Modeling Higher-Order Bias via the Cosmic Web

Implications for DESI Full-Shape Analyses

Samuel Brieden

based on: the WHM-PBS paper (arXiv:2607.21334)
and the Web–Halo Model: Brieden, Beutler & Pellejero-Ibañez (arXiv:2508.10902)

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Web–Halo Model Peak–Background Split (WHM-PBS): halo bias as a distribution, not a number

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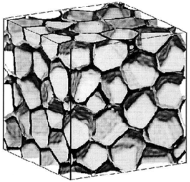
A tale of two rival theories

Bond, Kofman & Pogosyan, Nature 380, 603 (1996):
"How filaments of galaxies are woven into the cosmic web"

"the (Russian) pancaking picture" meets "the (Western) hierarchical clustering picture"

top-down: the (Russian) pancake picture



Zel'dovich 70: pancakes → filaments → clusters

"honeycomb / Swiss cheese"; Gott, *The Cosmic Web*, Fig. 4.1

bottom-up: (Western) hierarchical clustering

Gott & Rees 75; Peebles 80: merging haloes at *rare peaks*

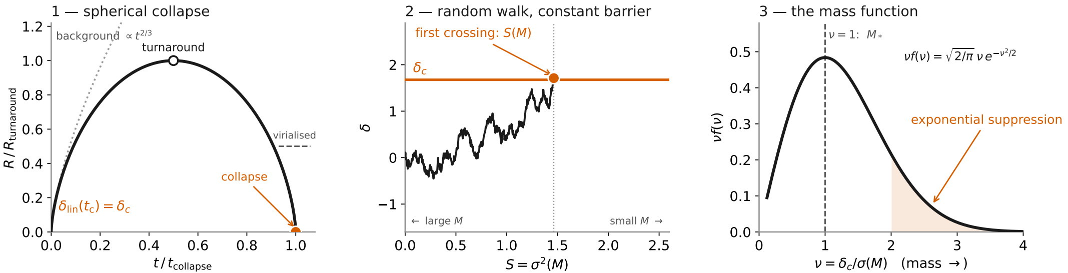
"meatballs / halo model"

Roadmap

1. **Act I —** spherical collapse: excursion set & bias expansion
2. **Act II —** the cosmic web enters the excursion set
3. **Act III —** WHM-PBS: bias becomes a *distribution*
4. **Act IV —** distributions become *priors* $\rightarrow$ DESI

Act I — spherical collapse: excursion set & bias expansion

Press–Schechter: collapse → barrier → mass function



- top-hat collapse: $\delta_c \simeq 1.686$ — **every mass** (Gunn & Gott 72)
- random walk $\delta(S)$, $S = \sigma^2(M)$: halo = **first crossing** (Bond, Cole, Efstathiou & Kaiser 91)
- **mass function** (Press & Schechter 74): $\nu f(\nu) = \sqrt{2/\pi}\,\nu\,\mathrm{e}^{-\nu^2/2}$, $\nu = \delta_c/\sigma(M)$,
  $$n(M) = \frac{\bar\rho}{M}\,\nu f(\nu)\,\frac{\mathrm{d}\ln\nu}{\mathrm{d}\ln M}$$

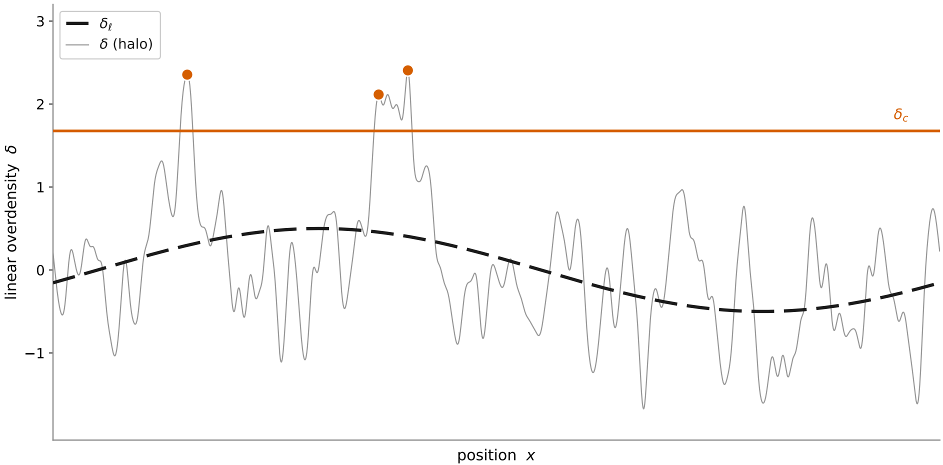
The bias expansion

$$\delta_h = \underbrace{\boxed{b_1\,\delta + \tfrac{b_2}{2}\,\delta^2 + \tfrac{b_3}{6}\,\delta^3}}_{\text{density sector}} + \underbrace{\boxed{b_{s^2}\,s^2}}_{\text{tidal sector}} + \ldots + \underbrace{\boxed{\varepsilon}}_{\text{stochasticity}}$$

- **Taylor-expand the abundance** $n_h(\delta_\ell)$ in the long mode δ_ℓ:

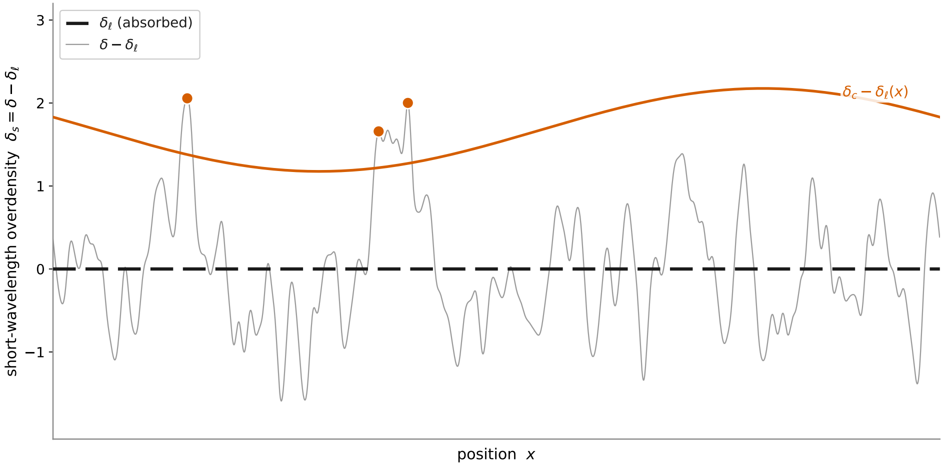
$$\delta_h \equiv \frac{n_h(\delta_\ell)}{n_h} - 1 \quad \Longrightarrow \quad b_N \equiv \frac{1}{n_h}\,\frac{\partial^N n_h}{\partial \delta_\ell^N}\bigg|_{\delta_\ell=0}$$

The peak–background split



a long mode δ_ℓ: crest — more haloes, trough — fewer (after Schmidt 13; Desjacques, Jeong & Schmidt 18)

The peak–background split



$\Rightarrow$ a *barrier shift*: $\quad n_h(M_h|\delta_\ell) = n_h(M_h)\big|_{\delta_c \to \delta_c - \delta_\ell}$

The peak–background split: the response

$$b_N^L(M_h) = \frac{1}{n_h(M_h)} \frac{\partial^N n_h(M_h|\delta_\ell)}{\partial \delta_\ell^N}\bigg|_{\delta_\ell=0}$$

the mass function determines the bias

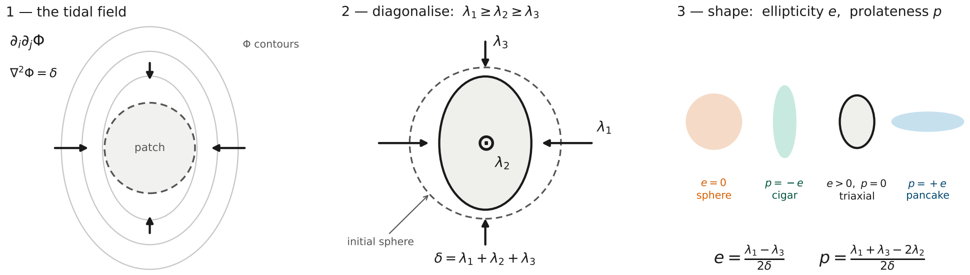
$n_h \propto \nu f(\nu)$ with $\nu = (\delta_c - \delta_\ell)/\sigma(M_h)$:

$$\boxed{b_1^L = \frac{\partial \ln n_h}{\partial \delta_\ell}\bigg|_{\delta_\ell=0} = -\frac{1}{\sigma}\frac{\partial \ln[\nu f(\nu)]}{\partial \nu} \overset{\text{PS}}{=} \frac{\nu^2 - 1}{\delta_c}}$$

deterministic $b_N(M)$ — bias is a number

Act II — the cosmic web enters the excursion set

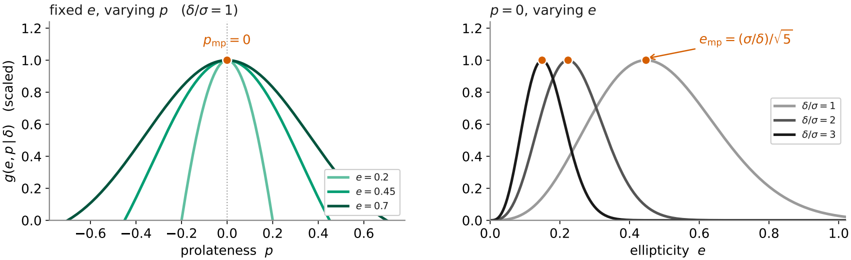
Beyond spherical: the tidal field and (δ, e, p)



- **tidal tensor** $\partial_i\partial_j\Phi$ — Φ: gravitational potential, $\nabla^2\Phi = \delta$
- initially spherical patch, sheared: axis rates $\lambda_1 \geq \lambda_2 \geq \lambda_3$; $\delta = \lambda_1 + \lambda_2 + \lambda_3$
- shape: $e = \frac{\lambda_1 - \lambda_3}{2\delta}$, $p = \frac{\lambda_1 + \lambda_3 - 2\lambda_2}{2\delta}$; spherical: $e = p = 0$
- **the traceless tidal shear** (Sheth, Mo & Tormen 01): $s_{ij} = \partial_i\partial_j\Phi - \frac{\delta}{3}\,\delta^{\mathrm{K}}_{ij}$,

$$s^2 = \sum_k \left(\lambda_k - \tfrac{\delta}{3}\right)^2 = \tfrac{2}{3}\,\delta^2\left(3e^2 + p^2\right) \xrightarrow{\langle\,\cdot\,\rangle_{g(e,p|\delta)}} \left\langle s^2 \right\rangle = \tfrac{2}{3}\,S$$

The shape distribution $g(e, p \,|\, \delta)$



- Gaussian ICs (Doroshkevich 70):

$$g(e,p|\delta) \propto e\,(e^2 - p^2)\left(\frac{\delta}{\sigma}\right)^5 \mathrm{e}^{-\frac{5}{2}\frac{\delta^2}{\sigma^2}(3e^2+p^2)}$$

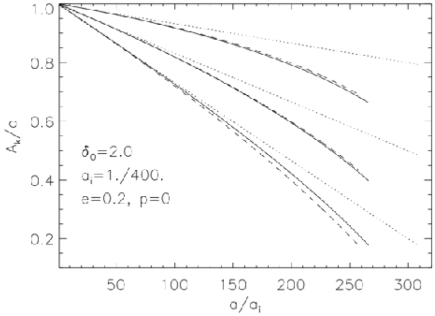
- $p_{\mathrm{mp}} = 0$; $e_{\mathrm{mp}} = (\sigma/\delta)/\sqrt{5}$ — rare peaks: **spherical**; late crossings: **sheared**

Ellipsoidal collapse: one clock per axis

- axes $A_k(t)$ (Bond & Myers 96):

$$\frac{\mathrm{d}^2 A_k}{\mathrm{d}t^2} = -4\pi G\bar{\rho}\, A_k \left[\frac{1+\delta}{3} + \frac{b'_k}{2}\,\delta + \lambda'_{\mathrm{ext},k}\right]$$

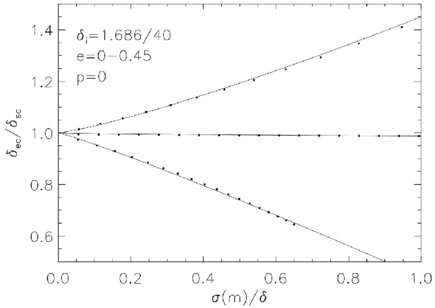
- axis collapsed at $A_k/a = 0.17$ — yields $\Delta_{\mathrm{vir}} \sim 200$
- **one patch, three collapse epochs**
- collapsed axes = morphology (Shen+06):
  1 $\rightarrow$ sheet, 2 $\rightarrow$ filament, 3 $\rightarrow$ halo



$A_k(t)$, $e = 0.2$ (Shen, Abel, Mo & Sheth 06)

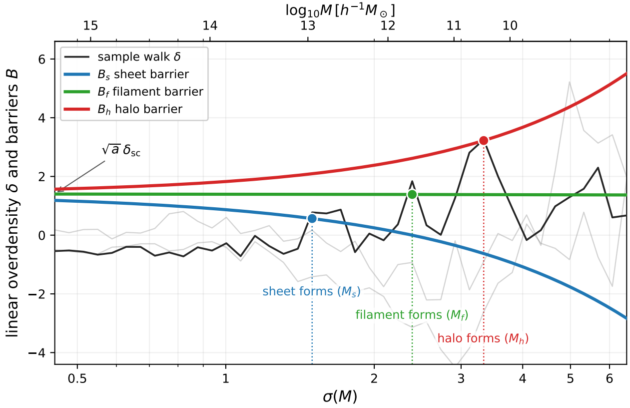
Why the barrier moves

- representative IC: $(e_{\rm mp}, 0)$ (Sheth et al. 01): $(e, p) \to (\delta, S)$
- three variables $\to$ **one walk** + **moving barrier** $B(S)$
- 3rd / 2nd / 1st axis $\Rightarrow$ **halo** / **filament** / **sheet**:
  rising / **flat** / **falling**



$\delta_{\rm ec}/\delta_{\rm sc}$ (Shen+06)

Moving barriers for the web



WHM-PBS Fig. 1 (arXiv:2607.21334)

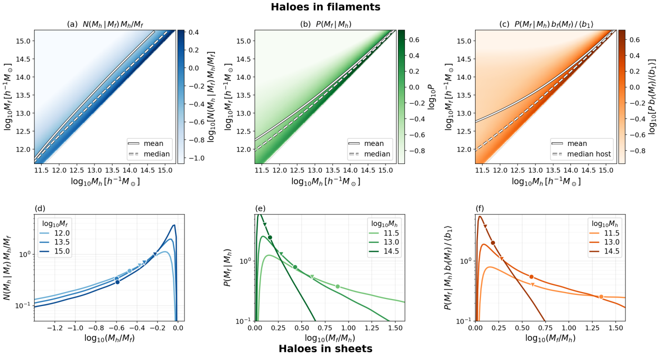
Shen et al. (2006), $a = 0.707$:

$$B(S) = \sqrt{a}\,\delta_{\rm sc}\left[1 + \beta\left(\frac{S}{a\,\delta_{\rm sc}^2}\right)^{\alpha}\right]$$

(β, α): halo $(0.45, 0.61)$ steep — filament $(-0.012, 0.28)$ flat — sheet $(-0.56, 0.55)$ falling

- nesting $B_s < B_f < B_h$: **sheet** $\to$ **filament** $\to$ **halo**, $M_s > M_f > M_h$

The host distribution $P(M_f|M_h)$

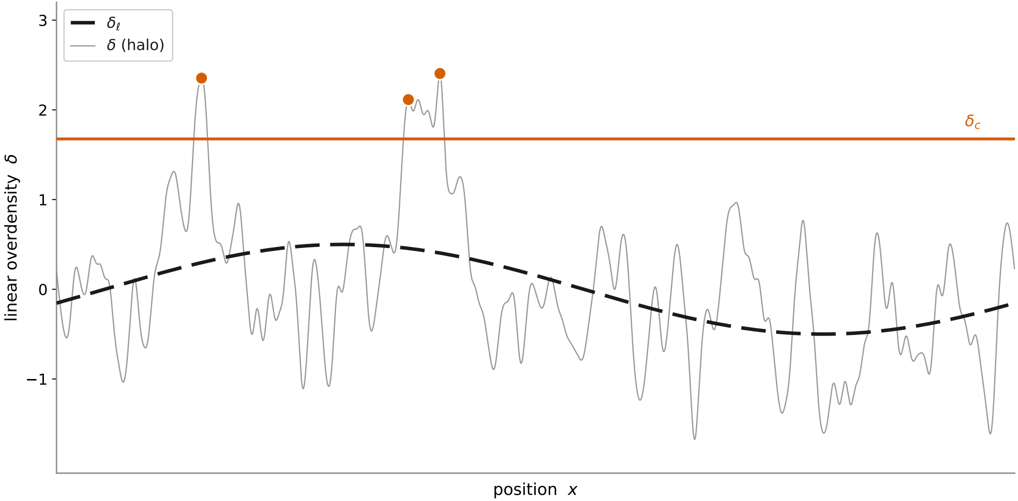


WHM-PBS Fig. 2

- conditional crossing $N(M_h|M_f)$
- Chapman–Kolmogorov: $n_h = \int N(M_h|M_f)\, n_f \,\mathrm{d}M_f$
- Bayes:
  $$P(M_f|M_h) = \frac{N(M_h|M_f)\, n_f}{n_h}$$
  $\Rightarrow$ **closure**
- skewed distribution

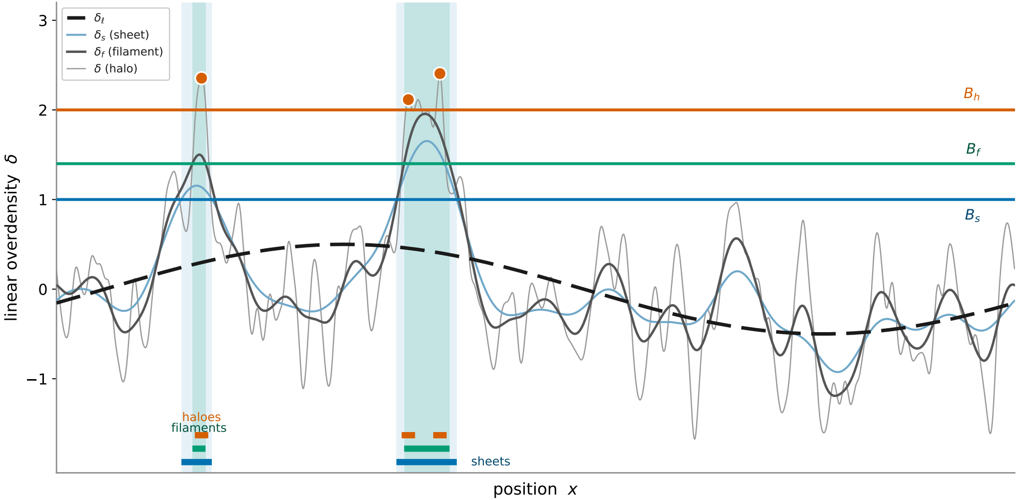
Act III — WHM-PBS: bias as a distribution

The Web–Halo Model Peak–Background Split (WHM-PBS)



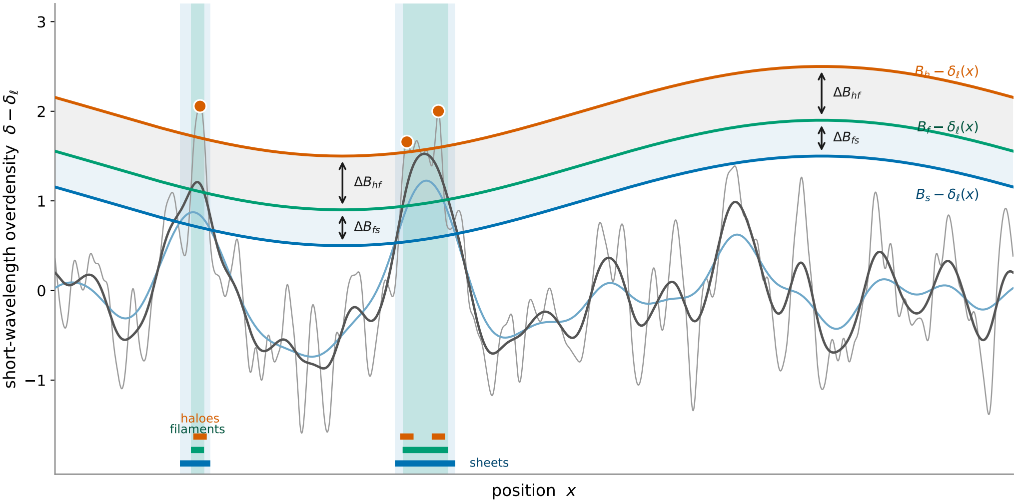
textbook: one barrier, one long mode

The Web–Halo Model Peak–Background Split (WHM-PBS)



nested fields, elevated barriers

The Web–Halo Model Peak–Background Split (WHM-PBS)



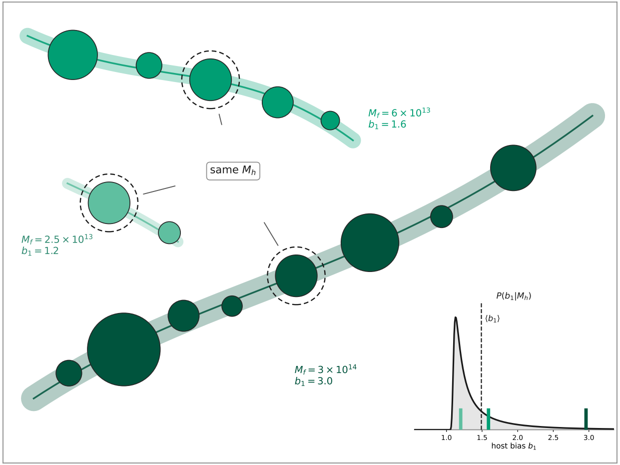
δ_ℓ into the barriers: **all three shift alike** — gaps unchanged

The environment-averaged bias: the calculation

$$\begin{aligned}
b^L_{h,N}(M_h) &= \frac{1}{n_h(M_h)} \left.\frac{\partial^N n_h(M_h|\delta_\ell)}{\partial \delta_\ell^N}\right|_{\delta_\ell=0} \\
&= \frac{1}{n_h(M_h)} \frac{\partial^N}{\partial \delta_\ell^N} \int N(M_h|M_f)\, n_f(M_f|\delta_\ell)\, \mathrm{d}M_f \\
&= \frac{1}{n_h(M_h)} \int N(M_h|M_f) \left.\frac{\partial^N n_f(M_f|\delta_\ell)}{\partial \delta_\ell^N}\right|_{\delta_\ell=0} \mathrm{d}M_f \\
&= \frac{1}{n_h(M_h)} \int N(M_h|M_f)\, n_f(M_f) \underbrace{\frac{1}{n_f(M_f)} \left.\frac{\partial^N n_f(M_f|\delta_\ell)}{\partial \delta_\ell^N}\right|_{\delta_\ell=0}}_{=\, b^L_{f,N}(M_f)} \mathrm{d}M_f \\
&= \int \underbrace{\frac{N(M_h|M_f)\, n_f(M_f)}{n_h(M_h)}}_{=\, P(M_f|M_h)} b^L_{f,N}(M_f)\, \mathrm{d}M_f = \boxed{\left\langle\, b^L_{f,N} \,\right\rangle_{M_f|M_h}}
\end{aligned}$$

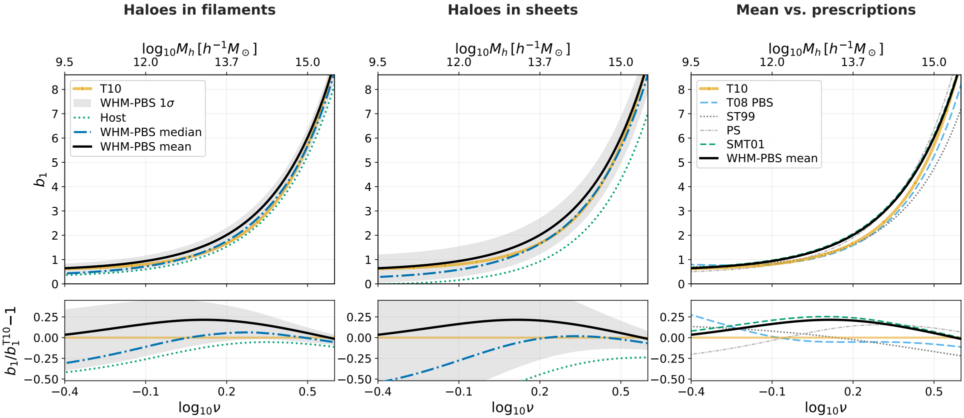
mean $=$ direct halo bias (**closure**); host random $\Rightarrow$ a distribution, not a number

A halo inherits its host's bias



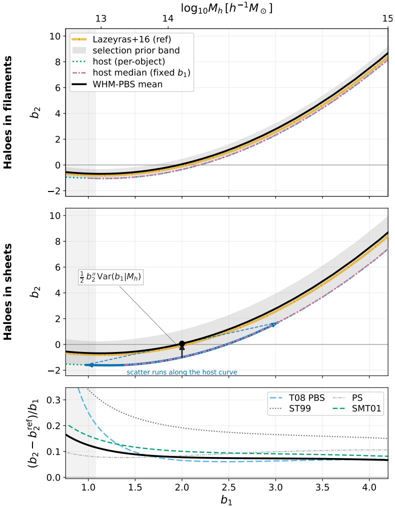
$b_{1,h} = \langle\, b_{1,f}(M_f)\, \rangle_{M_f\,|\,M_h}$ — same M_h, different hosts: **different bias**

Linear Halo Bias: a distribution, not a number



- mean: **closure-protected**; width: $\sigma(b_1|M_h) \approx 0.6\text{–}1.0$
- mean $\sim 18\%$ high (Tinker+10) — but the relations $b_N(b_1)$ are **what matters**

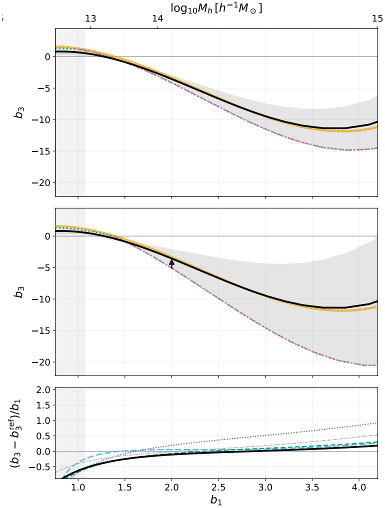
Density sector: the $b_2(b_1)$ relation and prior band



- one host fixes *all* b_N: haloes slide *along* $b_N(b_1)$
- aggregation: $b_2^S = b_2^{\mathrm{host}}(b_1^S) + \frac{1}{2}\, b_2^{\mathrm{host}\,\prime\prime}\,\mathrm{Var}(b_1|S)$; host curve = **lower envelope**
- **prior band**: width $\mathrm{Var}(b_1|M_h) \simeq 0.4$; tracks Lazeyras+16

Lazeyras+16: $\quad b_2(b_1) = 0.412 - 2.143\, b_1 + 0.929\, b_1^2 + 0.008\, b_1^3$

Density sector: the $b_3(b_1)$ relation



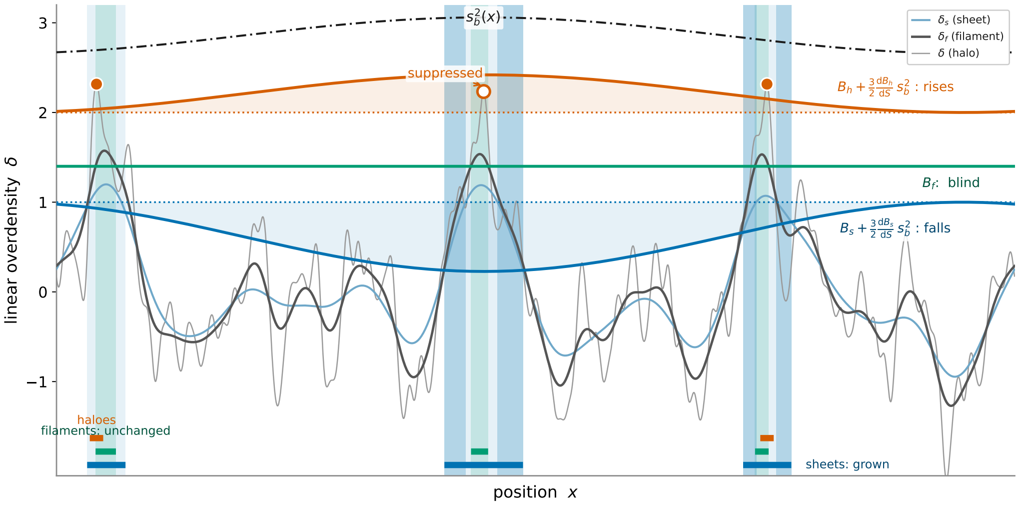
- cubic host curve — the skewness term enters:

$$b_3^S = b_3^{\rm host}(b_1^S) + \tfrac{1}{2}\, b_3^{\rm host\,\prime\prime}\, {\rm Var}(b_1|S) + \tfrac{1}{6}\, b_3^{\rm host\,\prime\prime\prime}\, \mu_3(b_1|S)$$

- the μ_3 term flips the lift at low b_1
- same band recipe; tracks Lazeyras+16

Lazeyras+16:
$b_3(b_1) = -1.028 + 7.646\, b_1 - 6.227\, b_1^2 + 0.912\, b_1^3$

The tidal sector: one shear, three responses



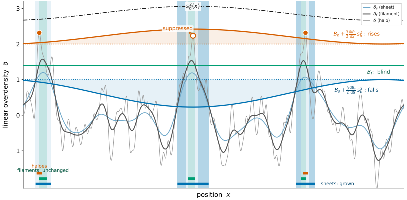
$\delta B = \frac{3}{2}(\mathrm{d}B/\mathrm{d}S)\, s_b^2$: sheet ↓ — filament blind — halo ↑

The tidal derivation

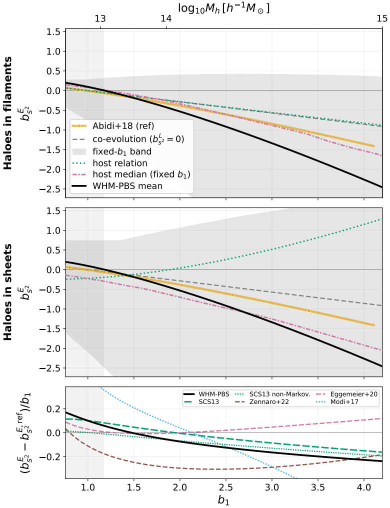
1. definition: $b^L_{s^2} = \frac{1}{n_x} \frac{\partial n_x(M|s_b^2)}{\partial s_b^2}\Big|_0$
2. **the barrier is the shear**: $\langle s^2 \rangle = \frac{2}{3} S \Rightarrow \delta B(S) = \frac{3}{2} \frac{\mathrm{d}B}{\mathrm{d}S} s_b^2$ — non-uniform
3. spherical: $\mathrm{d}B/\mathrm{d}S = 0 \Rightarrow b^L_{s^2} = 0$
4. $\mathrm{d}B_f/\mathrm{d}S \approx 0$: n_f blind, $N(M_h|M_f)$ responds

$$\boxed{b^L_{s^2}(M_h) = \left\langle\, b^L_{s^2}(M_h|M_f) \,\right\rangle_{M_f|M_h}}$$

genuinely broken: scatter at fixed b_1

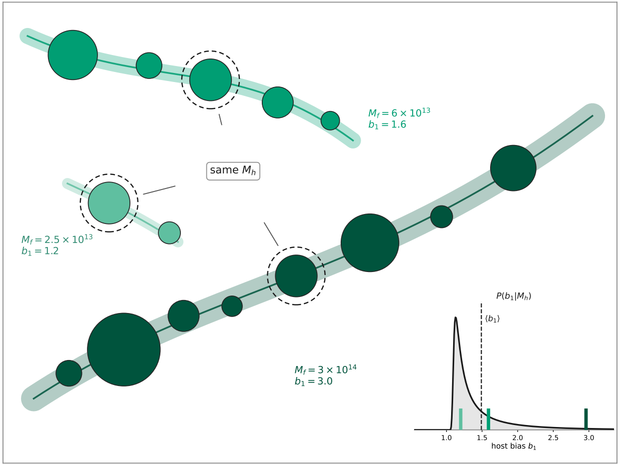


Tidal relation: $b_{s^2}(b_1)$



- band brackets the N-body spread (Abidi–Baldauf ↔ Zennaro, Eggemeier, Modi)
- typical-host track: RMS 0.09 — the mean: 0.51 (vs Abidi–Baldauf 18)
- same $P(M_f|M_h)$, no new parameter
- prior band? host–halo curve difference + assembly bias (see later)

One mass, many hosts



same M_h, different hosts: $\varepsilon^{\text{clust}}$ drawn from $P(b_1|M_h)$

Stochasticity from the same host variance

- deterministic + noise: $\delta_h = b_1\,\delta + \varepsilon$, $\langle \varepsilon\delta\rangle = 0 \;\Rightarrow\; P_\varepsilon = P_{hh} - P_{hm}^2/P_{mm}$
- the host makes ε physical — a bias *field*:

$$b_1 \to b_1(M_f(\boldsymbol{x})) \quad \Rightarrow \quad \varepsilon^{\rm clust} = \left[b_1(M_f(\boldsymbol{x})) - b_1\right]\delta(\boldsymbol{x})$$

- across mass bins: the **stochasticity matrix**,

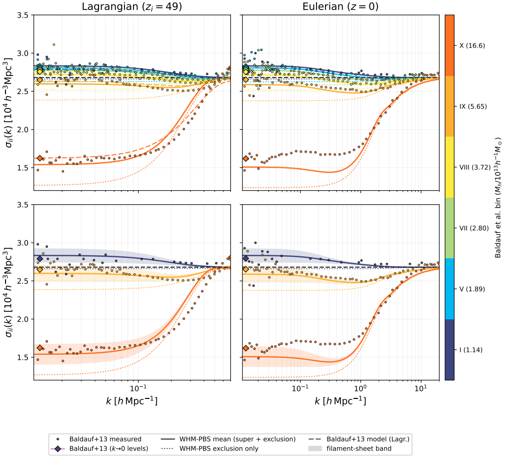
$$\sigma_{ij}(k) = \big\langle(\delta_{h,i} - b_{1,i}\delta)(\delta_{h,j} - b_{1,j}\delta)\big\rangle = \frac{\delta_{ij}}{\bar{n}_i} + P^{\rm clust}_{\varepsilon,ij} + P^{\rm excl}_{\varepsilon,ij}$$

- the host fixes $P^{\rm clust}_\varepsilon$ completely:

$$P_{\delta b}(q) = \int {\rm d}M_f\, P(M_f|M_h)\left[b_1(M_f) - \langle b_1\rangle_{M_f|M_h}\right]^2 V_\star(M_f)\, W^2_{R_\star}(q)$$

$$P^{\rm clust}_\varepsilon(k) = \int \frac{{\rm d}^3q}{(2\pi)^3}\, P_{\delta b}(q)\, P_{mm}(|\boldsymbol{k}-\boldsymbol{q}|)$$

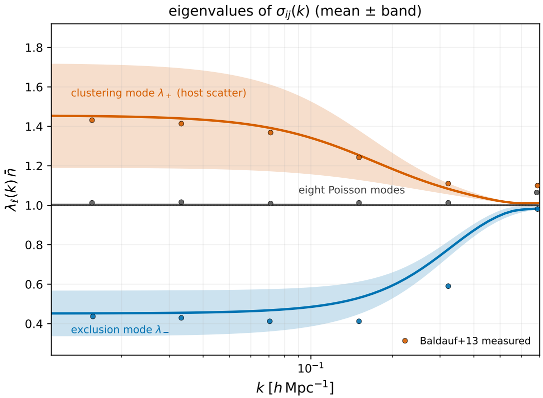
The diagonal: super- and sub-Poisson



WHM-PBS Fig. 7 vs Baldauf+13

- **coherent over the host**: every halo in one host shares the *same* offset δb across its size $R_\star(M_f)$ — the window $W_{R_\star}$ in $P_{\delta b}$
- $\Rightarrow$ white **super-Poisson** plateau at $k \ll 1/R_\star$
- exclusion $\Rightarrow$ **sub-Poisson**

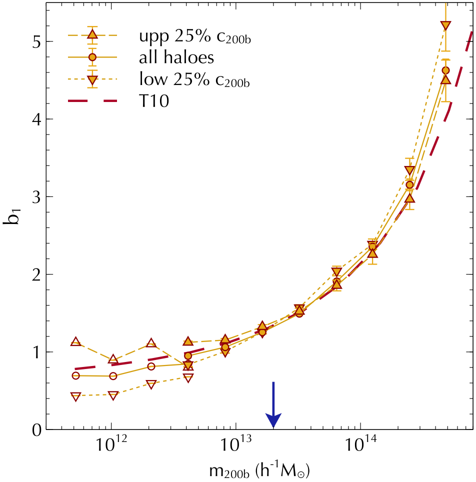
The eigenvalues of the stochasticity matrix



WHM-PBS Fig. 9

- band = **filament** and **sheet** predictions, thick line = their mean
- exactly **two** non-Poisson modes: clustering ↑, exclusion ↓

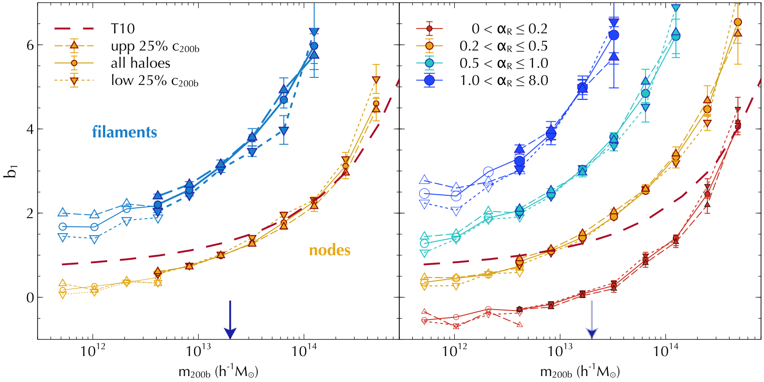
Assembly bias: the classic signal



Paranjape, Hahn & Sheth 18, Fig. 13

- split by concentration at fixed mass: **clustering differs**
- high mass: low-c more clustered; low mass: **inverted**
- inversion at $m_* \simeq 2 \times 10^{13}\, h^{-1} M_\odot$

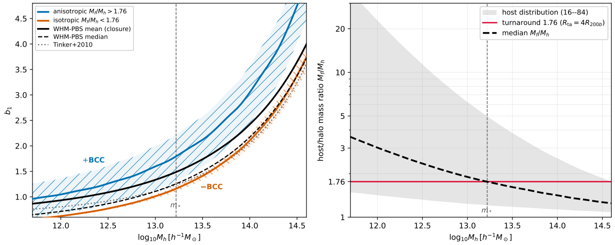
Assembly bias: it is the environment



Paranjape, Hahn & Sheth 18, Fig. 14

- the low-mass inversion comes from haloes in *filaments*
- smooth in the tidal anisotropy α_R at $R = R_{\rm ta} \simeq 4\,R_{200b}$

Assembly bias: the inversion as population mixing



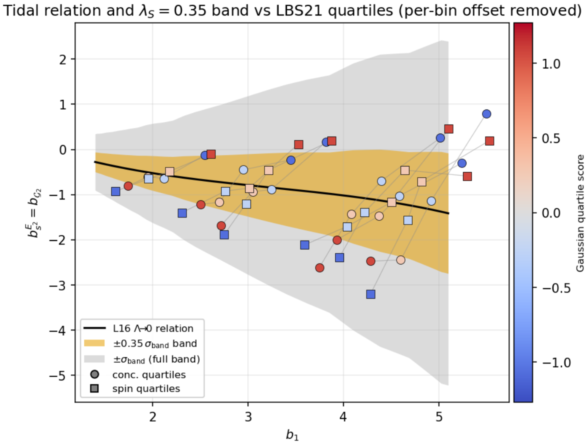
WHM-PBS Fig. 10

- turnaround host:

$$\frac{M_f}{M_h} = \frac{\bar{\rho}_{\rm ta}}{200\,\bar{\rho}}\left(\frac{R_{\rm ta}}{R_{200b}}\right)^3 \simeq \frac{5.5}{200}4^3 = 1.76$$

- $M_f/M_h > 1.76$: filament-fed; $M_f \approx M_h$: isotropic proto-halo
- median host crosses 1.76 at $m_* \simeq 1.7\times 10^{13}\,h^{-1}M_\odot$: $\sim$5% from Paranjape+18
- $\Delta b_1 \simeq 0.6$: one $\mathrm{Var}(b_1)$ drives **both**

Tidal-sector assembly bias



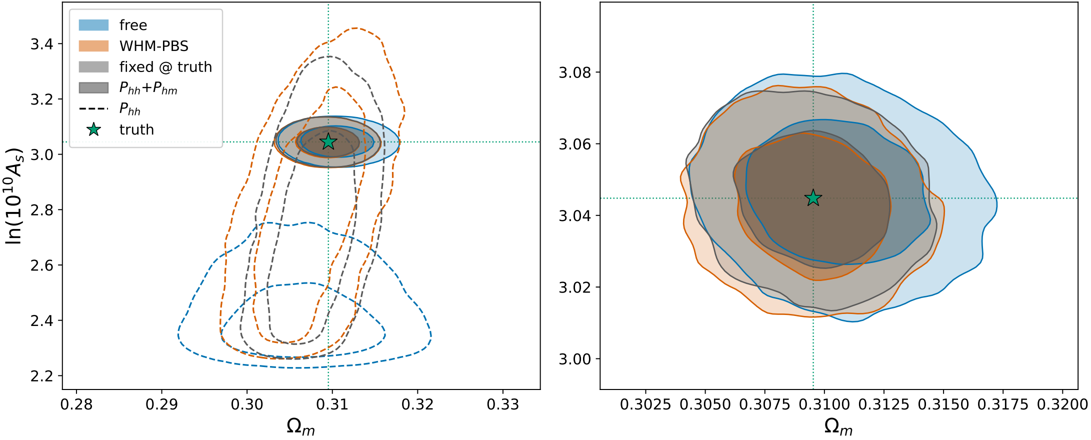
- concentration- and spin-quartile splits at fixed mass (Lazeyras, Barreira & Schmidt 21, Fig. 5)
- displace $b^E_{s^2}$ by $\simeq 0.26$–$0.44\,\sigma_{\rm band}$ — about $1/3$ of the full band
- adopted as the tidal prior width: $\lambda_S \simeq 0.35$

Act IV — implications for DESI

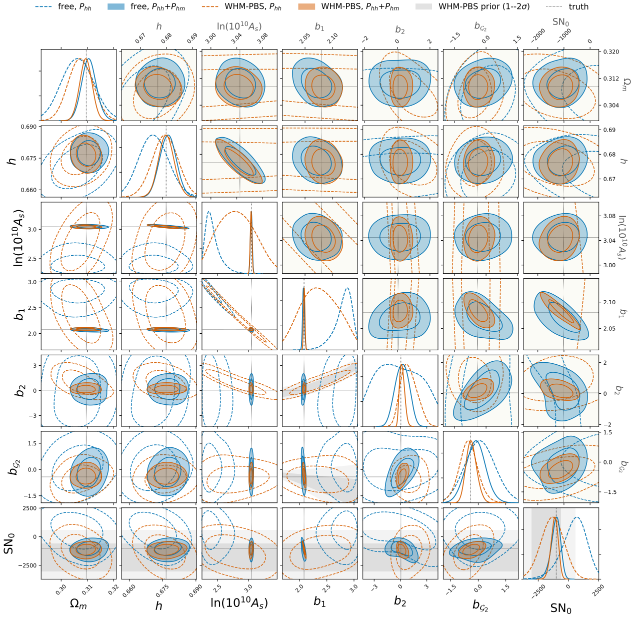
From theory to priors: three rules

1. $b_2(b_1)$, $b_3(b_1)$: Gaussian band, width $\simeq 0.4$, hard floor
2. $b_{s^2}(b_1)$: broad Gaussian, $\sigma^2 = [\text{mean}{-}\text{host}]^2 + [0.35\,\sigma_{\text{band}}]^2$
3. SN_0: Gaussian prior with width set by the difference between sheet and filament prediction

The $b_1^2 A_s$ valley



WHM-PBS Fig. 11; dashed = P_{hh}, filled = $P_{hh} + P_{hm}$



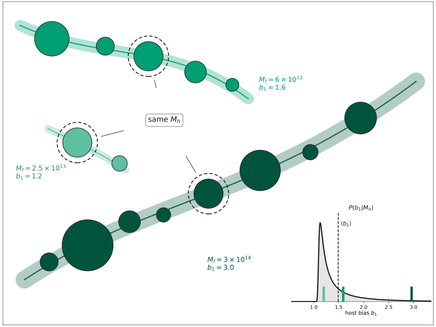
free, P_{hh}
free, P_{hh}+P_{hm}
WHM-PBS, P_{hh}
WHM-PBS, P_{hh}+P_{hm}
WHM-PBS prior (1--2σ)
truth
Ω_m
h
$\ln(10^{10}A_s)$
b_1
b_2
$b_{\mathcal{G}_2}$
SN_0

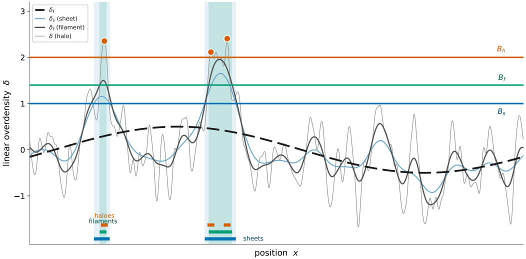
Conclusions

- **bias is a distribution, not a number**: mean = standard PBS, **the width is new physics**
- one $P(M_f|M_h)$:
  - $b_N(b_1)$ bands
  - broken tidal relation
  - stochasticity
  - assembly inversion

This presentation and all figures were created with help of Claude Code.

Thank you!





a halo clusters as its host does: $b_{1,h} = \langle\, b_{1,f}(M_f)\,\rangle_{M_f\,|\,M_h}$