UNIVERSITÀ DI TRENTO

PhD SST
Space Science and Technology

Università degli Studi di Ferrara

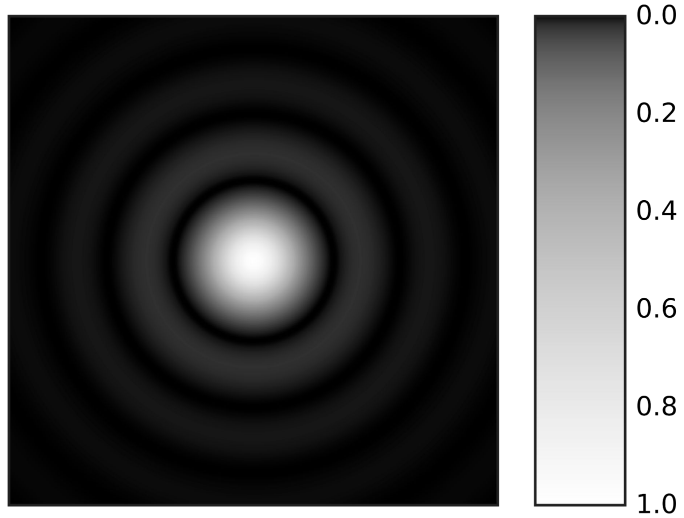
An efficient way to compute beam convolution at the map level

Challenges at the CMB Frontier - 01/07/26

Camille Demay

Introduction

- When observing the sky, telescope optics induce diffraction patterns:
  - Signal = Sky$\otimes$ Point Spread Function
  - In CMB experiments, *point spread functions* are called *beams*
  - If axially symmetric → Airy disc → gaussian near the center
  - If not → leakage of power between intensity and polarization



Source: Wikimedia Commons

Goal: Simulates intensity and polarization leakages at the map level

Fast systematic simulations

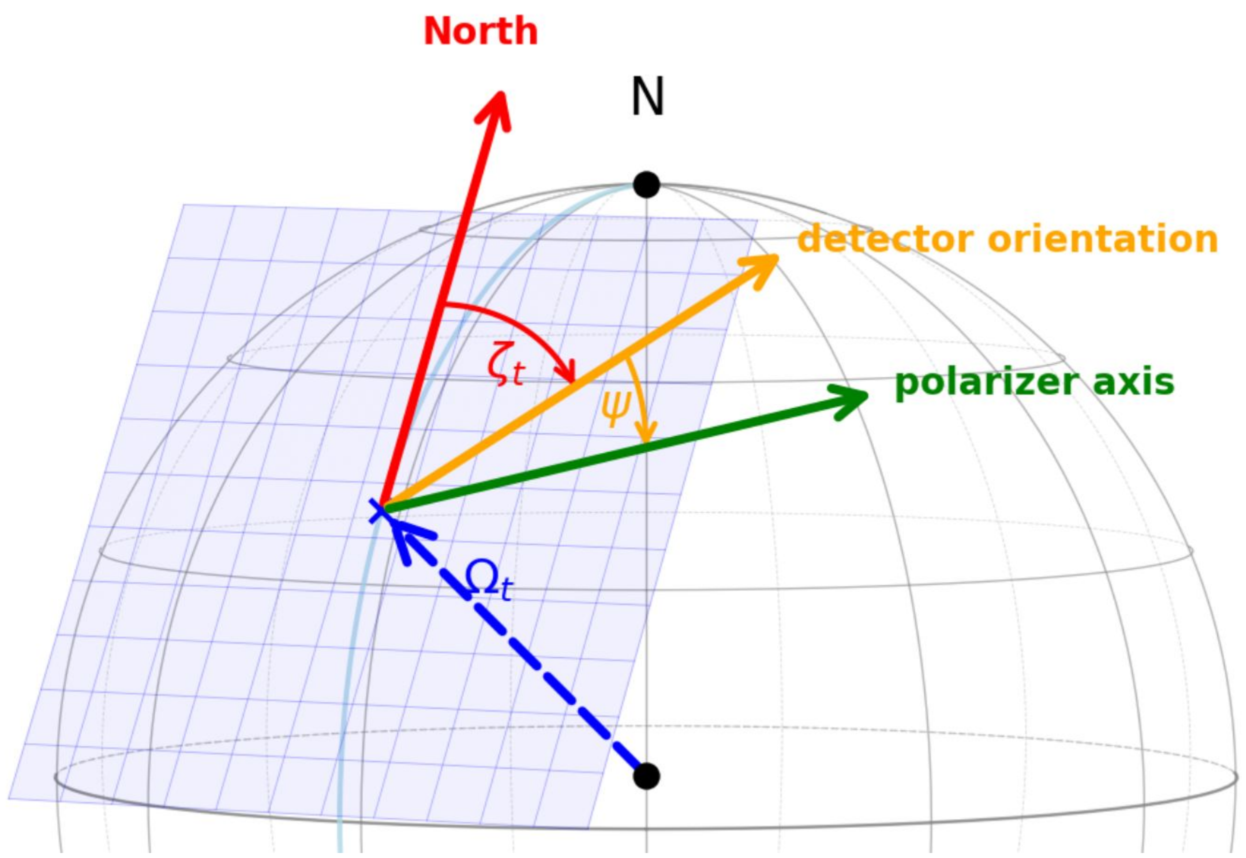
Upcoming CMB experiments aiming at measuring primordial B-modes will need an exquisite control on systematics effects. Having access to *fast* and *precise* simulations is a powerful asset:

- Produce many simulations, to better assess the impact of systematics
- Produce precises systematic templates to marginalize over

- Typical pipeline to propagate systematics: Detectors timestreams → Frequency Maps → Component separation → MCMC sampling
- Compute time:
  - 75 Hz sampling rate, 3 years → **7×10^9** samples
  - $n_{side} = 2048$ → **50×10^6** pixels

Going directly to the map level can save a lot of compute time

From timestreams to sky-maps



- Ω: Pointing direction
- ζ: Parallactic angle
- ψ: polarization angle

- Ideal detector: $$d(\Omega_t,\zeta_t) = \underbrace{\mathcal{I}}_{\text{spin 0}} + \underbrace{\left(\frac{\mathcal{Q}+i\mathcal{U}}{2}\right)}_{\text{spin 2}} e^{-2i(\zeta_t+\psi)} + \underbrace{\left(\frac{\mathcal{Q}-i\mathcal{U}}{2}\right)}_{\text{spin -2}} e^{2i(\zeta_t+\psi)}$$

- Binned mapmaking:

$$\begin{pmatrix}\hat{I}\\ \hat{Q}\\ \hat{U}\end{pmatrix} = \begin{pmatrix} 1 & \langle\cos(2\,[\zeta_t+\psi])\rangle & \langle\sin(2\,[\zeta_t+\psi])\rangle \\ \langle\cos(2\,[\zeta_t+\psi])\rangle & \langle\cos^2(2\,[\zeta_t+\psi])\rangle & \langle\cos(2\,[\zeta_t+\psi])\sin(2\,[\zeta_t+\psi])\rangle \\ \langle\sin(2\,[\zeta_t+\psi])\rangle & \langle\sin(2\,[\zeta_t+\psi])\cos(2\,[\zeta_t+\psi])\rangle & \langle\sin^2(2\,[\zeta_t+\psi])\rangle \end{pmatrix}^{-1} \begin{pmatrix}\langle d_t\rangle \\ \langle d_t\cos(2\,[\zeta_t+\psi])\rangle \\ \langle d_t\sin(2\,[\zeta_t+\psi])\rangle\end{pmatrix}$$

- Systematics ⇒ Additional terms in the timestream ⇒ biased maps

Spin mapmaking

McCallum et al. 2021b, extended to scanning with an half wave plate in *Takase et al. 2024*, for a multi-detector setup in *Morshed et al. In prep:*

$$\begin{pmatrix} \hat{I} \\ \hat{Q} - i\hat{U} \\ \hat{Q} + i\hat{U} \end{pmatrix} = \begin{pmatrix} \tilde{h}_0^{tot} & \frac{1}{2}\tilde{h}_2^{tot} & \frac{1}{2}\tilde{h}_{-2}^{tot} \\ \frac{1}{2}\tilde{h}_2^{tot} & \frac{1}{4}\tilde{h}_4^{tot} & \frac{1}{4} \\ \frac{1}{2}\tilde{h}_{-2}^{tot} & \frac{1}{4} & \frac{1}{4}\tilde{h}_{-4}^{tot} \end{pmatrix}^{-1} \begin{pmatrix} \sum_{j=1}^{N_{\det}} \frac{N_{hits}^j}{N_{hits}^{tot}} \sum_{k'=-\infty}^{\infty} \tilde{h}_{0-k'}^j \tilde{S}_{k'}^j \\ \frac{1}{2}\sum_{j=1}^{N_{\det}} e^{i2\psi_j} \frac{N_{hits}^j}{N_{hits}^{tot}} \sum_{k'=-\infty}^{\infty} \tilde{h}_{2-k'}^j \tilde{S}_{k'}^j \\ \frac{1}{2}\sum_{j=1}^{N_{\det}} e^{-i2\psi_j} \frac{N_{hits}^j}{N_{hits}^{tot}} \sum_{k'=-\infty}^{\infty} \tilde{h}_{-2-k'}^j \tilde{S}_{k'}^j \end{pmatrix}$$

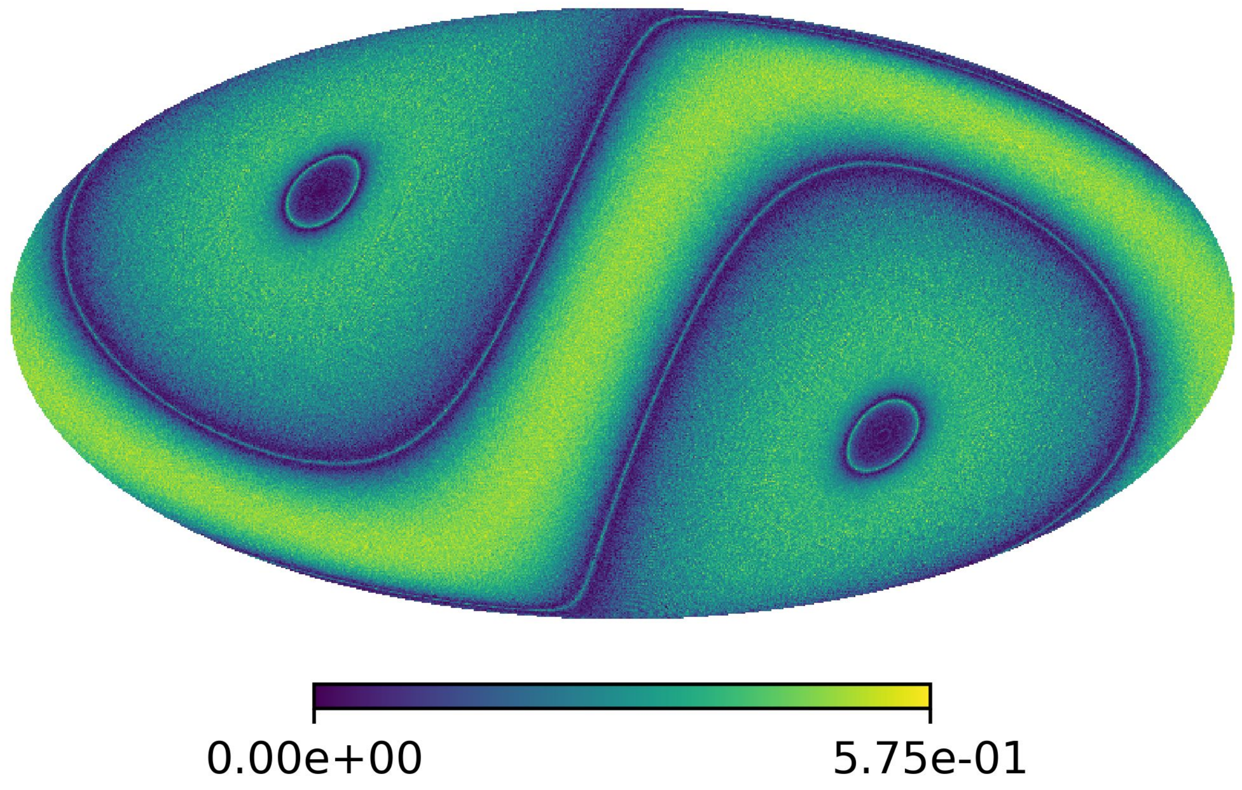
Estimated maps

Pointing information

Input sky maps, with systematics contaminants

- h-*maps:* $\tilde{h}_n^j(\Omega) \triangleq \frac{1}{N_{hits}} \sum_{i=1}^{N_{hits}} e^{in\zeta_i}$, **computed once**
- Systematic effect ⟶ set of $\tilde{S}_n$

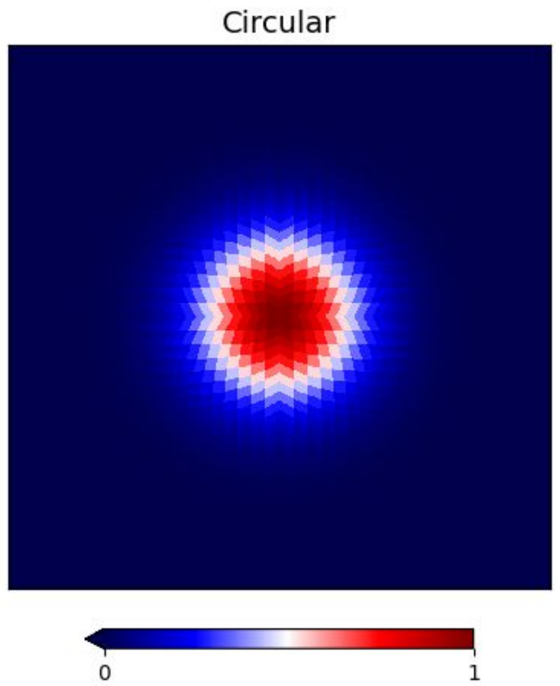
$\tilde{h}_2$



LiteBIRD scanning strategy

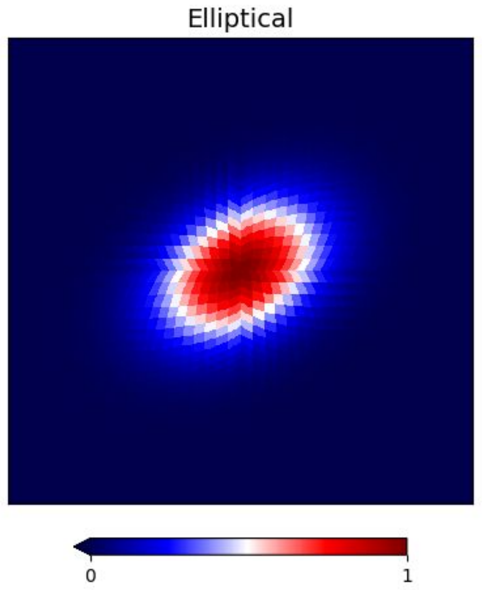
Beam convolution

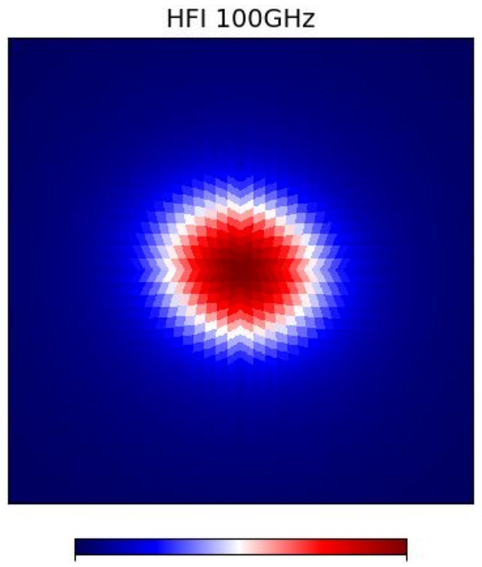
- Circular Gaussian beam: $$d(\Omega_t, \zeta_t) = \mathcal{I} + \left(\frac{\mathcal{Q}+i\mathcal{U}}{2}\right) e^{-2i(\zeta+\psi)} + \left(\frac{\mathcal{Q}-i\mathcal{U}}{2}\right) e^{2i(\zeta+\psi)}$$



    - Correspond to an ideal detector
    - The sky maps are smoothed

- Generic beam:





4π convolved timestream: $$d(\Omega_t, \zeta_t) = \int_{4\pi} d\vec{r}\,[I(\vec{r})B_I(\vec{r}-\Omega_t, \zeta_t+\psi)] + \text{polarization}$$

How to associate the convolved timestream to spin maps ?

Spin maps for beam convolution

- Generic beam:
  (*Hivon et al. 2016*)

$$d(\Omega_t,\zeta_t)=\sum_{\ell=0}^{+\infty}\sum_{m=-l}^{\ell}\sum_{s=-\ell}^{\ell}\left[\underbrace{{}_0a_{\ell m}\,{}_0b^*_{\ell s}}_{\textit{Intensity}}+\frac{1}{2}\underbrace{\left({}_2a_{\ell m}\,{}_2b^*_{\ell s}+{}_{-2}\,a_{\ell m}\,{}_{-2}b^*_{\ell s}\right)}_{\textit{Polarisation}}\right](-1)^s\sqrt{\frac{4\pi}{2\ell+1}}e^{is\zeta_t}\,{}_{-s}Y_{\ell m}(\Omega_t)$$

need $d(\Omega_t,\zeta_t)=\sum_{n\in\mathbb{Z}}\tilde{S}^j_n(\Omega_t)e^{-in\zeta_t}$ to use spin-mapmaking

$$\begin{aligned}
\tilde{S}_0 &= \mathcal{I}+{}^I_0Z+{}^P_0Z,\\
\tilde{S}_2 &= \left(\frac{\mathcal{Q}+i\mathcal{U}}{2}+{}^P_2Z\right)e^{-i2\psi}+{}^I_2Z,\\
\tilde{S}_{-2} &= \left(\frac{\mathcal{Q}-i\mathcal{U}}{2}+{}^P_{-2}Z\right)e^{i2\psi}+{}^I_{-2}Z,\\
\tilde{S}_n &= {}^I_nZ+{}^P_nZe^{-in\psi},\quad n\in\mathbb{Z}\backslash\{0,2,-2\}
\end{aligned}$$

$${}^I_nZ=\sum_{l=0}^{+\infty}\sum_{m=-\ell}^{\ell}(-1)^n\sqrt{\frac{4\pi}{2\ell+1}}\;{}_0a_{\ell m}\,{}_0b'^*_{\ell-n}\,{}_nY_{\ell m}$$

$${}^P_nZ=\sum_{l=0}^{+\infty}\sum_{m=-\ell}^{\ell}(-1)^n\sqrt{\frac{\pi}{2\ell+1}}\left({}_2a_{\ell m}\,{}_2b'^*_{\ell-n}+{}_{-2}a_{\ell m}\,{}_{-2}b'^*_{\ell-n}\right)e^{in\psi}\,{}_nY_{\ell m}$$

$$b'_{\ell m}=b_{\ell m}-b^{\text{circular gaussian}}_{\ell m}$$

With these spin maps, we have a new way to efficiently propagate systematics from b_{lm} to the map level !

Spin maps for beam convolution

- $$\begin{pmatrix} \hat{I} \\ \hat{Q} - i\hat{U} \\ \hat{Q} + i\hat{U} \end{pmatrix} = \begin{pmatrix} \tilde{h}_0^{tot} & \frac{1}{2}\tilde{h}_2^{tot} & \frac{1}{2}\tilde{h}_{-2}^{tot} \\ \frac{1}{2}\tilde{h}_2^{tot} & \frac{1}{4}\tilde{h}_4^{tot} & \frac{1}{4} \\ \frac{1}{2}\tilde{h}_{-2}^{tot} & \frac{1}{4} & \frac{1}{4}\tilde{h}_{-4}^{tot} \end{pmatrix}^{-1} \begin{pmatrix} \sum_{j=1}^{N_{\rm det}} \frac{N^j_{hits}}{N^{tot}_{hits}} \sum_{k'=-\infty}^{\infty} \tilde{h}^j_{0-k'} \tilde{S}^j_{k'} \\ \frac{1}{2} \sum_{j=1}^{N_{\rm det}} e^{i2\psi_j} \frac{N^j_{hits}}{N^{tot}_{hits}} \sum_{k'=-\infty}^{\infty} \tilde{h}^j_{2-k'} \tilde{S}^j_{k'} \\ \frac{1}{2} \sum_{j=1}^{N_{\rm det}} e^{-i2\psi_j} \frac{N^j_{hits}}{N^{tot}_{hits}} \sum_{k'=-\infty}^{\infty} \tilde{h}^j_{-2-k'} \tilde{S}^j_{k'} \end{pmatrix}$$

$$\left.\begin{aligned} \tilde{S}_0 &= \mathcal{I} + {}^I_0Z + {}^P_0Z, \\ \tilde{S}_2 &= \left(\frac{\mathcal{Q}+i\mathcal{U}}{2} + {}^P_2Z\right)e^{-i2\psi} + {}^I_2Z, \\ \tilde{S}_{-2} &= \left(\frac{\mathcal{Q}-i\mathcal{U}}{2} + {}^P_{-2}Z\right)e^{i2\psi} + {}^I_{-2}Z, \end{aligned}\right\}$$

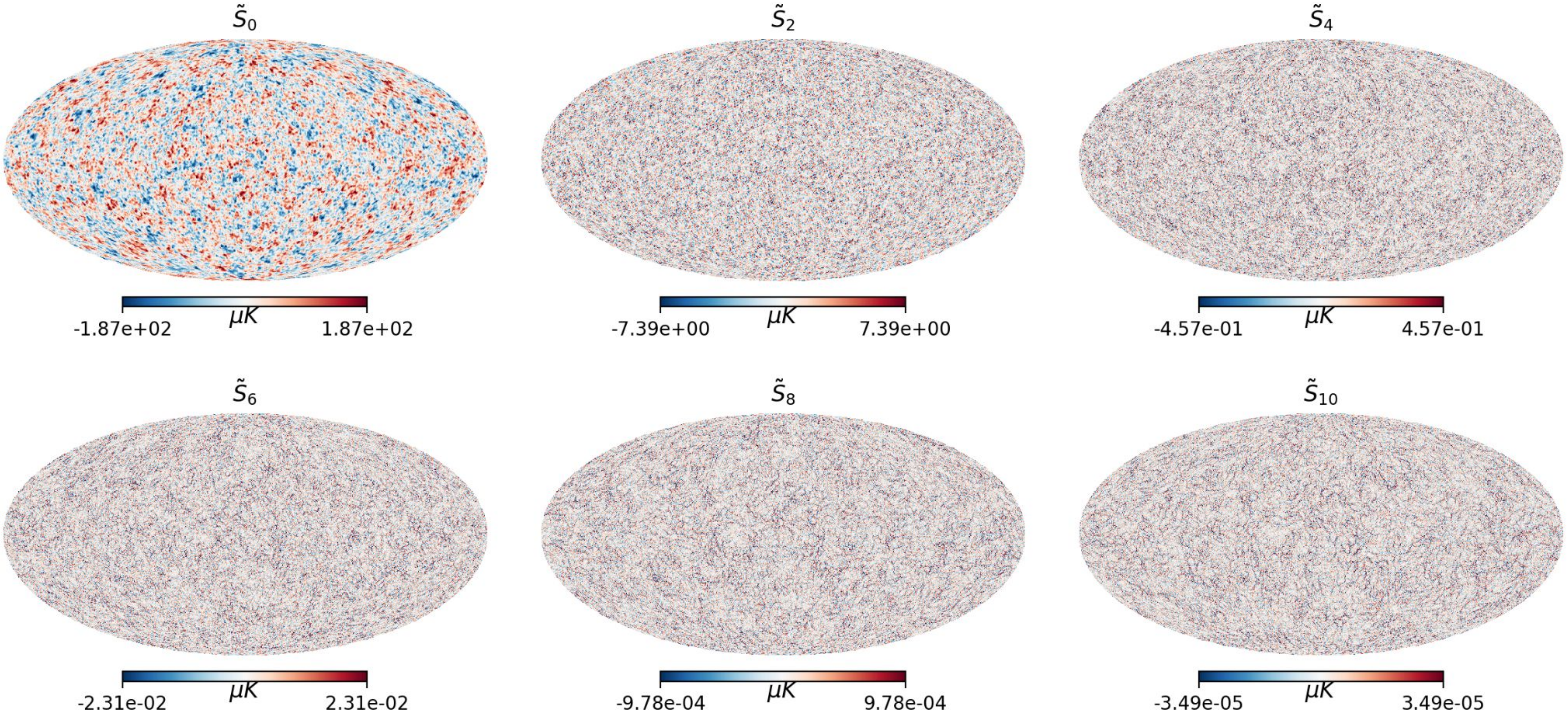
Theses leakages terms can't be suppress by a more uniform scanning

$$\tilde{S}_n = {}^I_nZ + {}^P_nZe^{-in\psi}, \quad n \in \mathbb{Z}\backslash\{0, 2, -2\} \Rightarrow \quad {}^P_nZ \propto {}_{\pm 2}b'_{\ell-n} \qquad {}^I_nZ \propto {}_0b'_{\ell-n}$$

$\Rightarrow$ the b_{lm} should have a small m_{max}

$\Rightarrow$ center of the beam: the main beam

Elliptical beam:

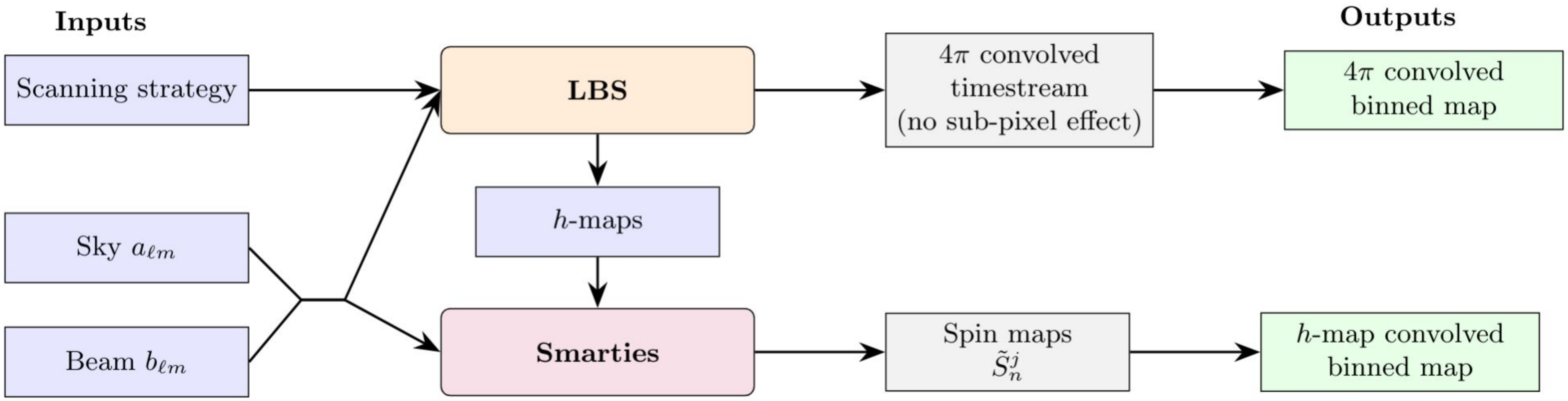


Spin maps from the main beam convolution are dominated by low spin orders

Validation against 4π convolution

- Baseline: The LiteBIRD simulation framework (LBS), *Tomasi et al. 2025* :
  - 4π convolution, using ducc:
    - $$d(\Omega_t,\zeta_t)=\sum_{\ell=0}^{+\infty}\sum_{m=-l}^{\ell}\sum_{s=-\ell}^{\ell}\left[{}_0a_{\ell m}\,{}_0b^*_{\ell s}+\frac{1}{2}\left({}_2a_{\ell m}\,{}_2b^*_{\ell s}+{}_{-2}\,a_{\ell m}\,{}_{-2}b^*_{\ell s}\right)\right](-1)^s\sqrt{\frac{4\pi}{2\ell+1}}e^{is\zeta_t}\,{}_{-s}Y_{\ell m}(\Omega_t)$$
    - No sub-pixel effects
- Spin-mapmaking: Smarties, *Morshed et al. In prep*
- Beams:
  - Circular, Elliptic and Planck HFI simulated ones
  - Truncated at m_{max}= 4 or 6
  - 2 pairs of orthogonal detectors

Validation against 4π convolution

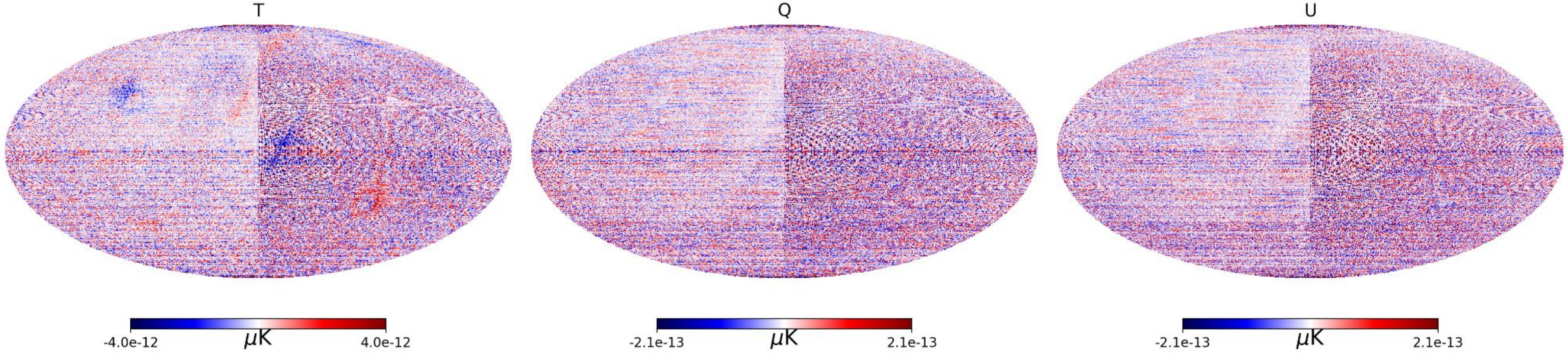


Same sets of harmonic coefficients → We expect perfect agreement between Smarties and LBS

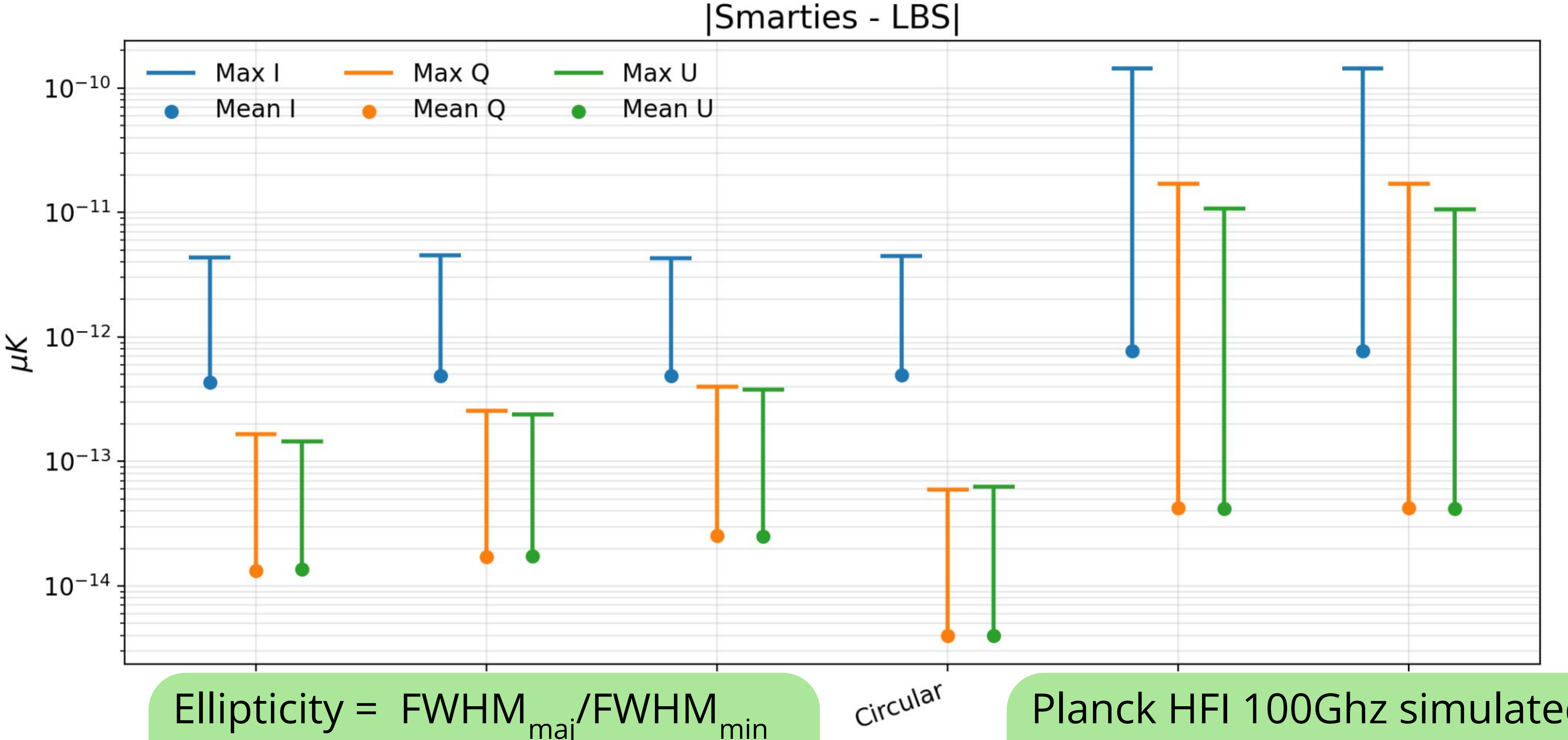
Validation results

- Planck HFI beam:
  m_{max}=6
  n_{side}=1024

Residual maps: Smarties - LBS



- Summary:



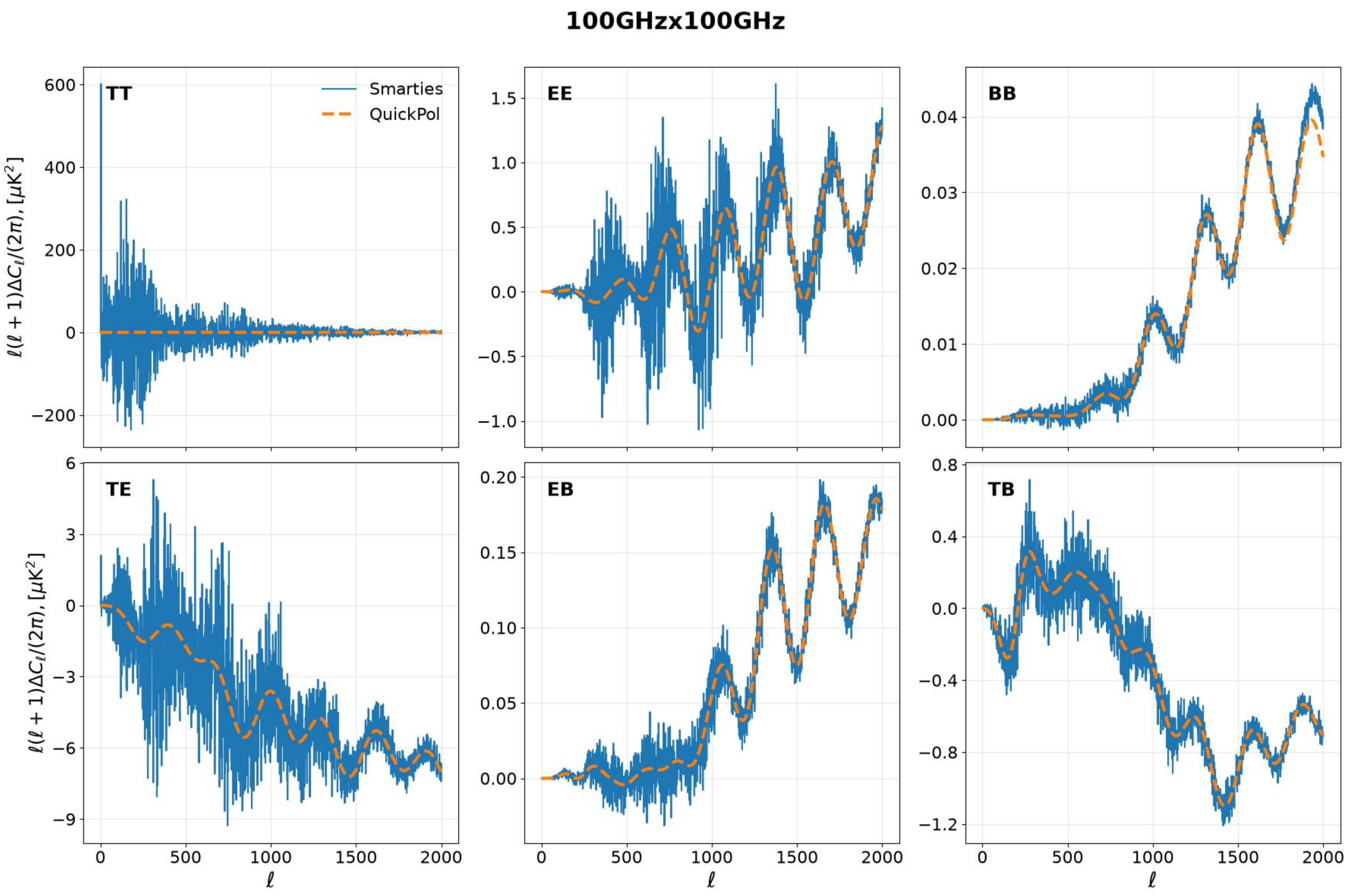
Ellipticity = $FWHM_{maj}/FWHM_{min}$ going from 1.1 to 1.3
m_{max} = 4
n_{side}=128

Planck HFI 100Ghz simulated beams
m_{max} = 4 and 6
n_{side}=1024

Generating convolved Planck maps

- Quickpol (*Hivon et al. 2016*):
  - Used in Planck
  - Power spectrum level
  - Inputs: h-*maps* and b_{lm}
- HFI 100 GHz detectors:
  - h-*maps*
  - Simulated b_{lm}
- $$\Delta\hat{C}_\ell^{XY} = \frac{\hat{C}_\ell^{XY}}{W_\ell^{XYXY}} - C_\ell^{XY}$$

Smarties can simulate beam convolved Planck maps, with power spectra in agreement with Quickpol !



Conclusion

- Spin-mapmaking can be used to produces convolved maps in a precise and fast way:
  - Perfect agreement with pixel centered TOD 4π convolution when using the same b_{lm}.
  - On a laptop: ≃ 1s at $n_{nside} = 128$, grows linearly with the number of pixels

- Practical case : We can simulate Planck-like maps much faster than with 4π convolution

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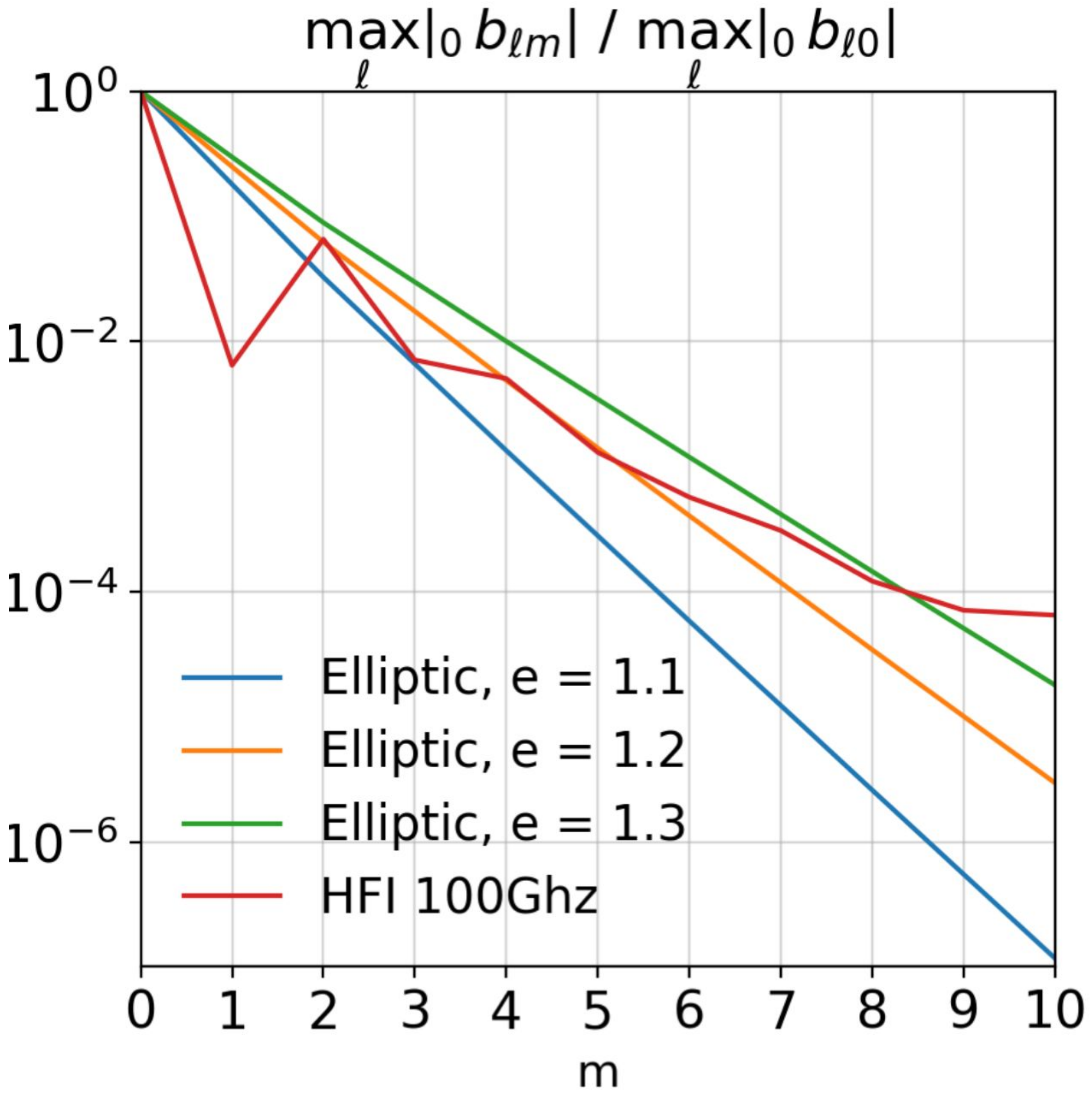
Thanks for your attention !

BACKUPS

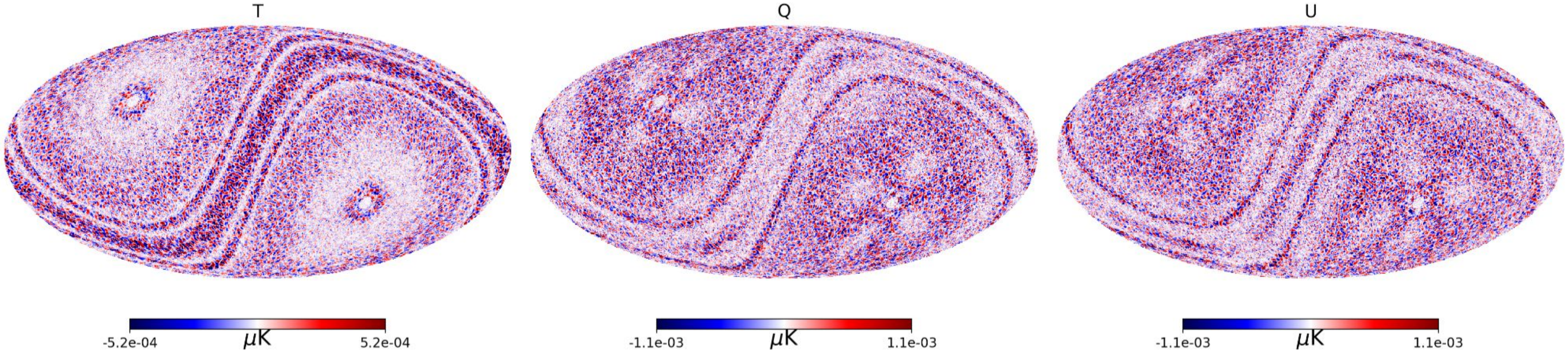


$\max_\ell |{}_0 b_{\ell m}| \,/\, \max_\ell |{}_0 b_{\ell 0}|$
10^0
10^{-2}
10^{-4}
10^{-6}
0 1 2 3 4 5 6 7 8 9 10
m
Elliptic, e = 1.1
Elliptic, e = 1.2
Elliptic, e = 1.3
HFI 100Ghz

Testing the m_{max} truncation

High m_{max} LBS simulations *VS* low m_{max} Smarties simulations.

Residual maps

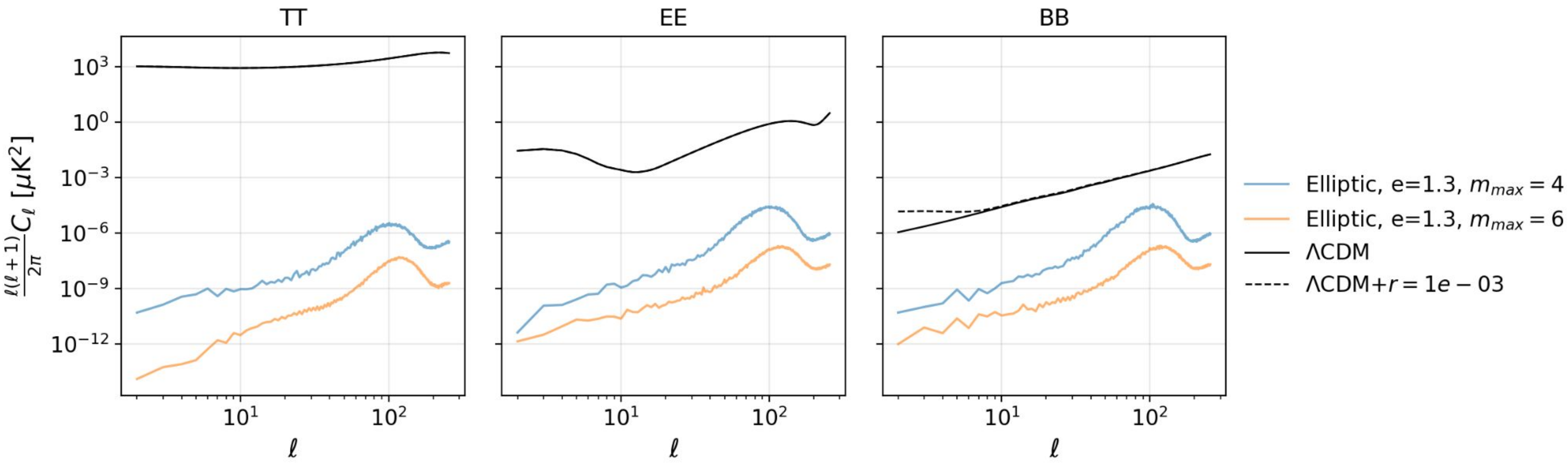


- $m_{max,LBS} = 15$
- $m_{max,Smarties} = 6$
- Ellipticity = 1.3

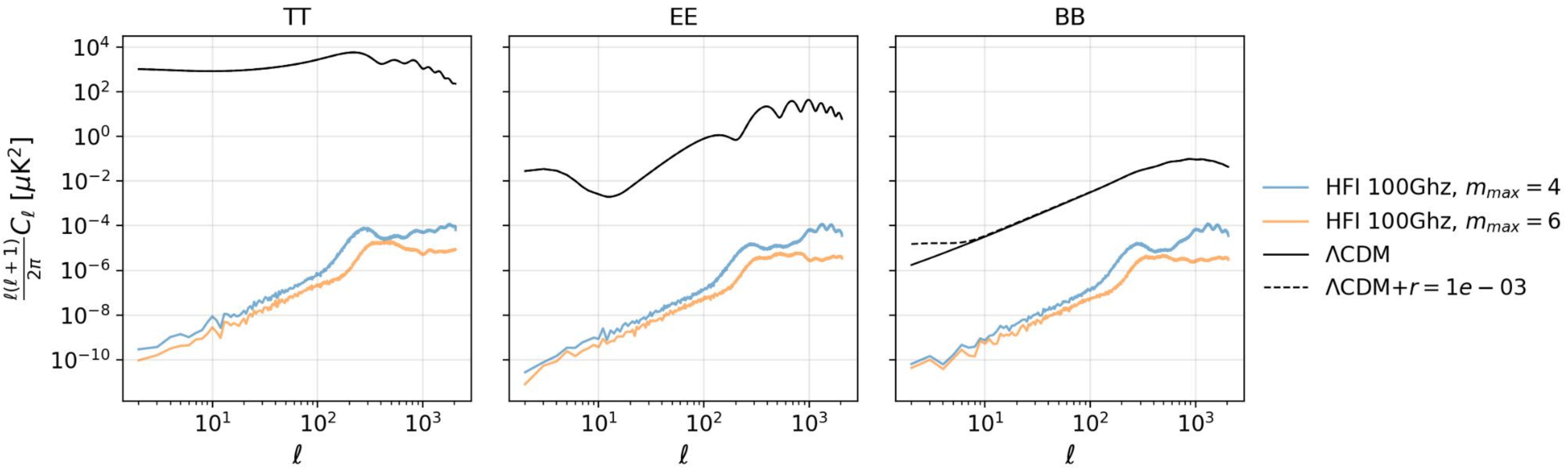
1.3 is in the higher range of ellipticities measured on Planck LFI

Testing the m_{max} truncation

Power spectra of the residual maps

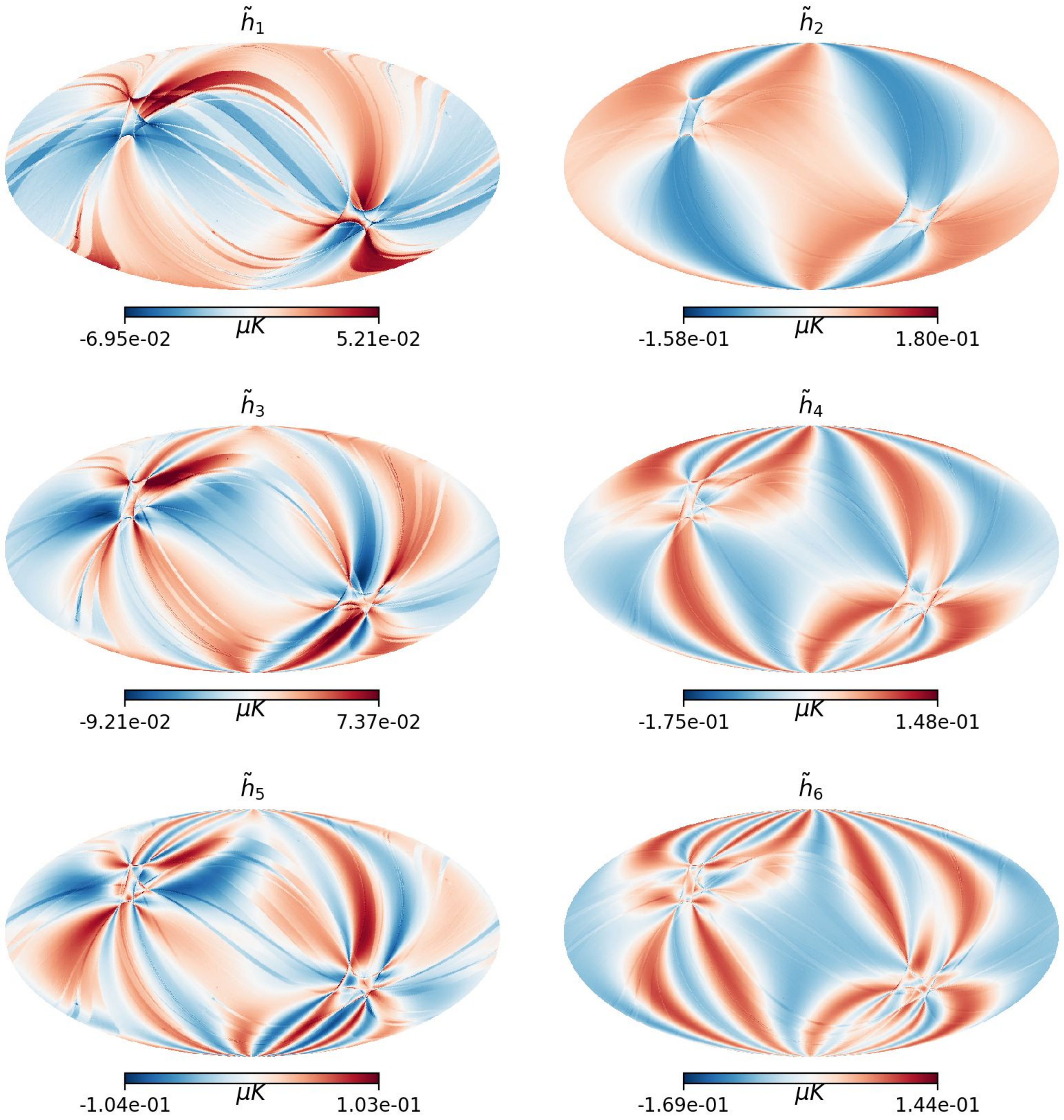


Truncating at m_{max} = 4 suffices to obtain residual power spectra several orders of magnitudes below the CMB ones.


TT
EE
BB
10^4
10^2
10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
$\frac{\ell(\ell+1)}{2\pi}C_\ell\ [\mu K^2]$
10^1
10^2
10^3
ℓ
HFI 100Ghz, $m_{max} = 4$
HFI 100Ghz, $m_{max} = 6$
ΛCDM
$\Lambda CDM + r = 1e-03$

Generating convolved Planck maps

- Quickpol (*Hivon et al. 2016*), used in Planck:
  - $$\widetilde{C}_\ell^{XY} = \sum_{X'Y'} W_\ell^{XY,\, X'Y'} C_\ell^{X'Y'}$$
  - Inputs: h-*maps* and b_{lm}



h-*maps* of a Planck HFI 100Ghz detector