Tutorials for today

- Repo 1: https://github.com/apalmese/GW_teaching_material/
    - Read readme and install relevant conda environment (you could try Google Colab if this doesn't work)
    - GW data analysis: Retrieve and use events strain and frequency
    - GW data analysis: Retrieve and use GW posteriors
    - A simple bright and dark standard siren analysis with GW170817

Advanced dark siren analysis

- Repo 2: https://github.com/apalmese/hitchhiker_guide_dark_sirens/
  - Based on our Hitchhiker's guide

The Hitchhiker's Guide to the Galaxy Catalog Approach for Dark Siren Gravitational-wave Cosmology

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Abstract

We outline the "dark siren" galaxy catalog method for cosmological inference using gravitational wave (GW) standard sirens, clarifying some common misconceptions in the implementation of this method. When a confident transient electromagnetic counterpart to a GW event is unavailable, the identification of a unique host galaxy is in general challenging. Instead, as originally proposed by Schutz, one can consult a galaxy catalog and implement a dark siren statistical approach incorporating all potential host galaxies within the localization volume. Trott & Huterer recently claimed that this approach results in a biased estimate of the Hubble constant, H_0, when implemented on mock data, even if optimistic assumptions are made. We demonstrate explicitly that, as previously shown by multiple independent groups, the dark siren statistical method leads to an unbiased posterior when the method is applied to the data correctly. We highlight common sources of error possible to make in the generation of mock data and implementation of the statistical framework, including the mismodeling of selection effects and inconsistent implementations of the Bayesian framework, which can lead to a spurious bias.

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- Simplified "toy model" version of dark siren, only 1 pixel and only z and d and explicit. Ppop/pCBC approximated - this is fine as long as it is done consistently in mock data and inference:

$$\mathcal{L}(x_i|H_0) = \frac{\int dz\ \mathcal{L}_{\rm GW}(x_i|d_L(z, H_0))\, p_{\rm CBC}(z)}{\int dz\ P_{\rm det}^{\rm GW}(z, H_0)\, p_{\rm CBC}(z)},$$

$$p_{\rm CBC}(z) \approx p_{\rm cat}(z).$$

- Redshifts zg perfectly known:

$$\mathcal{L}(x_i|H_0) = \frac{\sum_i^{N_{\rm gal}} \mathcal{L}_{\rm GW}(x_i|d_L(\hat{z}_g^i, H_0))}{\sum_i^{N_{\rm gal}} P_{\rm det}^{\rm GW}(\hat{z}_g^i, H_0)}.$$

- With uncertainties, assume a Gaussian into eq. 16:

$$\mathcal{L}(\hat{z}_g^i|z) = \frac{1}{\sqrt{2\pi}\,\sigma_z} e^{-\frac{(\hat{z}_g^i - z)^2}{2\sigma_z^2}},$$

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- In both cases, assume simple GW likelihood (note: Gaussian in measured d, not true, because of denominator):

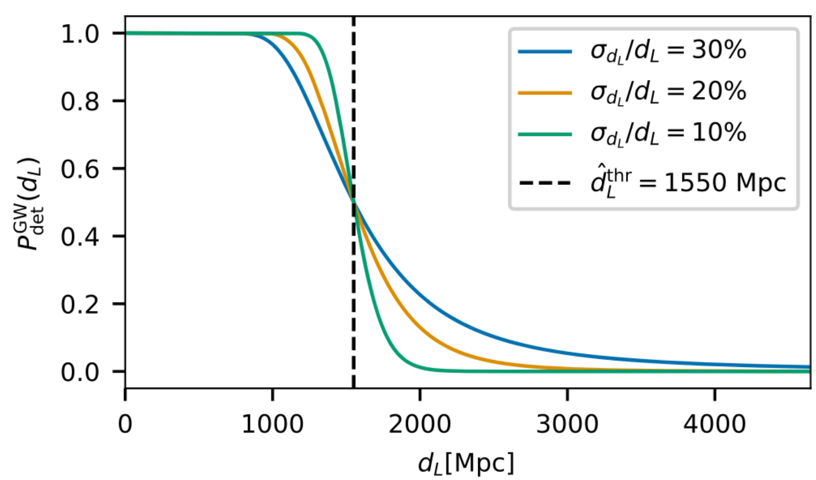
$$\mathcal{L}_{\mathrm{GW}}(\hat{d}_L^i | d_L(z, H_0)) = \frac{1}{\sqrt{2\pi}\,\sigma_{d_L}} e^{-\frac{(\hat{d}_L^i - d_L(z,H_0))^2}{2\sigma_{d_L}^2}}$$

- Detection probability for denominator:

$$P_{\mathrm{det}}^{\mathrm{GW}}(z, H_0) = \int_{\mathrm{detectable}} \mathcal{L}_{\mathrm{GW}}(x | d_L(z, H_0))\, dx,$$

  - Assume simple detectability for : $\hat{d}_L < \hat{d}_L^{\mathrm{thr}} = 1550\ \mathrm{Mpc}$

$$\begin{aligned} P_{\mathrm{det}}^{\mathrm{GW}}(z, H_0) &= \int_{-\infty}^{\infty} \Theta(\hat{d}_L;\ \hat{d}_L^{\mathrm{thr}}) \mathcal{L}_{\mathrm{GW}}(\hat{d}_L | d_L(z, H_0))\, d\hat{d}_L \\ &= \int_{-\infty}^{\hat{d}_L^{\mathrm{thr}}} \mathcal{L}_{\mathrm{GW}}(\hat{d}_L | d_L(z, H_0))\, d\hat{d}_L \\ &= \frac{1}{2}\left[1 + \mathrm{erf}\left[\frac{\hat{d}_L^{\mathrm{thr}} - d_L(z, H_0)}{\sqrt{2} A d_L(z, H_0)}\right]\right]. \end{aligned}$$



- Note Pdet is not a step function! (Unless dL is perfectly known - never)

Advanced dark siren analysis

- Repo 2: https://github.com/apalmese/hitchhiker_guide_dark_sirens/
  - Read README
  - Start from `H0analysis_perfect_z.ipynb` and `H0analysis.ipynb`
  - There are challenges in there but you can come up with your own and play with all notebooks, e.g.
    - explore the effect of common inconsistencies described in the paper in H0analysis_4inconsistencies.ipynb
      - Considering pdet a step function in the presence of dL uncertainties
      - Ignoring true d dependance in GW likelihood
    - Anything else one could do wrong and how you would fix it?

Mismodeling, misinterpretation of observed versus true variables, implementations of the Bayesian framework which are inconsistent with the mock data lead to biases

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- There are 2 lines of sight, each with a 1 sq.deg. and a 5 sq. Deg. Size, mimicking a possible well-localized dark siren sky localization.
- Galaxy catalog from MICEcat http://maia.ice.cat/mice/
- You can assume an uncertainty of your choice for the redshifts of the galaxies. One of the notebooks assumes no z uncertainty for simplicity
- You can pick an uncertainty for the GW distance estimate which is some fraction of the true distance of the event (here taken 10-30%).

Other Tutorials

- Other useful tutorials/codes:
    - Gravitational Wave Open Data Workshop (ODW) organized by the Gravitional Wave Open Science Center (GWOSC) has many tutorials on how to use the LVK data: https://github.com/gw-odw/odw/tree/main
    - gwcosmo https://git.ligo.org/lscsoft/gwcosmo
    - icarogw https://github.com/simone-mastrogiovanni/icarogw

Tutorials tips

```
GW frequency notebook
GW170817
**Tips**:
* There is a glitch in L1!
* Try adjusting the time slice from -30 +2
* Try adjusting qrange=(20, 150), frange=(30, 500)
```

```
Posterior samples notebook
**Tips**:
* A uniform prior in comoving volume in redshift terms can be computed with `Planck15.differential_comoving_volume(z)`
* Remove the $d^2_L$ prior and generate $d_L$ posterior samples.
* Convert the $d_L$ samples to redshift using the function `z_at_value` above
* Use the differential in comoving volume to reweight the samples
```