

Lensing from amplitudes beyond GR

Niccolò Feci

Work in collaboration with Adam Falkowski

IJCLab – Université Paris-Saclay (Orsay, France)

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Overview

➤ Introduction to the topic

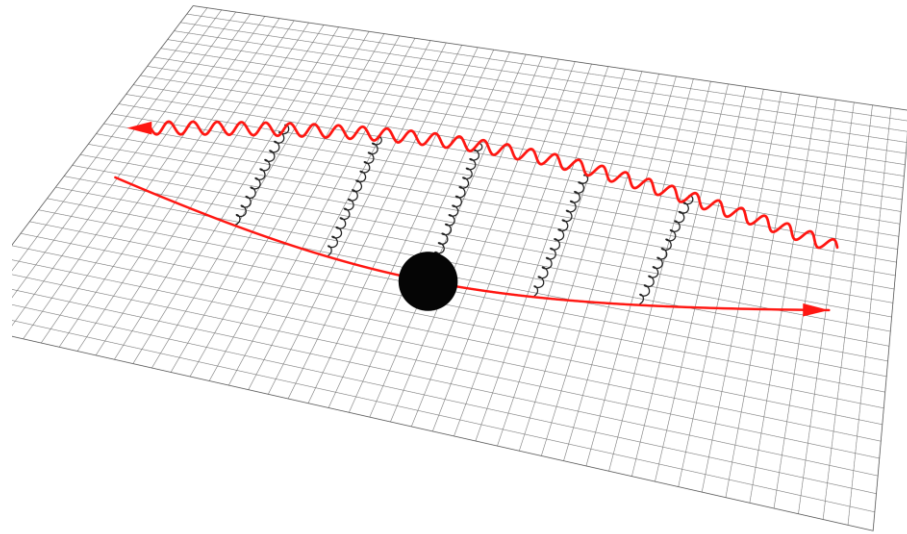
- Introduction to classical limits and motivations
- Lippmann-Schwinger equation $\longrightarrow$ 2PM potentials from amplitudes
- Long range interactions and on-shell amplitudes

➤ Computation of amplitudes and potentials

- One-loop amplitudes in GR
- **Results** for the 2PM potentials in GR
- Beyond GR amplitudes on-shell: FFR, $\Psi\Psi R$ and R^3
- **Results** for the 2PM potentials beyond GR

➤ Observables in gravitational wave (GW) lensing

Introduction to the problem



Flat space scattering

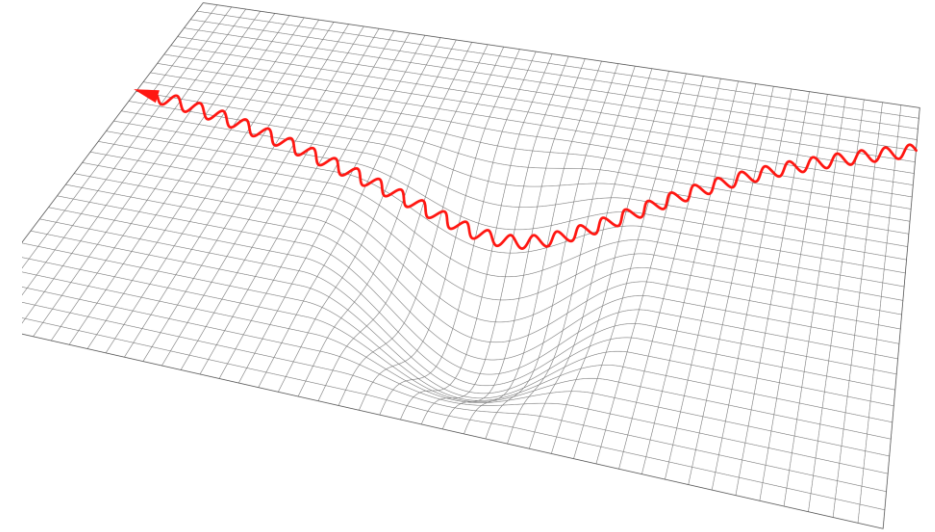


QFT amplitude

LS equation



Classical limit



Wave deflection by a potential



Solution of EOM

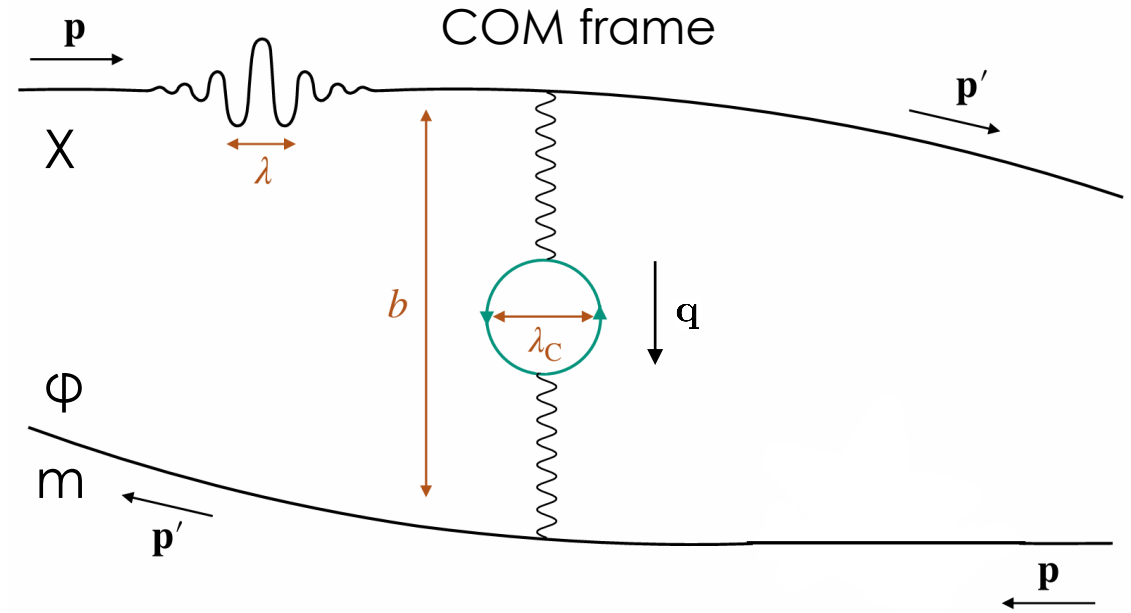
Why?

Well known techniques for analytical amplitude computations, extensible to higher PM orders (higher loops), spinning and beyond GR scenarios.

Definition of classical limits (Eikonal vs Born)

Scales of the problem [Correia, Isabella, 2025]

- Impact parameter: $b \sim \frac{1}{|\mathbf{q}|}$
- Massless wavelength: $\lambda \sim \frac{1}{\omega}$
- Compton wavelength: $\lambda_C \sim \frac{1}{m}$



Classical limit: $b \gg \lambda_C$

Eikonal limit $b \gg \lambda \gg \lambda_C, \quad |\mathbf{q}| \ll \omega \ll m$

Born limit $b \sim \lambda \gg \lambda_C, \quad |\mathbf{q}| \sim \omega \ll m$

$$\left(\frac{\mathbf{q}^2}{\omega^2} \right)^n$$

n-th Born correction

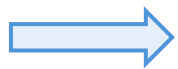
Motivations

In this work we
will compute:

2PM (one-loop) potentials from the gravity and beyond gravity scattering
of a helicity- λ wave and a massive scalar in the Born regime.

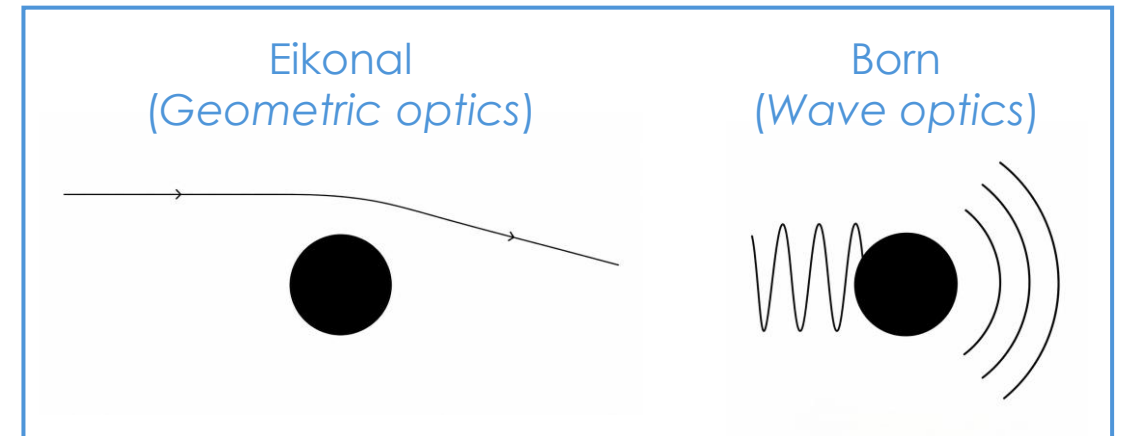
Why the Born limit?

- Less explored compared to the eikonal regime, more complicated classical limit
- Interesting features : breaking of gravitational coupling universality (GCU), helicity mixing effects.

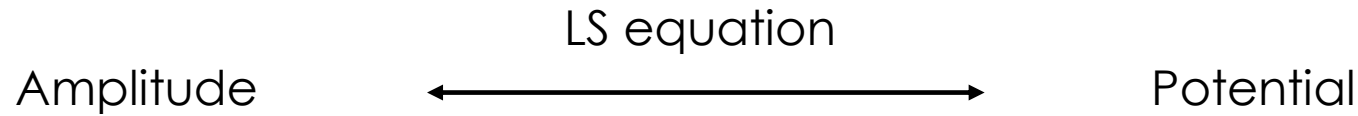


Wave optics regime of GW lensing

[C. Gonzalez, Parra-Martinez, et al., 2026]
[Nakamura, Deguchi, 1999]



Lippmann-Schwinger (LS) equation

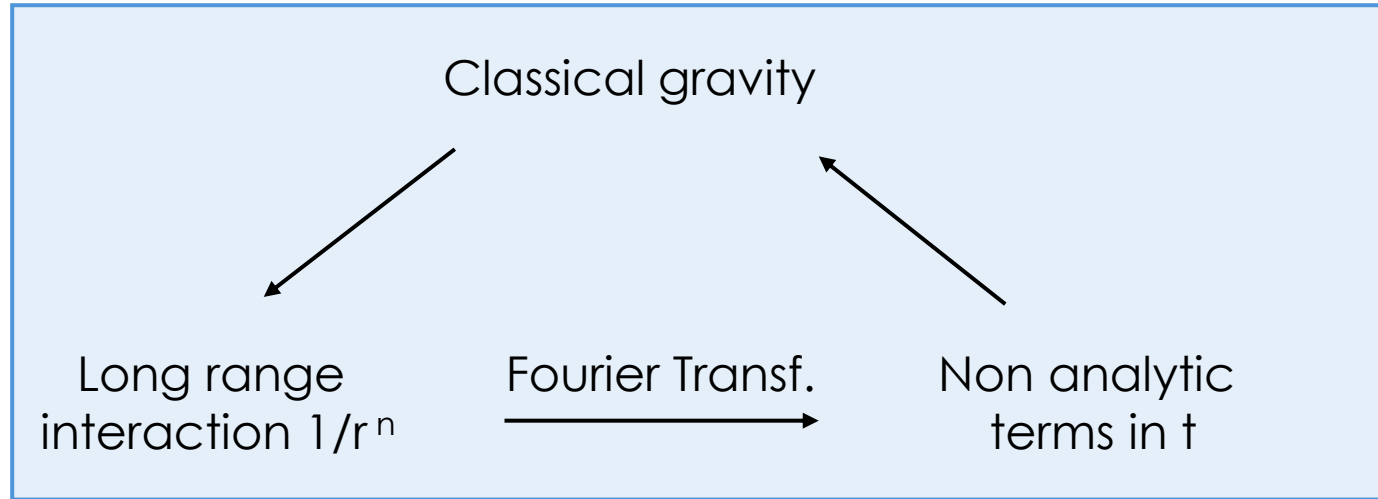


Lippmann-Schwinger (LS) eq. $\mathcal{M}(\mathbf{p}, \mathbf{p}') = V(\mathbf{p}, \mathbf{p}') + \int d^3\mathbf{l} \mathcal{M}(\mathbf{p}, \mathbf{l}) G(\mathbf{p}, \mathbf{l}) V(\mathbf{l}, \mathbf{p}')$

Real potential + Unitarity $\longrightarrow G(\mathbf{p}, \mathbf{k}) = \frac{1}{2m(2\pi)^3} \frac{1}{\mathbf{k}^2 - \mathbf{p}^2 - i\epsilon}$
[Todorov, 1971]

- 1PM (tree-level) potential $V^{(0)}(\mathbf{p}, \mathbf{p}') = \mathcal{M}^{(0)}(\mathbf{p}, \mathbf{p}')$
- 2PM (one-loop) potential $V^{(1)}(\mathbf{p}, \mathbf{p}') = \mathcal{M}^{(1)}(\mathbf{p}, \mathbf{p}') - \frac{1}{2m} \int \frac{d^3\mathbf{l}}{(2\pi)^3} \frac{\mathcal{M}^{(0)}(\mathbf{p}, \mathbf{l}) \mathcal{M}^{(0)}(\mathbf{l}, \mathbf{p}')}{\mathbf{l}^2 - \mathbf{p}^2}$

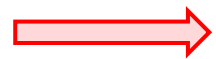
Long range interactions and on-shell amplitudes



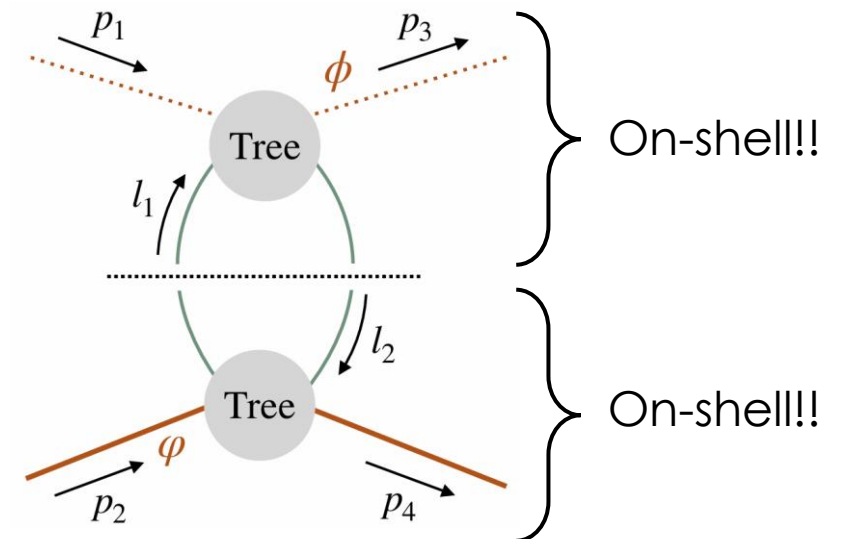
$$t = (p_1 + p_3)^2 = -|\mathbf{p}' - \mathbf{p}|^2 = -|\mathbf{q}|^2$$



Problem fully described by the t-channel discontinuity!
[Bjerrum-Bohr, Donoghue, Vanhove, 2014]



We can use the Spinor-helicity formalism!!



Master integrals decomposition

Decomposition in master integrals:

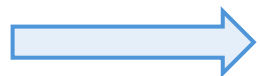
$$\mathcal{M}_\lambda^{(1)}(1_X^- 2_\varphi 3_X^+ 4_\varphi) \Big|_{\text{t-cut}} = \frac{\langle 1|p_2\sigma|3\rangle^{2\lambda}}{M_{Pl}^4 m^{4\lambda} (4\omega^2 - \mathbf{q}^2)^{2\lambda}} [c_\square(I_\square + I_X) + c_\triangle^m I_\triangle^m + c_\nabla^0 I_\nabla^0 + c_\circ I_\circ]$$

➤ Classical vs quantum:

$$\left. \begin{array}{c} c_\square(I_\square + I_X) \\ c_\triangle^m I_\triangle^m \end{array} \right\} \text{ Classical} \qquad \left. \begin{array}{c} c_\nabla^0 I_\nabla^0 \\ c_\circ I_\circ \end{array} \right\} \text{ Quantum}$$

➤ Poles for $\mathbf{q}^2 = 4\omega^2$, backward scattering $\cos(\theta) = -1$ $\cos \theta = 1 - \frac{\mathbf{q}^2}{2\omega^2}$ 

➤ $c_\square(I_\square + I_X) \sim \frac{ic_\square}{\mathbf{q}^2} \left(\frac{1}{\epsilon} - \log \mathbf{q}^2 \right) \longrightarrow$ Imaginary and IR divergent! $\longrightarrow$ It must cancel for a real and finite potential



Classical potential comes from $c_\triangle^m I_\triangle^m$ (and from iteration terms)

2PM helicity- λ potentials in the Born limit in GR

2PM graviton helicity flipping potential:

$$V_{\pm\mp}^h = \frac{m^2 \mathbf{q}_{\pm}^4}{4M_{Pl}^2 \omega^2 \mathbf{q}^2} - \frac{m^3 \mathbf{q}_{\pm}^4}{M_{Pl}^4 \omega^2 |\mathbf{q}|} \left(\frac{35}{512} + \frac{49 \mathbf{q}^2}{2048 \omega^2} \right) \quad \lambda = 2$$

2PM helicity conserving potentials:

$$\begin{aligned}
 V_{\pm\pm}^{\phi} &= \frac{4m^2 \omega^2}{M_{Pl}^2 \mathbf{q}^2} + \frac{15}{32} \frac{m^3 \omega^2}{M_{Pl}^4 |\mathbf{q}|} - \frac{1}{128} \frac{m^3 |\mathbf{q}|}{M_{Pl}^4} & \lambda = 0 \\
 V_{\pm\pm}^{\psi} &= \frac{4m^2 \omega^2}{M_{Pl}^2 \mathbf{q}^2} + \frac{15}{32} \frac{m^3 \omega^2}{M_{Pl}^4 |\mathbf{q}|} - \frac{11}{256} \frac{m^3 |\mathbf{q}|}{M_{Pl}^4} & \lambda = 1/2 \\
 V_{\pm\pm}^{\gamma} &= \frac{4m^2 \omega^2}{M_{Pl}^2 \mathbf{q}^2} + \frac{15}{32} \frac{m^3 \omega^2}{M_{Pl}^4 |\mathbf{q}|} - \frac{7}{128} \frac{m^3 |\mathbf{q}|}{M_{Pl}^4} & \lambda = 1 \\
 V_{\pm\pm}^{\Psi} &= \frac{4m^2 \omega^2}{M_{Pl}^2 \mathbf{q}^2} + \frac{15}{32} \frac{m^3 \omega^2}{M_{Pl}^4 |\mathbf{q}|} - \frac{11}{256} \frac{m^3 |\mathbf{q}|}{M_{Pl}^4} & \lambda = 3/2 \\
 V_{\pm\pm}^h &= \frac{4m^2 \omega^2}{M_{Pl}^2 \mathbf{q}^2} + \frac{15}{32} \frac{m^3 \omega^2}{M_{Pl}^4 |\mathbf{q}|} - \frac{1}{128} \frac{m^3 |\mathbf{q}|}{M_{Pl}^4} & \lambda = 2
 \end{aligned}$$

$\underbrace{\hspace{1.5cm}}_{1\text{PM}} \quad \underbrace{\hspace{1.5cm}}_{2\text{PM eikonal}} \quad \underbrace{\hspace{1.5cm}}_{2\text{PM Born}}$



- Non-zero helicity flipping Born potential!
- Fully real potential with no spurious poles
- Different helicity $\longrightarrow$ same eikonal pot. (GCU) but different Born pot. (breaking of GCU)

Beyond GR amplitudes

Higher dimensional operators of the form XXR, simplest beyond GR model:

$$\Delta\mathcal{L}_{XXR} = \sqrt{-g} \left(\frac{\mathcal{C}_{R^3}}{M_{Pl}^2} \mathcal{O}_{R^3} + \mathcal{C}_{FFR} \mathcal{O}_{FFR} + \mathcal{C}_{\Psi\Psi R} \mathcal{O}_{\Psi\Psi R} \right)$$

$$\mathcal{O}_{R^3} = R_{\mu\nu\alpha\beta} R^{\alpha\beta\rho\sigma} R_{\rho\sigma}^{\mu\nu}, \quad \mathcal{O}_{FFR} = F^{\mu\nu} F^{\alpha\beta} R_{\mu\nu\alpha\beta}, \quad \mathcal{O}_{\Psi\Psi R} = \Psi^{\mu\nu} \bar{\Psi}^{\alpha\beta} R_{\mu\nu\alpha\beta}$$

$$[\mathcal{C}_{R^3}] = -2, \quad [\mathcal{C}_{FFR}] = -2, \quad [\mathcal{C}_{\Psi\Psi R}] = -3$$

On-shell new 3pt vertices only for all-minus/all-plus helicity configuration

$$\mathcal{M}[1_X^- 2_X^- 3_X^-] = \frac{\mathcal{C}_X}{M_{Pl}} \langle 12 \rangle^{2\lambda-2} \langle 13 \rangle^2 \langle 23 \rangle^2, \quad \mathcal{M}[1_X^+ 2_X^+ 3_X^+] = \frac{\mathcal{C}_X}{M_{Pl}} [12]^{2\lambda-2} [13]^2 [23]^2$$

2PM potentials in the Born limit beyond GR

2PM helicity flipping potentials:

$$V_{\pm\mp}^{FFR} = -\frac{C_{FFR}}{M_{Pl}^2} \frac{4m^2\omega^2\mathbf{q}_{\pm}^2}{\mathbf{q}^2} \left[1 - \frac{m|\mathbf{q}|}{4M_{Pl}^2} \left(\frac{9}{32} + \frac{7\mathbf{q}^2}{64\omega^2} \right) \right]$$

$$V_{\pm\mp}^{\Psi\Psi R} = -\frac{C_{\Psi\Psi R}}{M_{Pl}^2} \frac{4m^2\omega^2\mathbf{q}_{\pm}^3}{\mathbf{q}^2} \left[1 - \frac{m|\mathbf{q}|}{4M_{Pl}^2} \left(\frac{15}{32} + \frac{75\mathbf{q}^2}{512\omega^2} \right) \right]$$

$$V_{\pm\mp}^{R^3} = -\frac{C_{R^3}}{M_{Pl}^2} \frac{4m^2\omega^2\mathbf{q}_{\pm}^4}{\mathbf{q}^2} \left[1 - \frac{m|\mathbf{q}|}{4M_{Pl}^2} \left(\frac{5}{8} + \frac{3\mathbf{q}^2}{16\omega^2} \right) \right]$$

2PM helicity conserving potentials:

$$V_{\pm\pm}^{FFR} = 0$$

$$V_{\pm\pm}^{\Psi\Psi R} = 0$$

$$V_{\pm\pm}^{R^3} = \frac{C_{R^3}}{M_{Pl}^4} \frac{m^3\omega^2\mathbf{q}^4}{32|\mathbf{q}|}$$

- Beyond GR corrections are mainly for helicity flipping potentials $\longrightarrow$ opposite of GR
- Fully real potential with no spurious poles
- Different helicity $\longrightarrow$ (almost) same eikonal pot. but different Born pot.
- Only helicity conserving potential appears for R^3 at one-loop and second Born correction!

GW lensing (1)

- GW lensing happens in triple systems

➡ One of the goals of the LISA mission

- Standard signature of GW lensing

➡ Multiple copies of the same GW

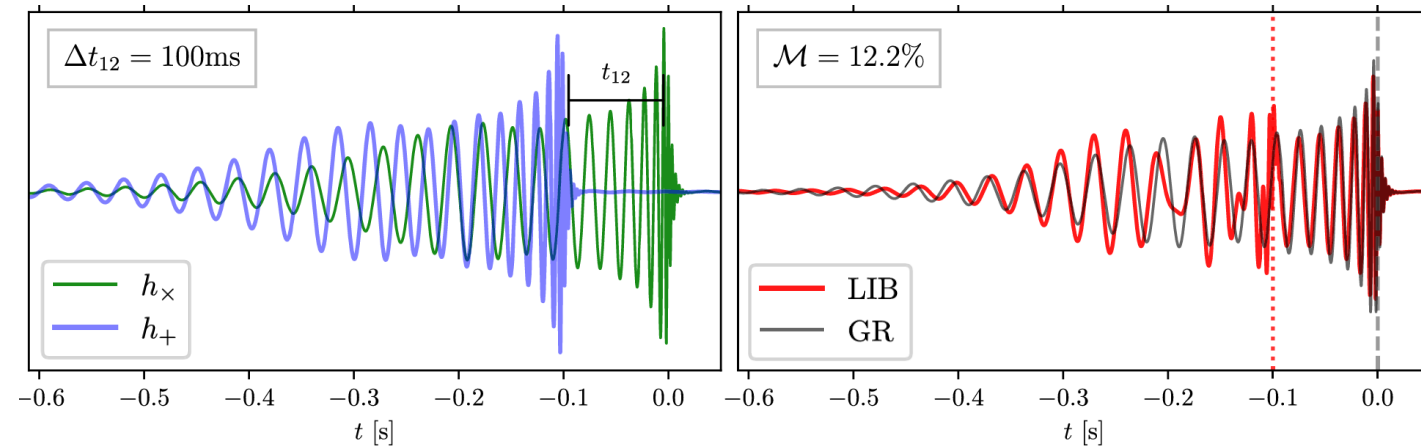
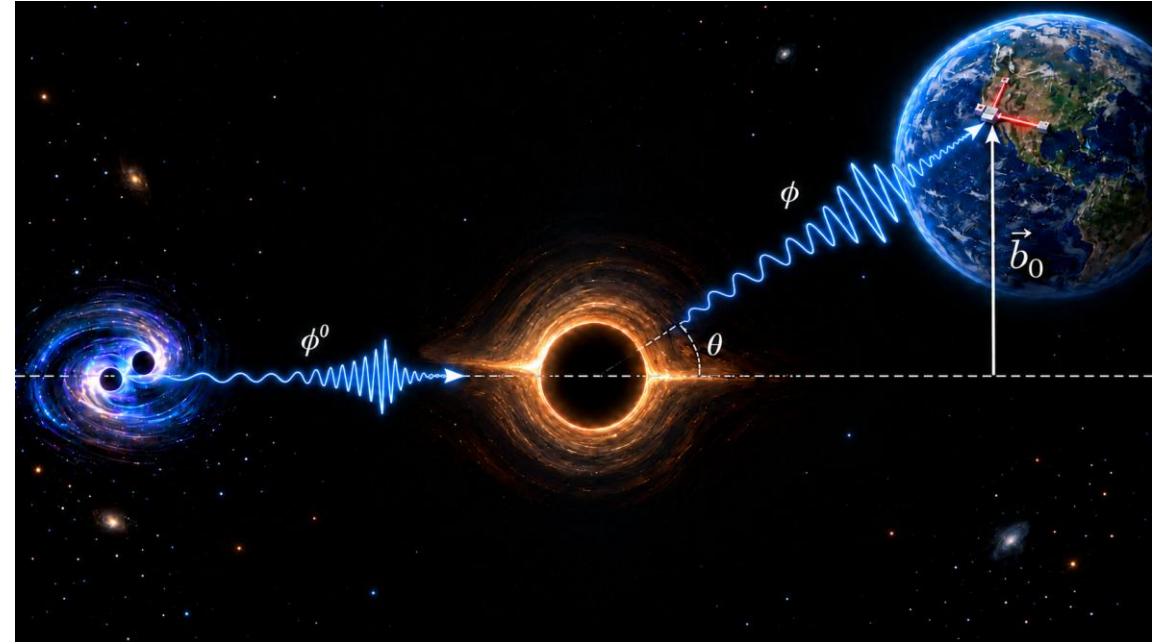
- Wave optics (Born limit) effects are expected to appear in LISA

[Pijnenburg, Cusin, Pitrou, Uzan, 2024]

- Beyond GR effects in GW lensing

➡ Time delay of the +,x polarizations

[Ezquiaga, Zumalacárregui 2020][Goyal, Vijaykumar et al. 2023]



Picture from [Goyal, Vijaykumar et al. 2023]

GW lensing (2)

Lensed outgoing wave $\phi(\mathbf{b}) = F_{diff}(\mathbf{b}_0)\phi^0(\mathbf{b})$

Diffraction integral $F_{diff}(\mathbf{b}_0) \propto \int d^2\mathbf{b} e^{i\omega t_d(\mathbf{b}, \mathbf{b}_0) - i\omega\psi(\mathbf{b})}$

where: $t_d(\mathbf{b}, \mathbf{b}_0) = \frac{z_{so}}{2z_{sl}z_{lo}} \left(\mathbf{b} - \frac{z_{sl}}{z_{so}} \mathbf{b}_0 \right)^2$ and $\psi(\mathbf{b}) \sim \int \frac{d^2\mathbf{q}}{(2\pi)^2} e^{i\mathbf{q}\cdot\mathbf{b}} V(\mathbf{p}, \mathbf{p}')$

Born potential
(previous slides)

Potential from the LS equation



Diffraction integral (DI) GW lensing

For more details [Nakamura, Deguchi, 1999] [C. Gonzalez, Parra-Martinez, et al., 2026]

GW lensing (3)

In our case the potential is a matrix!!



Diffraction integral is a matrix

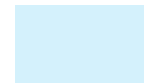
$$F = \begin{pmatrix} F_{++} & F_{+-} \\ F_{-+} & F_{--} \end{pmatrix}$$

Photon:

$$V^\gamma = \begin{pmatrix} V_{++}^\gamma & V_{-+}^{FFR} \\ V_{+-}^{FFR} & V_{--}^\gamma \end{pmatrix}$$

Graviton:

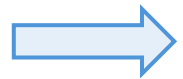
$$V^h = \begin{pmatrix} V_{++}^h + V_{++}^{R^3} & V_{-+}^h + V_{-+}^{R^3} \\ V_{+-}^h + V_{+-}^{R^3} & V_{--}^h + V_{--}^{R^3} \end{pmatrix}$$



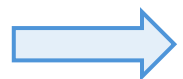
EH GR



Beyond GR



Helicity conserving F_{++} and F_{--} : beyond GR corrections very suppressed



$$F_{\pm\mp}^\gamma(\mathbf{b}_0) \sim c_{FFR,hb}^{(0,b)} \mathcal{C}_{FFR} \int d^2\mathbf{b} e^{i\omega[t_d(\mathbf{b},\mathbf{b}_0) - \log \frac{b}{b_0}]} \frac{b_\pm^2}{b^4}$$

$$F_{\pm\mp}^h(\mathbf{b}_0) \sim \left(\frac{c_{EH,hb}^{(0,b)}}{\omega^4} + c_{R^3,hb}^{(0,b)} \mathcal{C}_{R^3} \right) \int d^2\mathbf{b} e^{i\omega[t_d(\mathbf{b},\mathbf{b}_0) - \log \frac{b}{b_0}]} \frac{b_\pm^4}{b^8}$$

Helicity breaking F_{+-} and F_{-+}
sensible to beyond GR
corrections!!

GW lensing (4)

What happens in the +,x basis?

$$\mathbb{h} = h_{+, \times}$$

$$\mathbb{h} = \mathbb{F} h^0$$



$$\mathbb{F} \simeq \begin{pmatrix} \mathcal{F}_+ & 0 \\ 0 & \mathcal{F}_- \end{pmatrix}$$

$$\mathcal{F}_{\pm} \simeq \sum_j \mu_j e^{i\omega \left[\frac{b_j^2 - b_0^2}{2} - f(b_j) \mp g(b_j) \right]}$$

Beyond GR



General incoming GW:

$$\mathbb{h}^0(\omega) = A_+ \mathbb{h}_+(\omega) + A_{\times} \mathbb{h}_{\times}(\omega)$$

Outgoing GW in time domain:

$$\tilde{\mathbb{h}}(t) = \sum_j \mu_j \left[\tilde{A}_+(t - t_{fj} - t_{gj}) \mathbb{h}_+ + \tilde{A}_{\times}(t - t_{fj} + t_{gj}) \mathbb{h}_{\times} \right]$$



Time delay of the + and x polarizations, sensible to beyond GR $\Delta_t = 2t_g \propto g(b)$

- Same effect analysed in [Ezquiaga, Zumalacárregui 2020] [Goyal, Vijaykumar et al. 2023]
- Same conclusion for R³ and FFR

Summary and future outlooks

We computed the 2PM potentials from the scattering in GR and beyond of a massive scalar and an helicity- λ wave, with $\lambda = 0, \frac{1}{2}, 1, \frac{3}{2}, 2$, in the Born limit



Observables in GW lensing beyond GR

- Consistency of our Born limit potentials: reality, poles free and correct eikonal limit.
- GW lensing observables: beyond GR time delays.

Future outlooks:

- Computation for massive spinning particles (Kerr BH)
- Extension to scalar tensor theories

Thank you!!

Bibliography

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Backup slides

Kinematics of the process

Gravitational scattering of a massless helicity- λ particle X and a massive scalar ϕ

Momenta (all-incoming convention) COM frame:

$$p_1^\mu = (\omega, \mathbf{p}) \qquad p_2^\mu = (\sqrt{|\mathbf{p}|^2 + m^2}, -\mathbf{p})$$
$$p_3^\mu = (-\omega, -\mathbf{p}') \qquad p_4^\mu = (\sqrt{|\mathbf{p}|^2 + m^2}, \mathbf{p}')$$

3-momentum exchanged: $\mathbf{q} = \mathbf{p}' - \mathbf{p}$

The Mandelstam t -variable: $t = (p_1 + p_3)^2 = -|\mathbf{p}' - \mathbf{p}|^2 = -|\mathbf{q}|^2$

Kinematic constraints: $|\mathbf{p}| = |\mathbf{p}'| = \omega$ and $\mathbf{p} \cdot \mathbf{q} = -\frac{|\mathbf{q}|^2}{2}$

Spinor helicity-formalism (1)

The complexified Lorentz group $SO(1, 3)_{\mathbb{C}}$ is isomorphic to $SL(2, \mathbb{C}) \times SL(2, \mathbb{C})$



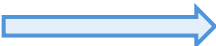
We can define the map for massless particles: $p_{\alpha\dot{\alpha}} = p_{\mu}\sigma^{\mu}_{\alpha\dot{\alpha}} = \lambda_{\alpha}\tilde{\lambda}_{\dot{\alpha}}$

λ and $\tilde{\lambda}$ are Weyl spinors $(\lambda, \tilde{\lambda}) \in SL(2, \mathbb{C}) \times SL(2, \mathbb{C})$

The spinors satisfy: $\lambda\lambda = 0 = \tilde{\lambda}\tilde{\lambda}$ which indeed implies: $p^2 = 0$

Spinor helicity-formalism (2)

Consider the spinor-transformation: $\lambda_\alpha \rightarrow \omega \lambda_\alpha$ and $\tilde{\lambda}_{\dot{\alpha}} \rightarrow \omega^{-1} \tilde{\lambda}_{\dot{\alpha}}$ $\longrightarrow$ $p_{\alpha\dot{\alpha}}$ left invariant!!

 Spinor transformations are little group transformations

The amplitude transforms as $\mathcal{M}_{h_1, \dots, h_n}(\{\omega_i |i\rangle, \omega_i^{-1} |i]\}) = (\omega_i)^{-2h_i} \mathcal{M}_{h_1, \dots, h_n}(\{|i\rangle, |i]\}) \quad \forall i = 1, \dots, n$

 Possible to compute 3-point tree-level amplitudes

↓

Using unitarity we can compute 4-point amplitudes

↓

Gluing tree amplitudes we can compute loop amplitudes

Simplified computation
doesn't consider
off-shell pieces

↓

Main source of
complexity in
Feynmann diagrams

Lippmann-Schwinger (LS) equation (1)

Lippmann-Schwinger (LS) eq. $\mathcal{M}(\mathbf{p}, \mathbf{p}') = V(\mathbf{p}, \mathbf{p}') + \int d^3\mathbf{l} \mathcal{M}(\mathbf{p}, \mathbf{l}) G(\mathbf{p}, \mathbf{l}) V(\mathbf{l}, \mathbf{p}')$

Following [\[Todorov, 1971\]](#) we require:

- Reality of $V(\mathbf{p}, \mathbf{p}')$
- Unitarity of the amplitude

Born series solution of the LS: $\mathcal{M} = V + VGV + VGVGV + \dots = \frac{V}{1 - GV}$

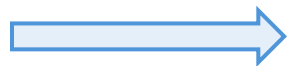
Real potential
+
unitarity $\longrightarrow \text{Im } G(\mathbf{p}, \mathbf{k}) = \frac{\delta(|\mathbf{k}|^2 - |\mathbf{p}|^2)}{16\pi^2 \sqrt{s}} \longrightarrow G(\mathbf{p}, \mathbf{k}) = \frac{1}{2m(2\pi)^3} \frac{1}{\mathbf{k}^2 - \mathbf{p}^2 - i\epsilon}$



Possible to add regular (real) terms to $G(\mathbf{p}, \mathbf{k})$



Change coordinates
of the potential!



This choice of $G(\mathbf{p}, \mathbf{k})$ corresponds to isotropic coordinates [\[Correia, Isabella, 2025\]](#)

On-shell t-channel discontinuity

One-loop t-channel discontinuity helicity- λ particle – massive scalar mediated by gravity for $\lambda = 0, 1, 2$

$$\text{Disc}_t \mathcal{M}_\lambda^{(1)}(1_X^- 2_\varphi^+ 3_X^+ 4_\varphi^-) = \frac{1}{2t^2 M_{Pl}^4} \int d\Pi_{l_1 l_2} \frac{\text{Tr}[l_1 \bar{\sigma} p_1 \sigma l_2 \bar{\sigma} p_2 \sigma]^{4-2\lambda} \langle 1 | l_1 \sigma p_2 \bar{\sigma} l_2 \sigma | 3 \rangle^{2\lambda} + (l_1 \leftrightarrow l_2)}{(2p_1 l_1)(2p_1 l_2)(2p_2 l_1)(2p_2 l_2)}$$

Very easy and compact result with spinor helicity formalism



Much easier computation than using Feynmann diagrams!!!



Next step: discontinuity in terms of discontinuities of scalar master integrals.

Graviton t-discontinuity computation

Tree-level amplitudes: graviton – massive scalar in gravity

$$\mathcal{M}^{(0)}(1_h^-, 2_h^+, 3_h^-, 4_h^+) = \frac{(\lambda_3 \lambda_1)^4 (\tilde{\lambda}_4 \tilde{\lambda}_2)^4}{M_{Pl}^2 (2p_1 p_2) (2p_1 p_3) (2p_1 p_4)}$$

$$\mathcal{M}^{(0)}[1_h^- 2_\varphi 3_h^- 4_\varphi] = -\frac{m^4 (\lambda_1 \lambda_3)^4}{M_{Pl}^2 t (s - m^2) (u - m^2)}$$

$$\mathcal{M}^{(0)}(1_h^-, 2_h^+, 3_h^+, 4_h^-) = \frac{(\lambda_4 \lambda_1)^4 (\tilde{\lambda}_3 \tilde{\lambda}_2)^4}{M_{Pl}^2 (2p_1 p_2) (2p_1 p_3) (2p_1 p_4)}$$

$$\mathcal{M}^{(0)}[1_h^- 2_\varphi 3_h^- 4_\varphi] = -\frac{m^4 (\tilde{\lambda}_1 \tilde{\lambda}_3)^4}{M_{Pl}^2 t (s - m^2) (u - m^2)}$$

t-channel discontinuity

$$\begin{aligned} \text{Disc}_t \mathcal{M}^{(1)}(1_h^- 2_\varphi 3_h^+ 4_\varphi) &= \frac{1}{2} \int d\Pi_{l_1 l_2} \left(\mathcal{M}^{(0)}(1_h^-, 3_h^+, (-l_1)_h^-, (-l_2)_h^+) \mathcal{M}^{(0)}(2_\varphi, 4_\varphi, l_{1h}^+, l_{2h}^-) + \right. \\ &\quad \left. + \mathcal{M}^{(0)}(1_h^-, 3_h^+, (-l_1)_h^+, (-l_2)_h^-) \mathcal{M}^{(0)}(2_\varphi, 4_\varphi, l_{1h}^-, l_{2h}^+) \right) = \\ &= \frac{1}{2M_{Pl}^4 t^2} \int d\Pi_{l_1 l_2} \frac{(\lambda_{l_1} \lambda_1)^4 (\tilde{\lambda}_{l_2} \tilde{\lambda}_3)^4 (\lambda_{l_2} p_2 \sigma \tilde{\lambda}_{l_1})^4 + (\lambda_{l_2} \lambda_1)^4 (\tilde{\lambda}_{l_1} \tilde{\lambda}_3)^4 (\lambda_{l_1} p_2 \sigma \tilde{\lambda}_{l_2})^4}{(2p_1 l_1) (2p_1 l_2) (2p_2 l_1) (2p_2 l_2)} = \\ &= \frac{1}{2M_{Pl}^4 t^2} \int d\Pi_{l_1 l_2} \frac{(\lambda_1 l_1 \sigma p_2 \bar{\sigma} l_2 \sigma \tilde{\lambda}_3)^4 + (l_1 \leftrightarrow l_2)}{(2p_1 l_1) (2p_1 l_2) (2p_2 l_1) (2p_2 l_2)} \end{aligned}$$

PM/quantum corrections (scalar case)

Considering the different contributions to the amplitude:

$$\begin{aligned}
 & \left. \begin{aligned}
 c_{\square}(I_{\square}^{st} + I_{\square}^{tu}) &\sim \frac{m^3 \omega^3}{M_{Pl}^4 \mathbf{q}^2} \sim \mathcal{O}\left(\mathcal{M}_{tree} \frac{m\omega}{M_{Pl}^2}\right) \sim \mathcal{O}\left(\mathcal{M}_{tree} \frac{R_s}{b} \frac{\omega}{|\mathbf{q}|}\right) \sim \mathcal{O}\left(\mathcal{M}_{tree} \frac{R_s}{b}\right) \\
 c_{\Delta} I_{\Delta} &\sim \frac{m^3 |\mathbf{q}|}{M_{Pl}^4} \left(1 + \# \frac{\omega^2}{\mathbf{q}^2}\right) \sim \left(\mathcal{M}_{tree} \frac{m|\mathbf{q}|}{M_{Pl}^2}\right) \sim \mathcal{O}\left(\mathcal{M}_{tree} \frac{R_s}{b}\right)
 \end{aligned} \right\} \text{PM corrections} \\
 & \left. \begin{aligned}
 c_{\circ} I_{\circ} &\sim \frac{m^2 \mathbf{q}^2}{M_{Pl}^4} \left(1 + \# \frac{\omega^2}{\mathbf{q}^2}\right) \sim \mathcal{O}\left(\mathcal{M}_{tree} \frac{m|\mathbf{q}|}{M_{Pl}^2} \frac{|\mathbf{q}|}{m}\right) \sim \mathcal{O}\left(\mathcal{M}_{tree} \frac{R_s}{b} \frac{l_c}{b}\right) \\
 c_{\nabla} I_{\nabla} &\sim \frac{m^2 \omega^2}{M_{Pl}^4} \sim \mathcal{O}\left(\mathcal{M}_{tree} \frac{m|\mathbf{q}|}{M_{Pl}^2} \frac{|\mathbf{q}|}{m}\right) \sim \mathcal{O}\left(\mathcal{M}_{tree} \frac{R_s}{b} \frac{l_c}{b}\right)
 \end{aligned} \right\} \text{Quantum corrections}
 \end{aligned}$$

Tree level amplitude: $\mathcal{M}(1_{\phi} 2_{\varphi} 3_{\phi} 4_{\varphi})_{tree} = \frac{(2m\omega)^2}{M_{Pl}^2 |\mathbf{q}|^2}$

PM parameter: $\frac{R_s}{b} = \frac{m|\mathbf{q}|}{M_{Pl}^2}$

Master scalar integrals dictionary

$$I_{\circ}^t = \int \frac{d^4 k}{i(2\pi)^4} \frac{1}{k^2 (k + p_1 + p_3)^2}$$

$$I_{\nabla}^t = \int \frac{d^4 k}{i(2\pi)^4} \frac{1}{k^2 (k + p_1)^2 (k + p_1 + p_3)^2}$$

$$I_{\triangle}^t = \int \frac{d^4 k}{i(2\pi)^4} \frac{1}{k^2 [(k + p_2)^2 - m^2] (k + p_2 + p_4)^2}$$

$$I_{\square} = \int \frac{d^4 k}{i(2\pi)^4} \frac{1}{k^2 (k + p_1)^2 (k + p_1 + p_3)^2 [(k - p_2)^2 - m^2]}$$

$$I_X = \int \frac{d^4 k}{i(2\pi)^4} \frac{1}{k^2 (k + p_1)^2 (k + p_1 + p_3)^2 [(k - p_4)^2 - m^2]}$$

Box and triangle master scalar integrals

Sum of box and crossed-box diagram:

$$I_{\square} = I_X \quad I_{\square} + I_X = \frac{1}{2m} \int \frac{d^3\mathbf{l}}{(2\pi)^3} \frac{1}{|\mathbf{l} - \mathbf{p}|^2 (\mathbf{l}^2 - \mathbf{p}^2) |\mathbf{l} - \mathbf{p} - \mathbf{q}|^2} = \underbrace{-\frac{i}{16\pi m \mathbf{q}^2 |\mathbf{p}|} \left[\frac{1}{\epsilon} - \log \mathbf{q}^2 \right]}_{\text{Imaginary}}$$

Massive triangle integral

$$I_{\triangle} = \frac{1}{2m} \int \frac{d^3\mathbf{l}}{(2\pi)^3} \frac{1}{|\mathbf{l} - \mathbf{p}|^2 |\mathbf{l} - \mathbf{p} - \mathbf{q}|^2} = -\frac{1}{32m |\mathbf{q}|}$$

Same result of [\[Cheung, Clifford et al., 2018\]](#)

Graviton amplitude in the Born limit

Amplitude in the Born limit

$$\mathcal{M}^{(1)}(1_h^- 2_\varphi 3_h^+ 4_\varphi) \Big|_{\text{t-cut}} = \frac{1}{M_{Pl}^4} \frac{\langle 1 | p_2 \sigma | 3 \rangle^4}{m^8 (4\omega^2 - \mathbf{q}^2)^4} [c_\square (I_\square + I_X) + c_\triangle^m I_\triangle^m + c_\nabla^0 I_\nabla^0 + c_\circ I_\circ]$$

The classical coefficients are:

$$c_\square = M^8 (\mathbf{q}^4 - 4\mathbf{q}^2 \omega^2 + 16\omega^4)^2 \longrightarrow \text{Should cancel for a real and finite potential!!}$$

$$c_\triangle^m = -\frac{M^8}{4} (\mathbf{q}^2 + 4\omega^2) (15\mathbf{q}^4 - 88\mathbf{q}^2 \omega^2 + 240\omega^4)$$



Poles for $\mathbf{q}^2 = 4\omega^2$, backward scattering $\cos(\theta) = -1$ $\cos \theta = 1 - \frac{\mathbf{q}^2}{2\omega^2}$

Graviton iteration term (1)

One-loop helicity conserving potential, graviton scattering:

$$\begin{aligned}
 V_{--}^{h,(1)}(\mathbf{p}, \mathbf{p}') &= \mathcal{M}^{(1)}(1_h^- 2_\varphi 3_h^+ 4_\varphi) - \frac{1}{2m} \int \frac{d^d \mathbf{l}}{(2\pi)^d} \sum_{k=\pm} \frac{\mathcal{M}^{(0)}(1_h^- 2_\varphi l_h^{-k} l'_\varphi) \mathcal{M}^{(0)}(3_h^- 4_\varphi l_h^{-k} l'_\varphi)^*}{\mathbf{l}^2 - \mathbf{p}^2} = \\
 &= \mathcal{M}^{(1)}(1_h^- 2_\varphi 3_h^+ 4_\varphi) - \frac{1}{2m} \int \frac{d^d \mathbf{l}}{(2\pi)^d} \left[\frac{\mathcal{M}^{(0)}(1_h^- 2_\varphi l_h^+ l'_\varphi) \mathcal{M}^{(0)}(3_h^- 4_\varphi l_h^+ l'_\varphi)^*}{\mathbf{l}^2 - \mathbf{p}^2} + \frac{\mathcal{M}^{(0)}(1_h^- 2_\varphi l_h^- l'_\varphi) \mathcal{M}^{(0)}(3_h^- 4_\varphi l_h^- l'_\varphi)^*}{\mathbf{l}^2 - \mathbf{p}^2} \right]
 \end{aligned}$$

Helicity breaking tree-level amplitudes
contribute (non-zero only in the Born limit)



Fundamental to cancel spurious poles!

$$\mathcal{M}^{(0)}(1_h^- 2_\varphi 3_h^+ 4_\varphi) \simeq \frac{\langle 1 | p_2 \sigma | 3 \rangle^4}{M_{Pl}^2 (2m\omega)^2 \mathbf{q}^2}$$

$$\underbrace{\mathcal{M}^{(0)}(1_h^- 2_\varphi 3_h^- 4_\varphi)} \simeq \frac{m^4 \langle 13 \rangle^4}{M_{Pl}^2 \mathbf{q}^2 (2m\omega)^2}$$

Non zero only for gravitons

Graviton iteration term (2)

Iteration term

$$\begin{aligned}
 I_{IT}^h &= \int \frac{d^d \mathbf{l}}{(2\pi)^d} \left[\overset{\text{Helicity conserving}}{\frac{\langle 1|p_2 \sigma l \bar{\sigma} p_4 \sigma|3\rangle^4}{(\mathbf{l}^2 - \mathbf{p}^2)(1 - \mathbf{p})^2(1 - \mathbf{p}')^2}} + m^8 \overset{\text{Helicity breaking}}{\frac{\langle 1|l \sigma|3\rangle^4}{(\mathbf{l}^2 - \mathbf{p}^2)(1 - \mathbf{p})^2(1 - \mathbf{p}')^2}} \right] = \\
 &= 2m^5 \langle 1|p_2 \sigma|3\rangle^4 \left[\frac{16\omega^4(\mathbf{q}^4 - 4\mathbf{q}^2\omega^2 + 16\omega^4)^2}{(4\omega^2 - \mathbf{q}^2)^4} (I_{\square} + I_X) - \right. \\
 &\quad \left. - \frac{(32768\omega^{10} - 19456\omega^8\mathbf{q}^2 + 6336\omega^6\mathbf{q}^4 - 944\omega^4\mathbf{q}^6 + 84\omega^2\mathbf{q}^8 - \mathbf{q}^{10})}{4(4\omega^2 - \mathbf{q}^2)^4} I_{\Delta} + \omega^2 \left(32 + \frac{7\mathbf{q}^2}{\omega^2} + \frac{35\mathbf{q}^4}{16\omega^4} \right) I_{\Delta} \right]
 \end{aligned}$$

Precisely the boxes
coefficient
of the discontinuity!

Only iteration term
In Eikonal limit!

Iteration term is a function of l_0 : $l_0 = \sqrt{\mathbf{l}^2} = \sqrt{\omega^2 + \mathbf{l}^2 - \omega^2} \simeq \omega + \frac{\mathbf{l}^2 - \omega^2}{2\omega} - \frac{(\mathbf{l}^2 - \omega^2)^2}{8\omega^3} + \dots$

2PM potential: $V_{--}^{h,(1)}(\mathbf{p}, \mathbf{p}') = \mathcal{M}^{(1)}(1_h^- 2_\varphi 3_h^+ 4_\varphi) - \mathcal{M}_{IT}(1_h^- 2_\varphi 3_h^+ 4_\varphi) = -\frac{\langle 1|p_2 \sigma|3\rangle^4}{M_{Pl}^4 \omega^2} \left(\frac{15}{16} + \frac{29\mathbf{q}^2}{64\omega^2} + \frac{35\mathbf{q}^4}{256\omega^4} \right) I_{\Delta}$

Potential poles-free and fully real!

Spinor factor: $|\langle 1|p_2 \sigma|3\rangle| = 2m\omega \sqrt{1 - \frac{\mathbf{q}^2}{4\omega^2}}$

Wave equation

Discussion from [\[Correia, Isabella, 2025\]](#)

Regge-Wheeler equation for a scalar in a Schwarzschild BH:

$$\partial_\mu(\sqrt{-g} g^{\mu\nu} \partial_\nu \phi) = 0 \quad g_{\mu\nu} = \text{diag}(A(r), -B(r), -B(r), -B(r))$$

After some manipulations it becomes:

$$(\nabla^2 + \omega^2)\phi = \left(\left(1 - \frac{\left(1 + \frac{GM}{r}\right)^6}{\left(1 - \frac{GM}{r}\right)^2}\right)\omega^2 + \frac{2G^2 M^2}{G^2 M^2 r - r^3} \partial_r \right) \phi = V \phi$$

With the potential up to $\mathcal{O}(G^2)$:

$$V = -\left(\frac{4MG}{r} + \frac{15G^2 M^2}{2r^2}\right)\omega^2 - \frac{G^2 M^2}{2r^3} \partial_r + \mathcal{O}(G^3)$$

FT of the scalar potential

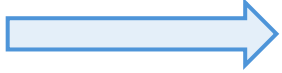
$$V(\mathbf{x}) = -\frac{1}{2M} \int \frac{d^3 \mathbf{k}}{(2\pi)^3} e^{i\mathbf{k} \cdot \mathbf{x}} V(\mathbf{p}, \mathbf{p}') = -\left(\frac{4GM}{|\mathbf{x}|} + \frac{15G^2 M^2}{2|\mathbf{x}|^2}\right)\omega^2 + \frac{G^2 M^2}{4|\mathbf{x}|^4}$$

Equal for an on-shell potential

Helicity- λ amplitudes in the eikonal limit

Spinor factor in the eikonal limit: $|\langle 1|p_2\sigma|3\rangle| = 2m\omega$

Amplitude: $\mathcal{M}^{(1)}(1_X^- 2_\varphi 3_X^+ 4_\varphi) = \frac{1}{M_{Pl}^4} \left[[c_X]_\square (I_\square + I_X) + [c_X]_\triangle^M I_\triangle^M + [c_X]_\nabla^0 I_\nabla^0 + [c_X]_\circ I_\circ \right]$

Superclassical	$\left\{ \begin{array}{ll} [c_X]_\square = 16M^4\omega^4, & X = \phi, \psi, \gamma, h \\ [c_X]_\triangle^M = -15M^4\omega^2, & X = \phi, \psi, \gamma, h \end{array} \right.$		Gravitational coupling universality!! [Bjerrum-Bohr, Donoghue et al., 2015]
Classical			

Quantum

$$\left\{ \begin{array}{llll} [c_X]_\nabla^0 = -12\mathbf{q}^2 M^2 \omega^2, & [c_h]_\nabla^0 = -16\mathbf{q}^2 M^2 \omega^2, & X = \phi, \psi, \gamma \\ [c_\phi]_\circ = \frac{3}{10} M^2 \omega^2, & [c_\psi]_\circ = -\frac{17}{10} M^2 \omega^2, & [c_\gamma]_\circ = -\frac{161}{30} M^2 \omega^2, & [c_h]_\circ = -\frac{29}{2} M^2 \omega^2 \end{array} \right.$$

Observables in GW lensing - eikonal

Matching diffraction integral and classical amplitude

[C. Gonzalez, Parra-Martinez, et al., 2026]

$$\frac{z_{so}}{z_{sl}z_{lo}} e^{i\frac{\mathbf{b}_0^2}{2r_F^2}} \frac{\mathcal{M}}{8\pi\sqrt{s}} = F_{diff} - 1$$

Eikonal limit

Partial wave amplitude: $\mathcal{M} = \mathcal{N} \sum_{l=0}^{\infty} (2l+1)(e^{2i\delta_l} - 1)P_l(\cos\theta) \xrightarrow{l \gg 1} \mathcal{N} \frac{\omega^2}{\pi} \int d^2\mathbf{b} e^{-i\mathbf{q}\cdot\mathbf{p}} \left(e^{i\delta_{eik}(\mathbf{b})} - 1 \right)$

Eikonal phase

- Impact parameter: $b = \frac{l+1/2}{\omega}$
- Legendre polynomials asymptotic expansion $P_l(\cos\theta) \xrightarrow{l \gg 1} J_0(\omega b\theta)$

Matching relation



$$\delta_{eik}(\mathbf{b}) = -\omega\psi(\mathbf{b})$$

Next step: extend this matching to include the first Born correction

Gravitational lensing in detail (1)

In our case the potential is a matrix!!

Photon:

$$V^\gamma = \begin{pmatrix} V_{++}^\gamma & V_{-+}^{FFR} \\ V_{+-}^{FFR} & V_{--}^\gamma \end{pmatrix}$$

Graviton:

$$V^h = \begin{pmatrix} V_{++}^h + V_{++}^{R^3} & V_{-+}^h + V_{-+}^{R^3} \\ V_{+-}^h + V_{+-}^{R^3} & V_{--}^h + V_{--}^{R^3} \end{pmatrix}$$

After some manipulations the diffraction integral matrix:

$$F_{\pm\pm}(\mathbf{b}_0) \sim \int d^2\mathbf{b} e^{i\omega t_d(\mathbf{b}, \mathbf{b}_0)} \frac{\left(e^{-i\omega\xi_+(b)} \overset{\downarrow}{+} e^{-i\omega\xi_-(b)} \right)}{2}$$

$$F_{\pm\mp}(\mathbf{b}_0) \sim \int d^2\mathbf{b} e^{i\omega t_d(\mathbf{b}, \mathbf{b}_0)} \frac{\left(e^{-i\omega\xi_+(b)} \overset{\downarrow}{-} e^{-i\omega\xi_-(b)} \right)}{2} \frac{\mathbf{b}_\pm^4}{b^4}$$

ξ_i eigenvalues
of the ψ matrix:

$$\xi_\pm^h \sim \log \frac{b}{b_{IR}} + \frac{m}{M_{Pl}^2} \frac{c_{EH,hc}^{(1,e)}}{b} + \frac{m}{M_{Pl}^2} \frac{c_{EH,hc}^{(1,1b)}}{\omega^2 b^3} \overset{\downarrow}{\pm} \left(\frac{c_{EH,hb}^{(0,b)}}{\omega^4} + c_{R^3,hb}^{(0,b)} c_{R^3} \right) \frac{1}{b^4} + \frac{m}{M_{Pl}^2} \left(c_{R^3,hc}^{(1,2b)} + \dots \right) \frac{1}{b^5}$$

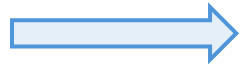
$$\xi_\pm^\gamma \sim \log \frac{b}{b_{IR}} + \frac{m}{M_{Pl}^2} \frac{c_{\gamma,hc}^{(1,e)}}{b} + \frac{m}{M_{Pl}^2} \frac{c_{\gamma,hc}^{(1,1b)}}{\omega^2 b^3} \overset{\downarrow}{\pm} \left(\frac{c_{FFR,hb}^{(0,b)}}{b^2} + \frac{m}{M_{Pl}^2} \frac{c_{FFR,hb}^{(1,e)}}{b^3} \right)$$

Gravitational lensing in detail (2)

Born corrections << Eikonal corrections



Helicity breaking contributions
(including beyond GR) very suppressed.



$$F_{\pm\pm}(\mathbf{b}_0) \sim \int d^2\mathbf{b} e^{i\omega[t_d(\mathbf{b}, \mathbf{b}_0) - \xi_{\pm}(b)]} \quad \text{where:} \quad \xi_{\pm}^X = \xi_{\mp}^X \sim \log \frac{b}{b_{IR}} + \frac{m}{M_{Pl}^2} \frac{c_{EH,hc}^{(1,e)}}{b} + \frac{m}{M_{Pl}^2} \frac{[c_{EH,hc}^{(1,1b)}]_X}{\omega^2 b^3}, \quad X = \gamma, h$$

Same result of [C. Gonzalez, Parra-Martinez, et al., 2026]

But



$$F_{\pm\mp}^{\gamma}(\mathbf{b}_0) \sim c_{FFR,hb}^{(0,b)} \mathcal{C}_{FFR} \int d^2\mathbf{b} e^{i\omega[t_d(\mathbf{b}, \mathbf{b}_0) - \log \frac{b}{b_0}]} \frac{b_{\pm}^2}{b^4}$$

$$F_{\pm\mp}^h(\mathbf{b}_0) \sim \left(\frac{c_{EH,hb}^{(0,b)}}{\omega^4} + c_{R^3,hb}^{(0,b)} \mathcal{C}_{R^3} \right) \int d^2\mathbf{b} e^{i\omega[t_d(\mathbf{b}, \mathbf{b}_0) - \log \frac{b}{b_0}]} \frac{b_{\pm}^4}{b^8}$$



Helicity breaking DI
sensible do beyond GR
corrections!!

For graviton lensing: $\omega^4 \mathcal{C}_{R^3} \sim \omega^4 l_{R^3}^4$ with $\omega l_{R^3} = \frac{2\pi f}{c} \mathcal{O}(10 \text{ km}) \simeq 2 \times 10^{-4} \left(\frac{f}{\text{Hz}} \right)$

Can be O(1)!

Gravitational lensing in detail (3)

What happens in the +,x basis?

$$h = h_{\pm 2}, \quad \mathbb{h} = h_{+, \times}$$

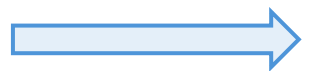
$$\mathbb{h} = \mathbb{F} h^0, \quad \mathbb{F} = \mathbf{U}^{-1} \mathbf{F} \mathbf{U} \quad \longrightarrow \quad \mathbb{F} \simeq \begin{pmatrix} \mathcal{F}_+ & 0 \\ 0 & \mathcal{F}_- \end{pmatrix}$$

$$\mathcal{F}_{\pm} = \frac{\nu}{2\pi i} \int d^2\varphi e^{i\nu \left(\frac{(\varphi - \varphi_o)^2}{2} - \tilde{\xi}_{\pm} \right)} \simeq \sum_j \mu_j e^{i\nu \left[\frac{\varphi_j^2 - \varphi_0^2}{2} - f(\varphi_j) \mp g(\varphi_j) \right] + i\tilde{n}_j} \quad \tilde{\xi}_{\pm}(\varphi) = f(\varphi) \pm g(\varphi)$$

General incoming GW: $\mathbb{h}^0(\omega) = A_+ \mathbb{h}_+(\omega) + A_{\times} \mathbb{h}_{\times}(\omega)$

Outgoing GW: $\mathbb{h}(\omega) = \mathbb{F}(\omega) \mathbb{h}^0 = \mathcal{F}_+(\omega) A_+(\omega) \mathbb{h}_+ + \mathcal{F}_-(\omega) A_{\times}(\omega) \mathbb{h}_{\times}$

Outgoing GW in time domain: $\tilde{\mathbb{h}}(t) = \sum_j \mu_j \left[\tilde{A}_+(t - t_{fj} - t_{gj}) \mathbb{h}_+ + \tilde{A}_-(t - t_{fj} + t_{gj}) \mathbb{h}_{\times} \right]$



Time delay difference of the +,x polarizations, sensible to beyond GR