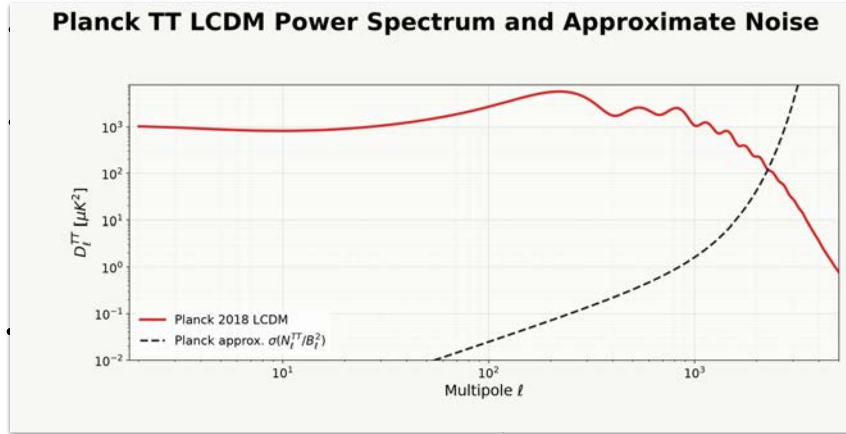
Introduction to the angular power spectrum

- The variance within a patch of radius θ is $\sigma^2(\theta) \simeq \int_{\ell_{\min}}^{1/\theta} d\ln\ell \, \frac{\ell^2 C_\ell}{2\pi}$
- Each pixel will have some (uncorrelated) noise (n = rms pixel noise):

$$\sigma^2_{\text{noise}}(\theta) = \frac{\text{Variance per pixel}}{\text{\# of pixels in patch}} = \frac{n^2}{\pi\theta^2/\Omega_{\text{pix}}} \quad \Rightarrow \quad N_\ell = n^2\Omega_{\text{pix}}$$

- The telescope has some finite resolution, σ_b (often called a 'beam')
- Approximating it as a Gaussian we have $B_\ell = e^{-\frac{1}{2}\ell(\ell+1)\sigma_b^2}$
- The *observed* power spectrum is: $C_{\ell,\text{obs}} = C_\ell B_\ell^2 + N_\ell$

A (theorist's) guide to CMB sensitivity



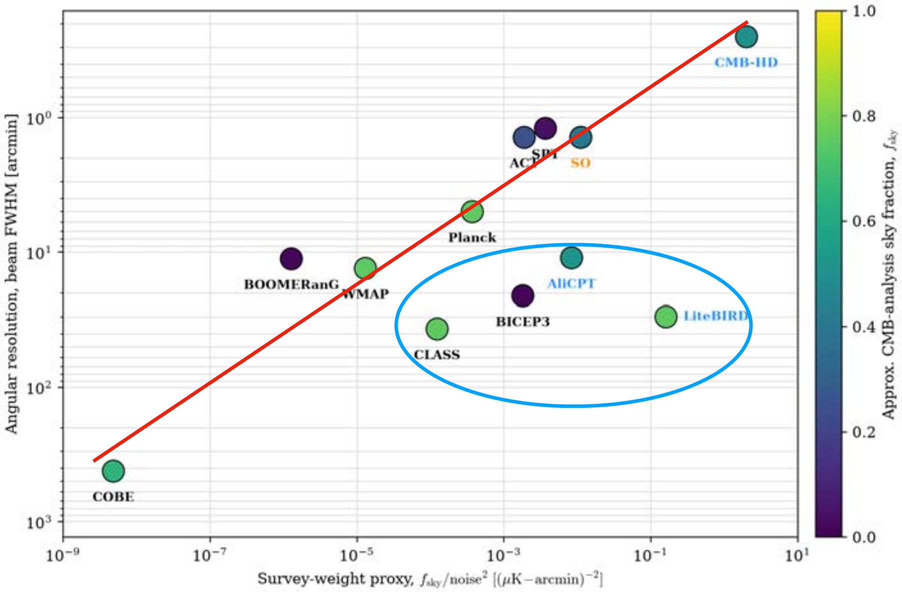
- The variance in this estimator is

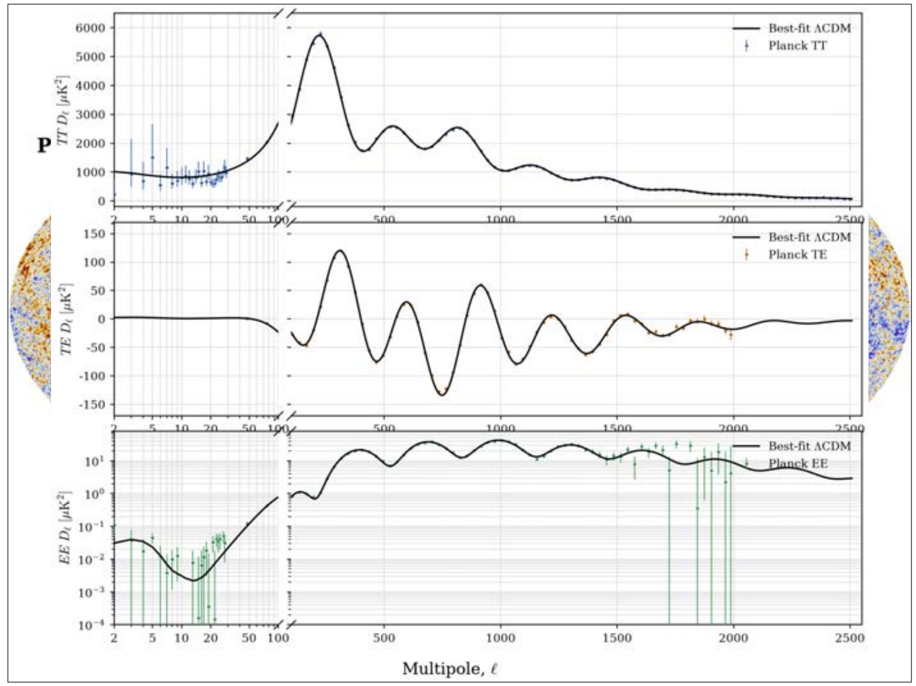
$$\text{Var}\left(\hat{C}_\ell^{TT,\text{dec}}\right) = \frac{2}{(2\ell+1)f_{\text{sky}}}\left(C_\ell^{TT} + N_\ell^{TT} e^{\ell(\ell+1)\sigma_b^2}\right)^2$$

noise amplitude $= n^2/f_{\text{sky}}$

$\ell_{\min} \simeq f_{\text{sky}}^{-1/2}$

A (theorist's) guide to CMB sensitivity

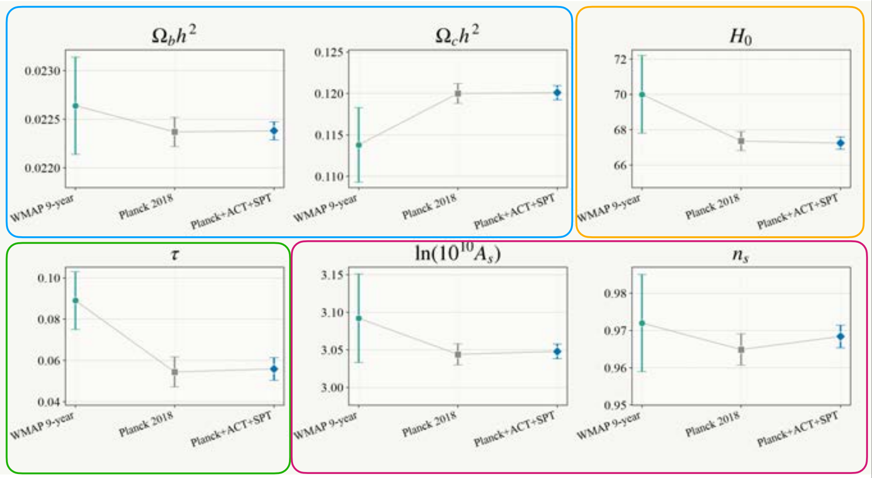




Current LCDM parameter constraints

What the universe is made of

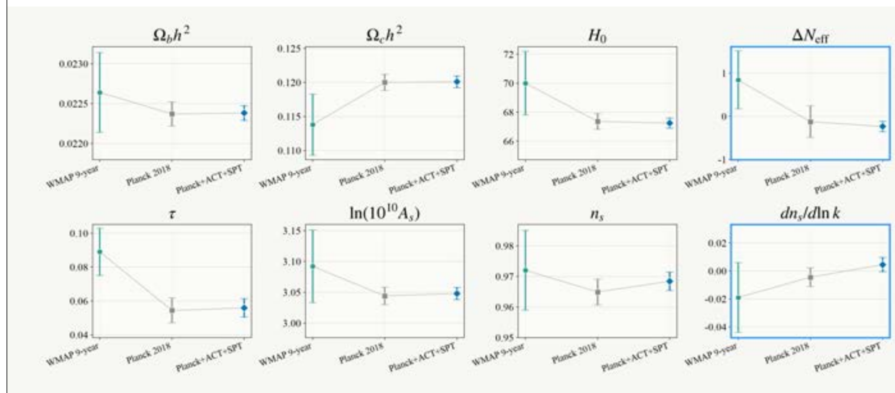
Current expansion rate of the universe



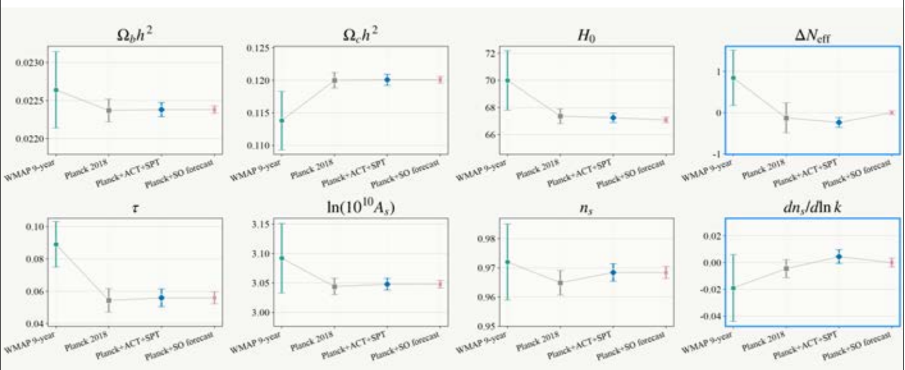
Late-time re-ionization of the universe

Cosmic inflation

Current LCDM+ parameter constraints

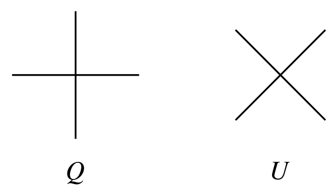


Forecasted LCDM+ parameter constraints



A bit more on polarization

- CMB light has fluctuations in its polarization from place to place
- The detectors measure linear polarization in perpendicular directions



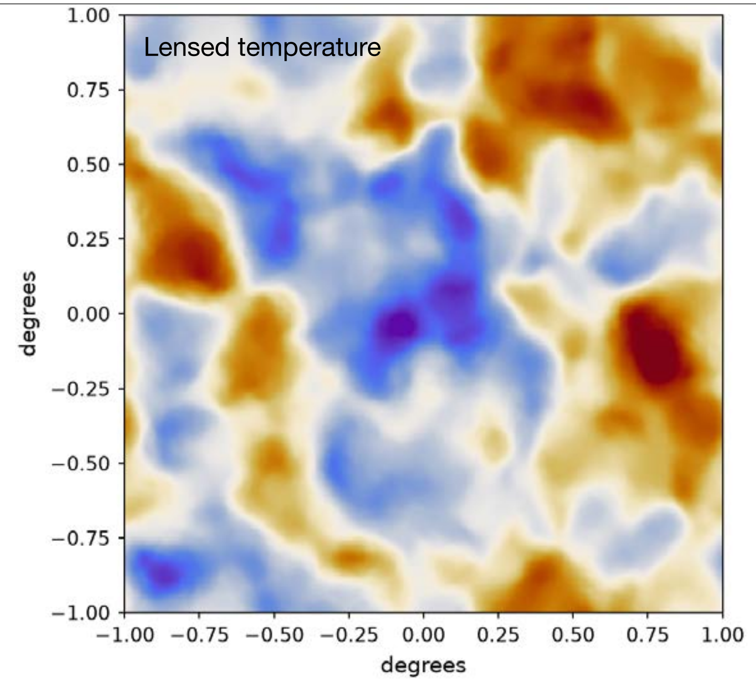
- These polarizations do not have definite parity… but physics cares a lot about parity, so *(in harmonic space)* we define linear combinations:

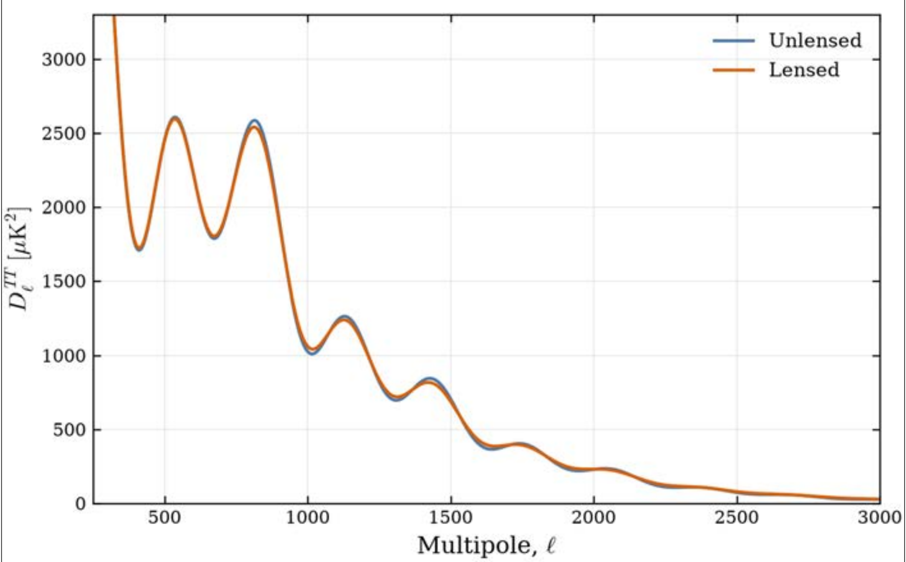
$E(\vec{\ell}) \equiv Q(\vec{\ell})\cos 2\phi_\ell + U(\vec{\ell})\sin 2\phi_\ell$ → Even parity (scalar perts)

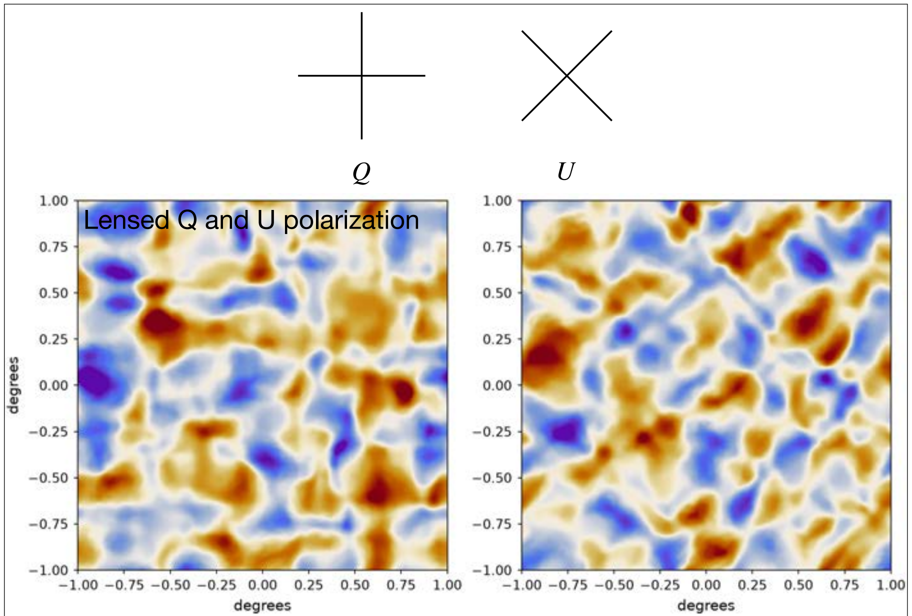
$B(\vec{\ell}) \equiv -Q(\vec{\ell})\sin 2\phi_\ell + U(\vec{\ell})\cos 2\phi_\ell$ → Odd parity (vector/GWs)

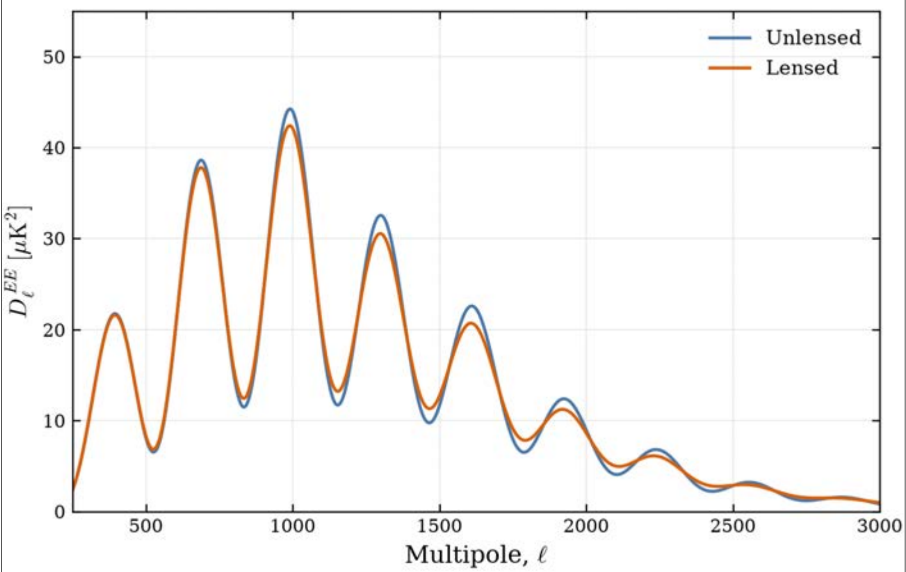
Beyond the 'primary' power spectra

- If the CMB fluctuations were purely Gaussian, the power spectrum would contain all of the statistical information….
- Lensing of CMB photons by intervening clumps of matter introduces a non-Gaussian contribution
- (There are more exotic— hypothesized— sources of non-Gaussian features that I won't discuss)
- The photons are deflected: $T(\hat{n}) \rightarrow T(\hat{n} + \vec{\nabla}\phi)$
- Modulates the power spectra, mixes the two polarization modes, and generates a non-Gaussian contribution to higher order correlations





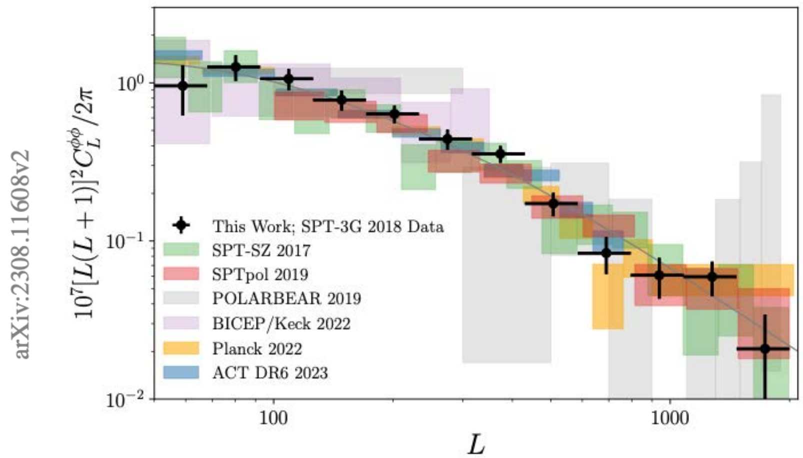




Lensing potential power spectrum

- Lensing potential power spectrum is estimated from the four point correlation function

$$\langle a^T_{\ell_1 m_1} a^T_{\ell_2 m_2} a^T_{\ell_3 m_3} a^T_{\ell_4 m_4} \rangle = \langle a^T_{\ell_1 m_1} a^T_{\ell_2 m_2} a^T_{\ell_3 m_3} a^T_{\ell_4 m_4} \rangle_G + \langle a^T_{\ell_1 m_1} a^T_{\ell_2 m_2} a^T_{\ell_3 m_3} a^T_{\ell_4 m_4} \rangle_c^{\phi\phi}$$



Forecasted LCDM+ parameter constraints

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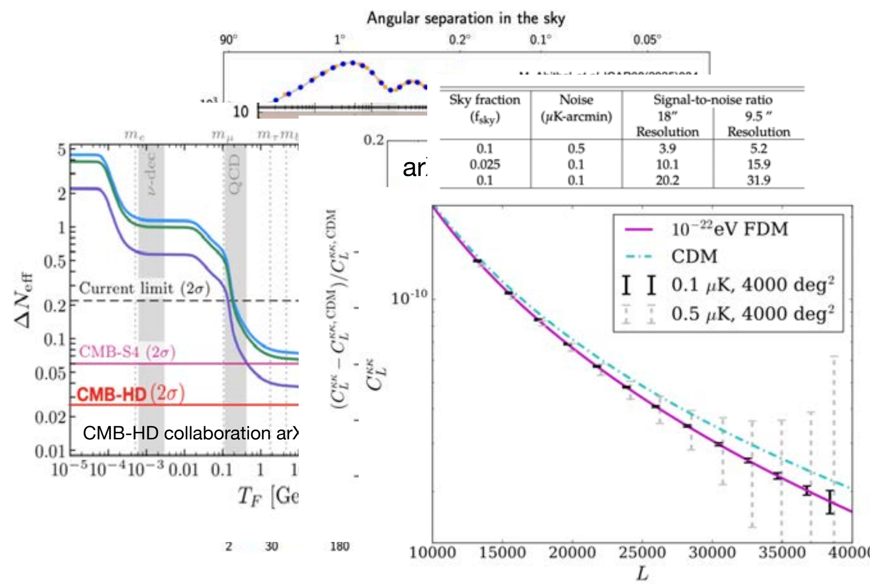
Parameter $\Lambda\mathrm{CDM} + N_{\mathrm{eff}} + \sum m_\nu$	Ratio of 1σ errors CMB-HD/SO	CMB-HD/ CMB-S4
$\Omega_b h^2$	0.46	0.67
$\Omega_c h^2$	0.55	0.73
$\ln(10^{10} A_s)$	0.84	0.86
n_s	0.43	0.52
τ	0.85	0.87
H_0 [km s^{-1} Mpc^{-1}]	0.81	0.91
N_{eff}	0.33	0.47
$\sum m_\nu$ [eV]	0.83	0.86

Forecasted LCDM+ parameter constraints



Information matrix calculations

- A very useful way to estimate constraints on parameters without the real data
- There are a few different ways to discuss this— see Tegmark et al. ApJ 480:22-35, 1997
- Here we will do the simple thing: imagine our data are the power spectra and that they are Gaussian distributed:

$$\vec{C}_\ell \equiv \begin{pmatrix} C_\ell^{TT} \\ C_\ell^{TE} \\ C_\ell^{EE} \end{pmatrix} \quad \mathcal{L}(\vec{\theta}) = \exp\left[-\frac{1}{2}\sum_{\ell,\ell'} \Delta\vec{C}_\ell \cdot \mathbf{Cov}^{-1}_{\ell\ell'} \cdot \Delta\vec{C}_{\ell'}\right]$$

$$\Delta\vec{C}_\ell \equiv \widehat{\vec{C}}_\ell - \vec{C}_\ell(\vec{\theta})$$

Information matrix calculations

$$\mathcal{L}(\vec{\theta}) = \exp\left[-\frac{1}{2}\sum_{\ell,\ell'} \Delta\vec{C}_\ell \cdot \mathbf{Cov}^{-1}_{\ell\ell'} \cdot \Delta\vec{C}_{\ell'}\right]$$

$$\Delta\vec{C}_\ell \equiv \widehat{\vec{C}}_\ell - \vec{C}_\ell(\vec{\theta})$$

- Imagine varying the parameters by some small amount about their best fit values: $\vec{\theta} \to \vec{\theta}_* + \delta\vec{\theta}$

Vanishes at max

$$\ln\mathcal{L}(\vec{\theta}) \simeq \ln\mathcal{L}(\vec{\theta}_*) + \left.\frac{\partial \ln\mathcal{L}}{\partial\theta_i}\right|_* \delta\theta_i + \left.\frac{\partial^2 \ln\mathcal{L}}{\partial\theta_i\theta_j}\right|_* \delta\theta_i\delta\theta_j$$

- The likelihood can then be written approximately as

$$\mathcal{L}(\vec{\theta}) \simeq \mathcal{L}(\vec{\theta}_*)\exp\left[-\frac{1}{2}\delta\vec{\theta}_i F_{ij}\delta\theta_j\right] \quad F_{ij} \equiv -\left.\frac{\partial^2 \ln\mathcal{L}}{\partial\theta_i\partial\theta_j}\right|_*$$

Information matrix calculations

- The likelihood can then be written approximately as

$$\mathcal{L}(\vec{\theta}) \simeq \mathcal{L}(\vec{\theta}_*)\exp\left[-\frac{1}{2}\delta\vec{\theta}_i F_{ij}\delta\theta_j\right]$$

$$\mathbf{Cov}^{-1}(\theta_i,\theta_j) = F_{ij} \equiv -\left.\frac{\partial^2 \ln\mathcal{L}}{\partial\theta_i\partial\theta_j}\right|_*$$

Vanishes at max

$$\frac{\partial^2 \ln\mathcal{L}}{\partial\theta_i\partial\theta_j} = -\sum_{\ell,\ell'}\frac{\partial\vec{C}_\ell}{\partial\theta_i}\mathbf{Cov}^{-1}_{\ell\ell'}\frac{\partial\vec{C}_{\ell'}}{\partial\theta_j} + \sum_{\ell,\ell'}\frac{\partial^2\vec{C}_\ell}{\partial\theta_i\partial\theta_j}\mathbf{Cov}^{-1}_{\ell\ell'}\Delta\vec{C}_{\ell'}.$$

$$\mathbf{Cov}_{\ell\ell'} = \frac{\delta_{\ell\ell'}}{(2\ell+1)f_{\rm sky}}\begin{pmatrix} 2\tilde{C}^2_{TT} & 2\tilde{C}_{TT}C_{TE} & 2C^2_{TE} & 0 \\ 2\tilde{C}_{TT}C_{TE} & \tilde{C}_{TT}\tilde{C}_{EE} + C^2_{TE} & 2\tilde{C}_{EE}C_{TE} & 0 \\ 2C^2_{TE} & 2C_{EE}C_{TE} & 2C^2_{EE} & 0 \\ 0 & 0 & 0 & 2\tilde{C}^2_{BB} \end{pmatrix}$$

A useful way to re-write the Information Matrix

$$F_{ij} = \frac{1}{2}\sum_{\ell=\ell_{\rm min}}^{\ell_{\rm max}}(2\ell+1)f_{\rm sky}\,\mathrm{Tr}\left[\mathbf{C}_\ell^{-1}\frac{\partial\mathbf{C}_\ell}{\partial\theta_i}\mathbf{C}_\ell^{-1}\frac{\partial\mathbf{C}_\ell}{\partial\theta_j}\right]$$

$$\mathbf{C}_\ell = \begin{pmatrix} \tilde{C}_\ell^{TT} & \tilde{C}_\ell^{TE} & 0 \\ \tilde{C}_\ell^{TE} & \tilde{C}_\ell^{EE} & 0 \\ 0 & 0 & \tilde{C}_\ell^{BB} \end{pmatrix}$$

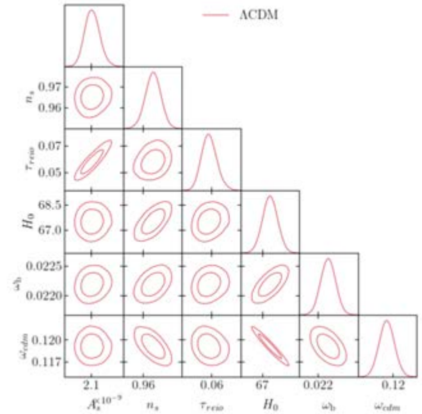
$$\tilde{C}_\ell^{XY} \equiv C_\ell^{XY} + N_\ell^{XY}$$

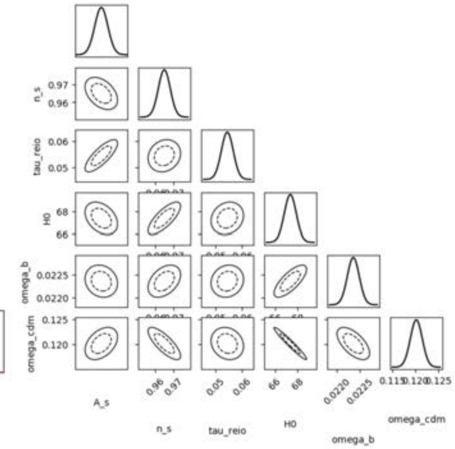
https://github.com/tsmith2/GGI_cosmology_notebooks

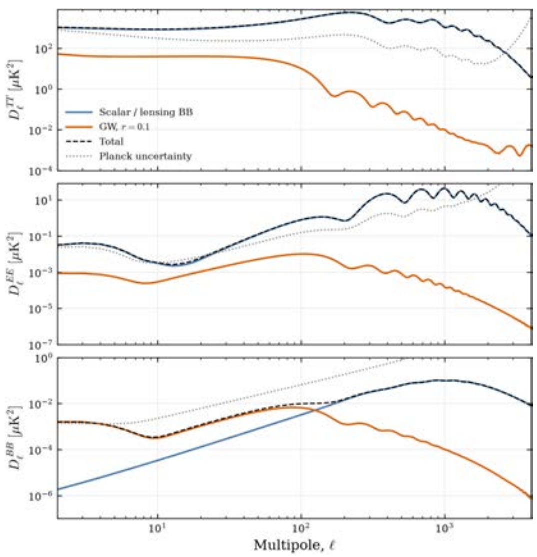
Notebook 2

Parameter	Planck-only constraint
A_s	$(2.101 \pm 0.034) \times 10^{-9}$
n_s	0.9649 ± 0.0042
$\tau_{\rm reio}$	0.0544 ± 0.0073
$H_0\ [{\rm km\,s^{-1}\,Mpc^{-1}}]$	67.36 ± 0.54
$\omega_b \equiv \Omega_b h^2$	0.02236 ± 0.00015
$\omega_{\rm cdm} \equiv \Omega_c h^2$	0.1202 ± 0.0014

```
A_s       = 2.101e-09 +/- 1.16e-11
n_s       = 0.9649 +/- 0.00359
tau_reio = 0.0544 +/- 0.00249
H0        = 67.36 +/- 0.583
omega_b  = 0.02236 +/- 0.000143
omega_cdm = 0.1202 +/- 0.00129
```



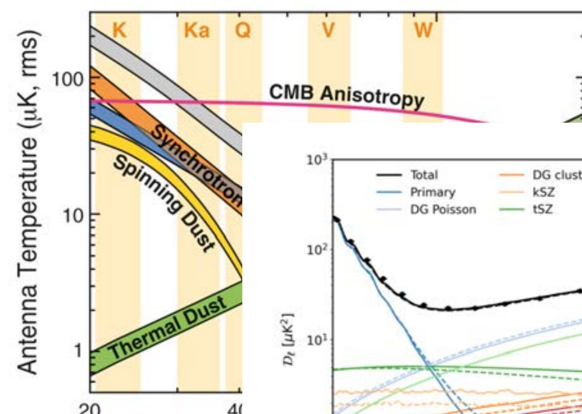




https://github.com/tsmith2/GGI_cosmology_notebooks

Notebook 3

We are neglecting the effects of foregrounds



https://lambda.gsf