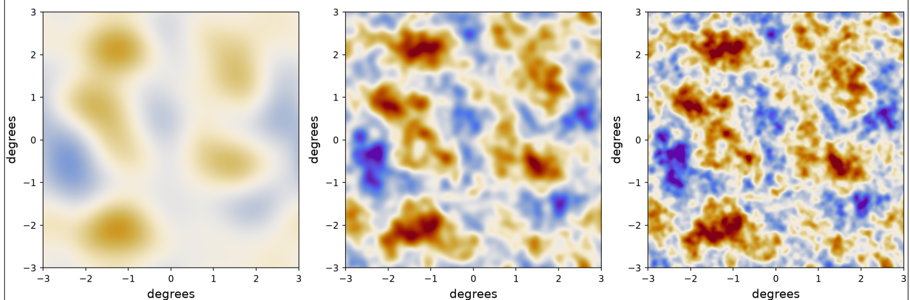
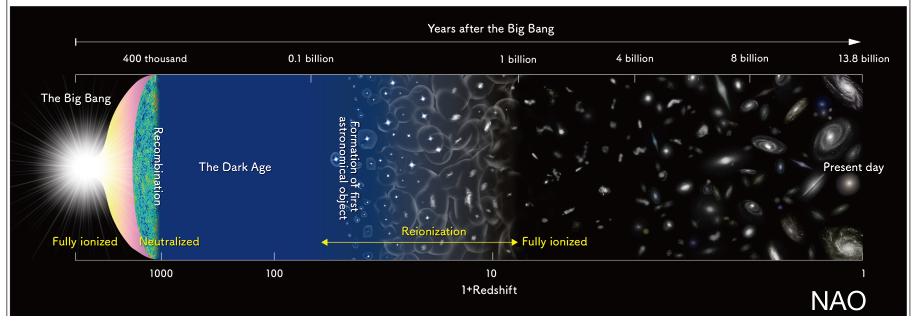
CMB and the next generation observations



Tristan L. Smith, Swarthmore College

Covered topics



Who am I?

- Faculty at Swarthmore College near Philadelphia
- My research mainly focuses on constraining early universe extensions of LCDM
- Models that attempt to address the Hubble tension (mostly in collaboration with Vivian Poulin)
- Recently I have been very interested in understanding whether near future CMB measurements will be decisive for pre-recombination extensions
- Another part of my research is on pulsar timing arrays, searches for anisotropies, and constraints to modified gravity
- I love film photography
- I've been to GGI once before (2008 I think…) and didn't go to any museums! But I did read Machiavelli's Florentine Histories….

Who are you?

- Introduce yourselves to your neighbors (you will be working with them a bit today)
- Suggested discussion topics
  - Your home institution
  - What got you interested in cosmology
  - Non-physics things you plan to do while in Florence
  - The World Cup

Cosmology 101

- Homogeneous evolution governed by the Friedman equation

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$$

- In LCDM universe is filled with radiation (photons+neutrinos), matter (baryons+cold dark matter), and a cosmological constant

Cosmology 101

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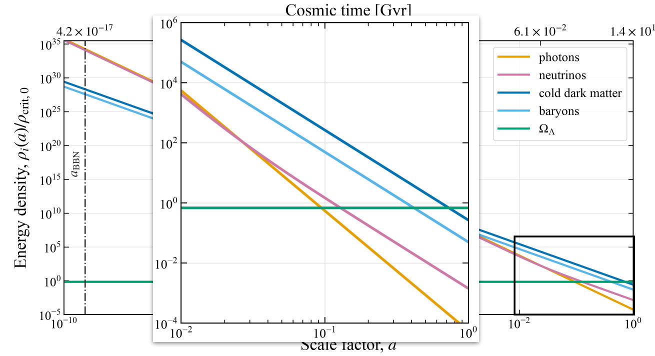


Gaussian statistics 101

- Imagine a Gaussian random variable x with mean μ and variance σ^2

$$\langle x \rangle = \mu$$

$$\mathrm{Var}(x) \equiv \langle (x - \langle x \rangle)^2 \rangle = \langle x^2 \rangle - \langle x \rangle^2 = \sigma^2$$

- If we have a collection of statistically independent measurements from this distribution, $\{x_i\}$, we have

$$\langle x_i \rangle = \mu$$

$$\langle x_i x_j \rangle = \mu^2 + \delta_{ij}\sigma^2$$

- Using these measurements we can write down an estimator for the mean

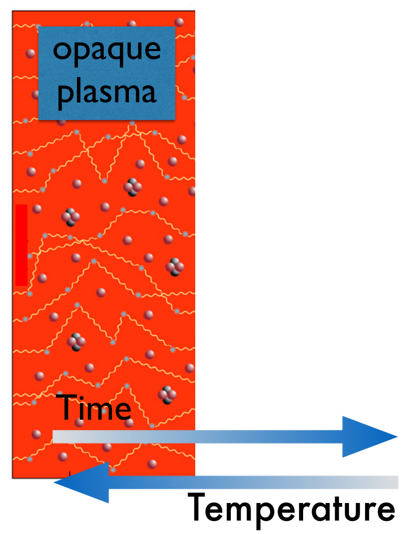
$$\hat{\mu} = \frac{1}{N}\sum_{i=1}^{N} x_i \qquad \langle \hat{\mu} \rangle = \frac{1}{N}\sum_{i=1}^{N} \mu = \mu$$

'Unbiased estimator'

- What is the variance of the estimator, $\hat{\mu}$? σ^2/N

Thermal history of the universe

(From the perspective of a CMB photon)



A brief history of CMB science

- In the late 1940s-1960s there was an intense debate between whether the universe changes in time (Fred Hoyle vs. George Gamov; steady state vs big bang)
- In a paper by Alpher and Herman ("Remarks on the evolution of the expanding universe" 1949)
- They use BBN to *predict* that the CMB temperature today is about 5 K!

PHYSICAL REVIEW   VOLUME 75, NUMBER 7   APRIL 1, 1949

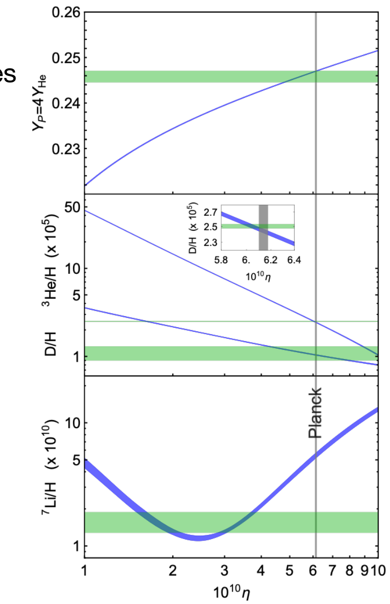
Remarks on the Evolution of the Expanding Universe*,†

RALPH A. ALPHER AND ROBERT C. HERMAN
Applied Physics Laboratory, The Johns Hopkins University, Silver Spring, Maryland
(Received December 27, 1948)

The relativistic energy equation for an expanding universe of non-interconverting matter and radiation is integrated. The above result, together with a knowledge of the physical conditions that prevailed during the element forming process in the early stages of the expansion, is used to determine the time dependences of proper distance as well as of the densities of matter and radiation. These relationships are employed to determine the mean galactic diameter and mass when formed as 2.1×10^3 light years and 3.8×10^7 sun masses, respectively. Galactic separations are computed to be of the order of 10^6 light years at the present time.

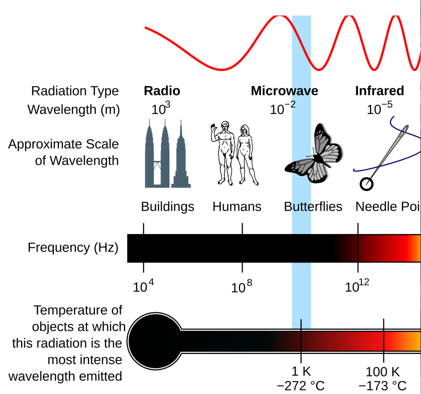
A brief history of CMB science

- A more precise treatment uses measurements of D/H which gives $\eta \simeq 6\times 10^{10}$
- The *Planck* line is constructed from the measured CMB temperature and $\rho_{b,0}$
- The consistency is one of the major triumphs of the standard cosmological model



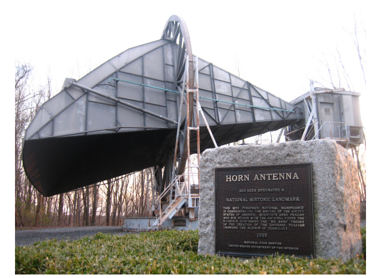
A brief history of CMB science

- Standard expectation: CMB photons will have a blackbody spectrum
- Peak will be $\nu_{\rm peak} = 2.82\dfrac{k_B T_{\rm CMB,0}}{h} \simeq 100$ GHz



A brief history of CMB science

- 1965 Penzias and Wilson report 'excess noise temperature' of 'about 3.5 K at $\nu_{\rm PW} \simeq 4$ GHz'
- We know that $\nu_{\rm PW} \ll \nu_{\rm CMB,peak}$ so this measurement was made in the Rayleigh-Jeans tail
- $I_\nu = 2k_B T\nu^2/c^2$ — this made the inference of the temperature straightforward



- Excess radiation was 'isotropic, unpolarized, and free from seasonal variations'.
- "At one point, new suspects emerged. Two pigeons had set up housekeeping inside the guts of the antenna. Maybe their droppings were causing the noise? Wilson and Penzias had the birds trapped and then cleaned the equipment, but the signals continued."

A brief history of CMB science

- 1992 COBE satellite was the first to measure the CMB black body spectrum

FIXSEN ET AL.

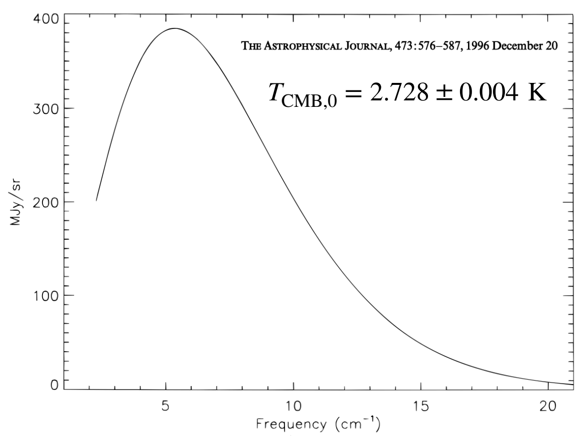
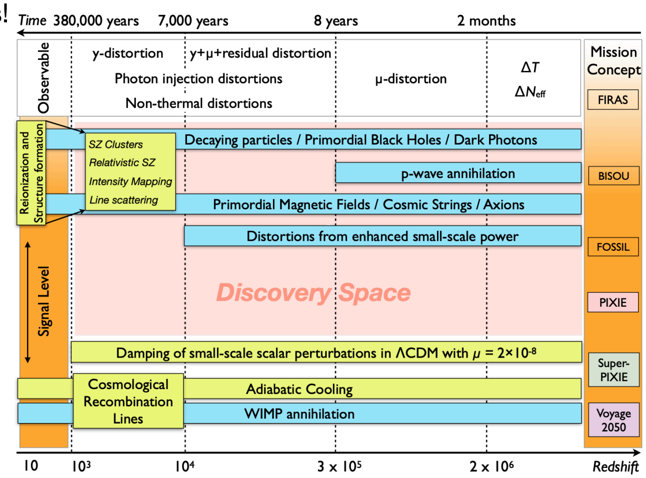


FIG. 4.—Uniform spectrum and fit to Planck blackbody (T). Uncertainties are a small fraction of the line thickness.

A brief history of CMB science

- Measurement of the spectrum has interesting implications for new physics!

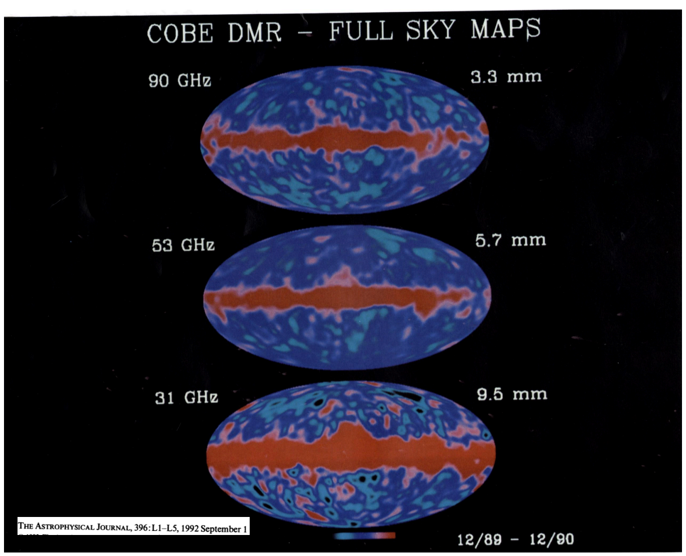


Chluba arXiv:2502.05188

- But, I am far from an expert in this…

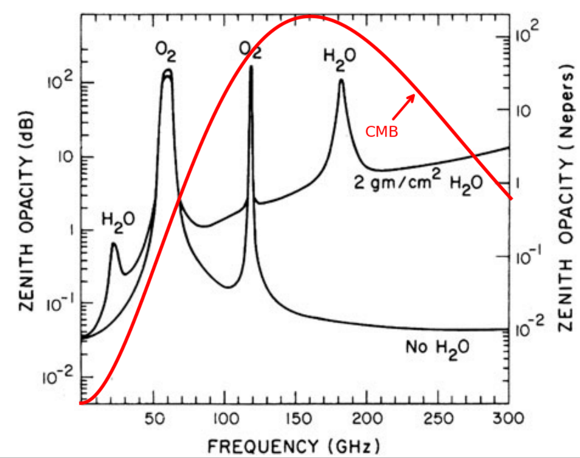
A brief history of CMB science

- COBE also the first to measure temperature anisotropies in the CMB



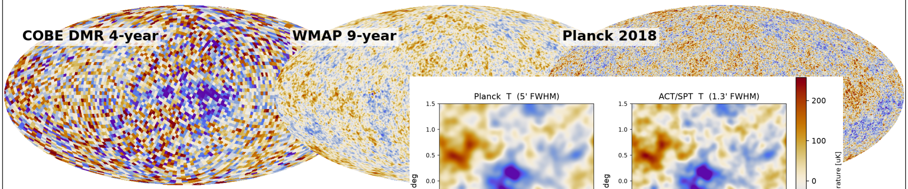
A brief history of CMB science

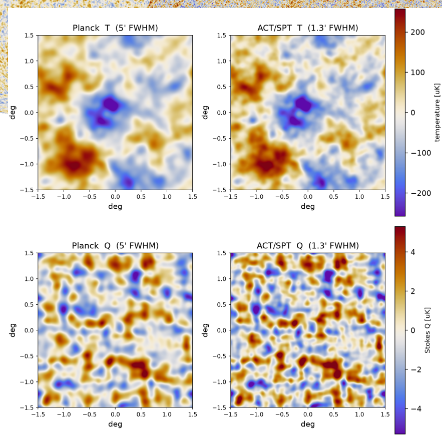
- Why go to space?
- First, to get a 'full sky' map
- Second, water vapor:



A brief history of CMB science

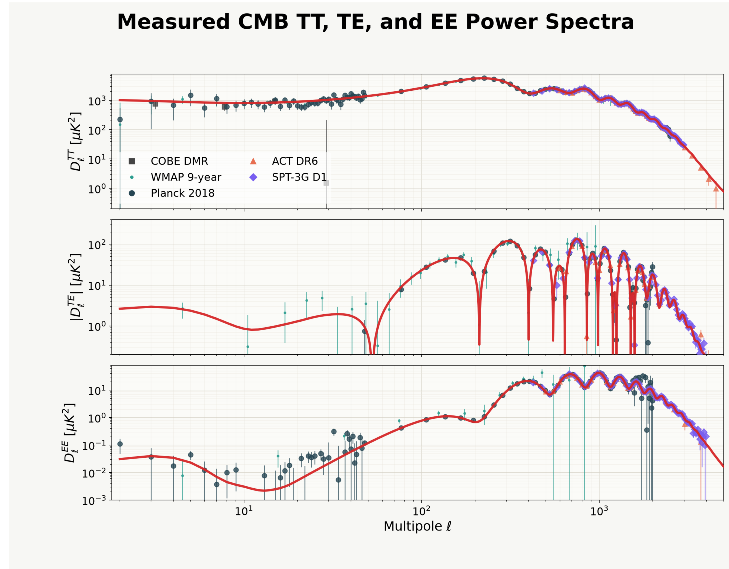
- WMAP 2003 and Planck 2008… improvements in resolution and noise



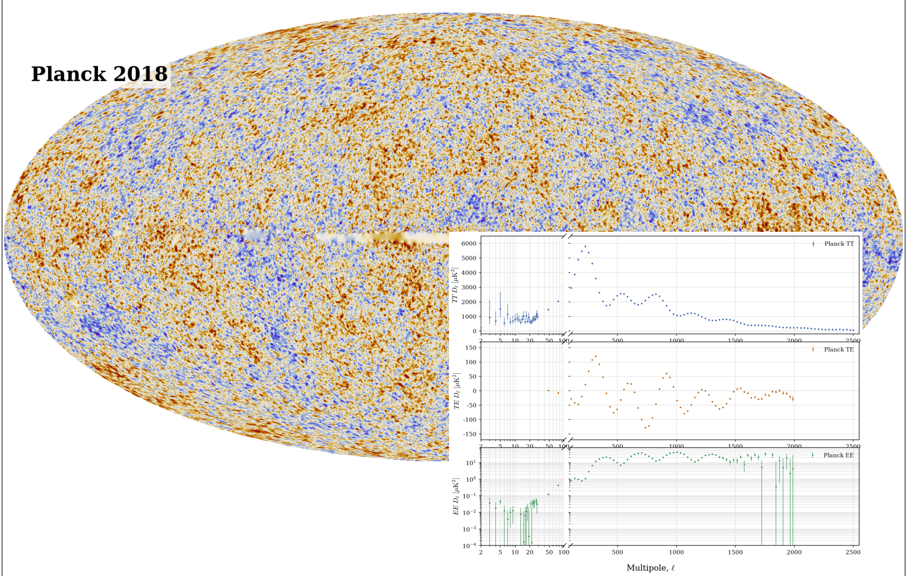


- Atacama Cosmology Telescope (ACT) and South Pole Telescope are in high and dry places
- CMB has fluctuations in both temperature and polarization

A brief history of CMB science

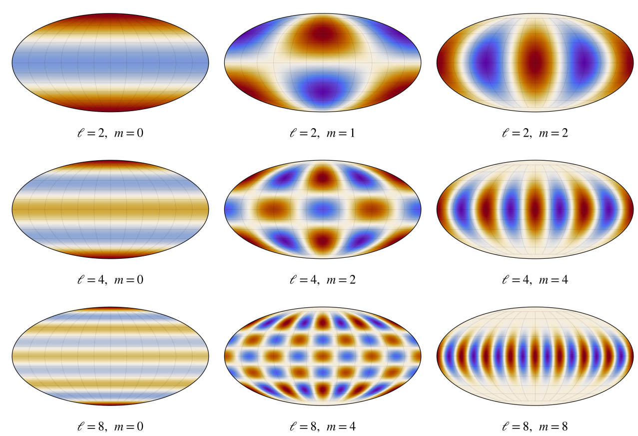


From maps to spectra…



Brief introduction to the angular power spectrum

$$T(\hat{n}) \leftrightarrow a^T_{\ell m} \equiv \int d^2 n T(\hat{n}) Y^*_{\ell m}(\hat{n})$$



$$\theta \simeq \frac{\pi}{\ell}$$

$\ell = 2$ roughly divides the sky into four parts: $2\pi/4 = \pi/2$

Introduction to the angular power spectrum

$$T(\hat{n}) = \sum_{\ell,m} a^T_{\ell m} Y_{\ell m}(\hat{n}) \quad \Longleftrightarrow \quad a^T_{\ell m} = \int d^2n T(\hat{n}) Y^*_{\ell m}(\hat{n})$$

$$\theta \simeq \frac{\pi}{\ell}$$

(Analogous to Fourier series: $x = 2\pi/k$)

- The CMB fluctuations are stochastic and expected to be statistically homogeneous and isotropic

Rotationally invariant

$$\langle T(\hat{n})T(\hat{n}')\rangle = C(\hat{n}\cdot\hat{n}')$$

$$\updownarrow$$

$$\langle a^T_{\ell m} a^{*T}_{\ell' m'}\rangle = \delta_{\ell,\ell'}\delta_{m,m'}C^{TT}_\ell$$

- On a small patch of the sky it is a Fourier transform:

$$T(\vec{\theta}) \simeq \int \frac{d^2\ell}{(2\pi)^2} e^{i\vec{\ell}\cdot\vec{\theta}}$$

Introduction to the angular power spectrum

- When we look at a CMB map our eyes are drawn to the patches with the highest variance (contrast)
- The variance within a flat patch of radius θ is

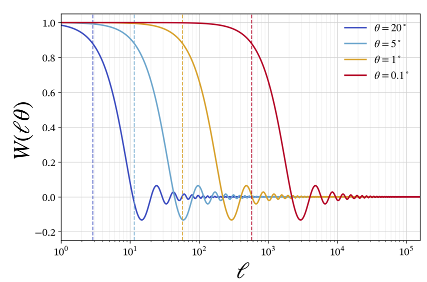
$$\sigma^2(\theta) = \int_0^\infty d\ln\ell \frac{\ell^2 C_\ell}{2\pi} W^2(\ell\theta) \qquad W(\ell\theta) = 2\frac{J_1(\ell\theta)}{\ell\theta}$$

Introduction to the angular power spectrum

- The variance within a patch of radius θ is

$$\sigma^2(\theta) = \int_{\ell_{\rm min}}^\infty d\ln\ell \frac{\ell^2 C_\ell}{2\pi} W^2(\ell\theta)$$

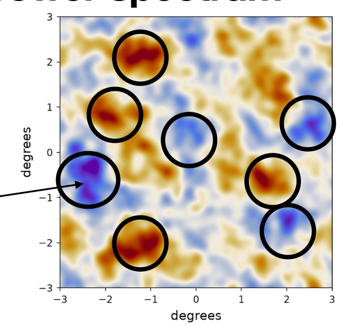
$$\Rightarrow \quad \sigma^2(\theta) \simeq \int_{\ell_{\rm min}}^{1/\theta} d\ln\ell \frac{\ell^2 C_\ell}{2\pi}$$



- This gives us a way to understand how detector noise appears in the observed CMB power spectrum

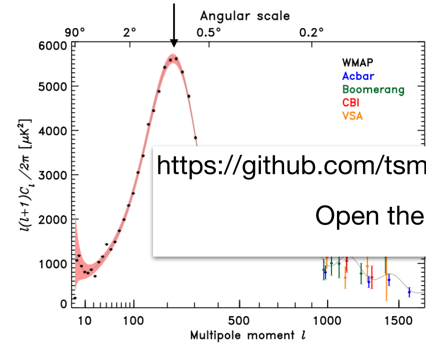
Introduction to the angular power spectrum

$$\sigma^2(\theta) \simeq \int_{\ell_{\rm min}}^{1/\theta} d\ln\ell \frac{\ell^2 C_\ell}{2\pi}$$



- We can now try to read off what kind of structure(s) we might see in a map from the power spectrum that generated it

$\theta \simeq \pi/\ell \simeq 1^\circ$



https://github.com/tsmith2/GGI_cosmology_notebooks

Open then copy Notebook 1

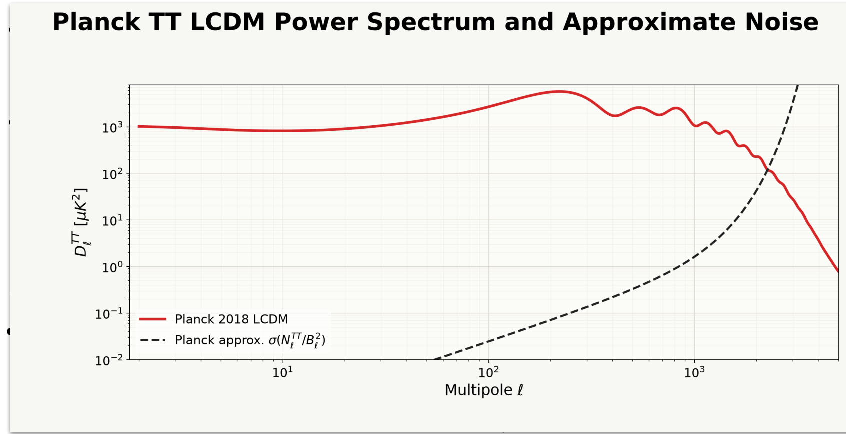
Introduction to the angular power spectrum

- The variance within a patch of radius θ is $\sigma^2(\theta) \simeq \int_{\ell_{\min}}^{1/\theta} d\ln\ell \, \frac{\ell^2 C_\ell}{2\pi}$
- Each pixel will have some (uncorrelated) noise (n = rms pixel noise):

$$\sigma^2_{\rm noise}(\theta) = \frac{\text{Variance per pixel}}{\text{\# of pixels in patch}} = \frac{n^2}{\pi\theta^2/\Omega_{\rm pix}} \Rightarrow N_\ell = n^2\Omega_{\rm pix}$$

- The telescope has some finite resolution, σ_b (often called a 'beam')
- Approximating it as a Gaussian we have $B_\ell = e^{-\frac{1}{2}\ell(\ell+1)\sigma_b^2}$
- The *observed* power spectrum is: $C_{\ell,\rm obs} = C_\ell B_\ell^2 + N_\ell$

A (theorist's) guide to CMB sensitivity



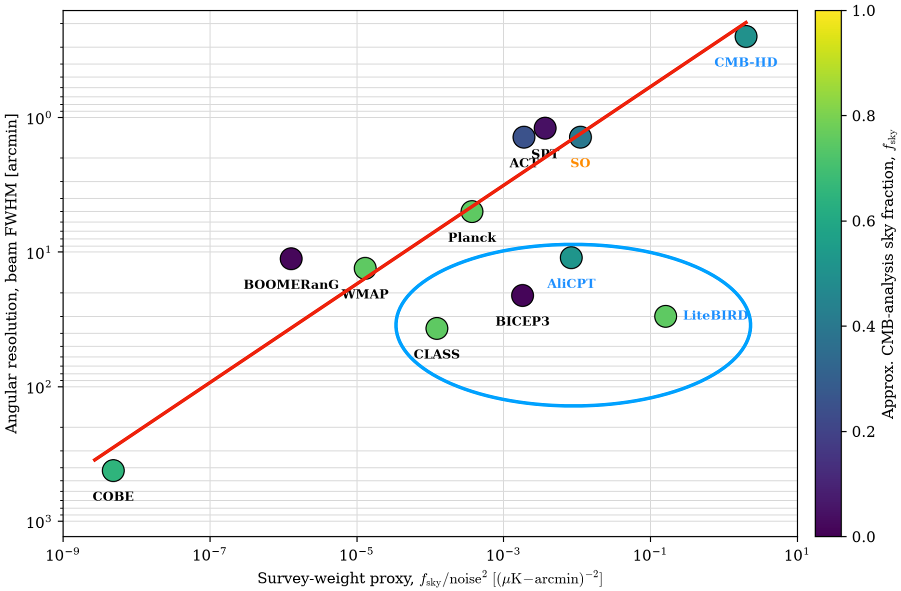
- The variance in this estimator is

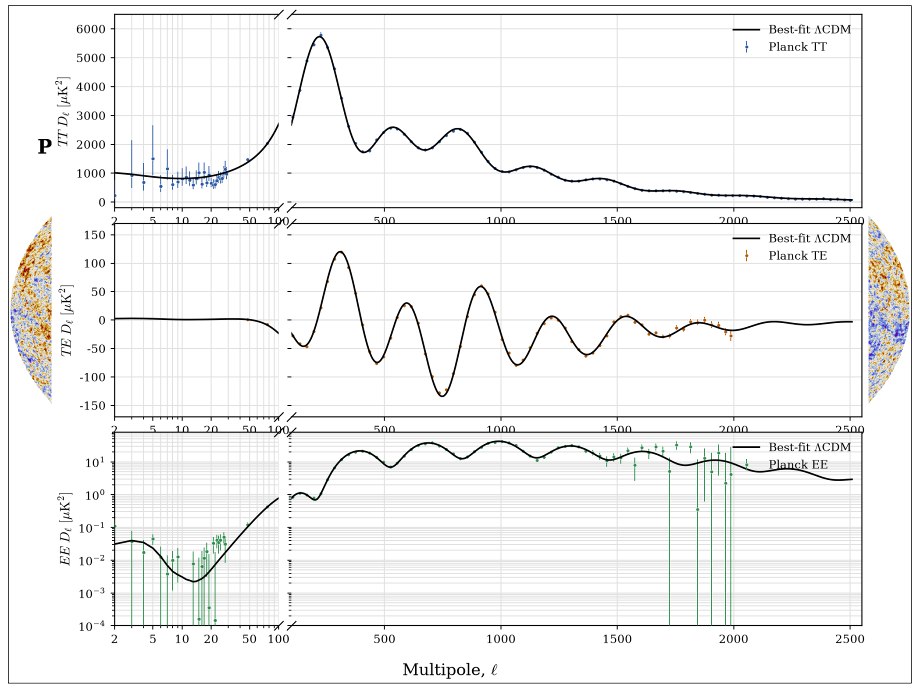
$$\mathrm{Var}\left(\hat{C}_\ell^{TT,\rm dec}\right) = \frac{2}{(2\ell+1)f_{\rm sky}}\left(C_\ell^{TT} + N_\ell^{TT} e^{\ell(\ell+1)\sigma_b^2}\right)^2$$

noise amplitude $= n^2/f_{\rm sky}$

$\ell_{\rm min} \simeq f_{\rm sky}^{-1/2}$

A (theorist's) guide to CMB sensitivity

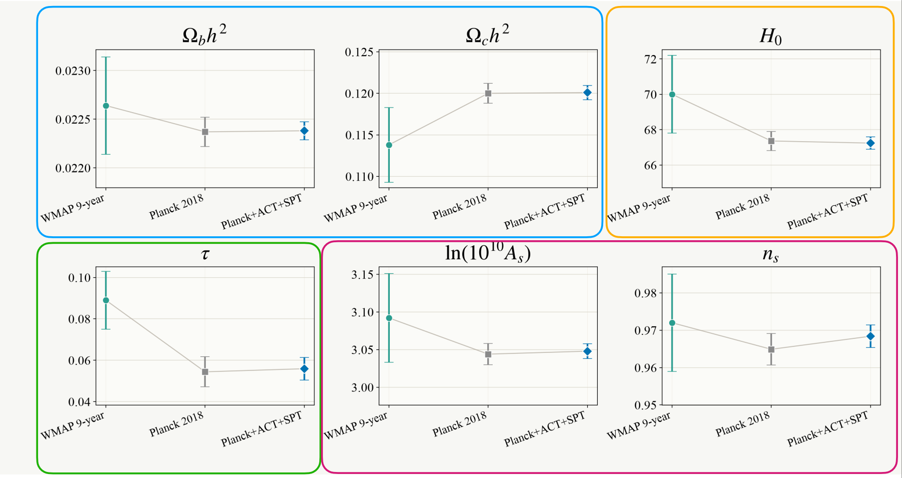




Current LCDM parameter constraints

What the universe is made of

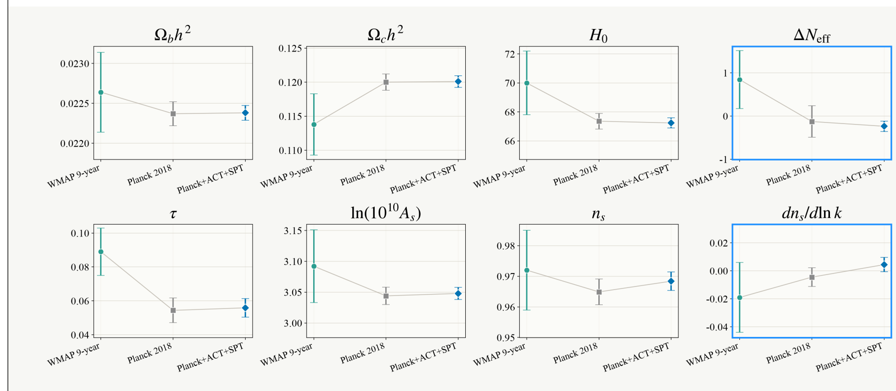
Current expansion rate of the universe



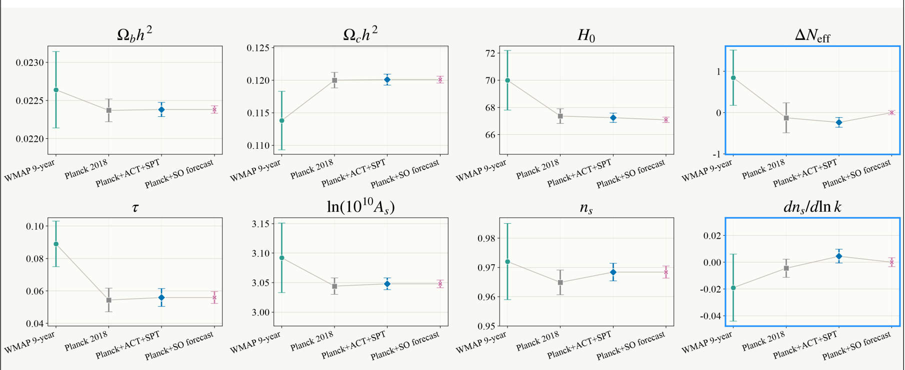
Late-time re-ionization of the universe

Cosmic inflation

Current LCDM+ parameter constraints

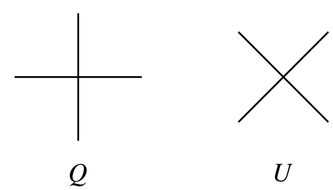


Forecasted LCDM+ parameter constraints



A bit more on polarization

- CMB light has fluctuations in its polarization from place to place
- The detectors measure linear polarization in perpendicular directions



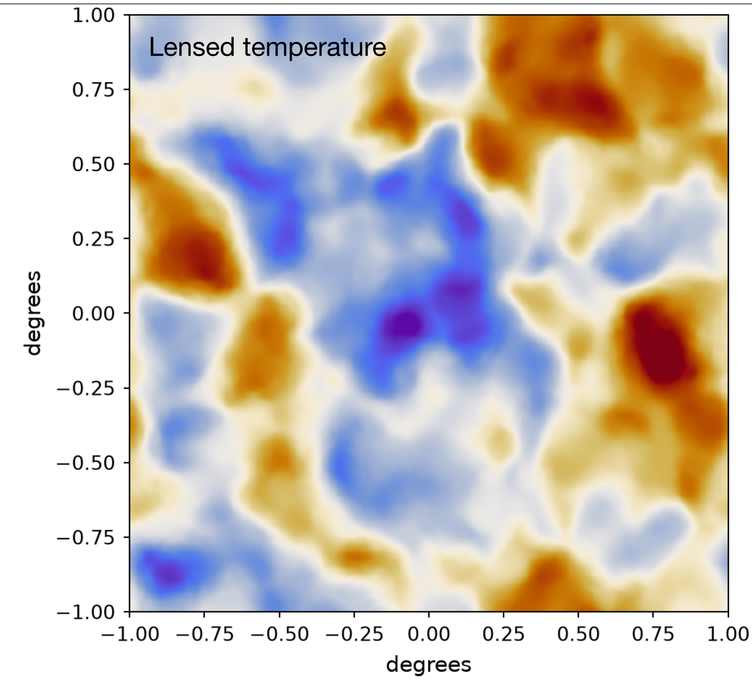
- These polarizations do not have definite parity… but physics cares a lot about parity, so *(in harmonic space)* we define linear combinations:

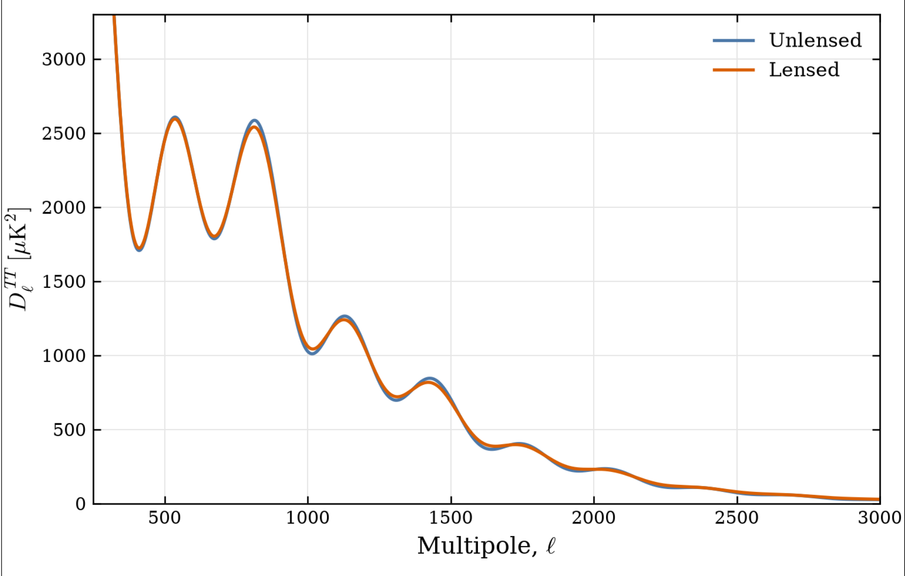
$E(\vec{\ell}) \equiv Q(\vec{\ell})\cos 2\phi_\ell + U(\vec{\ell})\sin 2\phi_\ell$ → Even parity (scalar perts)

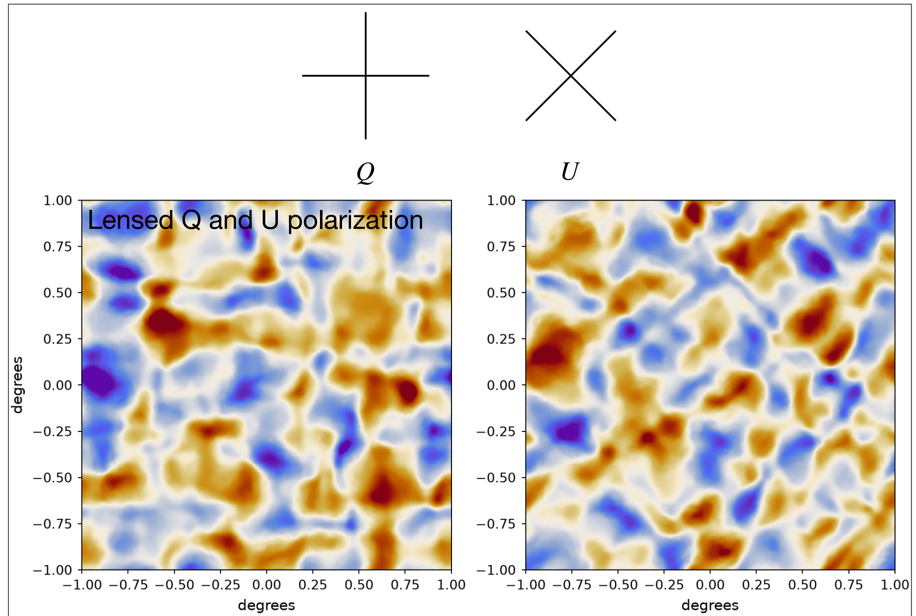
$B(\vec{\ell}) \equiv -Q(\vec{\ell})\sin 2\phi_\ell + U(\vec{\ell})\cos 2\phi_\ell$ → Odd parity (vector/GWs)

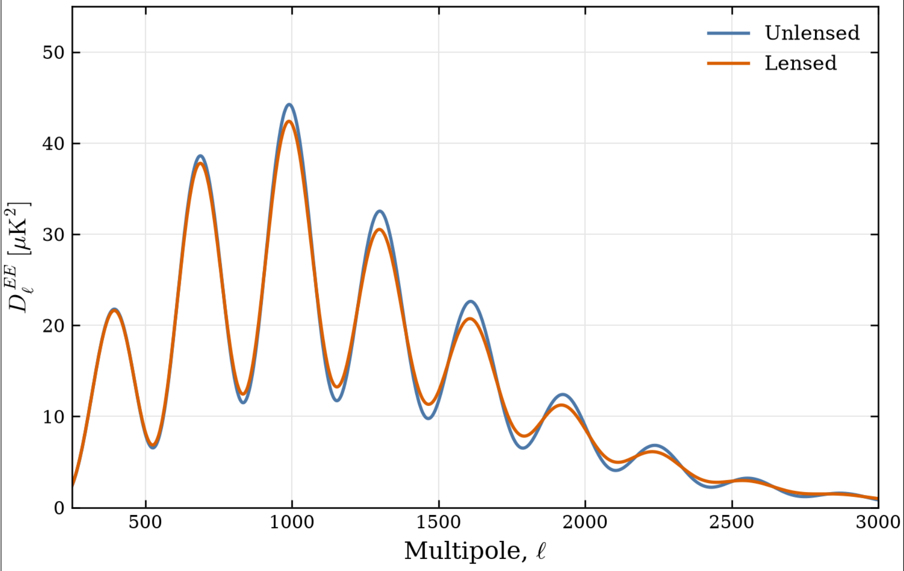
Beyond the 'primary' power spectra

- If the CMB fluctuations were purely Gaussian, the power spectrum would contain all of the statistical information….
- Lensing of CMB photons by intervening clumps of matter introduces a non-Gaussian contribution
- (There are more exotic— hypothesized— sources of non-Gaussian features that I won't discuss)
- The photons are deflected: $T(\hat{n}) \rightarrow T(\hat{n} + \overrightarrow{\nabla}\phi)$
- Modulates the power spectra, mixes the two polarization modes, and generates a non-Gaussian contribution to higher order correlations





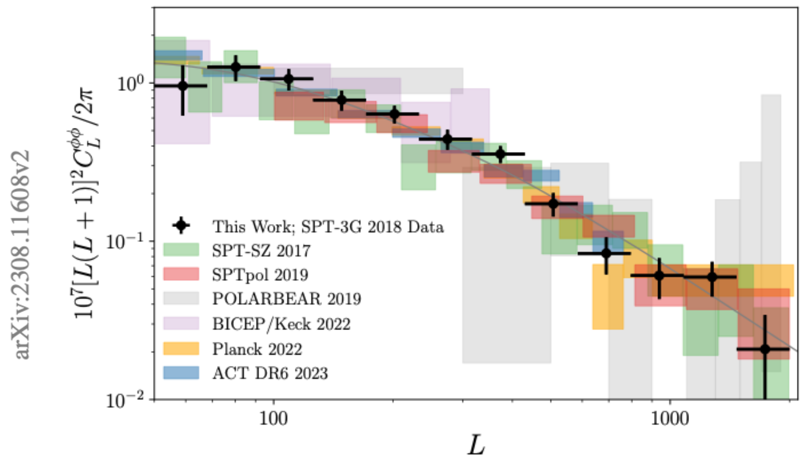




Lensing potential power spectrum

- Lensing potential power spectrum is estimated from the four point correlation function

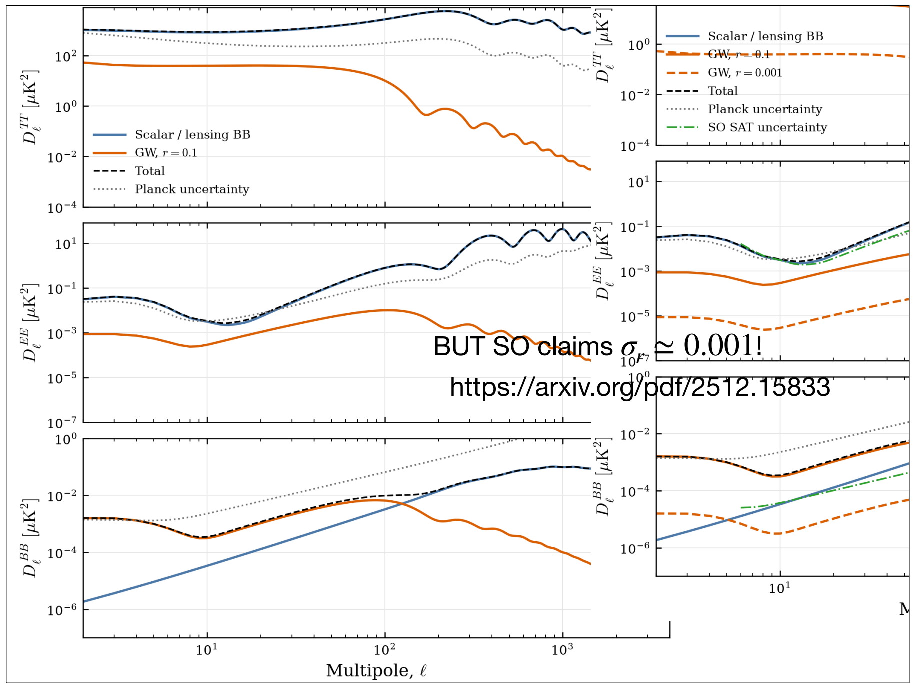
$$\langle a^T_{\ell_1 m_1} a^T_{\ell_2 m_2} a^T_{\ell_3 m_3} a^T_{\ell_4 m_4} \rangle = \langle a^T_{\ell_1 m_1} a^T_{\ell_2 m_2} a^T_{\ell_3 m_3} a^T_{\ell_4 m_4} \rangle_{\rm G} + \langle a^T_{\ell_1 m_1} a^T_{\ell_2 m_2} a^T_{\ell_3 m_3} a^T_{\ell_4 m_4} \rangle_c^{\phi\phi}$$



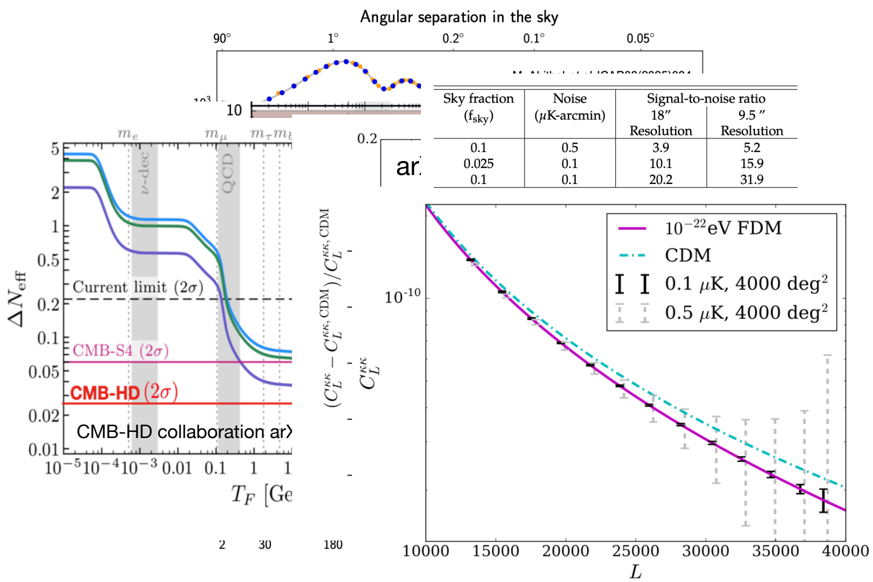
Forecasted LCDM+ parameter constraints

PHYS. REV. D **109**, 063527 (2024)

Parameter ΛCDM $+ N_{\rm eff} + \sum m_\nu$	Ratio of 1σ errors: CMB-HD/SO	Ratio of 1σ errors: CMB-HD/CMB-S4
$\Omega_b h^2$	0.46	0.67
$\Omega_c h^2$	0.55	0.73
$\ln(10^{10} A_s)$	0.84	0.86
n_s	0.43	0.52
τ	0.85	0.87
H_0 [km s^{-1} Mpc^{-1}]	0.81	0.91
$N_{\rm eff}$	0.33	0.47
$\sum m_\nu$ [eV]	0.83	0.86



Forecasted LCDM+ parameter constraints



Information matrix calculations

- A very useful way to estimate constraints on parameters without the real data
- There are a few different ways to discuss this— see Tegmark et al. ApJ 480:22-35, 1997
- Here we will do the simple thing: imagine our data are the power spectra and that they are Gaussian distributed:

$$\vec{C}_\ell \equiv \begin{pmatrix} C_\ell^{TT} \\ C_\ell^{TE} \\ C_\ell^{EE} \end{pmatrix} \quad \mathcal{L}(\vec{\theta}) = \exp\left[-\frac{1}{2}\sum_{\ell,\ell'} \Delta\vec{C}_\ell \cdot \mathbf{Cov}^{-1}_{\ell\ell'} \cdot \Delta\vec{C}_{\ell'}\right]$$

$$\Delta\vec{C}_\ell \equiv \hat{\vec{C}}_\ell - \vec{C}_\ell(\vec{\theta})$$

Information matrix calculations

$$\mathcal{L}(\vec{\theta}) = \exp\left[-\frac{1}{2}\sum_{\ell,\ell'} \Delta\vec{C}_\ell \cdot \mathbf{Cov}^{-1}_{\ell\ell'} \cdot \Delta\vec{C}_{\ell'}\right]$$

$$\Delta\vec{C}_\ell \equiv \hat{\vec{C}}_\ell - \vec{C}_\ell(\vec{\theta})$$

- Imagine varying the parameters by some small amount about their best fit values: $\vec{\theta} \to \vec{\theta}_* + \delta\vec{\theta}$

Vanishes at max

$$\ln\mathcal{L}(\vec{\theta}) \simeq \ln\mathcal{L}(\vec{\theta}_*) + \left.\frac{\partial \ln\mathcal{L}}{\partial\theta_i}\right|_* \delta\theta_i + \left.\frac{\partial^2 \ln\mathcal{L}}{\partial\theta_i\theta_j}\right|_* \delta\theta_i\delta\theta_j$$

- The likelihood can then be written approximately as

$$\mathcal{L}(\vec{\theta}) \simeq \mathcal{L}(\vec{\theta}_*) \exp\left[-\frac{1}{2}\delta\vec{\theta}_i F_{ij}\delta\theta_j\right] \quad F_{ij} \equiv -\left.\frac{\partial^2\ln\mathcal{L}}{\partial\theta_i\partial\theta_j}\right|_*$$

Information matrix calculations

- The likelihood can then be written approximately as

$$\mathcal{L}(\vec{\theta}) \simeq \mathcal{L}(\vec{\theta}_*) \exp\left[-\frac{1}{2}\delta\vec{\theta}_i F_{ij}\delta\theta_j\right]$$

$$\mathbf{Cov}^{-1}(\theta_i,\theta_j) = F_{ij} \equiv -\left.\frac{\partial^2\ln\mathcal{L}}{\partial\theta_i\partial\theta_j}\right|_*$$

Vanishes at max

$$\frac{\partial^2\ln\mathcal{L}}{\partial\theta_i\partial\theta_j} = -\sum_{\ell,\ell'}\frac{\partial\vec{C}_\ell}{\partial\theta_i}\mathbf{Cov}^{-1}_{\ell\ell'}\frac{\partial\vec{C}_{\ell'}}{\partial\theta_j} + \sum_{\ell,\ell'}\frac{\partial^2\vec{C}_\ell}{\partial\theta_i\partial\theta_j}\mathbf{Cov}^{-1}_{\ell\ell'}\Delta\vec{C}_{\ell'}.$$

$$\mathbf{Cov}_{\ell\ell'} = \frac{\delta_{\ell\ell'}}{(2\ell+1)f_{\rm sky}}\begin{pmatrix} 2\tilde{C}_{TT}^2 & 2\tilde{C}_{TT}C_{TE} & 2C_{TE}^2 & 0 \\ 2\tilde{C}_{TT}C_{TE} & \tilde{C}_{TT}\tilde{C}_{EE}+C_{TE}^2 & 2\tilde{C}_{EE}C_{TE} & 0 \\ 2C_{TE}^2 & 2\tilde{C}_{EE}C_{TE} & 2\tilde{C}_{EE}^2 & 0 \\ 0 & 0 & 0 & 2\tilde{C}_{BB}^2 \end{pmatrix}$$

A useful way to re-write the Information Matrix

$$F_{ij} = \frac{1}{2} \sum_{\ell=\ell_{\rm min}}^{\ell_{\rm max}} (2\ell+1) f_{\rm sky}\, {\rm Tr}\left[\mathbf{C}_\ell^{-1} \frac{\partial \mathbf{C}_\ell}{\partial \theta_i} \mathbf{C}_\ell^{-1} \frac{\partial \mathbf{C}_\ell}{\partial \theta_j}\right]$$

$$\mathbf{C}_\ell = \begin{pmatrix} \tilde{C}_\ell^{TT} & \tilde{C}_\ell^{TE} & 0 \\ \tilde{C}_\ell^{TE} & \tilde{C}_\ell^{EE} & 0 \\ 0 & 0 & \tilde{C}_\ell^{BB} \end{pmatrix}$$

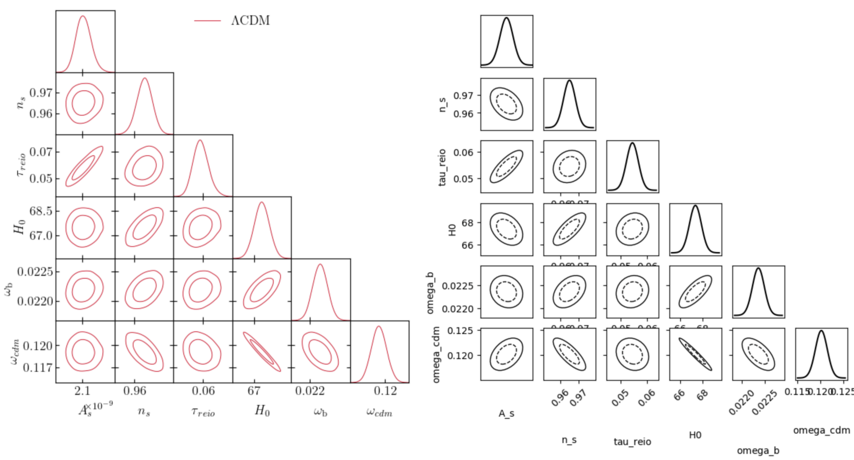
$$\tilde{C}_\ell^{XY} \equiv C_\ell^{XY} + N_\ell^{XY}$$

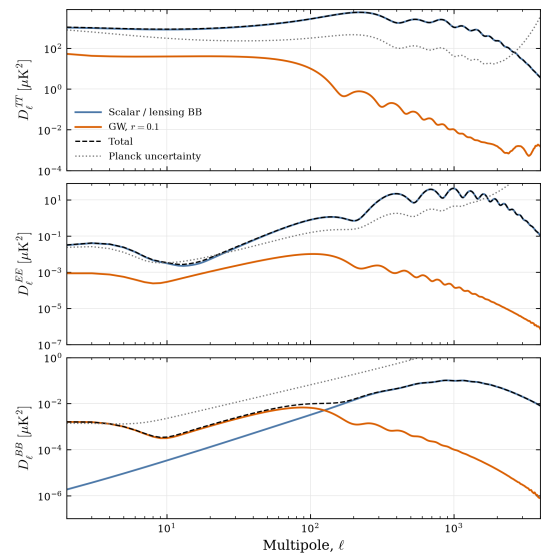
https://github.com/tsmith2/GGI_cosmology_notebooks

Notebook 2

Parameter	Planck-only constraint
A_s	$(2.101 \pm 0.034) \times 10^{-9}$
n_s	0.9649 ± 0.0042
$\tau_{\rm reio}$	0.0544 ± 0.0073
H_0 [km s^{-1} Mpc^{-1}]	67.36 ± 0.54
$\omega_b \equiv \Omega_b h^2$	0.02236 ± 0.00015
$\omega_{\rm cdm} \equiv \Omega_c h^2$	0.1202 ± 0.0014

```
A_s       = 2.101e-09 +/- 1.16e-11
n_s       = 0.9649 +/- 0.00359
tau_reio = 0.0544 +/- 0.00249
H0        = 67.36 +/- 0.583
omega_b   = 0.02236 +/- 0.000143
omega_cdm = 0.1202 +/- 0.00129
```





https://github.com/tsmith2/GGI_cosmology_notebooks

Notebook 3

We are neglecting the effects of foregrounds