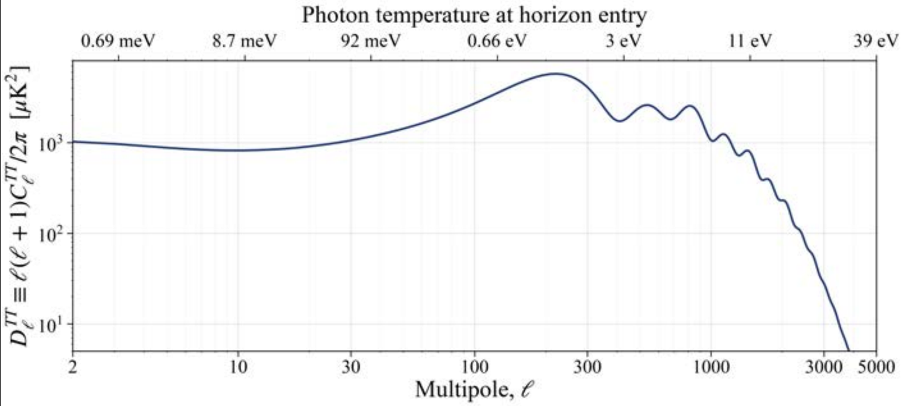
The CMB as a movie in time and space

- You have probably heard someone say: the CMB gives us a snap-shot of what the universe looked like 400,000 years after the big bang….

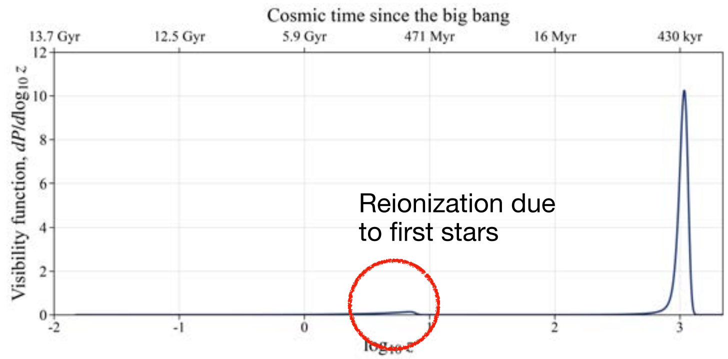
Projection relates multipole to wavenumber $\ell \simeq k\eta_0$

Wavenumber enters Hubble horizon at a certain time $k = aH$



Temperature anisotropies

- CMB photons are flying through the universe; we only see those moving into our telescopes
- The temperature is a function of position and time: $T(\vec{x}, t)$
- Most of the photons that we see last scattered off of free electrons a long time ago (hence the phrase 'CMB is a snapshot')



Temperature anisotropies

- There will be intrinsic anisotropies at the peak of the visibility function

$$\frac{\delta\rho_\gamma}{\bar{\rho}_\gamma} = 4\frac{\delta T}{\bar{T}} \quad \Rightarrow \quad \left(\frac{\delta T}{\bar{T}}\right)_{\rm intrinsic} = \frac{1}{4}\delta_\gamma$$

- Photons will also gain/lose relative energy as they red/blue shift from over/underdensities

$$\left(\frac{\delta T}{\bar{T}}\right)_{\rm grav} = \Phi$$

- Photons are Doppler shifted according to the velocity of the electron the photon last scattered from

$$\left(\frac{\delta T}{\bar{T}}\right)_{\rm Dop} = \hat{n}\cdot\vec{v}_b$$

- After photon decoupling their energies redshift when they pass through time-varying gravitational potentials

$$\left(\frac{\delta T}{\bar{T}}\right)_{\rm ISW} = 2\int_{\tau_*}^{\tau_0} d\tau\dot{\Phi}$$

Temperature anisotropies

$$\frac{\delta T}{\bar{T}} = \frac{1}{4}\delta_\gamma + \delta\Phi + \hat{n}\cdot\vec{v}_b + 2\int_{\eta_*}^{\eta_0}\dot{\Phi}$$

- The dynamical equations are solved in Fourier space which means that

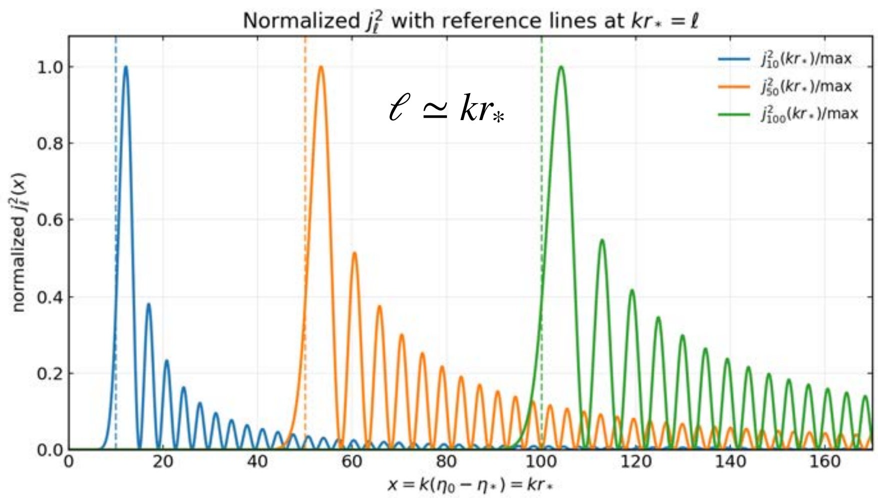
$$\delta T(\vec{x}, t) = \int \frac{d^3k}{(2\pi)^3}\delta T(\vec{k}, t)e^{-i\vec{k}\cdot\vec{x}}$$

- Approximating the visibility function as a Dirac delta at a comoving distance r_* we then have the temperature fluctuations today

$$\delta T(\hat{n}, t_0) = \int \frac{d^3k}{(2\pi)^3}\delta T(\vec{k}, t_0)e^{-ir_*\vec{k}\cdot\hat{n}}$$

- And we want to expand this into spherical harmonic coefficients…

Temperature anisotropies



$$\langle a_{\ell m} a^*_{\ell' m'} \rangle = \delta_{\ell \ell'} \delta_{mm'} \frac{2}{\pi} \int d \ln k \; k^3 P_T(k, t_0) j_\ell^2(kr_*)$$

C_ℓ

The physics of the CMB

- All are described as fluids, and hence follow (relativistic) fluid equations

$$\frac{\partial \rho}{\partial t} = -\vec{\nabla}(\rho \vec{v}) \qquad \frac{\partial \vec{v}}{\partial t} = -\frac{1}{2}\vec{\nabla} v^2 - \vec{\nabla}\Phi - \frac{\vec{\nabla} P}{\rho} \qquad \nabla^2 \Phi = 4\pi G \rho$$

Continuity — Euler ($\vec{F} = m\vec{a}$) — Gravity

- Combine these together to write a single second order equation for δ

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} = 4\pi G \bar{\rho}(t)\delta + \frac{c_s^2}{a^2}\nabla^2 \delta$$

- Damped, driven, harmonic oscillator!

The physics of the CMB

- General relativity changes the details but not the story
- Work with the divergence of the Euler equation where $\theta \equiv \vec{\nabla} \cdot \delta \vec{v}$
- Write time derivatives with respect to 'conformal time': $dt = a d\tau$

Friedman $$\mathcal{H}^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} a^2 \sum_i \bar{\rho}_i$$

Homogenous continuity $$\dot{\bar{\rho}}_i = -3(1 + w_i)\bar{\rho}_i \frac{\dot{a}}{a}$$

Perturbed continuity $$\dot{\delta}_i = -(1 + w_i)(\theta_i - 3\dot{\Phi})$$

$\vec{\nabla} \cdot$ Euler $$\dot{\theta}_i = -\frac{\dot{a}}{a}(1 - 3w_i)\theta_i - \nabla^2 \delta\Phi - \frac{c_{s,i}^2}{1 + w_i}\nabla^2 \delta_i$$

Gravity $$\nabla^2 \Phi + 3\frac{\dot{a}}{a}\left(\dot{\Phi} + \frac{\dot{a}}{a}\Phi\right) = 4\pi G \sum_i \bar{\rho}_i \delta_i$$

Thomson scattering

- For baryons and photons the Euler equation is modified since they interact through Thomson scattering
- The upshot is the two fluids make a singe effective fluid ('photon-baryon fluid') with

$$\theta_\gamma = \theta_b \equiv \theta_{\gamma b}$$

$$\delta_b = \frac{3}{4}\delta_\gamma$$

- Second order differential equation for the photon-baryon fluid:

$$\ddot{\delta}_\gamma + \frac{\dot{R}}{1+R}\dot{\delta}_\gamma + k^2 c_s^2 \delta_\gamma = -\frac{4}{3}k^2\Phi + 4\ddot{\Phi} + \frac{4\dot{R}}{1+R}\dot{\Phi}$$

$$R \equiv \frac{3}{4}\frac{\rho_b}{\rho_\gamma} \propto a \qquad c_s^2 \equiv \frac{1}{3(1+R)}$$

Mode evolution in CLASS

$$\ddot{\delta}_\gamma + \frac{\dot{R}}{1+R}\dot{\delta}_\gamma + k^2 c_s^2 \delta_\gamma = -\frac{4}{3}k^2\Phi + 4\ddot{\Phi} + \frac{4\dot{R}}{1+R}\dot{\Phi}$$

In radiation domination Φ goes to zero: drives δ_γ amplitude up

In matter domination Φ is constant: no driving

https://github.com/tsmith2/GGI_cosmology_notebooks

Open Notebook 4

Silk damping

- On scales below the photon mean free path the photons diffuse from hotter to colder regions, erasing the perturbations
- The instantaneous comoving mean free path is

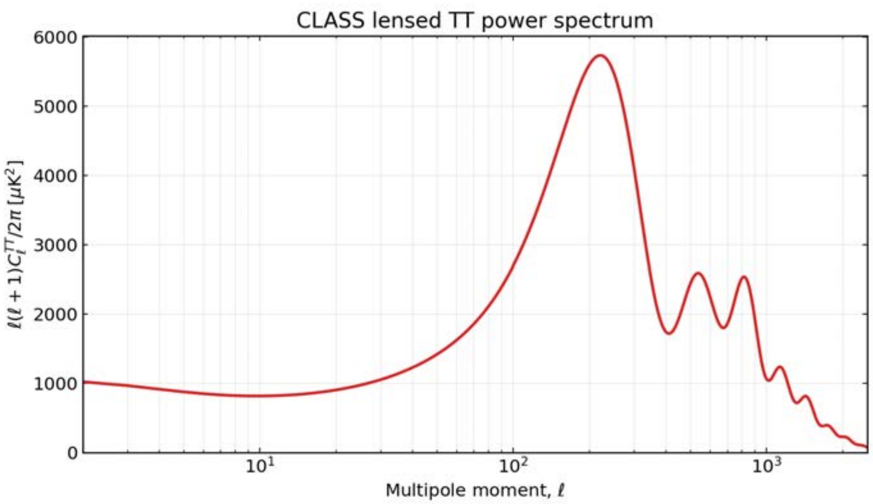
$$\lambda_{\rm mfp} = \frac{m_p a^2}{\rho_{\rm crit,0}\Omega_b \sigma_T}$$

- The full distance the photons travel is a random walk:

$$r_D^2 \sim \int_0^{\eta_*} d\eta \lambda_{\rm mfp}$$

- Leads to an exponential damping of CMB perturbations, e^{-k/k_D^2}

Primordial power spectrum



The key ingredients

- Continuity and Euler equations for all materials
- The evolution of the gravitational potential
- Thomson scattering between baryons and photons
- Projection effects on to the curved sky

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Open Notebook 5

Your task: take a look at the explanatory.md in the cpp directory of 'simplified_CMB' and modify the code to add neutrinos

Each multipole gives information about physics at different scales and times