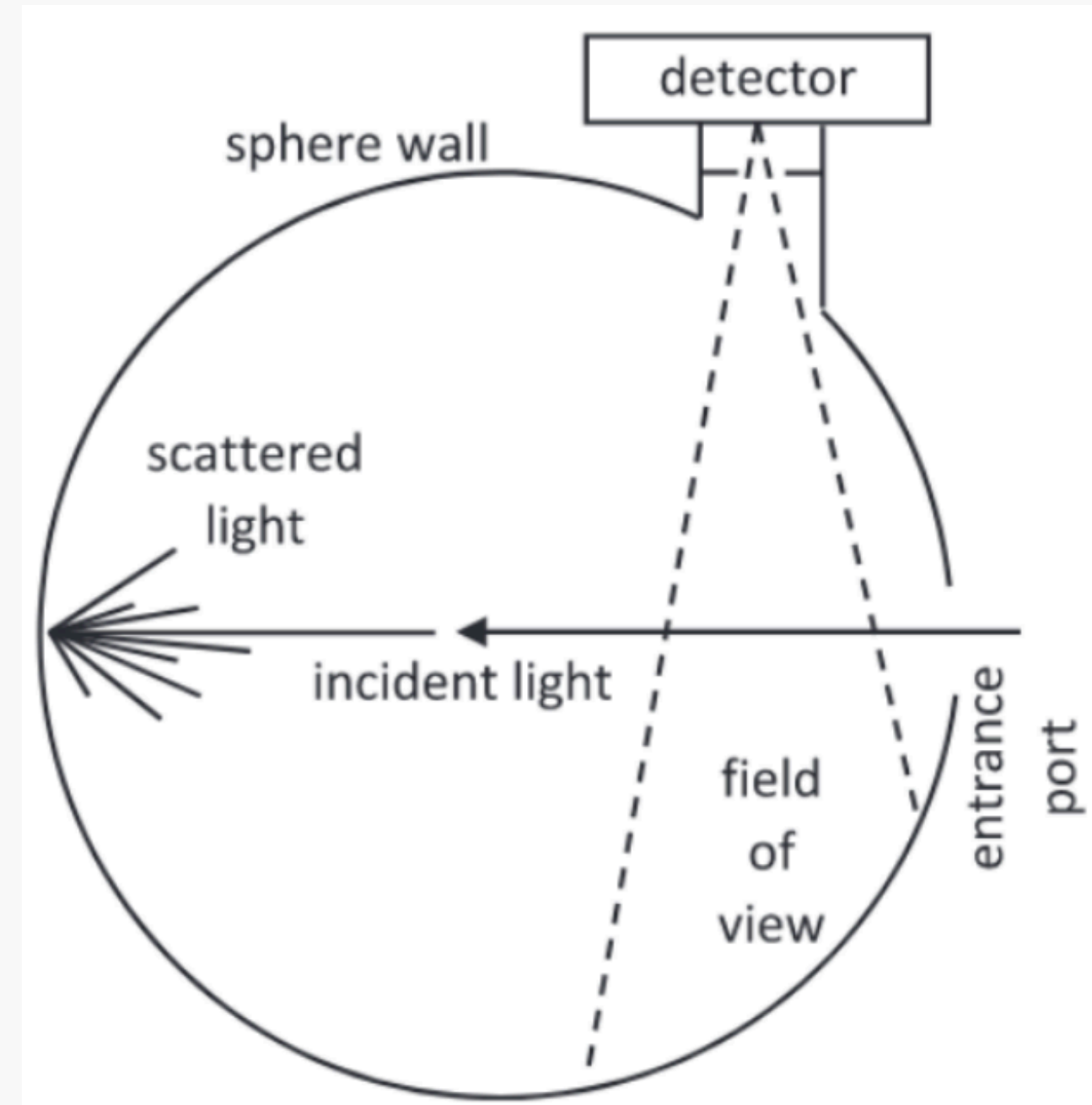
SOURCE - PhD thesis D. Estevez



$$\Delta x_{pend}(f) = -\frac{1}{M} \cdot \frac{2 \cos(\theta)}{c} \cdot \frac{\Delta P_{ref}(f)}{(2\pi f)^2}$$

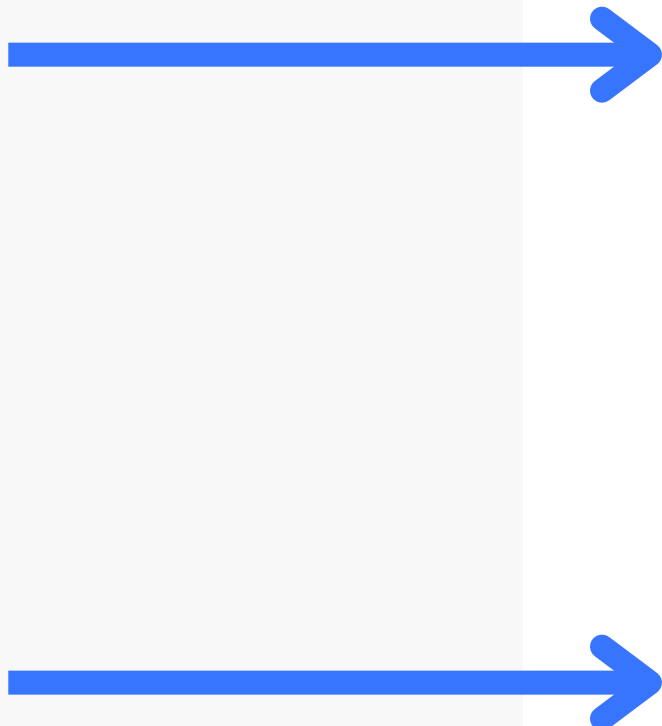


$$V_{output} = \rho \cdot P$$



$$V_{output} = \rho \cdot P$$

**Responsivity**

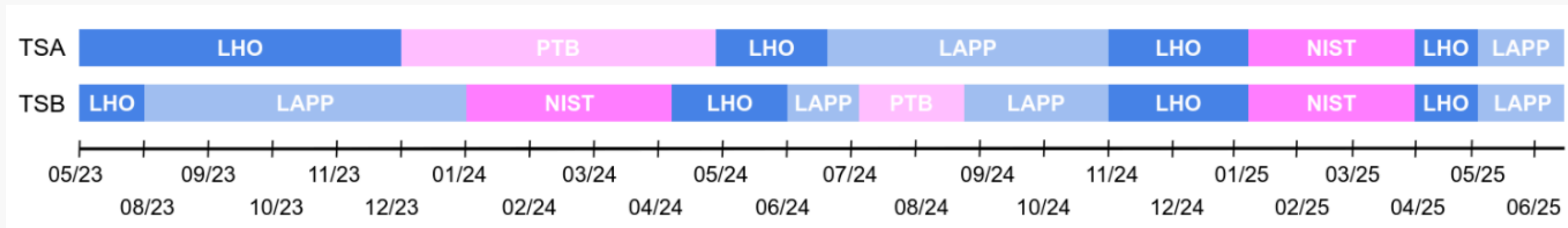
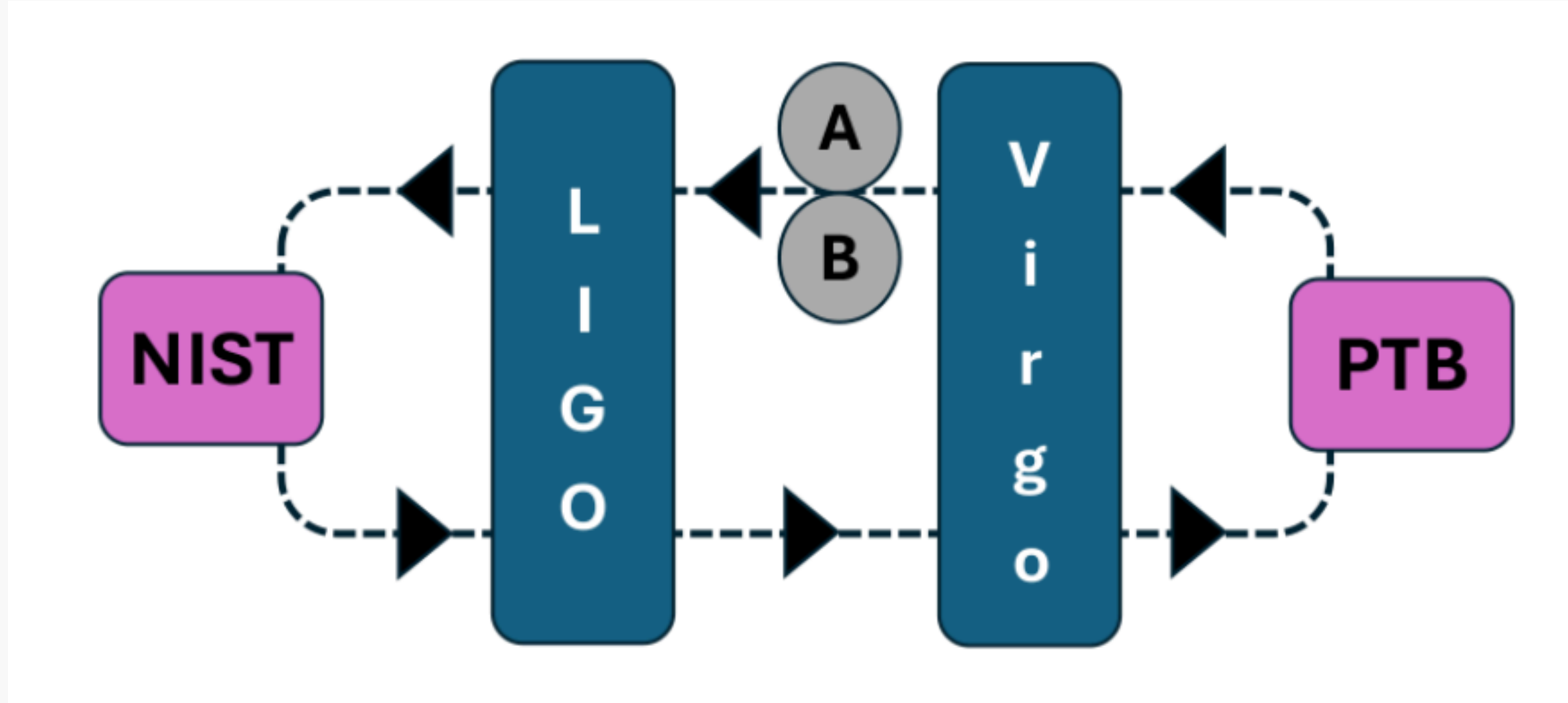


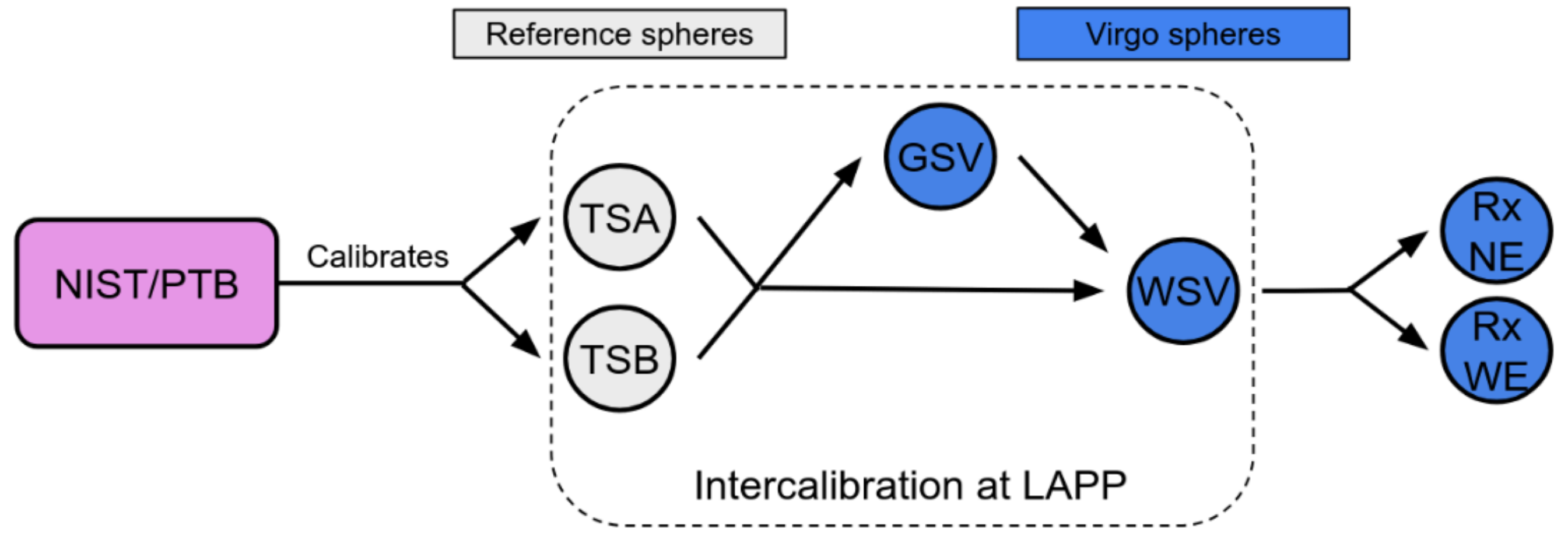
Source	Relative uncertainty [%]
Rx responsivity	0.22
Deformation model	0.38
Incident angle, $\cos(\theta)$	0.16
ETM mass, M	0.05
ETM rotation	0.09
Optical efficiency	0.10
<b>Total</b>	<b>0.49</b>



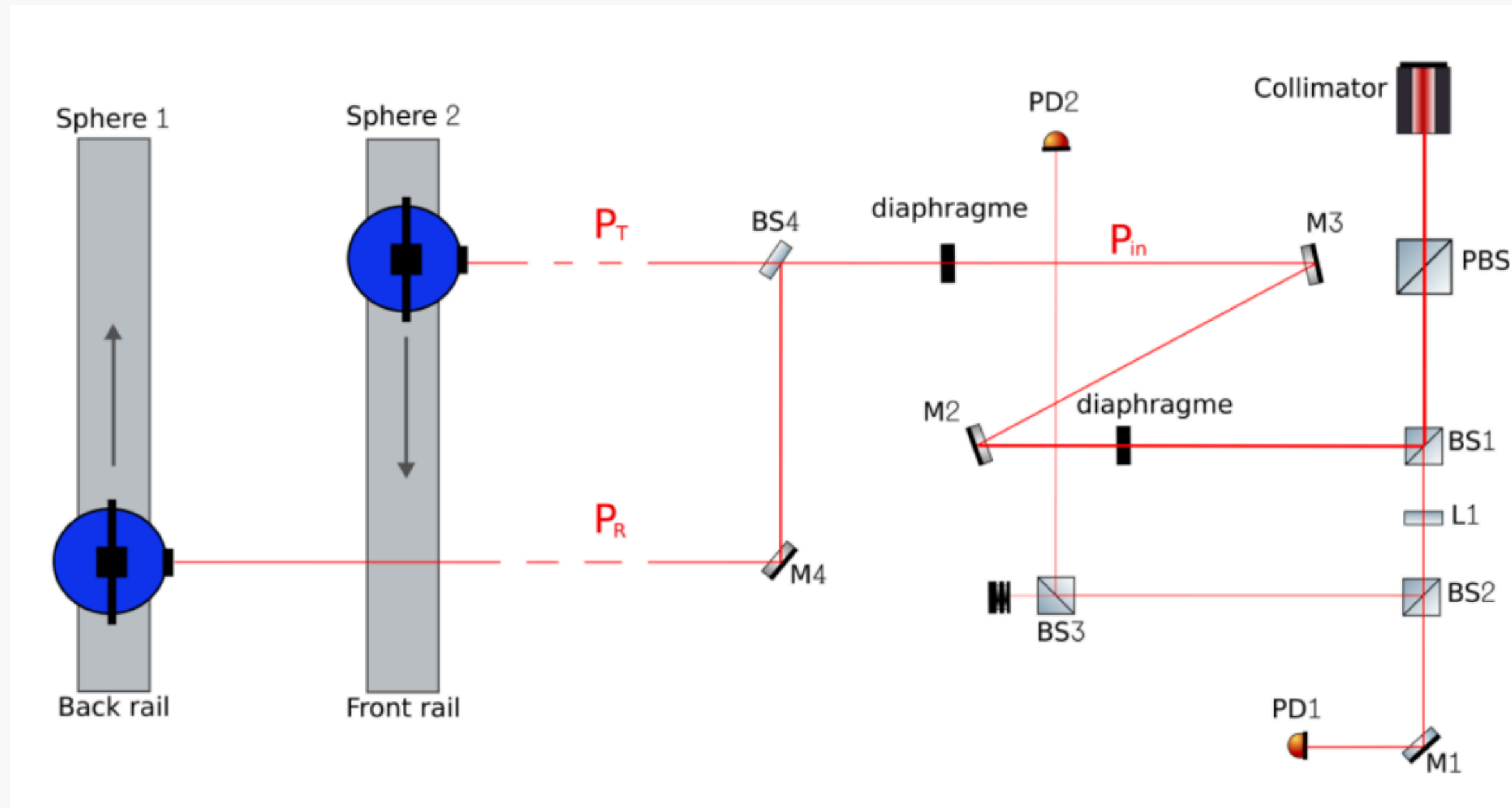
# *Intercalibration*







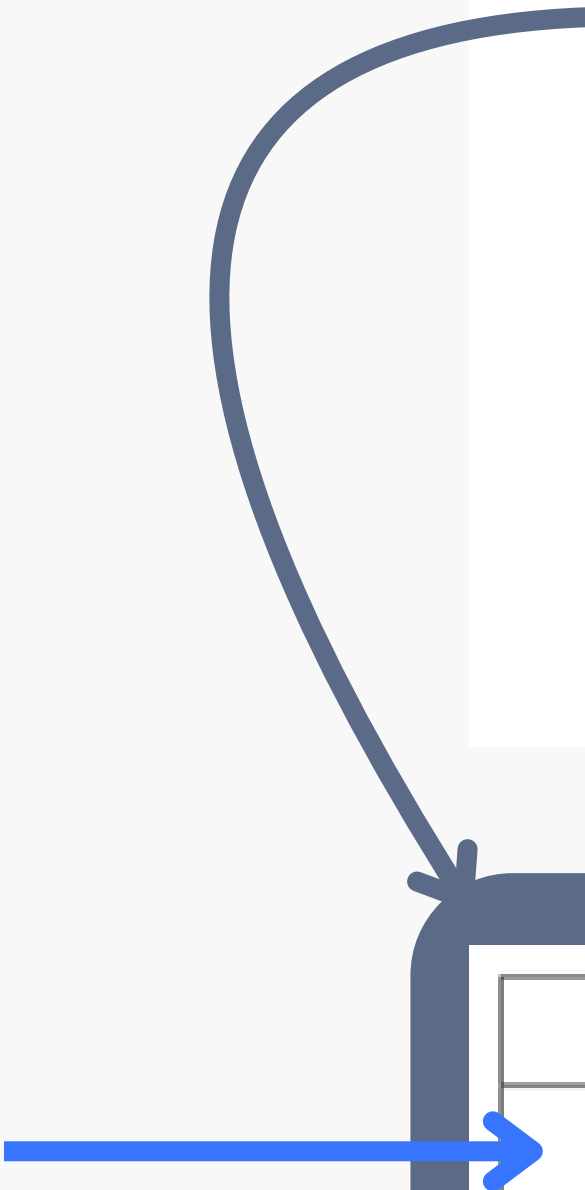
# Intercalibration setup



$$\alpha = \sqrt{\frac{V_{raw}(T.P_{in})}{V_{ref}(T.P_{in})} \frac{V_{raw}(R.P_{in})}{V_{ref}(R.P_{in})}}$$



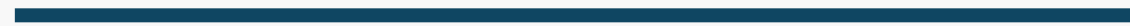
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Source	Relative uncertainty [%]
WSV responsivity	0.11
Rx calibration	0.19
<b>Total</b>	<b>0.22</b>



*Linearity measurements of  
integrating spheres*




Intercalibration

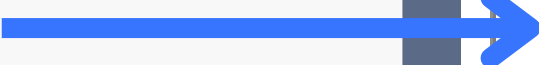
Done P\_laser = 0.3W

Calibration transfer from WSV to Rx  
(on site)

Done P\_laser = 1.3W



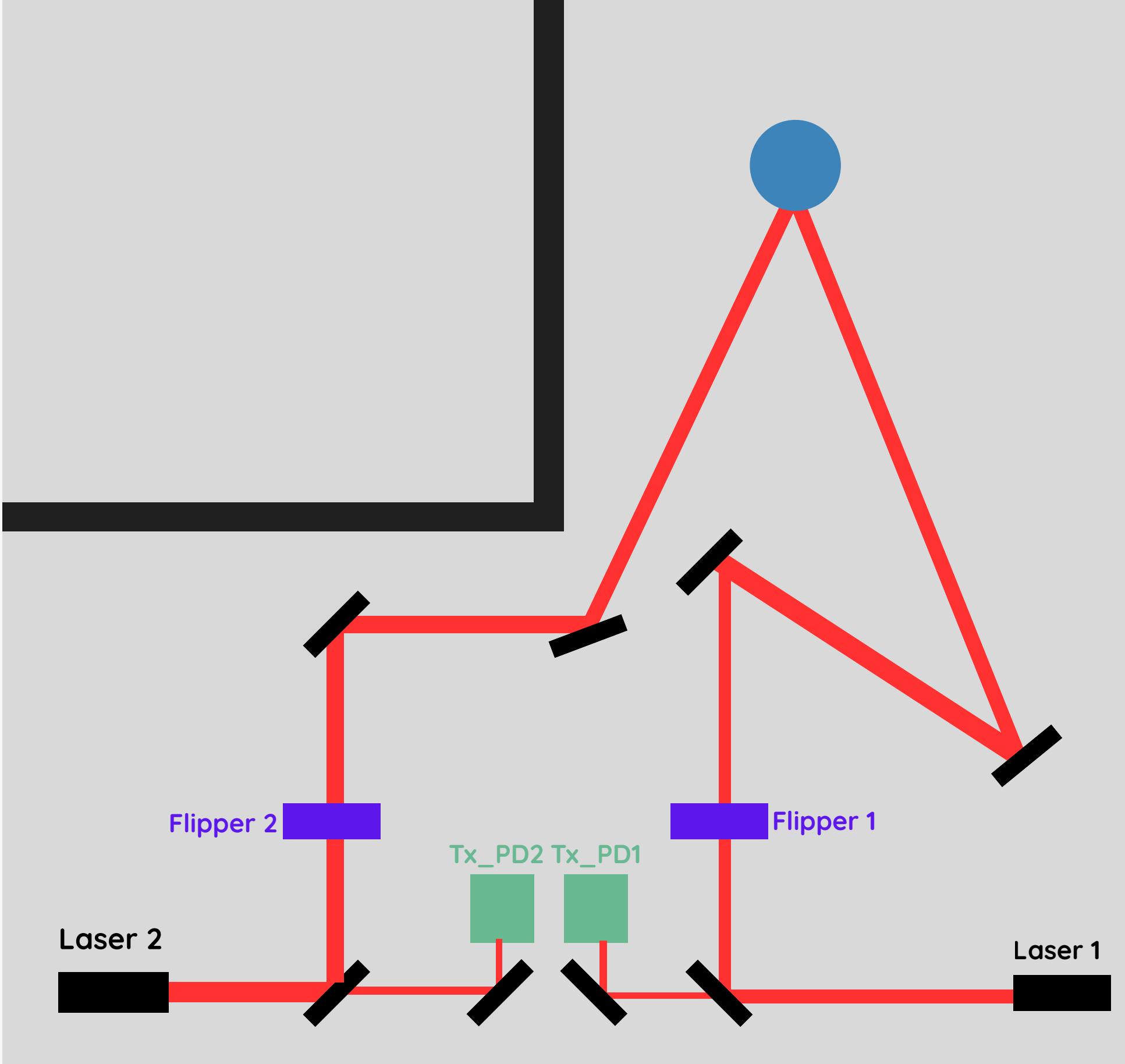
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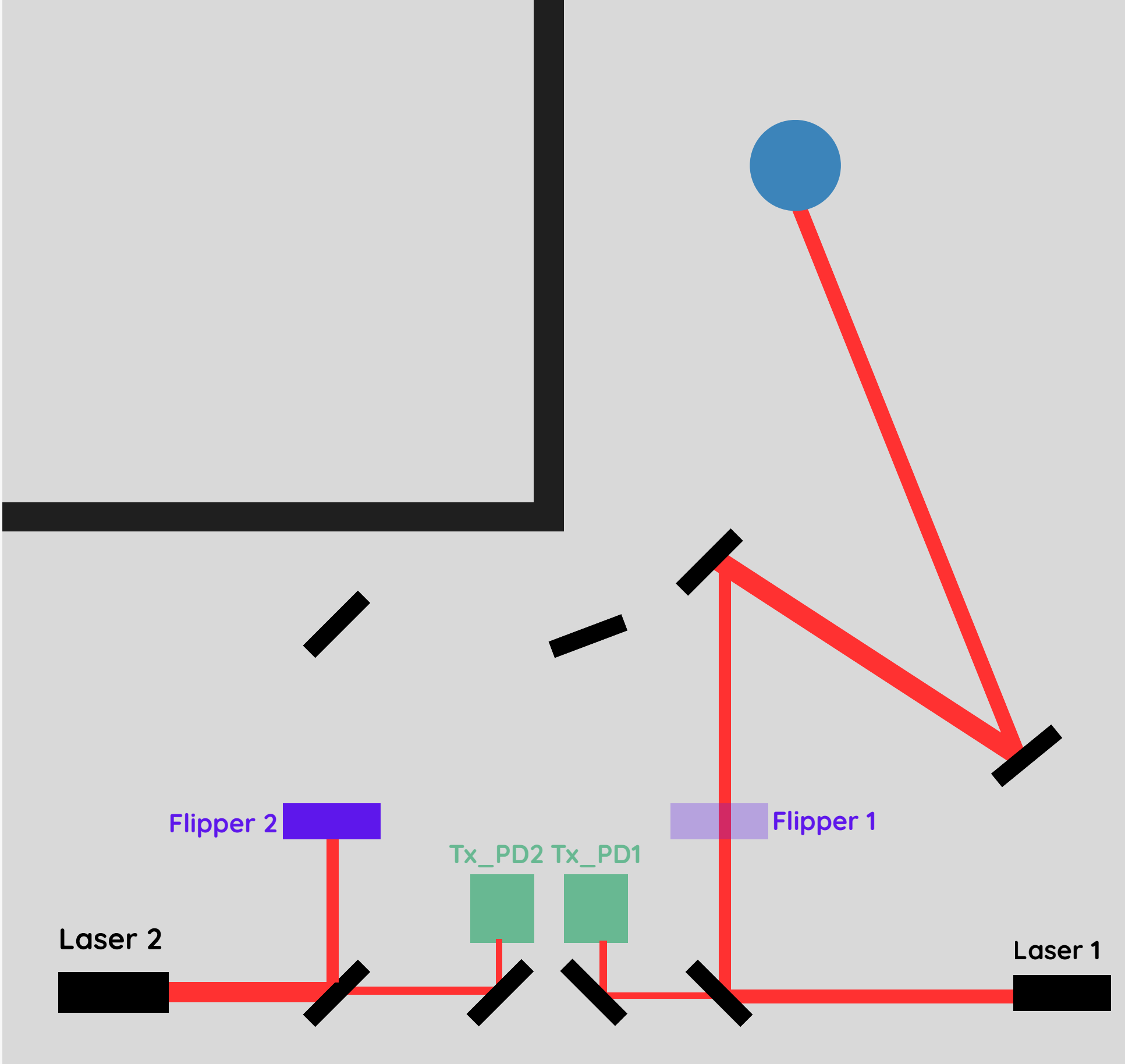
Source	Relative uncertainty [%]
WSV responsivity	0.11
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<b>Total</b>	<b>0.22</b>

$$V_{sphere}(\phi_1 + \phi_2) = V_{sphere}(\phi_1) + V_{sphere}(\phi_2)$$

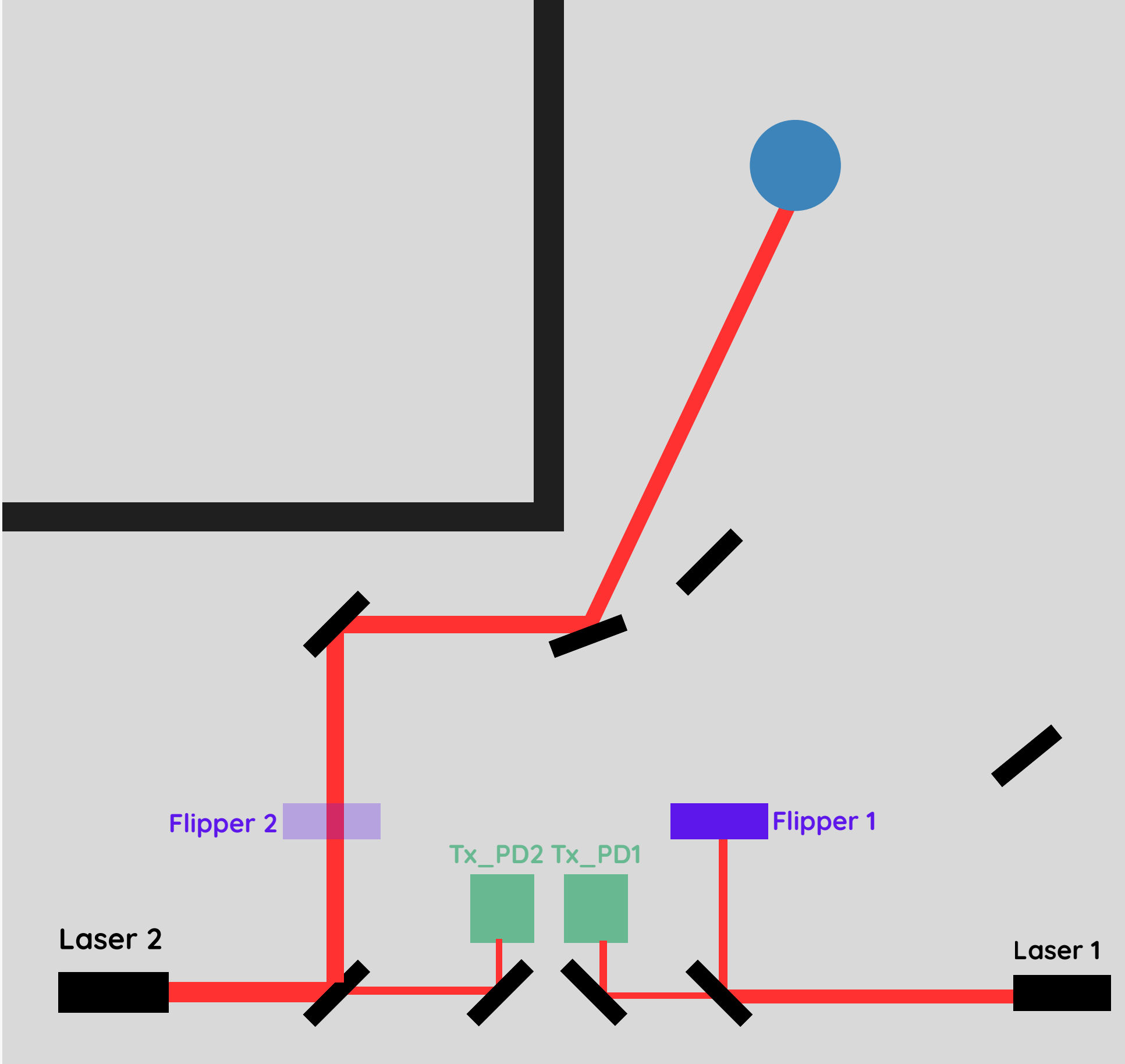
$$V_{sphere}(\phi_1 + \phi_2) = V_{sphere}(\phi_1) + V_{sphere}(\phi_2)$$



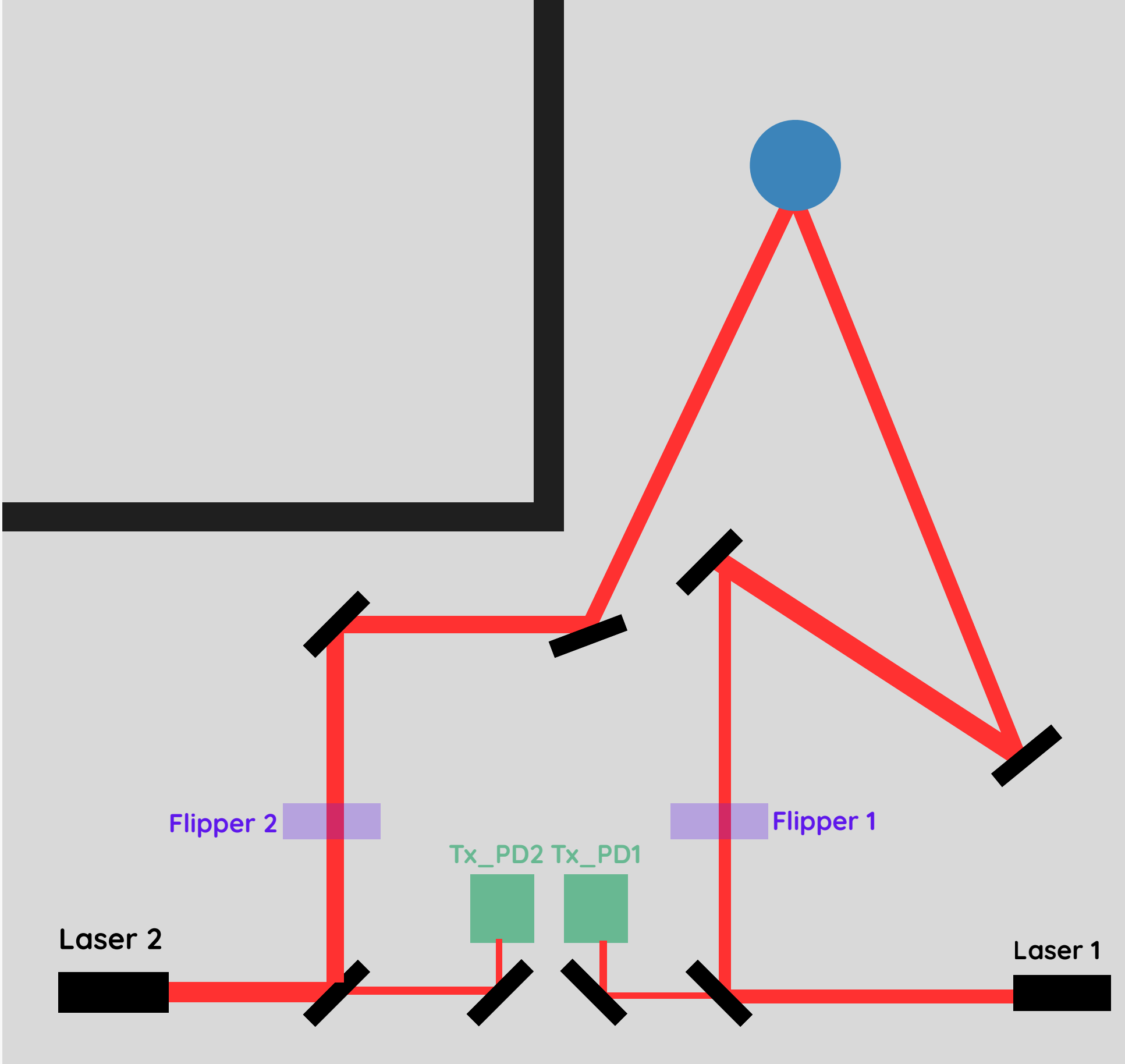
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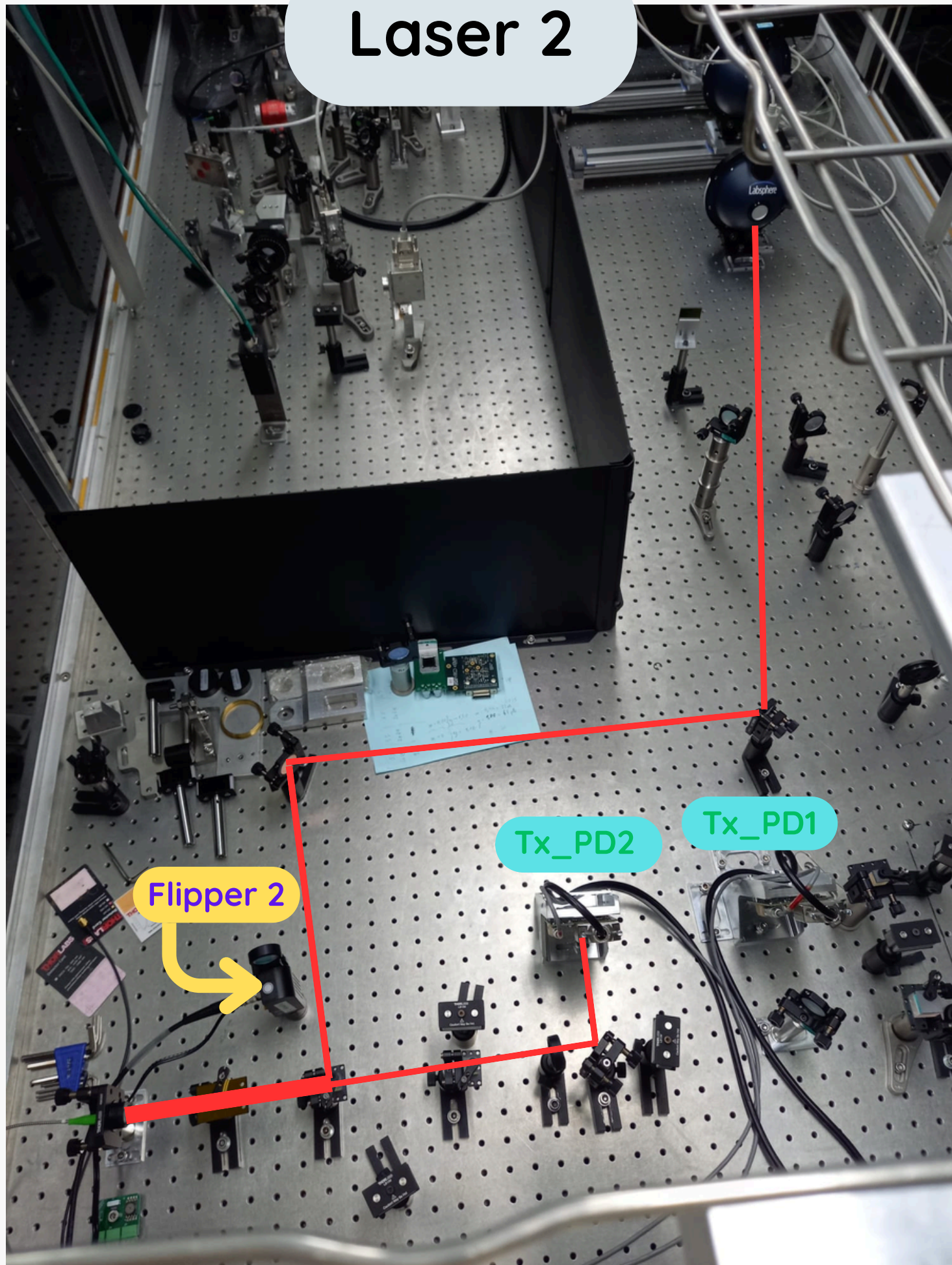
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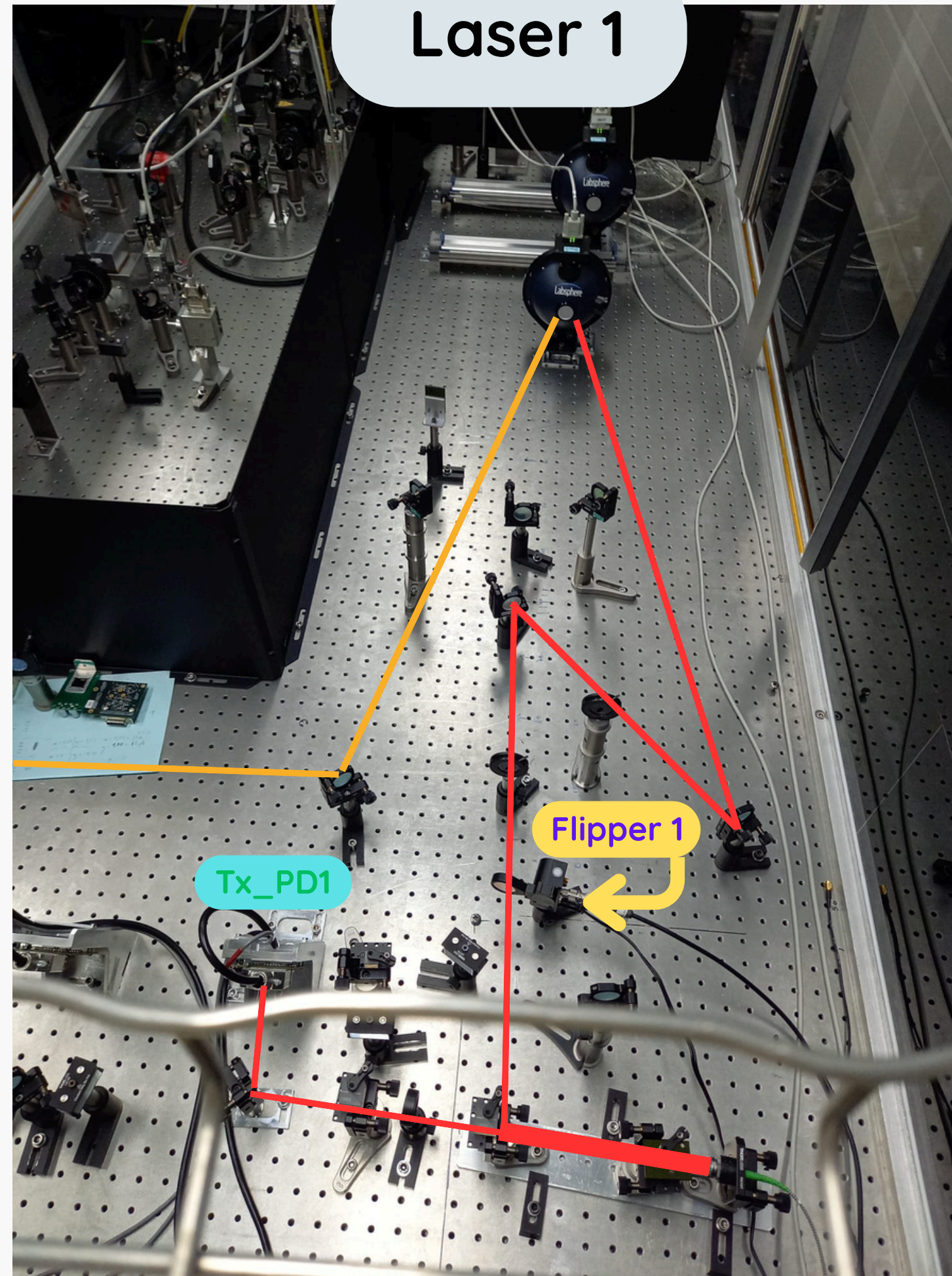
$$V_{sphere}(\phi_1 + \phi_2) = V_{sphere}(\phi_1) + V_{sphere}(\phi_2)$$



Laser 2



Laser 1

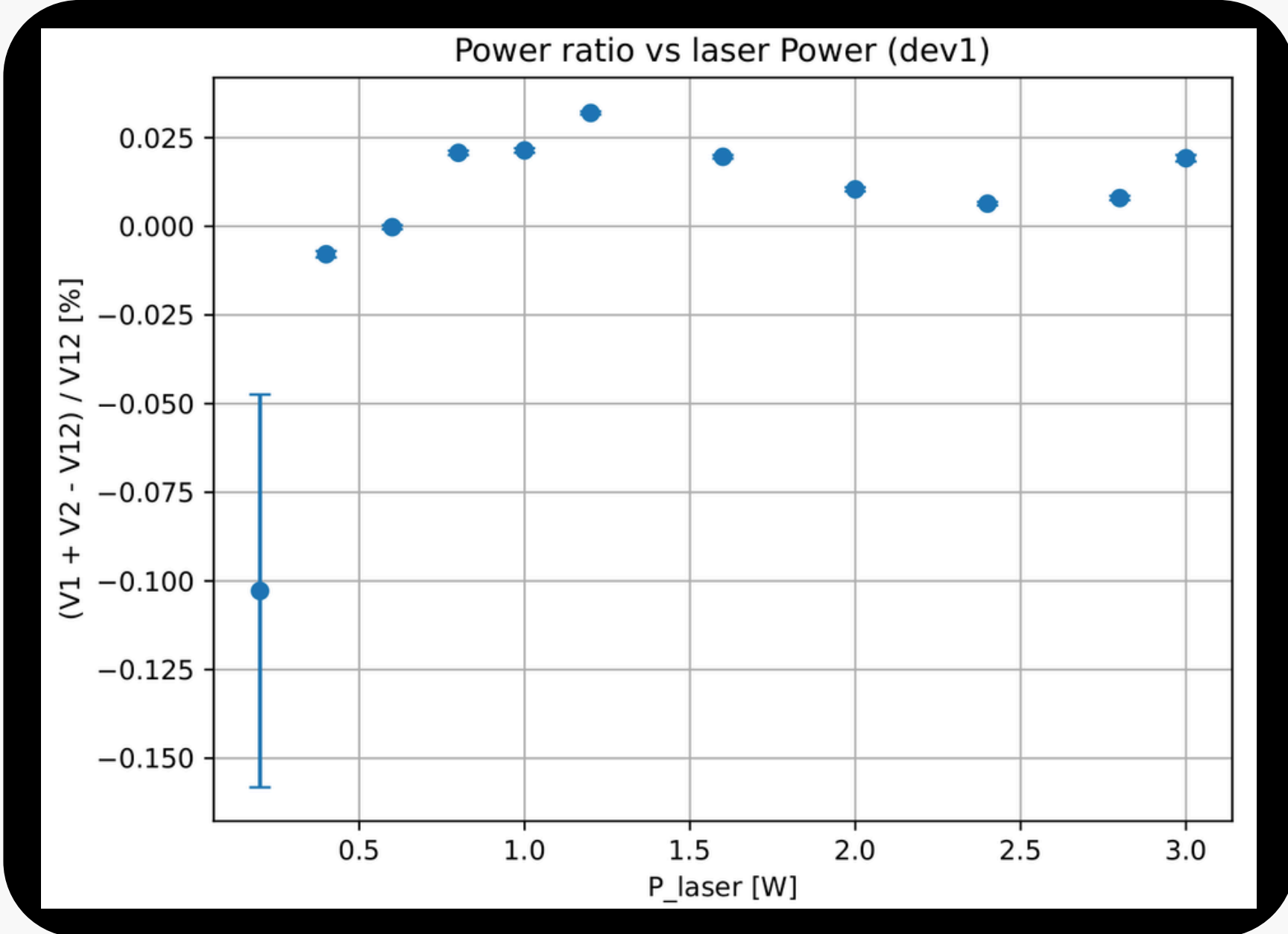


$$\frac{[V(\phi_1) + V(\phi_2)] - V(\phi_1 + \phi_2)}{V(\phi_1 + \phi_2)}$$

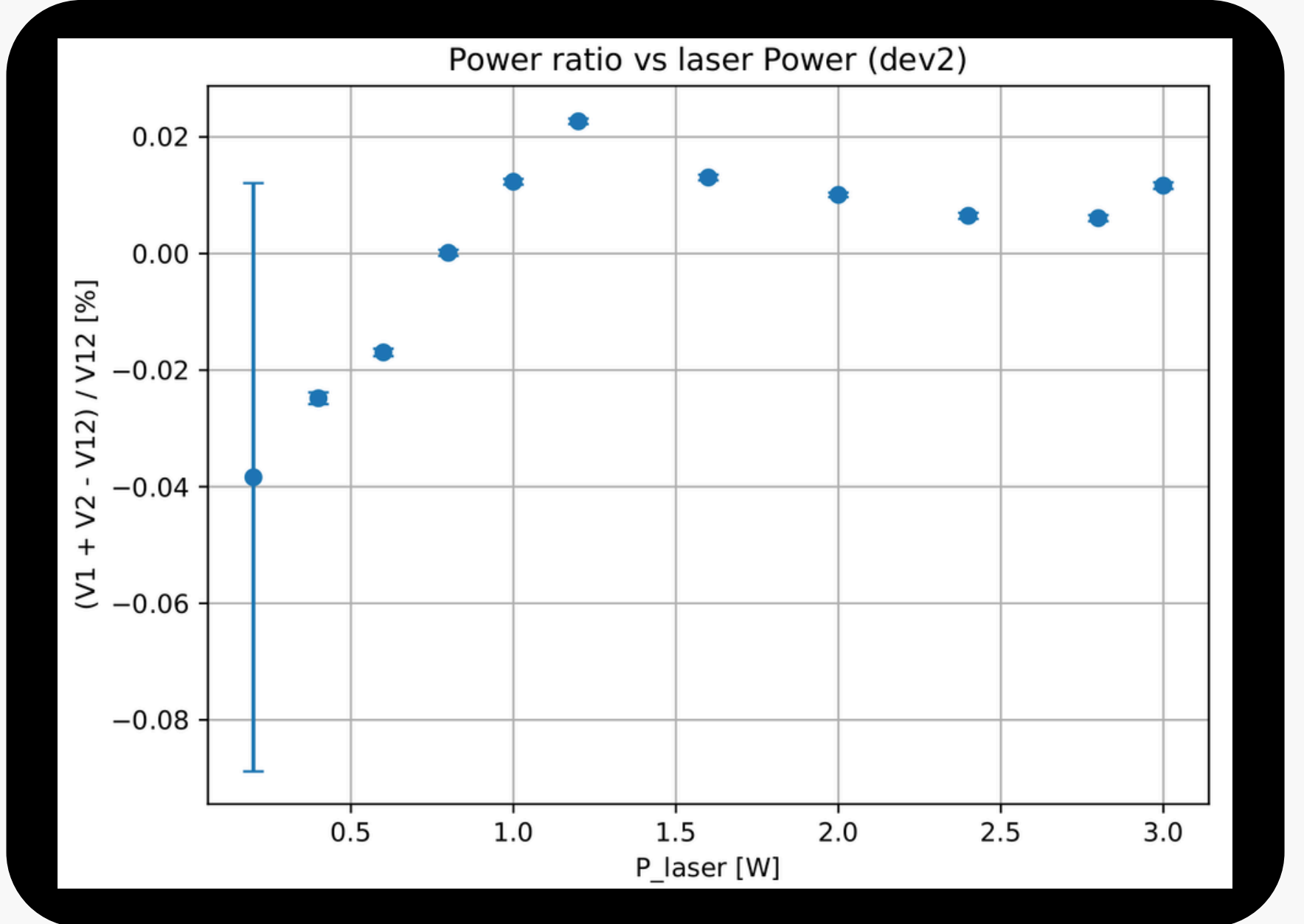
**Correction**  
Tension offset + Temperature

**WSV**

**GSV**



**Power (L1 + L2) [W]**

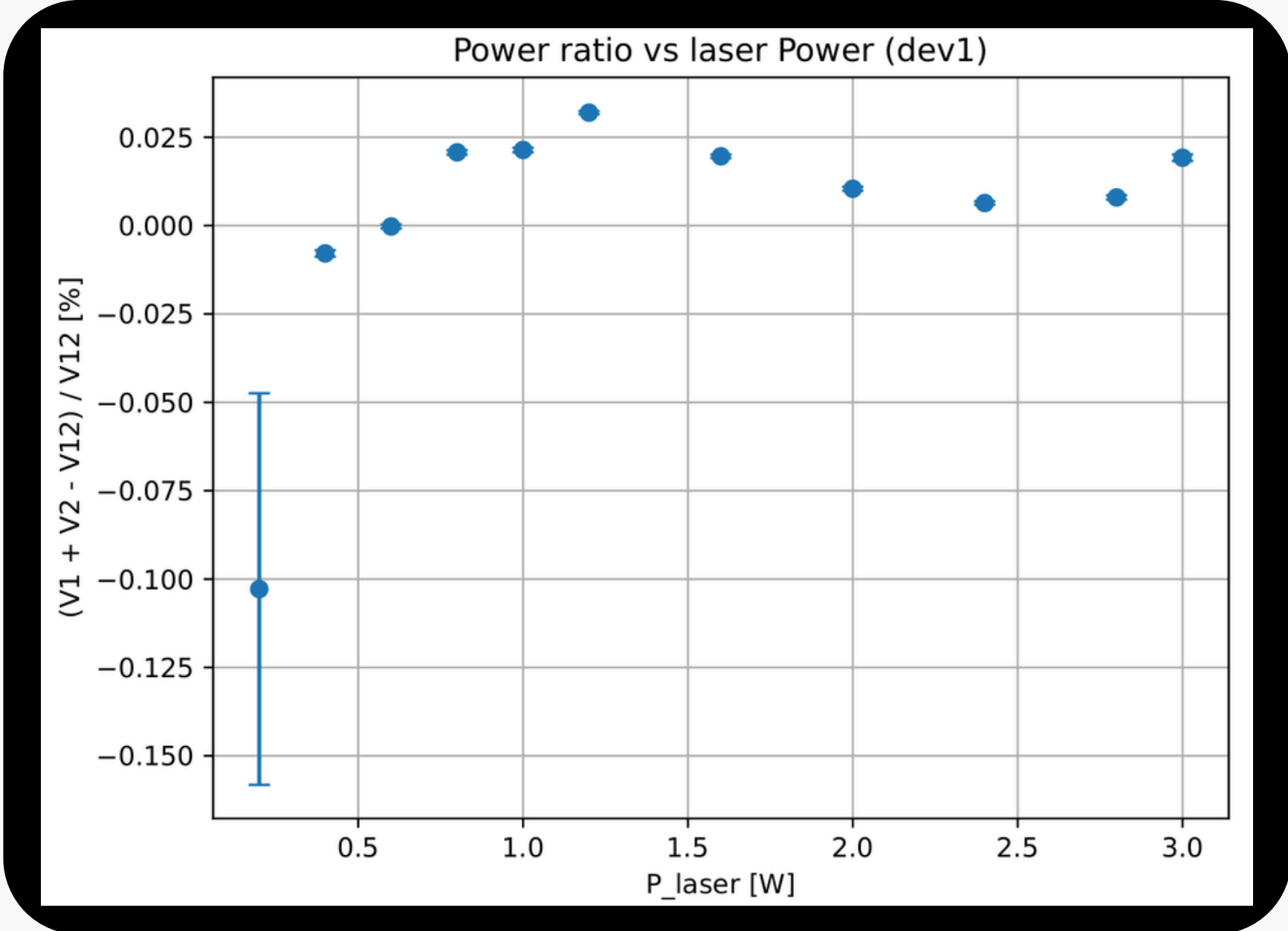


**Power (L1 + L2) [W]**

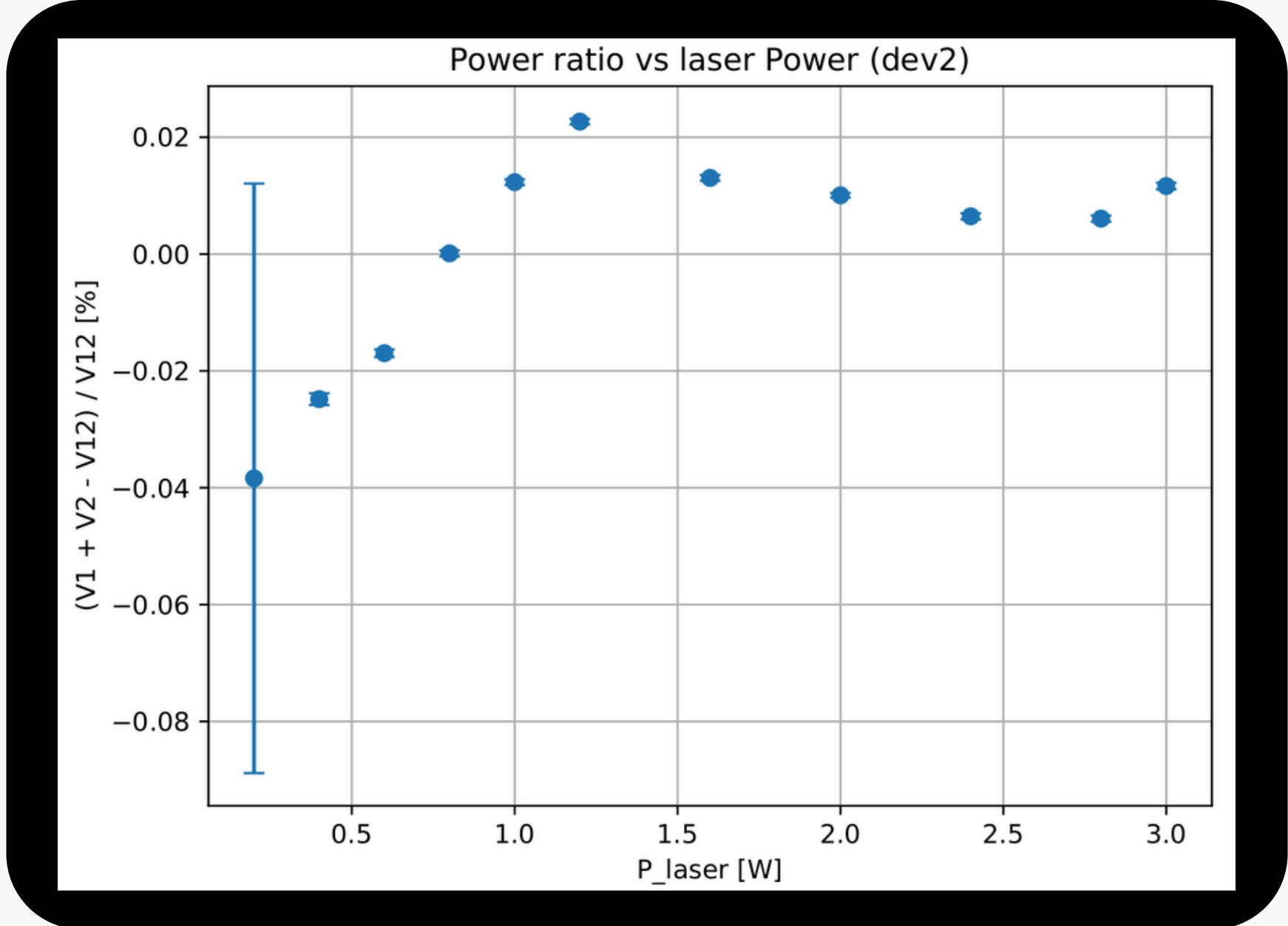
# Spheres are linear at <0.1%

WSV

GSV



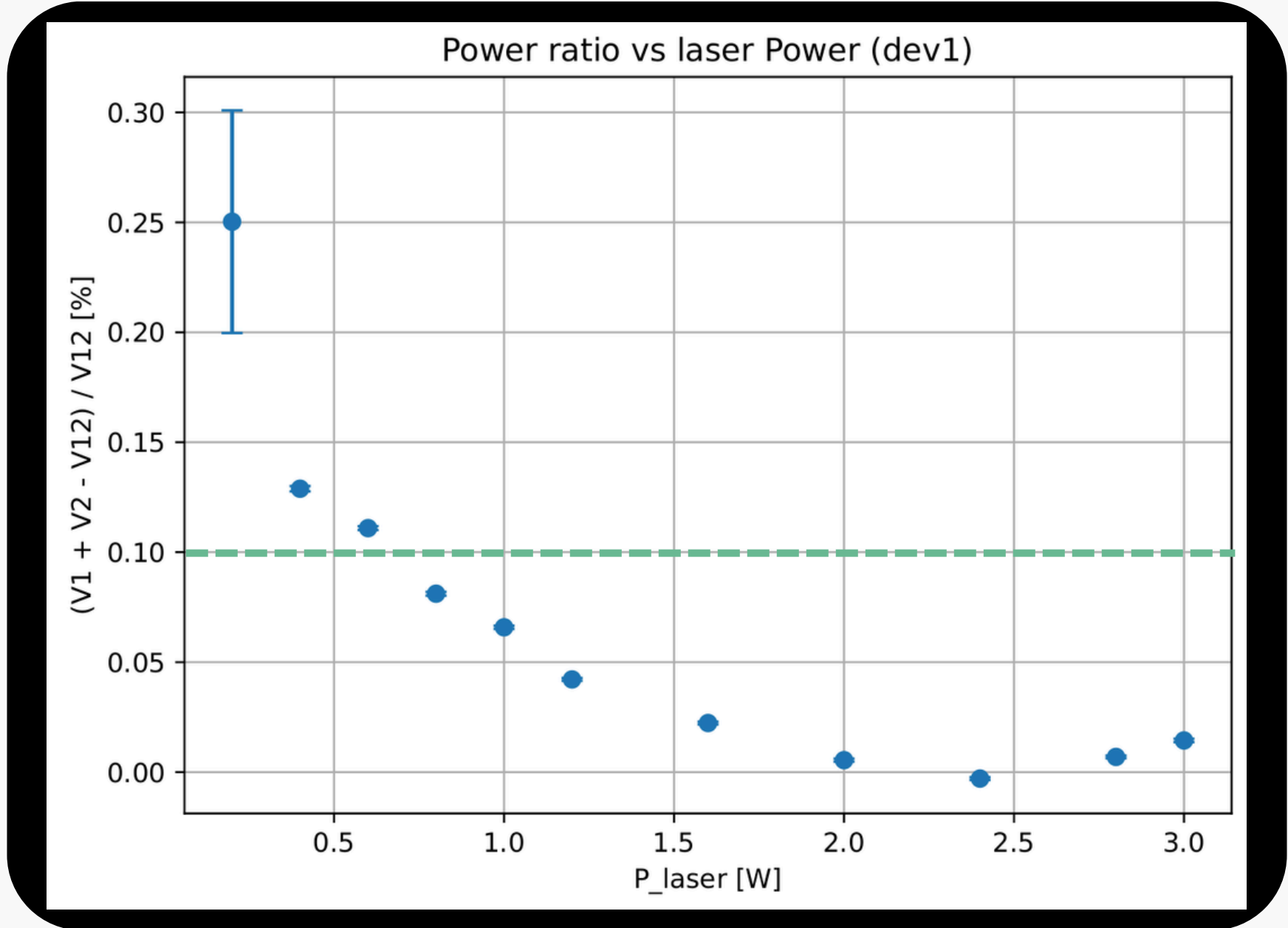
Power (L1 + L2) [W]



Power (L1 + L2) [W]

# RxNE not linear below 0.1% everywhere

## RxNE



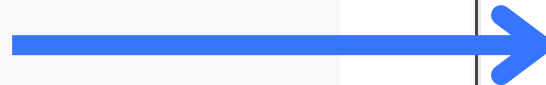
Power (L1 + L2) [W]

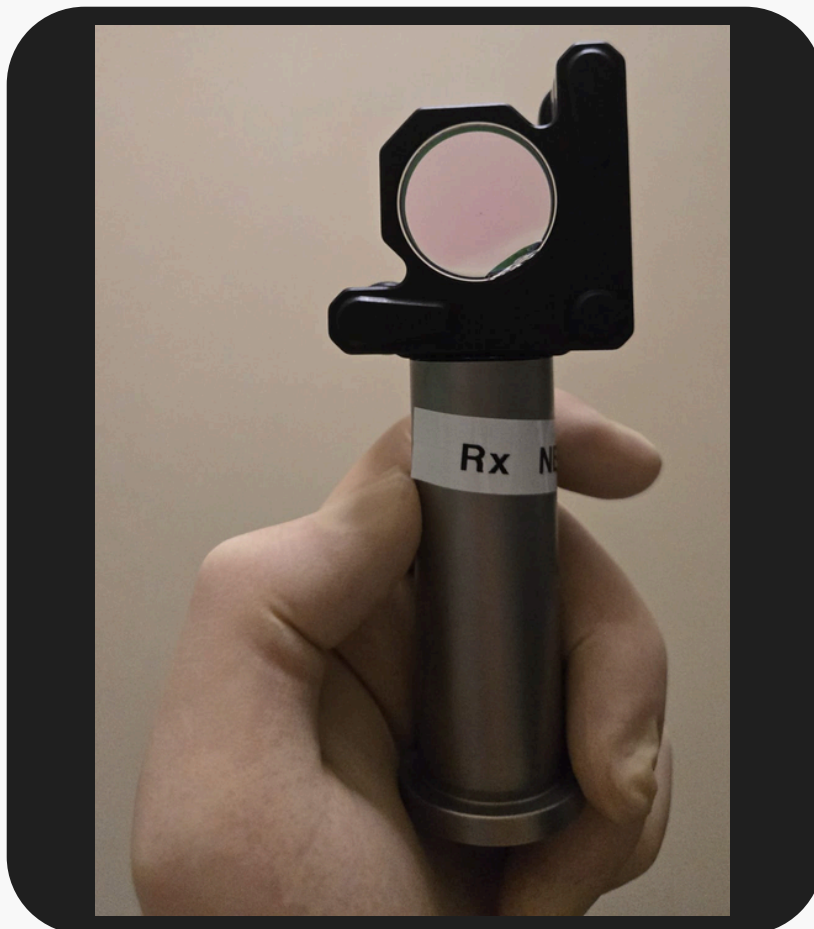
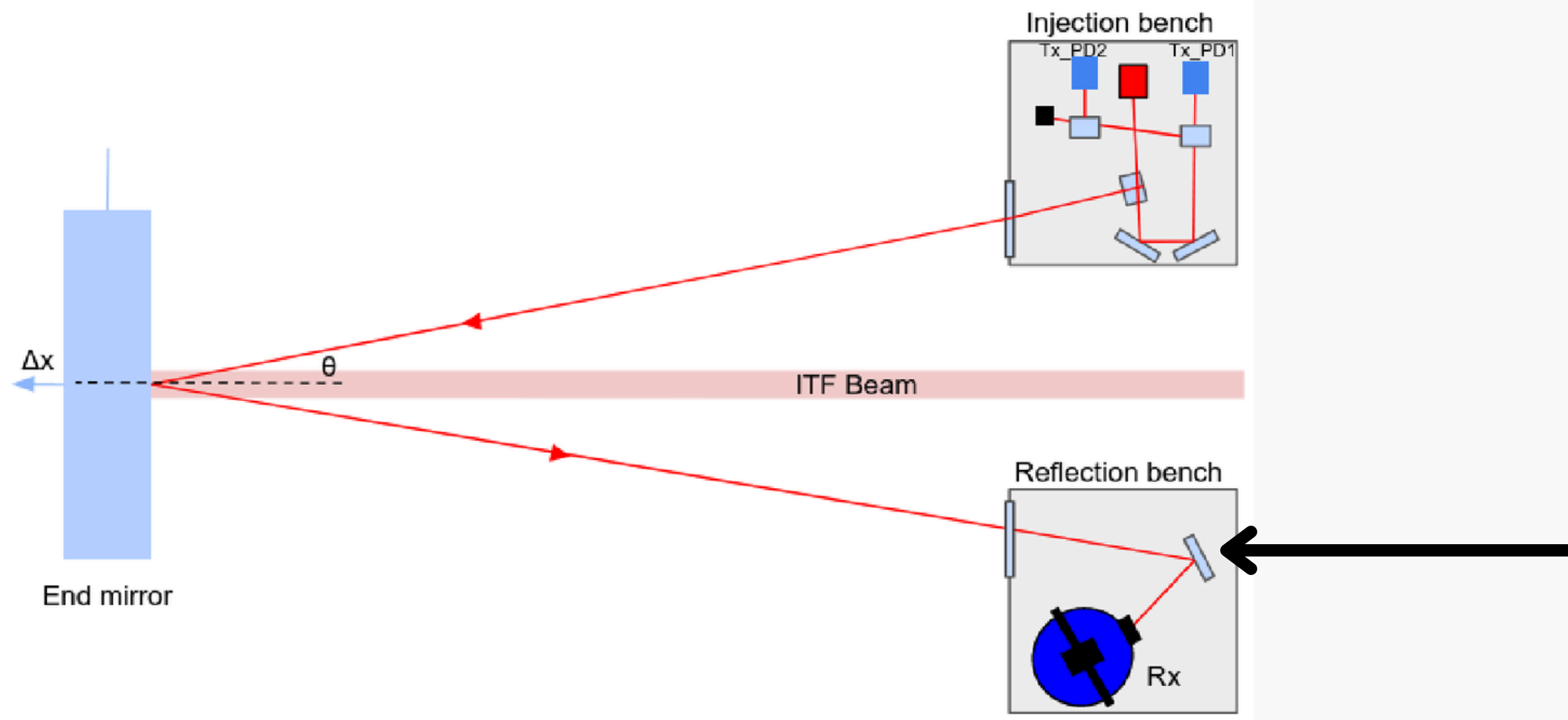


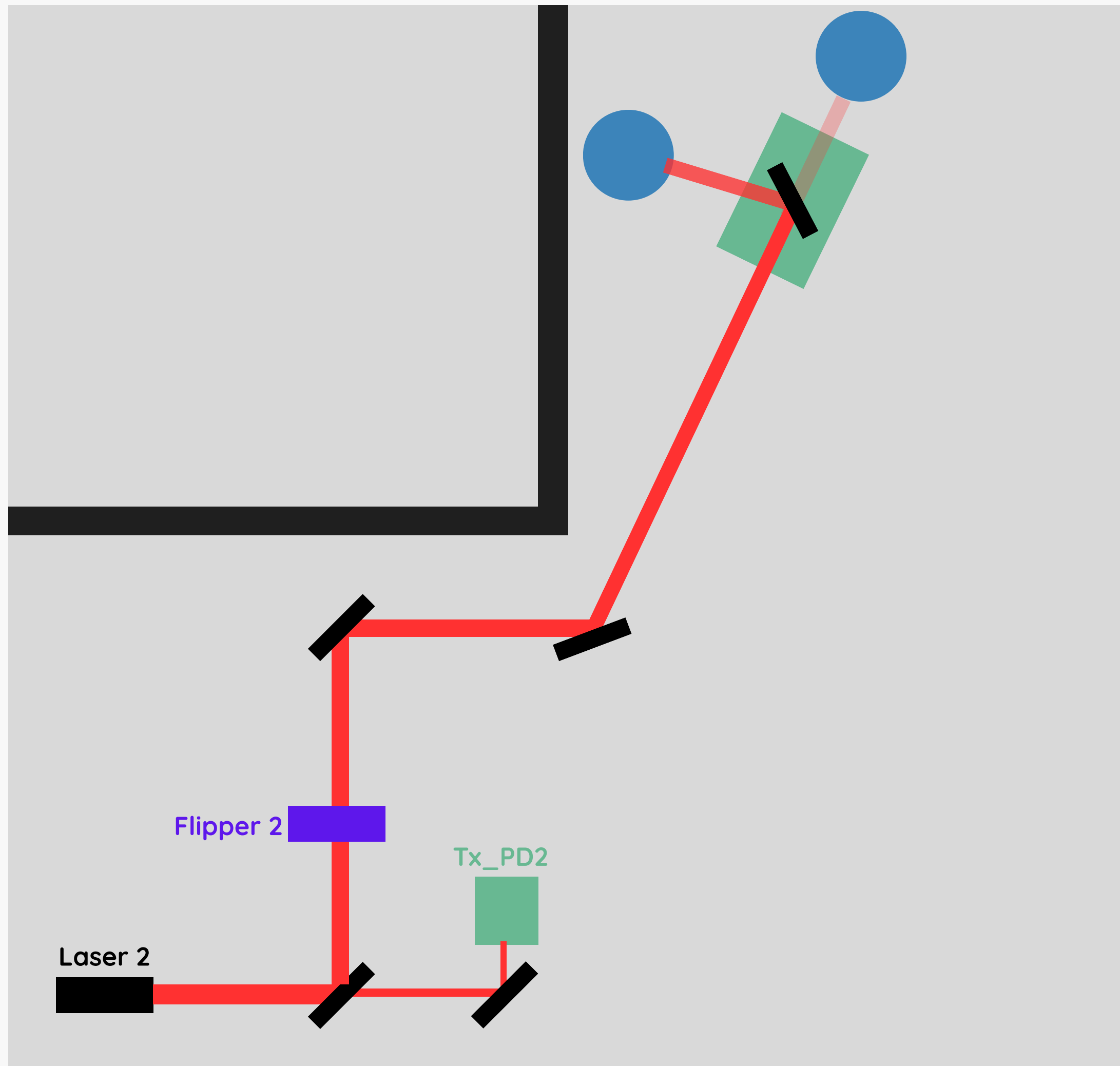
*NE bench mirror  
characterization*



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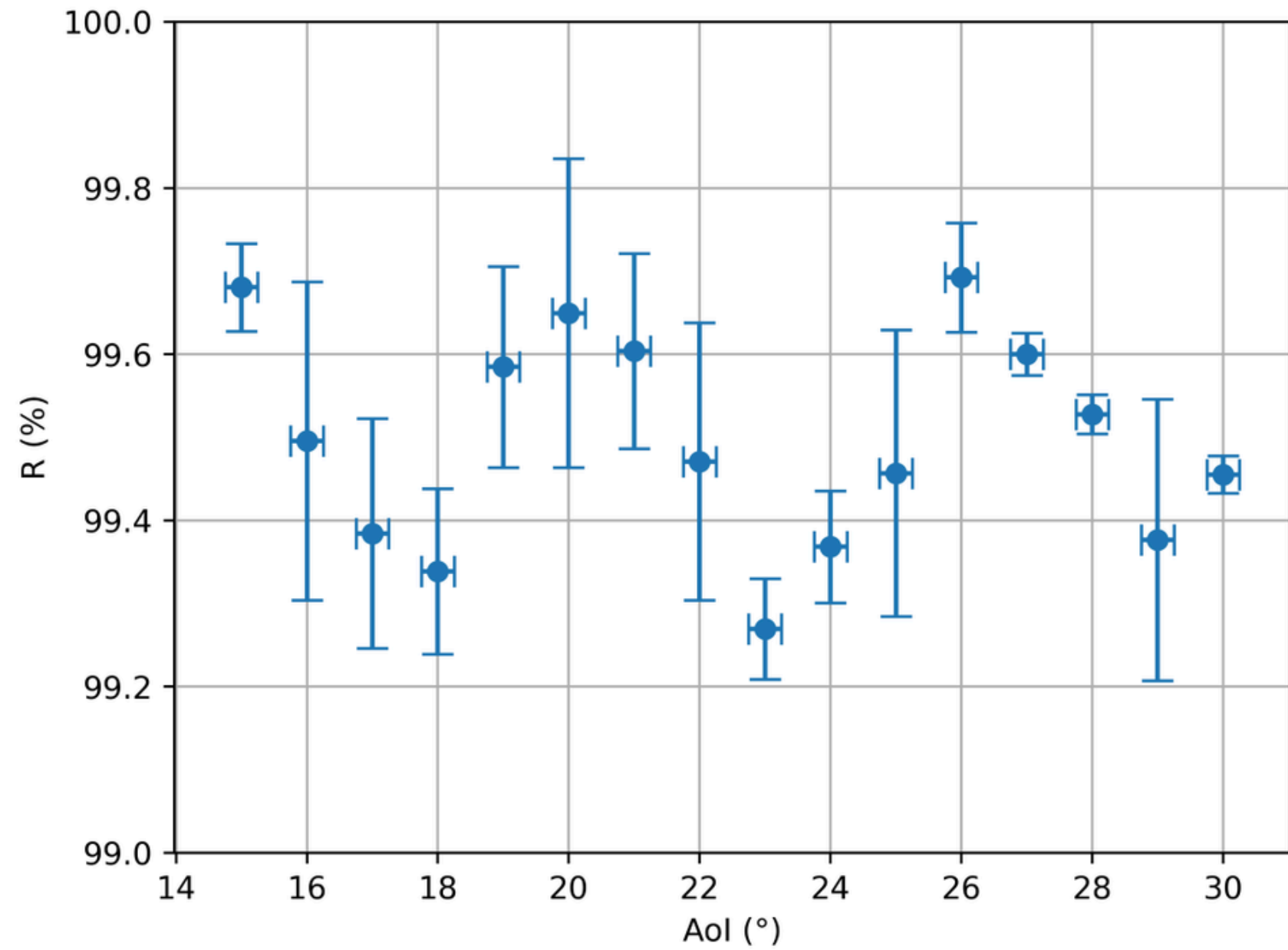


Figure 3: R as a function of AoI.

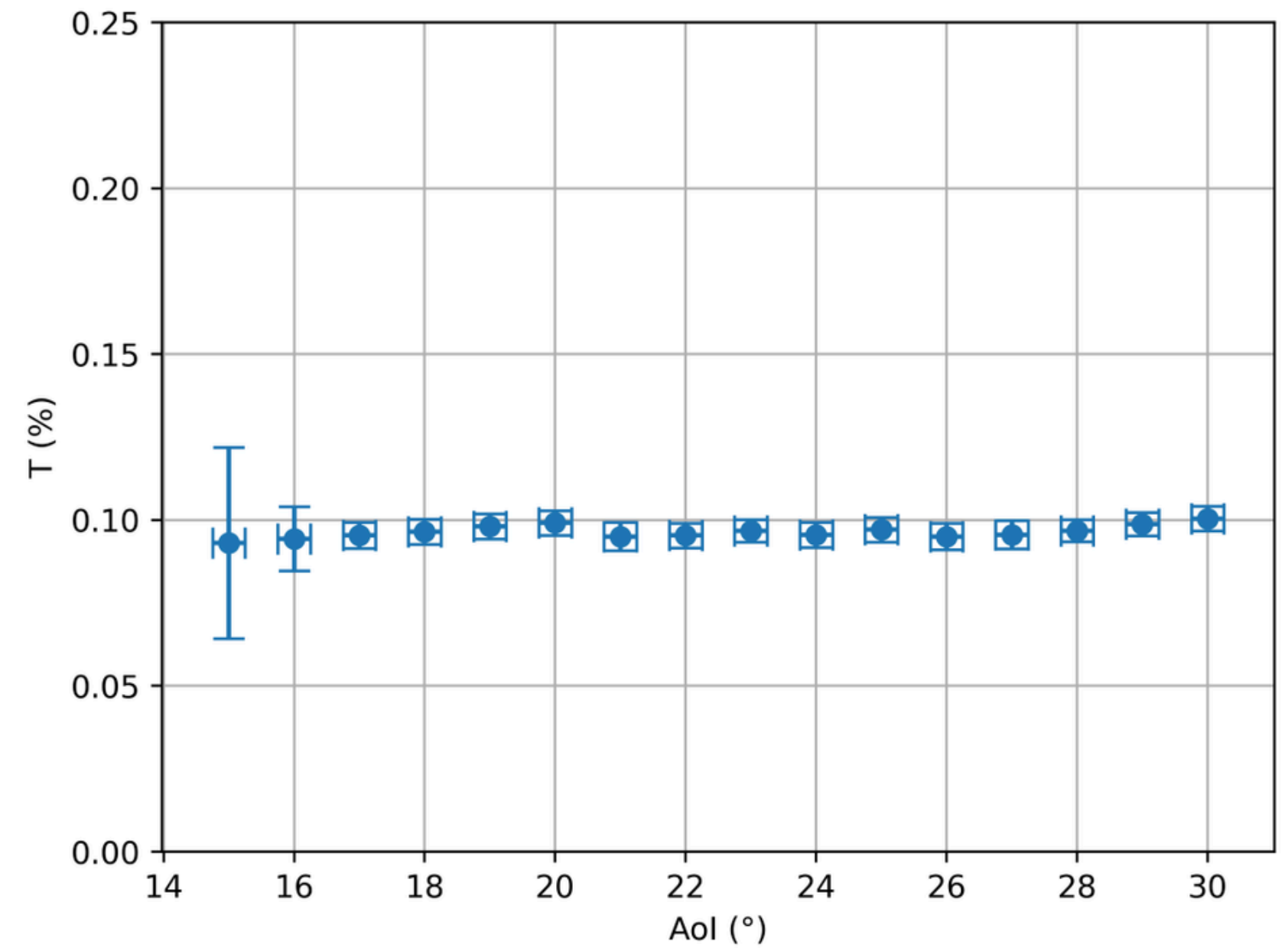
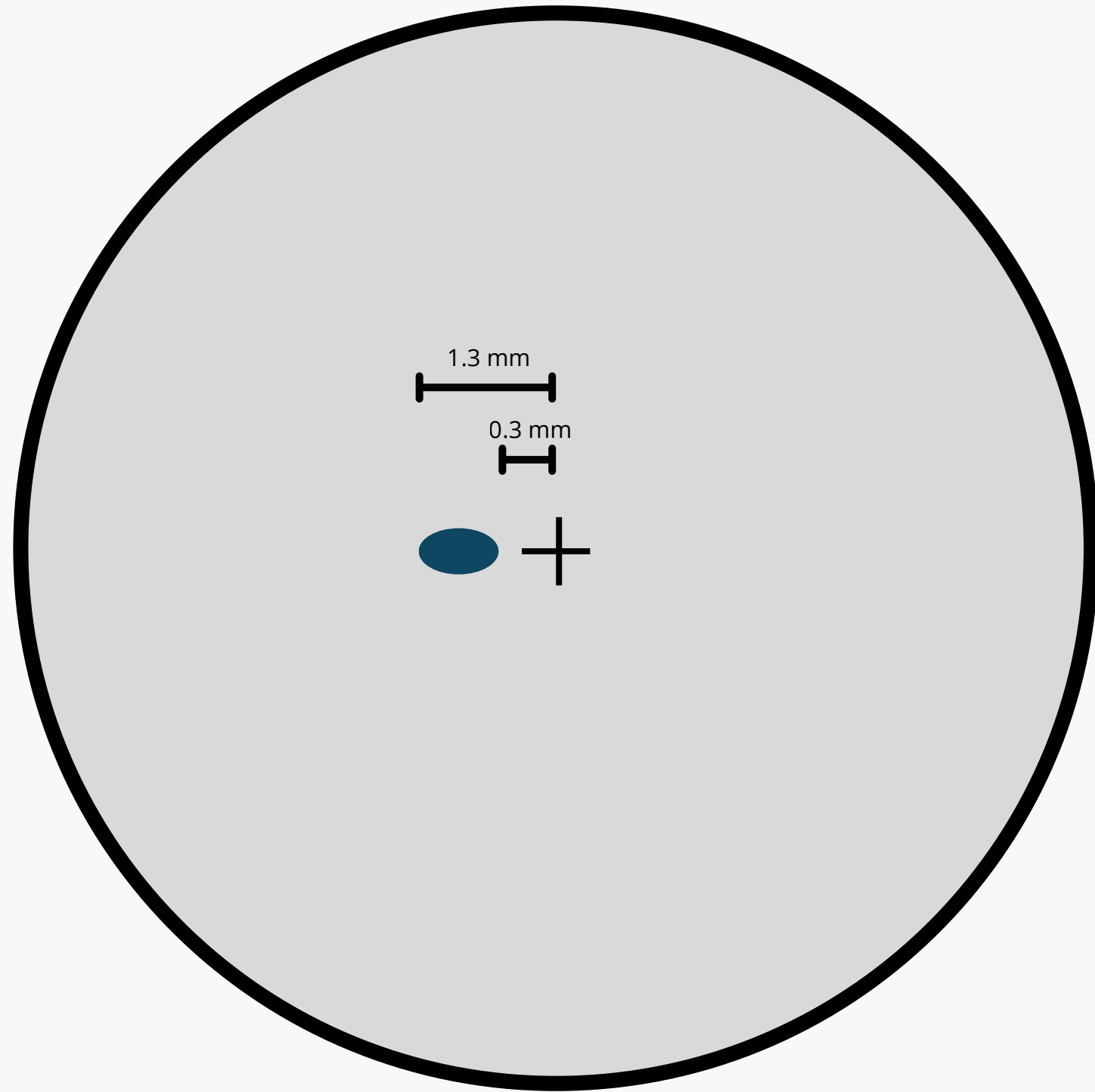


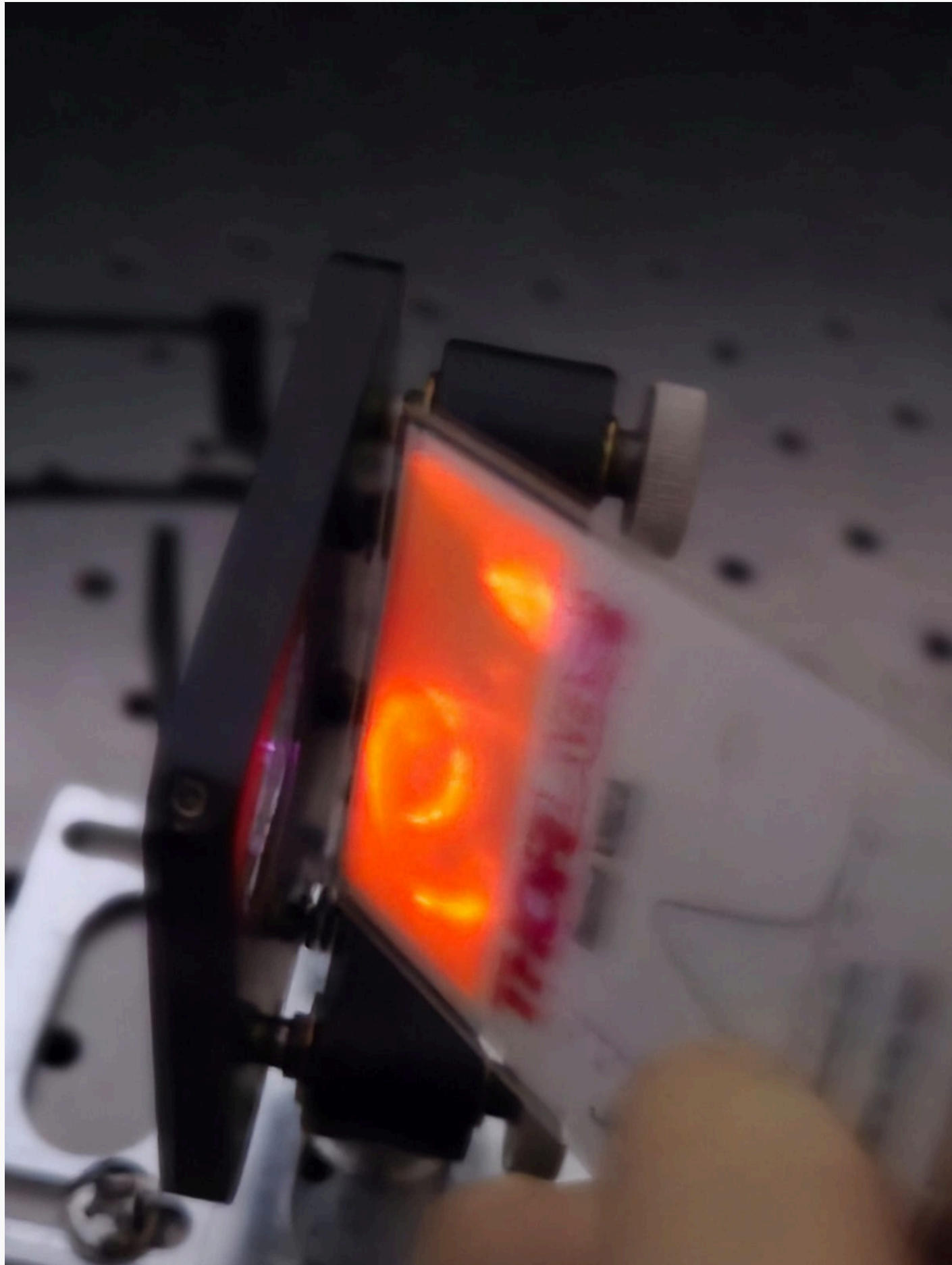
Figure 4: T as a function of AoI.

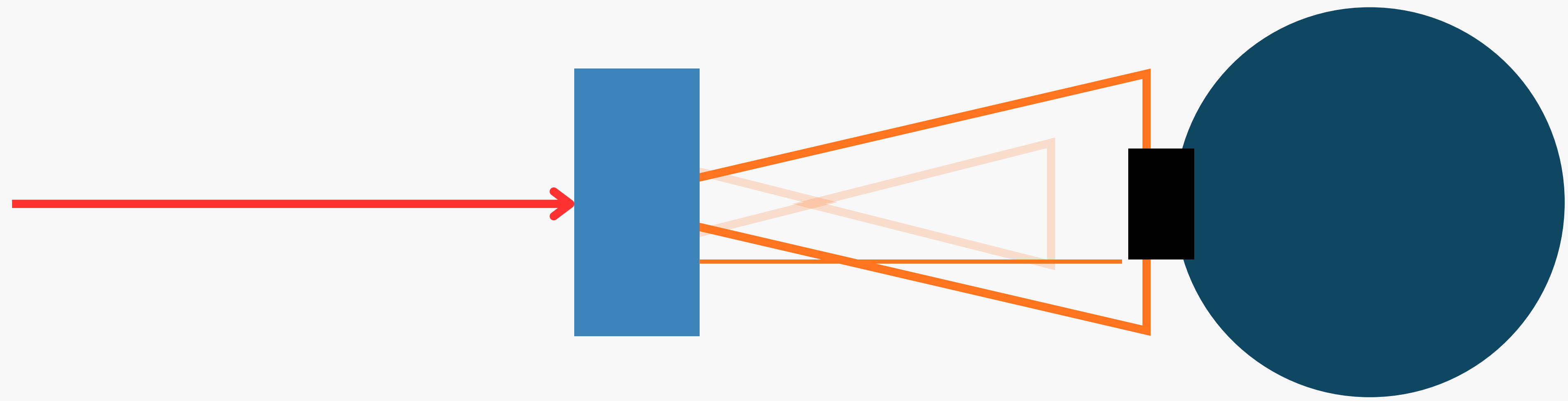
1 in = 25.4 mm



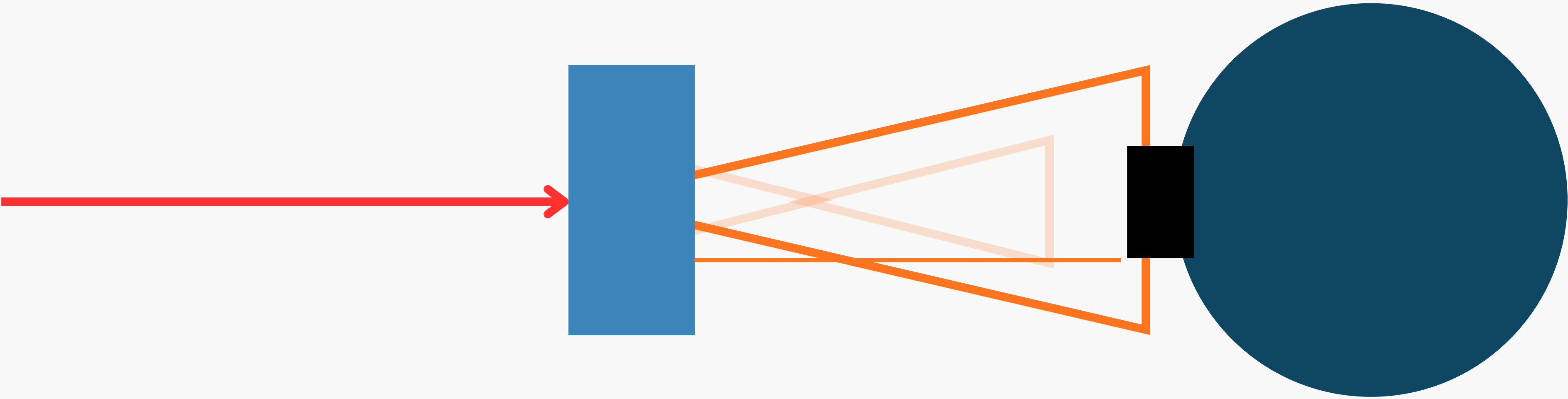
**Defect ~1mm on the left from the center**







Mirrors to be changed  
:(



## Conclusion

Intercalibration is going on well

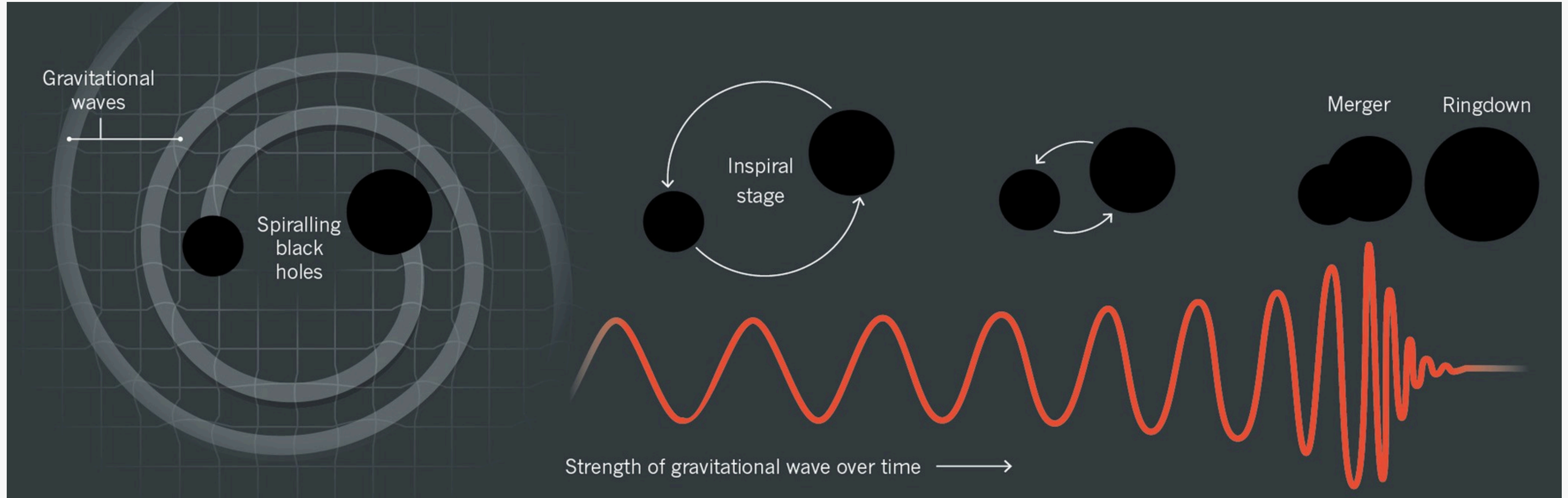
Integrating spheres are linear below 0.1% (except for RxNE)

PCal mirrors have to be changed

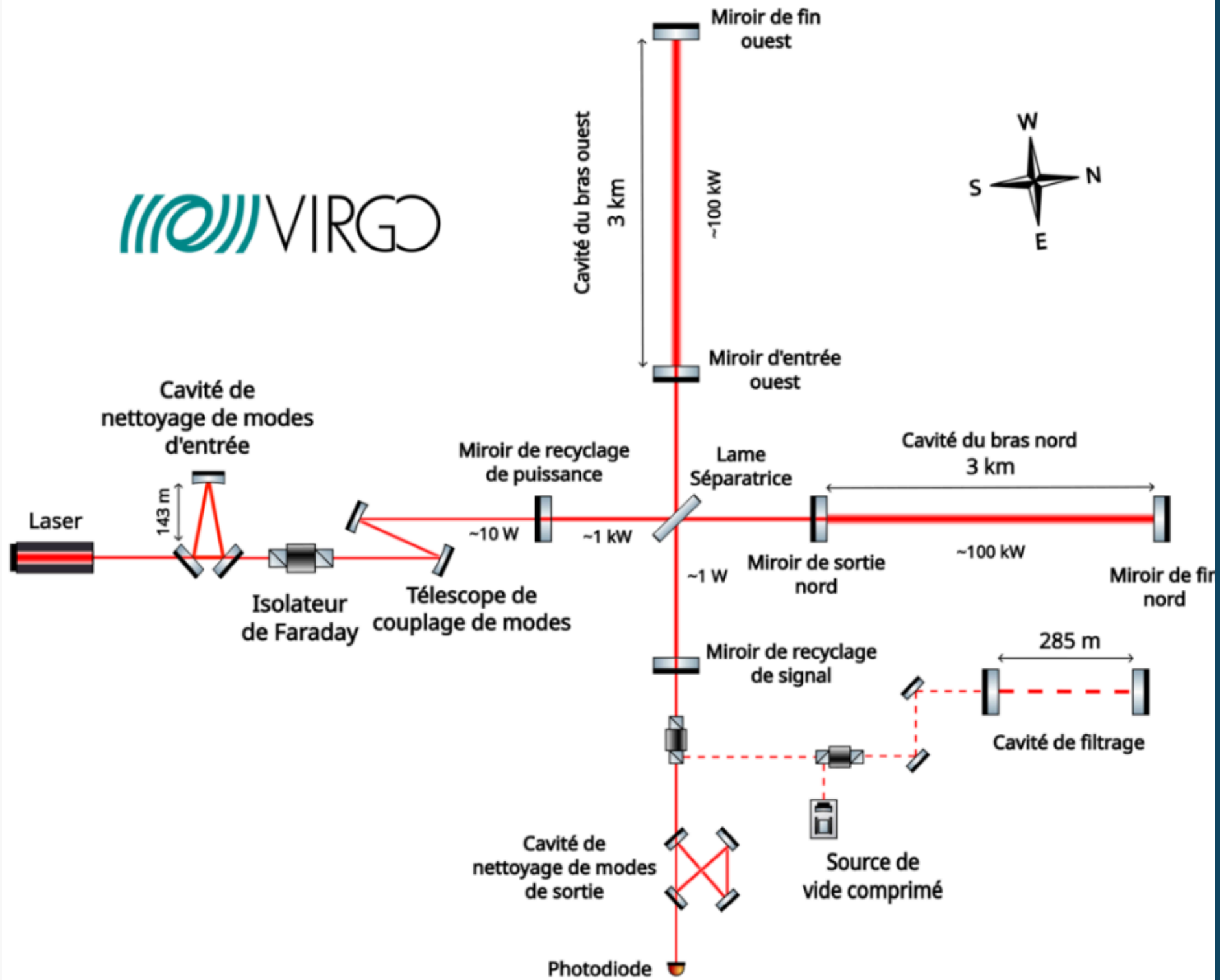


*Earth's rotation impact on  
ET*

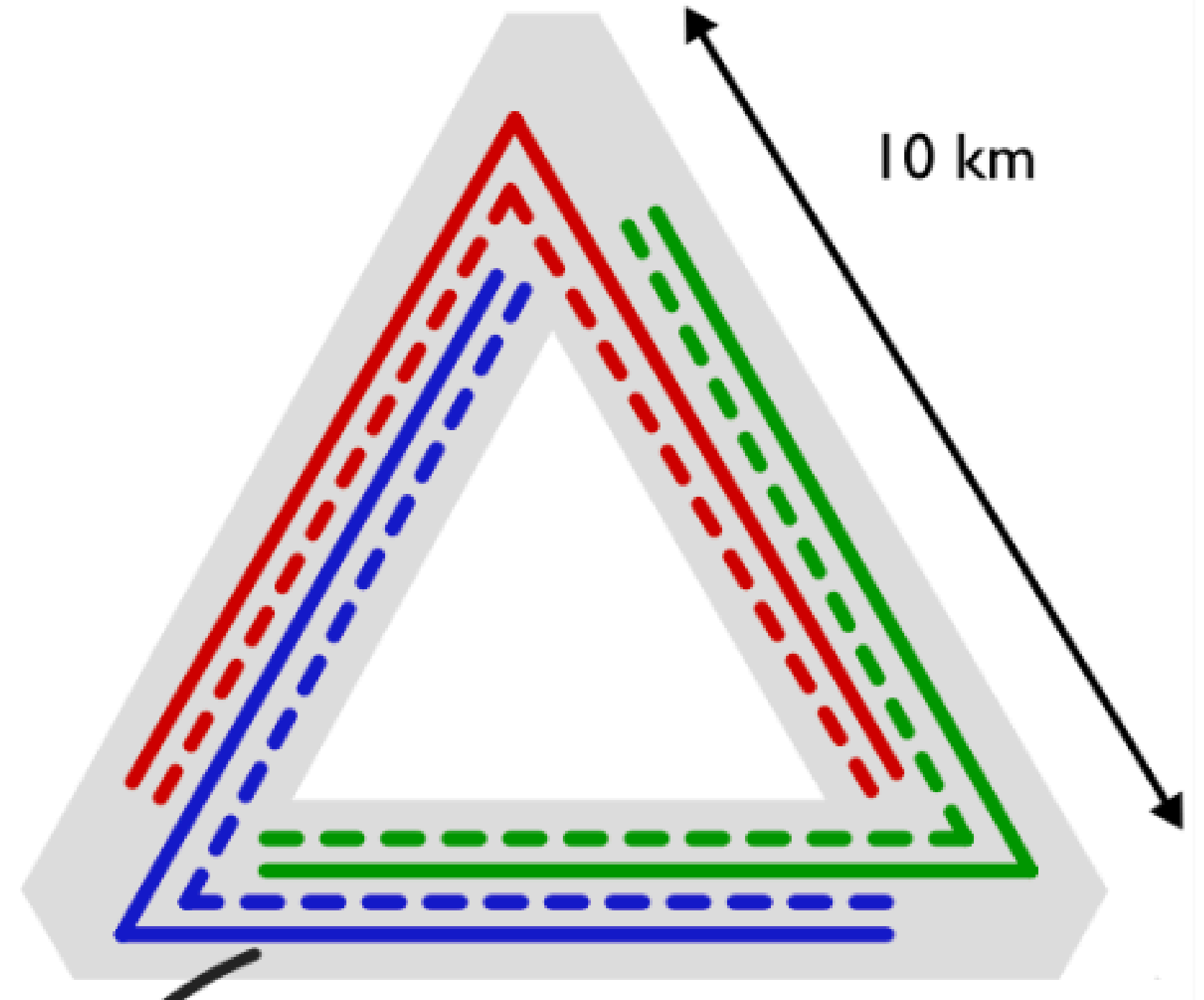




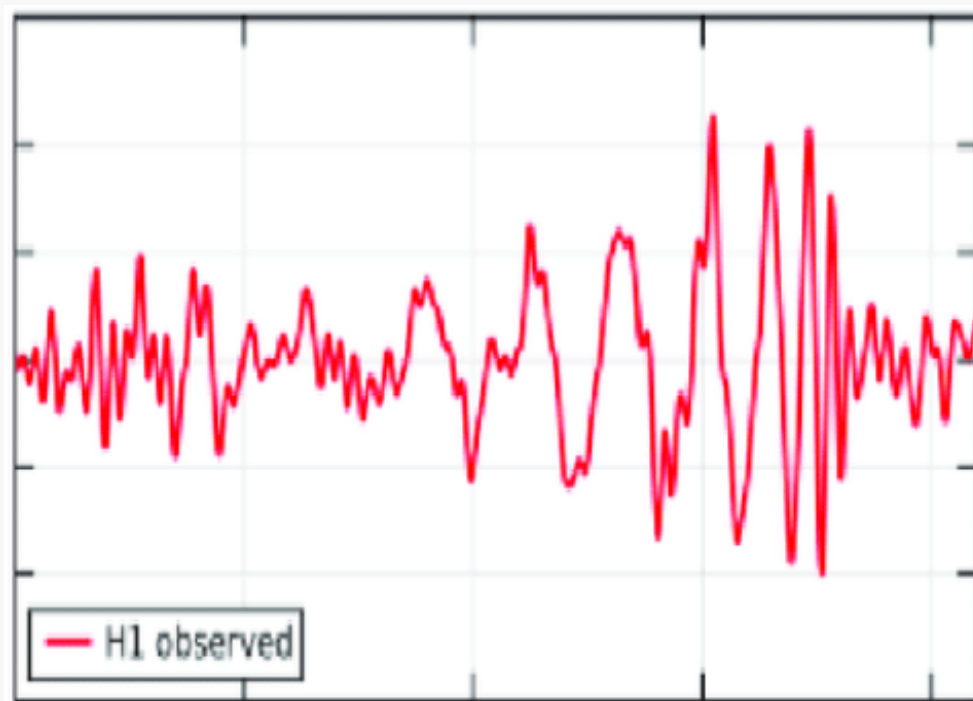
# Virgo



# Einstein Telescope



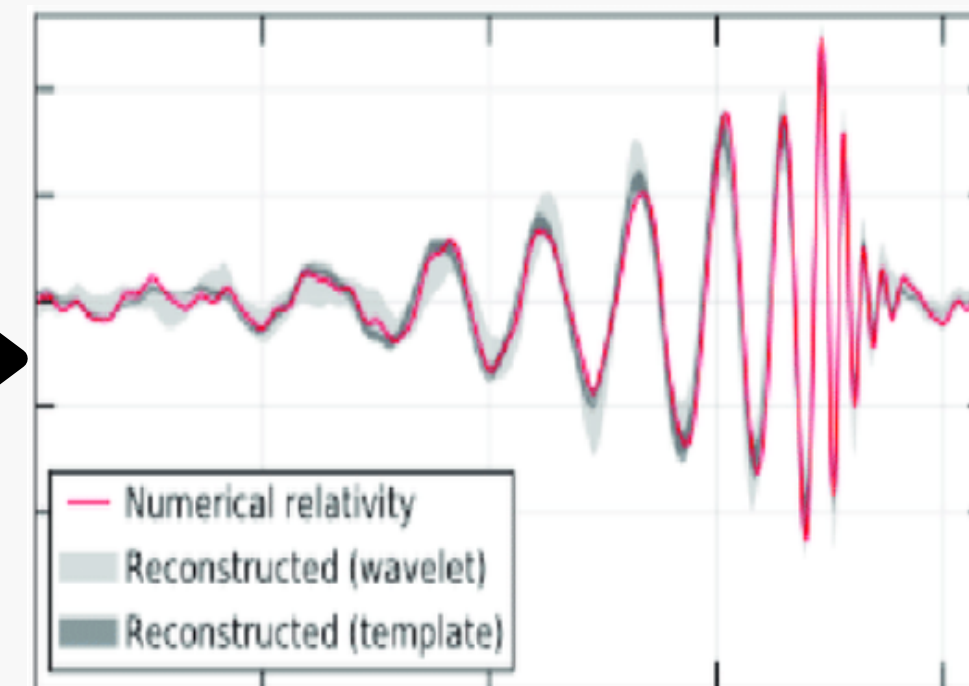
## Signal reception



## Analysis

Template bank  
(simulated GW, known parameters)

## Data output



Matching with  
templates → SNR

$$\mathcal{M}(s, T) = \max_{\Delta t, \Delta \phi} \frac{\langle s, T \rangle}{\|s\| \cdot \|T\|}$$

$$\rho = \frac{|(h|T)|}{\sqrt{(T|T)}}$$

Template  $Q(t)$  with  
highest SNR kept

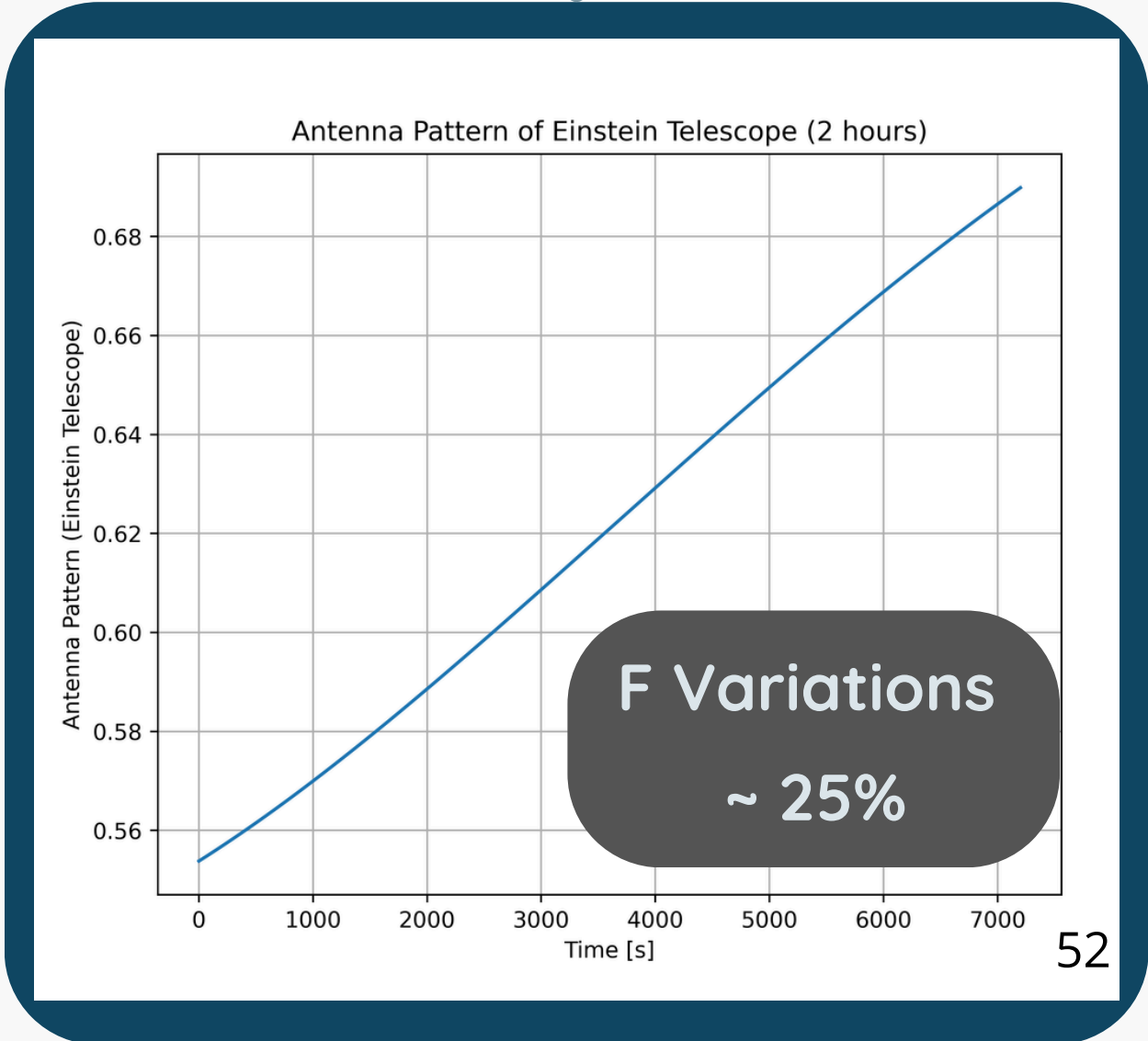
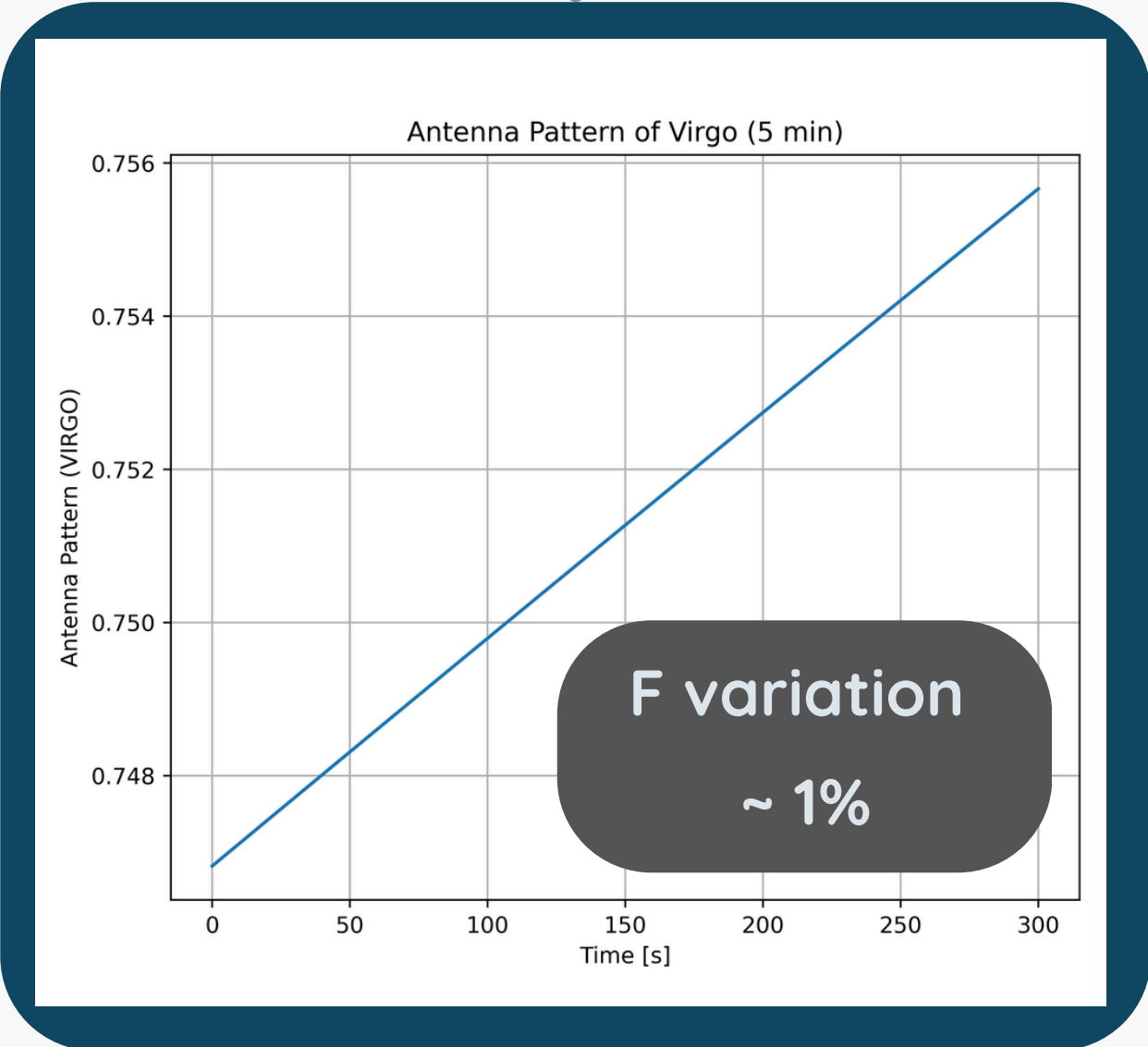
$$h(t) = h_+(t)F_+(\theta, \phi, \psi) + h_\times F_\times(\theta, \phi, \psi)$$

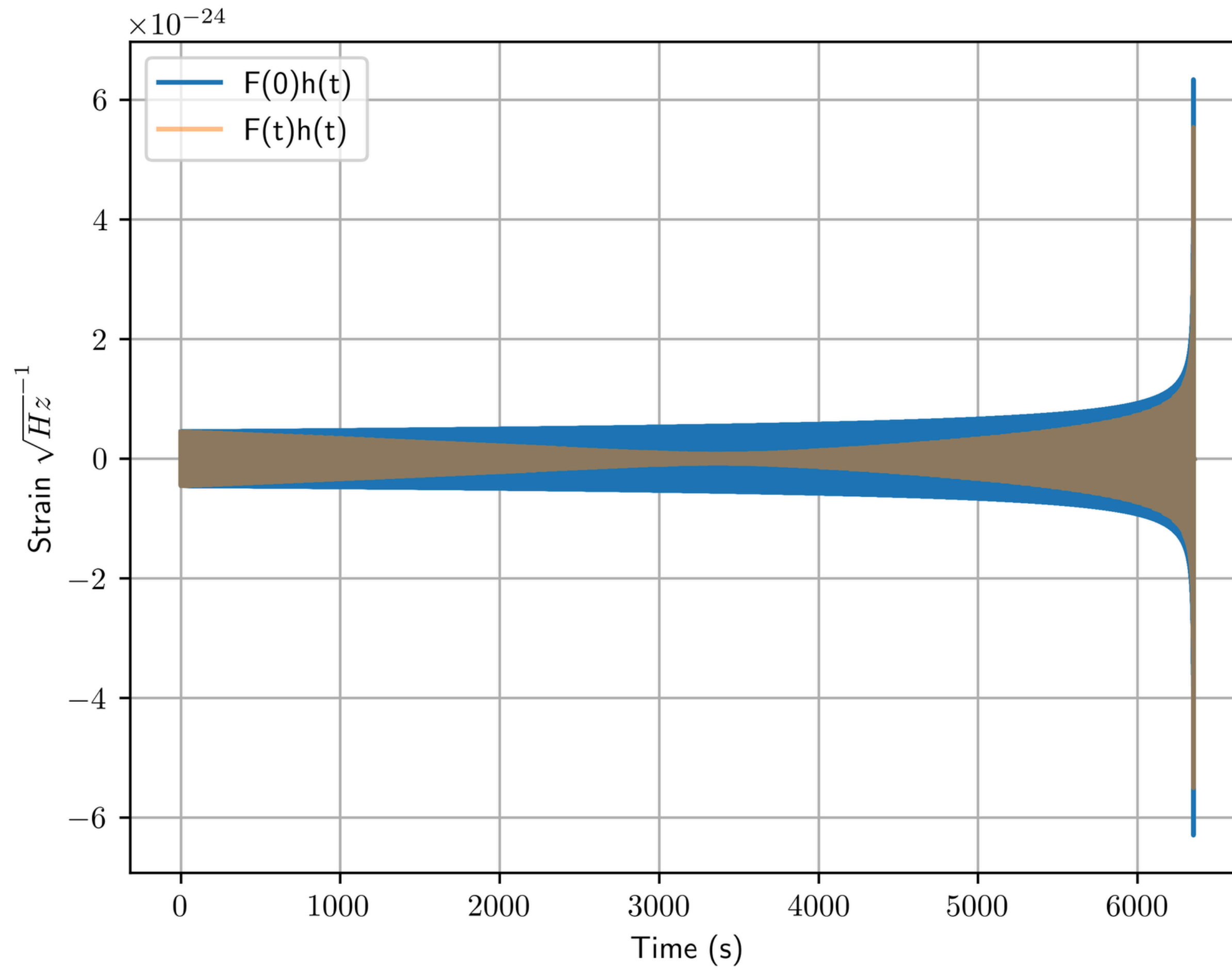
ET-Xylophone

### Signal duration

- Virgo : ~5 min
- ET : ~1-2h

Random example  
RA = 3.45 [rad]  
DEC = -0.408 [rad]





Incoming signal  
(Naturally modulated by  $F(t)$ )

Optimal template

=

Time-modulated template

$F(t)h(t)$

(Complexifies the bank)

Simple template

=

Non time-modulated template

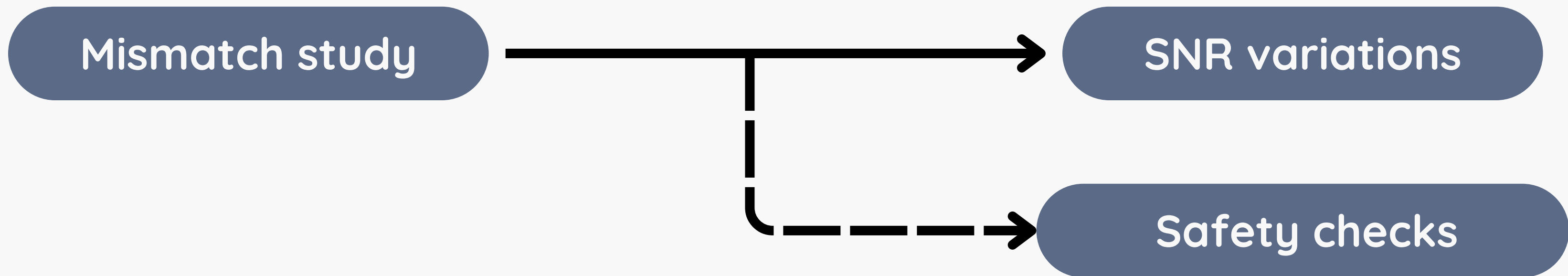
$F(0)h(t)$

(Virgo-like)

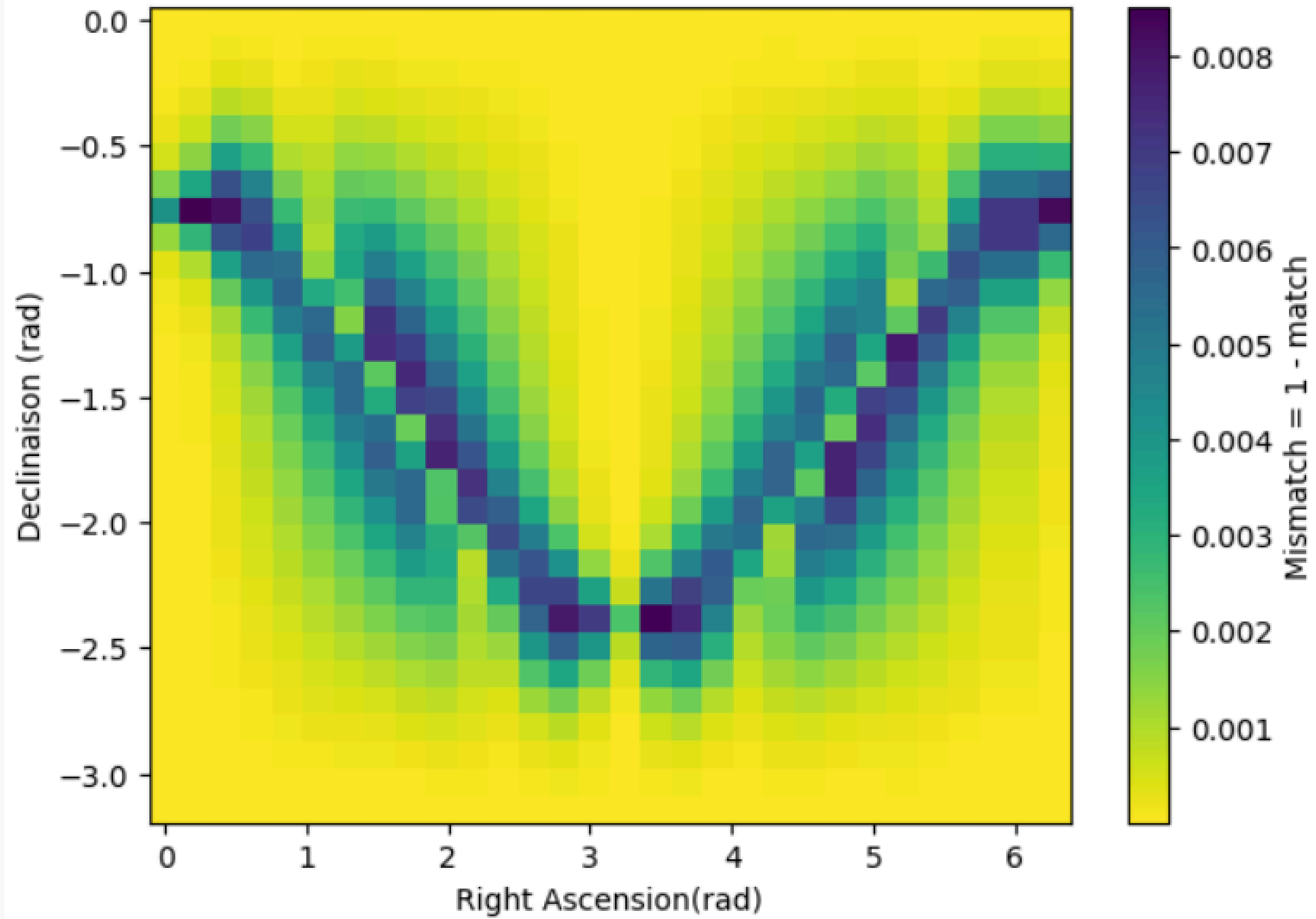
Which one do we  
choose ?

How much Match and SNR do we  
lose if E.R. neglected ?

$$\frac{\Delta\rho}{\rho} := \frac{\rho_{mod} - \rho_{const}}{\rho_{mod}} \approx 1 - \mathcal{M}[F(t)h(t), F(0)h(t)]$$



Mismatch map [ET - 5 Hz]



Max ~ 0.8%

Earth rotation can be neglected here

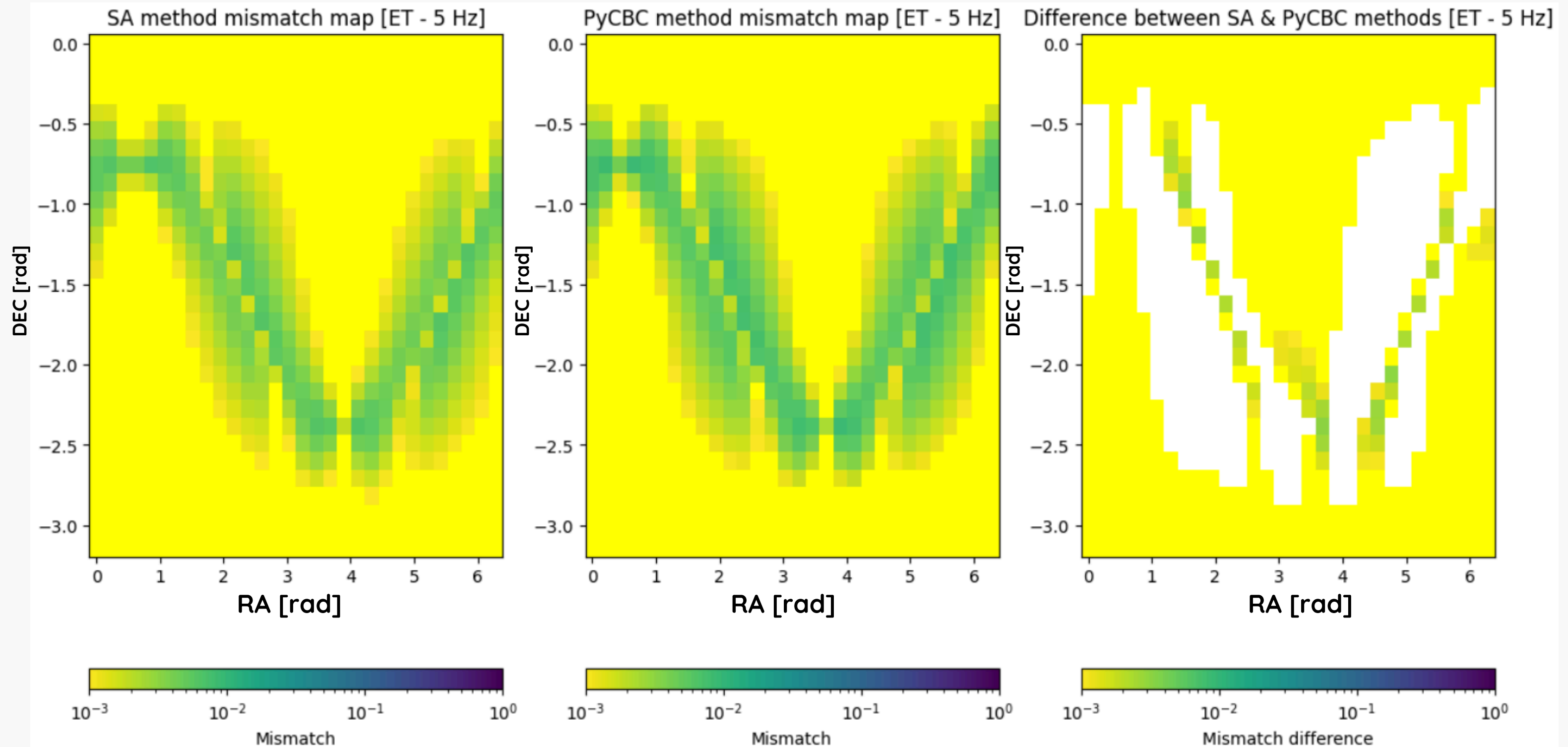
Slow computation

ET Xylo,  $f_{low} = 5 \text{ Hz}$ ,  $\iota = 0^\circ$ ,  $\Psi = 0^\circ$ ,  $M_{Chirp} = 1.2M_\odot$

SA method  
(Max = 0.006)

PyCBC  
(Max = 0.008)

Differences





*Global study of the mismatch  
Monte-Carlo simulation*



### Sky position

$$\alpha, \cos(\delta) \sim \mathcal{U}[0, 2\pi], \mathcal{U}[0, 1]$$

### Polarisation

$$\Psi \sim \mathcal{U}[0, 2\pi]$$

### Inclination

$$\cos(\iota) \sim \mathcal{U}[0, 1]$$

### Individual masses

$$M_i \sim \mathcal{N}(1.35, 0.1)$$

### Low frequency cutoff

$$f_{low} = 5 \text{ Hz}$$

### Coalescence time

$$t_c \sim \mathcal{U}[0, 86400]$$

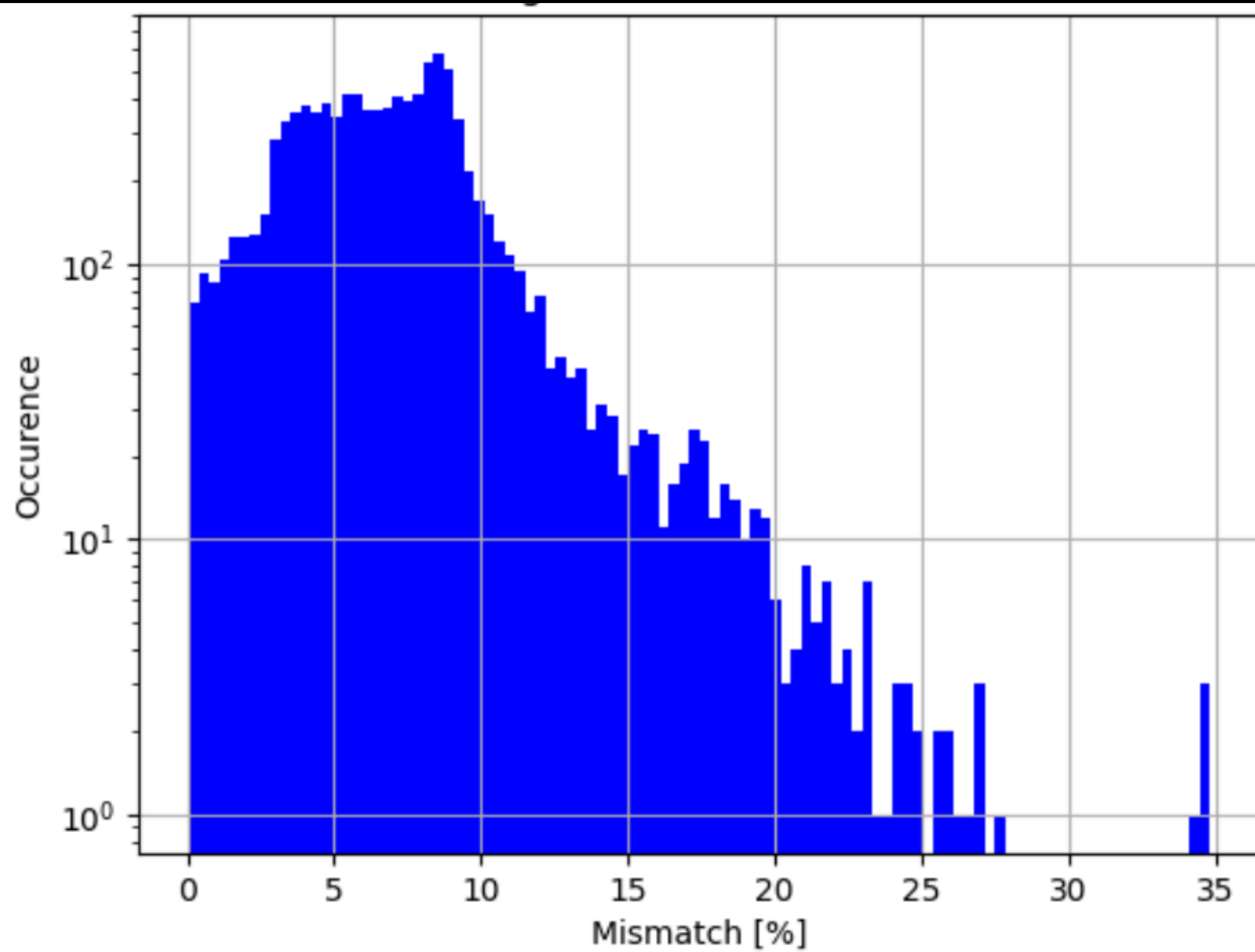
### Redshift

$$p(z) \propto \frac{dV_c}{dz} \mathcal{R}(z)$$

Tidal and spin  
neglected

(10.000 sources)

## Mismatch histogram



Up to **40%** Mismatch

~**13%** of sources have  
Mismatch > **10%**

~**70%** of sources have  
Mismatch > **5%**

~**90%** of sources have  
Mismatch > **3%**



*Global study of the*  
***SNR***



We necessarily loose SNR

$$\rho_{const} \leq ||h|| = \rho_{opt,mod}$$

Let's do a Monte-Carlo !

### Sky position

$$\alpha, \cos(\delta) \sim \mathcal{U}[0, 2\pi], \mathcal{U}[0, 1]$$

### Polarisation

$$\Psi \sim \mathcal{U}[0, 2\pi]$$

### Inclination

$$\cos(\iota) \sim \mathcal{U}[0, 1]$$

### Individual masses

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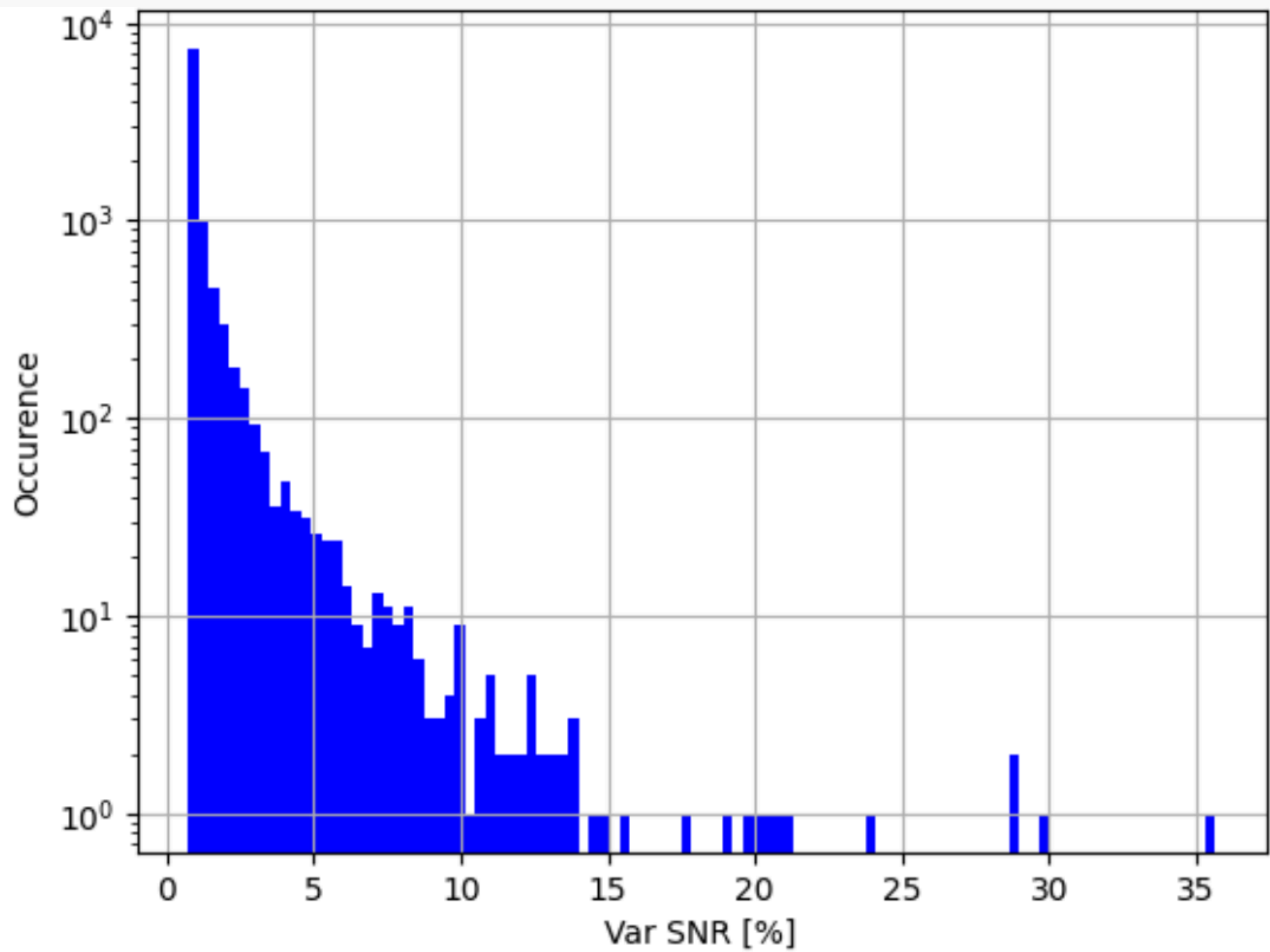
### Coalescence time

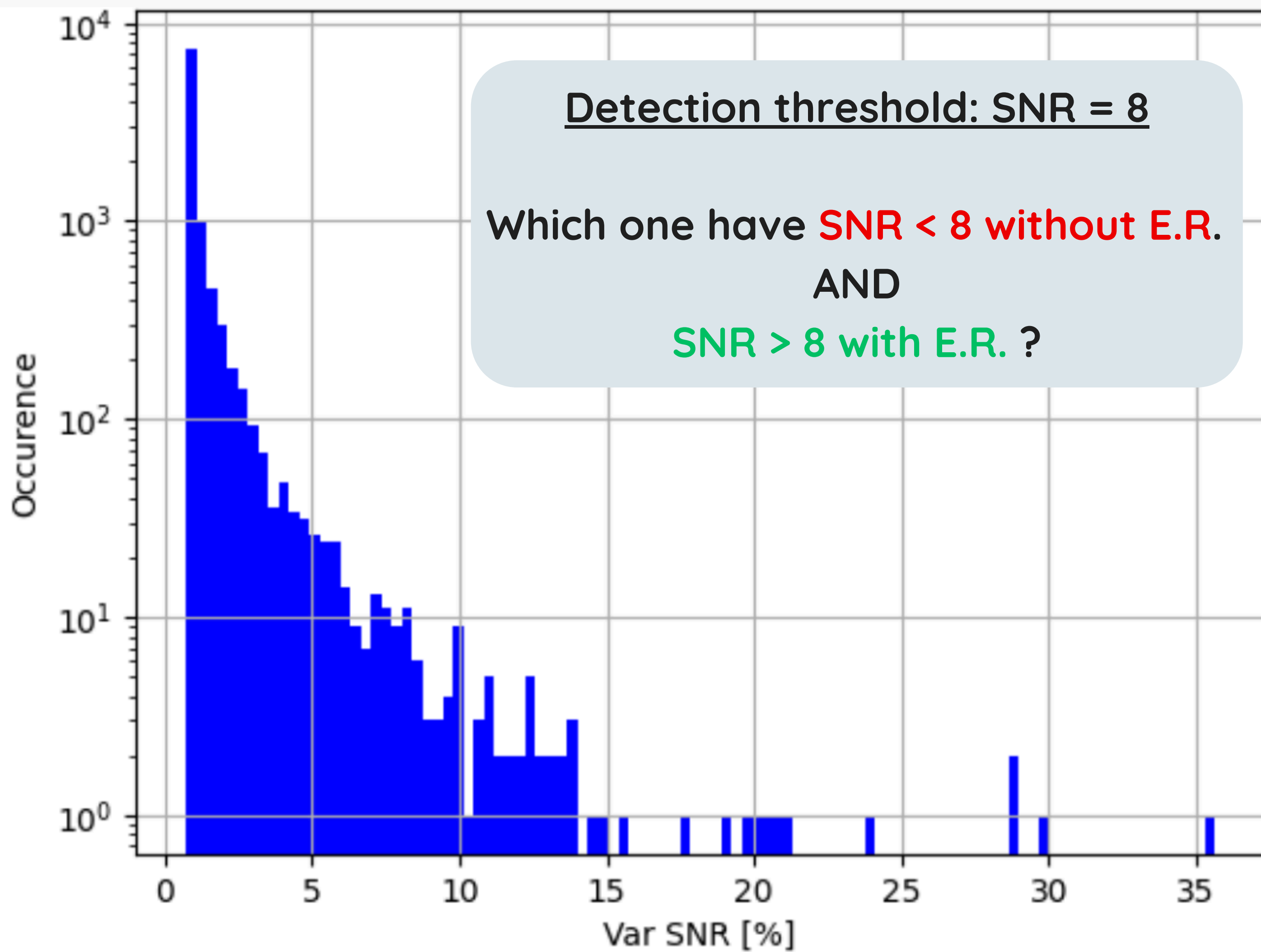
$$t_c \sim \mathcal{U}[0, 86400]$$

### Redshift

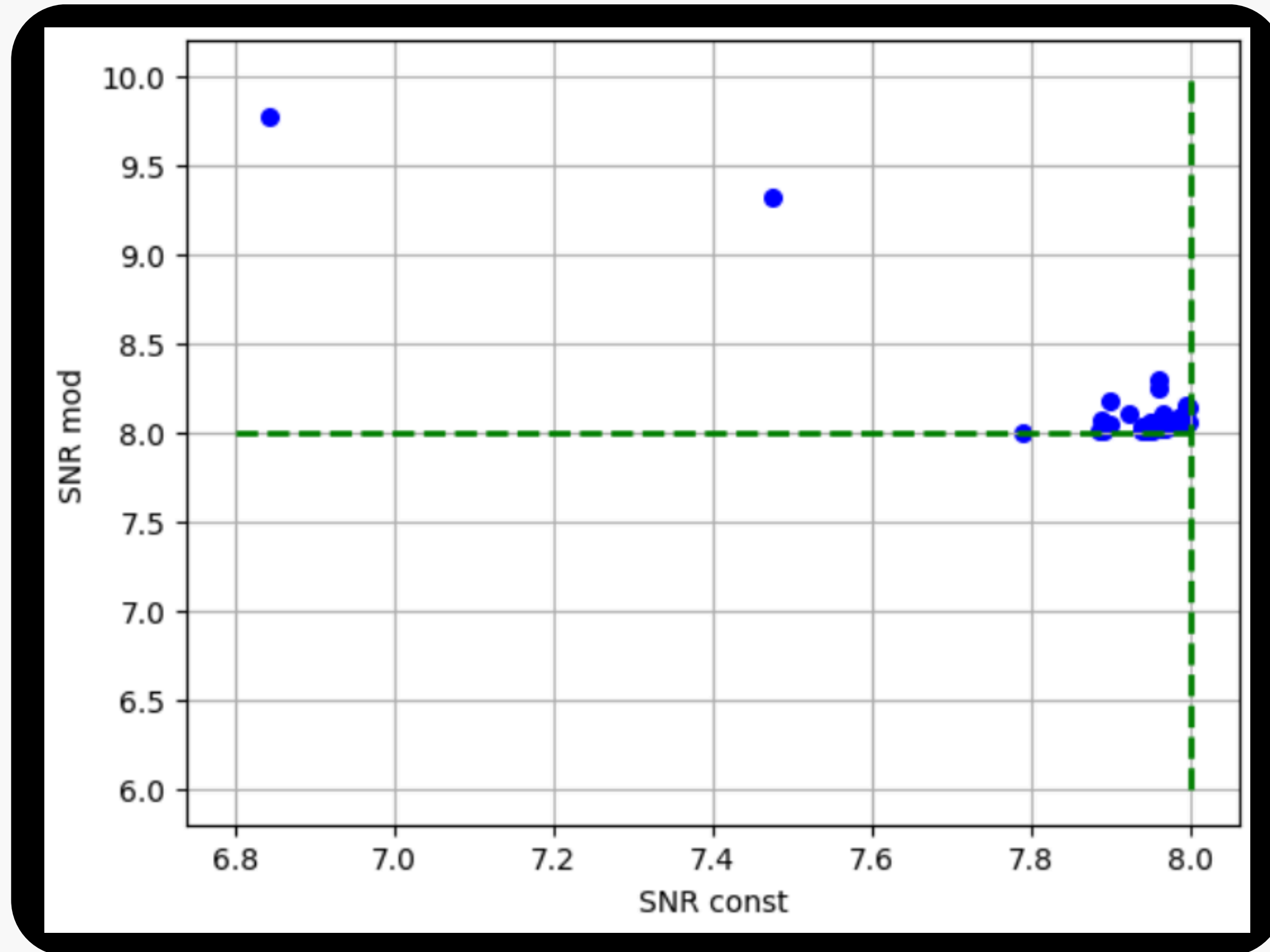
$$p(z) \propto \frac{dV_c}{dz} \mathcal{R}(z)$$

Tidal and spin  
neglected





# Recovered SNR stay around SNR = 8



## Statistics !

About ~ **0.6%** de ALL sources  
detected at 5 Hz

~ **25%** of ALL sources can be  
retrieved if starting at 2 Hz

## Conclusion

MC simulations show that  
**Mismatch can be severely  
affected**

Earth rotation induced SNR  
variations can be **neglected** for  
 $f_{low} > 5\text{Hz}$

Taking E.R. into account would complexify the  
Match Filtering process for **<1% of sources**



# *Conclusion - Miscellaneous*



## Virgo

- Intercalibration still going on without problems
- Integrating spheres are linear at  $<0.1\%$
- M3 mirror has a defect leading to a loss of  $<1\%$  of R

## ET

Earth's rotation not that easily negligible  
(at 5Hz,  $\sim 10\%$  of sources could be missed because  
of high mismatch)

## Scientific contributions

ET internal note  
about Earth rotation

Virgo technical notes x2  
(linearity + M3)

Calibration mission at Virgo  
(December)

PCal article  
co-author

ET France talk  
(Paris, April)

Poster at Workshop ET  
(Rome, February)

PhD Student: Mathieu SCHOOR (LAPP, Annecy, France)  
Supervised by: Damir BUSKULIC & Loïc ROLLAND (LAPP, Annecy, France)

## Why might this be important ?

We expect, in an optimistic case, that the ET's sensibility will allow the frequency range to start at  $\sim 5$ Hz [1]. If so, signals (especially BNS ones) would be around 1-2 hours long while today's signals do not last more than 5 minutes.

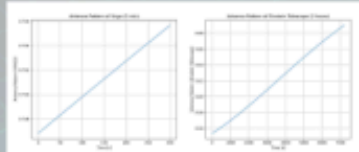


Fig 1: Comparison of the antenna pattern variation for a typical signal duration for Virgo (left) and ET-Xylophone (right)

As we can see on Fig [1], while the antenna pattern (noted F) of Virgo is almost constant for the whole signal's duration, we can have great variation for ET, such as 25% variation of F.

$$F_{\nu}(n, \delta, \psi, t) = \sin(\dots) \cos(2\pi\nu t) + \dots$$

Do we have to take into account the Earth's rotation induced modulation for the detection of long-duration signals ?

If neglected, how much Match do we loose ?

This kind of variation might severely influence the detected signal  $h(t)$ , since it directly depends on the values of the antenna pattern

$$h_{\text{total}}(n, \delta, \psi, t) = h_1(t)F_1(n, \delta, \psi, t) + h_2(t)F_2(n, \delta, \psi, t)$$

$$M = \frac{\text{MAX}_{\Delta\alpha, \Delta\beta} (|h_1|, |h_2|)}{\text{MAX}_{\Delta\alpha, \Delta\beta} (|h_1|, |h_2|)}$$

## Mismatch study

Since generating full waveform is very computationally demanding, we choose to use a semi-analytical method to compute the match between the waveform that takes into account the Earth's rotation ( $h_{rot}$ ) and the one that doesn't ( $h_{nonrot}$ ) using the Stationary Phase Approximation (SPA) [2].

$$h_{rot}(t) = A(t) (1 + \cos^2\iota) \cos(\Phi(t))$$

$$h_{nonrot}(t) = A(t) 2 \cos(\iota) \sin(\Phi(t))$$

$$A(t) = \frac{1}{D} \left( \frac{GM}{c^3} \right)^{5/6} \left( \frac{2\pi f(t)}{c} \right)^{5/6}$$

$$f_{rot}(t) = \frac{1}{2} \left( \frac{GM}{c^3} \right)^{5/6} \left( \frac{1}{2\pi} \left( \frac{1}{\delta} - \dot{\delta} \right) \right)^{5/6}$$

$$\Phi(t) = -2 \left( \frac{GM}{c^3} \right)^{5/6} \left( \frac{1}{\delta} - \dot{\delta} \right) t + \phi_0$$

$$M(h_{rot}, h_{nonrot}) = \frac{\int |h_{rot}(t) - h_{nonrot}(t)|^2 dt}{\int |h_{rot}(t)|^2 dt + \int |h_{nonrot}(t)|^2 dt}$$

- Our development is thus limited by the SPA, and we choose to focus on "major parameters" which are :
- $(\alpha, \delta)$  the sky position;
  - $f_{low}$  the lower frequency cutoff;
  - $M_{Chirp}$  the chirp mass;
  - $\psi$  the polarisation angle;
  - $\iota$  the source's inclination angle

While we can pretty easily understand the match's behavior when  $(M_{Chirp}, f_{low})$  vary, predicting it when  $(\psi, \iota)$  vary was not trivial. For that we thus developed equations using the previous approximations.

We introduce a complex antenna pattern function

$$F_{\nu}(n, \delta, \psi, t) = \dots$$

We can show that the match's behavior when  $(\psi, \iota)$  vary is only determined by the function L

$$L(n, \delta, \psi) = \dots$$

Then, extremizing L gives us the extrema of the match for  $(\psi, \iota)$

$$L(\psi) = \eta^2 f_{\nu}(\psi) + \epsilon^2 f_{\nu}(\psi)$$

## So, do we have to worry ?

$$(M_{Chirp}, f_{low})$$

For these parameters' impact, we fixed  $(\psi, \iota)$  to  $(0^\circ, 0^\circ)$ . Then, for each  $(M_{Chirp}, f_{low})$ , we plotted the sky map of the mismatch and saved the max value. Doing this for several  $(M_{Chirp}, f_{low})$  gives us Fig [2].

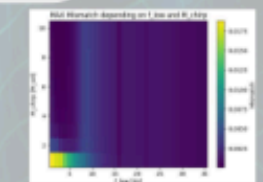


Fig 2: Mismatch map in  $(M_{Chirp}, f_{low})$  plan.

As expected, the Mismatch (1-Match) increases when  $M_{Chirp}$  and  $f_{low}$  decrease, because the signal becomes exponentially longer.

$$f_{low}(t_c - t) = \frac{1}{2} \left( \frac{GM}{c^3} \right)^{5/6} \left( \frac{5}{256} (t_c - t) \right)^{5/6}$$

While we can see in Fig. [2] that the mismatch does not exceed 1%, we can however see that if we change  $\iota$  to be at  $90^\circ$  we have high mismatch up to  $\sim 30\%$  as shown in Fig. [3]. These high values show that we have to study how the  $(\psi, \iota)$  angles affect the mismatch.

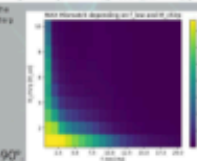


Fig 3: Mismatch map in  $(M_{Chirp}, f_{low})$  plan for  $\iota=90^\circ$ .

## How often would we have such extreme cases ?

We ran a Monte-Carlo simulation on  $(\alpha, \delta, M_{Chirp}, \psi, \iota)$  to see the mismatch's distribution over 3.000.000 samples. The Fig [6] shows an histogram of the mismatch's distribution.

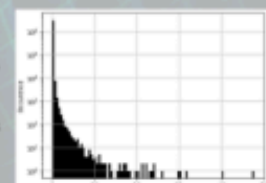


Fig 6: Mismatch distribution

If we define a "danger" threshold to 5% of mismatch, the MC simulation gives a probability to have such an event of... **0.00005%!** In other terms, if we detect  $\sim 70,000$  BNS per year, **only 4** would have such a mismatch!

However, some signals are heavily modulated... Can this affect their SNR ? If so, will we be able to still detect them ? To be continued.



Fig 7: Example of a heavily modulated signal. The blue one does not take into account the Earth's rotation.

$$(\psi, \iota)$$

We have a great variety of mismatch sky maps depending on the value of these two parameters (see Fig. [4]).

To determine the worst case, we use the equations we developed earlier.

- The results are:
- $\psi = 0$  or  $90^\circ$
  - $\iota = 0^\circ$

In that case, the mismatch goes up to 50%!

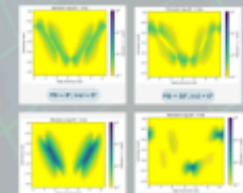


Fig 4: Four mismatch sky maps for different  $\psi$  and  $\iota$ .

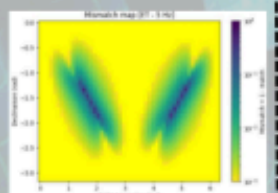


Fig 5: Mismatch sky map for  $\psi=0^\circ$  and  $\iota=90^\circ$ .

## Conclusion

Our study showed that the detection of long-duration signals will, overall, not be affected by the Earth's rotation induced amplitude modulation.

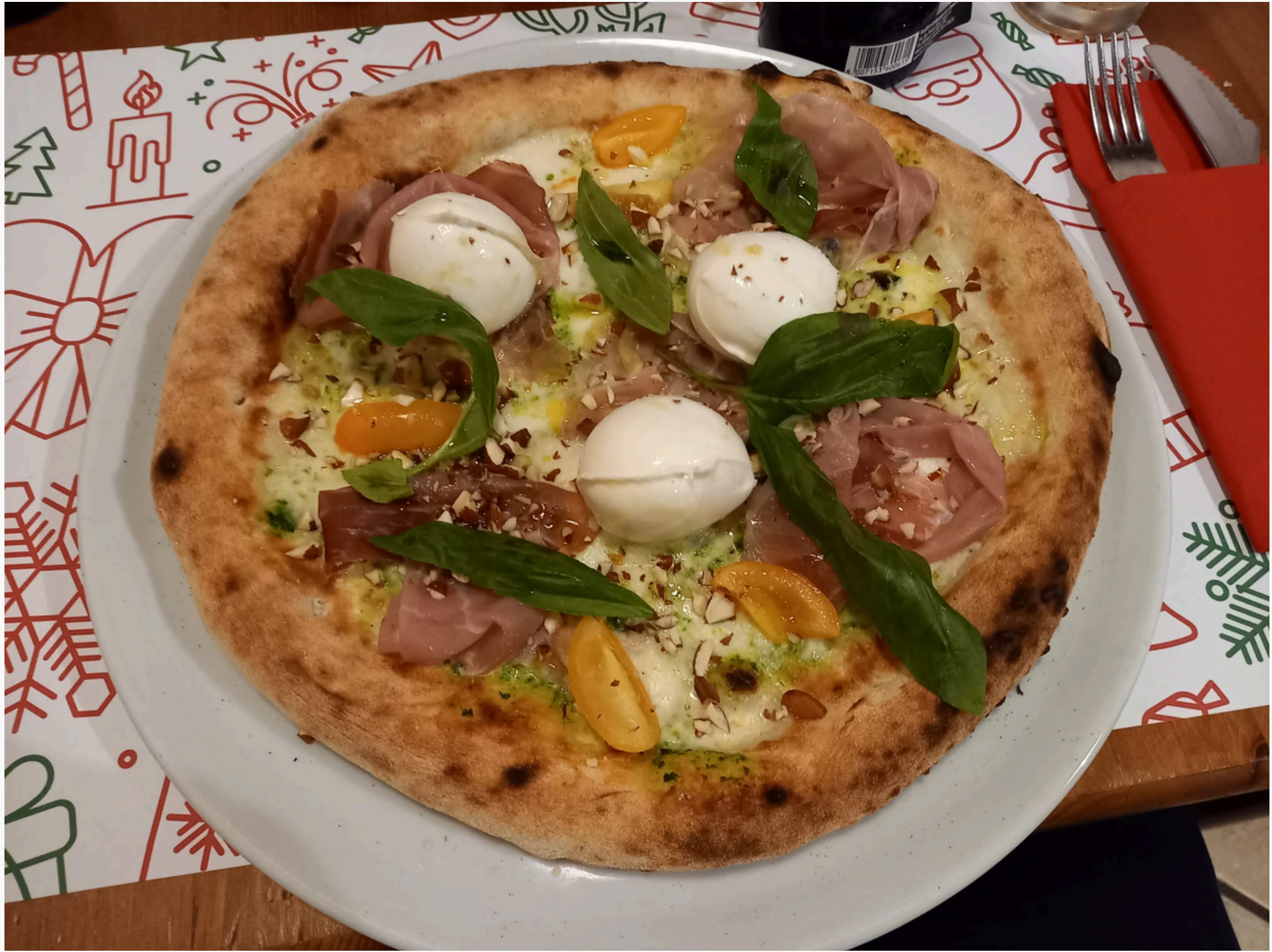
Depending on the "danger" threshold, one could however develop a method to compensate such an effect

Then, the next step of our study will be to quantify how much the SNR will be affected in cases such as Fig [7].

## Sources

- [1] The Science of the Einstein Telescope <https://arxiv.org/abs/2503.12263>
- [2] M. Pillas, "Exploring the physics of neutron star mergers with gravitational waves and gamma-ray bursts", 2023





## Contributions scientifiques

Calibration meetings

Electronics meeting

Optics meetings

MBTA meetings  
(Virgo)

Div10 ET meetings  
(data analysis)

## Next works

Study of MBTA for ET-like signals

Speed up the  $h(t)$  reconstruction algorithm

????

Virgo calibration for autumn observation run

## Divers

48h of teaching  
(Optics, Math, EM)

~40h ED formation:

- 15h Scientific journalism
- 15h Research ethics
- 4h VSS

## Pint of Science

ILE Project

Fête de la Science

End of the year Party organisation

Lab visits x7 (secondary school, high school,  
students, public)



*Thank you !*





# *Backup*

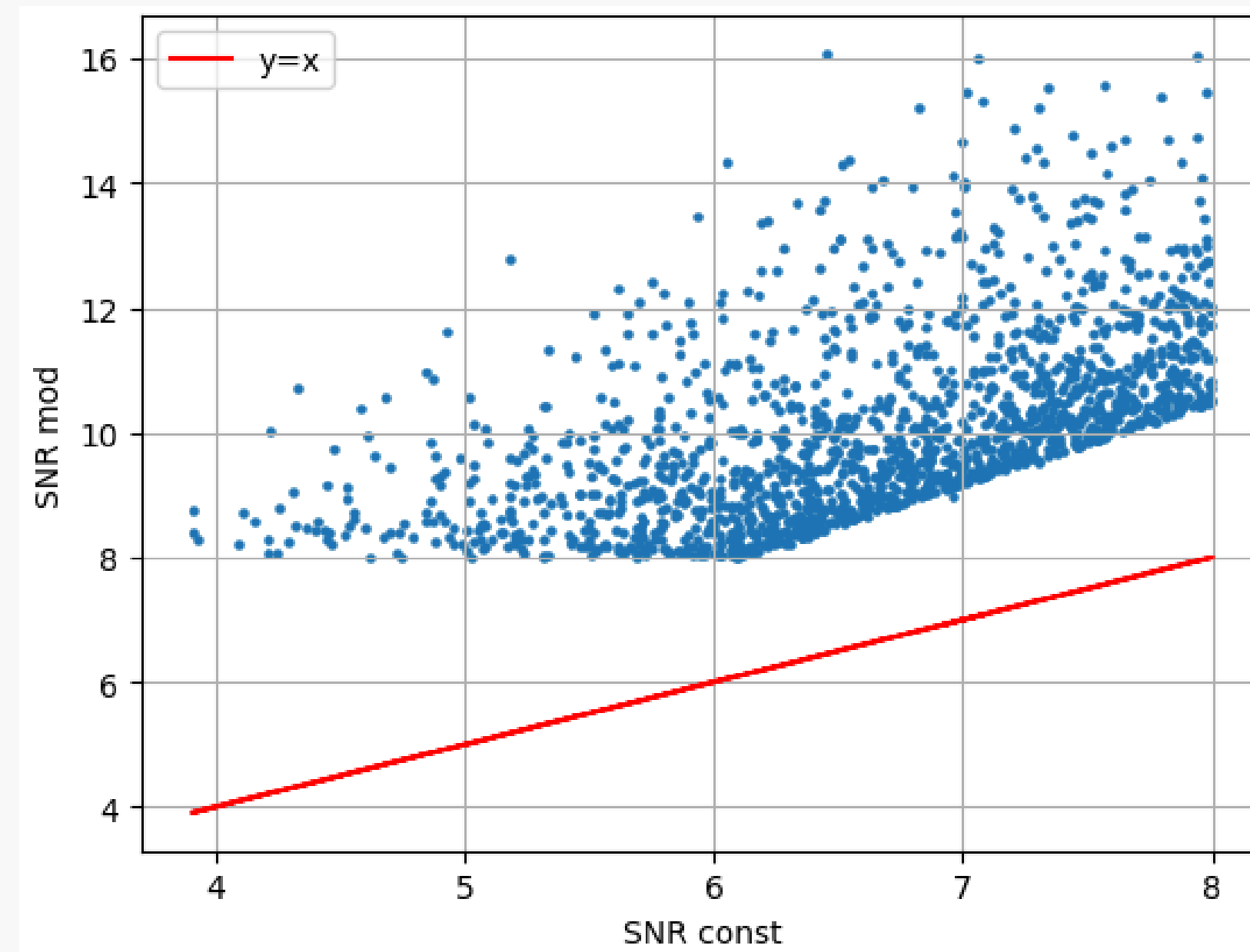
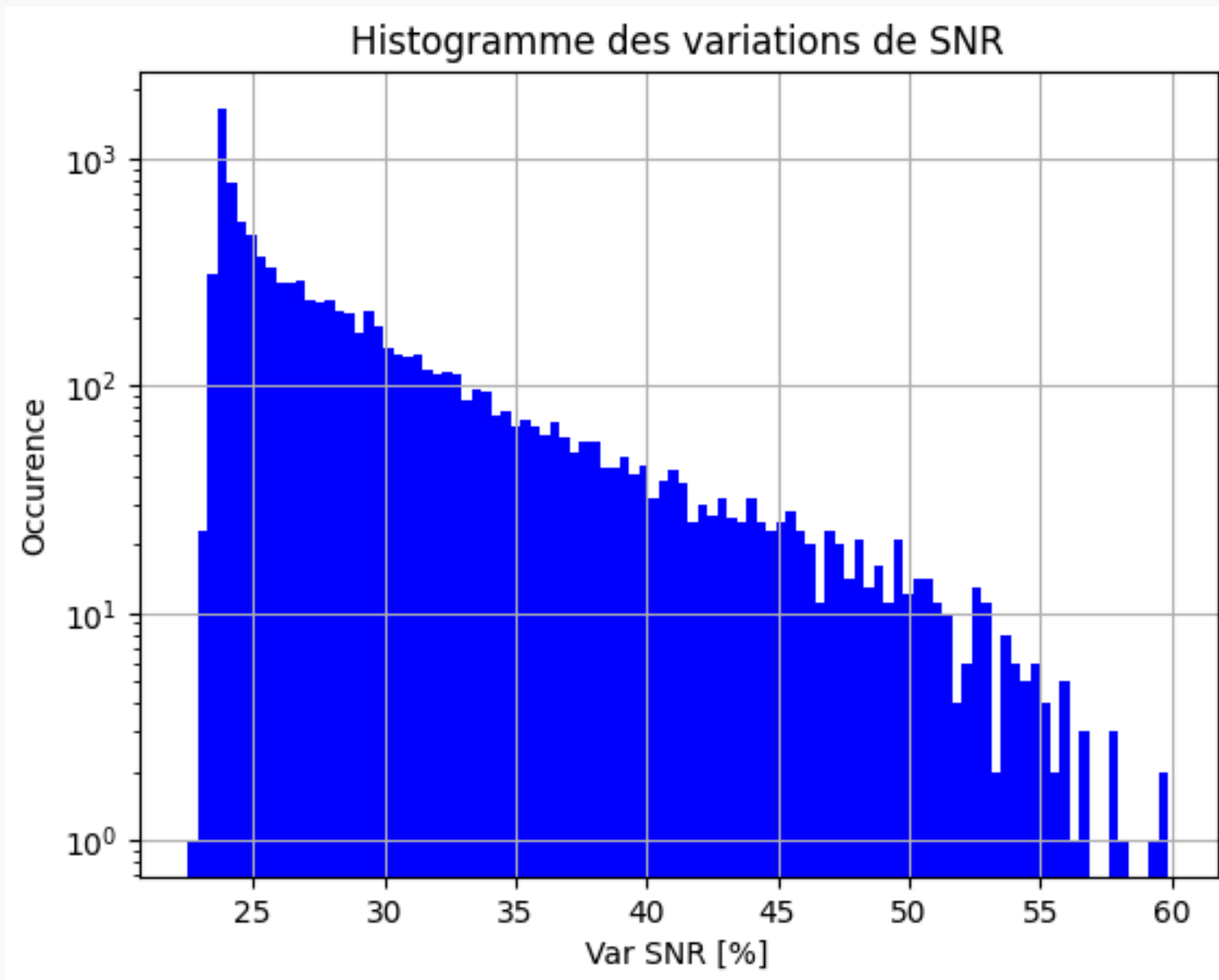


$$\begin{aligned}\mathcal{M}(\alpha, \delta, \psi) &\propto \sqrt{A + \mathcal{B} \sin(4\psi + \Phi_1) + \mathcal{C} \sin(8\psi + \Phi_2)} + \sqrt{A - \mathcal{B} \sin(4\psi + \Phi_1) + \mathcal{C} \sin(8\psi + \Phi_2)} \\ &= f_+(\psi) + f_\times(\psi)\end{aligned}$$

$$\mathcal{L}(\psi) = \eta^2 f_+(\psi) + \xi^2 f_\times(\psi)$$

$$F_{+,ET}(\alpha, \delta, \psi, t) = \sqrt{\sum_{i=1}^n F_{+,i}^2(\alpha, \delta, \psi, t)}$$

$$F_{\times,ET}(\alpha, \delta, \psi, t) = \sqrt{\sum_{i=1}^n F_{\times,i}^2(\alpha, \delta, \psi, t)}$$



~ 25% de TOUTES les sources peuvent être récupérées

Match Filtering Output (complexe)

$$\rho_{const} = \frac{\langle h, T \rangle}{\|T\|}$$

Cauchy-Schwarz

$$|\langle h, T \rangle| \leq \|h\| \cdot \|T\|$$

On perd nécessairement du SNR

$$\rho_{const} \leq \|h\| = \rho_{opt,mod}$$

Faisons une Monte-Carlo !

Codé en Python  
(PyCBC)

Pour chaque point (RA, DEC) dans le ciel

Génération de  $h(t)$  (+ et x)

Calcul de  $F(t)$  et  $F(0)$  (+ et x)

Calcul de  $F(t)h(t)$  et  $F(0)h(t)$

Calcul du Mismatch entre  
 $F(t)h(t)$  et  $F(0)h(t)$

$$\mathcal{M}(s, T) = \max_{\Delta t, \Delta \phi} \frac{\langle s, T \rangle}{\|s\| \cdot \|T\|}$$

Cas simple  
(2h)



$$M_1 = M_2 = 1.4 M_{\odot}$$

$$f_{\text{low}} = 5 \text{ Hz}$$

WF model:  
IMRPhenomD

$$\text{Distance} = 100 \text{ Mpc}$$

Pas de spin

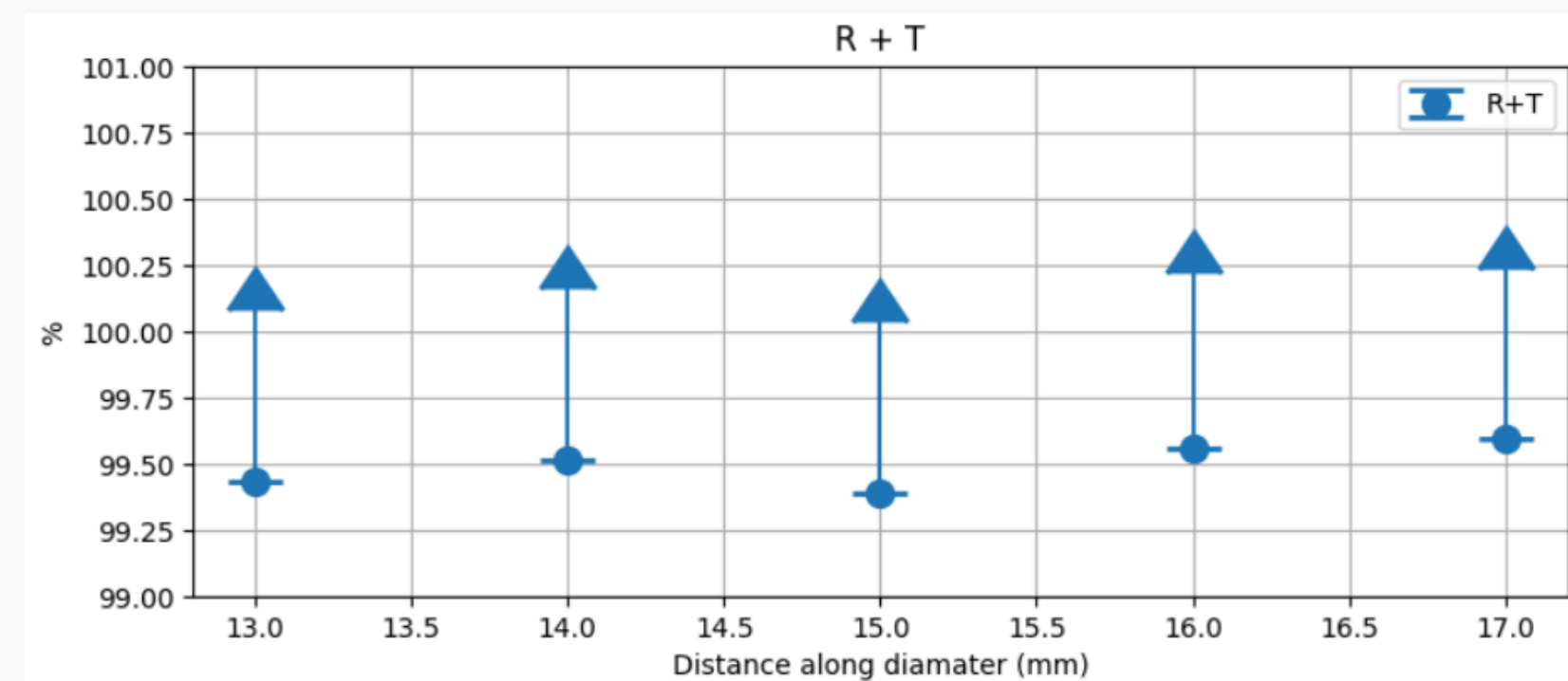
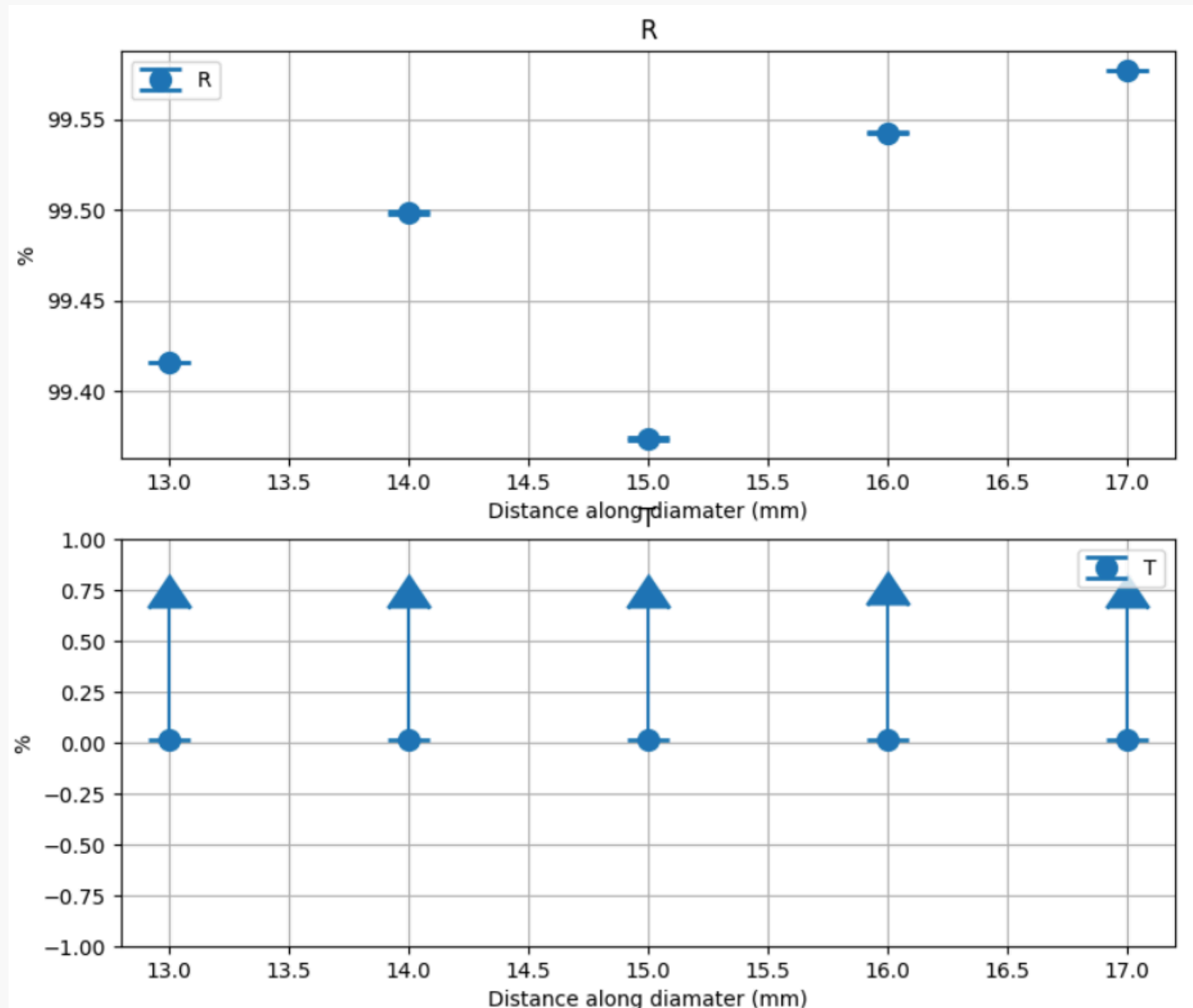
$$\text{Inclinaison} = 0^{\circ}$$

$$\text{PSI} = 0^*$$

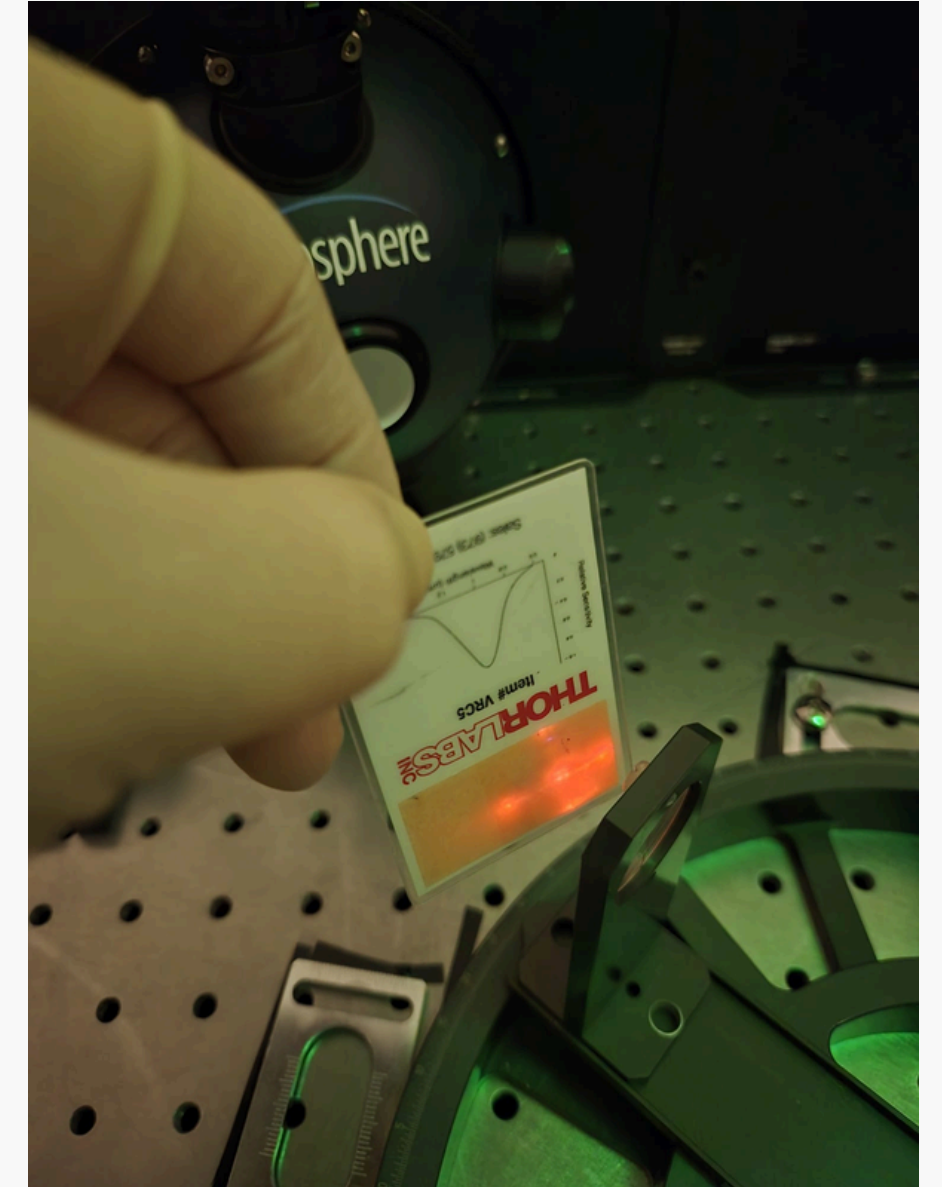
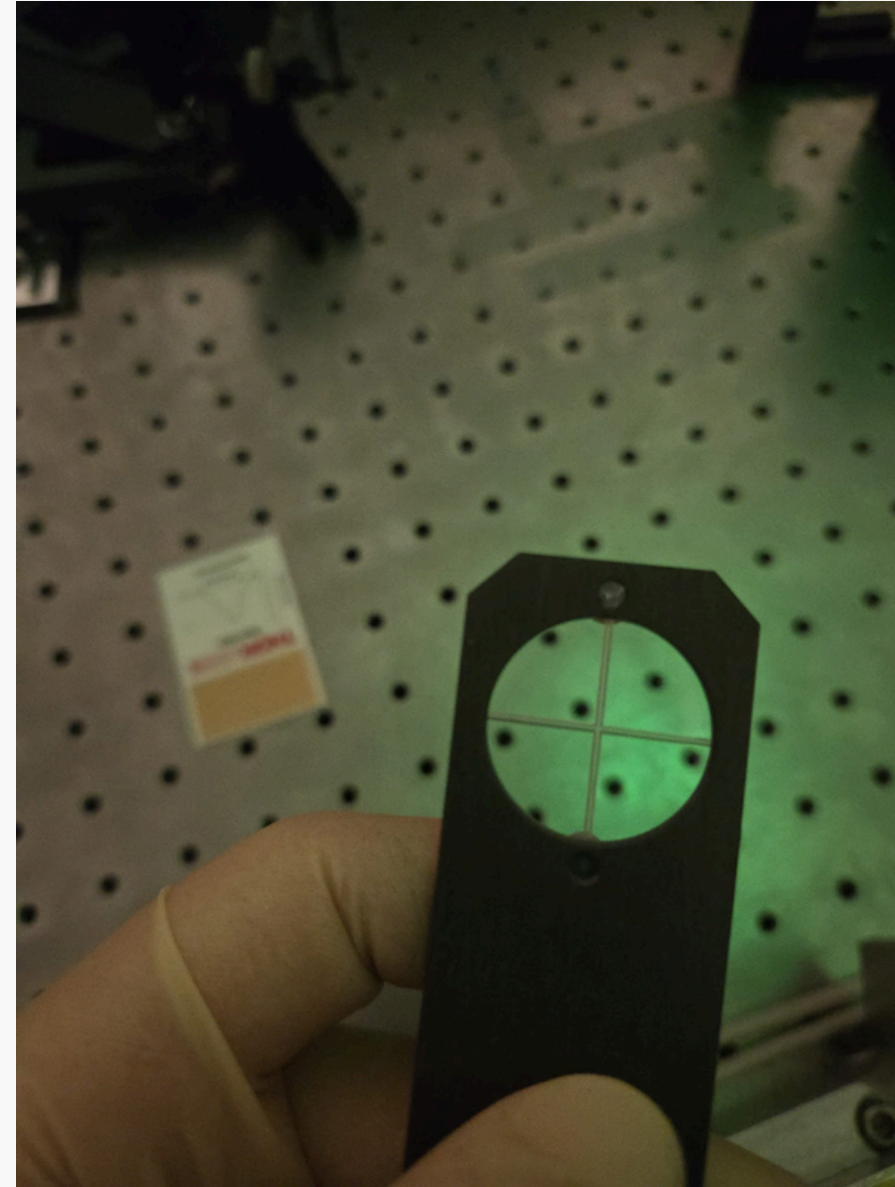
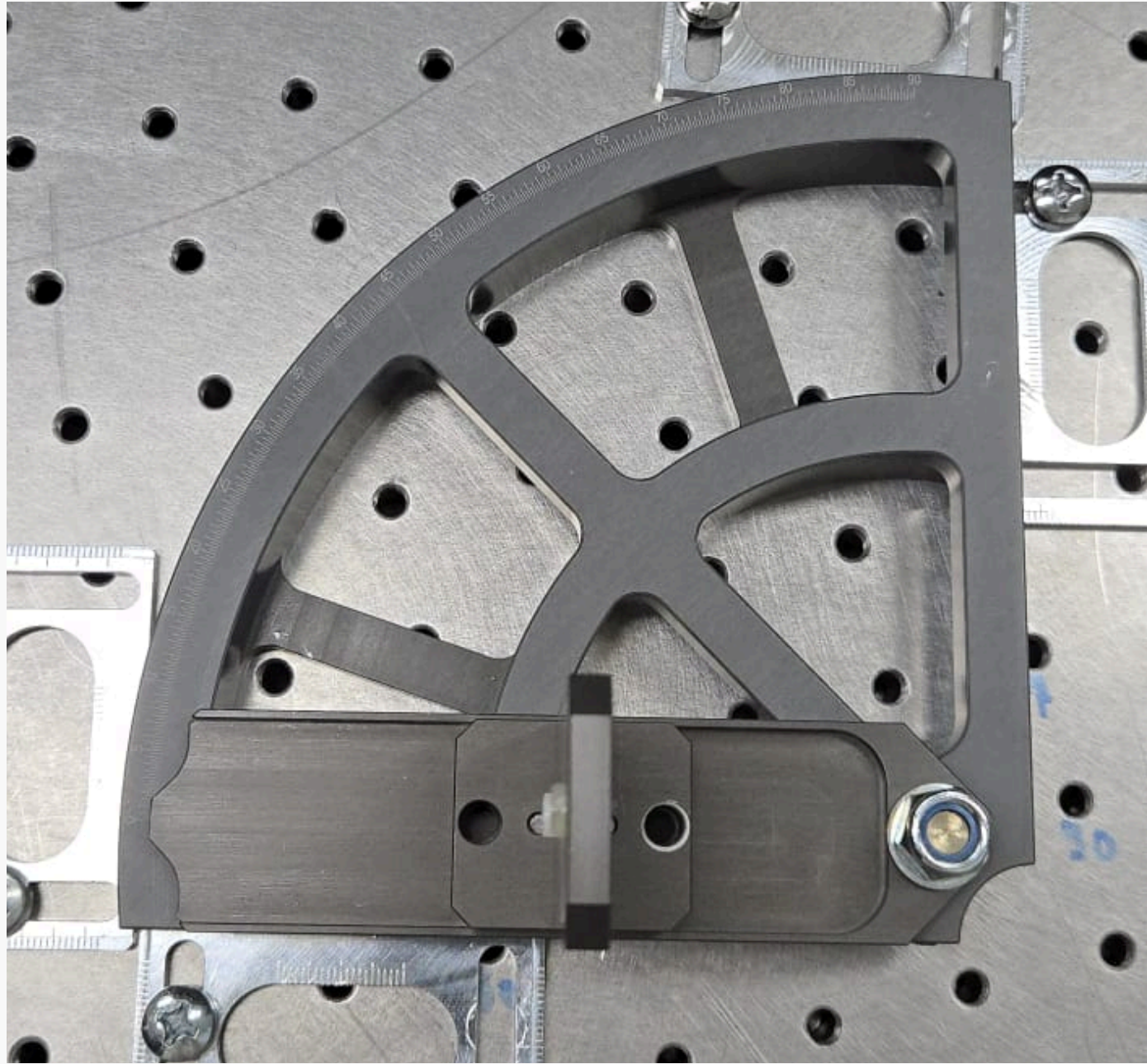
\*Contribution égale de  $F_+$  et  $F_x$   
dans le Match

Sphère T voit ~25% de la  
lumière transmise





Supposant que la sphère voit  
~25% de la lumière transmise



## Mesures

Measures jusqu'à 3W dans la sphere (1.5W chacun)

Mettre les lasers à 1.5W:

- Seulement laser 1
- Seulement laser 2
- Deux faisceaux

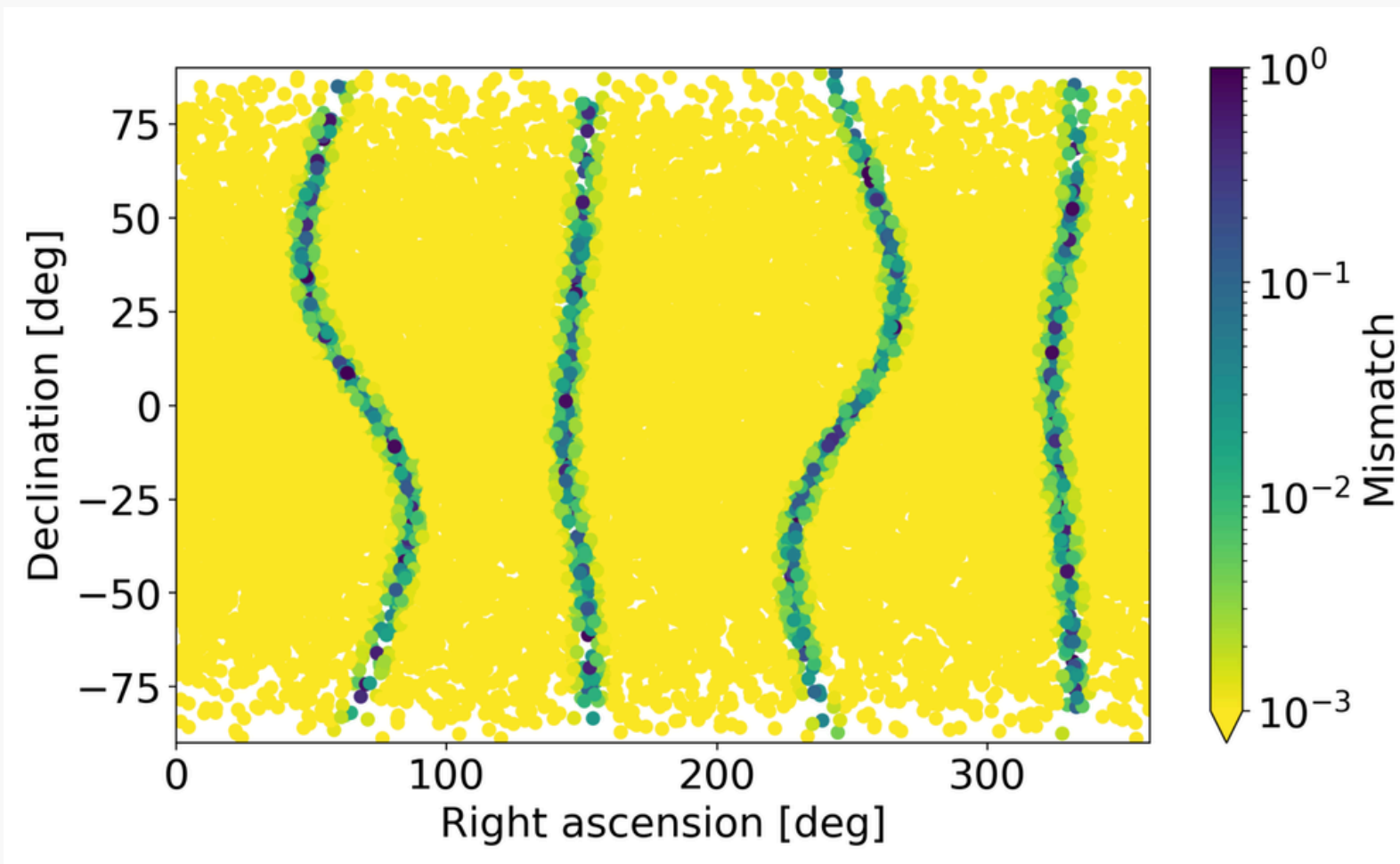
Recommencer en descendant  
la puissance à 0W

Recommencer en montant à  
1.5W

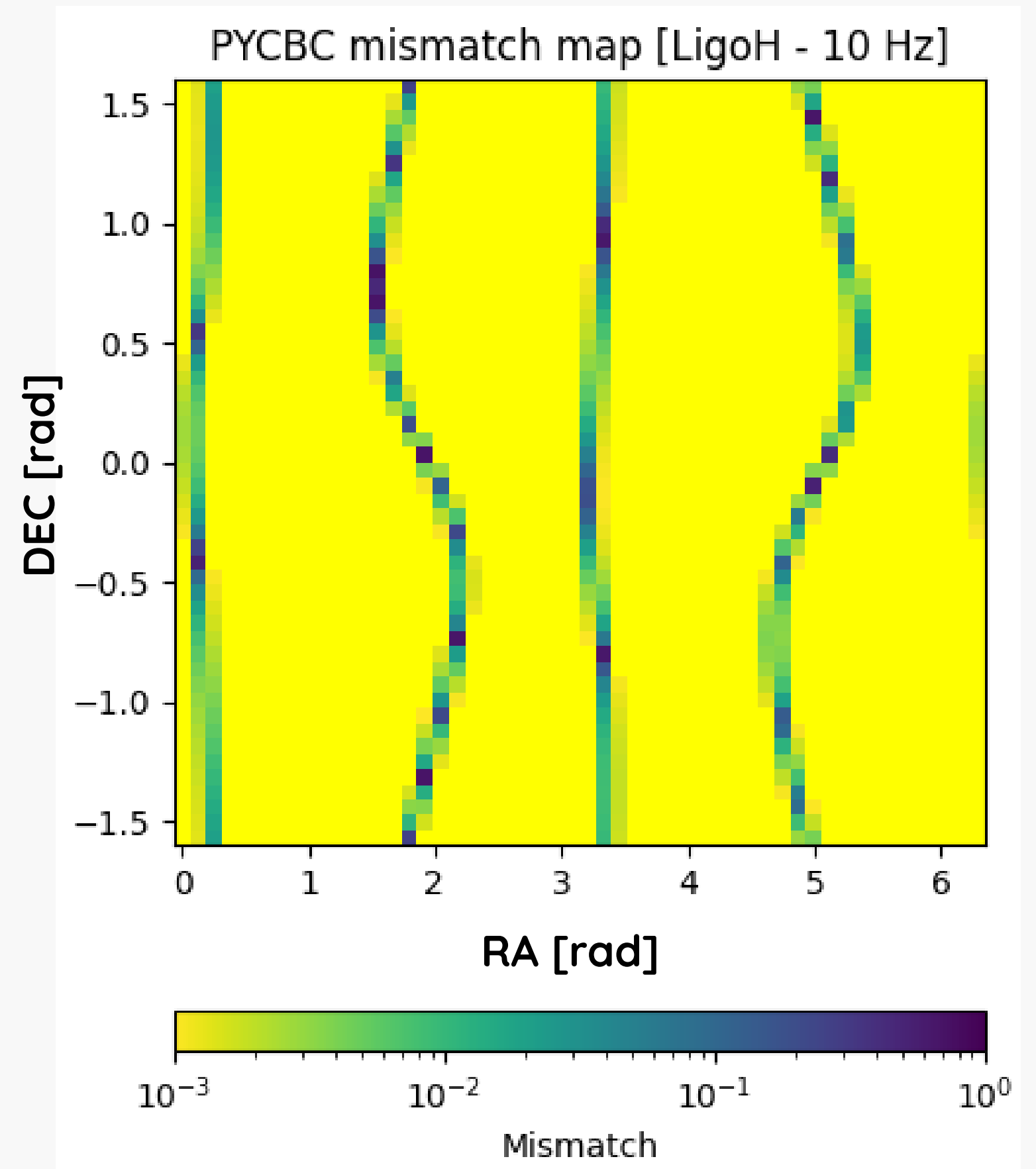
Faire plusieurs cycles

(mise en lumière d'éventuels effets tels la mémoire optique, paramètres variant avec le temps, etc...)

# Comparison avec ma méthode

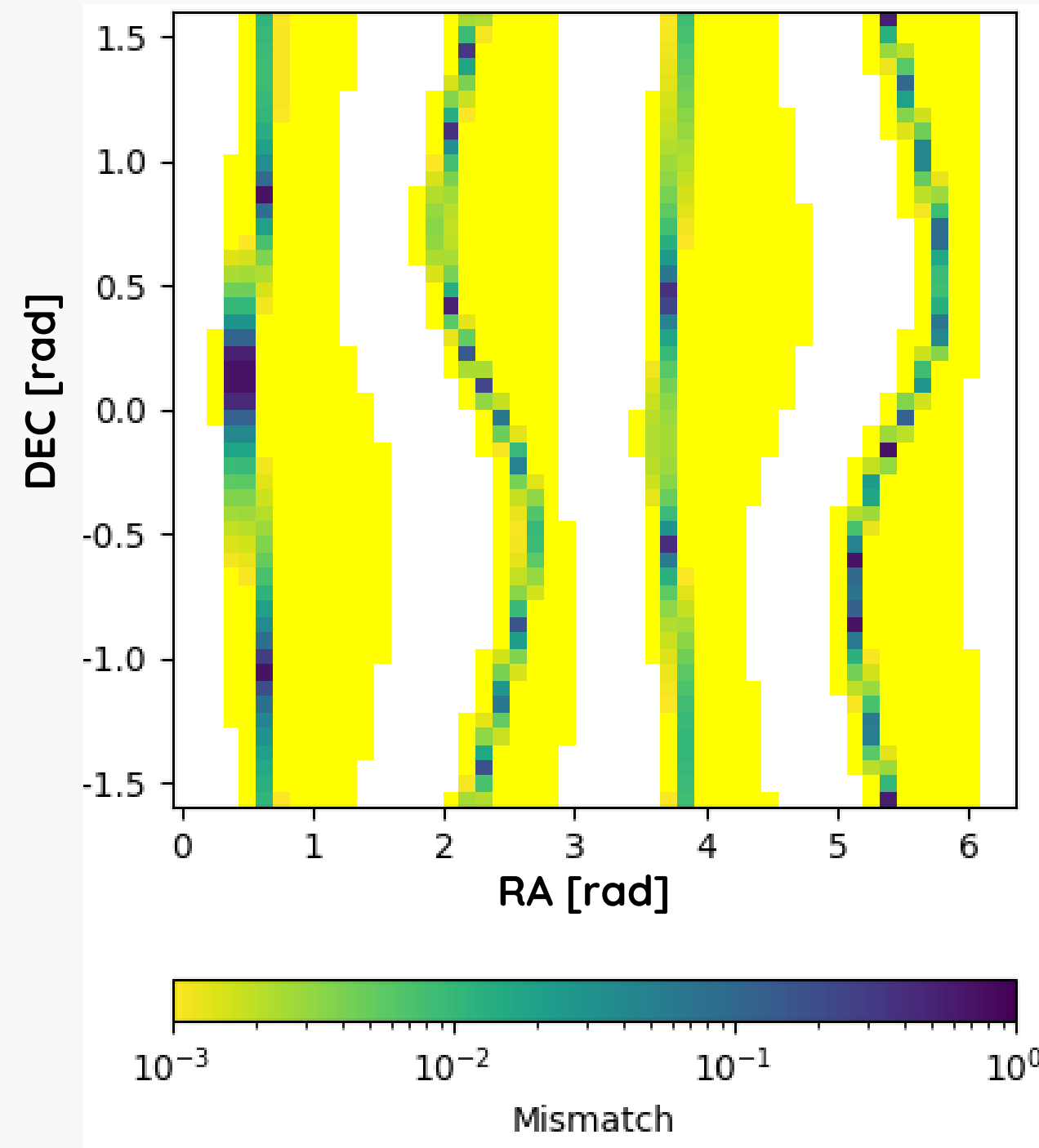
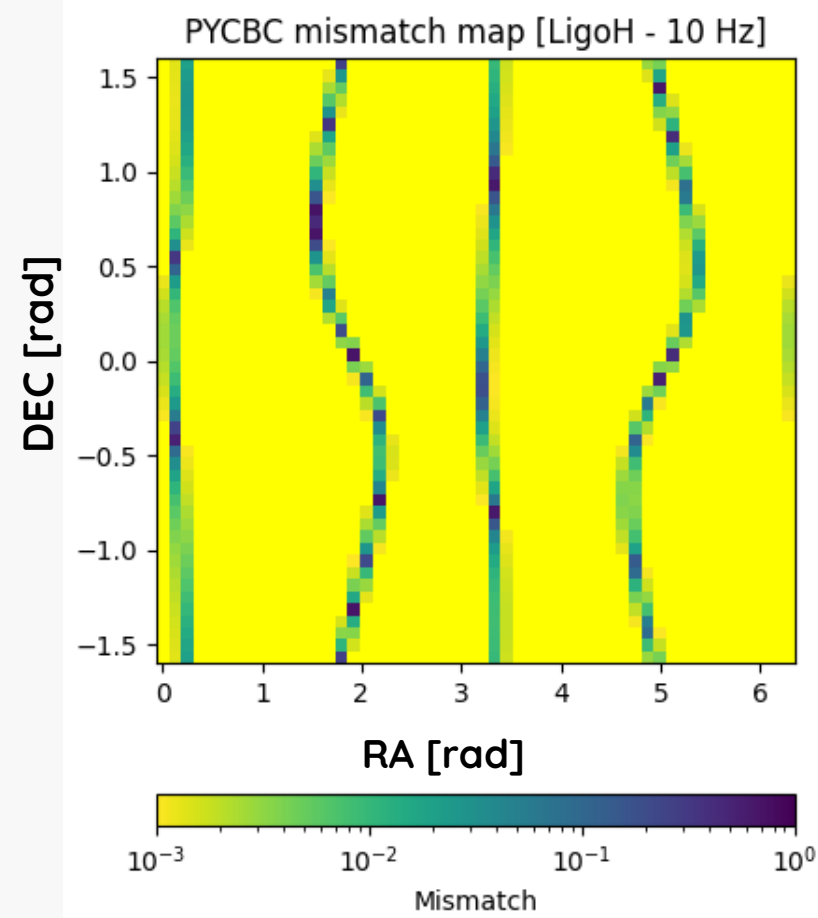
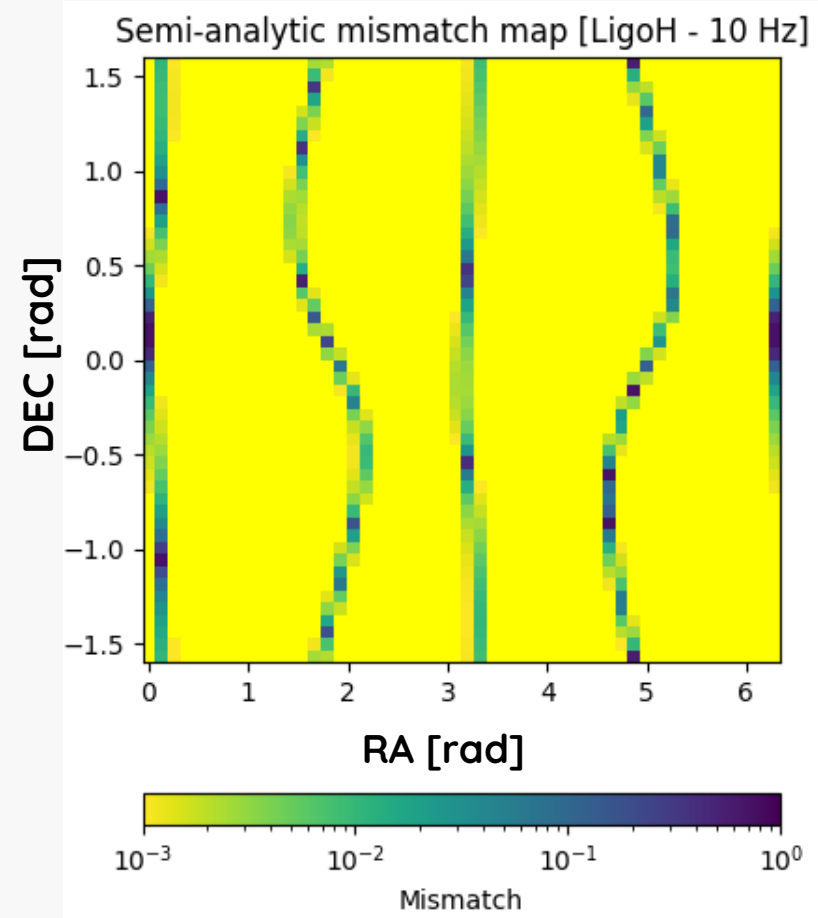


LigoH,  $f_{\text{low}} = 10$  Hz, inclination =  $90^\circ$   
 $M_{\text{chirp}} = 1.2 M_{\text{sol}}$

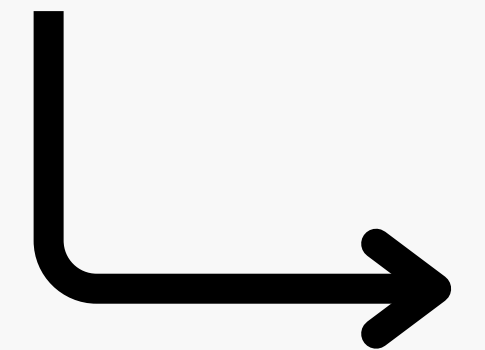


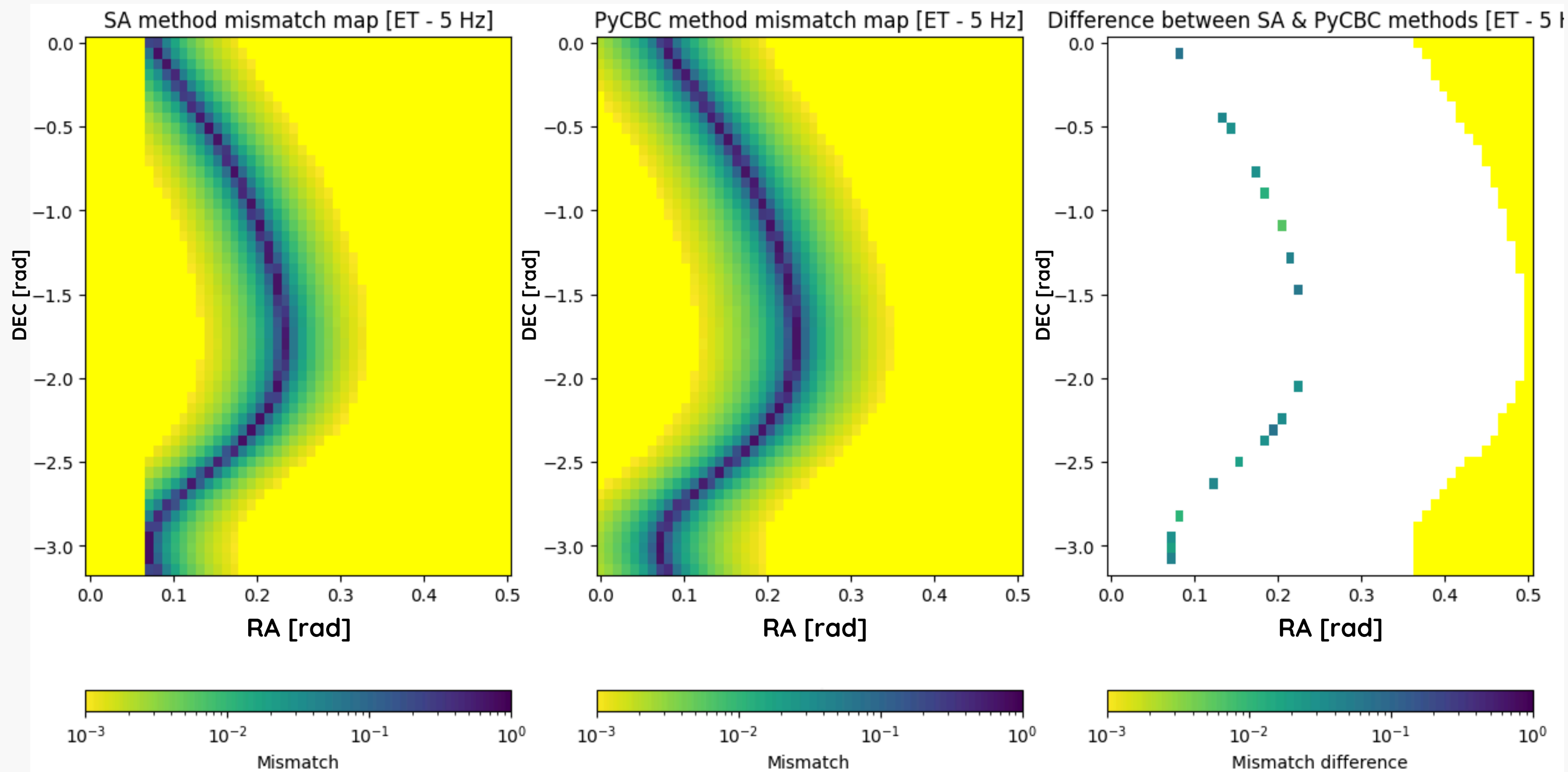
# Différence entre méthodes SA et PyCBC

SA - PyCBC



Causées par faible résolution de la carte





**Zoomer retire les divergences**

**Simulation de la même physique**

## Match Filtering Output (complex)

$$z(t_s) = 4 \int_0^\infty df \frac{\tilde{h}_{\text{data}}(f) \tilde{h}_{\text{template}}^*(f, t_s = 0, \phi_0 = 0)}{S_n(f)} e^{2\pi i f t_s}$$

SNR optimal

$$\rho_{\text{opt}}^2 = 4 \int_0^\infty df \frac{|\tilde{h}(f)|^2}{S_n(f)}$$

SNR (Time Serie)

$$\rho(t_s) = \frac{|z(t_s)|}{\rho_{\text{opt}}}$$

(Template's optimal SNR)

SNR (Time Serie)

$$\rho(t_s) = \frac{|z(t_s)|}{\rho_{opt}}$$

(Template's optimal SNR)

SNR

$$\rho = \max_{t_s} \rho(t_s)$$

SNR Total pour ET

$$\text{SNR}_{\text{tot}}^2 = \rho_1^2 + \rho_2^2 + \rho_3^2$$

$\rho_{const}$

$$\rho[F(t)h(t), F(0)h(t)]$$

$\rho_{mod}$

$$\rho[F(t)h(t), F(t)h(t)] = \rho_{opt}[F(t)h(t)]$$

## Développement d'une formule SA pour calculer le match dans cette situation

$$h_{+}(t) = A(t) (1 + \cos^2(\iota)) \cos(\Phi(t))$$

$$h_{\times}(t) = A(t) 2 \cos(\iota) \sin(\Phi(t))$$

$$A(t) = \frac{4}{D} \left( \frac{GM}{c^2} \right)^{5/3} \left( \frac{\pi f_{\text{gw}}(t)}{c} \right)^{2/3}$$

$$f_{\text{gw}}(t_c - t) = \frac{1}{\pi} \left( \frac{GM}{c^3} \right)^{-5/8} \left( \frac{5}{256} \frac{1}{(t_c - t)} \right)^{3/8}$$

$$\Phi(t) = -2 \left( \frac{5GM}{c^3} \right)^{-5/8} (t_c - t)^{5/8} + \phi_c$$

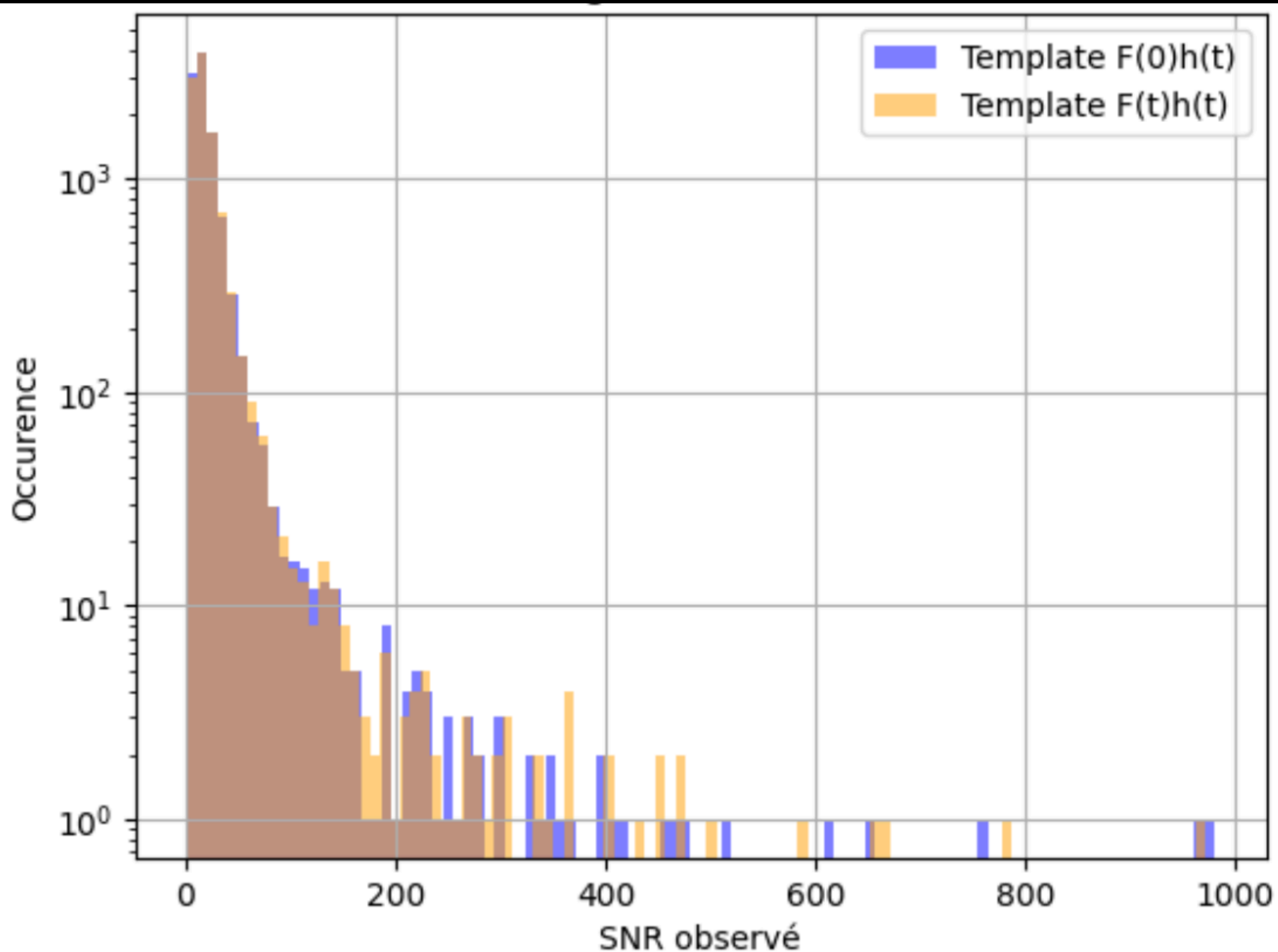
$$m(h_{\text{rot}}, h_{\text{const}}) = \max_{\Delta t_c, \Delta \phi_c} \frac{\Re \int \frac{f^{-7/3} |B(t_{\text{rot}}(f))| e^{-i\theta(t_{\text{rot}}(f))} e^{i(2\pi f \Delta t_c - \Delta \phi_c)} e^{i\phi_0}}{S_n(f)} df}{\sqrt{\int \frac{f^{-7/3} |B(t_{\text{rot}}(f))|^2}{S_n(f)} df \int \frac{f^{-7/3}}{S_n(f)} df}}$$

With:

$$B(t(f)) = \sqrt{F_+^2(t_{\text{rot}}(f))(1 + \cos^2(\iota))^2 + F_\times^2(t_{\text{rot}}(f)) 4 \cos^2(\iota)}$$

$$\theta(t(f)) = \arctan \left( \frac{-F_\times(t_{\text{rot}}(f)) 2 \cos(\iota)}{F_+(t_{\text{rot}}(f))(1 + \cos^2(\iota))} \right)$$

# SNR histogram



(10.000 sources)

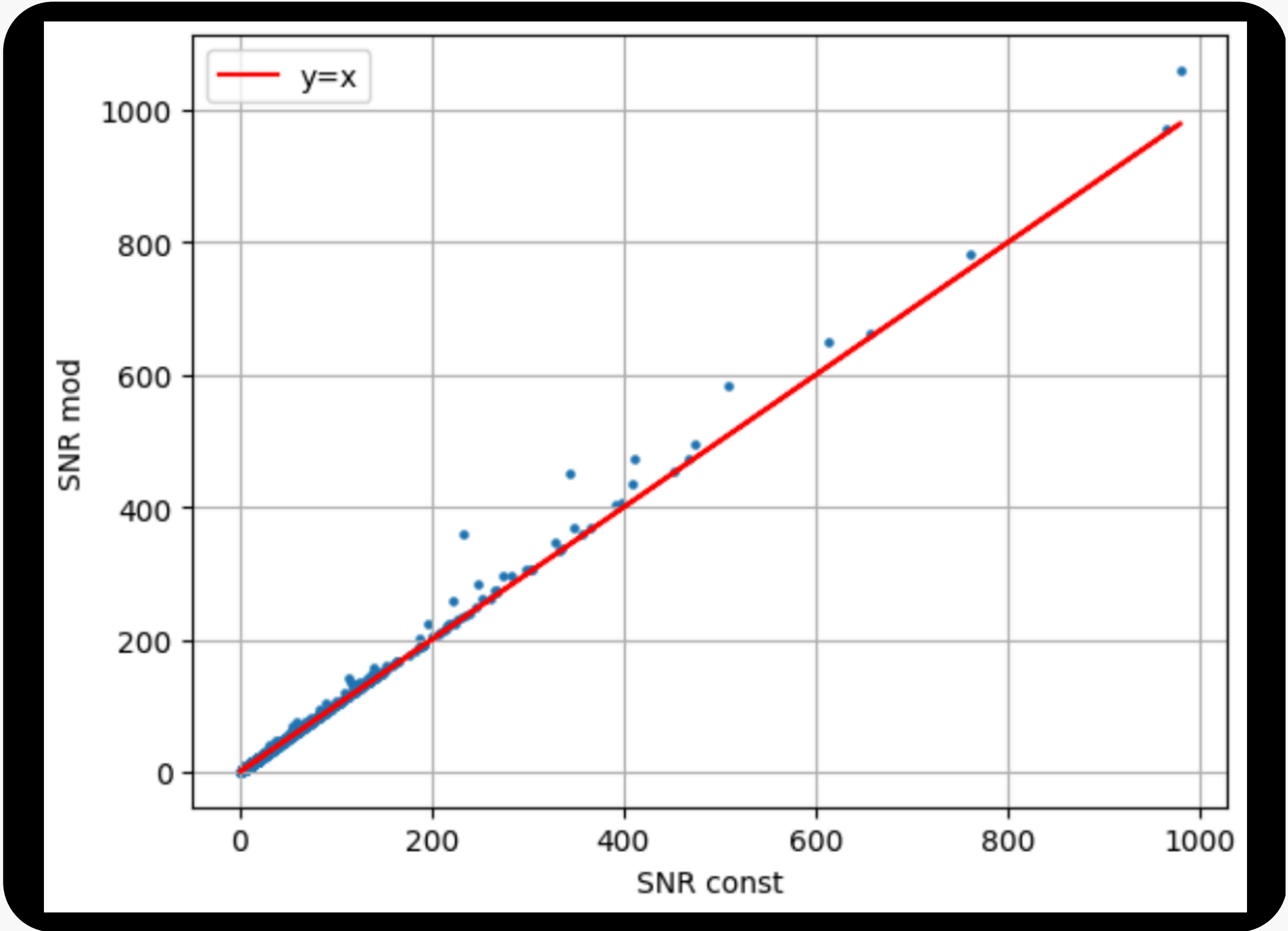
SNR entre 0 and 1000

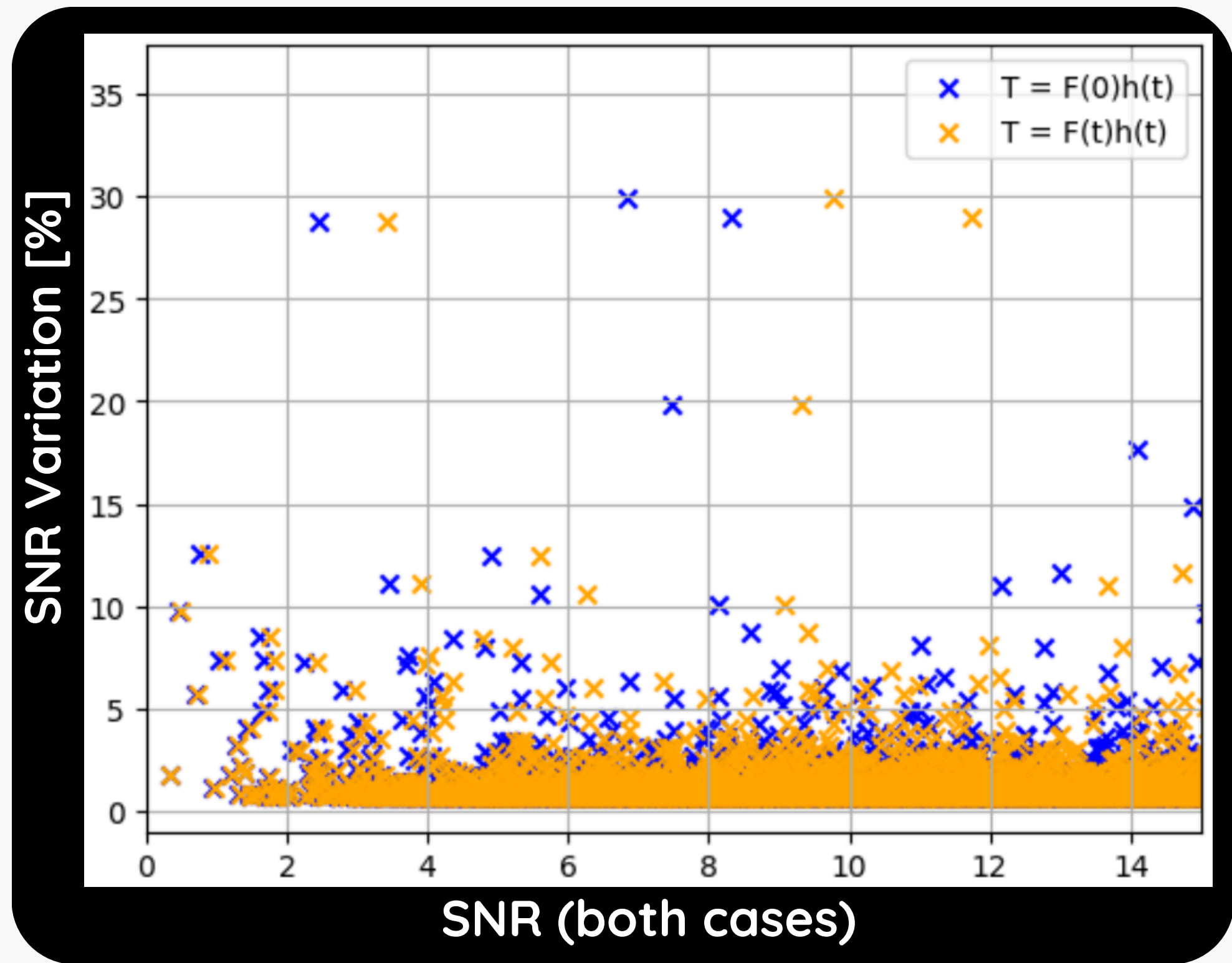
SNR moyen ~24

~75% des sources ont  
 $8 < \text{SNR} < 55$

Plus optimiste que le MDC

$\rho_{const}$  vs  $\rho_{mod}$





45 sources with  
 $SNR_{const} < 8$  BUT  $SNR_{mod} > 8$

(10.000 sources  
 9.200 detected)

$$\frac{\rho_{mod} - \rho_{const}}{\rho_{mod}} = 1 - \frac{\rho_{const}}{\rho_{mod}}$$

$$\frac{\rho_{mod} - \rho_{const}}{\rho_{mod}} = 1 - \frac{\rho_{const}}{\|s\|}$$

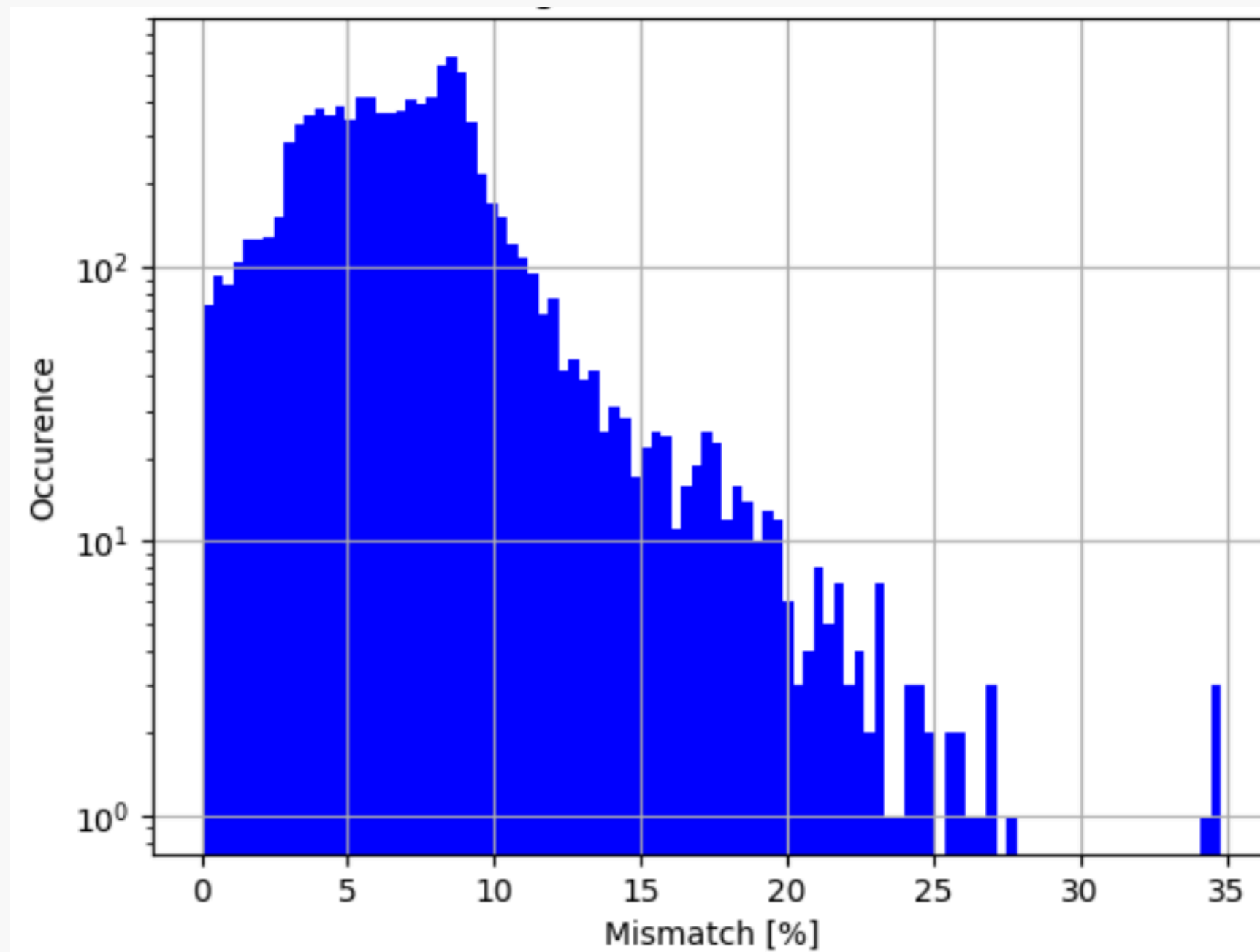
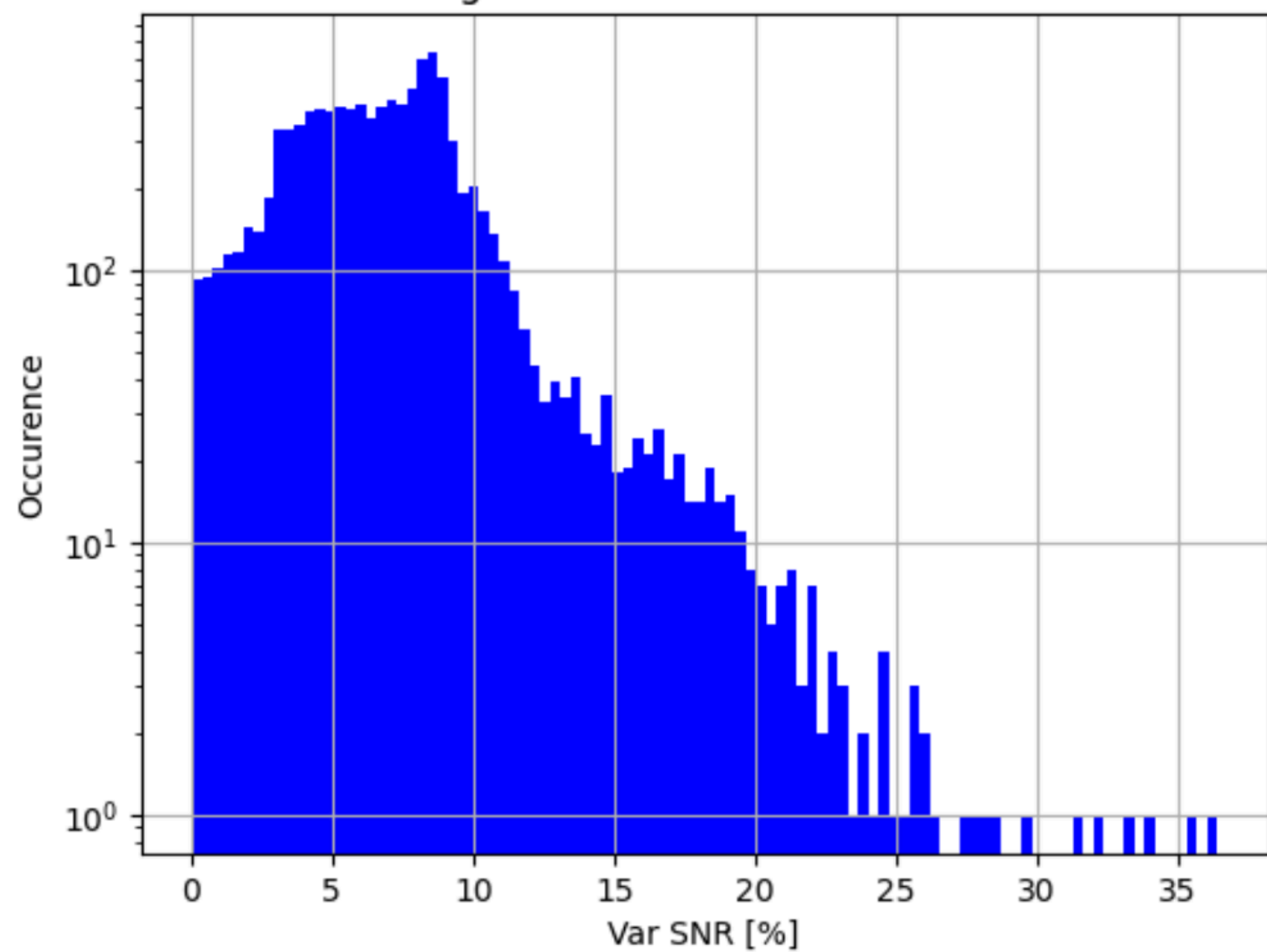
$$\mathcal{M}(s, T) = \max_{\Delta t, \Delta \phi} \frac{\langle s, T \rangle}{\|s\| \cdot \|T\|}$$

$$\mathcal{M}(s, T) \approx \frac{\langle s, T \rangle}{\|s\| \cdot \|T\|}$$

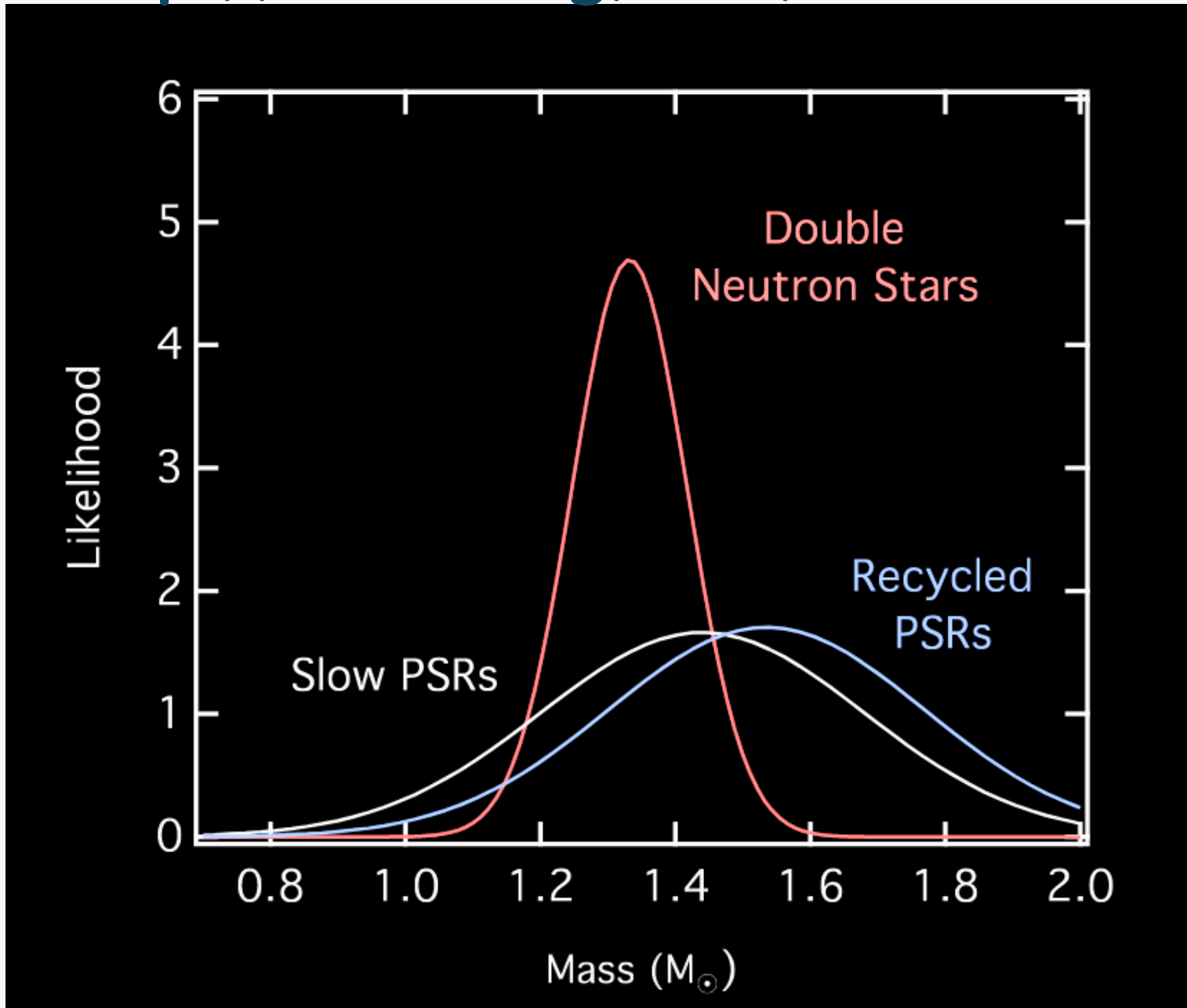
$$\mathcal{M}(s, T) \approx \frac{\rho_{const}}{\|s\|}$$

$$\rho_{const} = \frac{\langle h, T \rangle}{\|T\|}$$

Histogramme des variations de SNR

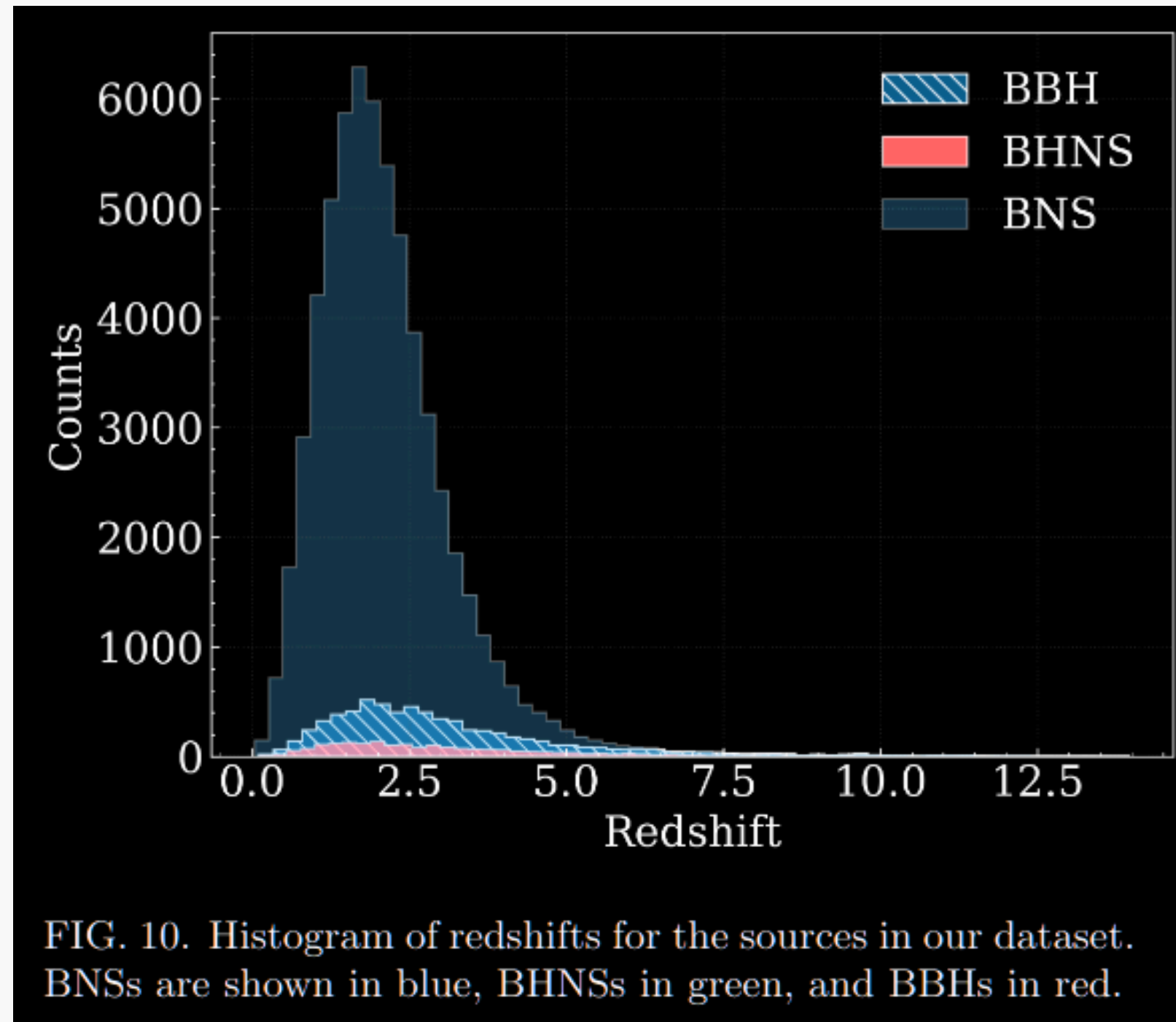


<http://arxiv.org/abs/1603.02698>

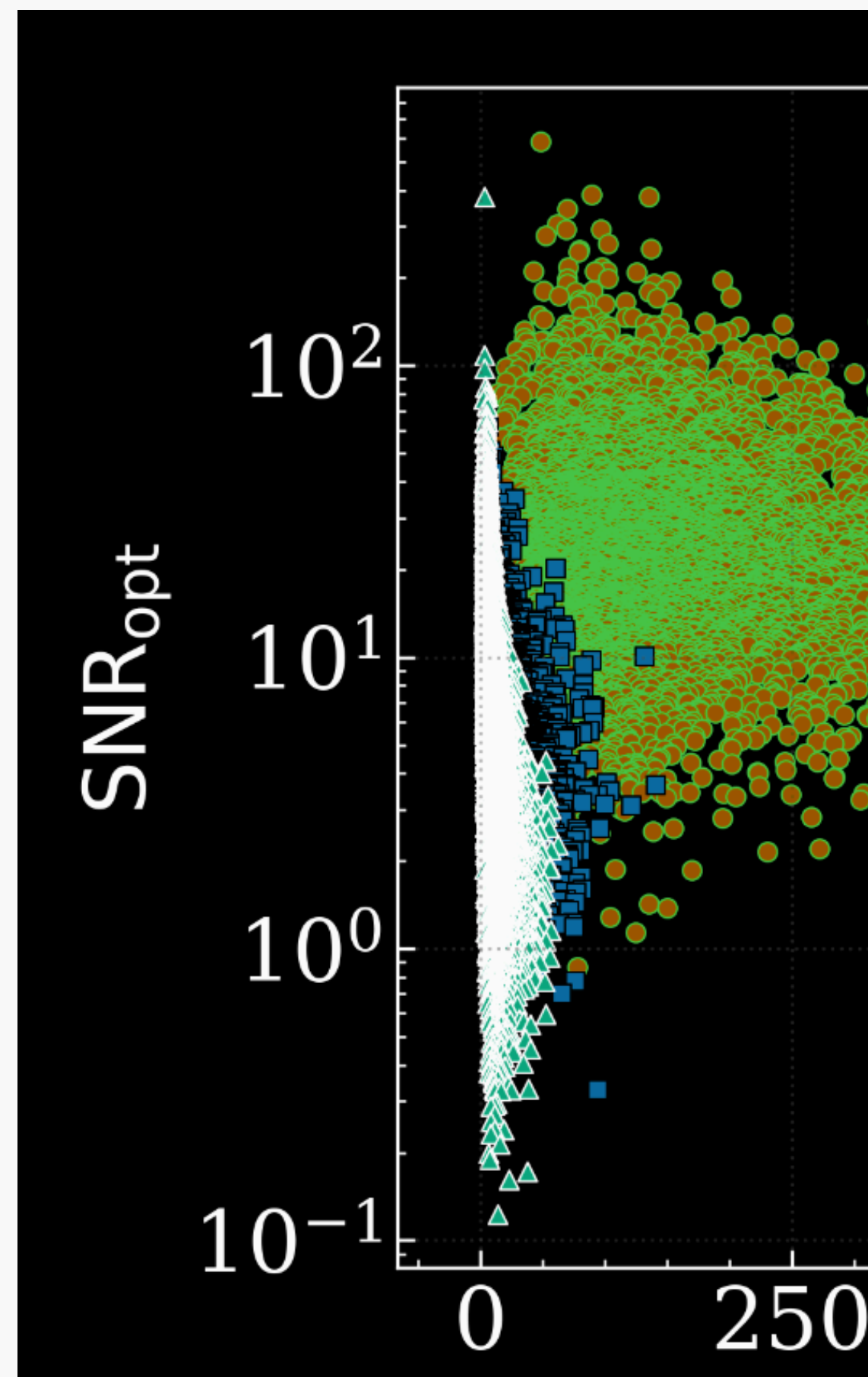
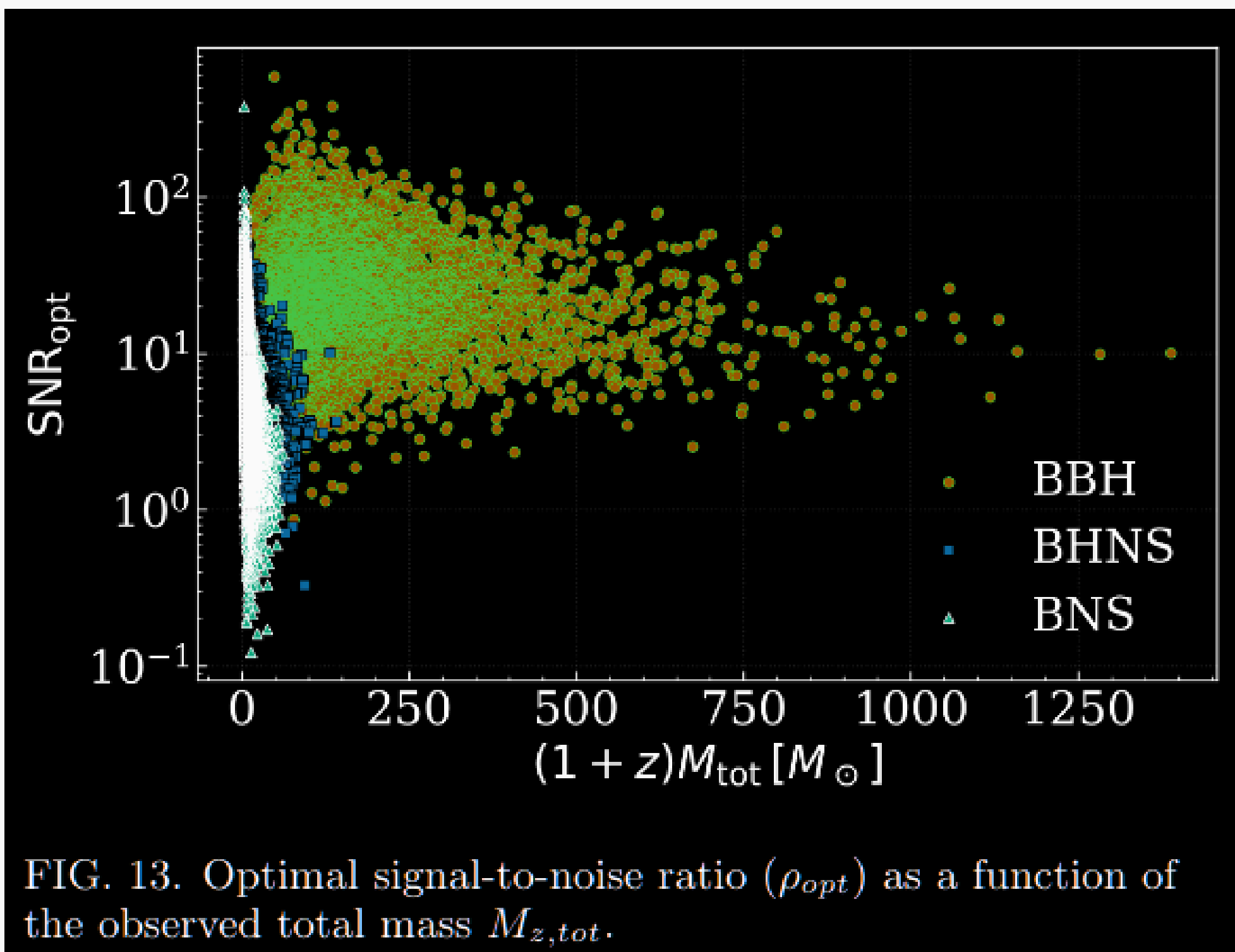


<http://arxiv.org/abs/1403.0007>

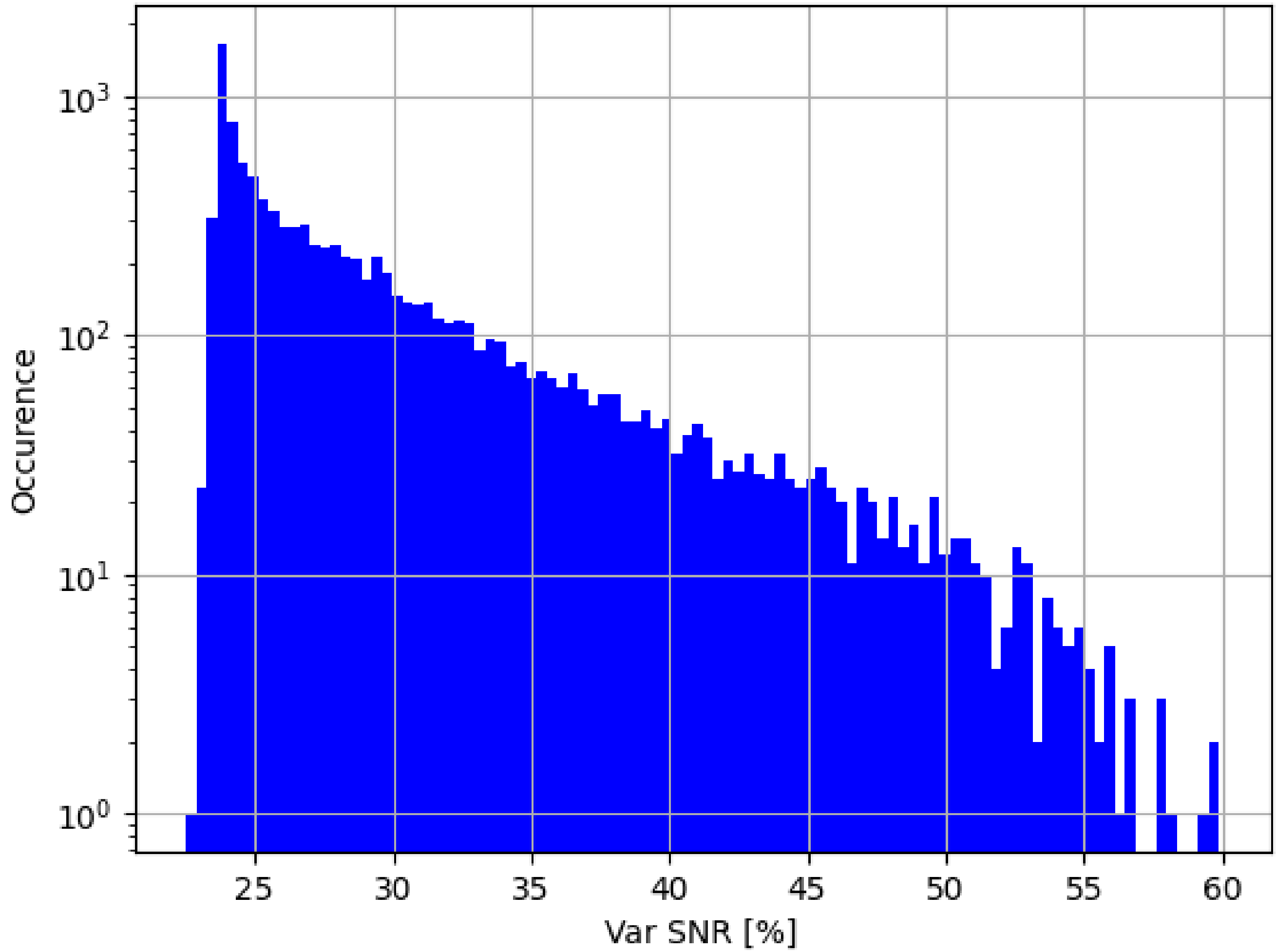
$$\psi(z) = 0.015 \frac{(1+z)^{2.7}}{1 + [(1+z)/2.9]^{5.6}} \text{ M}_{\odot} \text{ year}^{-1} \text{ Mpc}^{-3}.$$



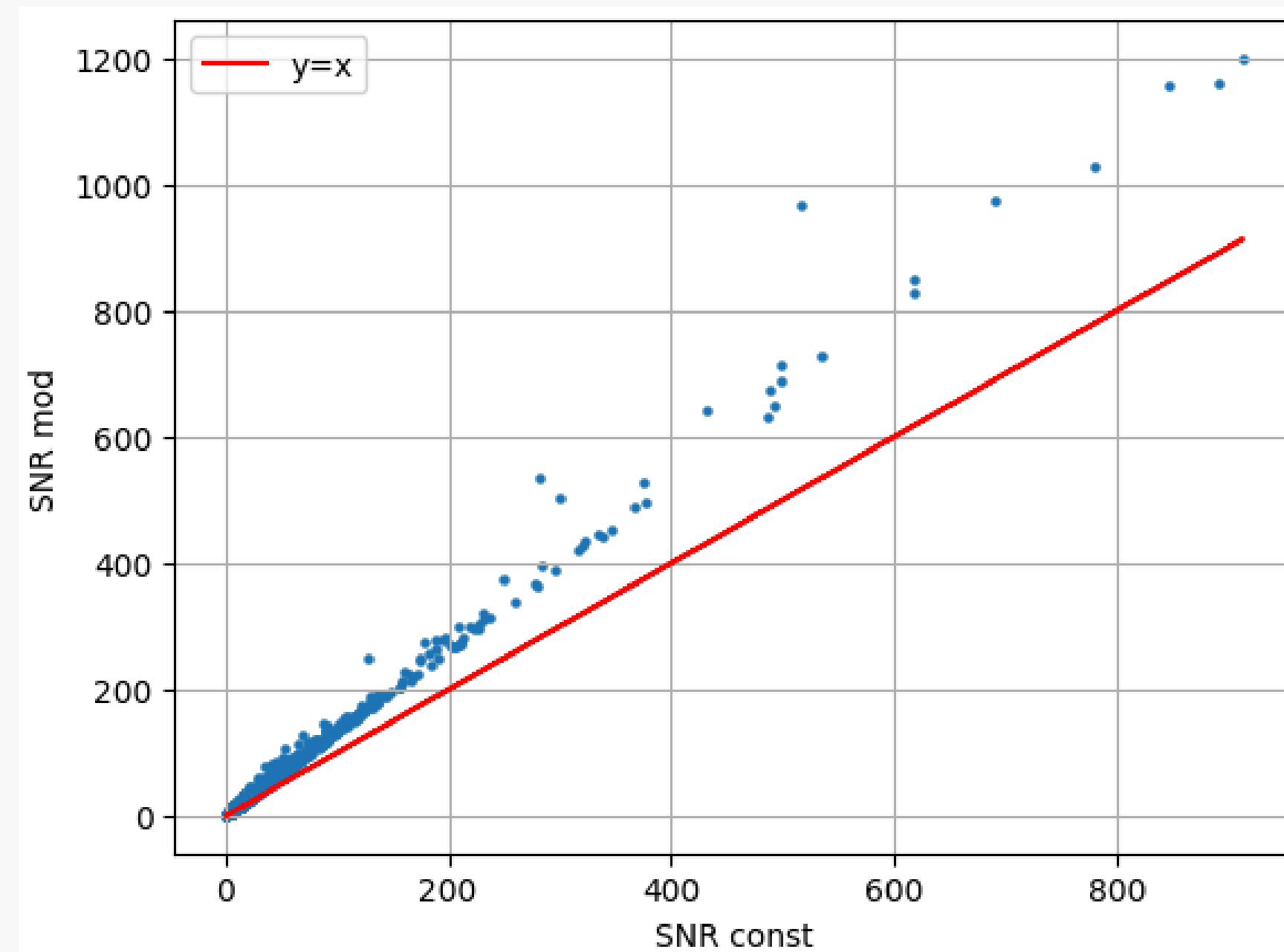
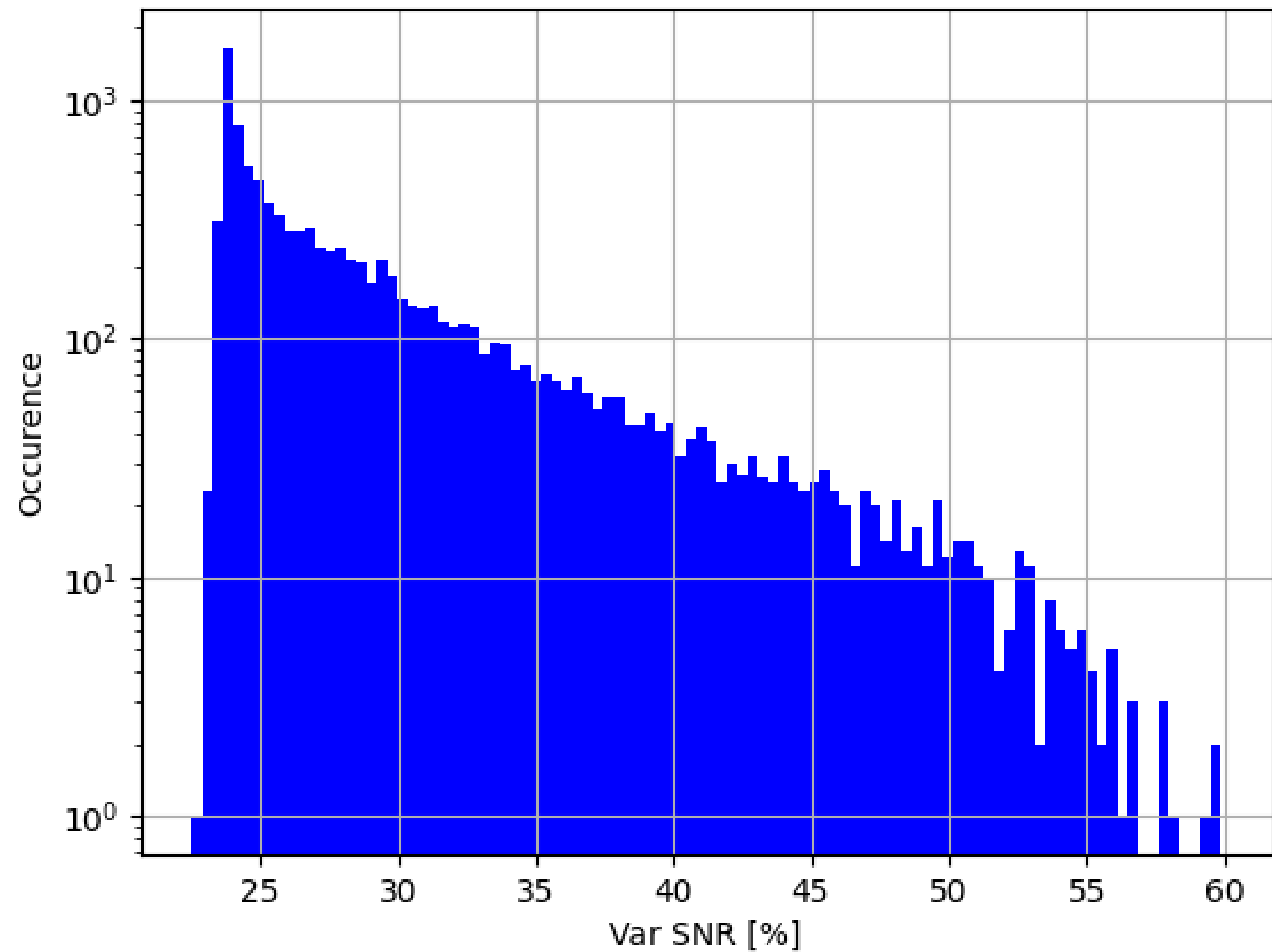
- 61031 BNSs: 19% have  $\text{SNR} > 8$ , 7% have  $\text{SNR} > 12$ .

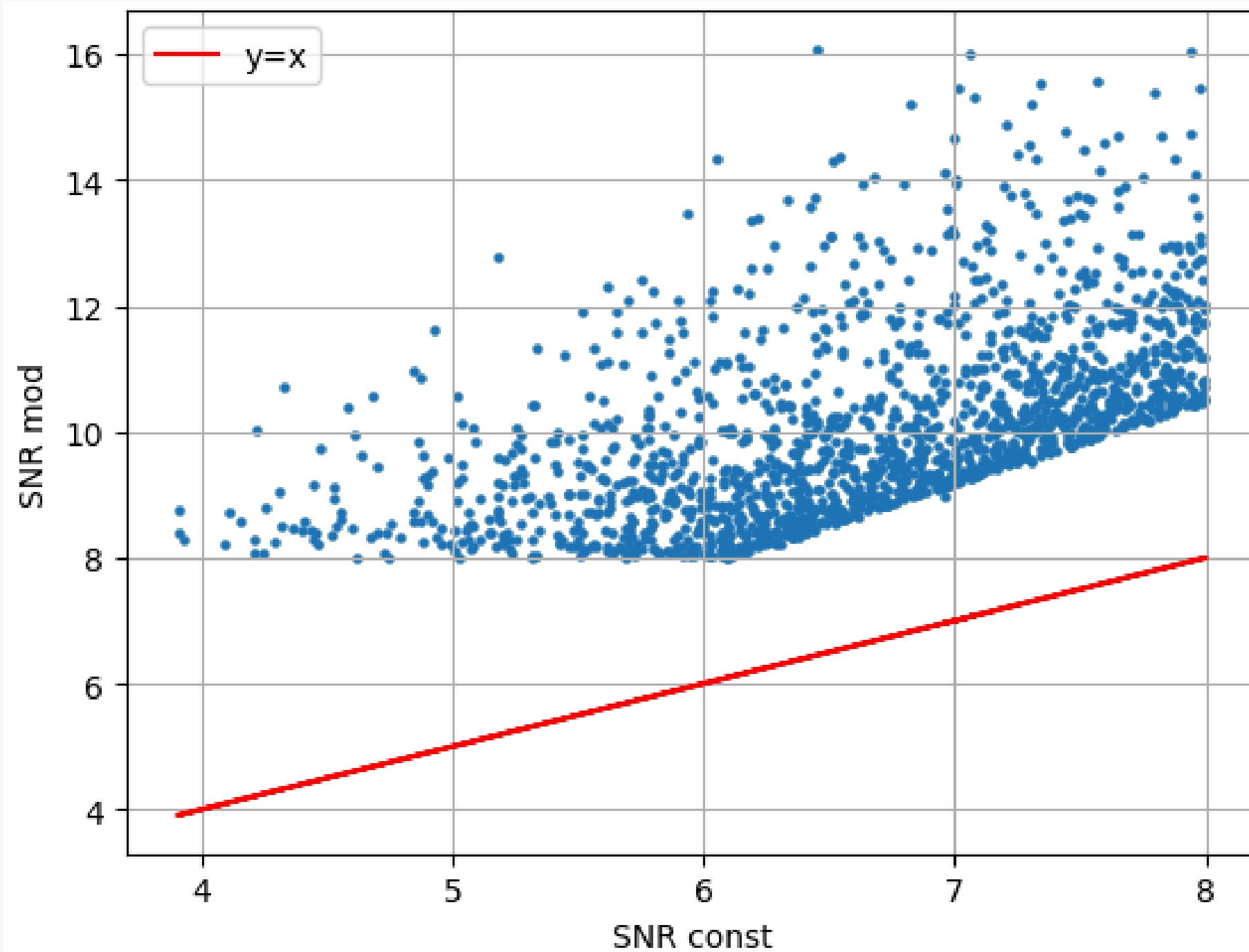


# Histogramme des variations de SNR



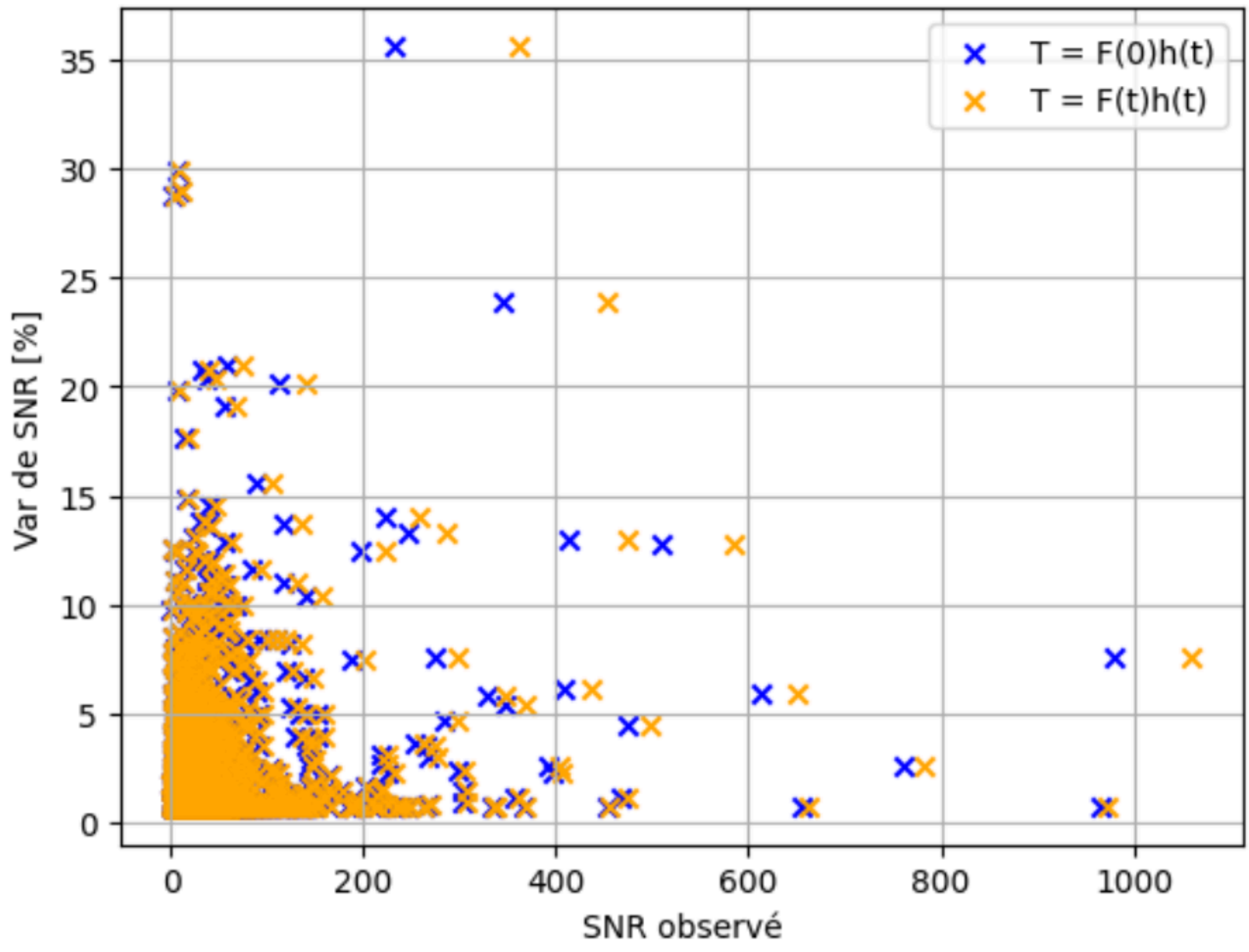
Histogramme des variations de SNR

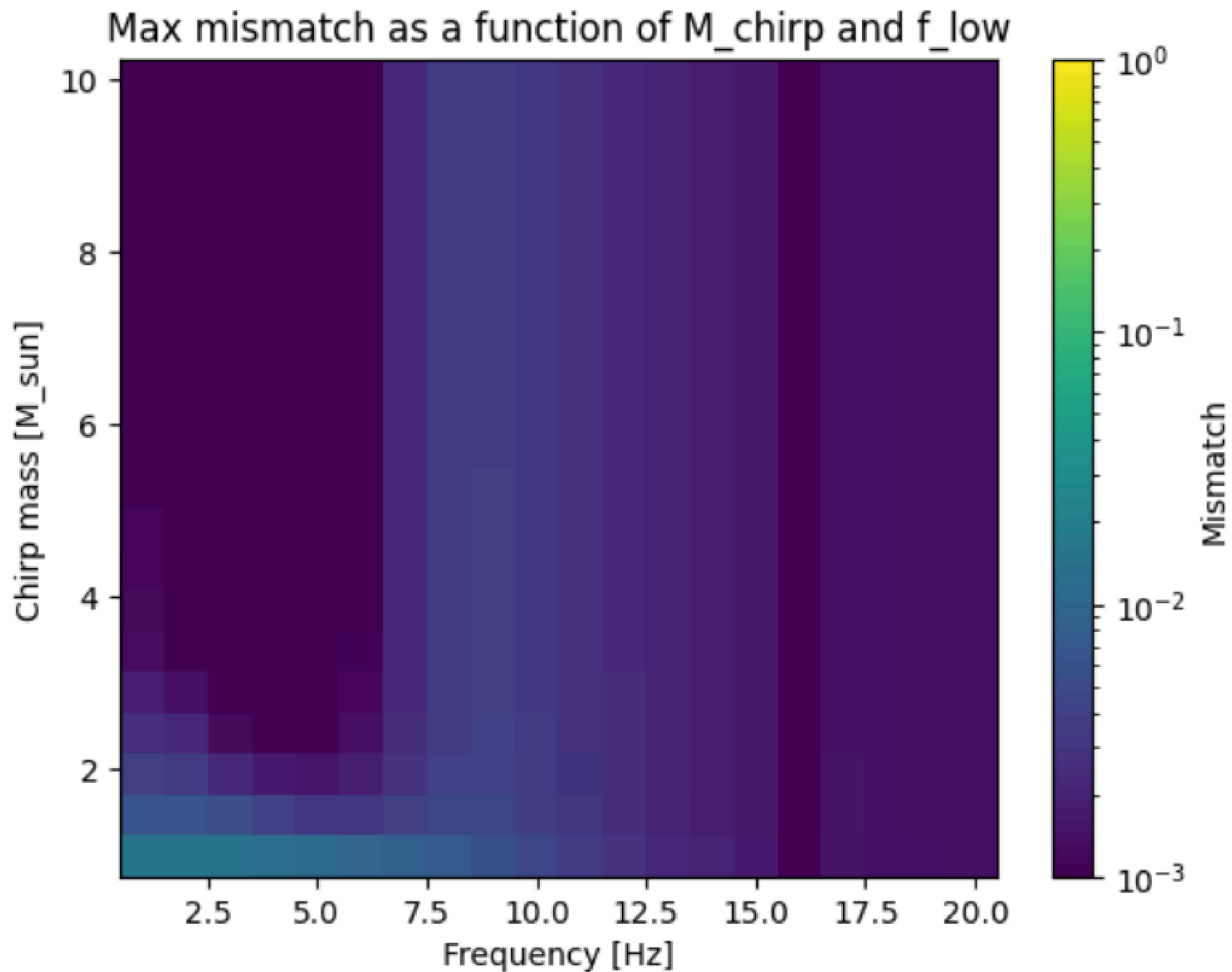




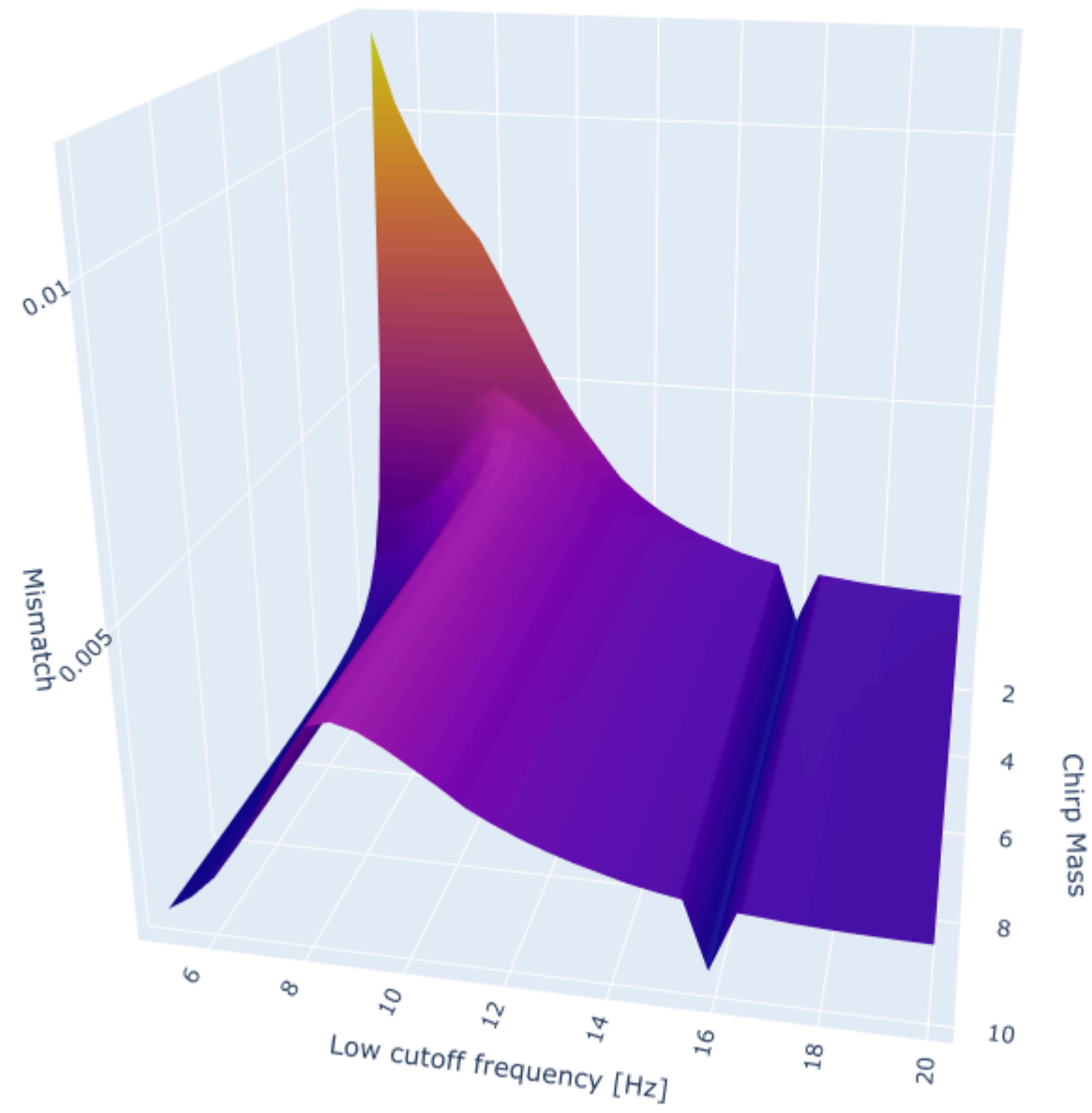
If  $f_{low} = 2\text{Hz}$

~ 25% of ALL detected sources  
can be retrieved

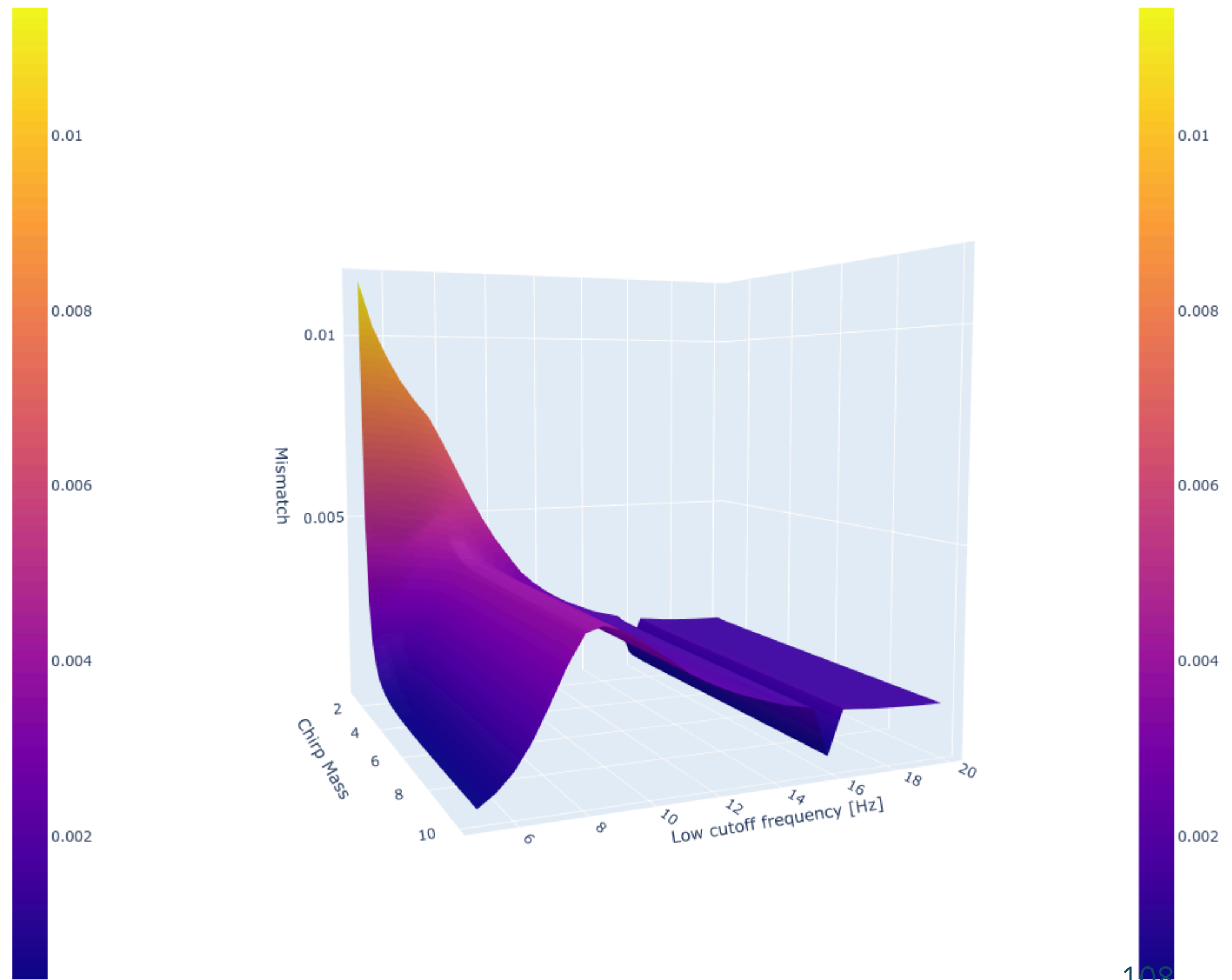




Mismatch as a function of M\_chirp and f\_low

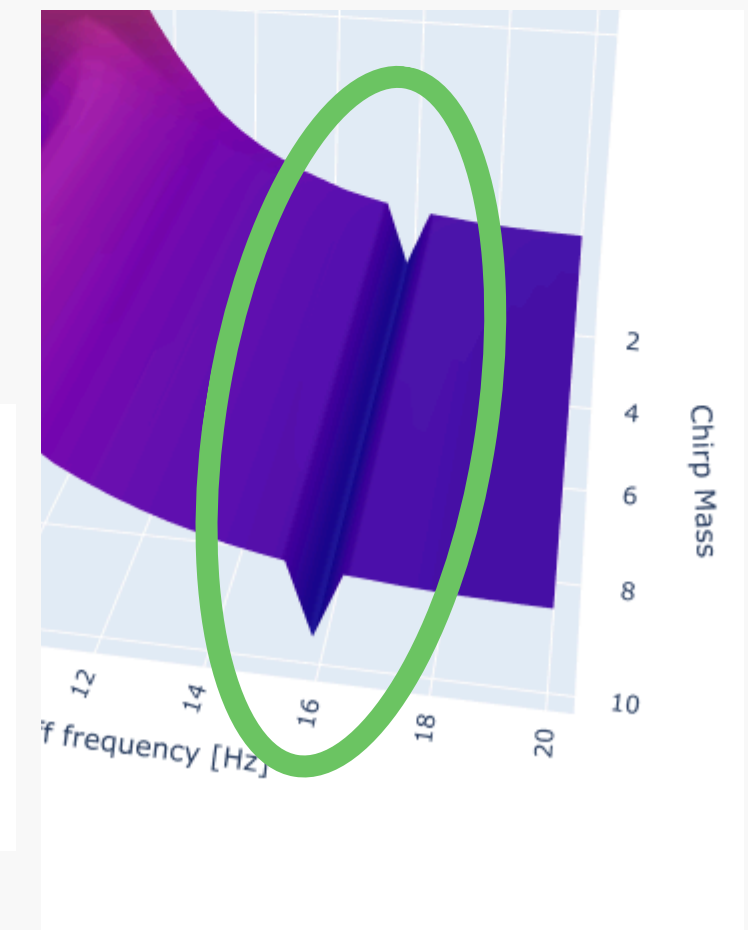


Mismatch as a function of M\_chirp and f\_low

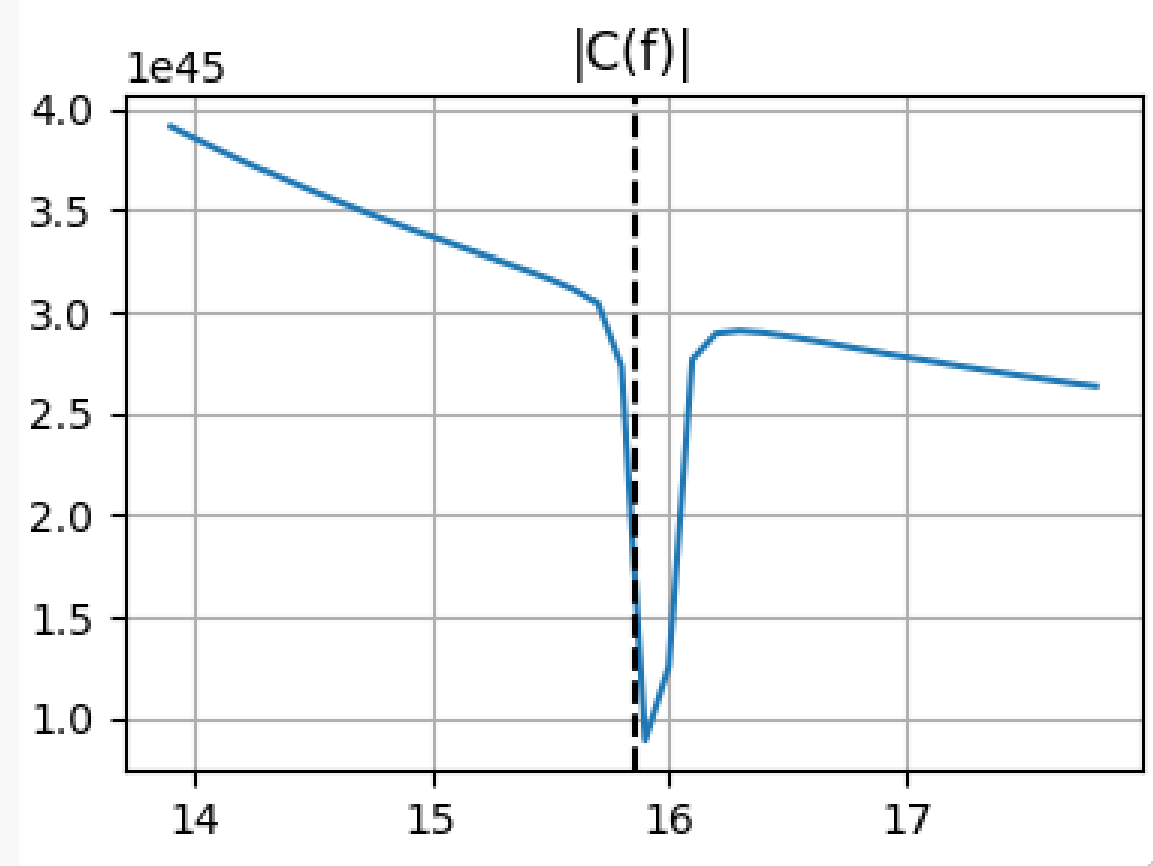


Caused by the PSD

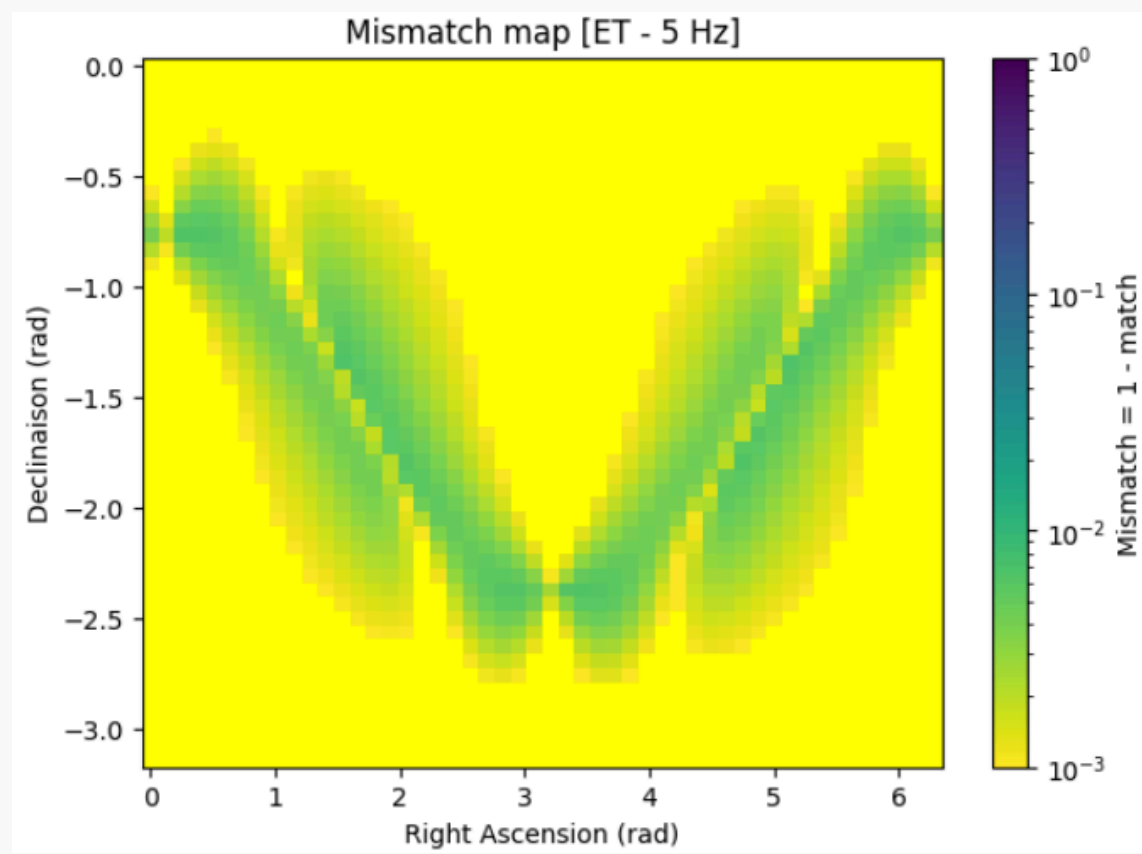
$$m(h_{\text{rot}}, h_{\text{const}}) = \max_{\Delta t_c, \Delta \phi_c} \frac{\Re \int \frac{f^{-7/3} |B(t_{\text{rot}}(f))| e^{-i\theta(t_{\text{rot}}(f))} e^{i(2\pi f \Delta t_c - \Delta \phi_c)} e^{i\phi_0}}{S_n(f)} df}{\sqrt{\int \frac{f^{-7/3} |B(t_{\text{rot}}(f))|^2}{S_n(f)} df \int \frac{f^{-7/3}}{S_n(f)} df}}$$



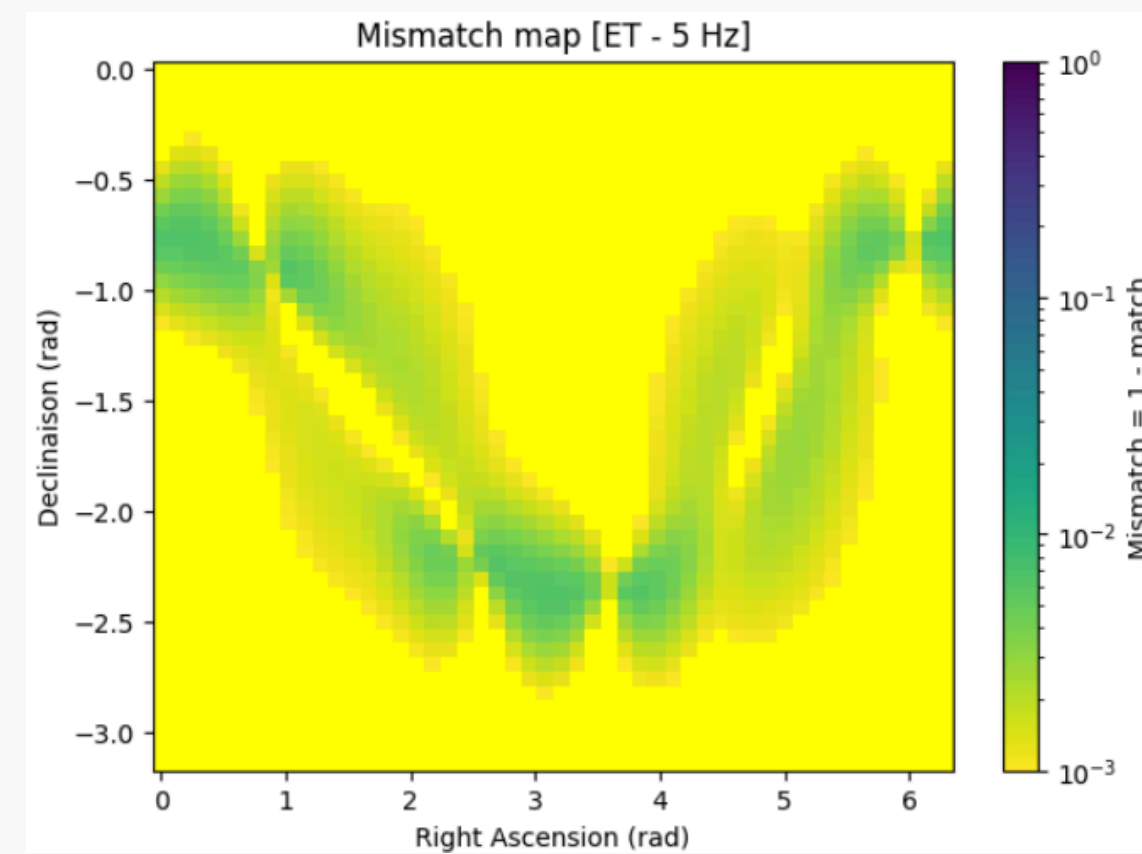
Modulus of numerator's integrand



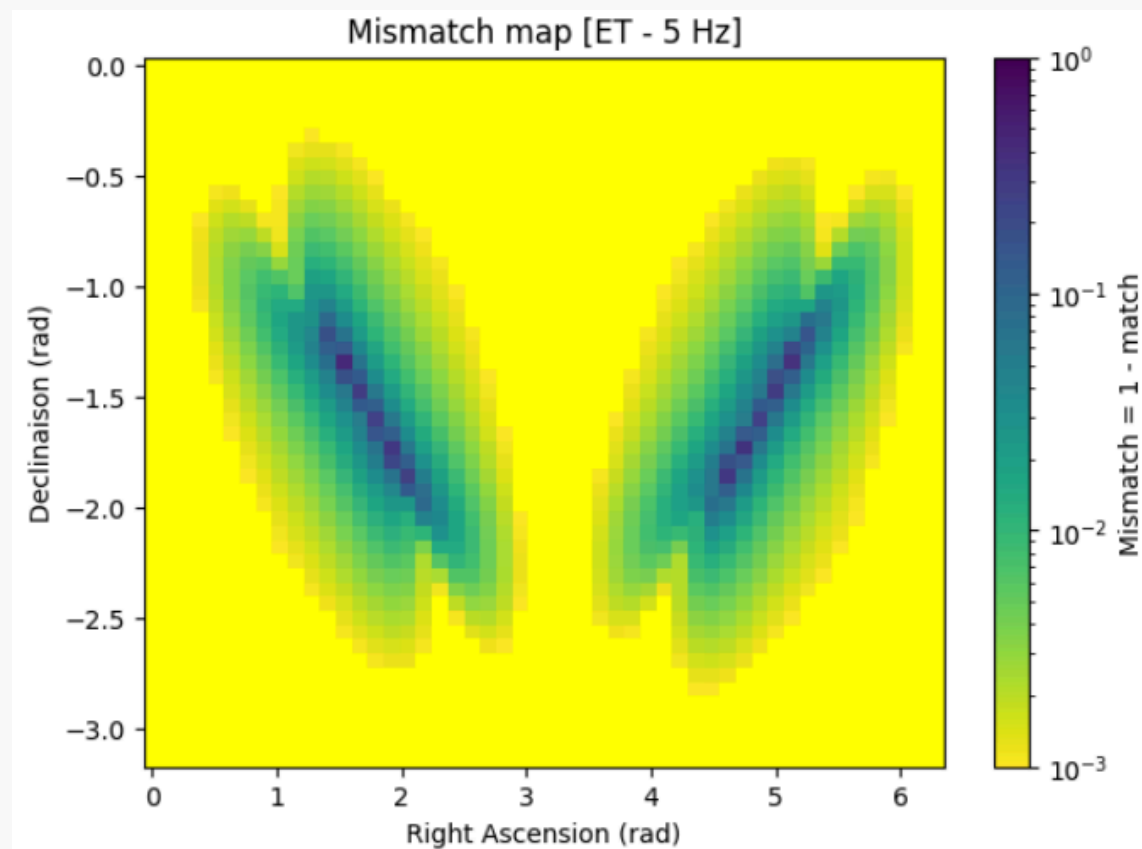
Physical interpretation yet to be given (if not an error)



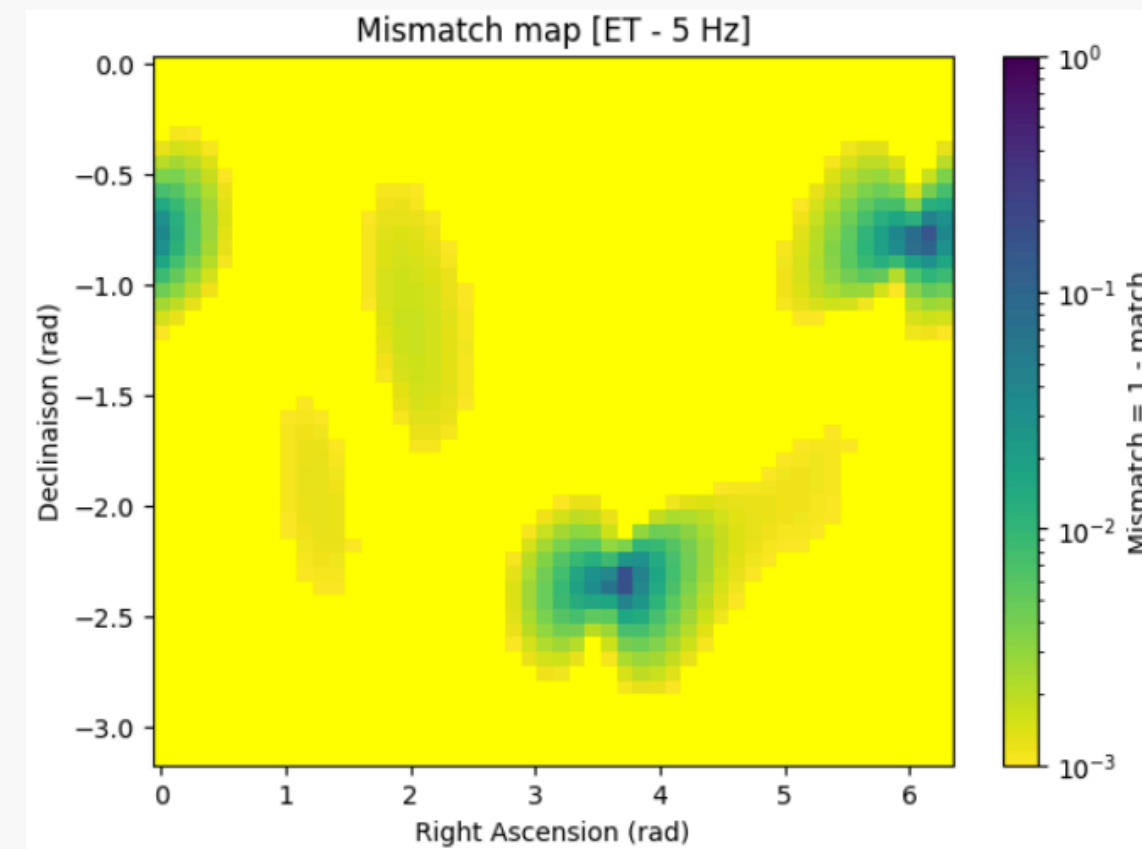
$\text{PSI} = 0^\circ, \text{incl} = 0^\circ$



$\text{PSI} = 30^\circ, \text{incl} = 0^\circ$



$\text{PSI} = 0^\circ, \text{incl} = 90^\circ$



$\text{PSI} = 30^\circ, \text{incl} = 90^\circ$

## Development of equations using Marion's formalism

$$\mathcal{F}(\alpha, \delta, \psi, t) = F_+(\alpha, \delta, \psi, t) + iF_\times(\alpha, \delta, \psi, t)$$

$$\begin{cases} h_{mod}(\alpha, \delta, \psi, t) = f^{-\frac{7}{6}}(t)(F_+(\alpha, \delta, \psi, t) + iF_\times(\alpha, \delta, \psi, t)) = f^{-\frac{7}{6}}\mathcal{F}(\alpha, \delta, \psi, t) \\ h_{const}(\alpha, \delta, \psi) = f^{-\frac{7}{6}}(t)(F_+(\alpha, \delta, \psi, 0) + iF_\times(\alpha, \delta, \psi, 0)) = f^{-\frac{7}{6}}\mathcal{F}(\alpha, \delta, \psi, 0) \end{cases}$$

For 3 detectors (ET-Xylophone)  
&  
inclination = 0°

$$\mathcal{M}(\alpha, \delta, \psi) \propto \max_{\Delta\tau} \int_{f_{low}}^{f_{high}} \frac{f^{-\frac{7}{6}}}{S_n(f)} \Re([\mathcal{F}_{ET}(\alpha, \delta, \psi, 0)][\mathcal{F}_{ET}^*(\alpha, \delta, \psi, t)]) df$$

$$\mathcal{L}(\psi) := \Re([\mathcal{F}_{ET}(\alpha, \delta, \psi, 0)][\mathcal{F}_{ET}^*(\alpha, \delta, \psi, t)])$$

$$\begin{aligned} \mathcal{L}(\psi) &= \sqrt{A + \mathcal{B} \sin(4\psi + \Phi_1) + \mathcal{C} \sin(8\psi + \Phi_2)} + \sqrt{A - \mathcal{B} \sin(4\psi + \Phi_1) + \mathcal{C} \sin(8\psi + \Phi_2)} \\ &= f_+(\psi) + f_-(\psi) \end{aligned}$$

Match has a non-trivial  
polarisation angle dependency

For 3 detectors (ET-Xylophone)  
&  
any inclination angle (iota)

$$\begin{aligned}\mathcal{F}_{ET}(\alpha, \delta, \psi, t) &= [1 + \cos^2(\iota)]F_{+,ET}(\alpha, \delta, \psi, t) + i[2 \cos(\iota)]F_{\times,ET}(\alpha, \delta, \psi, t) \\ &= \eta F_{+,ET}(\alpha, \delta, \psi, t) + i\xi F_{\times,ET}(\alpha, \delta, \psi, t)\end{aligned}$$

$$\mathcal{L}(\psi) = \eta^2 f_+(\psi) + \xi^2 f_\times(\psi)$$

Match has a non-trivial  
polarisation angle dependency

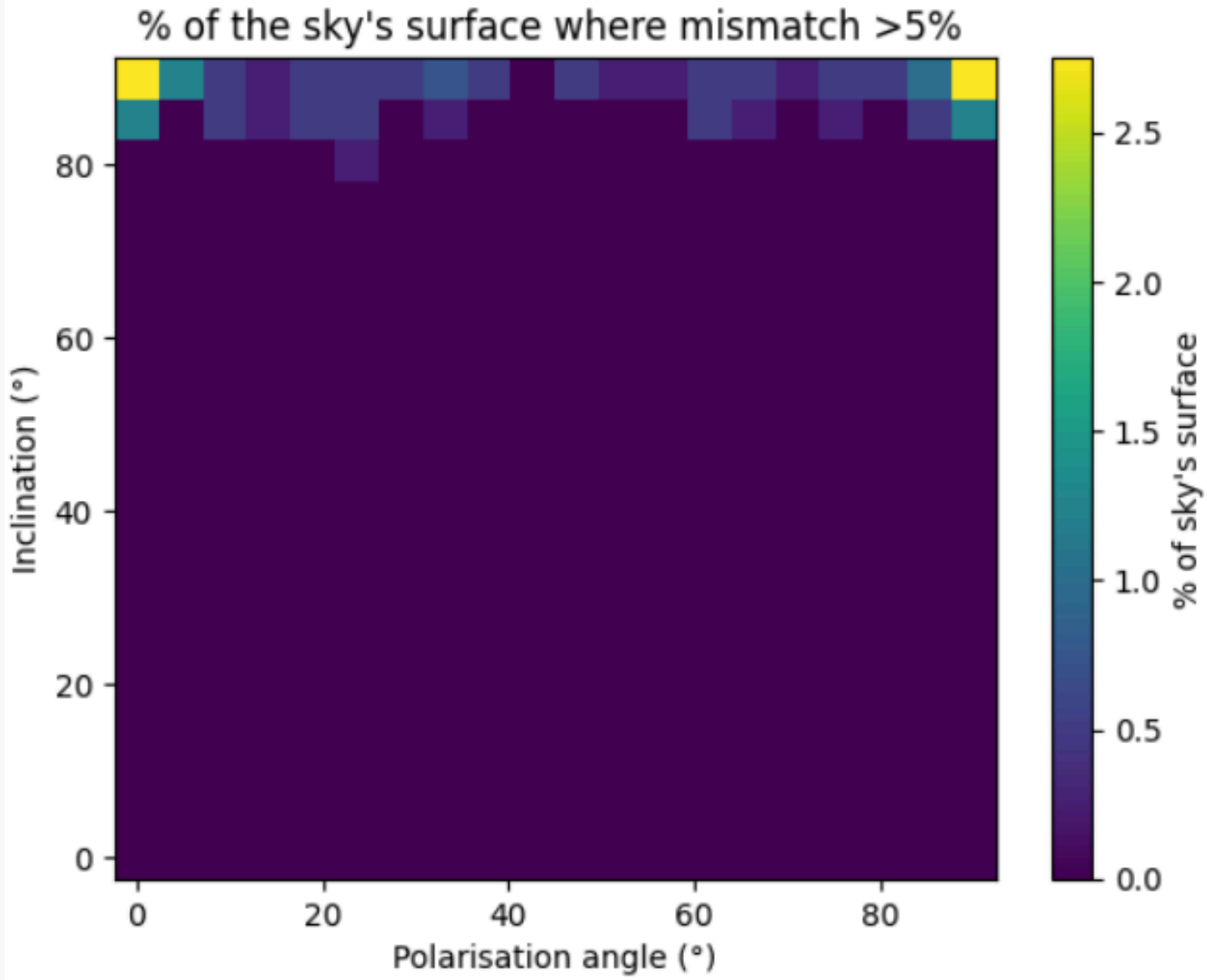
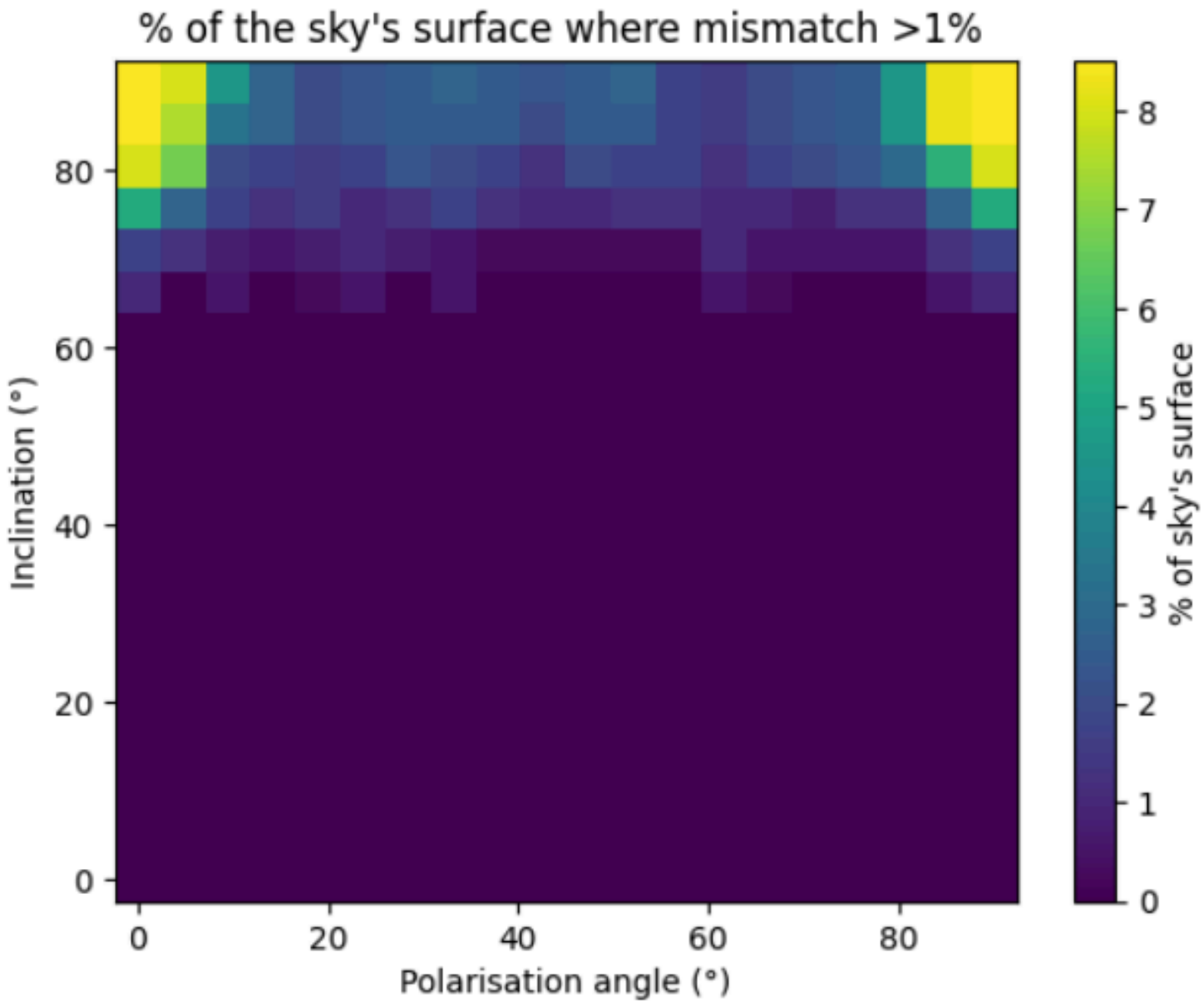
Studying the L function gives us the “geometric” behavior of the match

Extendable to any network of L-shaped detectors

$$\mathcal{L}(\psi) = \eta^2 f_+(\psi) + \xi^2 f_\times(\psi)$$

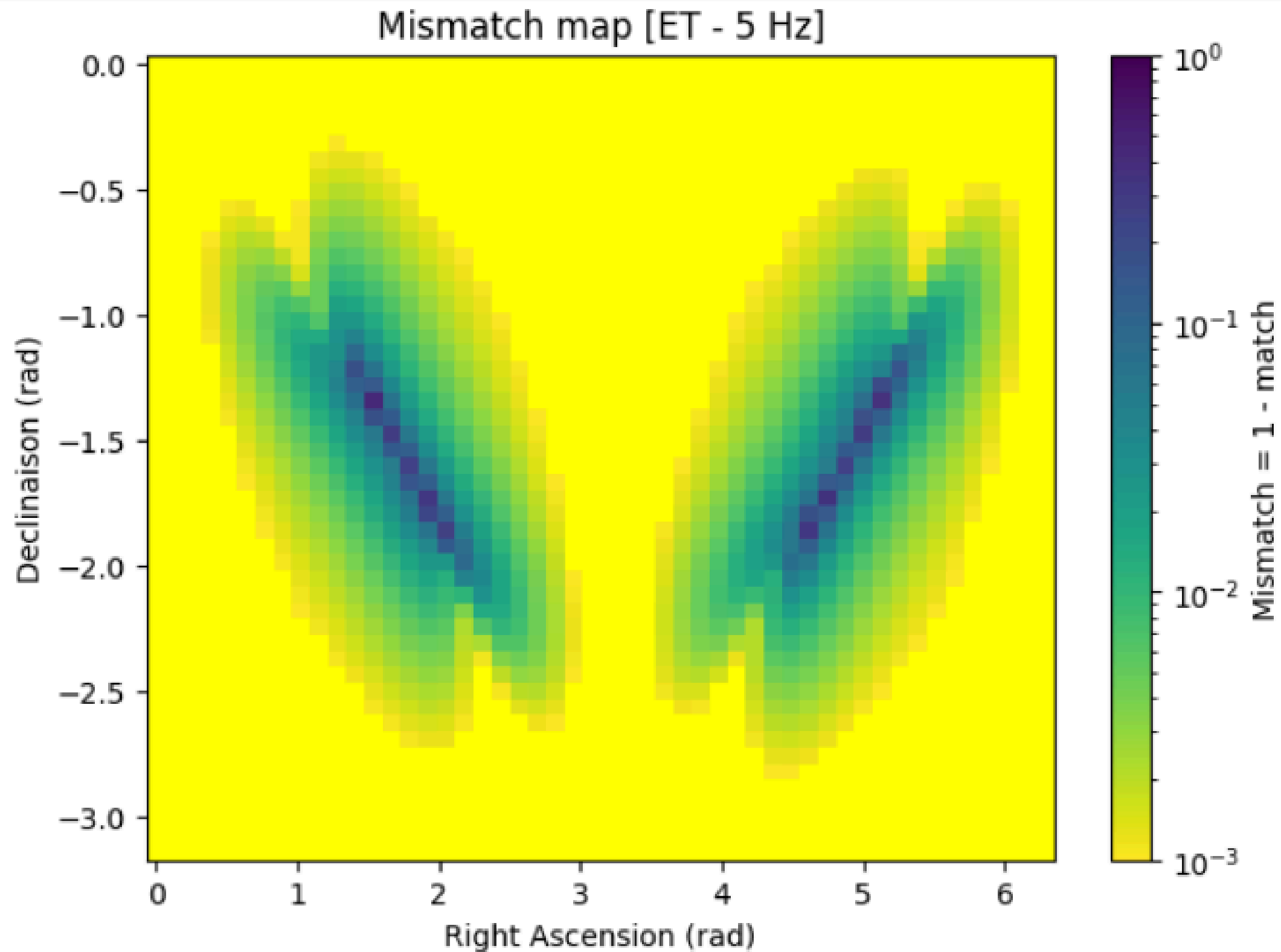
$$F_{+,\text{net}}(\alpha, \delta, \psi, t) = \sqrt{\sum_{i=1}^n F_{+,i}^2(\alpha, \delta, \psi, t)}$$
$$F_{\times,\text{net}}(\alpha, \delta, \psi, t) = \sqrt{\sum_{i=1}^n F_{\times,i}^2(\alpha, \delta, \psi, t)}$$

# Areas where mismatch >1% and 5%



Confirmed !

Inclination =  $90^\circ$ , PSI =  $90^\circ$



Max mismatch ~ 50%