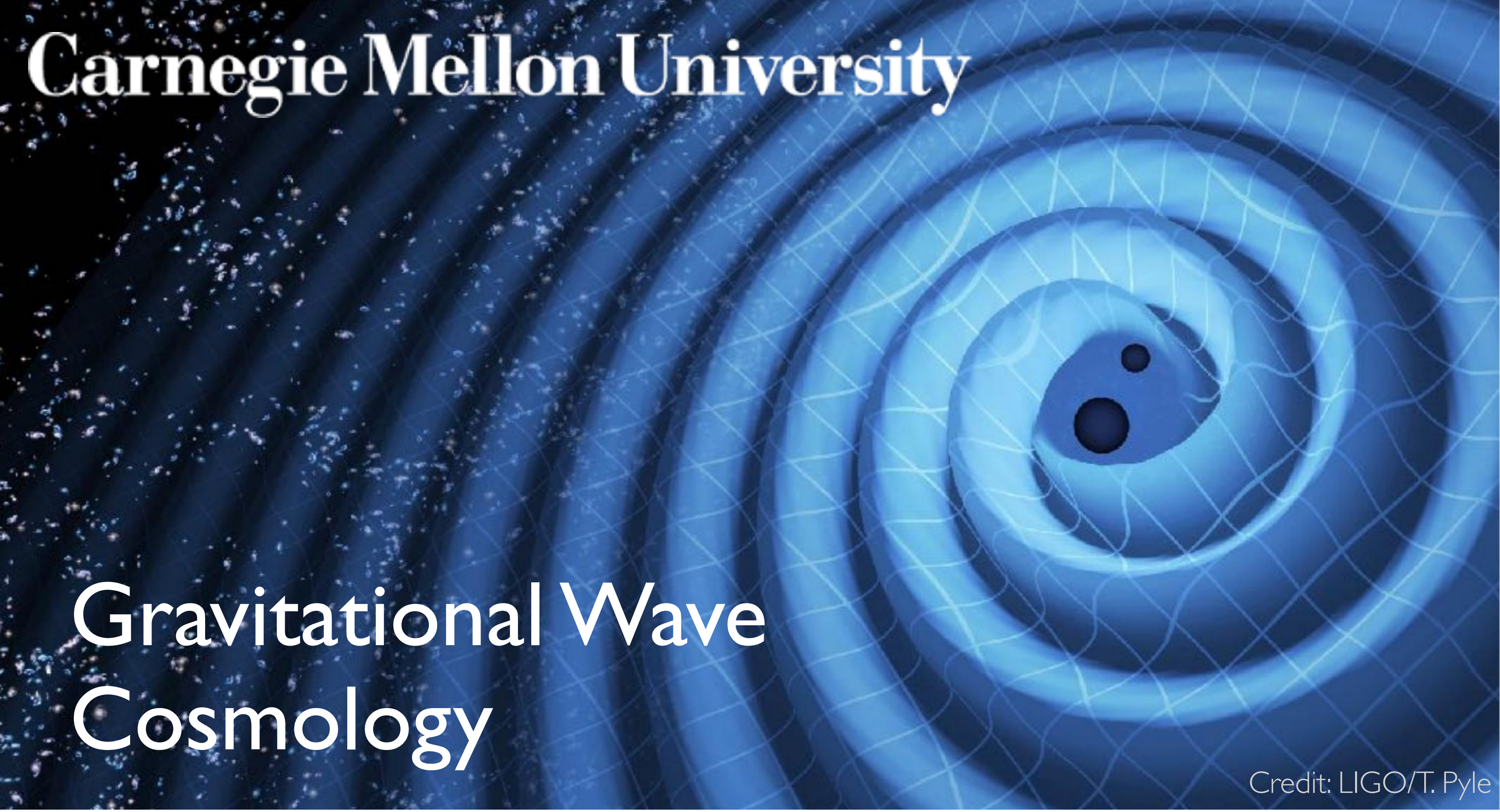


Carnegie Mellon University

Gravitational Wave Cosmology

Credit: LIGO/T. Pyle

Antonella Palmese

Exploring New Frontiers in Cosmology

July 2026

GGI - Florence

Overview - Practical Info

- The goal of these lectures is to give an overview of gravitational wave astronomy and cosmology, focused on applications with standard sirens
  - Focus on compact object binaries in LIGO/Virgo/KAGRA band
  - I will avoid technical derivations
- Lectures based on:
  - Jose Maria Ezquiaga (Niels Bohr Institute) material: https://github.com/ezquiaga/a-premiere-on-gw-cosmology/ - includes notes with derivations for the equations shown, references to seminal papers and textbooks
  - Encyclopedia of Astrophysics Gravitational wave cosmology Book Chapter: AP & Mastrogiovanni 2025 https://arxiv.org/abs/2502.00239
  - Links to relevant resources in each slide will be available in the pdf

Overview

- Origin of Compact Object Binaries

  Brief Intro

- Gravitational waves (GWs) basics

  Gravitational wave propagation, quadrupole formula, compact binary coalescence

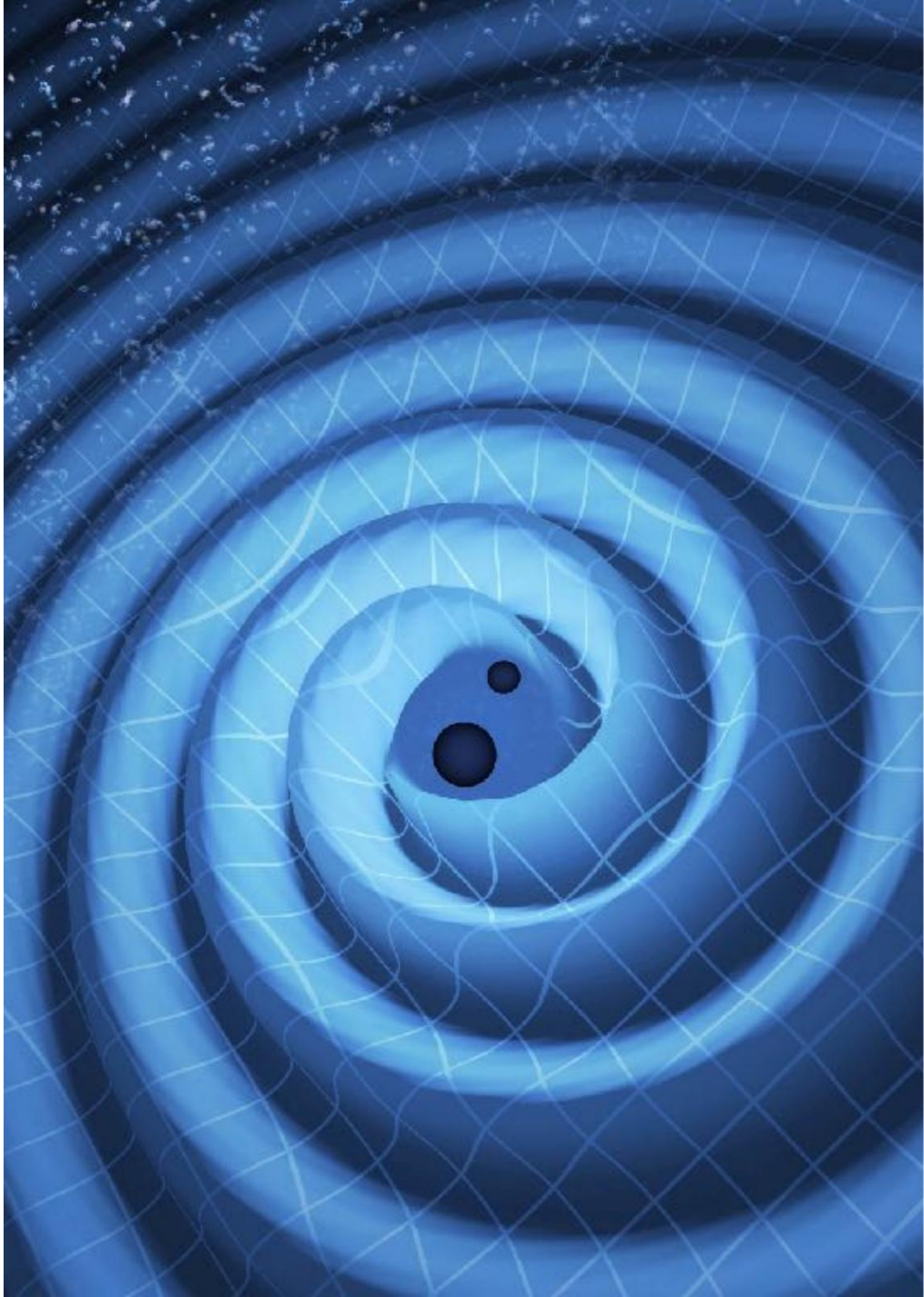
- Gravitational wave astronomy

  Detection, matched filtering, data analysis, current observations, future prospects

- Standard siren Cosmology

  Bright, dark, and spectral sirens, EM emission from GW sources, current measurements and prospects

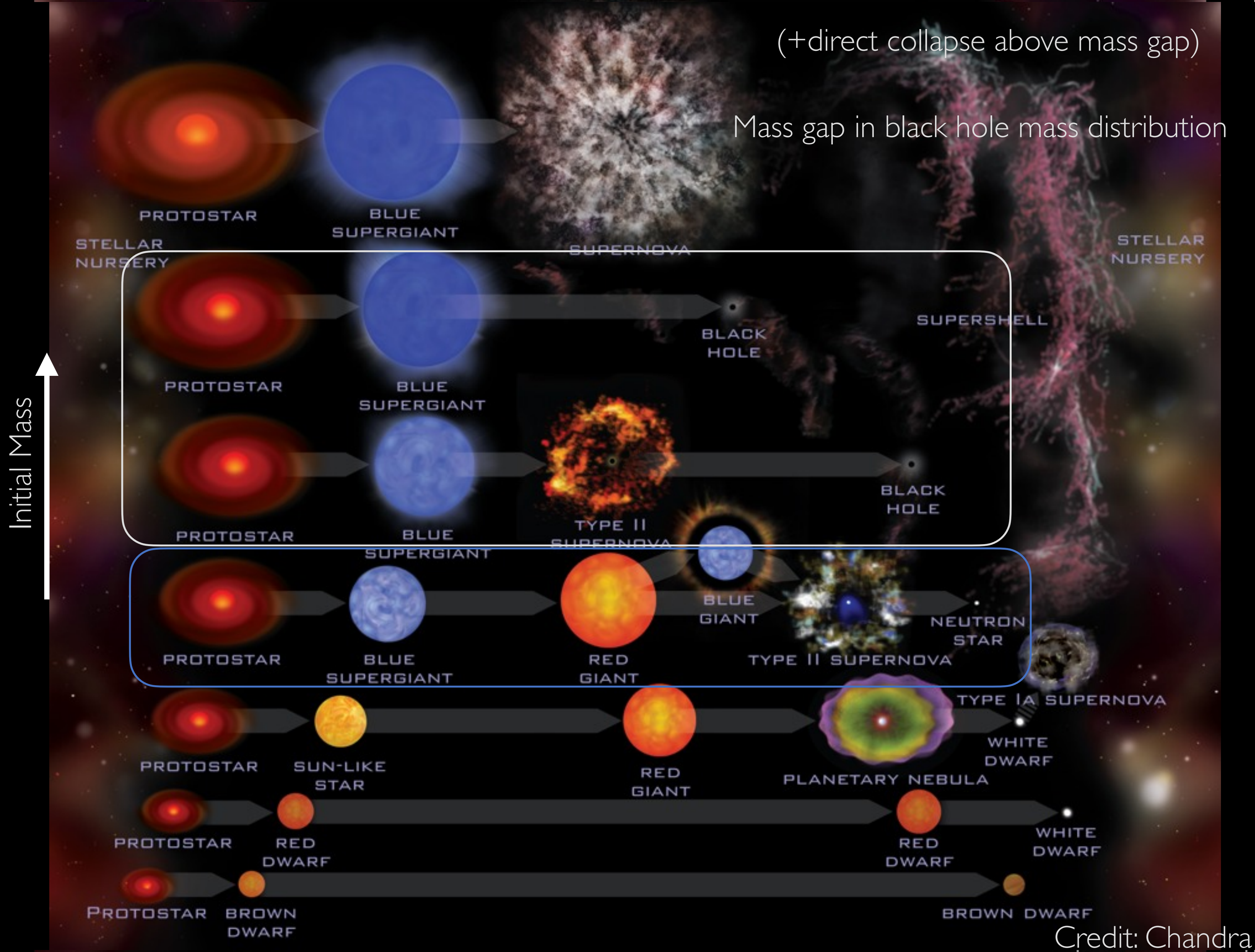
Origin of Compact Object Binaries



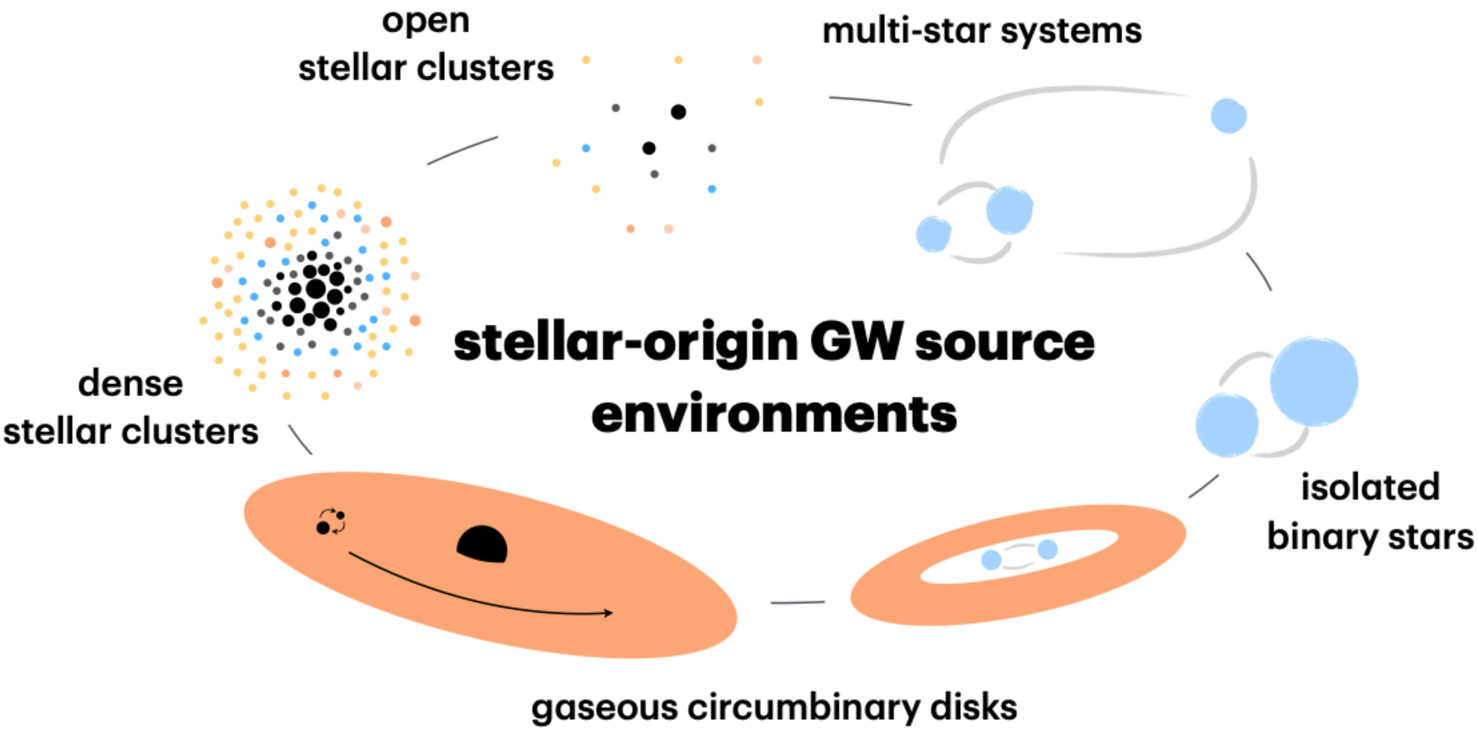
(+direct collapse above mass gap)

Mass gap in black hole mass distribution

Initial Mass



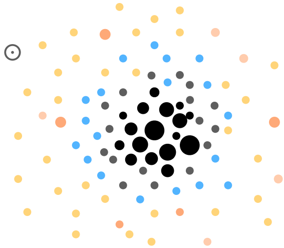
Credit: Chandra



+ primordial black holes
+ Central black holes in ultra-dwarf galaxies…

See Breivik 2025, Encyclopedia of Astrophysics

There are **many** formation environments!



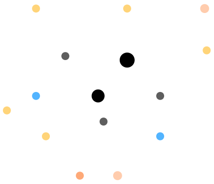
(many) $M > 45\ M_\odot$

$M_1/M_2{\sim}1$

~large e

symmetric effective spins

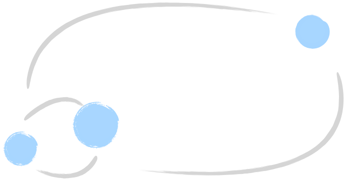
e.g. Antonini+16, Rodriguez+19, Kremer+19, Mapelli+21



few $M > 45\ M_\odot$

some $M_1/M_2<<1$

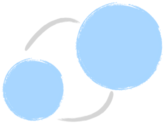
e.g. Di Carlo+19,20



~high e

no $> 45\ M_\odot$

e.g. Antonini+17, Martinez+20,21, Xuan+24, Dorozmai+24

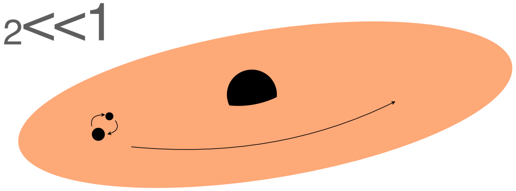


no $> 45\ M_\odot$

maybe large effective spins

mostly $M_1/M_2{\sim}1$

e.g. Kinugawa+14, Dominik+15, Mandel+16, Marchant+16, Chruslinska+18, Kruckow+18, Zevin+20, van Son+20 Santoloquido+21, Broekgaarden+21



many multi-gen: $> 45\ M_\odot$

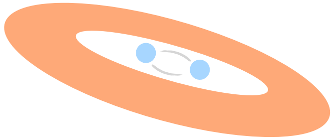
some $M_1/M_2<<1$

EM flares

~large? e

correlations in M & spins

e.g. McKernan+20,22, Tagawa+21, Dittmann+23, Grishin+24



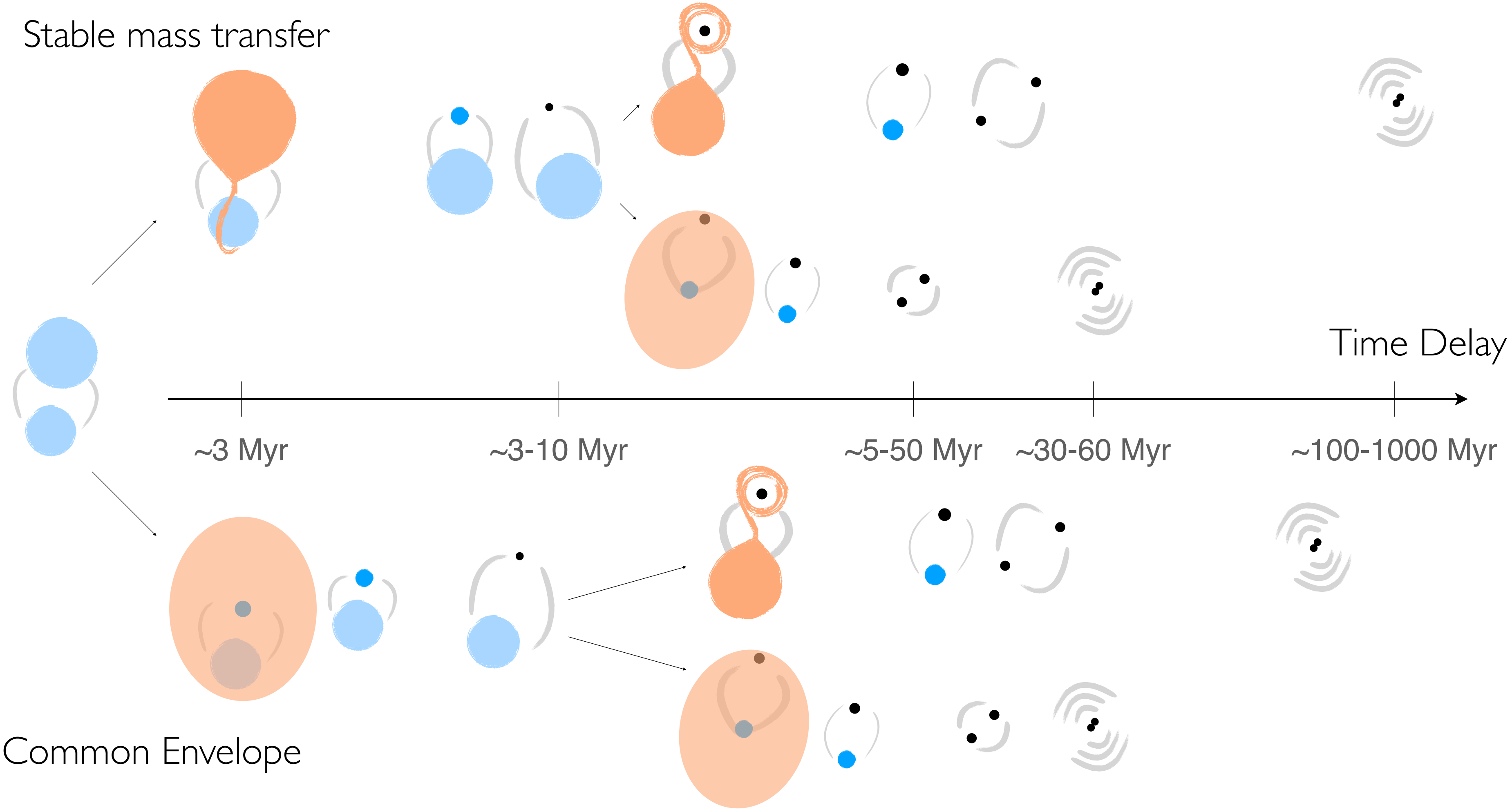
accelerated inspirals

e.g. DeLaurentiis+24, Dittmann+24

See Breivik 2025, Encyclopedia of Astrophysics

Mandel & Broekgaarden 2022, LLR

Isolated Binaries



Stable mass transfer

Time Delay

~3 Myr

~3-10 Myr

~5-50 Myr

~30-60 Myr

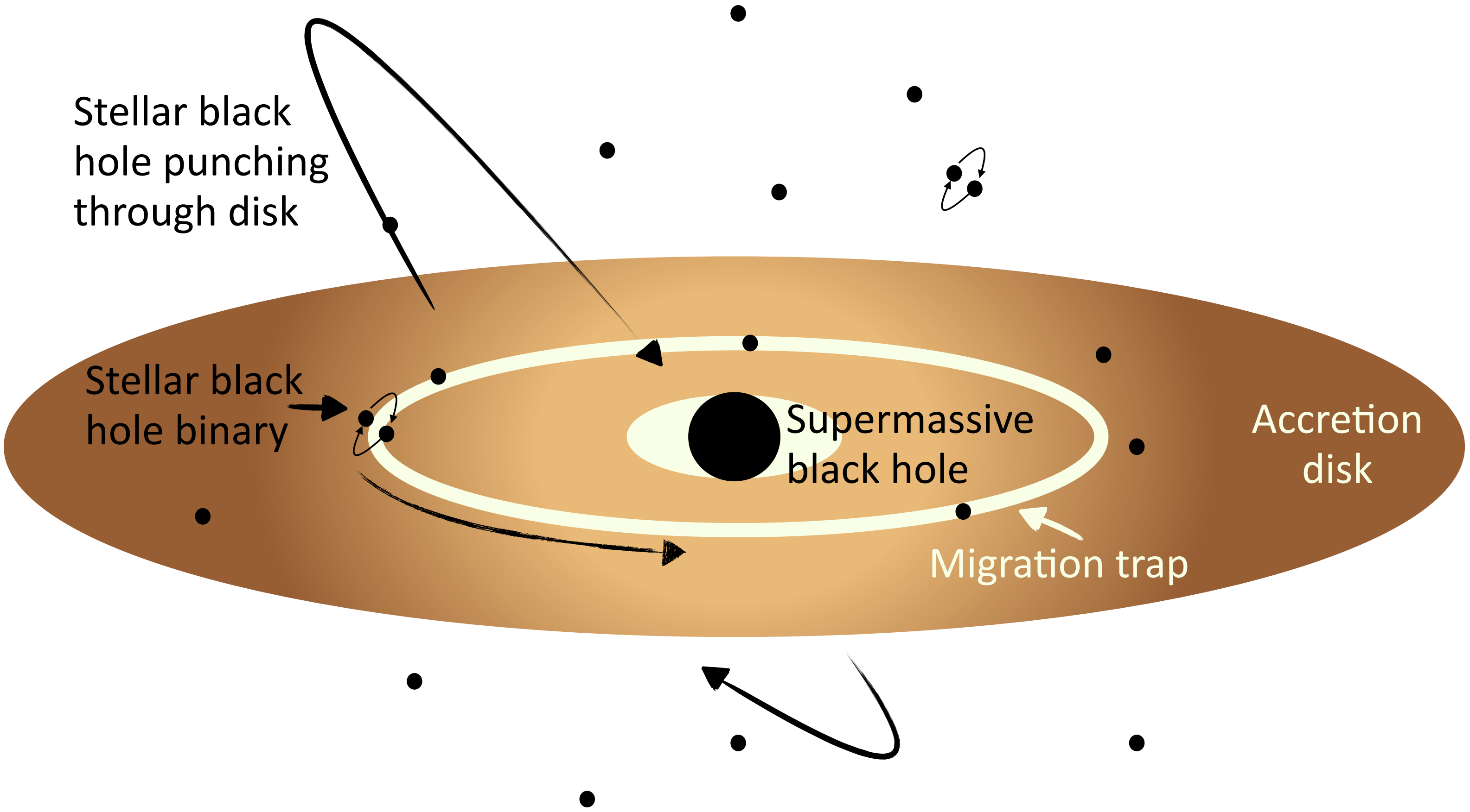
~100-1000 Myr

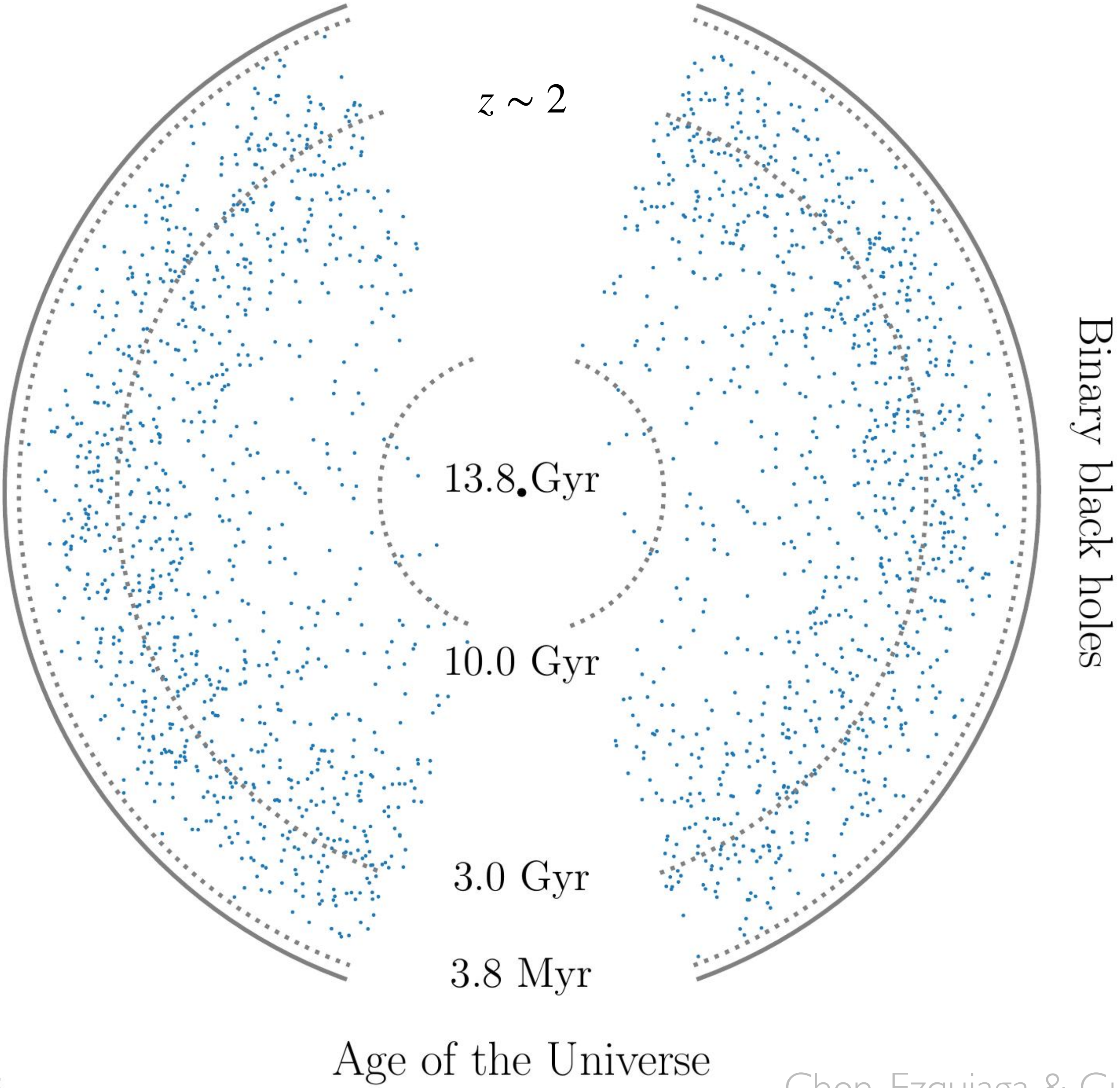
Common Envelope

[See Breivik 2025, Encyclopedia of Astrophysics](#)

[Mandel & Broekgaarden 2022, LLR](#)

Binaries in AGN disks

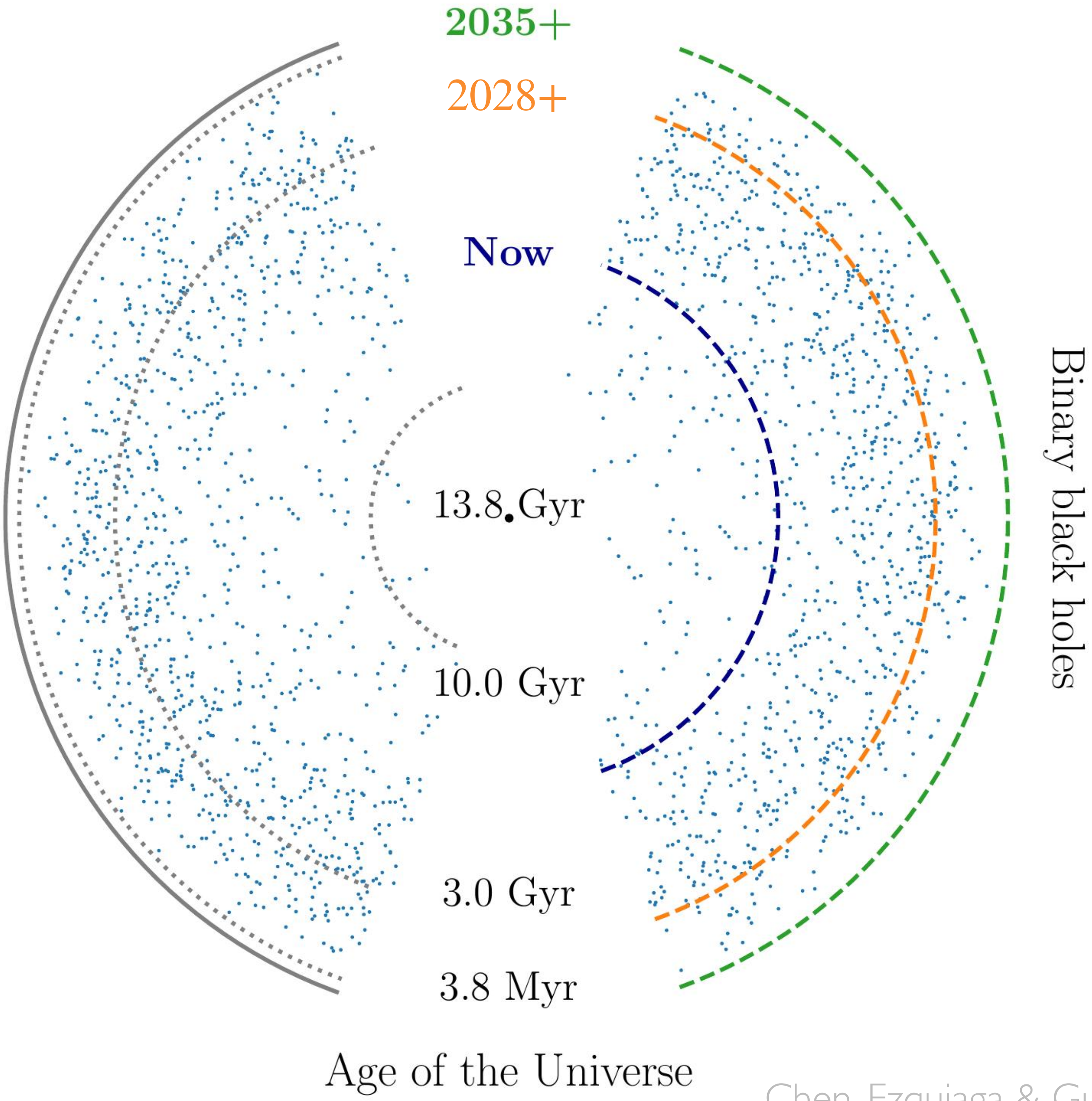




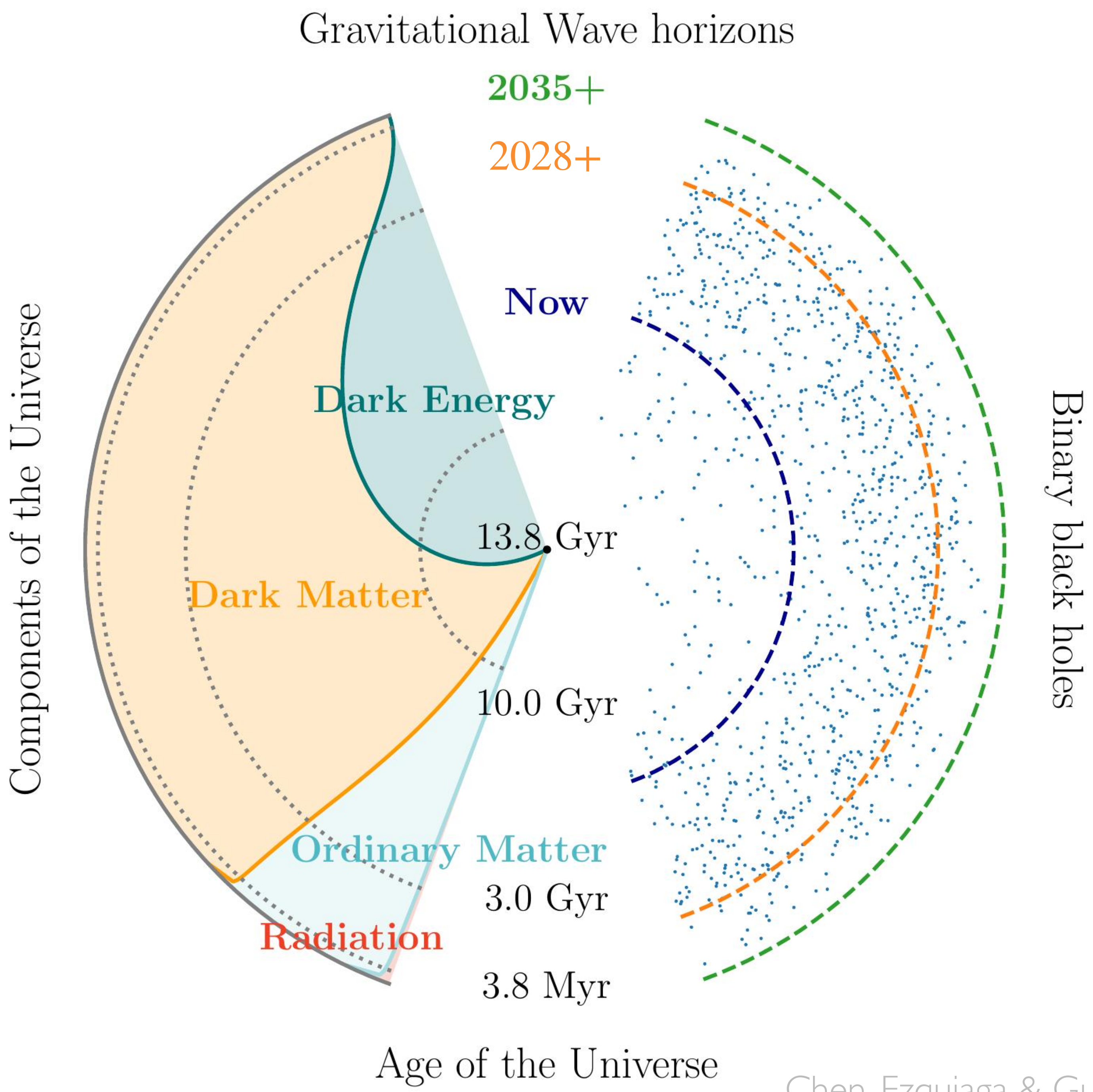
*stellar mass
binary black holes

Chen, Ezquiaga & Gupta 2024

Gravitational Wave horizons

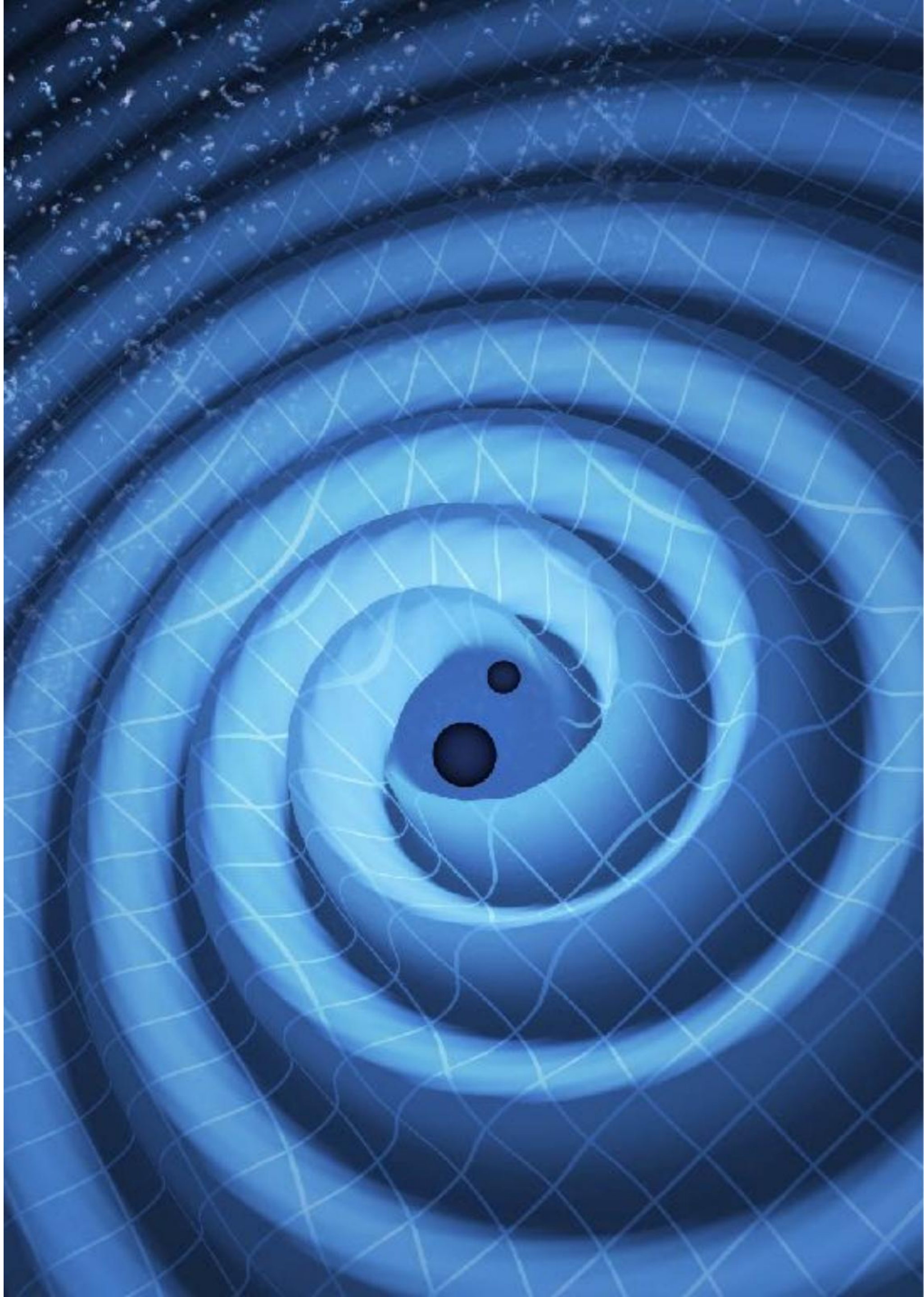


Chen, Ezquiaga & Gupta 2024



Chen, Ezquiaga & Gupta 2024

Gravitational Waves Basics



Gravitational waves in flat space

- Perturbations around Minkowski

$$g_{\mu\nu}(t,\vec{x}) = \eta_{\mu\nu} + h_{\mu\nu}(t,\vec{x})$$

$$|h_{\mu\nu}(t,\vec{x})| \ll 1$$

- Einstein field equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu}$$

- Gravitational wave propagation

$$\boxed{\Box h_{\mu\nu} = -16\pi G\left(T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}T\right)}$$

Gravitational wave properties

- Wave equation in vacuum $\Box h_{\mu\nu} = 0$
- Wave ansatz $h_{\mu\nu}(x) = \mathrm{Re}\left[A_{\mu\nu}(x)e^{i\theta(x)}\right]$

$$k_\mu \equiv \partial_\mu \theta$$

$$A_{\mu\nu} \equiv A\epsilon_{\mu\nu}$$

- Highly oscillatory phase: $\theta \to \theta/\varepsilon$
- Leading order: *gravitational wave follow null geodesics*

$$\eta_{\mu\nu}k^\mu k^\nu = 0$$

- Next to Leading order *parallel transport*

$$k^\alpha \nabla_\alpha \epsilon_{\mu\nu} = 0$$

Gravitational wave polarizations

- Counting degrees of freedom:

Symmetric 4D tensor $\epsilon_{\mu\nu} = \epsilon_{\nu\mu}$: **10**

Lorenz gauge $\nabla^{\mu} h_{\mu\nu} = 0$: **10 - 4 = 6**

Residual gauge $\epsilon_{0\mu} = 0$: **10 - 4 - 4 = 2**

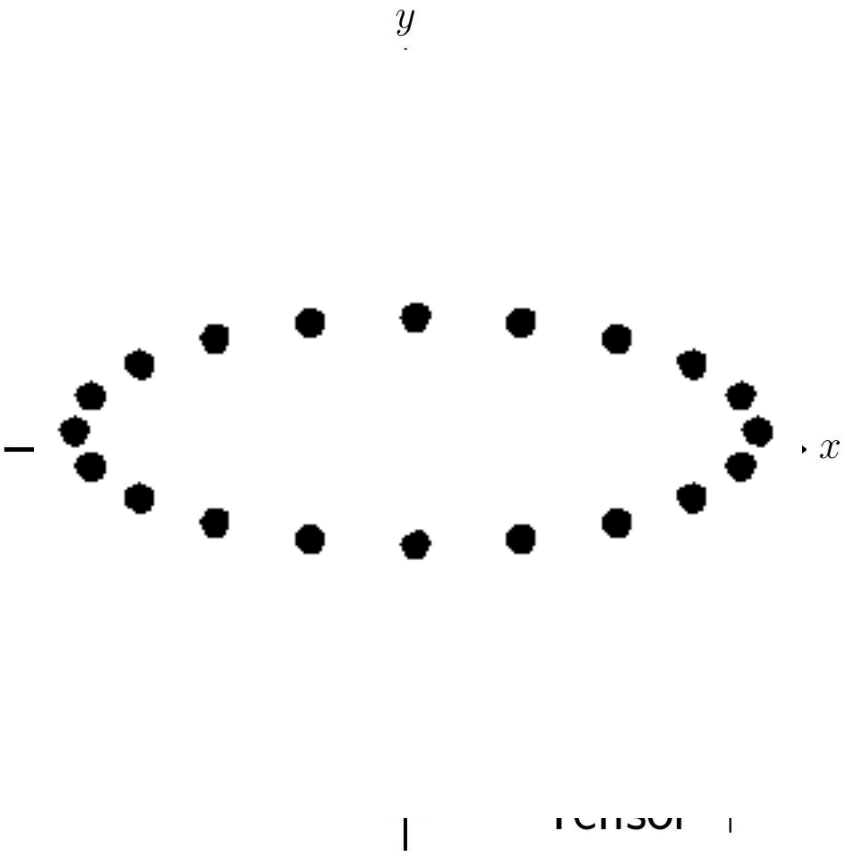
$\epsilon^{\mu}_{\mu} = 0$

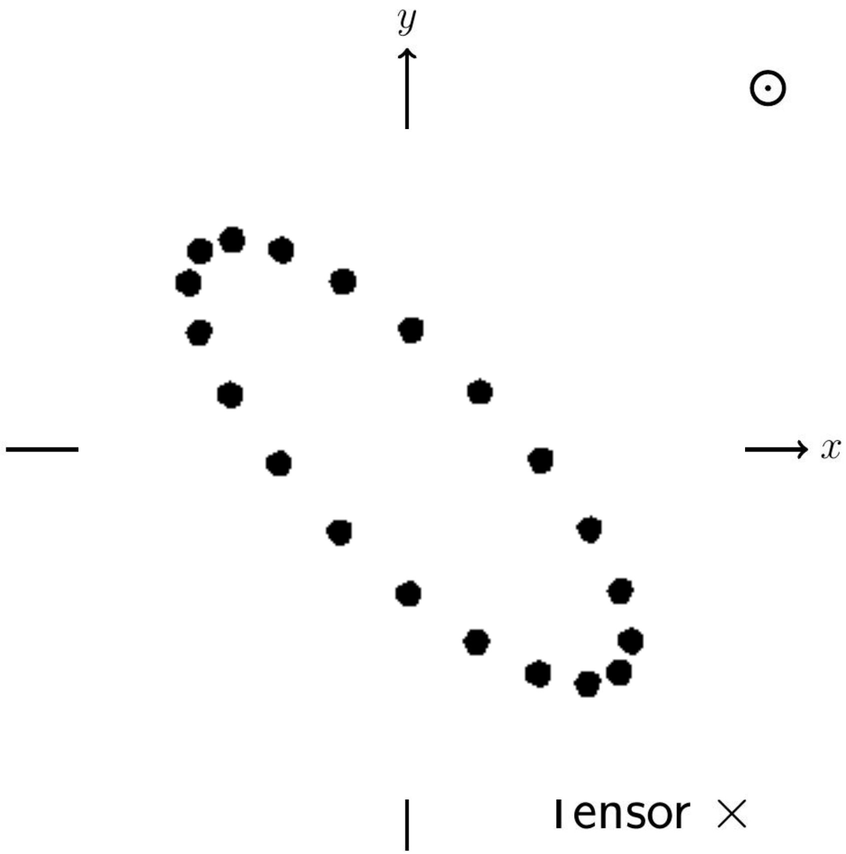
- Polarization decomposition:

$$\boxed{\epsilon_{\mu\nu}(x) = \epsilon_{+}(x)\hat{\epsilon}^{+}_{\mu\nu} + \epsilon_{\times}(x)\hat{\epsilon}^{\times}_{\mu\nu}}$$

$$\epsilon_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \epsilon_{+} & \epsilon_{\times} & 0 \\ 0 & \epsilon_{\times} & -\epsilon_{+} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Gravitational Wave Polarizations





— $\omega t = 0, \pi$ ---- $\omega t = \pi/2$ ······ $\omega t = 3\pi/2$

Gravitational waves in cosmology

- Perturbations around homogeneous and isotropic backgrounds

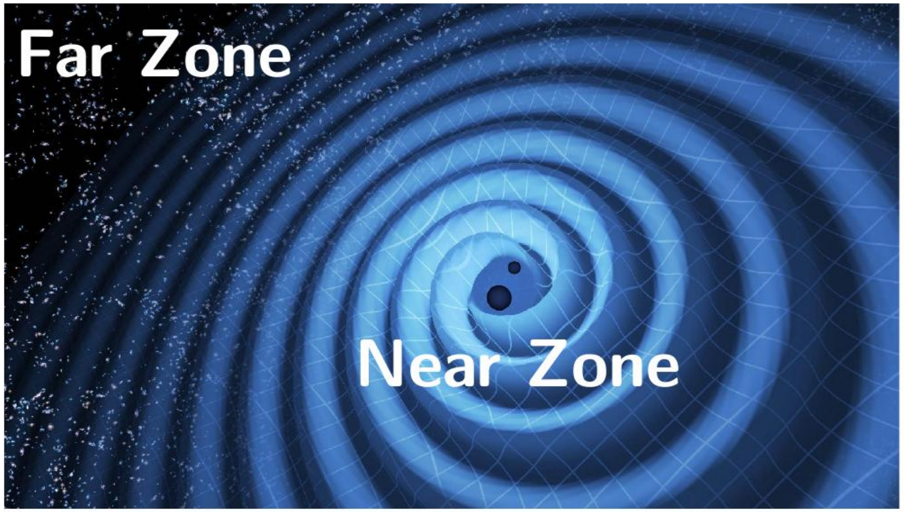
$$g_{\mu\nu} = g_{\mu\nu}^{\mathrm{FLRW}} + h_{\mu\nu}$$

- Wave equation in vacuum

$$\boxed{\begin{array}{c} \Box^{\mathrm{FLRW}} h_{ij} + 2R_{ijkl}^{\mathrm{FLRW}} h^{jl} = 0 \\ \Big\downarrow \\ h''_{ij} + 2\mathcal{H}h'_{ij} + \nabla^2 h_{ij} = 0 \\ \Big\downarrow \\ h_{ij}(\eta, \mathbf{x}) \simeq \dfrac{1}{a(\eta)} h_{ij}^{\mathrm{flat}}(\eta, \mathbf{x}) \end{array}}$$

Gravitational wave generation

- Different regimes



- Rewriting the field equations:

$$\Box\bar{h}_{\mu\nu} = -16\pi G T_{\mu\nu} + \mathcal{O}(h^2) \equiv -16\pi G \tau_{\mu\nu}$$

Trace reversed perturbations

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} g_{\mu\nu} h$$

- Green's function solution:

$$\boxed{\bar{h}_{\mu\nu}(t,\vec{x}) = 4G \int d^3x' \frac{\tau_{\mu\nu}(t - |\vec{x} - \vec{x}'|, \vec{x}')}{|\vec{x} - \vec{x}'|}}$$

Quadrupole formula

- Far zone solution: *expand large distances*
- Near zone solution: *expand small velocities v/c*
- Leading Newtonian limit: *match near and far zone solutions*

$$\boxed{h_{ij}^{TT}(t,\vec{x}) = \frac{2G}{c^4 r}\frac{d^2 Q_{ij}^{TT}(t-r/c)}{dt^2}}$$

Amplitude scales inversely with distance

Gravitational waves sourced by accelerated quadrupole moment

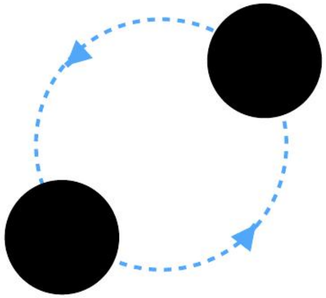
$$Q^{ij} \equiv \int d^3x\,\tau^{00}(x)\left(x^i x^j - \frac{1}{3}r^2\delta^{ij}\right)$$

Compact binary coalescence

- At leading order in post-Newtonian expansion

$$h_+(t) = h_c \left(\frac{1+\cos^2 \iota}{2} \right) \cos\left[\Phi(t)\right]$$

$$h_\times(t) = h_c \cos\iota \, \sin\left[\Phi(t)\right]$$



$$\mathcal{M}_c = (m_1 m_2)^{3/5}/(m_1+m_2)^{1/5}$$

- Amplitude

$$\boxed{h_c \sim \frac{\mathcal{M}_c^{5/3} f_{\rm gw}^{2/3}}{r}}$$

- Frequency

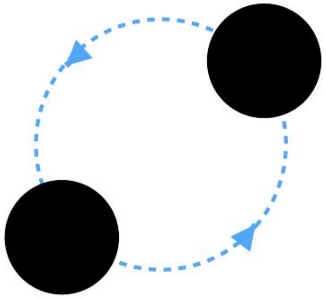
$$\boxed{\dot{f}_{\rm gw} \sim \mathcal{M}_c^{5/3} f_{\rm gw}^{11/3}}$$

Compact binary coalescence

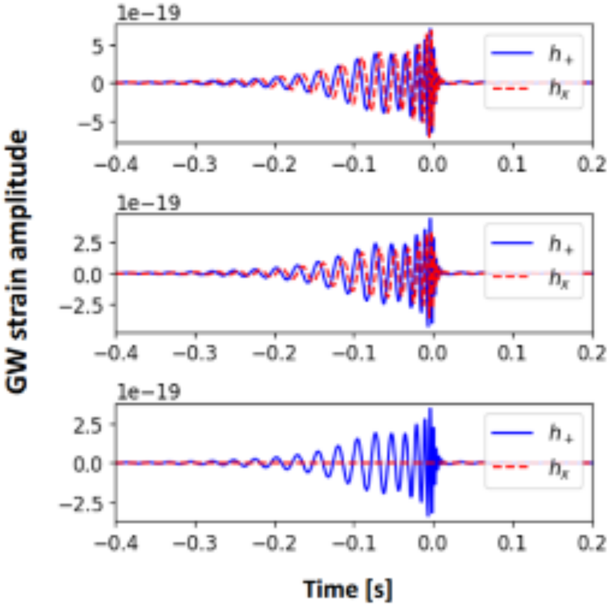
- At leading order in post-Newtonian expansion

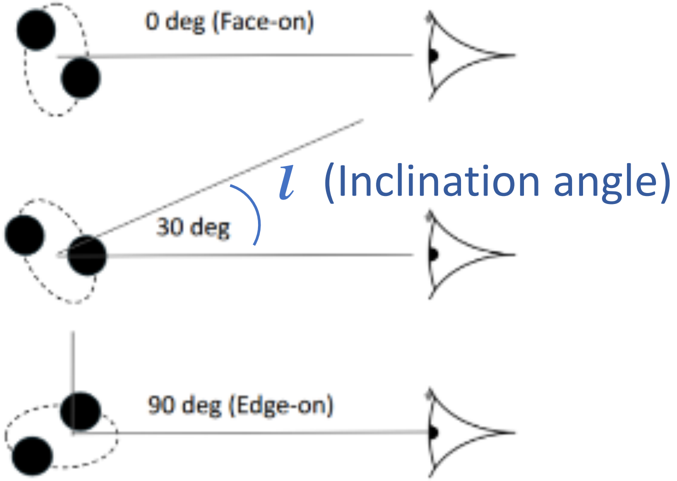
$$h_+(t) = h_c \left(\frac{1+\cos^2\iota}{2}\right)\cos\left[\Phi(t)\right]$$

$$h_\times(t) = h_c \cos\iota \, \sin\left[\Phi(t)\right]$$



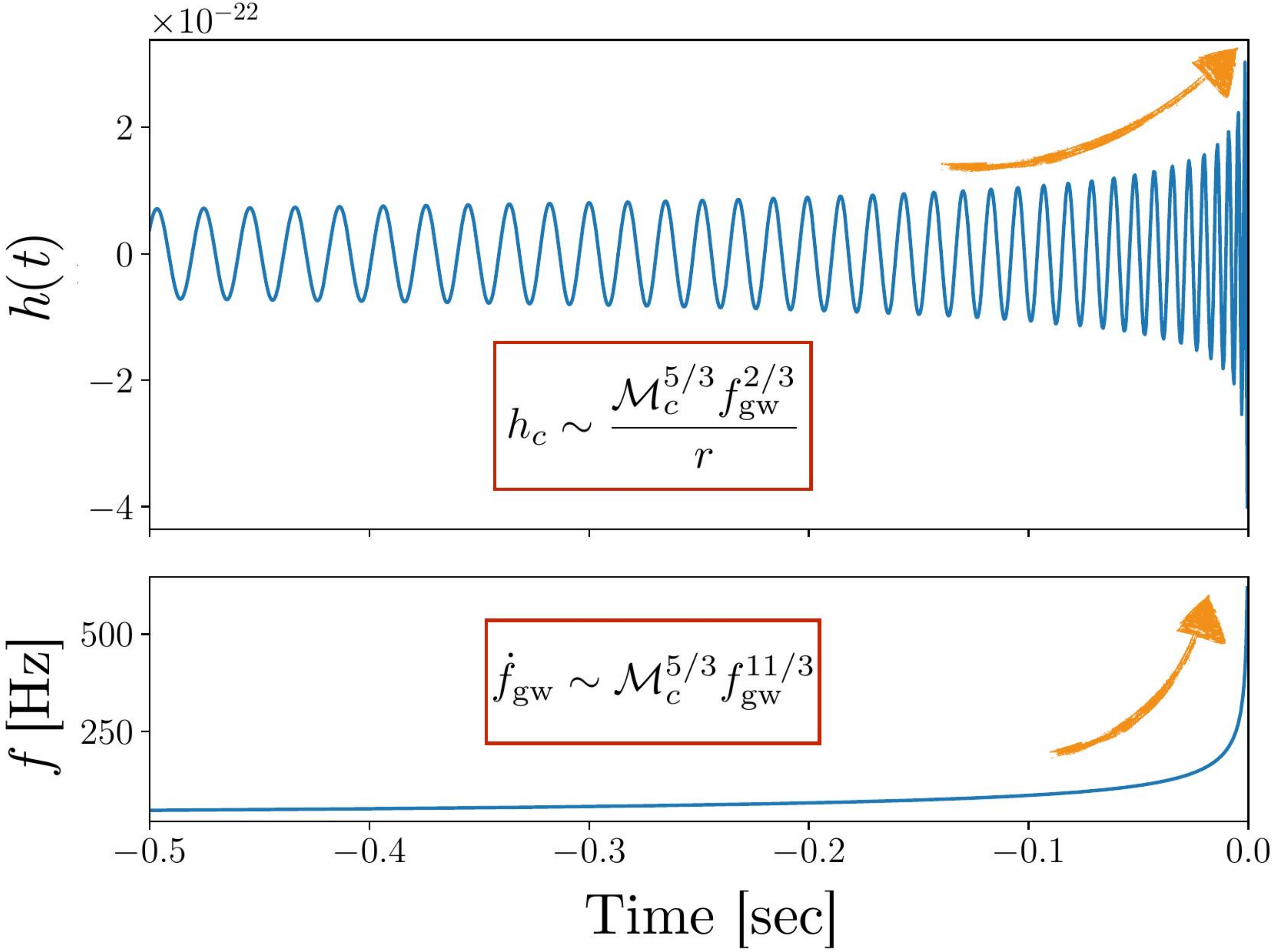
$$\mathcal{M}_c = (m_1 m_2)^{3/5}/(m_1+m_2)^{1/5}$$





Credit: S. Mastrogiovanni

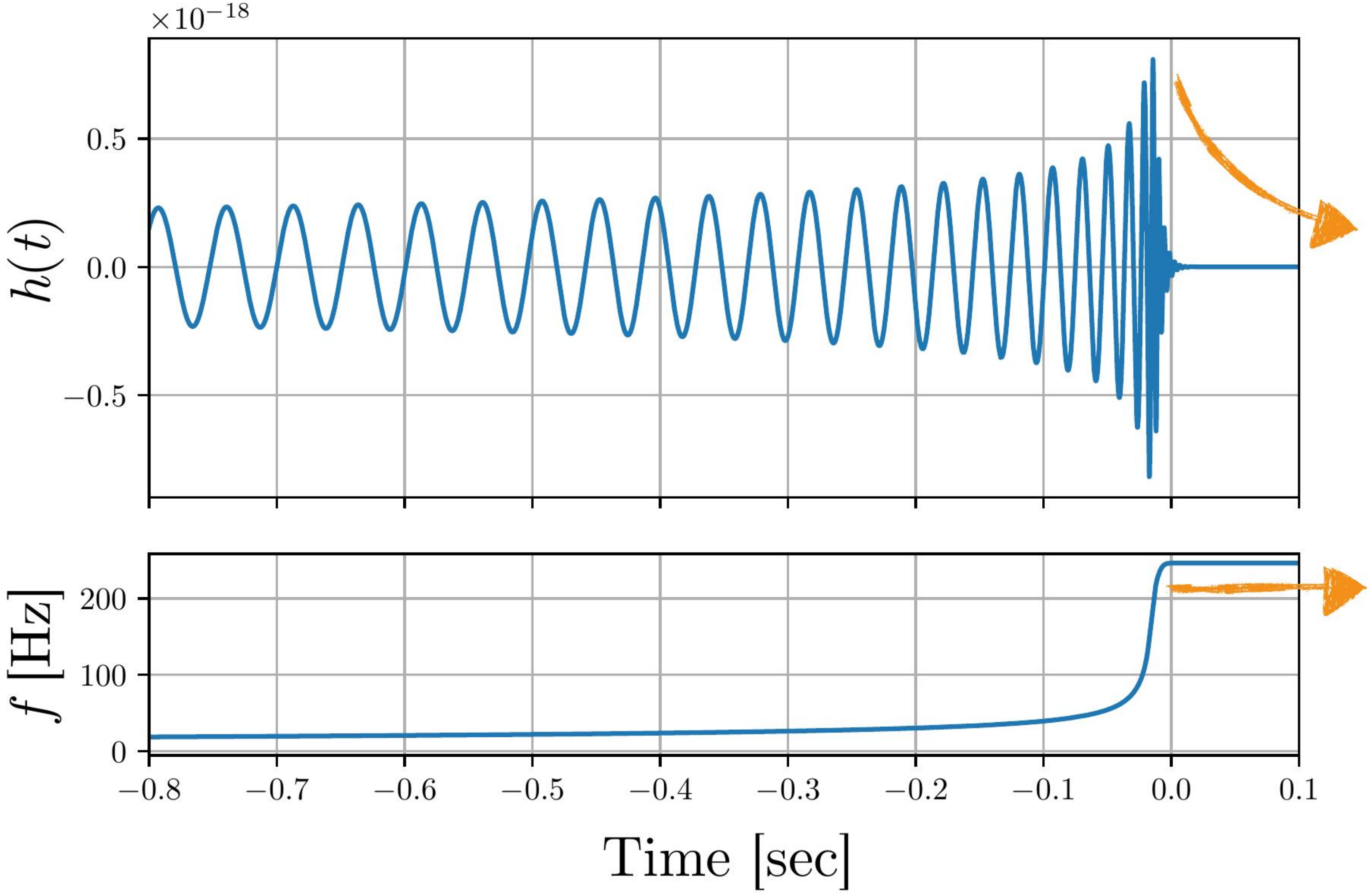
Inspiral - the “chirp”



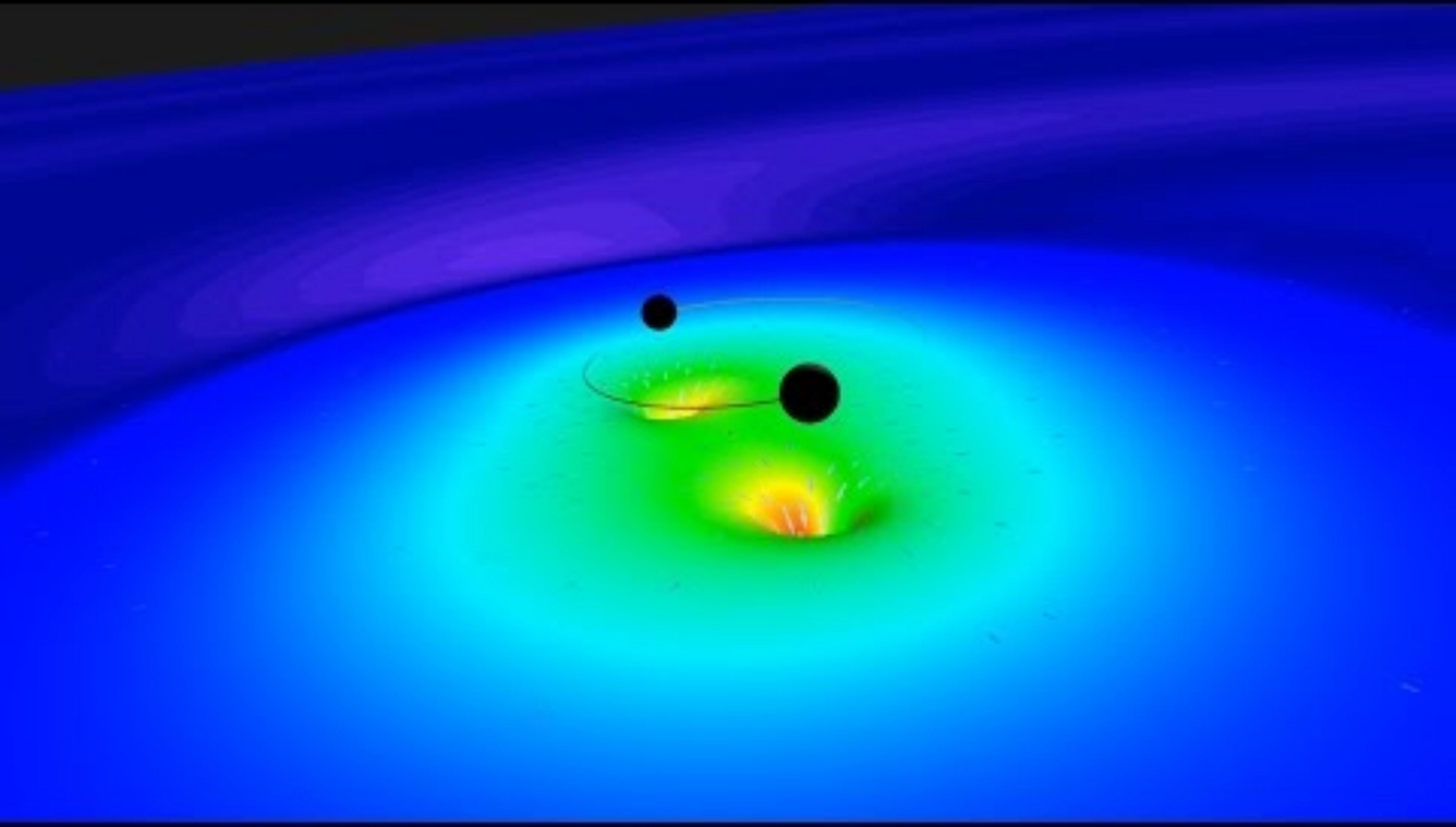
$$h_c \sim \frac{\mathcal{M}_c^{5/3} f_{\rm gw}^{2/3}}{r}$$

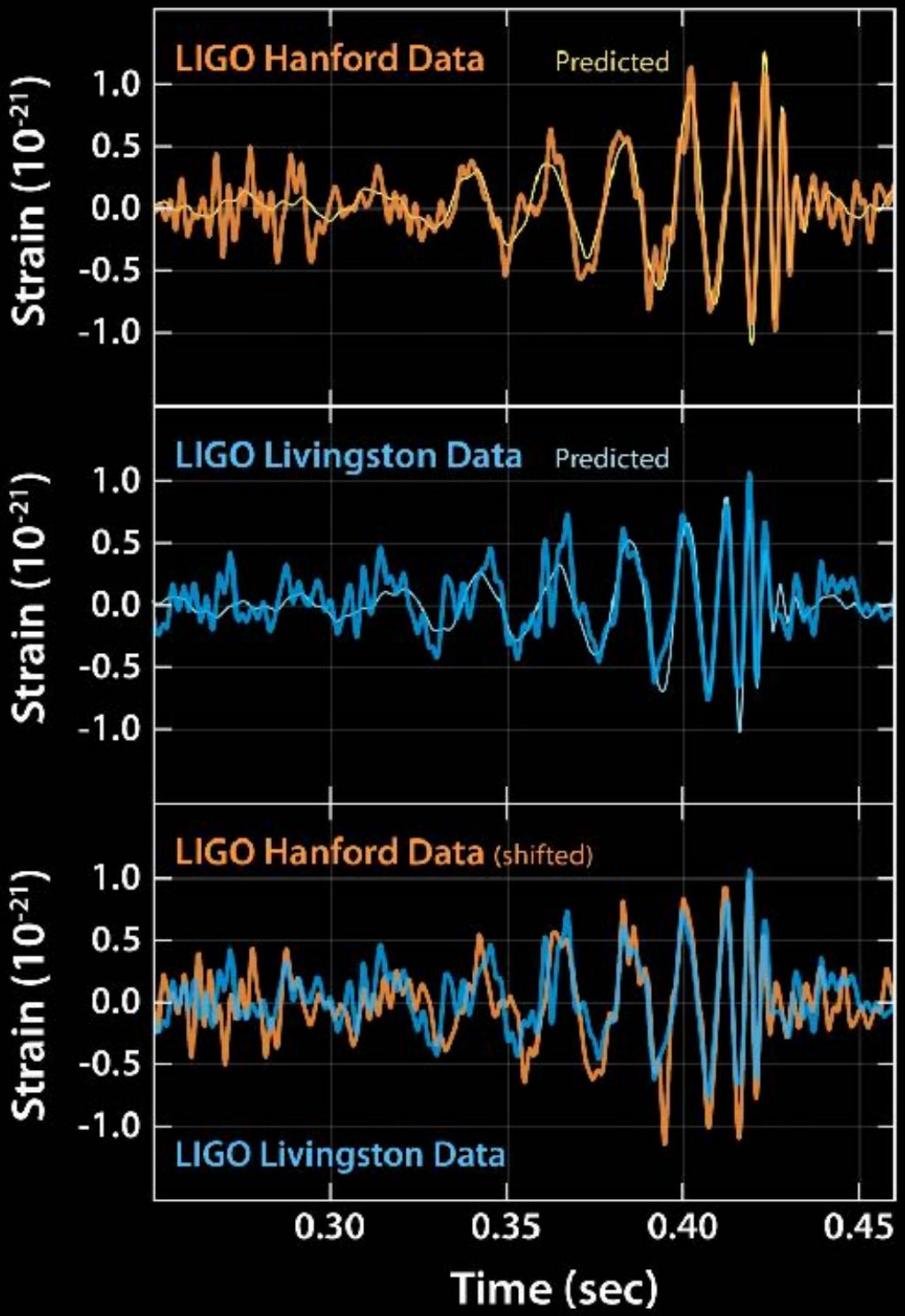
$$\dot{f}_{\rm gw} \sim \mathcal{M}_c^{5/3} f_{\rm gw}^{11/3}$$

Inspiral-Merger-Ringdown

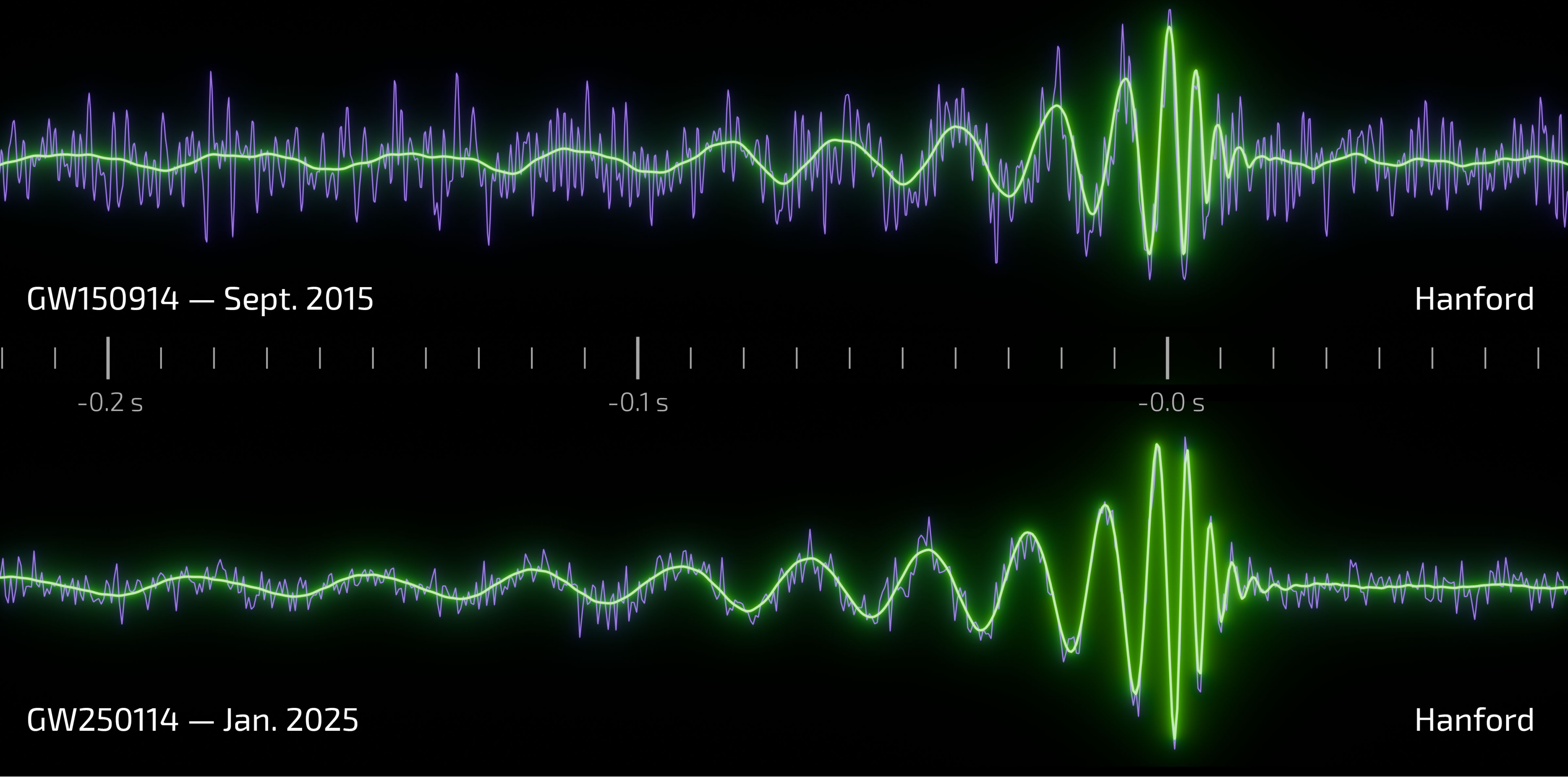


Numerical relativity simulations of binary black hole merger




LIGO Hanford Data
Predicted
LIGO Livingston Data
Predicted
LIGO Hanford Data (shifted)
LIGO Livingston Data
Strain (10^{-21})
1.0
0.5
0.0
-0.5
-1.0
0.30
0.35
0.40
0.45
Time (sec)

10 Years Later: LIGO Hears Loud and Clear



Caltech/MIT/LIGO Laboratory

Cosmological compact binary coalescence

- Compact binaries at cosmological distances

$$h_c(t_{\mathrm{obs}}) \sim \frac{\mathcal{M}_c^{5/3} f_{\mathrm{gw}}^{2/3}}{a(t_{\mathrm{obs}})\, r}$$

$$f_{\mathrm{gw}} = (1+z) f_{\mathrm{obs}}$$

$$\downarrow \qquad \mathcal{M}_z = (1+z)\mathcal{M}_c$$

$$\boxed{h_c(t_{\mathrm{obs}}) \sim \frac{\mathcal{M}_z^{5/3} f_{\mathrm{obs}}^{2/3}}{d_{\mathrm{L}}^{\mathrm{gw}}}}$$

$$\boxed{d_{\mathrm{L}}^{\mathrm{gw}} = d_{\mathrm{L}}^{\mathrm{em}} = a_0(1+z)\int_0^{z_{\mathrm{src}}} \frac{dz}{H(z)}}$$

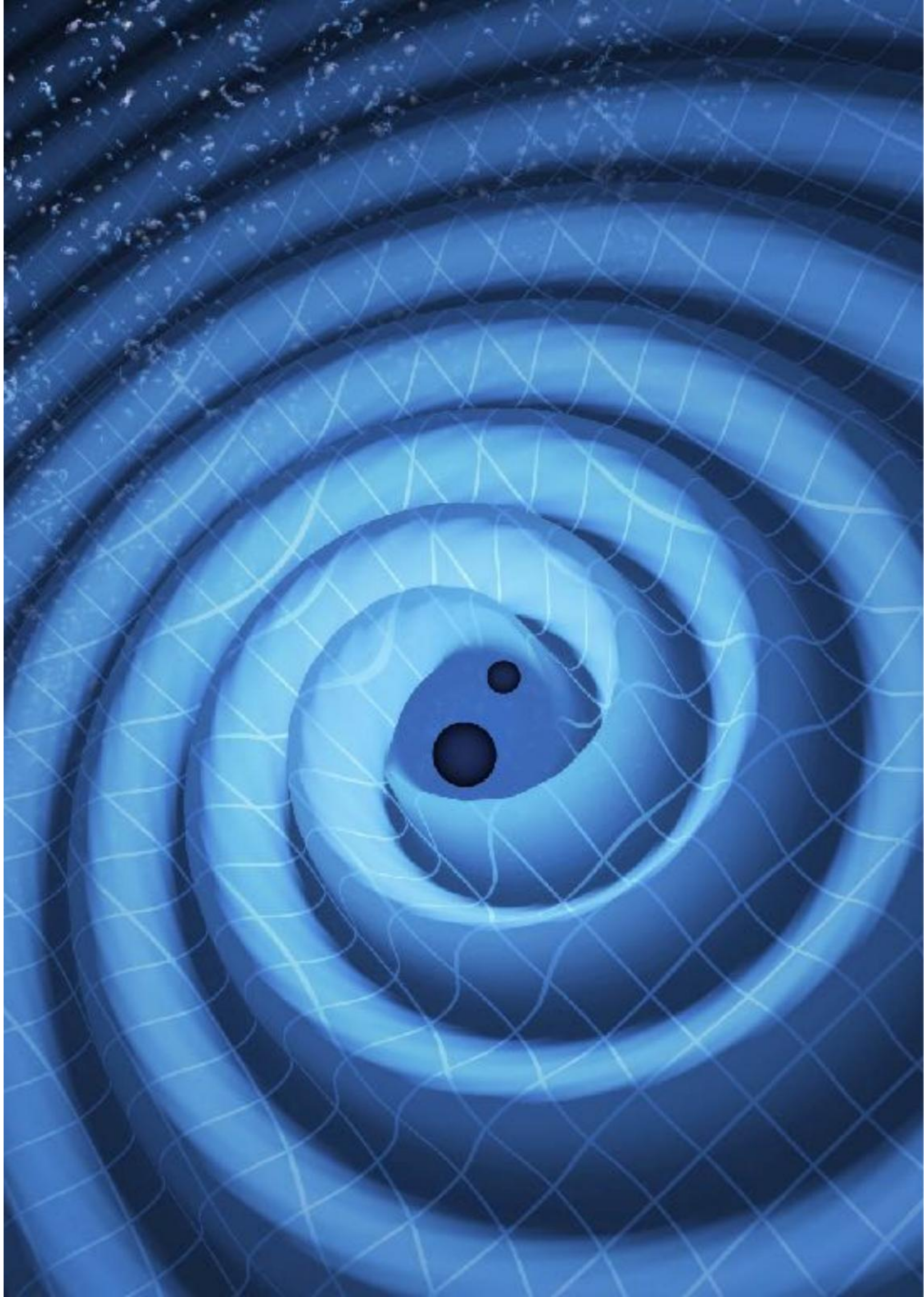
GW's amplitude scale with the inverse of the luminosity distance!

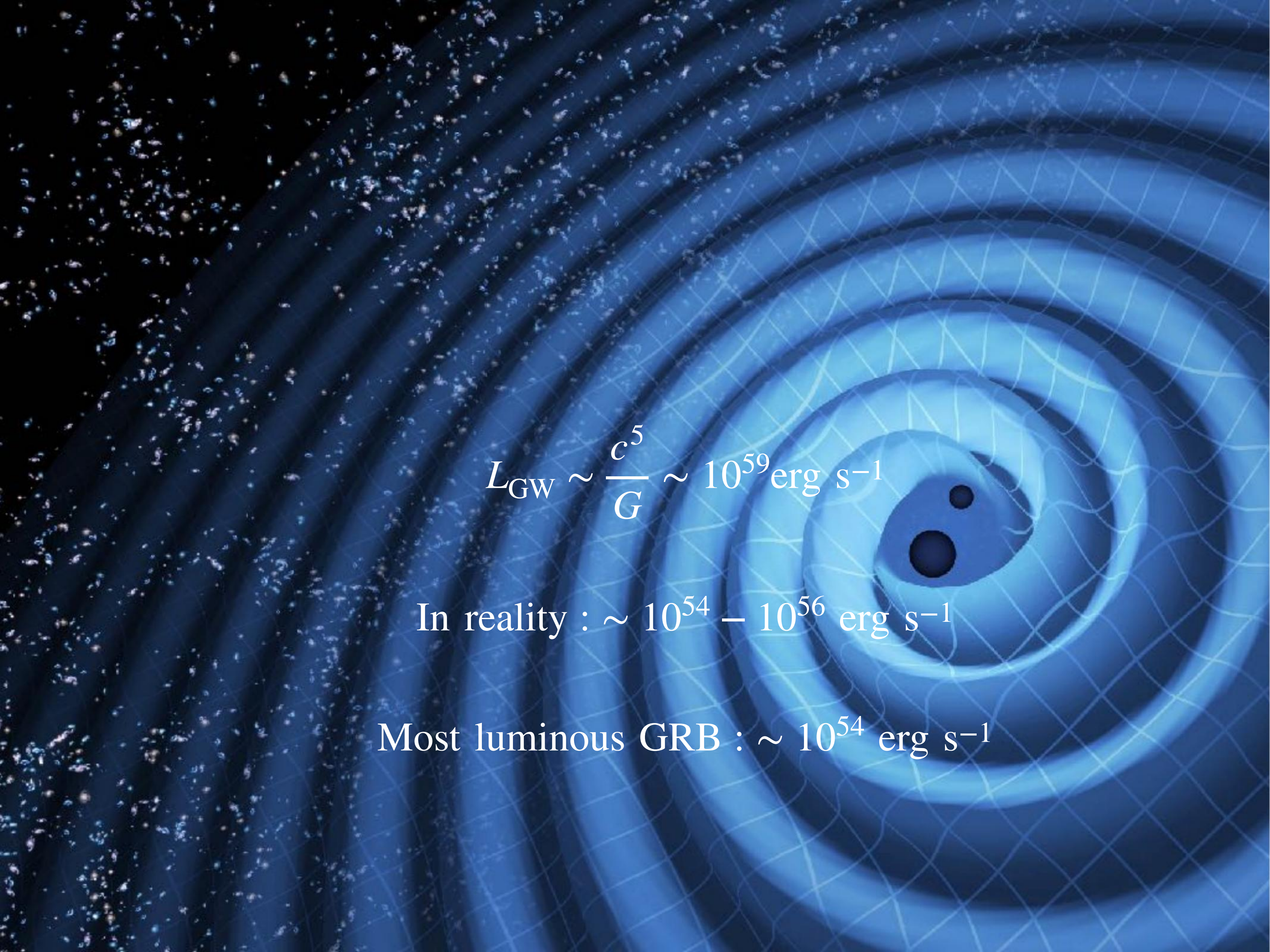
Their amplitude is sensitive to the expansion rate of the Universe!

1. Key takeaways

- Compact object binaries have many possible formation channels
- Gravitational waves are perturbations of space-time that propagate across the Universe
- They propagate along null geodesics and carry only two polarizations
- Gravitational waves are sourced by the second time derivative of the quadrupole moment
- Compact binary coalescences produce sizable gravitational waves with a chirping waveform
- On a cosmological background, amplitude scales inversely with the luminosity distance

Gravitational Wave Astronomy





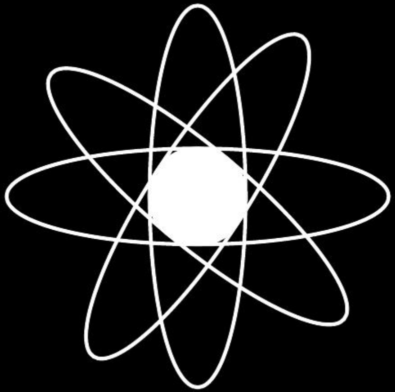
$$L_{\mathrm{GW}} \sim \frac{c^5}{G} \sim 10^{59} \mathrm{erg\ s}^{-1}$$

In reality : $\sim 10^{54} - 10^{56}$ erg s^{-1}

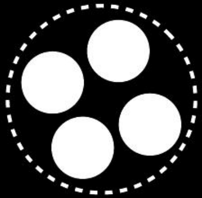
Most luminous GRB : $\sim 10^{54}$ erg s^{-1}

The variation in the distance is minuscule

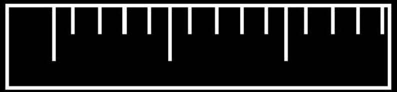
0.000000000000000001 meters



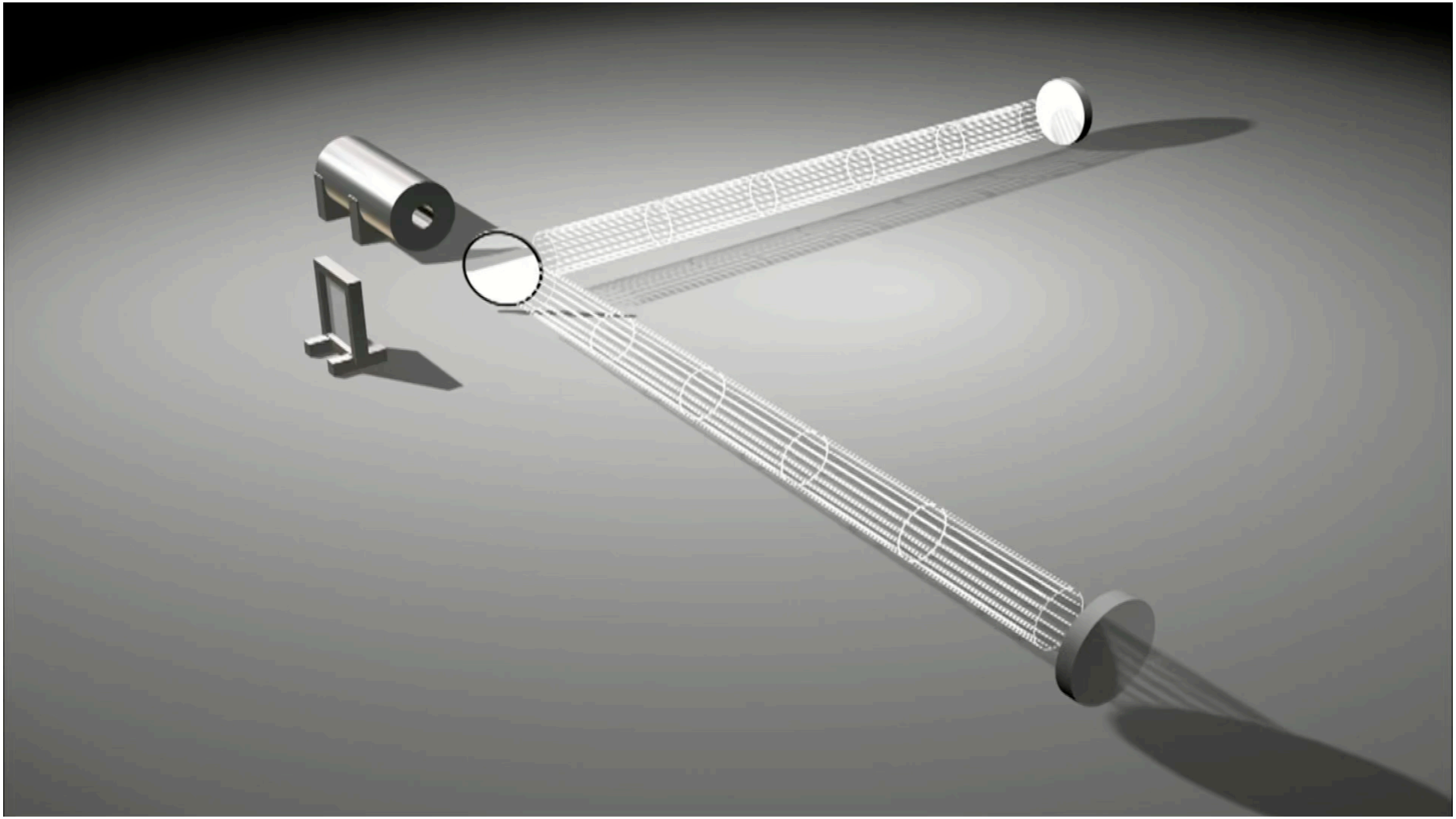
atom: 10^{-10} meters



nucleus: 10^{-15} meters



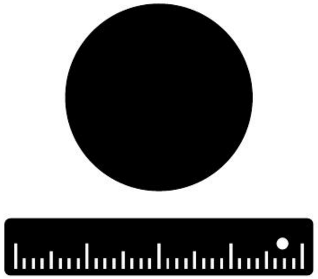
GW effect: 10^{-18} meters



Credit: NSF

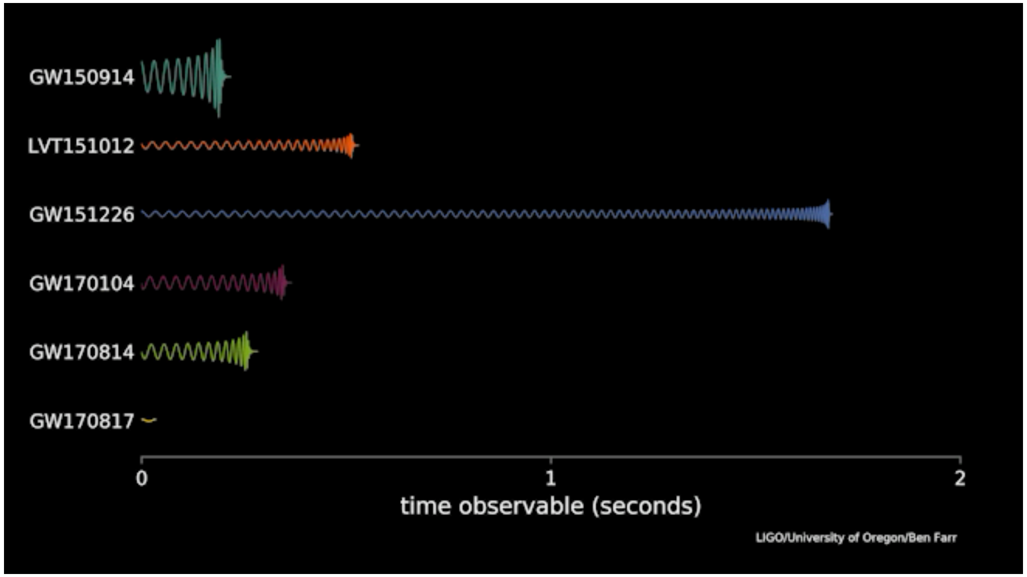
Tuned for detecting compact objects

$$f \sim \frac{1}{2\pi}\frac{1}{2t_{\mathrm{Sch}}} \sim 800\mathrm{Hz}\left(\frac{10M_\odot}{M}\right)$$



$$h \sim \mathcal{O}(1)\cdot\frac{r_{\mathrm{Sch}}}{r} \sim 10^{-23}\left(\frac{1\mathrm{Gpc}}{r}\right)\left(\frac{M}{10M_\odot}\right)$$

$$r_{\mathrm{Sch}} = 2GM/c^2$$



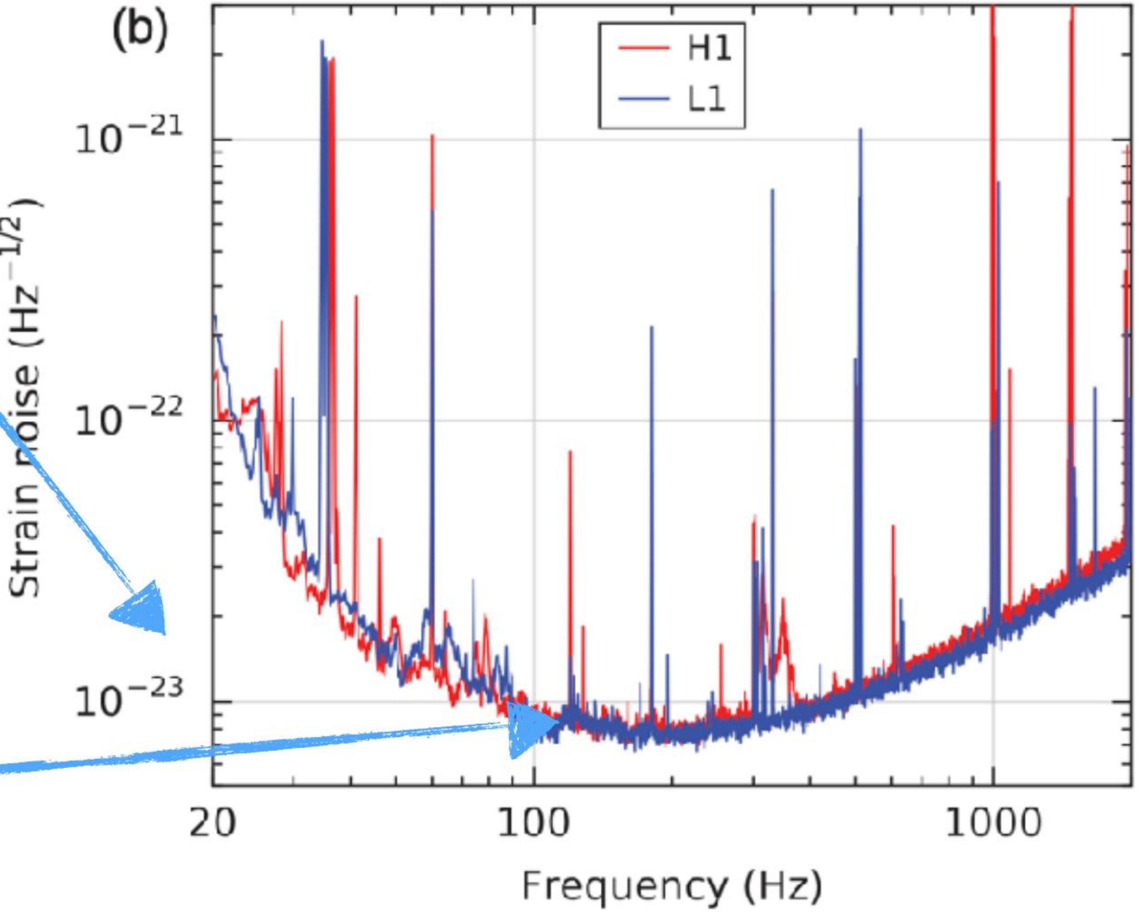
LIGO/University of Oregon/Ben Farr

Cosmological distance

$$\frac{1\mathrm{Gpc}}{c} \sim 3\mathrm{Gyr} \sim 0.2t_{\mathrm{Uni}}$$

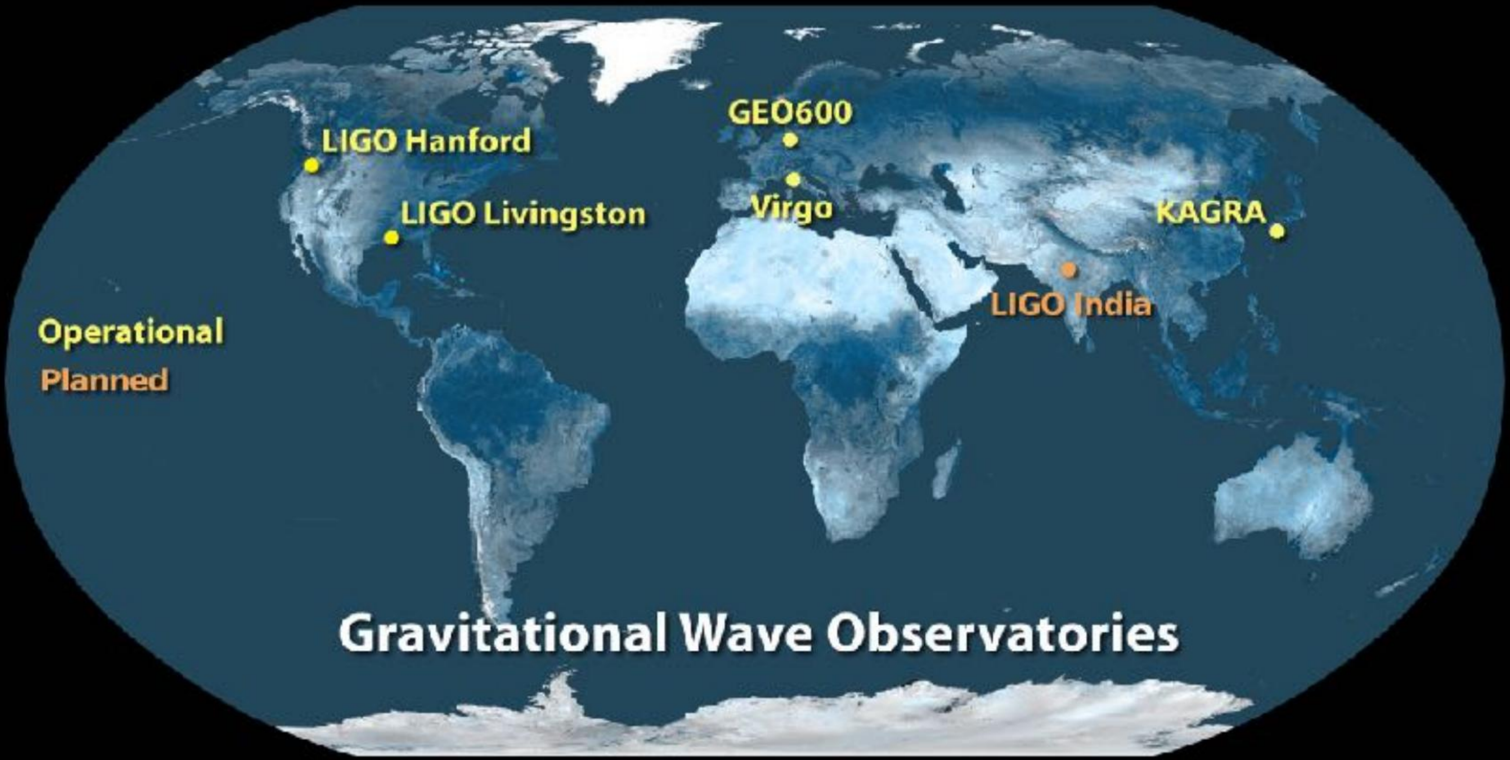
Tuned for detecting compact objects

$$h \sim 10^{-23} \left(\frac{1\mathrm{Gpc}}{r}\right)\left(\frac{M}{10M_\odot}\right)$$



$$f \sim 800\mathrm{Hz}\left(\frac{10M_\odot}{M}\right)$$

The era of gravitational wave astronomy is **here**!



[Hanford, US]

[Livingston, US]

[Virgo, Italy]

[KAGRA, Japan]

Gravitational wave detectors

- Detectors are defined by their *noise, n(t)*

- Some simplifying assumptions:

Stationary: $R(\tau) \equiv \langle n(t) n(t+\tau) \rangle$

Ergodic: $\langle n \rangle = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} n(t) dt$

Zero-mean: $\langle n(t) \rangle = 0$

Power spectral density (PSD)

Gaussian: $\langle \tilde{n}^*(f) \tilde{n}(f') \rangle = \frac{1}{2} S_n(f) \delta(f - f')$

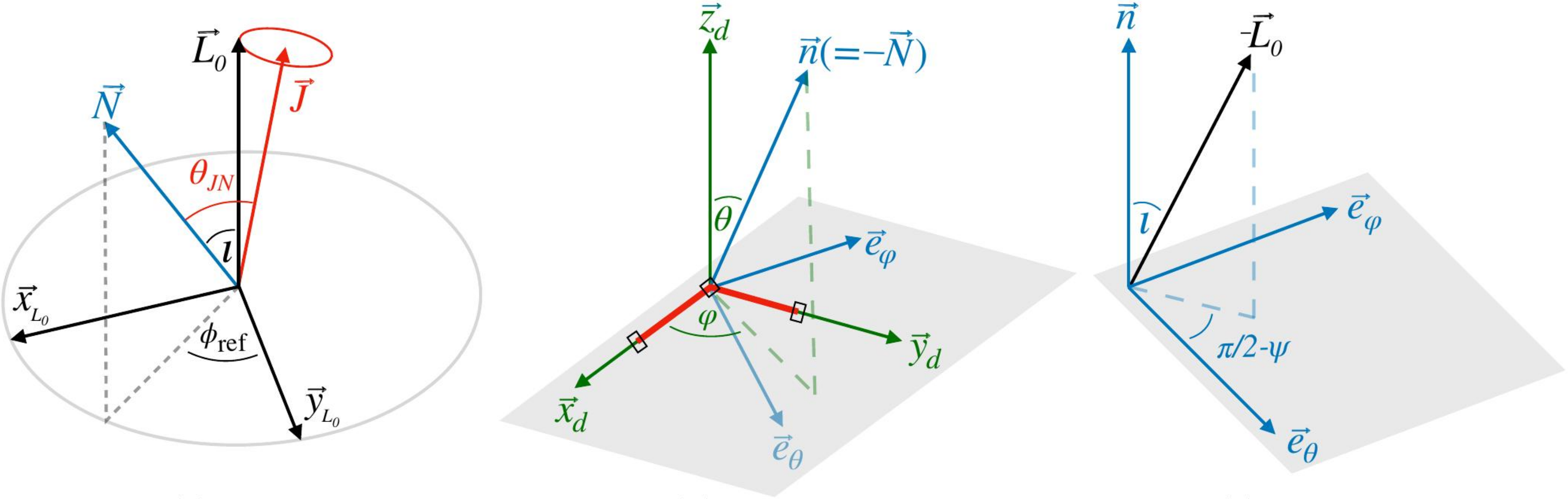
- Probability of noise realization *n(t)*

$$\boxed{p_n[n(t)] \propto \exp\left[-2 \int_0^\infty \frac{|\tilde{n}(f)|^2}{S_n(f)} df\right]}$$

Gravitational wave detectors

- Detectors are also defined by their *antenna response*

$$h(t) = h_+(t)F_+(\hat{n}) + h_\times(t)F_\times(\hat{n})$$

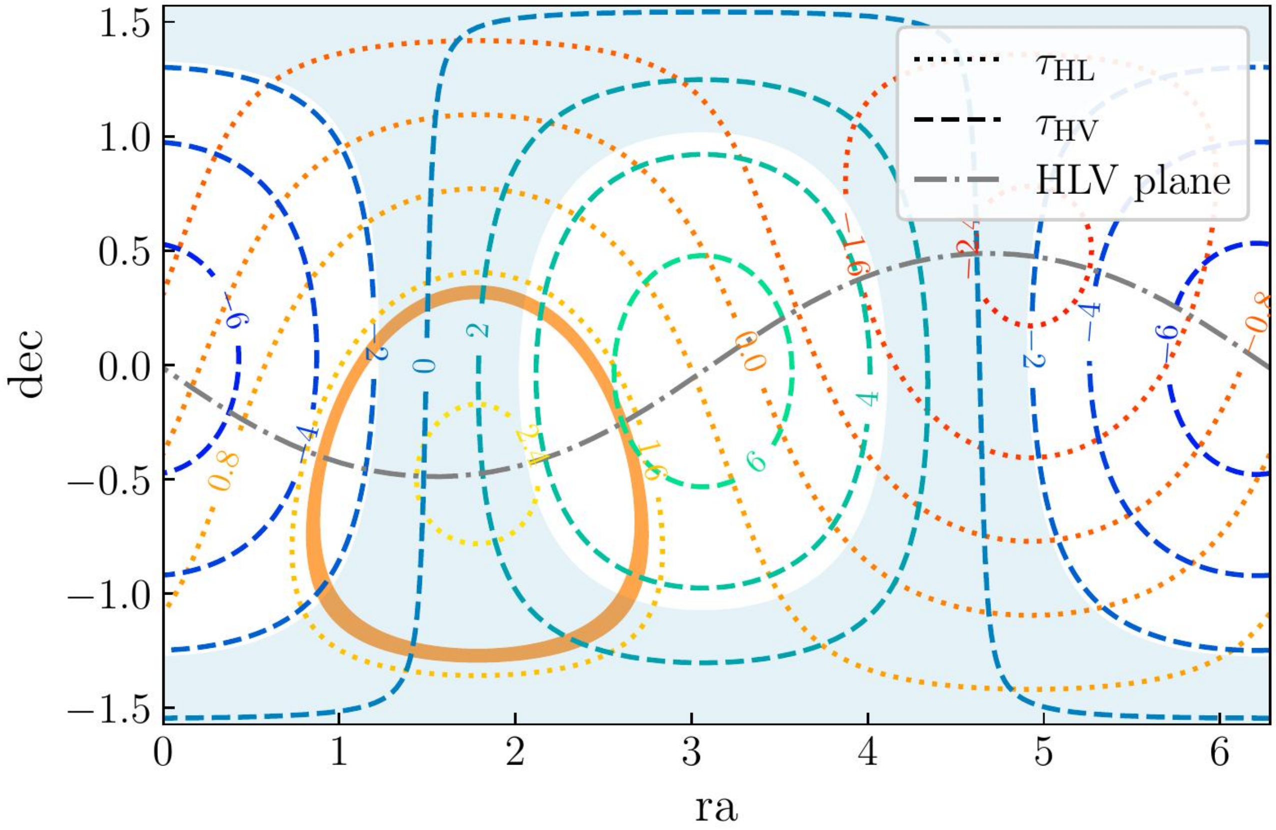


(a) Source (b) Earth detector (c) Sky

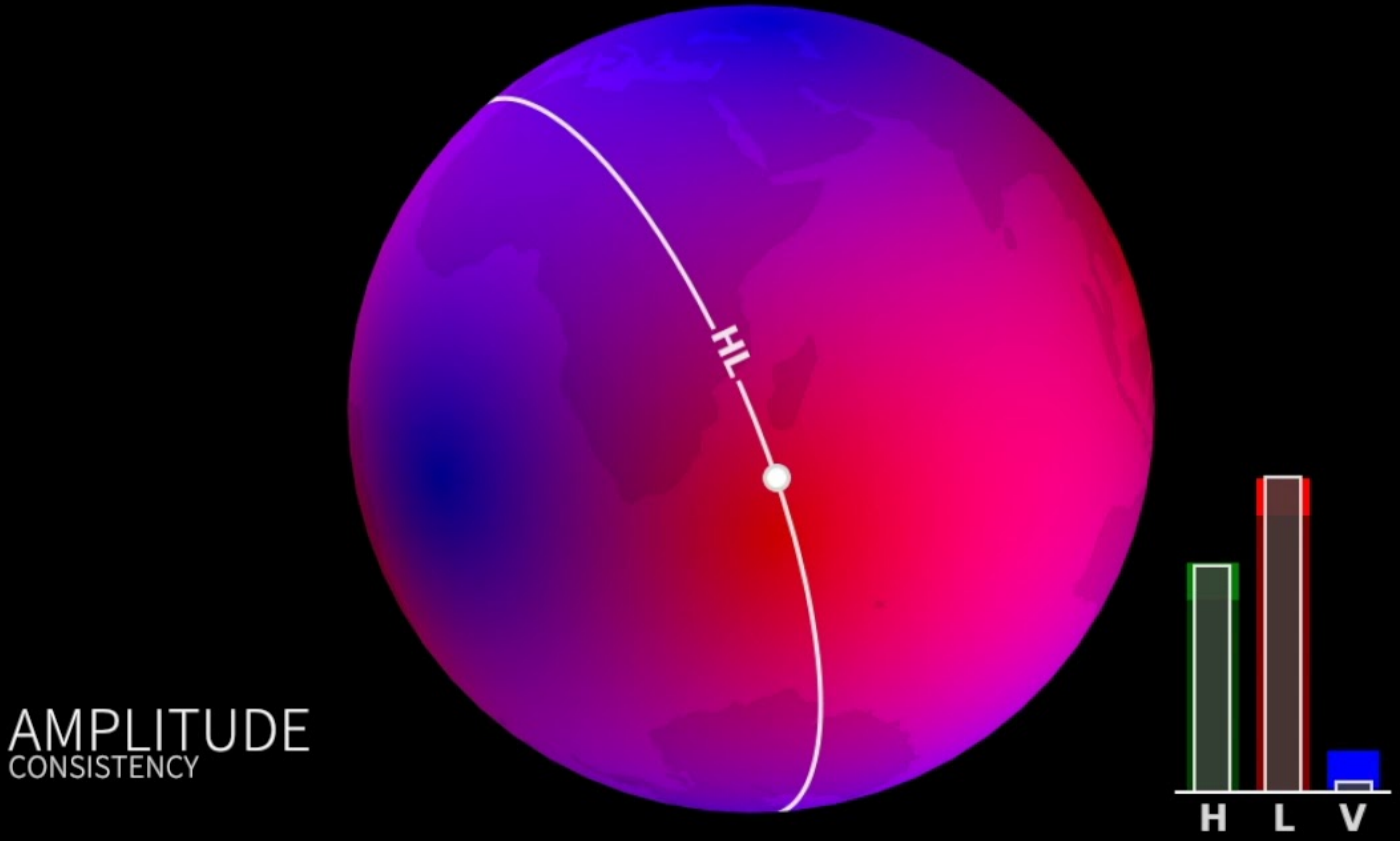
Sky localization

- The arrival time difference between two detectors defines a ring in the sky

$$\Delta t_{d_1 d_2} = \vec{n} \cdot \vec{r}_{d_1 d_2} / c$$



Sky localization



[Credit: Leo Singer]

Matched-filtering

Idea: use **waveform models** as **templates to match and filter** the data, digging up the signal from the noise

- The data stream: $d(t) = s(t) + n(t)$
- Filter data: $\hat{d} = \int_{-\infty}^{\infty} dt\, d(t) K(t)$
- Signal to noise:

$$S/N = \frac{\int_{-\infty}^{\infty} df\, \tilde{s}(f) \tilde{K}^*(f)}{\sqrt{\int_{-\infty}^{\infty} df\, \frac{1}{2} S_n(f) |\tilde{K}(f)|^2}}$$

- Define noise weighted inner product

$$(a|b) \equiv \mathrm{Re}\left[\int_{-\infty}^{\infty} \frac{\tilde{a}^*(f)\tilde{b}(f)}{S_n(f)/2}\right] = 4\mathrm{Re}\left[\int_{0}^{\infty} \frac{\tilde{a}^*(f)\tilde{b}(f)}{S_n(f)}\right],$$

- Rewrite S/N:

$$\boxed{S/N = \frac{(u|s)}{\sqrt{(u|u)}} \qquad \tilde{u}(f) = \frac{1}{2} S_n(f) \tilde{K}(f)}$$

Matched-filtering

- Signal to noise:

$$S/N = \frac{(u|s)}{\sqrt{(u|u)}}$$

- Optimal filter when *u* is parallel to *s*

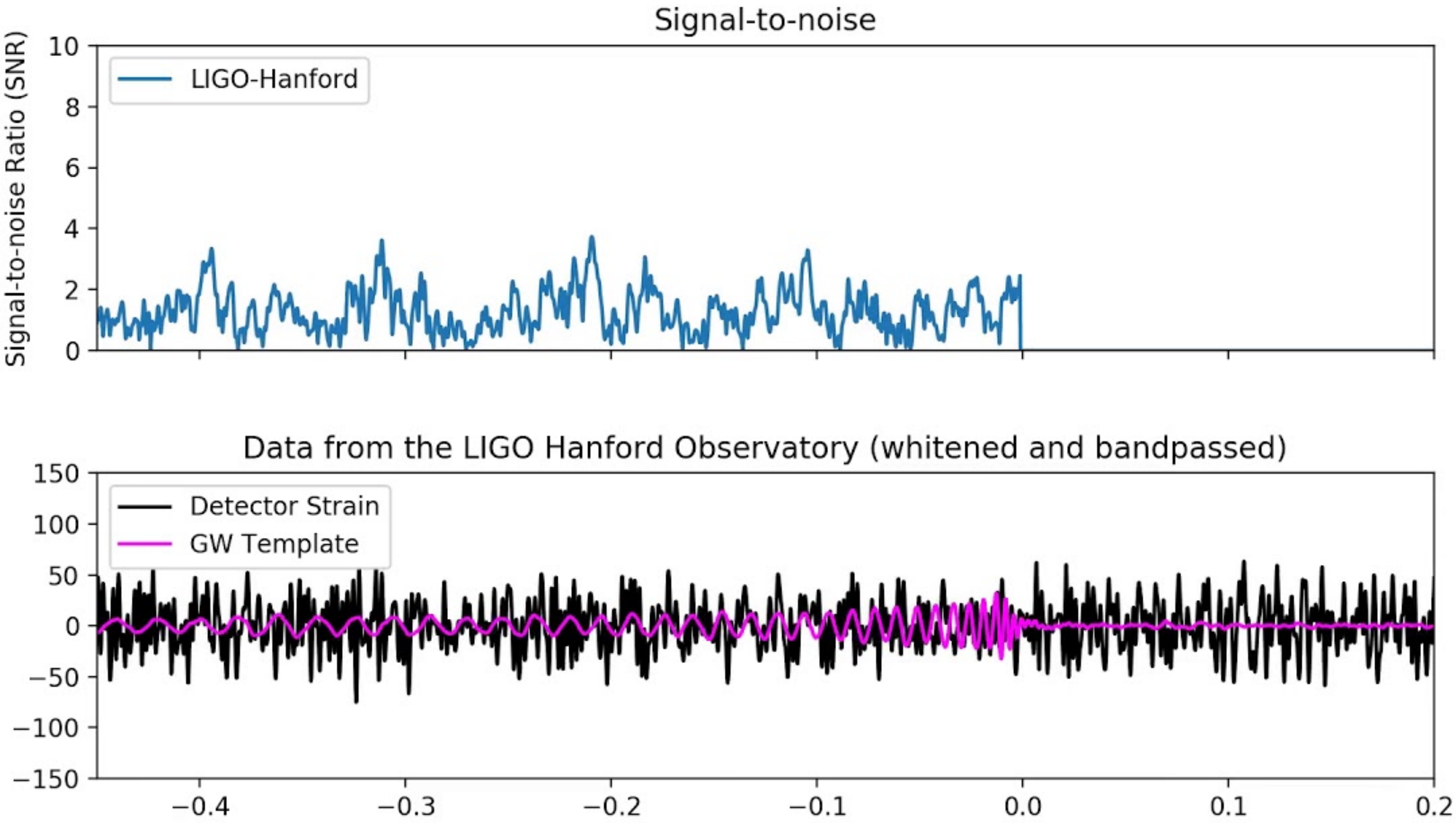
$$\tilde{K}(f) \propto \frac{\tilde{s}(f)}{S_n(f)}$$

- Optimal signal-to-noise ratio

$$\boxed{\rho^2_{\rm opt} = (h|h) = 4\mathrm{Re}\left[\int_0^\infty \frac{|\tilde{h}(f)|^2}{S_n(f)}\right]}$$

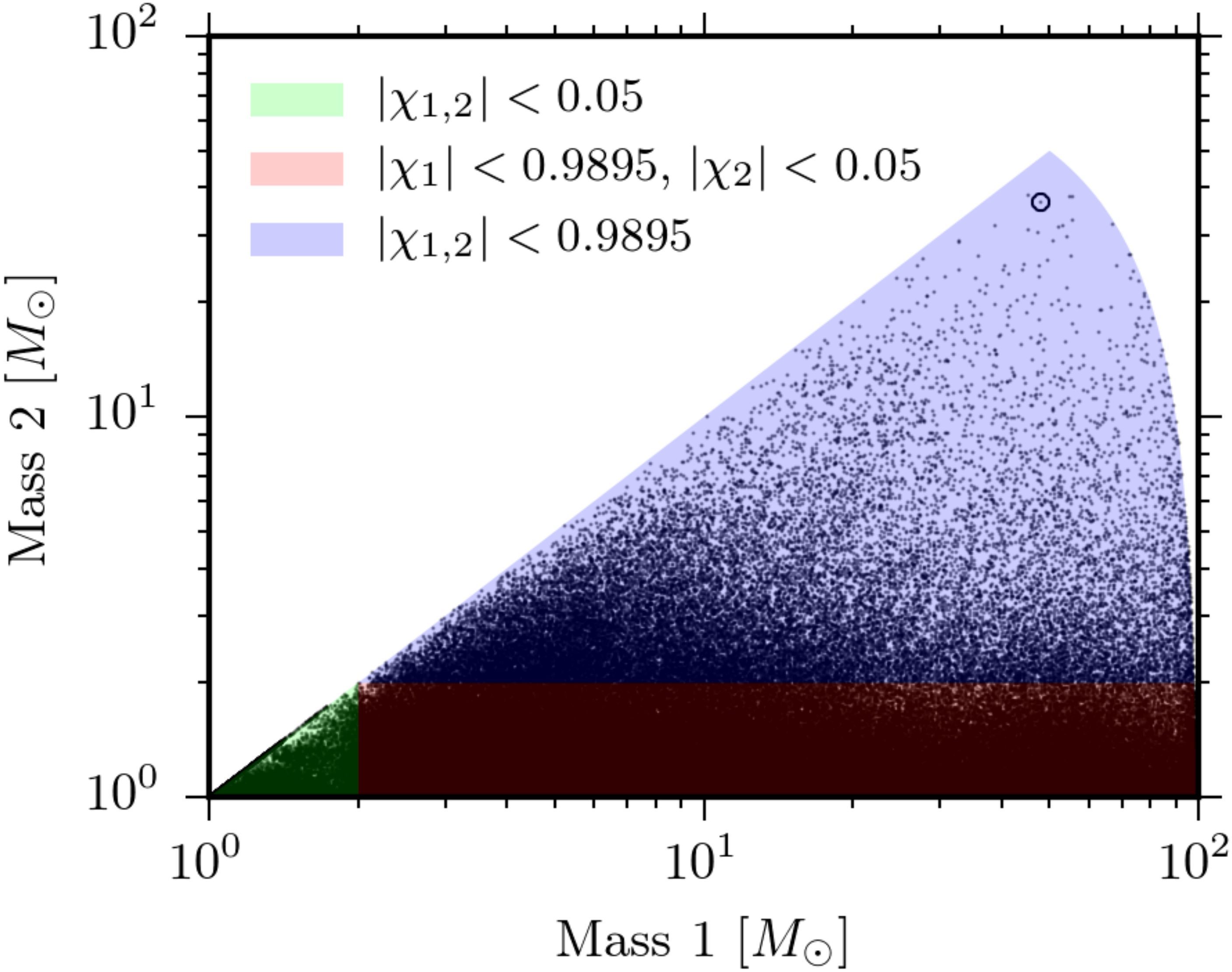
$$\rho_{\rm ntw} = \sqrt{\sum_i \rho_i^2}$$

Matched-filtering



[Credit: Alex Nitz]

Matched-filtering



[Credit: LVK]

Parameter estimation

- If we subtract the the right signal to the data, we should recover the noise

$$n(t) = d(t) - s(t)$$

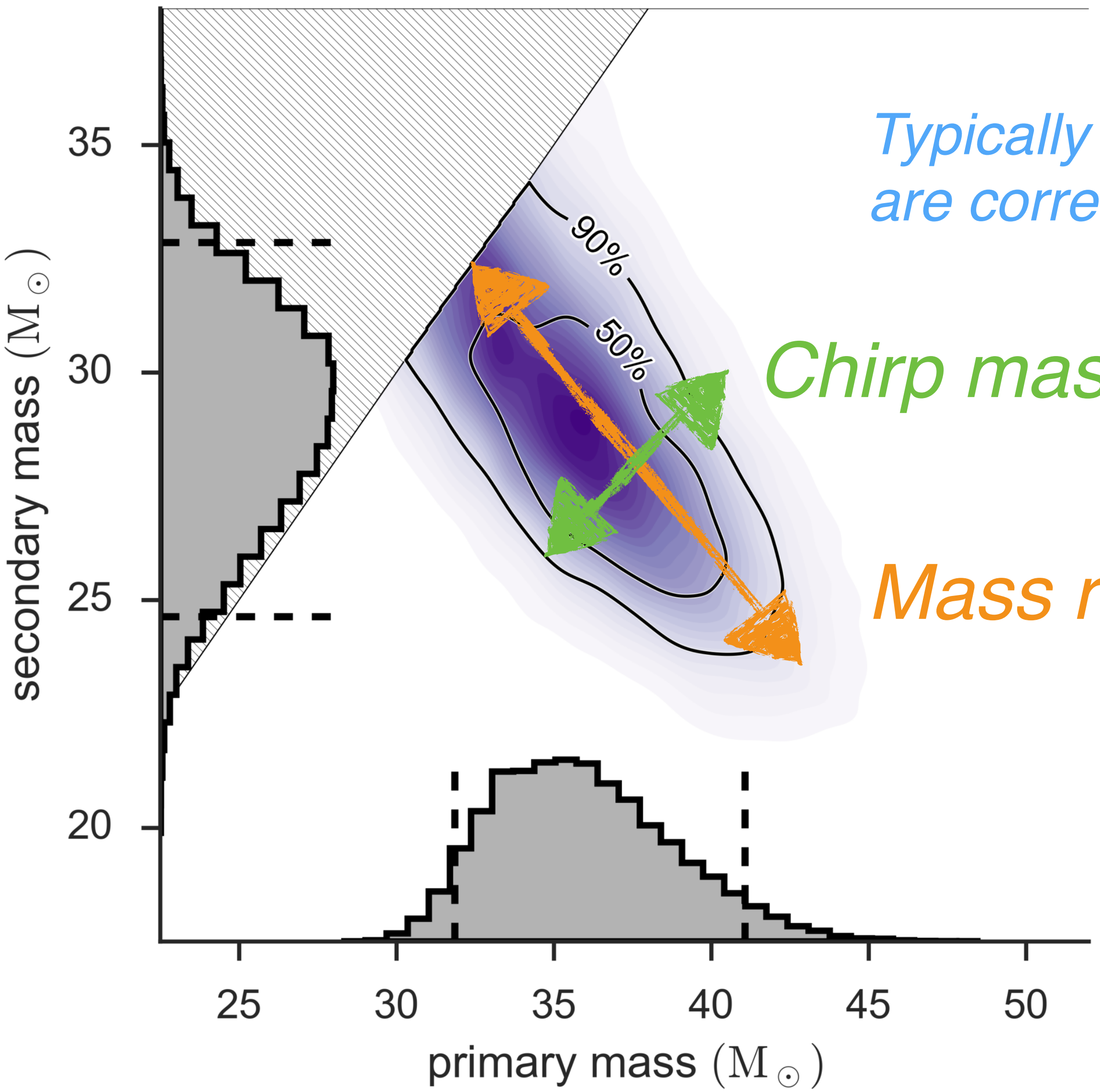
- Assuming Gaussian noise, the likelihood of the data is

$$\begin{aligned}\Lambda(d|\theta) &\propto \exp\left[-\frac{1}{2}(d - h(\theta)|d - h(\theta))\right] \\ &= \exp\left[(d|h(\theta)) - \frac{1}{2}(h(\theta)|h(\theta)) - \frac{1}{2}(d|d)\right]\end{aligned}$$

- The posterior distribution of a parameter (Bayes theorem)

$$\boxed{p(\theta|d) \propto p(\theta) \exp\left[(d|h(\theta)) - \frac{1}{2}(h(\theta)|h(\theta))\right]}$$

Parameter estimation



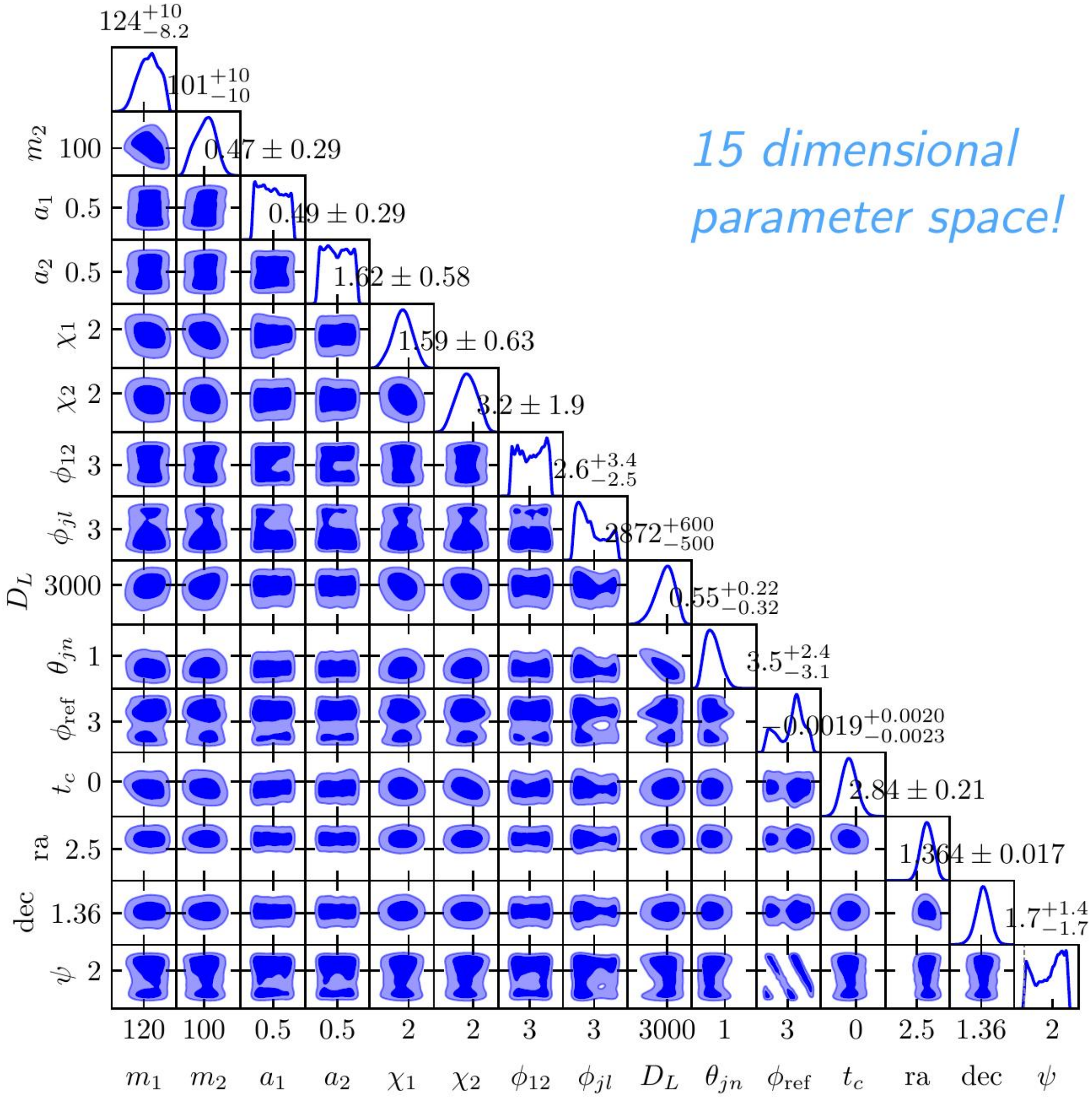
Typically parameters are correlated

Chirp mass

Mass ratio

[Credit: LVK]

Parameter estimation



15 dimensional parameter space!

Population inference

- Consider events collectively to learn about the population properties: formation channels, standard sirens
- *Astrophysical* *vs* *detected* population
- Individual detections can be fit jointly using **hierarchical Bayesian inference** to measure the mass (and other parameters) distribution of merging compact objects
  - Population model described by hyper-parameters $\lambda = \{\lambda\}$
  - GW data $\{d_i\}$ for Nobs events, with posteriors $p(\theta | d_i)$ on the parameters $\theta = \{\theta_i\}$
  - Hierarchical: we need $p(\theta | d_i)$ first before getting hyper-parameters posteriors

Population inference

- The posterior distribution of the hyper-parameters

$$p(\lambda|\{d_i\}) \propto p(\lambda)p(\{d_i\}|\lambda) = p(\lambda)\prod_{i=1}^{N_{\rm obs}} \frac{p_{\rm pop}(\theta_i|\lambda)}{\int d\theta p_{\rm pop}(\theta|\lambda)}$$

- Including selection effects

$$p(\{d_i\}|\lambda) = \prod_{i=1}^{N_{\rm obs}} \frac{p_{\rm pop}(\theta_i|\lambda)p_{\rm det}(\theta_i)}{\int d\theta p_{\rm pop}(\theta|\lambda)p_{\rm det}(\theta)} = \prod_{i=1}^{N_{\rm obs}} \frac{p_{\rm pop}(\theta_i|\lambda)}{\int d\theta p_{\rm pop}(\theta|\lambda)p_{\rm det}(\theta)}$$

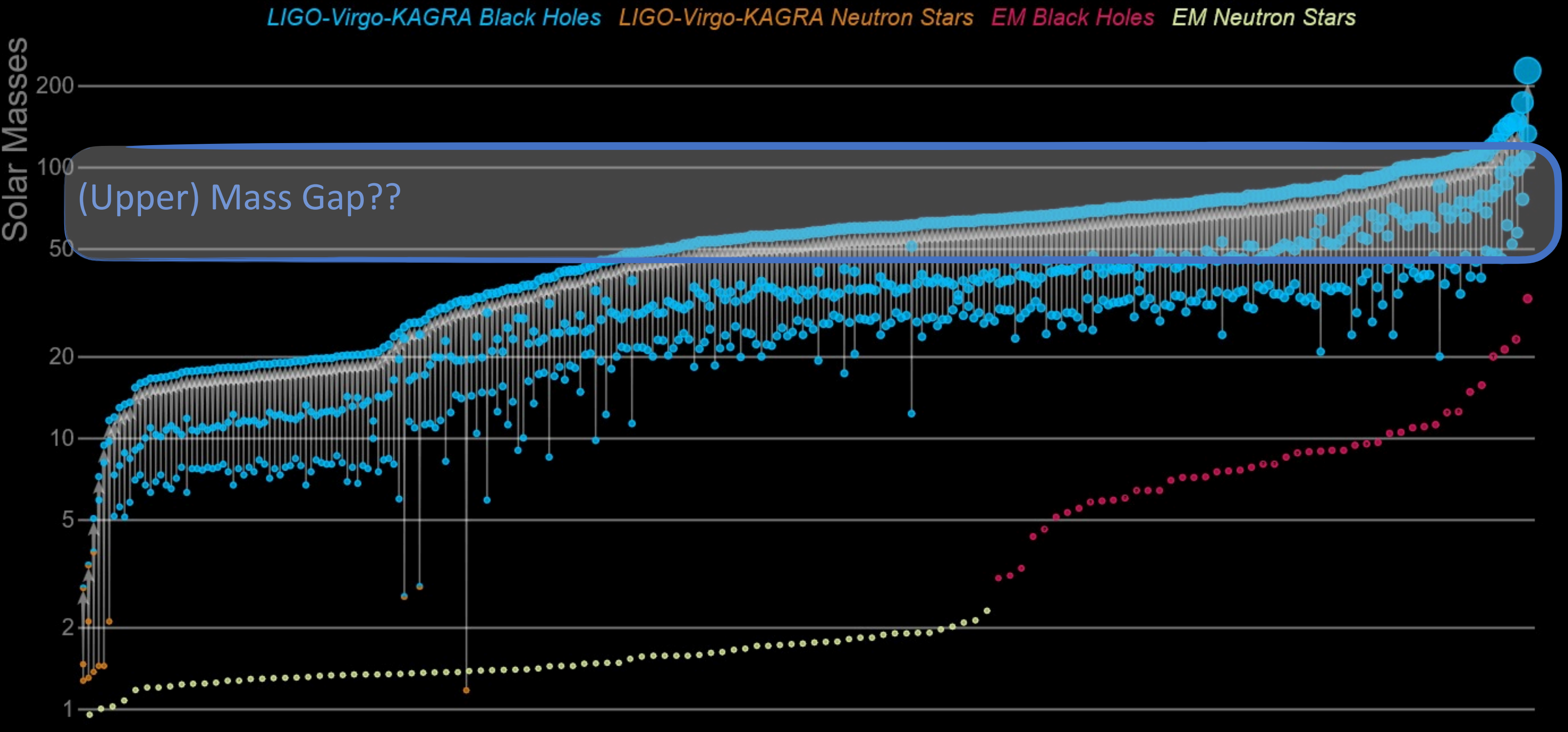
$p(d_i|\lambda)$ Likelihood of data given pop model for event i

- Including measurement uncertainties

$$p(\{d_i\}|\lambda) = \prod_{i=1}^{N_{\rm obs}} \frac{\int d\theta p(d_i|\theta)p_{\rm pop}(\theta|\lambda)}{\int d\theta p_{\rm pop}(\theta|\lambda)p_{\rm det}(\theta)}$$

See Mandel, Farr & Gair 2018, MNRAS; Callister 2025, Encyclopedia of Astrophysics

Masses in the Stellar Graveyard



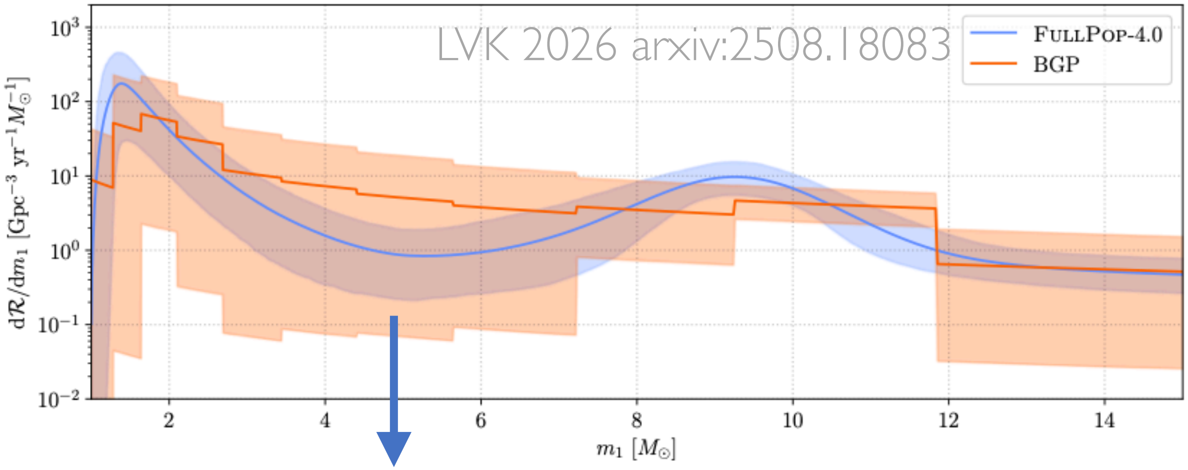
LIGO-Virgo-KAGRA | Aaron Geller | Northwestern

390 GW events

- 2-3 binary neutron stars
- 5-7 neutron star black holes

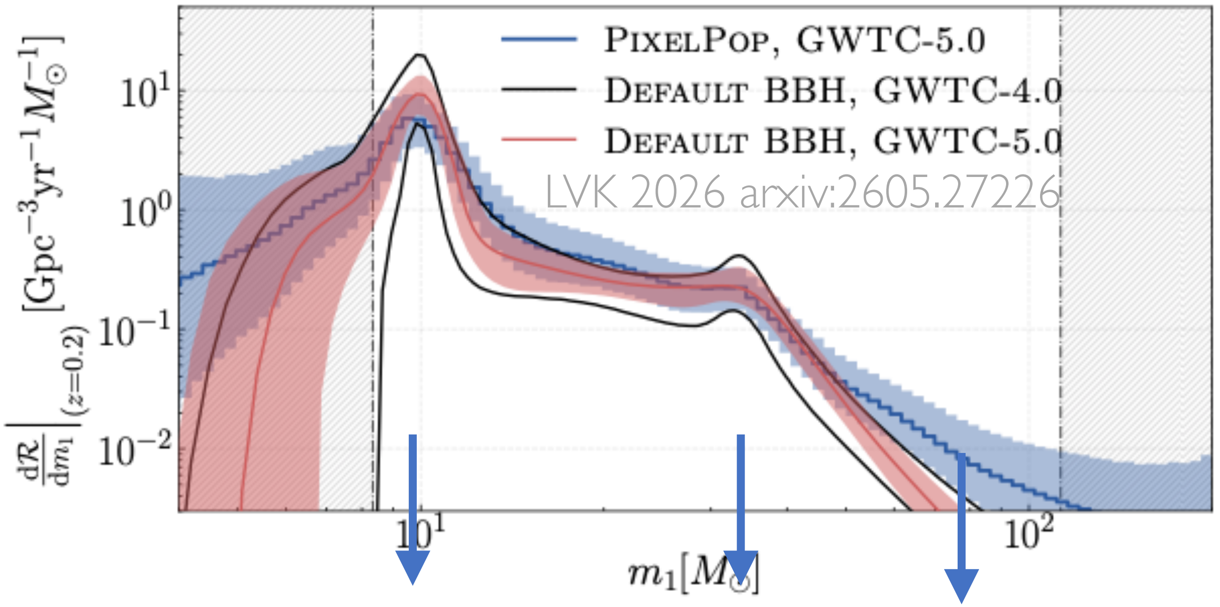
1 confirmed EM counterpart (a dozen candidates)

GWTC-4/5 Population



LVK 2026 arxiv:2508.18083

Drop



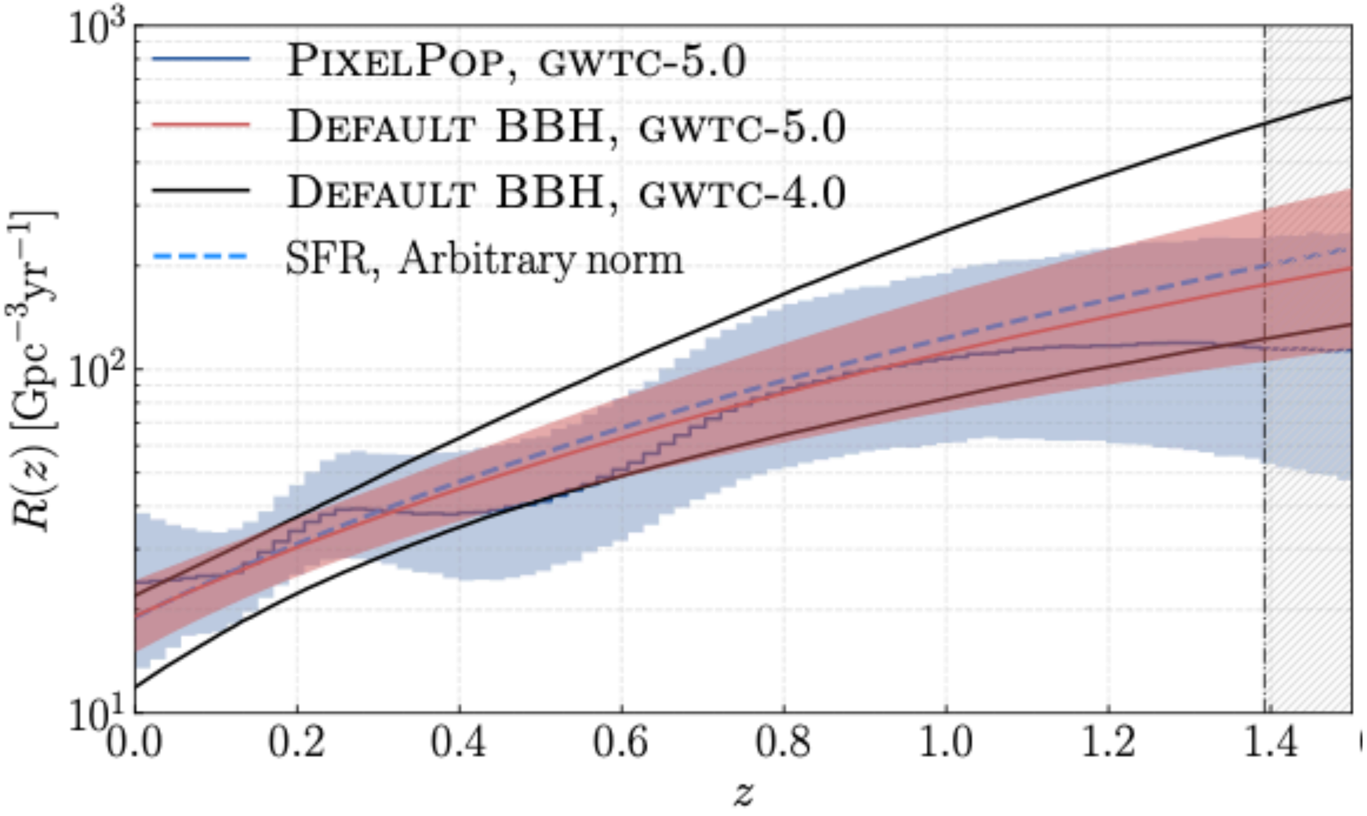
LVK 2026 arxiv:2605.27226

Peak

Build-up/Shoulder

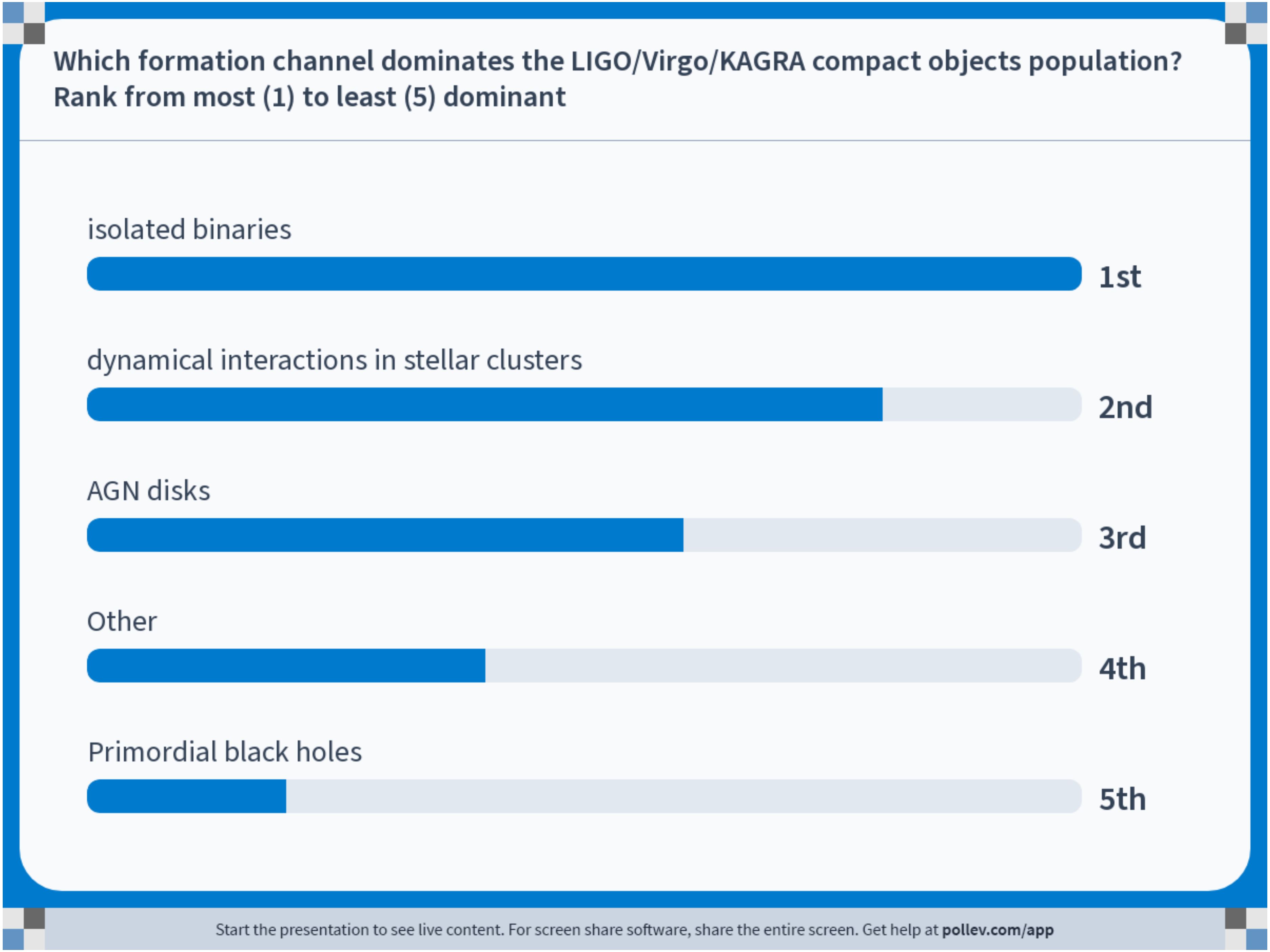
Gap?

GWTC-5 Population



LVK 2026 arxiv:2605.27226

Join by Web **PollEv.com/antonellapal851**



Which formation channel dominates the LIGO/Virgo/KAGRA compact objects population? Rank from most (1) to least (5) dominant

isolated binaries
1st

dynamical interactions in stellar clusters
2nd

AGN disks
3rd

Other
4th

Primordial black holes
5th

Start the presentation to see live content. For screen share software, share the entire screen. Get help at **pollev.com/app**