

Sign-Switching Λ_s CDM: Theoretical Realizations and Implications for Cosmological Tensions

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- DESI-era BAO+CMB+SN combinations revive **evolving dark energy**, usually phrased through $w_{\text{de}}(z)$ (CPL); inferred dynamics is sensitive to dataset choices and to the functional prior — and standard EoS parametrizations are **sign-preserving by construction**: density-level structure gets smeared into smoother $w_{\text{de}}(z)$.
- Independent threads point at **sign-changing effective DE densities**:
 - negative- Λ / $\rho_{\text{de}} < 0$ hints at high z ; Λ_{s} CDM-type transitions easing H_0 (& M_B), S_8 , γ , with $\sum m_\nu$ back in the physical region;
 - model-agnostic reconstructions: inferred density zero around $z \sim 1.5\text{--}2.5$, with localized intermediate-redshift structure.

The problem

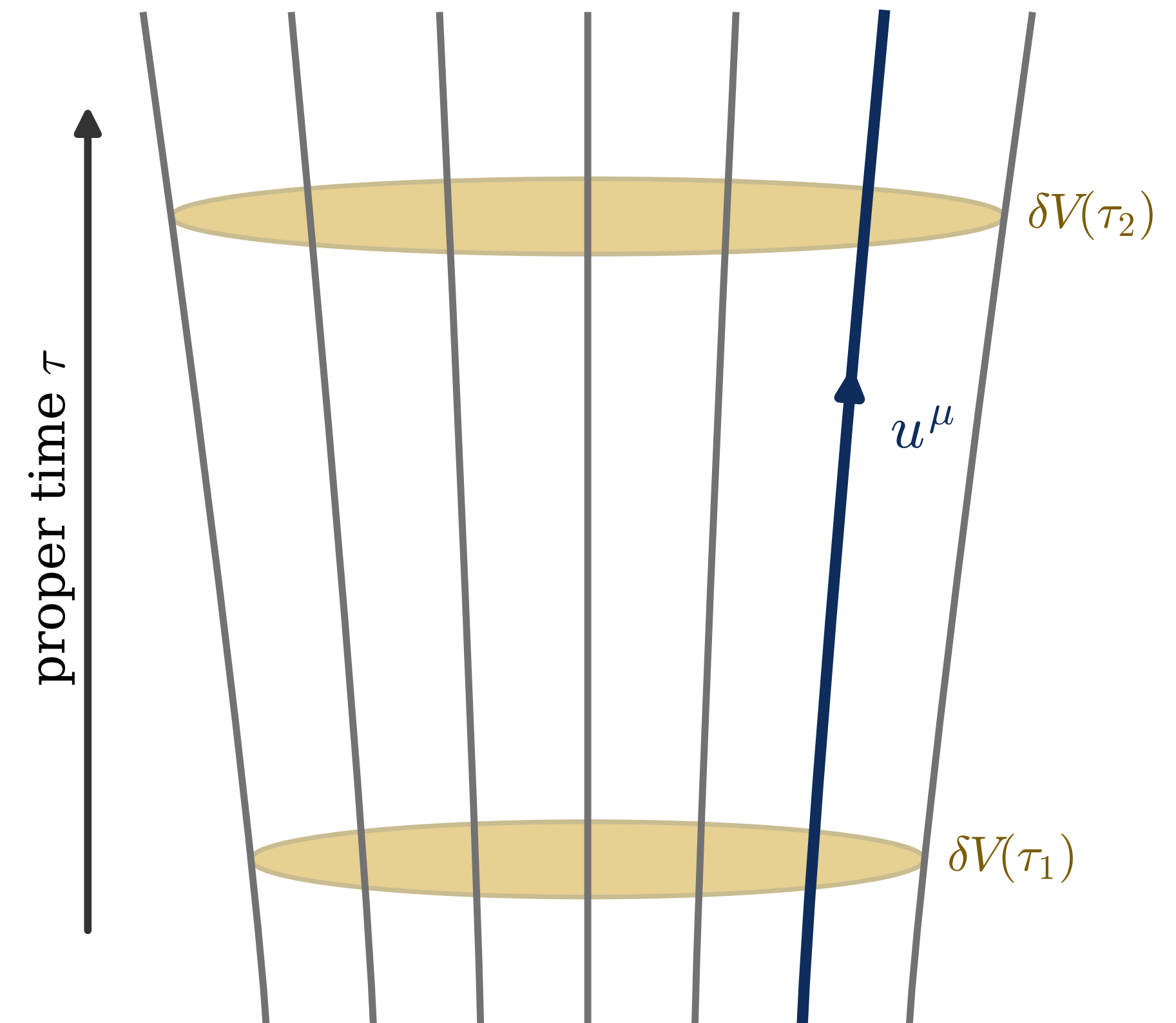
These histories enter exactly the regime where $w_{\text{de}} = p_{\text{de}}/\rho_{\text{de}}$ ceases to be a regular variable: the inequalities reverse for $\rho_{\text{de}} < 0$ and the ratio is undefined at $\rho_{\text{de}} = 0$.

Which variables should organize the description?

- A **congruence**: a family of timelike worldlines filling spacetime — exactly one through each event — with unit tangent u^μ . For cosmology: the comoving observers.
- Its **expansion scalar** is the fractional growth rate of a comoving volume element:

$$\Theta \equiv \nabla_\mu u^\mu = \frac{1}{\delta V} \frac{d\delta V}{d\tau}, \quad \Theta_{\text{FLRW}} = 3H.$$

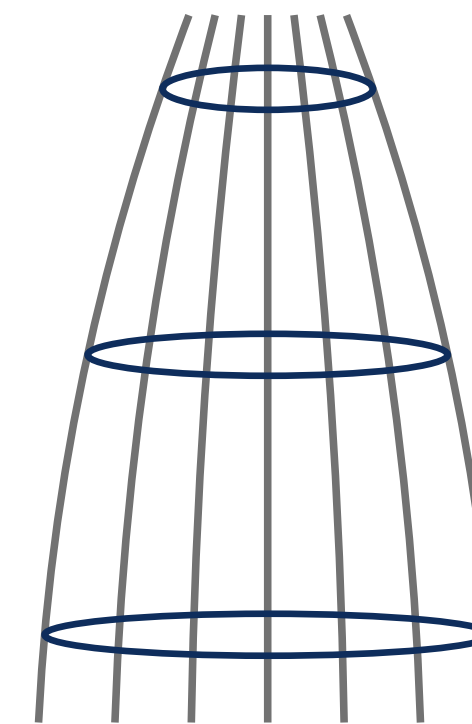
- Focusing / defocusing = whether neighboring worldlines are driven together or apart — a statement about *geometry*, before any matter content is specified.



$$\frac{d\Theta}{d\tau} = \underbrace{-\frac{1}{3}\Theta^2}_{\text{expansion self-interaction}} \underbrace{-\sigma_{\mu\nu}\sigma^{\mu\nu}}_{\text{shear: always focuses}} \underbrace{+\omega_{\mu\nu}\omega^{\mu\nu}}_{\text{rotation: resists focusing}} \underbrace{-R_{\mu\nu}u^\mu u^\nu}_{\text{gravity: matter enters only here}} \underbrace{+\nabla_\mu A^\mu}_{\text{non-geodesic (forces)}}$$

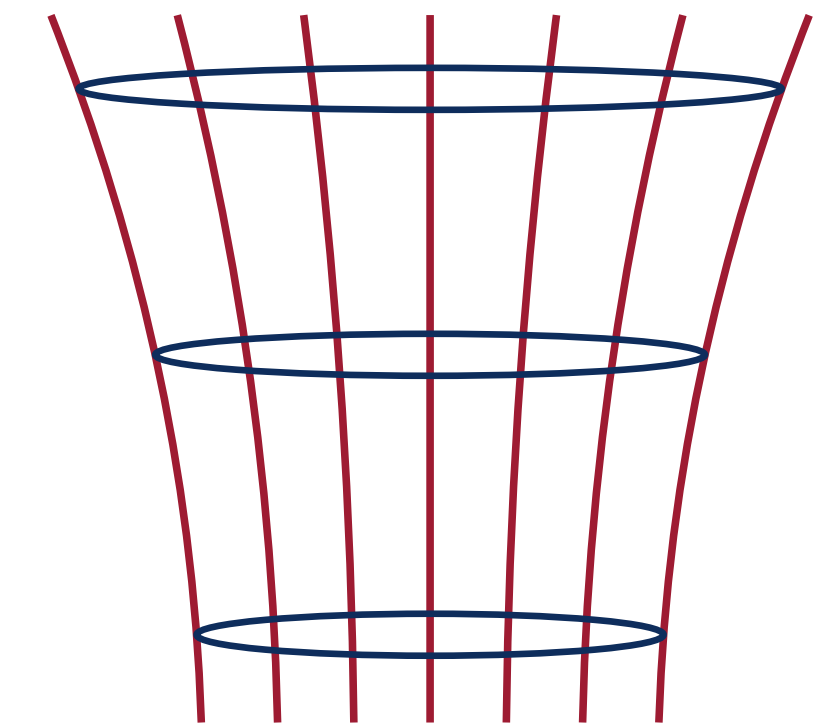
- A purely geometric identity: no EoS parameter, no stress–energy tensor appears.
- FLRW comoving congruence: $\sigma = \omega = A^\mu = 0$, $\Theta = 3H \Rightarrow \ddot{a}/a = -\frac{1}{3} R_{\mu\nu} u^\mu u^\nu$.
- **Acceleration = defocusing**, $R_{\mu\nu} u^\mu u^\nu < 0$ — it does *not* require $\dot{\Theta} > 0$: in flat FLRW $\dot{\Theta} = 3\dot{H} = -12\pi G \mathcal{I}_{\text{tot}}$, so $\dot{\Theta} < 0$ whenever the total NEC holds; acceleration needs only $\mathcal{M}_{\text{tot}} < 0$. GR maps it to matter:
 $R_{\mu\nu} u^\mu u^\nu = 4\pi G \sum_i \mathcal{M}_i$,
 $R_{\mu\nu} k^\mu k^\nu = 8\pi G \mathcal{E}^2 \sum_i \mathcal{I}_i$, and $\rho'_i = \frac{3}{1+z} \mathcal{I}_i$. The equations select $\mathcal{M} = \rho + 3p$, $\mathcal{I} = \rho + p$;
 $w = p/\rho$ is a derived, branch-dependent ratio.

focusing: $R_{\mu\nu} u^\mu u^\nu > 0$



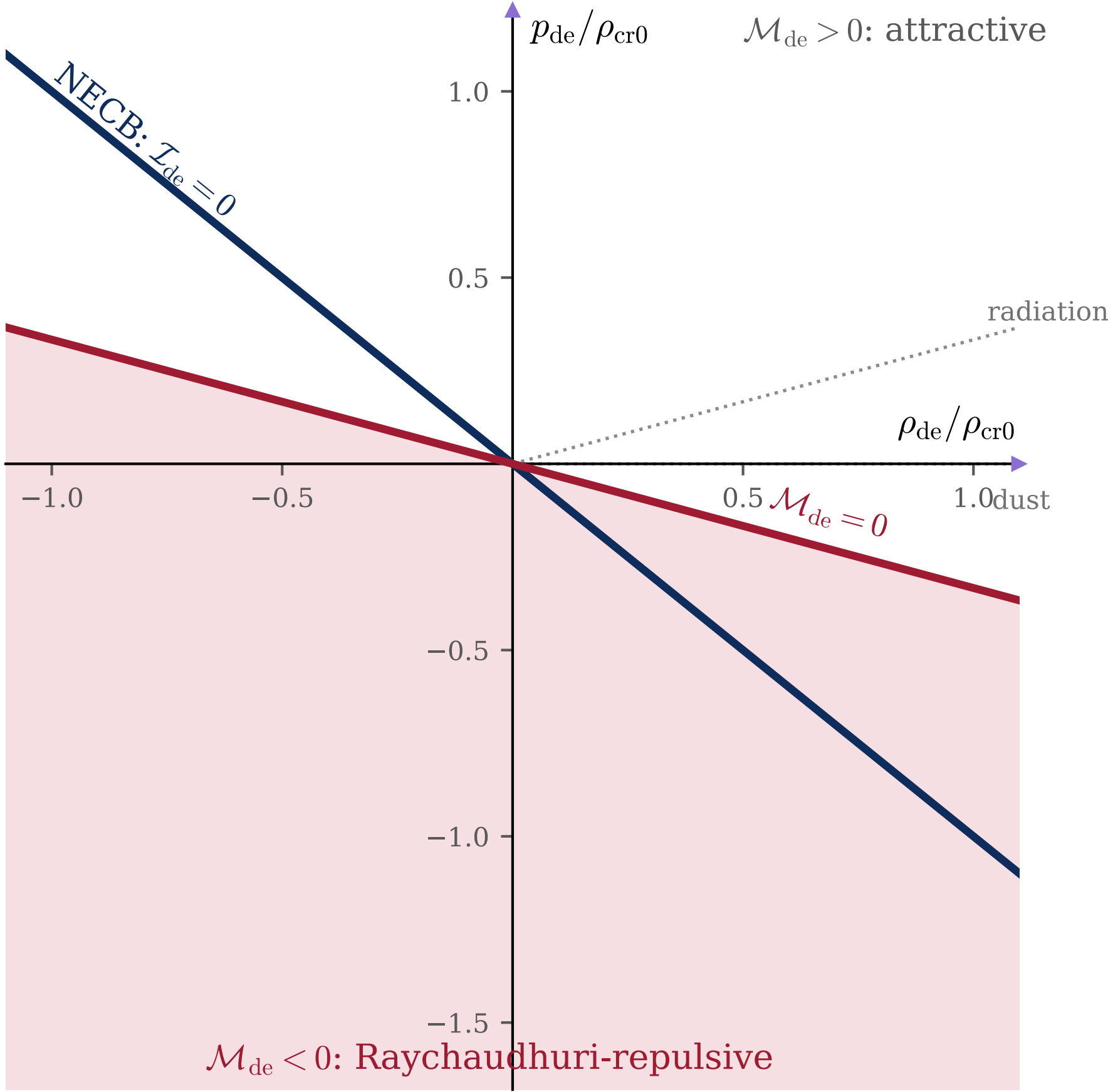
worldlines converge — gravity attracts

defocusing: $R_{\mu\nu} u^\mu u^\nu < 0$

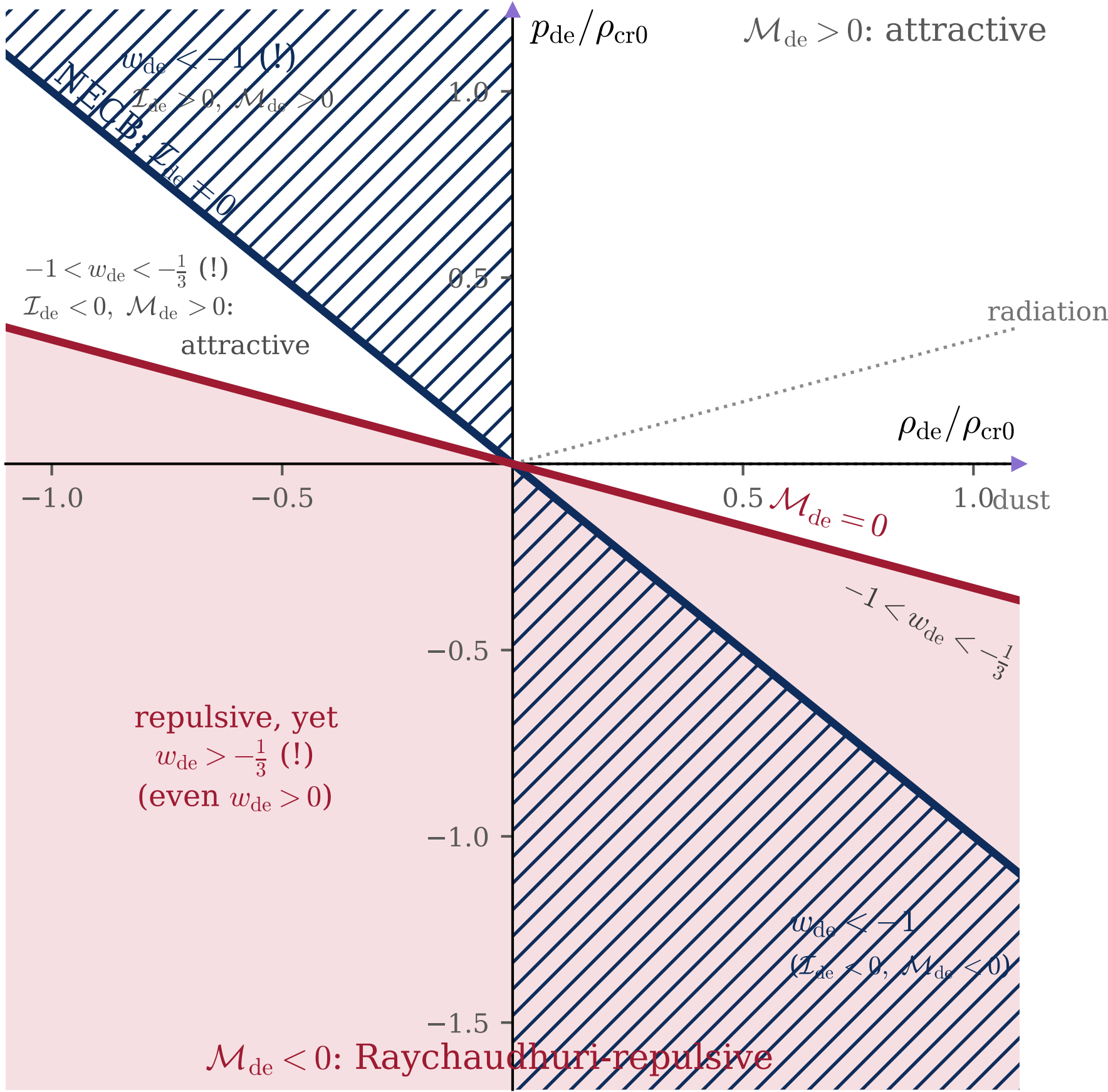


driven apart — cosmic acceleration ($\ddot{a} > 0$)

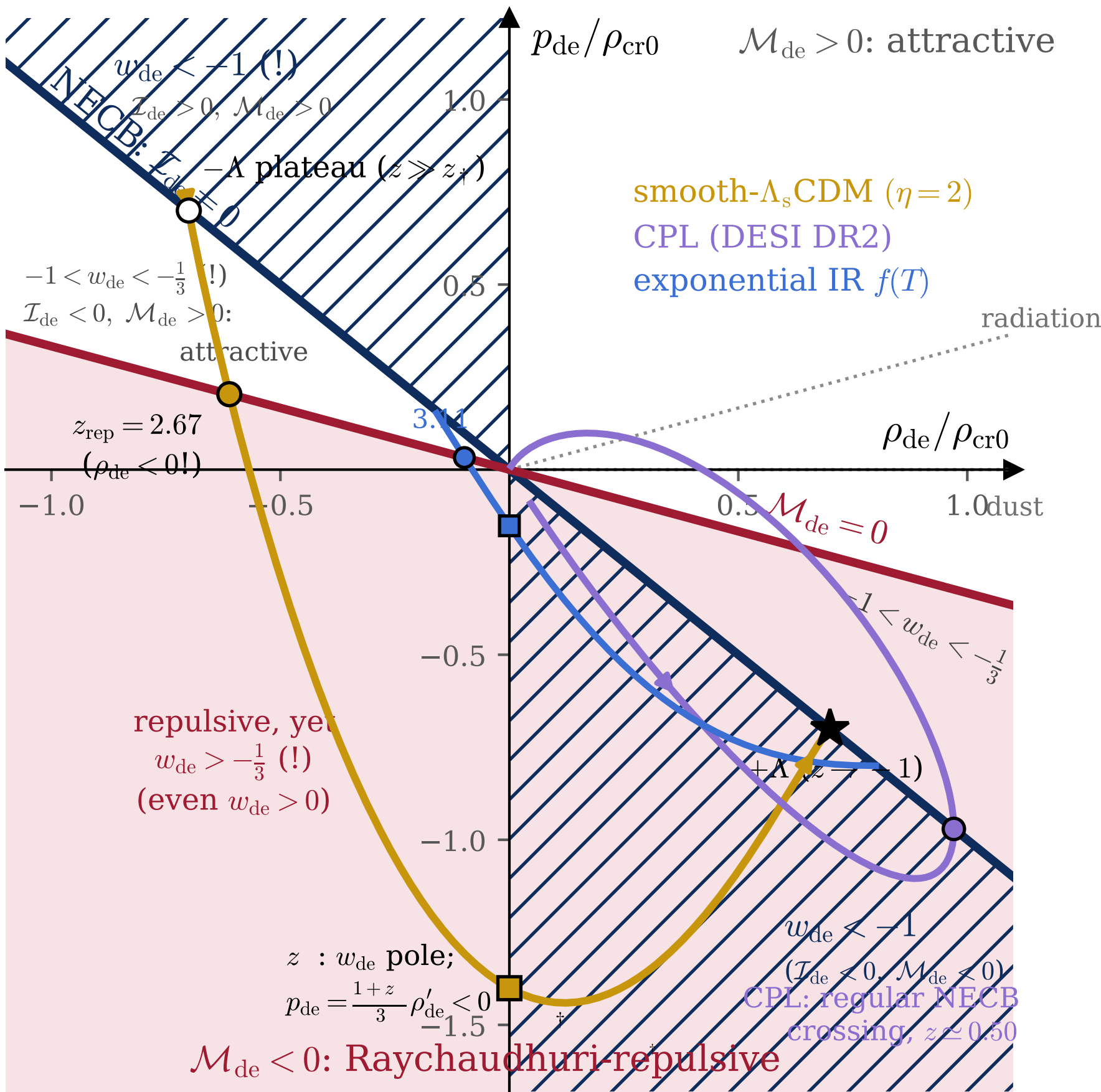
- **Convergence conditions:** gravity focuses timelike congruences iff $R_{\mu\nu} u^\mu u^\nu \geq 0$ (timelike CC), null ones iff $R_{\mu\nu} k^\mu k^\nu \geq 0$ (null CC). The field equations translate: TCC $\iff \mathcal{M}_{\text{tot}} \geq 0$; NCC $\iff \mathcal{I}_{\text{tot}} \geq 0$ (the total NEC).
- **Active gravitational mass density** $\mathcal{M}_i = \rho_i + 3p_i$: the Newtonian limit of Raychaudhuri is $\nabla^2 \Phi = 4\pi G \sum_i (\rho_i + 3p_i)$ — pressure gravitates, and $\mathcal{M}$ is precisely *what attracts*.
- **Inertial mass density** $\mathcal{I}_i = \rho_i + p_i$: the relativistic Euler equation, $(\rho + p) a^\mu = -\nabla^\mu_\perp p$, makes $\rho + p$ the inertia a pressure gradient must push; and continuity transports exactly it: $\dot{\rho}_i = -\Theta \mathcal{I}_i$, i.e. $\rho'_i = \frac{3}{1+z} \mathcal{I}_i$.



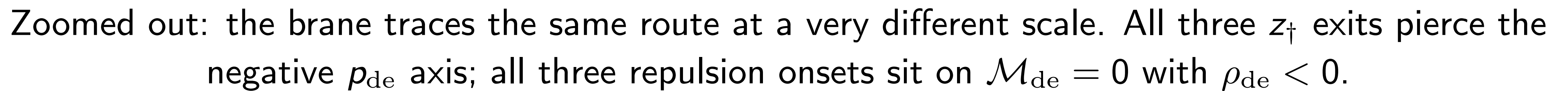
- The field equations draw **half-planes**: repulsive $\mathcal{M}_{\text{de}} < 0$; phantom-like $\mathcal{I}_{\text{de}} < 0$. Regular everywhere — including $\rho_{\text{de}} = 0$. Λ lives *on* the NECB: $T_{\mu\nu}^{(\Lambda)} = -\rho_{\Lambda}g_{\mu\nu} \Rightarrow \mathcal{I}_{\Lambda} \equiv 0$; $w_{\Lambda} = -1$ is only its ratio form.



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- Ratio language draws **bowties**: $\{w_{\text{de}} < -1\}$ is a union of two *opposite* sectors; on $\rho_{\text{de}} < 0$ even $w_{\text{de}} > 0$ can be repulsive, and $\mathcal{M}_{\text{de}} < 0 \Rightarrow \mathcal{I}_{\text{de}} < 0$ there.



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- Trajectories: smooth- Λ_s CDM enters on the NECB at the $-\Lambda$ plateau, crosses $\mathcal{M}_{\text{de}} = 0$ at z_{rep} (*while* $\rho_{\text{de}} < 0$), exits through the p_{de} axis *below* the origin at z_+ , lands on $+\Lambda$. CPL instead crosses the NECB regularly.

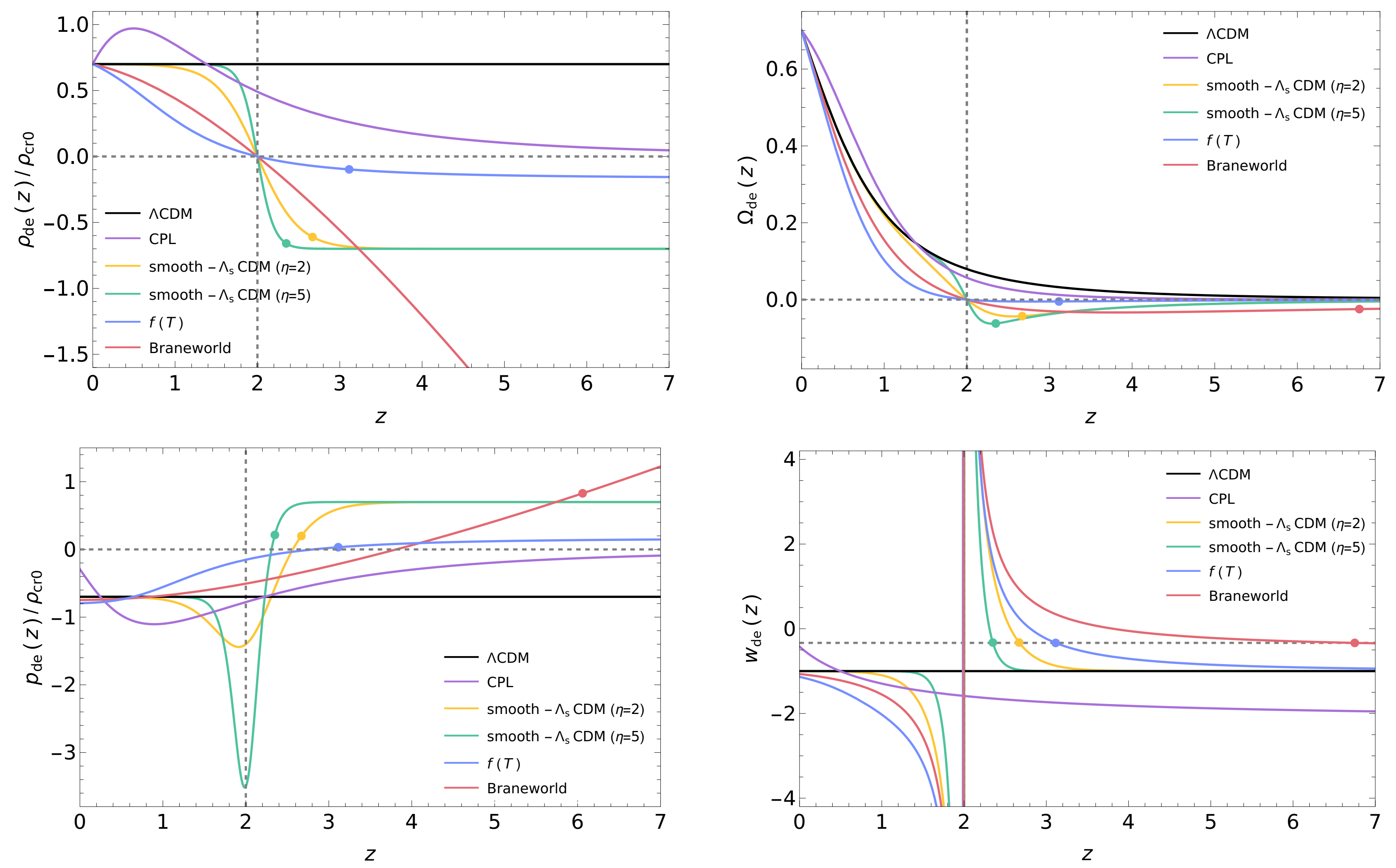


$$\mathcal{M}_{\text{de}} < 0 \iff \begin{cases} w_{\text{de}} < -\frac{1}{3}, & \rho_{\text{de}} > 0, \\ w_{\text{de}} > -\frac{1}{3}, & \rho_{\text{de}} < 0, \end{cases} \quad \mathcal{I}_{\text{de}} = p_{\text{de}}, \quad \mathcal{M}_{\text{de}} = 3p_{\text{de}} \text{ at } \rho_{\text{de}} = 0.$$

	$\mathcal{I}_{\text{de}} > 0$	$\mathcal{I}_{\text{de}} < 0$
$\rho_{\text{de}} > 0$	$w_{\text{de}} > -1$	$w_{\text{de}} < -1$
$\rho_{\text{de}} < 0$	$w_{\text{de}} < -1$	$w_{\text{de}} > -1$

The line $w_{\text{de}} = -1$ represents $\mathcal{I}_{\text{de}} = 0$ only on a nonzero-density branch, with the conventional side assignment valid only for $\rho_{\text{de}} > 0$.

Two frequent questions: (i) the Friedmann constraint bounds only the *total* source, $\rho_{\text{tot}} \geq 3K/(8\pi Ga^2)$ — no positivity condition follows for any individual sector; (ii) the opposite local assignment near a crossing ($\mathcal{I}_{\text{de}} > 0$, i.e. $w_{\text{de}} < -1$ on the $\rho_{\text{de}} < 0$ side) is impossible for *any* continuous zero — realizing it requires a sign flip *without* a zero (jump- or pole-like, hence singular).



ρ_{de} switches sign; Ω_{de} flips with it; p_{de} stays finite; only the ratio w_{de} diverges at $z_{\dagger}$ (dashed).

Separately conserved sector; smooth negative $\rightarrow$ positive crossing of finite odd order n at $z_{\dagger}$.

- $\mathcal{I}_{\text{de}}, \mathcal{M}_{\text{de}} < 0$ in a punctured neighborhood of $z_{\dagger}$; at the zero, ≤ 0 (strict for a simple zero), with $p_{\text{de}}(z_{\dagger}) = \frac{1+z_{\dagger}}{3} \rho'_{\text{de}}(z_{\dagger})$ — the slope at the zero fixes the pressure there: finite and negative.
- w_{de} develops a **kinematic and universal** pole at $z_{\dagger}$, fixed by $(n, z_{\dagger})$ alone — no amplitude, no model details.
- $w_{\text{de}} = -1$ is *never attained* near $z_{\dagger}$: the apparent quintessence $\rightarrow$ phantom transition **never occurs**; the two sides of the divide connect only through the pole.

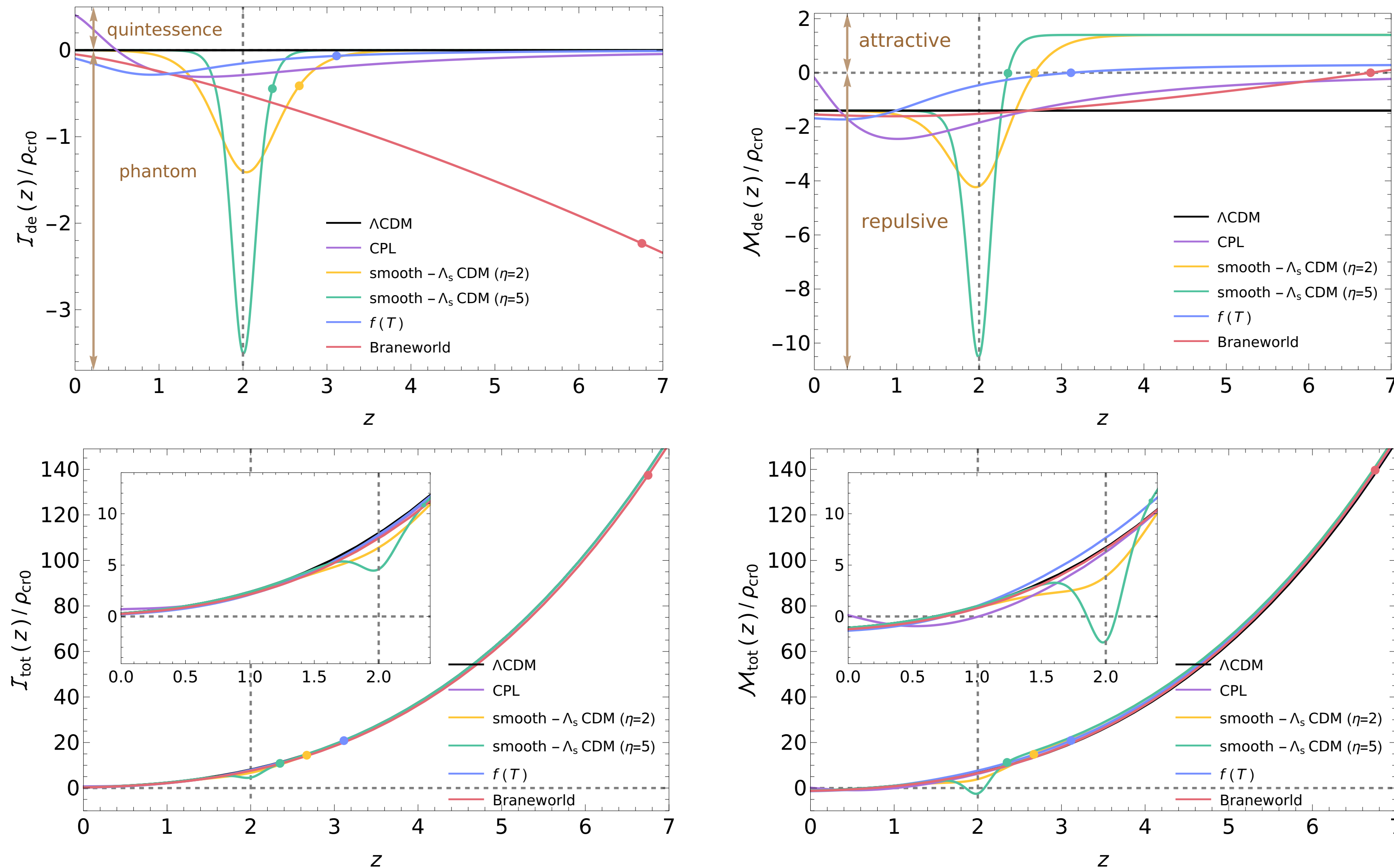
[zero $\leftrightarrow$ pole for conserved sectors: Özülker '22]

Theorem (continuity)

If $\mathcal{M}_{\text{de}} > 0$ at some $z_{\text{hi}} > z_{\dagger}$, there is *at least one* $z_{\text{rep}} \in (z_{\dagger}, z_{\text{hi}})$ with $\mathcal{M}_{\text{de}}(z_{\text{rep}}) = 0$: a finite interval where $\rho_{\text{de}} < 0$ *and* $\mathcal{M}_{\text{de}} < 0$.

$w_{\text{de}} < -1/3$ **misdiagnoses** this interval as non-repulsive.

Smooth- Λ_{s} CDM profile $\rho_{\Lambda_{\text{s}}} \propto \tanh[\eta(z_{\dagger} - z)]$: the onset is *unique*; for $z_{\dagger} = 2$: $z_{\text{rep}} = 2.67$ ($\eta=2$), 2.35 ($\eta=5$) — and 3.11 for $f(T)$, 6.75 for the brane: same pattern, widely different separations.



Top: sign-switchers keep $\mathcal{I}_{\text{de}} < 0$ (CPL crosses the NECB regularly); circles mark z_{rep} . Bottom: $\mathcal{I}_{\text{tot}} > 0$ throughout — no total NEC violation; $q < 0 \iff \mathcal{M}_{\text{tot}} < 0$, and only the $\eta = 5$ dip re-enters.

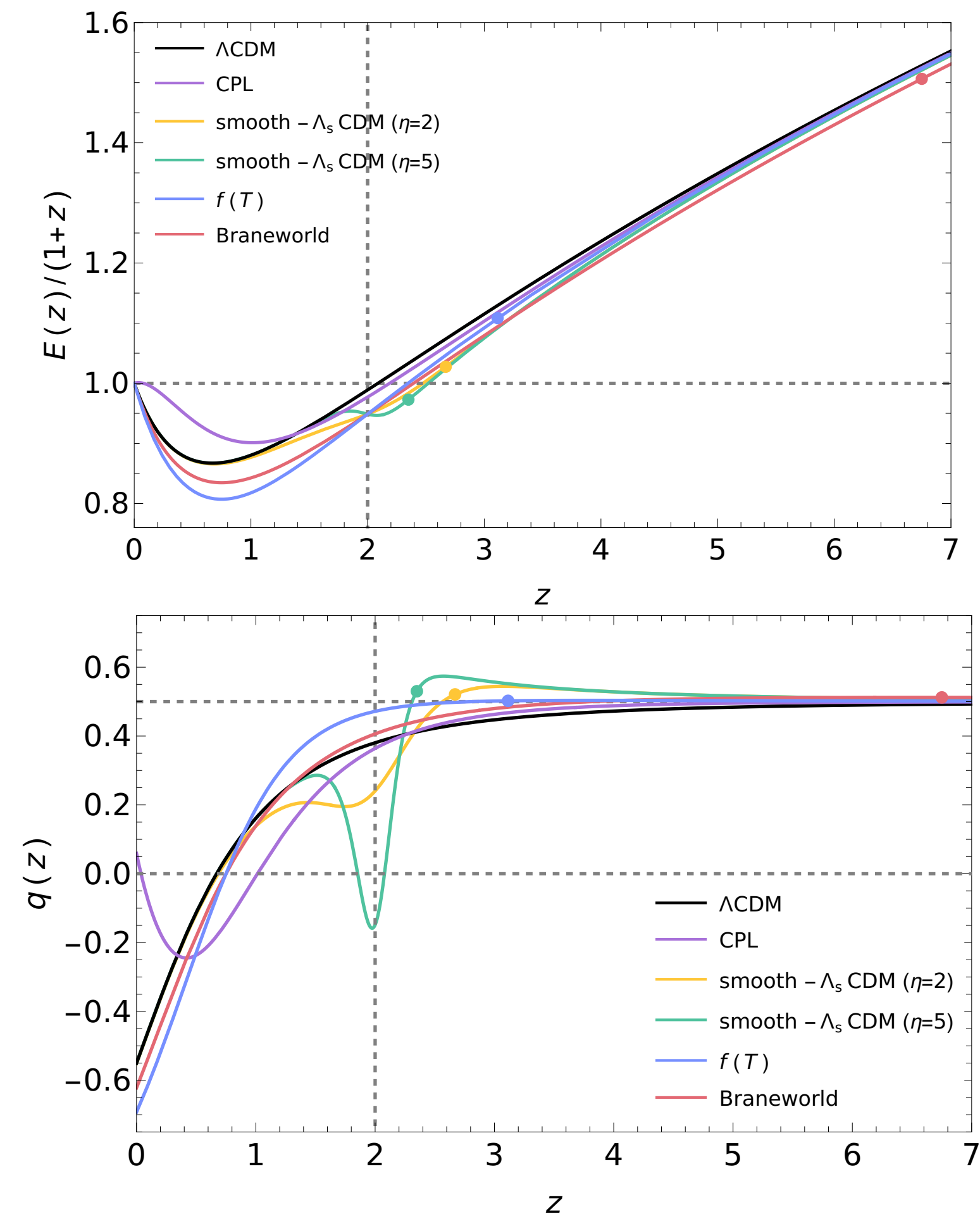
At $\rho_{\text{de}}(z_{\dagger}) = 0$ (flat, matter era):

$q(z_{\dagger}) < 0 \iff \rho'_{\text{de}}(z_{\dagger}) < -\rho_{\text{m}0}(1+z_{\dagger})^2$, and
the total NEC bounds the slope from below:

$$-3\rho_{\text{m}0}(1+z_{\dagger})^2 \leq \rho'_{\text{de}}(z_{\dagger}) < -\rho_{\text{m}0}(1+z_{\dagger})^2$$

acceleration at the crossing without total NEC violation.

$z_{\dagger} = 2$, $\Omega_{\text{m}0} = 0.3$: the dip must pass
 $\mathcal{M}_{\text{de}}(z_{\dagger}) < -8.1 \rho_{\text{cr}0}$ — $\eta=2$ gives -4.2 (fails),
 $\eta=5$ gives -10.5 (succeeds); window
 $3.9 \lesssim \eta \lesssim 11.6$.



The kinematics: transient $q < 0$ episode around $z_{\dagger}$
only for the sharper transition.

- $\mathcal{I}_{\text{de}}$ drives the **history**: $\rho'_{\text{de}} = \frac{3}{1+z} \mathcal{I}_{\text{de}}$ — a negative $\rightarrow$ positive crossing *requires* $\mathcal{I}_{\text{de}} < 0$ there; sign-switchers keep $\mathcal{I}_{\text{de}} < 0$ throughout (phantom-like).
- $\mathcal{M}_{\text{de}}$ drives **repulsion**, competing with matter in $\mathcal{M}_{\text{tot}} = \rho_{\text{m}} + \mathcal{M}_{\text{de}}$: on $\rho_{\text{de}} < 0$ the localized dip must beat $\rho_{\text{m}} \propto (1+z)^3 \Rightarrow$ at most a **transient intermediate** window; on $\rho_{\text{de}} > 0$, $\mathcal{M}_{\text{de}} < 0$ persists while matter dilutes $\Rightarrow$ the **late-time** epoch follows automatically.
- Counting (single impulse in $\mathcal{M}_{\text{de}}$; $\mathcal{M}_{\text{tot}} \in C^1$, at most two stationary points): Rolle $\Rightarrow$ exactly **one** or **three** sign-changing zeros of q ; $\eta=5$ realizes $z_{\text{rep}} > z_{\text{acc}}^{\text{int,b}} > z_{\dagger} > z_{\text{acc}}^{\text{int,e}} > z_{\text{acc}}^{\text{late}}$: $2.35 > 2.08 > 2 > 1.85 > 0.67$.

Density sign change $\neq$ sector repulsion $\neq$ cosmic acceleration $\neq$ total NEC violation.

$$\mathcal{L}_\phi = \epsilon X - V(\phi), \quad X = \dot{\phi}^2/2 \geq 0: \quad \mathcal{I}_\phi = \epsilon \dot{\phi}^2, \quad \mathcal{M}_\phi = 2(\epsilon \dot{\phi}^2 - V).$$

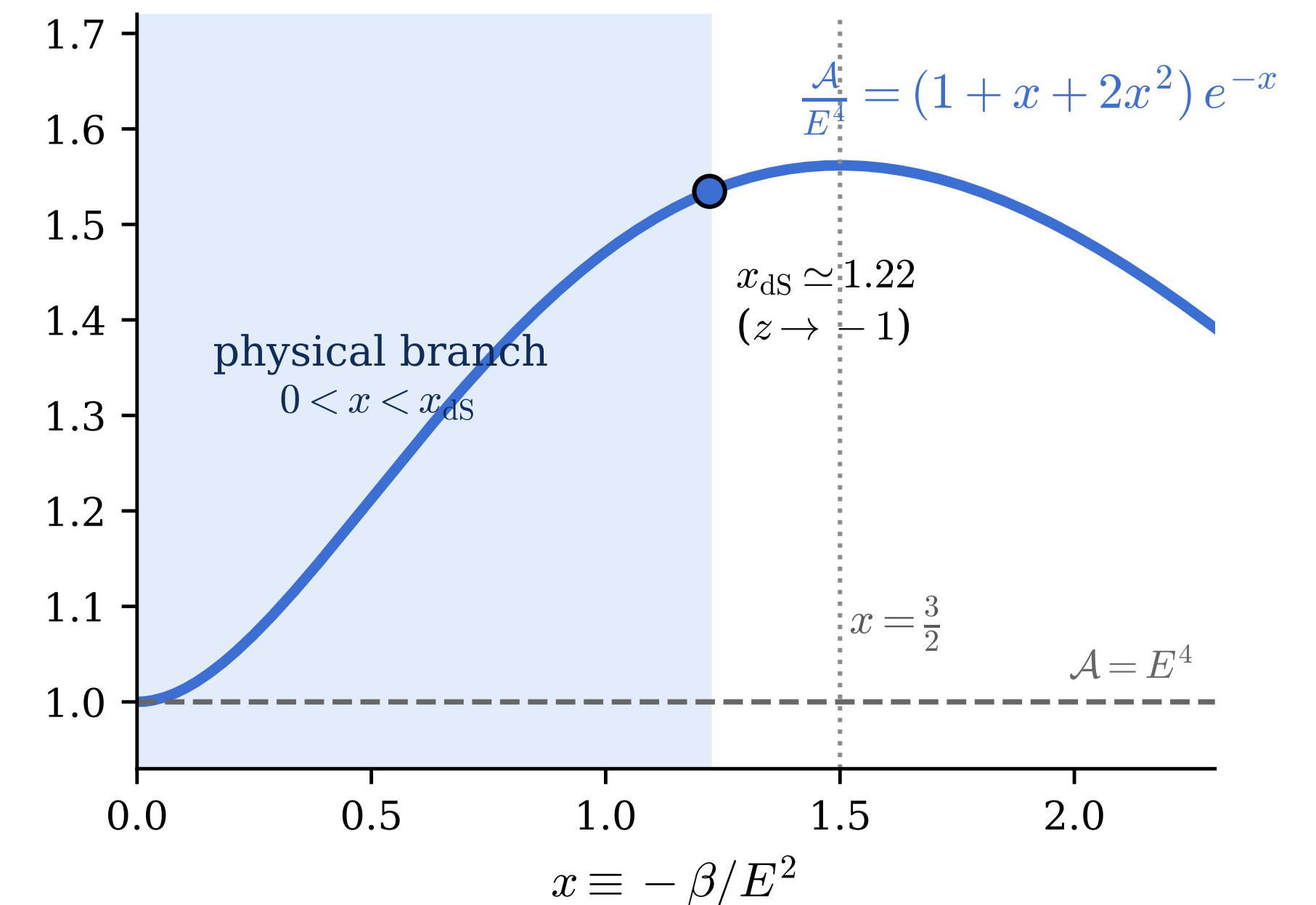
- **EoS labels mislead**: canonicity fixes $\text{sgn } \mathcal{I}_\phi = \epsilon$, *not* $\text{sgn } \rho_\phi$. A canonical field ($\epsilon = +1$) with V sufficiently negative has $\rho_\phi < 0$, $p_\phi > 0$: $w_\phi < -1$ with perfectly healthy kinetics.
- **Negative energy density is not forbidden**: no known physical principle prevents a canonical scalar from reaching $\rho_\phi < 0$.
- **The crossing is what costs**: $\dot{\rho}_\phi = -\Theta \mathcal{I}_\phi$, so climbing from $\rho_\phi < 0$ to $\rho_\phi > 0$ in an expanding universe requires $\mathcal{I}_\phi < 0$, i.e. $\epsilon = -1$: a phantom field climbing its potential. The minimal realization is a genuine ghost — kinematic $\neq$ microphysical.
- Fixed ϵ forbids an NECB crossing (touched only at $\dot{\phi} = 0$) — the single-field no-go. Stable embedding used in practice: $\Lambda_s \text{VCDM}$ (type-II minimally modified gravity; no extra propagating scalar mode).

$S = \frac{1}{2\kappa^2} \int d^4x |e| f(T) + S_m$, $f(T) = T e^{\beta T_0/T} + 2\Lambda$ ($T = 6H^2$), negative- β branch; GR-like split gives

$$\rho_{\text{tp}}(E) = \rho_{\text{cr0}} [E^2 - (E^2 - 2\beta)e^{\beta/E^2} + \Omega_{\Lambda 0}], \quad \text{sgn } \rho'_{\text{tp}} = \text{sgn} [E^4 - \mathcal{A}(E)].$$

- Whole-branch result: with $x \equiv -\beta/E^2$, $\mathcal{A}/E^4 = (1 + x + 2x^2)e^{-x}$ rises monotonically from 1 on $0 < x < x_{\text{dS}} \simeq 1.22 < \frac{3}{2} \Rightarrow E^4 < \mathcal{A} \Rightarrow \mathcal{I}_{\text{tp}} < 0$ for all $z > -1$.
- $z_{\dagger} = 2$, $\Omega_{m0} = 0.3 \Rightarrow \beta \simeq -0.977$, $\Omega_{\Lambda 0} \simeq 0.81$; high- z : $\rho_{\text{tp}} \rightarrow \rho_{\text{cr0}}(\beta + \Omega_{\Lambda 0}) < 0$, attractive ($\mathcal{M}_{\text{tp}} > 0$) $\Rightarrow z_{\text{rep}} = 3.11$; no intermediate window.
- Caveat: a recent perturbation-level analysis disfavors the *minimal* $\Lambda = 0$ branch; the Λ -extended branch here is not automatically subject — still, kinematic regularity $\neq$ viability.

[Awad+ '18; Hashim+ '21; Akarsu+ '25 · caveat: Hashim+ '26]



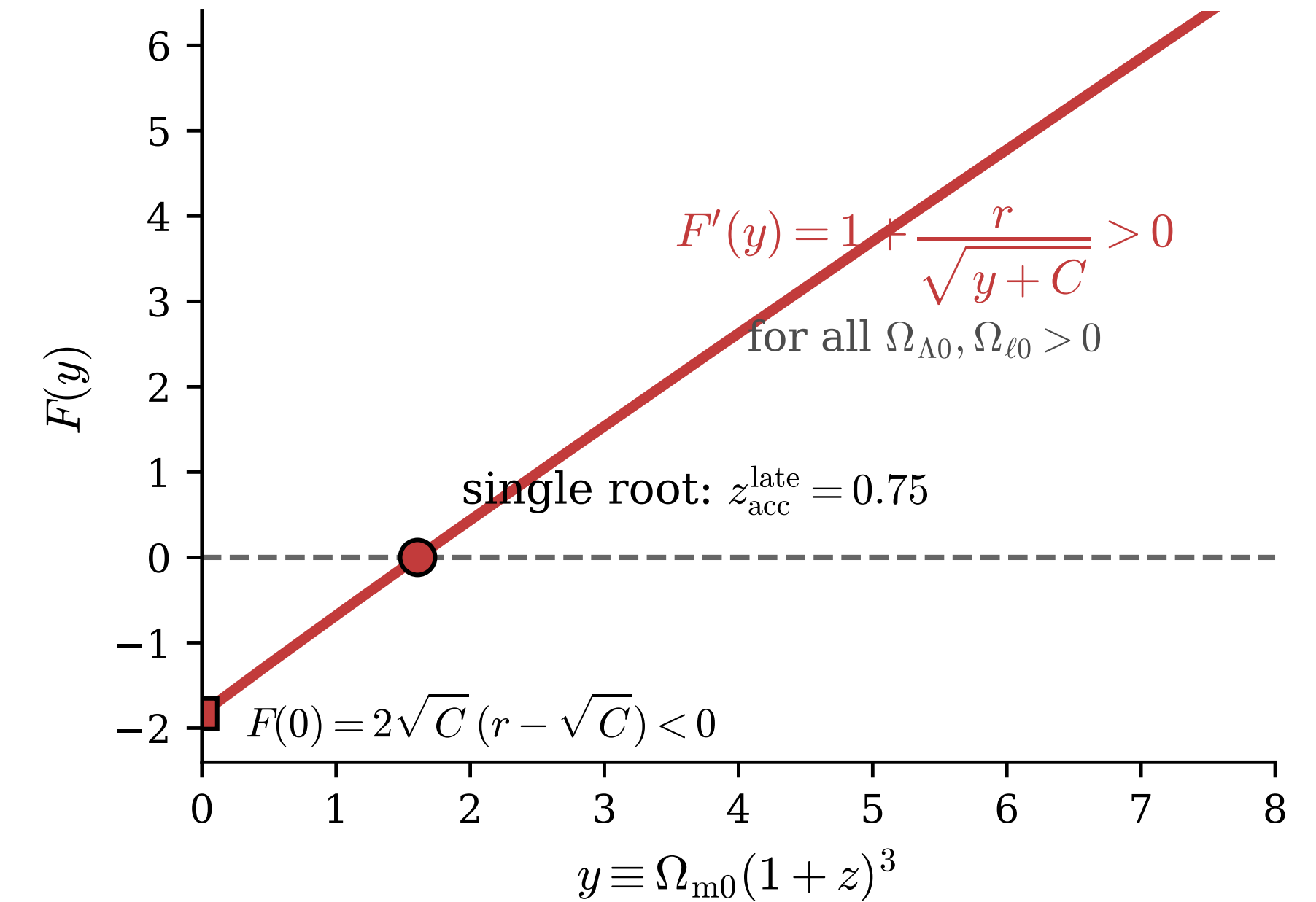
The whole-branch proof in one curve: $\mathcal{A}/E^4$ rises from unity and the physical branch never leaves $(0, \frac{3}{2})$.

Normal branch of the induced-gravity (DGP) braneworld, flat bulk

$$[S = M^3 \int_{\text{bulk}} \sqrt{-g} R + m^2 \int_{\text{brane}} \sqrt{-h} R - 2\sigma \int_{\text{brane}} \sqrt{-h} + S_m; \quad \ell = 2m^2/M^3, \Omega_{\ell 0} = 1/\ell^2 H_0^2]:$$

$$E(z) = \sqrt{\Sigma(z)} - \sqrt{\Omega_{\ell 0}}, \quad \Sigma = \Omega_{m0}(1+z)^3 + \Omega_{\Lambda 0} + \Omega_{\ell 0}, \quad E(0) = 1 \Rightarrow \Omega_{\Lambda 0} = 1 - \Omega_{m0} + 2\sqrt{\Omega_{\ell 0}}.$$

- The sign switch is **automatic** — for every parameter choice: $\rho_{\text{bw}}(0) = (1 - \Omega_{m0})\rho_{\text{cr0}} > 0$ while $\rho_{\text{bw}} \simeq -2\rho_{\text{cr0}}\sqrt{\Omega_{\ell 0}\Omega_{m0}}(1+z)^{3/2} < 0$ at high z .
- z_{rep} from an exact quadratic: $z_{\text{rep}} = 6.75$ for $\Omega_{\ell 0} = 0.036$, $\Omega_{\Lambda 0} = 1.08$ ($z_{\dagger} = 2$).
- $q = 0$ crossing **unique, analytically**, for all parameters; acceleration at the switch would need $6\Omega_{\ell 0}/(\Omega_{\Lambda 0} + 2\Omega_{\ell 0}) > 1$, i.e. $\Omega_{\ell 0} \gtrsim 0.54$ vs the adopted $\Omega_{\ell 0} = 0.036$ (LHS $\simeq 0.19$): repulsive while $\rho_{\text{de}} < 0$, yet uniqueness forbids a separate acceleration episode for every parameter choice.



Uniqueness in one curve: F starts negative and increases for every parameter choice.

Model	$z_{\dagger}$	z_{rep}	NEC character	Accelerating $q < 0$
Λ CDM	—	—	$\mathcal{I}_{\Lambda} \equiv 0$	$-1 < z < 0.67$
CPL (DESI DR2)	—	—	regular NECB crossing at $z \simeq 0.50$	$0.04 < z < 1.01$
Smooth- Λ_{s} CDM ($\eta=2$)	2	2.67	$\mathcal{I}_{\Lambda_{\text{s}}} < 0$ for all $z > -1$	$-1 < z < 0.68$
Smooth- Λ_{s} CDM ($\eta=5$)	2	2.35	$\mathcal{I}_{\Lambda_{\text{s}}} < 0$ for all $z > -1$	$-1 < z < 0.67$; $1.85 < z < 2.08$
Exponential infrared $f(T)$	2	3.11	$\mathcal{I}_{\text{tp}} < 0$ for all $z > -1$	$-1 < z < 0.75$
Minimal phantom brane	2	6.75	$\mathcal{I}_{\text{bw}} < 0$ for all $z > -1$	$-1 < z < 0.75$

Same qualitative pattern — density zero, phantom-like NEC character, $z_{\text{rep}} > z_{\dagger}$ — with widely different separations; only the sufficiently sharp $\eta = 5$ profile opens an intermediate acceleration window.

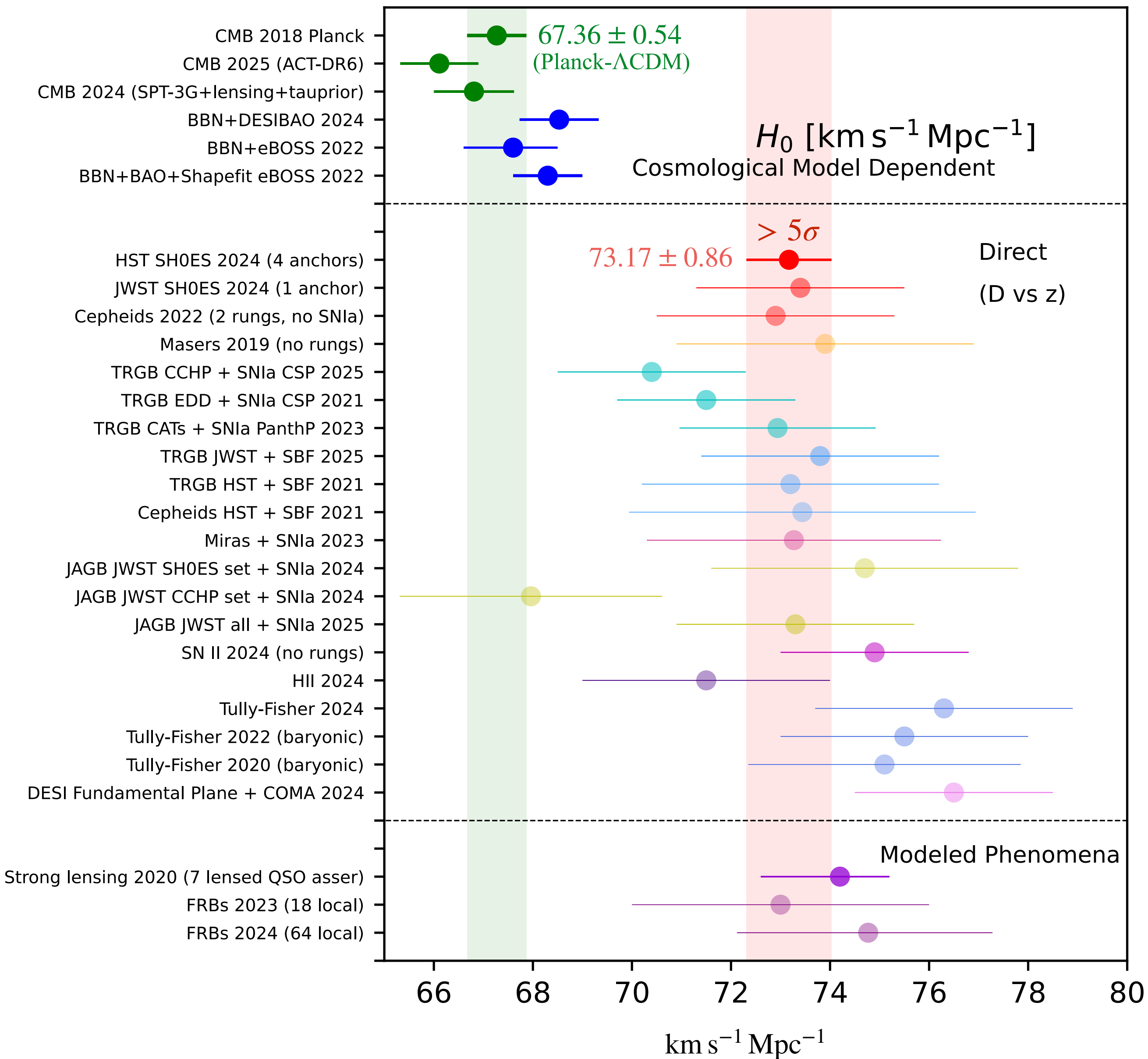
- Kinematic, from $H(z)$ alone (flat GR): $\mathcal{M}_{\text{tot}} = 2q \rho_{\text{cr}}$, $\mathcal{I}_{\text{tot}} = \frac{(1+z)HH'}{4\pi G}$ — no sector split needed; report these first.
- Sector-level quantities follow by subtracting matter+radiation: quote $z_{\dagger}$, z_{rep} **jointly with the adopted $\Omega_{\text{m}0}$** ($|\Delta z_{\dagger}| \simeq 0.19$ ($\eta=2$), 0.08 ($\eta=5$) per $\Delta\Omega_{\text{m}0} = 0.01$).
- Linear theory is **ratio-safe**: evolve unnormalized $(\delta\rho_i, \delta p_i, Q_i = \mathcal{I}_i\theta_i, \Pi_i)$ — no division by ρ_i or $\mathcal{I}_i$ anywhere.
- Stability and closure belong to the completion: stable embedding in practice is $\Lambda_{\text{s}}\text{VCDM}$ (no new propagating scalar mode); departure concentrated near/above the horizon $\Rightarrow$ late-time ISW $\times$ LSS as the natural discriminator.

- Density sign change, sector repulsion, cosmic acceleration, total NEC violation: **four distinct notions** — the Raychaudhuri pair $(\mathcal{I}_{\text{de}}, \mathcal{M}_{\text{de}})$ keeps them apart; w_{de} cannot.
- Exact results at a smooth crossing: $\mathcal{I}_{\text{de}}, \mathcal{M}_{\text{de}} \leq 0$; universal pole residue $n(1 + z_{\dagger})/3$; $z_{\text{rep}} > z_{\dagger}$; acceleration-with-NEC window; one-or-three $q = 0$ transitions.
- In practice: track $(\rho_{\text{de}}, p_{\text{de}}, \mathcal{I}_{\text{de}}, \mathcal{M}_{\text{de}})$; use parametrizations and **priors that admit both density branches** (at the density, pressure, or $\mathcal{I}$ level — $\mathcal{M}$ -level as a natural next step); read w_{de} poles as kinematic signatures; take the **NECB, not the PDL**, as the regular boundary; quote $z_{\dagger}, z_{\text{rep}}$ with $\Omega_{\text{m}0}$.

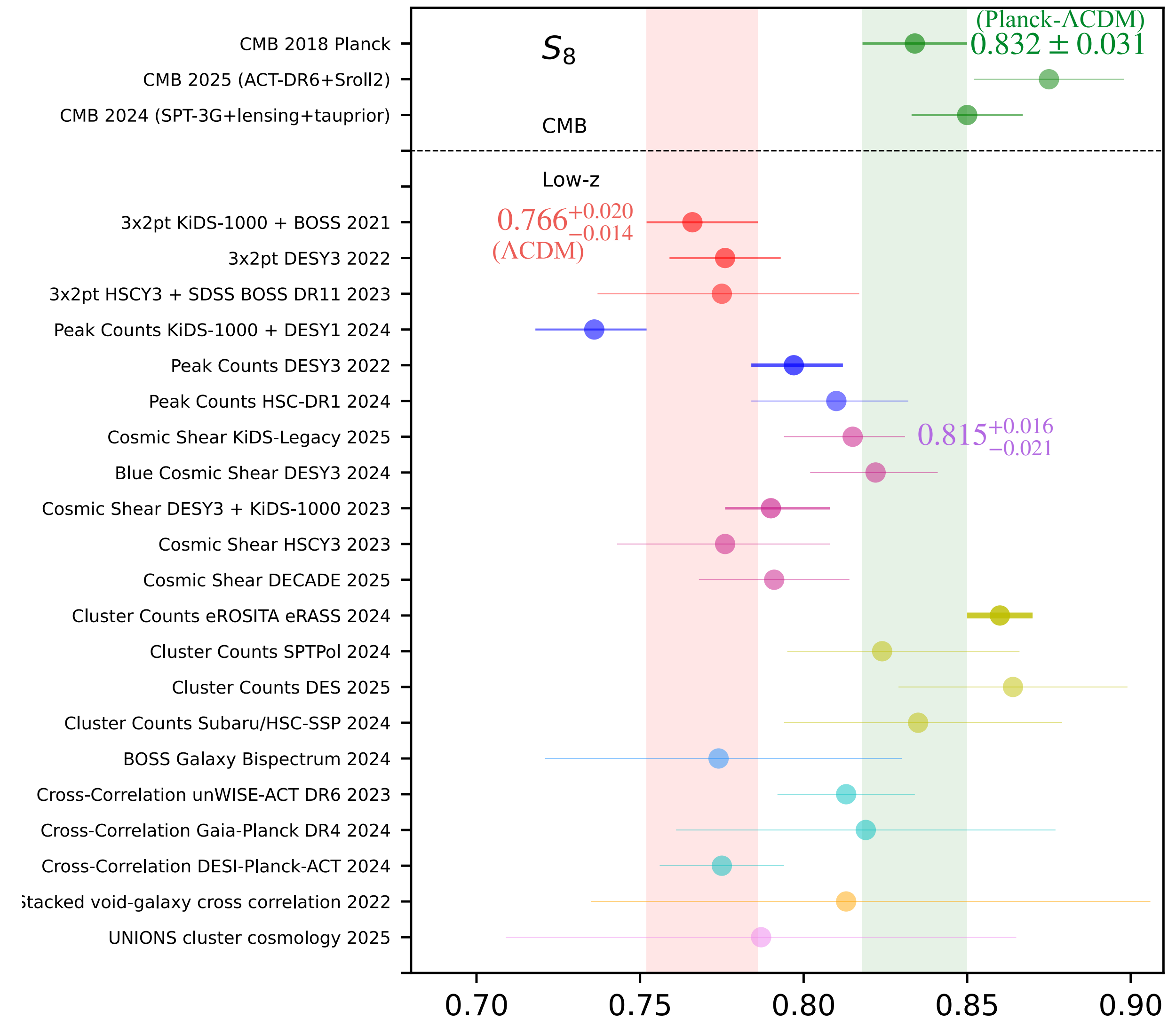
**The field equations already single out these combinations;
we propose only that inference follow them.**

[arXiv:2607.18008]

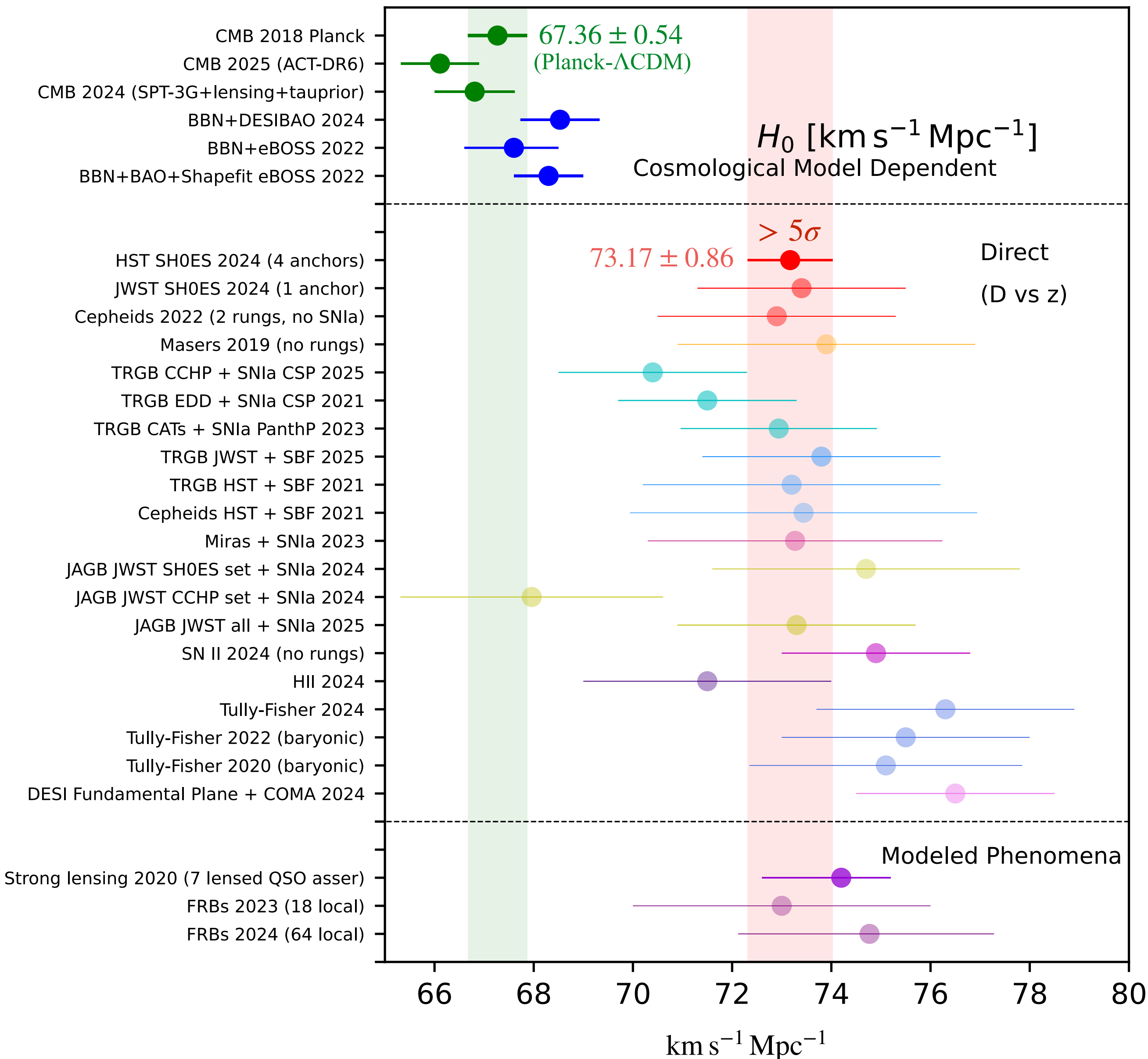
H_0 Tension



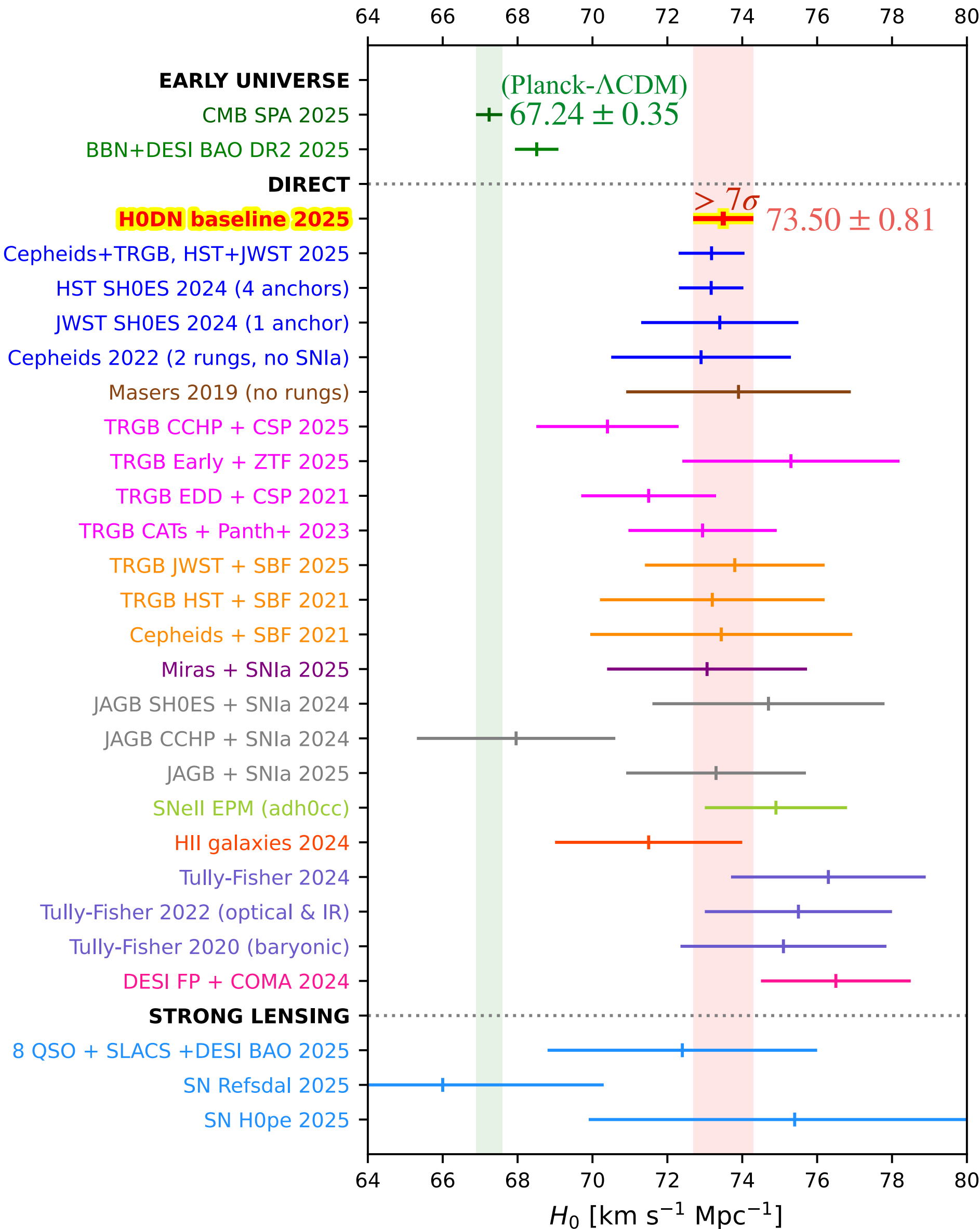
S_8 Tension



H_0 Tension

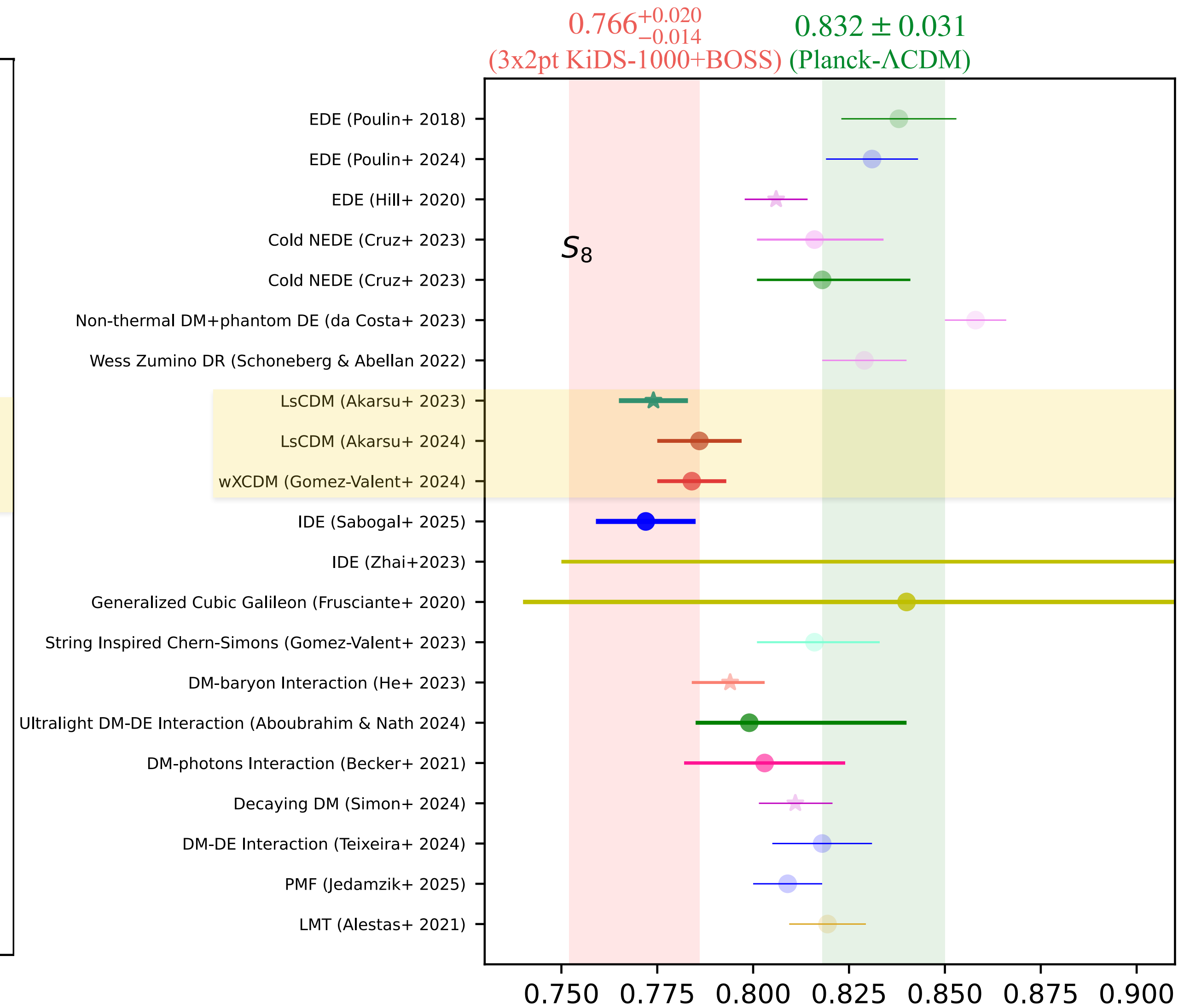
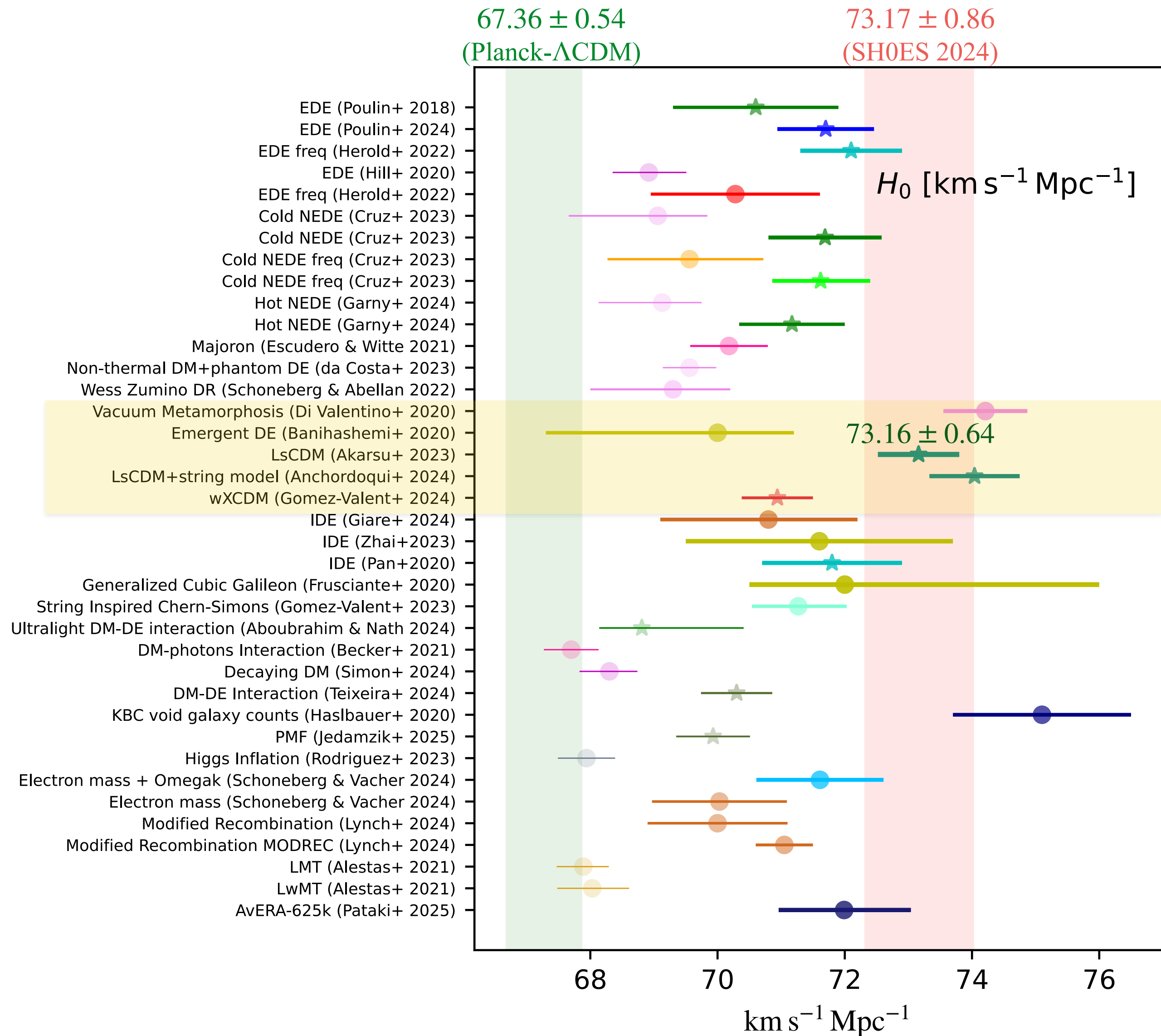


H_0 Tension (most recent)



Models proposed to solve the H_0 tension

Models proposed to solve the S_8 tension



$$\nabla_\mu G^{\mu\nu} = 0$$

$$\xrightarrow[G_{\mu\nu} = -T_{\mu\nu}]{\text{GR}}$$

$$\nabla_\mu T^{\mu\nu} = 0$$

$$\dot{\rho} + \Theta \varrho = 0$$

Local energy conservation
(Continuity equation)

$$D^\mu p + \varrho \dot{u}^\mu = 0$$

Local momentum conservation
(Euler equation/Newton's 2nd law of motion)

$$\varrho = \rho + p$$

(Inertial mass density)

Cosmological constant
or the usual vacuum energy of QFT
$$\varrho_\Lambda = 0$$

Simple-gDE:
$$\varrho = \text{constant}$$

Acquaviva, Akarsu, Katirci & Vazquez
PRD (2021), arXiv:2104.02623

gDE: Graduated dark energy
$$\varrho = \gamma \rho_0 \left(\frac{\rho}{\rho_0} \right)^\lambda$$

The simplest dynamical deviation from zero inertial mass density of the usual vacuum energy density. Inspired by *Graduated inflationary universes*, Barrow, PLB (1990).

$$\rho = \rho_0 [1 + 3\gamma(\lambda - 1) \ln a]^{\frac{1}{1-\lambda}}$$

$$\downarrow \text{EoS parameter}$$

$$w = -1 + \frac{\gamma}{1 + 3\gamma(\lambda - 1) \ln a}$$

$$\xrightarrow[m \text{ and } n \text{ are odd integers}]{\frac{1}{1-\lambda} = \frac{m}{n}}$$

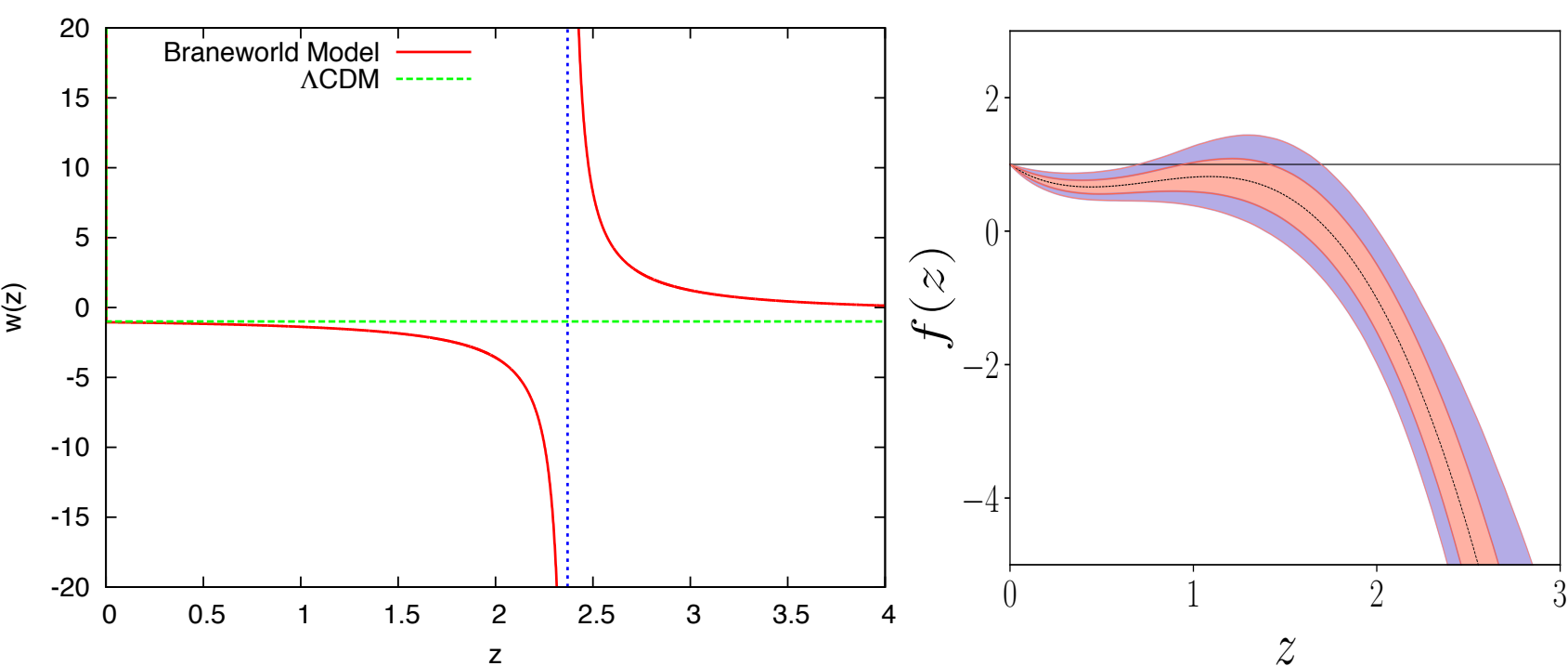
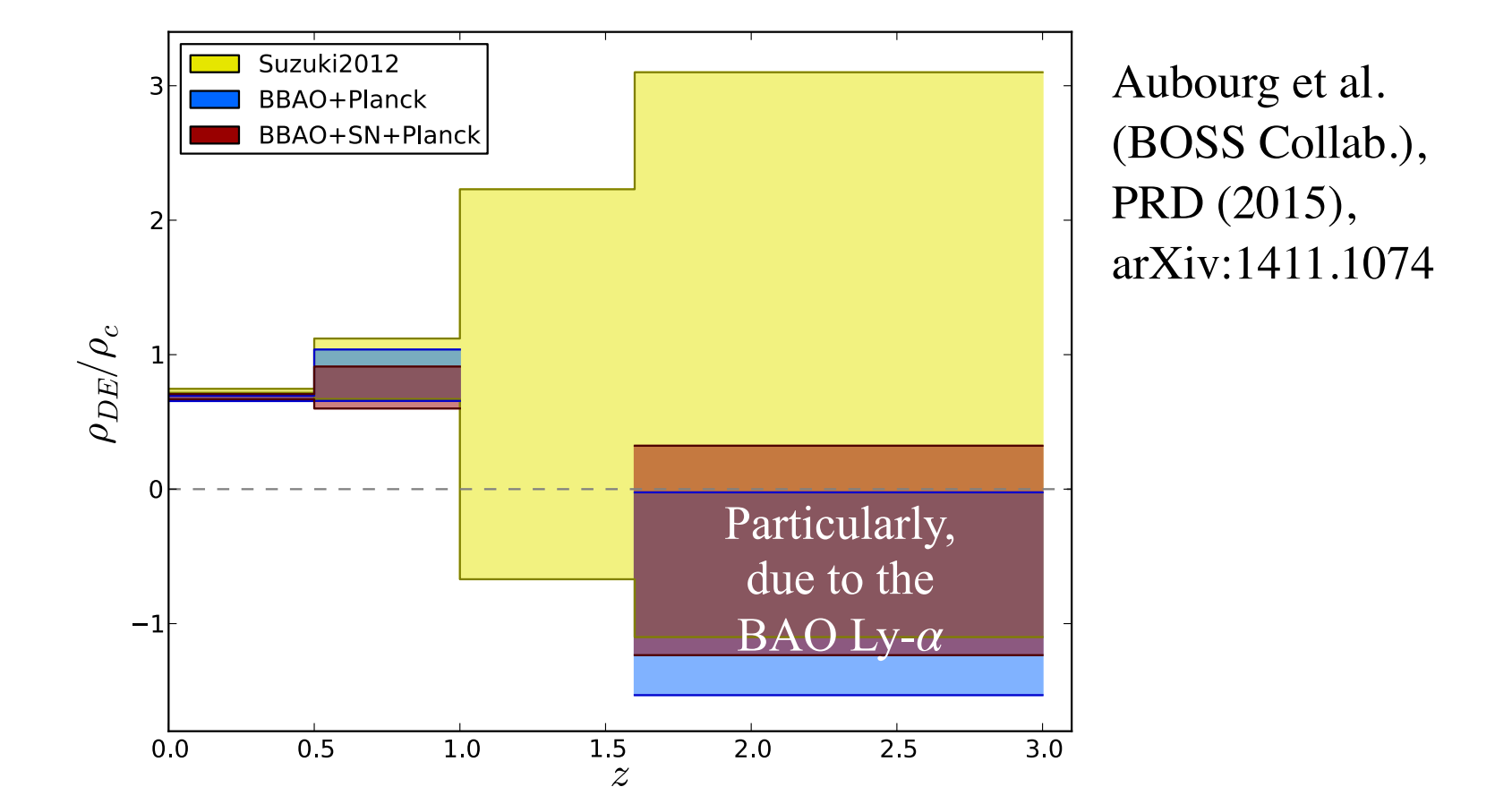
$$\rho = \rho_0 \operatorname{sgn}[1 - \Psi \ln a] |1 - \Psi \ln a|^{\frac{1}{1-\lambda}}$$

Otherwise, one encounters computer-related issues!

$$w = -1 + \gamma \text{ for } a = 1$$

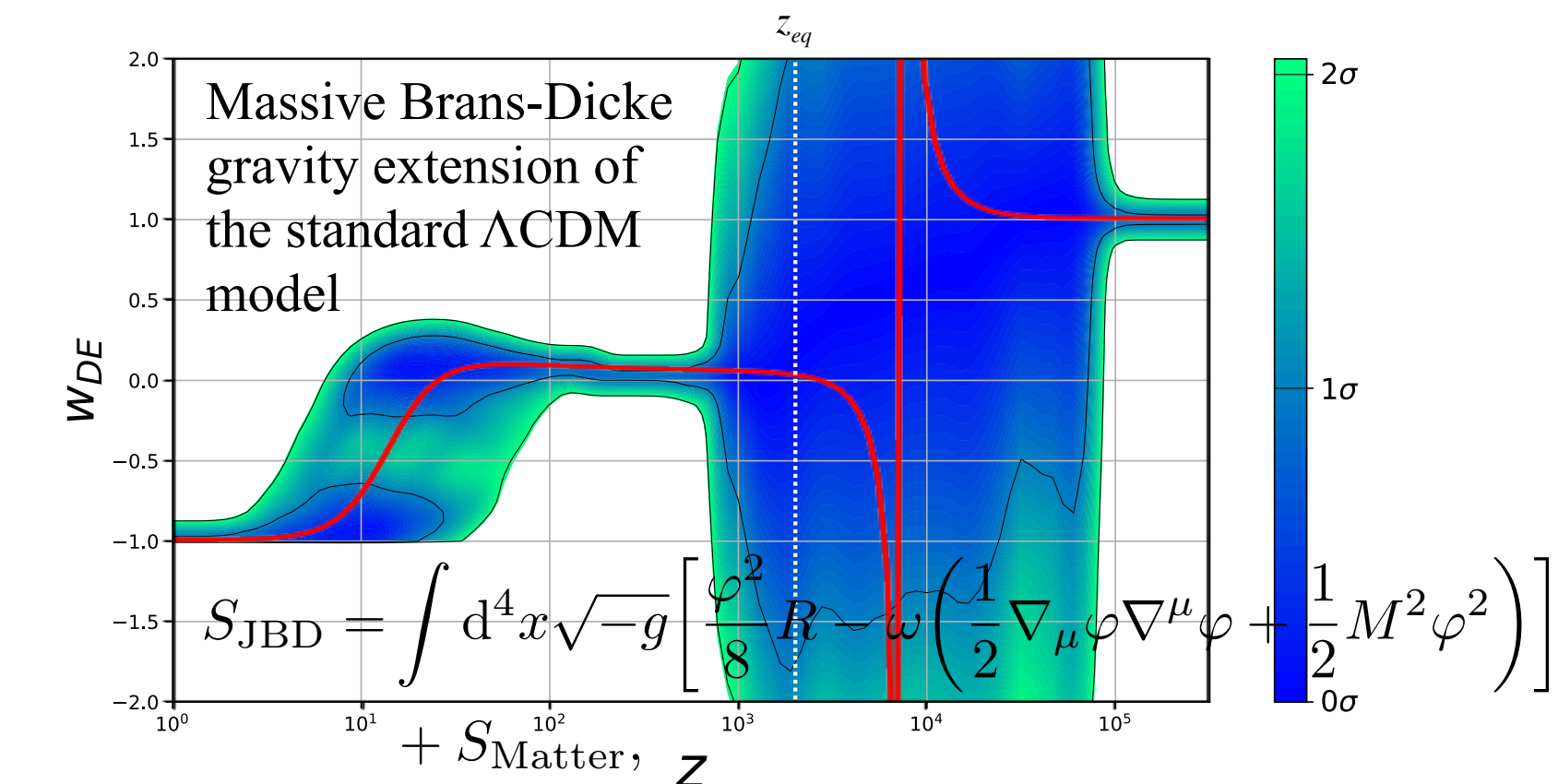
$$\lambda < 1 \text{ and } \gamma < 0 \longrightarrow \Psi \equiv -3\gamma(\lambda - 1) < 0$$

Sign changing energy density requires an EoS exhibiting a singularity/pole.
(Özülker, PRD 2022, arXiv:2203.04167)
Under these conditions, energy density takes negative values in the past, and the EoS exhibits a singularity/pole during its sign change!



Sahni, Shafieloo & Starobinsky, APJ Let (2014), arXiv:1406.2209

Dutta, Roy, Sen & Sheikh-Jabbari, GRG (2020), arXiv:1808.06623



Akarsu, Katırcı, Özdemir & Vazquez, EPJC (2020) arXiv:1903.06679

gDE-CDM, replaces the Λ of Λ CDM with the gDE

$$\frac{H^2}{H_0^2} = \Omega_{\text{r},0} a^{-4} + \Omega_{\text{m},0} a^{-3} + \Omega_{\text{DE},0} \text{sgn}[1 - \Psi \ln a] |1 - \Psi \ln a|^{\frac{1}{1-\lambda}}$$

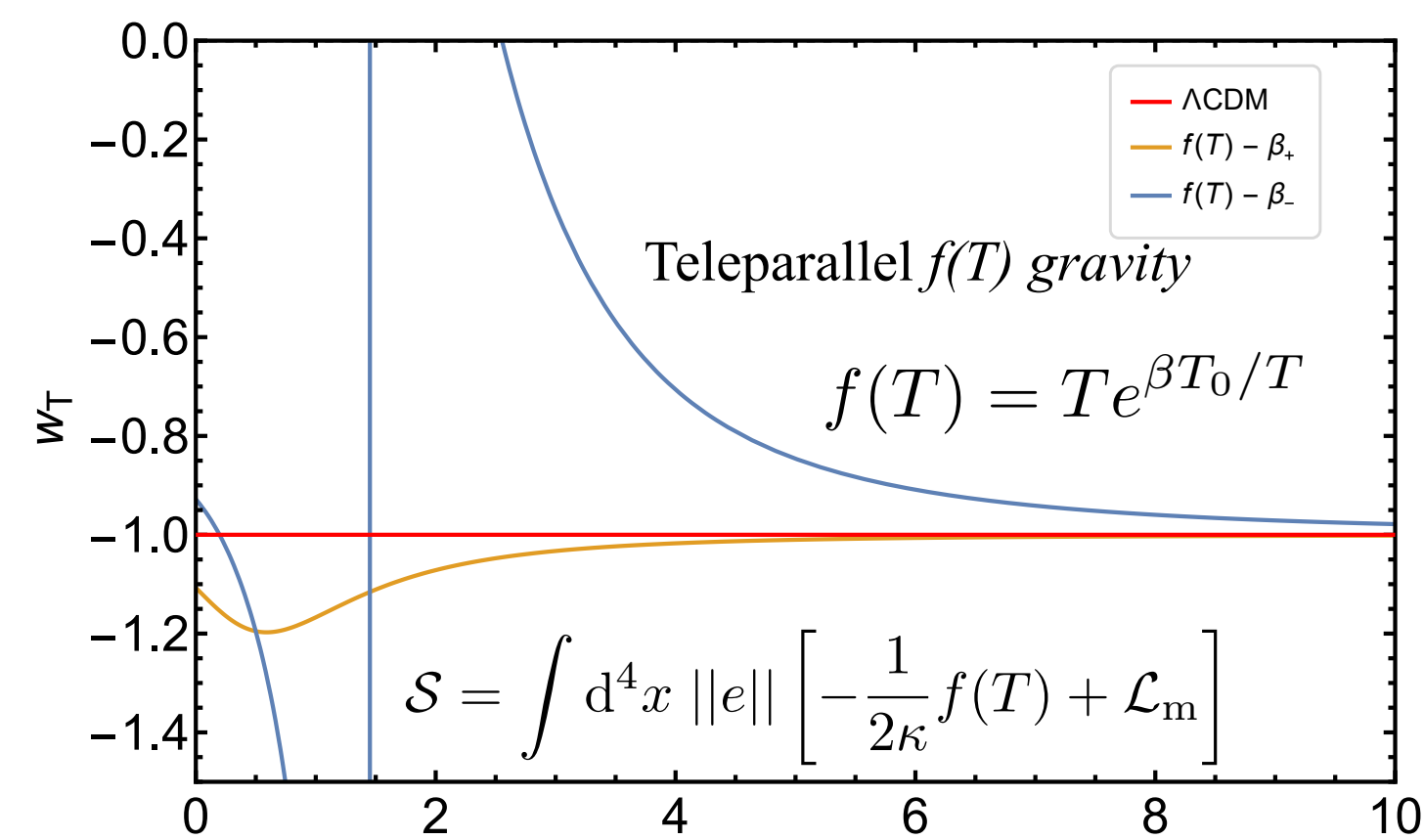
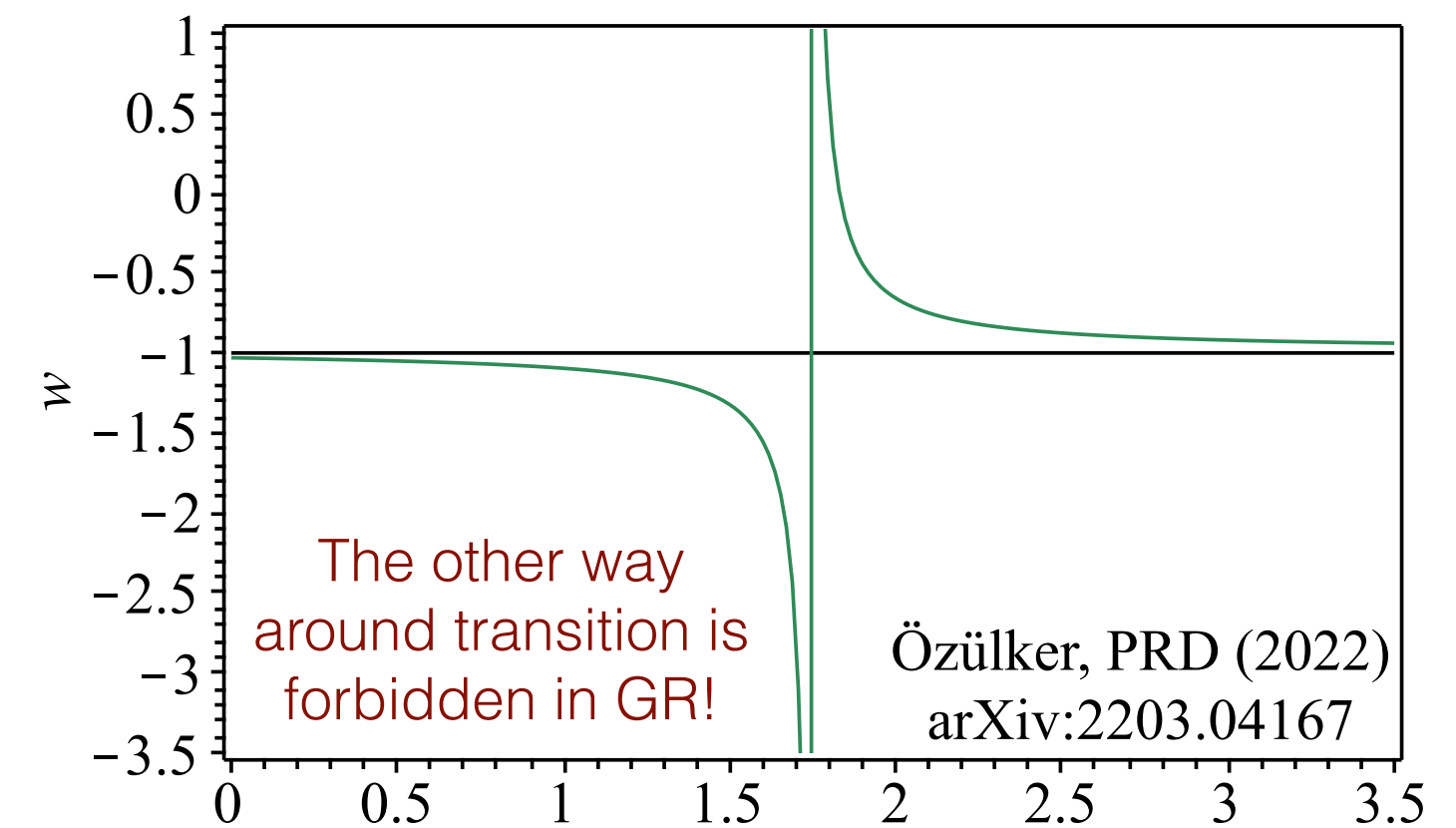
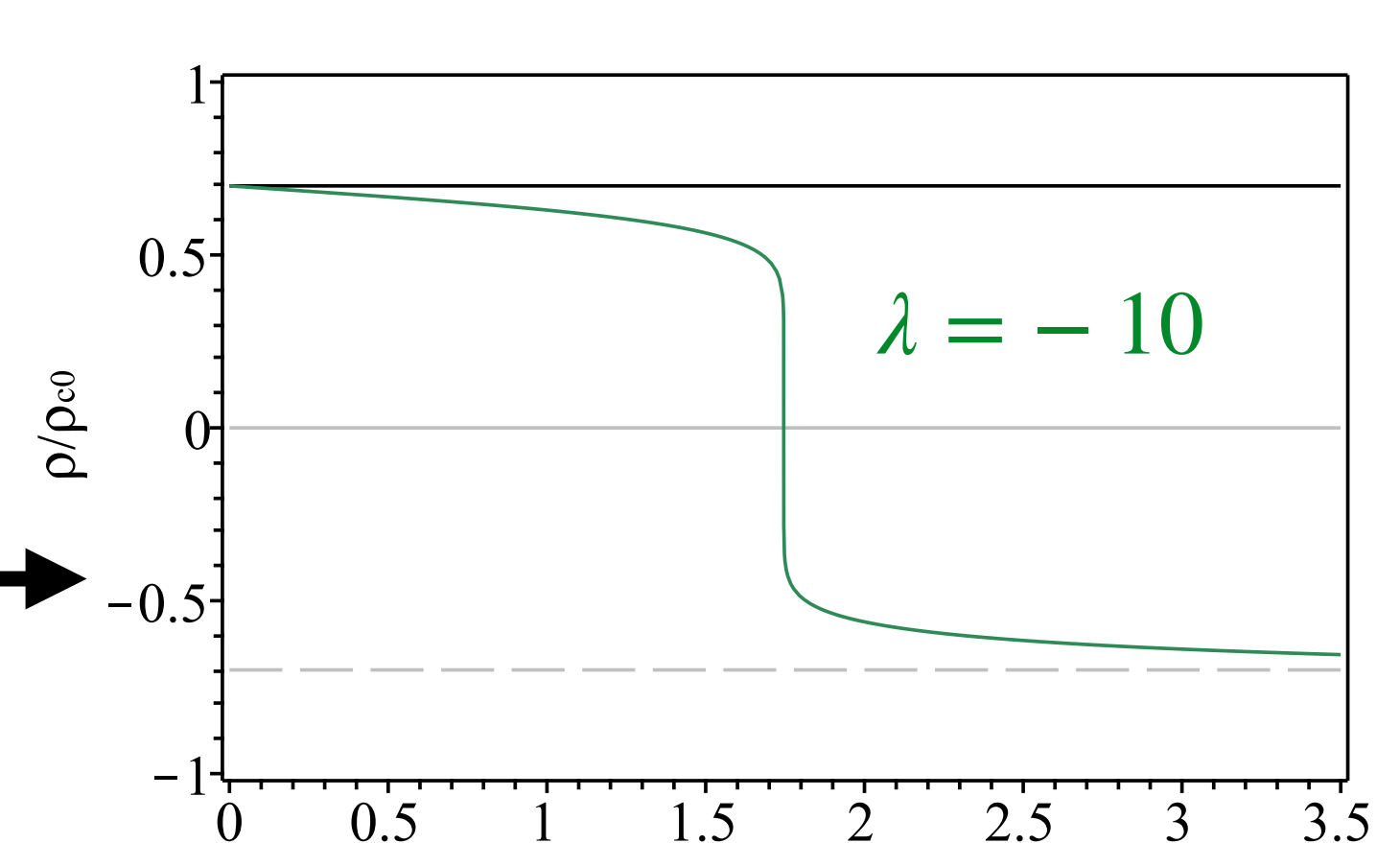
$$\frac{\rho_{\text{DE}}}{\rho_{\text{c},0}} = \Omega_{\text{DE},0} \text{sgn}[1 - \Psi \ln a] |1 - \Psi \ln a|^{\frac{1}{1-\lambda}}$$

$$\rho_{\text{gDE}} = \rho_{\text{gDE}0} \text{sgn}[1 + \Psi \ln(1 + z)] |1 + \Psi \ln(1 + z)|^{\frac{1}{1-\lambda}}$$

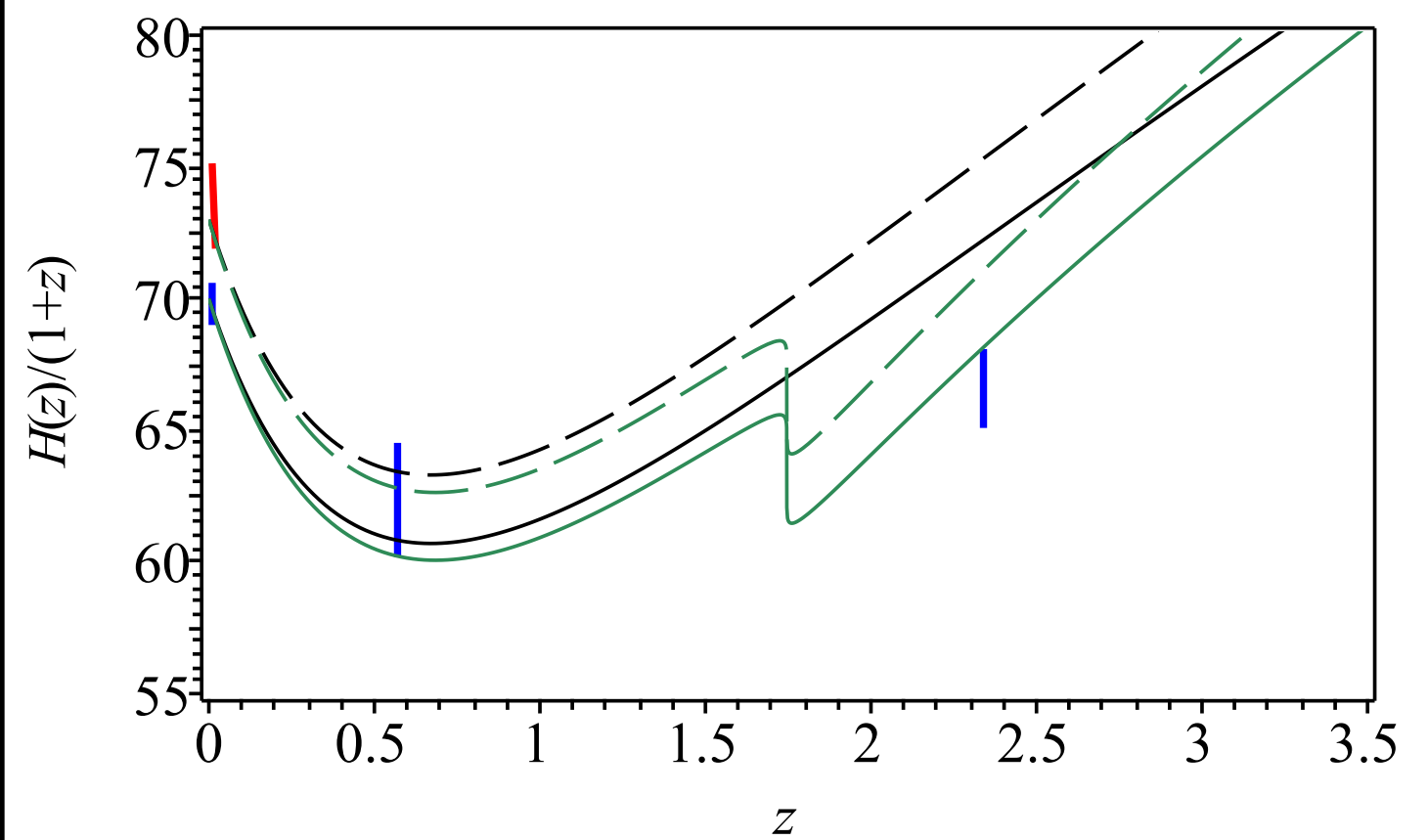
gDE density changes sign at $z_{\dagger} = e^{-\Psi^{-1}} - 1$

Λ_s CDM?

$$\frac{\rho_{\text{DE}}}{\rho_{\text{c},0}} \rightarrow \Omega_{\text{DE},0} \text{sgn}[1 - \Psi \ln a] \quad \text{as} \quad \lambda \rightarrow -\infty$$

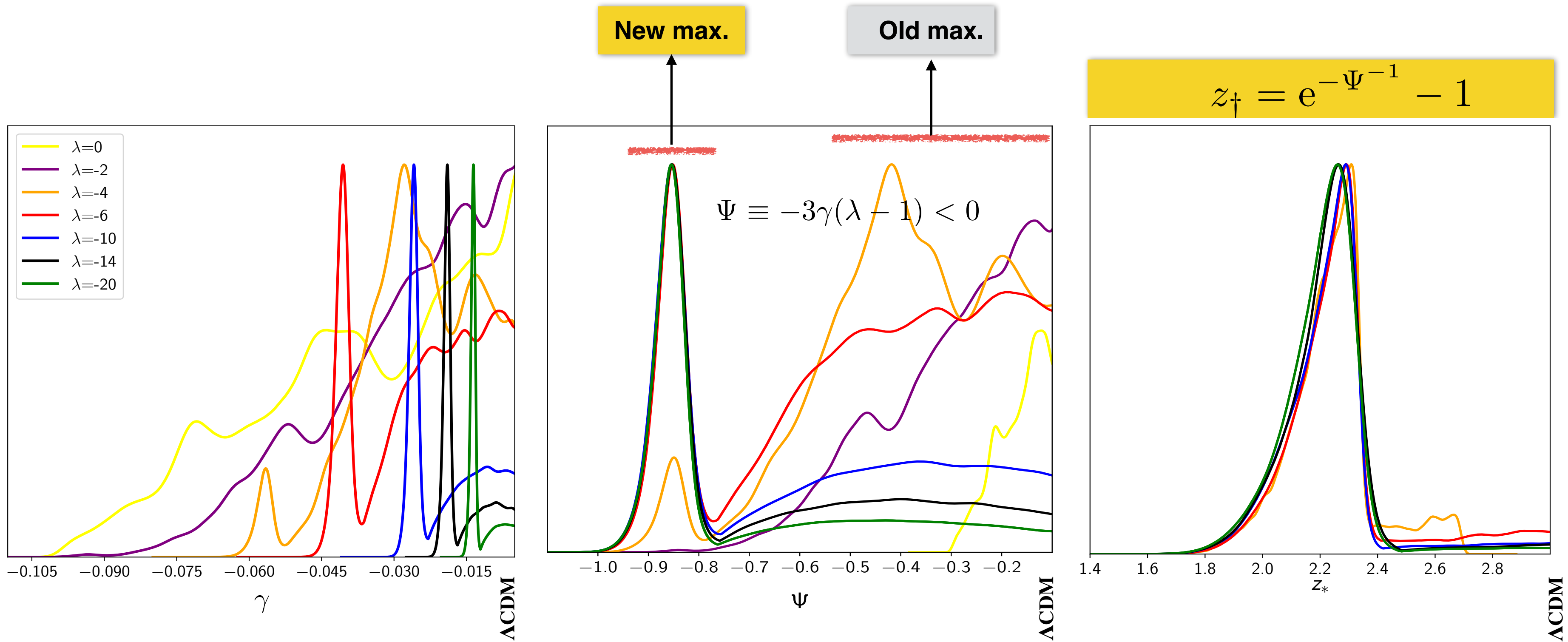
$$\Lambda \rightarrow \Lambda_s \equiv \Lambda_{\text{s}0} \text{sgn}[z_{\dagger} - z]$$


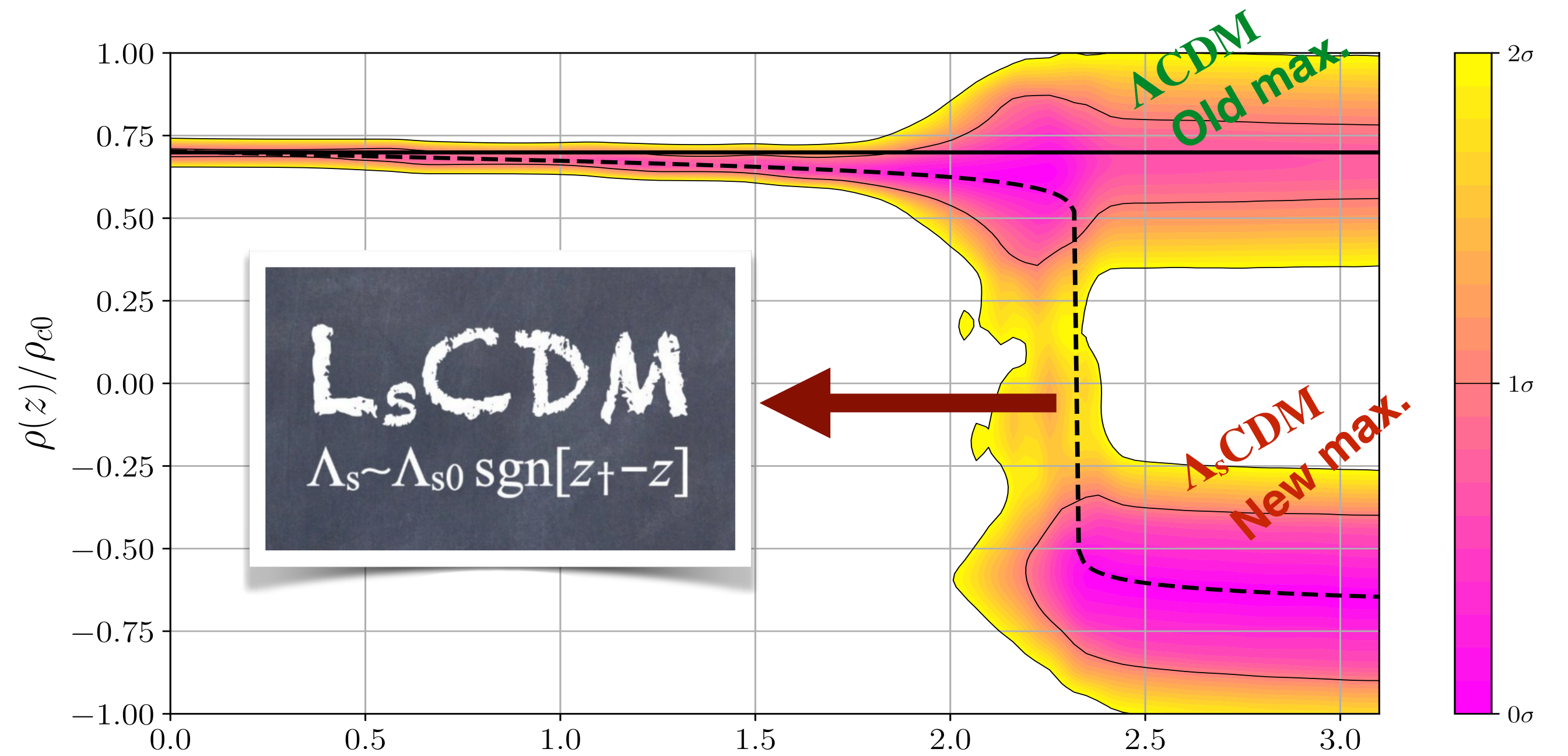
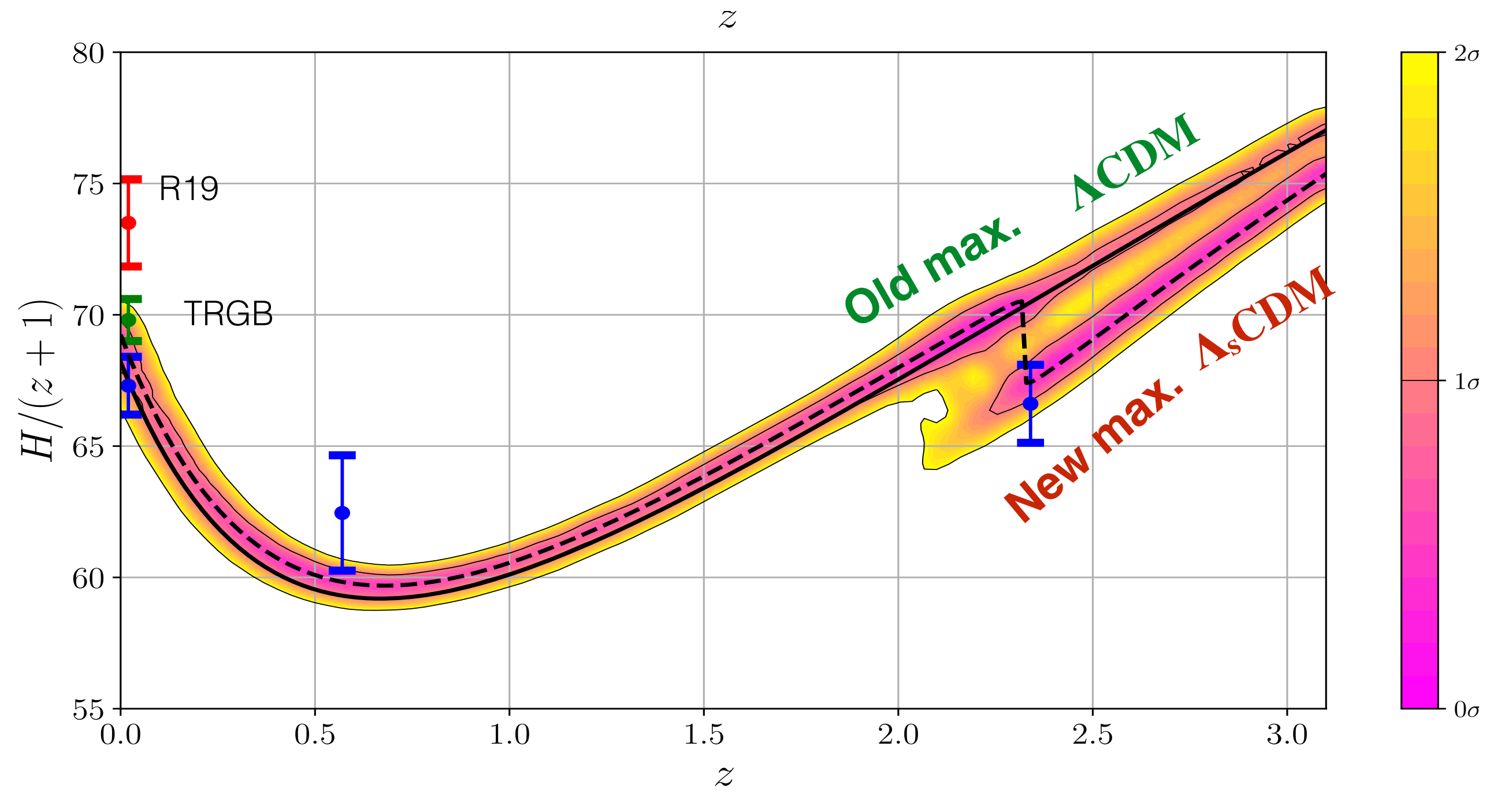
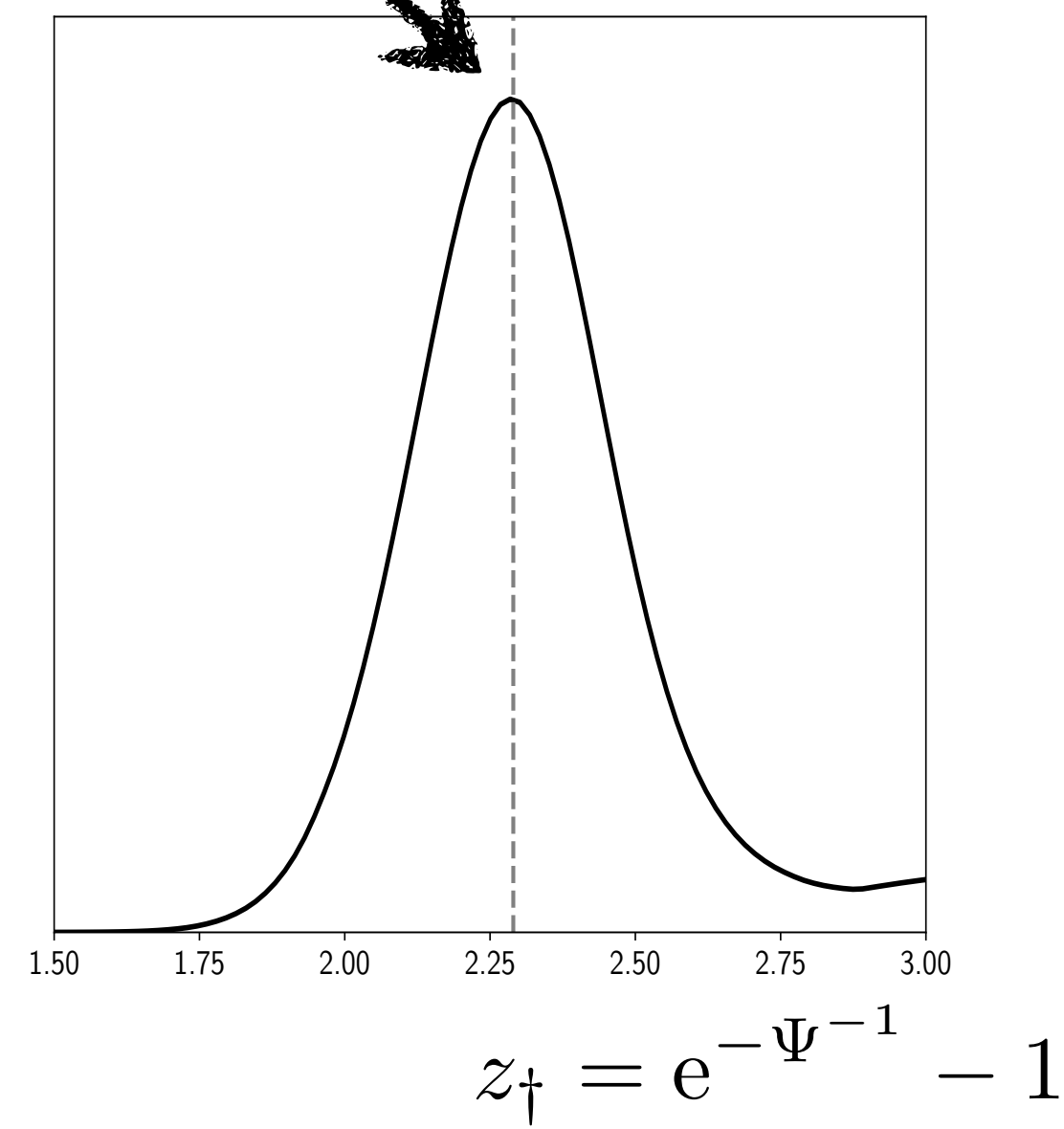
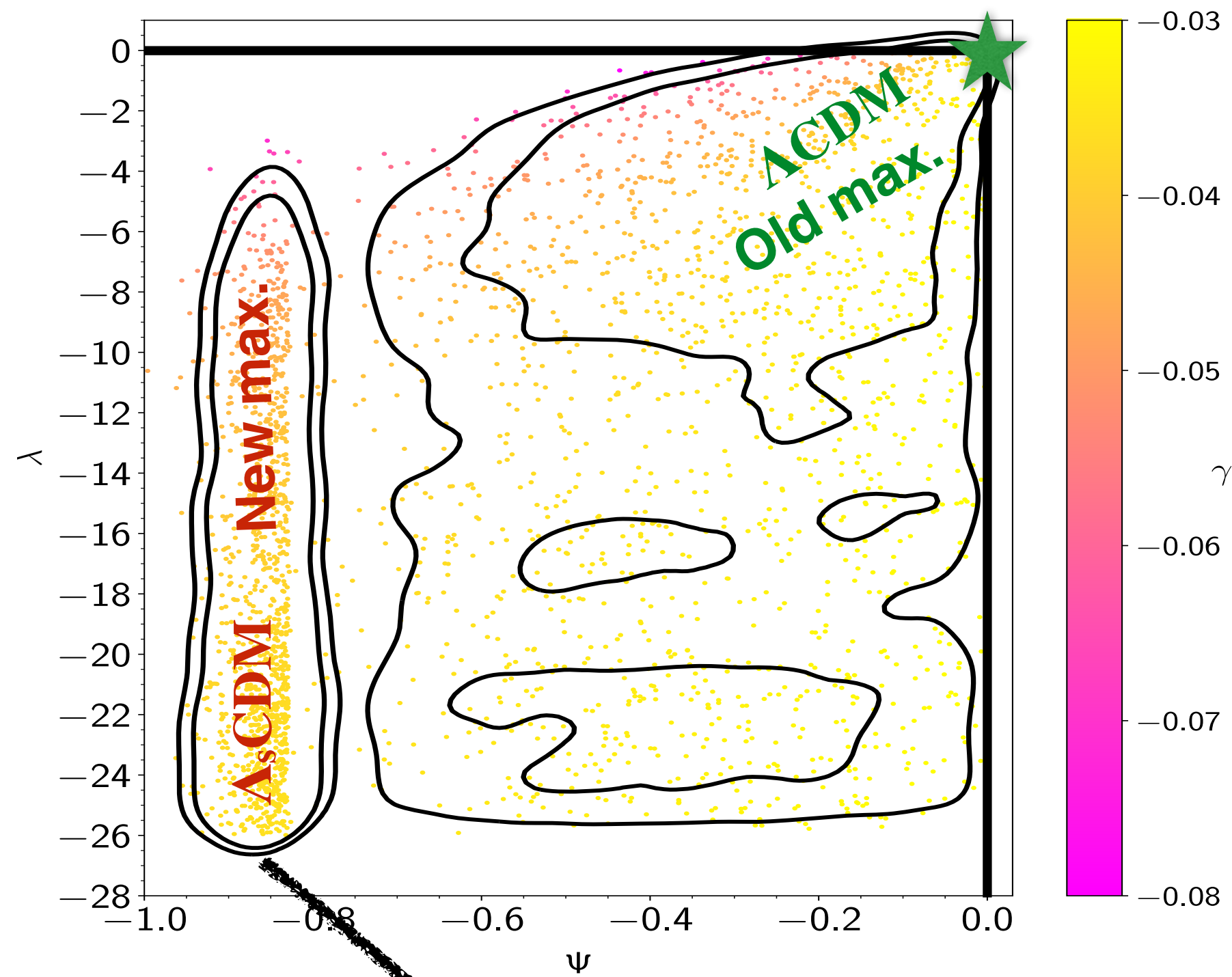
Akarsu, Bulduk, De Felice, Katırcı & Uzun, PRD (2025) arXiv:2410.23068



Constraints on gDE-CDM from PLK+BAO+SN (JLA)+ H (Cosmic Chronometers)

	λ	$\Omega_{\text{m},0}$	h_0	$\gamma = w_0 + 1$	Ψ	z_*	$t_0[\text{Gyr}]$	$-2\Delta \ln \mathcal{L}_{\text{max}}$
	ΛCDM	0.302(6)	0.682(5)	0	0	—	13.806(22)	0.0
Simple-gDE	0	0.297(7)	0.689(7)	> -0.08	> -0.25	—	13.796(24)	0.02
	−2	0.297(7)	0.688(7)	> -0.06	> -0.61	—	13.795(25)	0.02
	−4	0.289(6), 0.298(7)	0.700(9), 0.686(7)	$-0.057(2), > -0.048$	$-0.86(3), > -0.73$	2.31(12), —	13.714(25), 13.791(26)	1.0, 0.02
	−6	0.292(6), 0.299(6)	0.699(9), 0.685(7)	$-0.039(1), > -0.037$	$-0.86(3), > -0.77$	2.31(12), —	13.715(25), 13.792(27)	2.0, 0.01
	−10	0.294(6), 0.299(6)	0.696(8), 0.684(7)	$-0.025(1), > -0.021$	$-0.86(3), > -0.69$	2.32(12), —	13.722(27), 13.797(25)	4.4, 0.02
	−14	0.296(6), 0.300(6)	0.695(8), 0.683(7)	$-0.019(1), > -0.017$	$-0.86(3), > -0.76$	2.33(12), —	13.719(31), 13.794(27)	5.3, 0.01
	−20	0.297(6), 0.300(6)	0.696(9), 0.683(7)	$-0.013(1), > -0.012$	$-0.86(3), > -0.76$	2.32(12), —	13.718(31), 13.795(26)	6.0, 0.02
free λ	−17.9(5.8)	0.296(6), 0.299(7)	0.697(9), 0.684(8)	$-0.017(8), > -0.074$	$-0.85(4), > -0.69$	2.32(19), —	13.719(30), 13.795(24)	6.4, 0.01



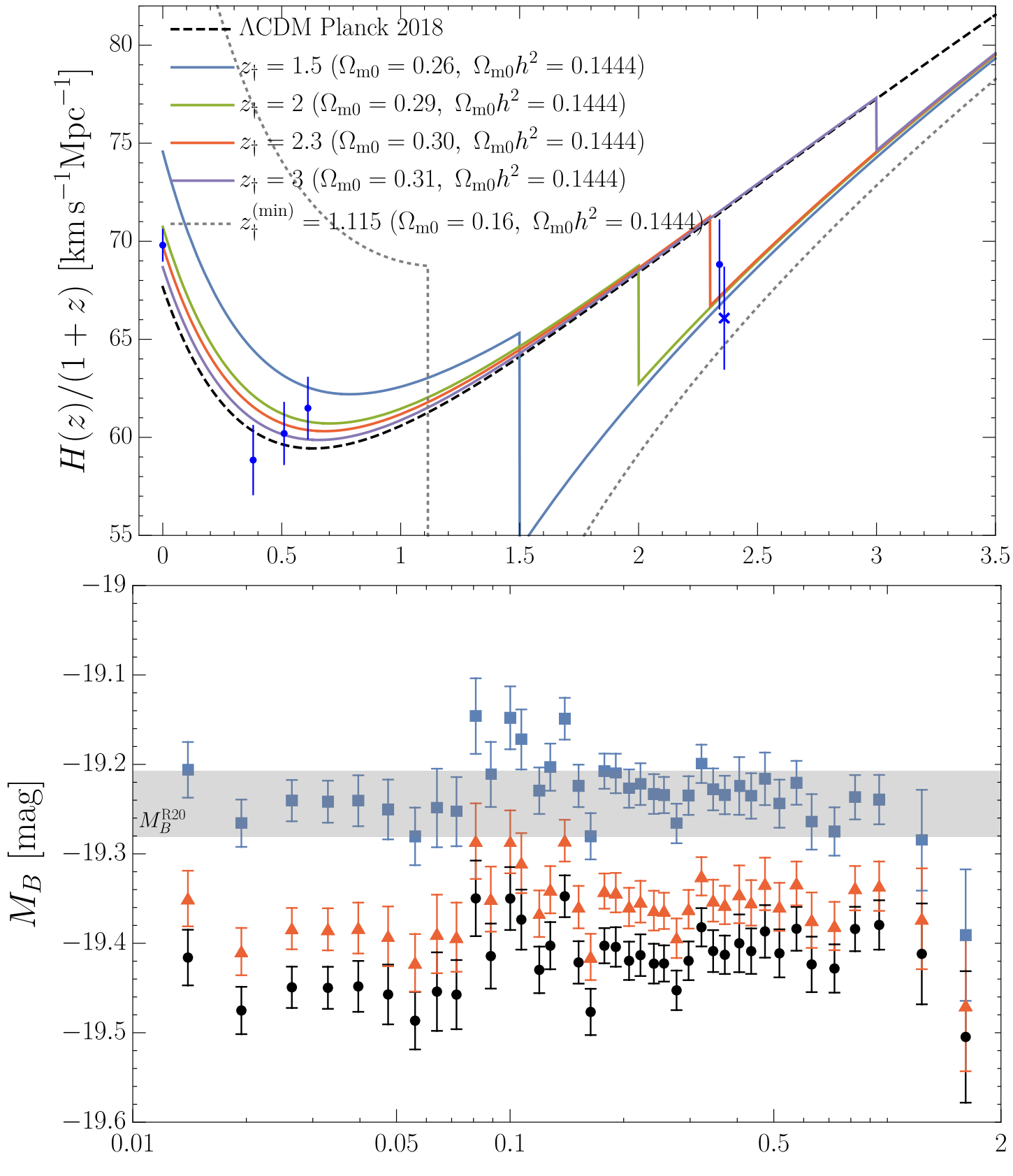
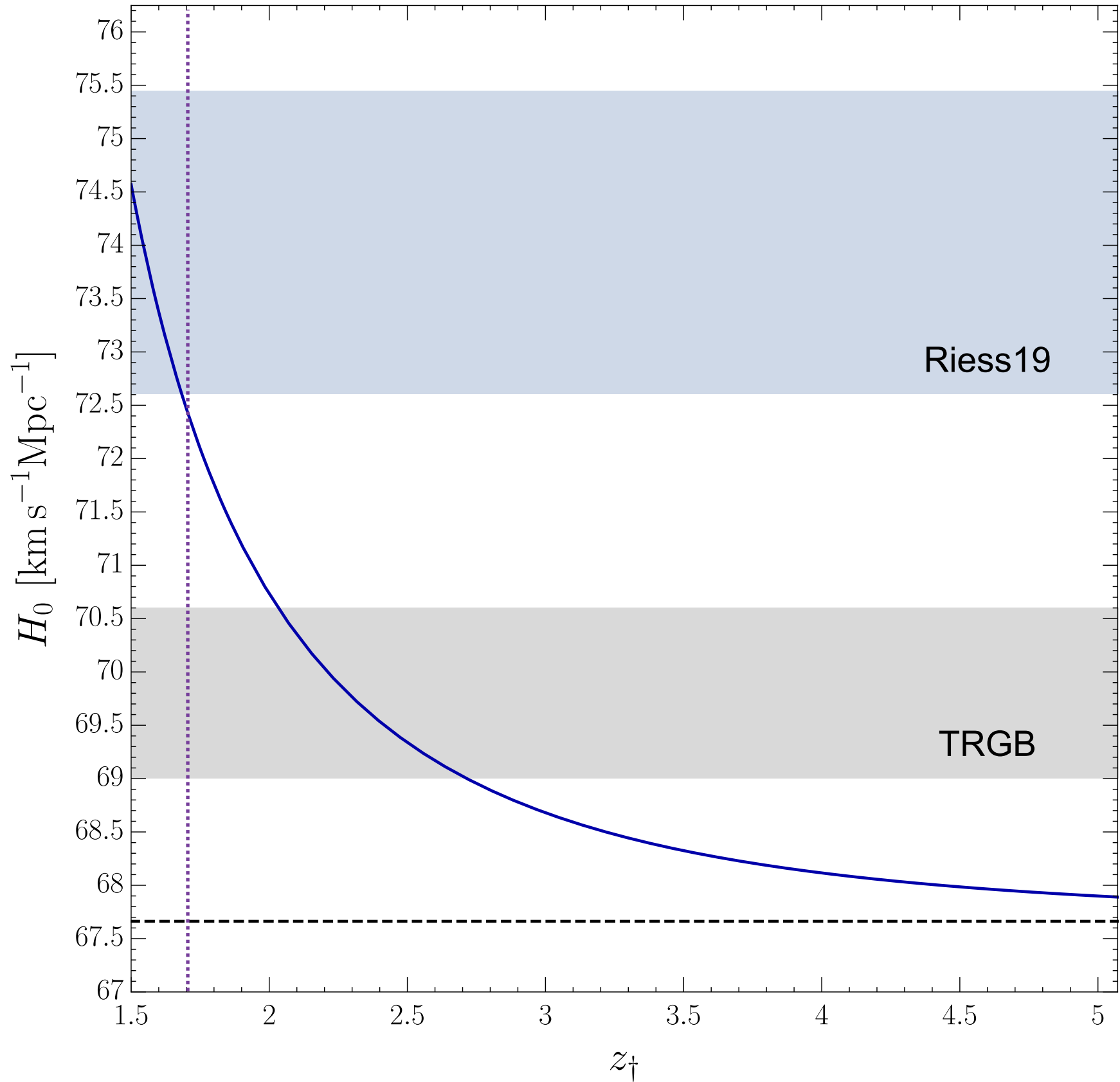


$\Lambda_{\text{s}}\text{CDM MODEL: SIGN-SWITCHING } \Lambda$

$$\frac{\rho_{\text{DE}}}{\rho_{\text{c},0}} \rightarrow \Omega_{\text{DE},0} \operatorname{sgn}[1 - \Psi \ln a] \quad \text{as} \quad \lambda \rightarrow -\infty$$

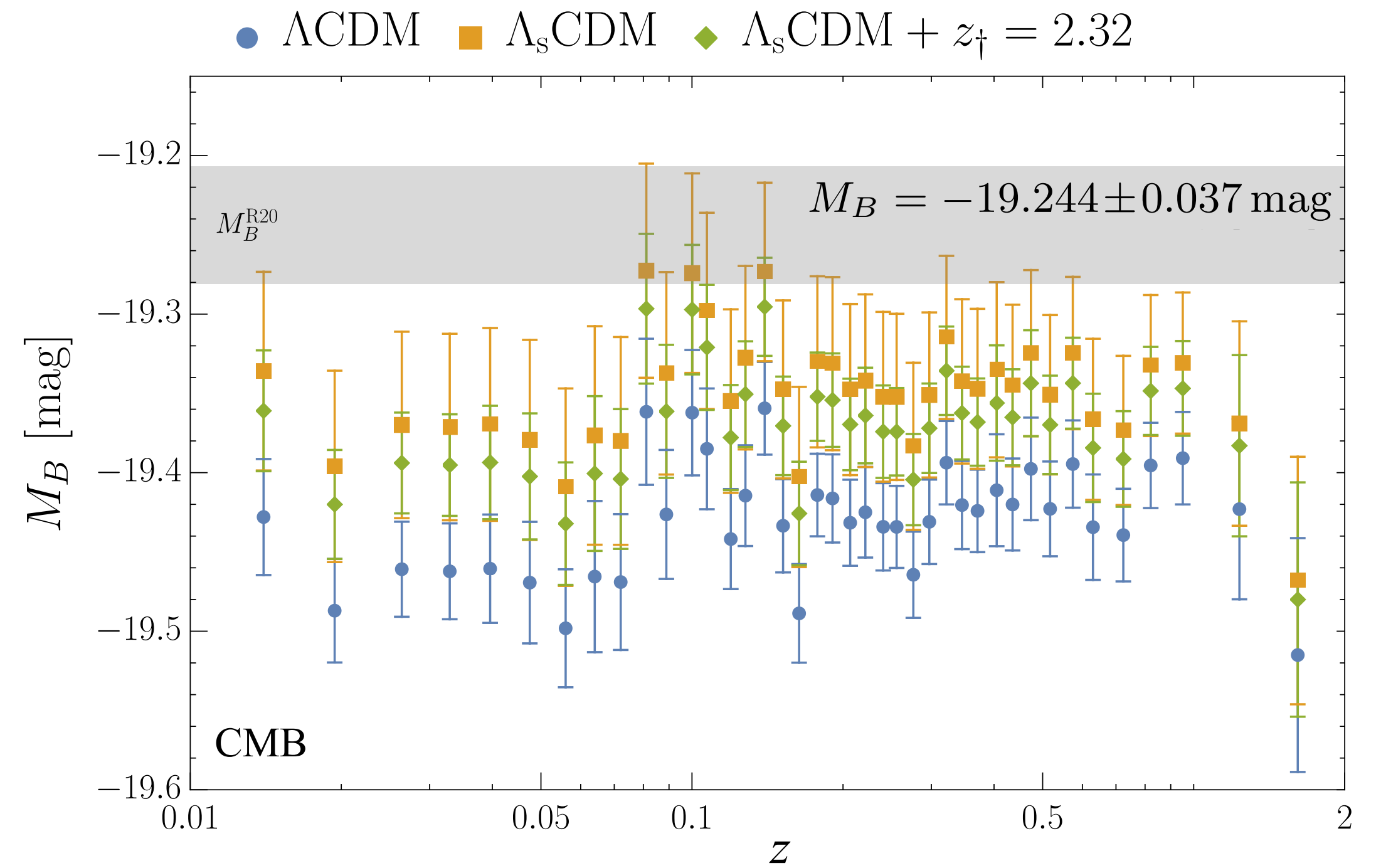
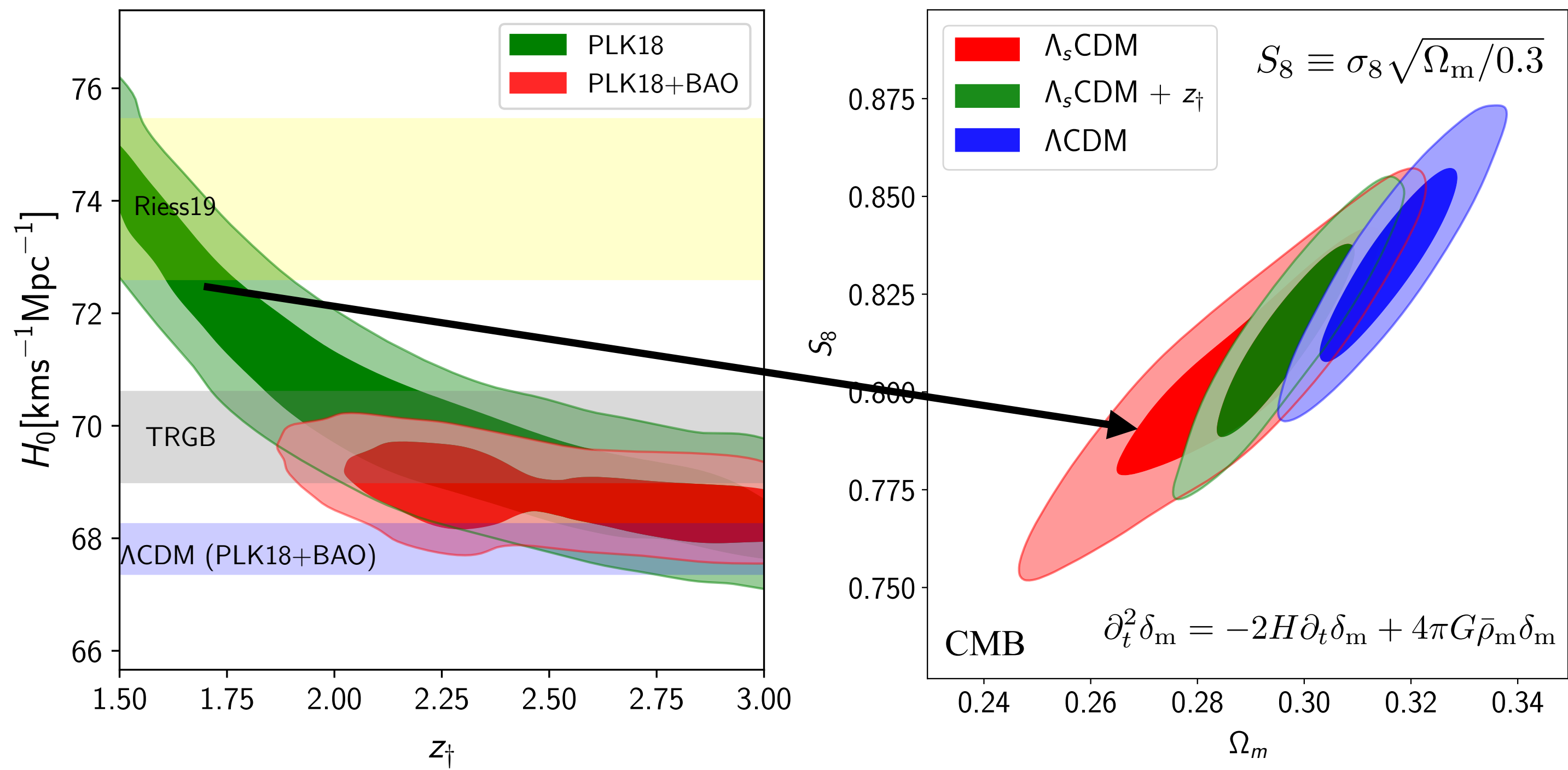
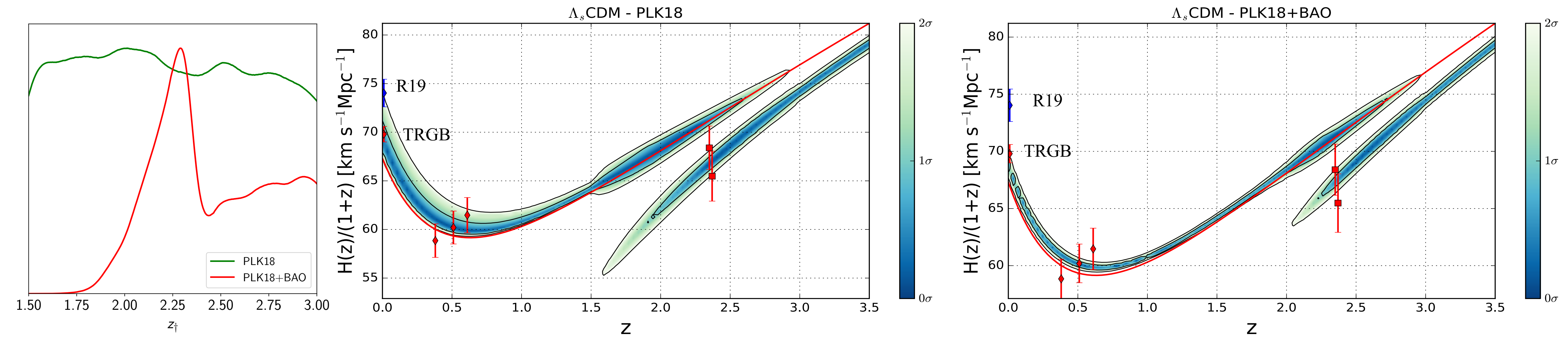


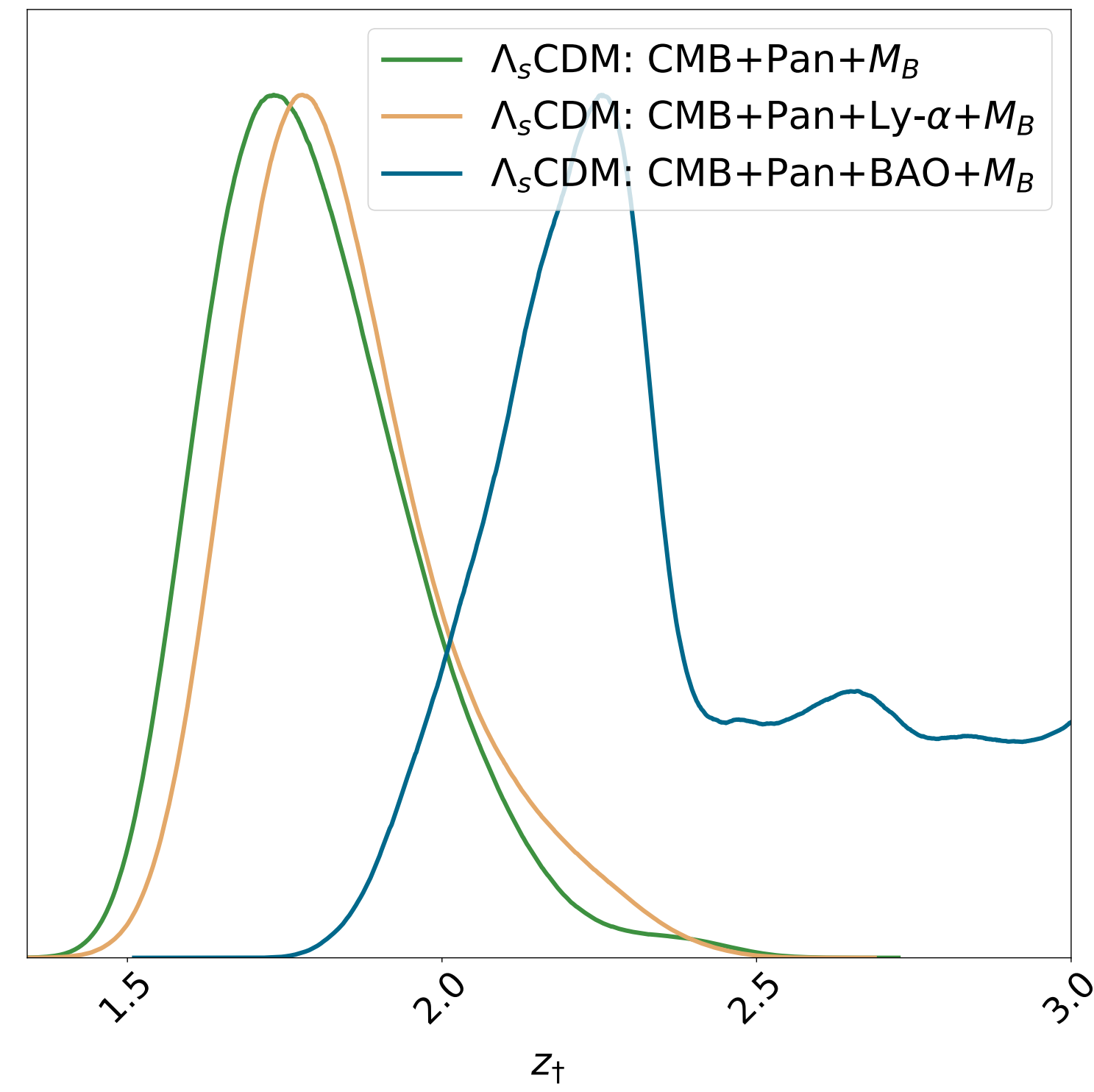
$$\Lambda \rightarrow \Lambda_{\text{s}} \equiv \Lambda_{\text{s}0} \operatorname{sgn}[z_{\dagger} - z]$$



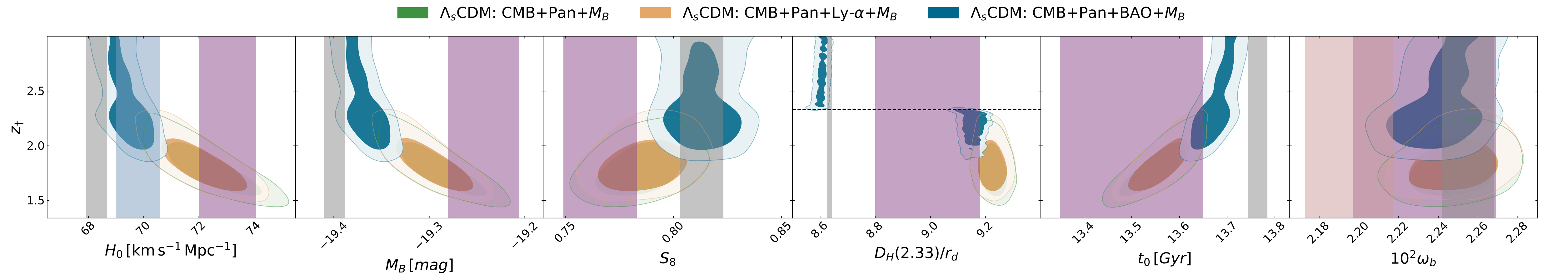
The figures are plotted by fixing $D_M(z_*)$ and $\rho_{\text{m}}(z_*)$ (and hence $\rho_{\text{m}0}$) to that of ΛCDM using the mean values of the *Planck* 2018 TT,TE,EE+lowE+lensing results.

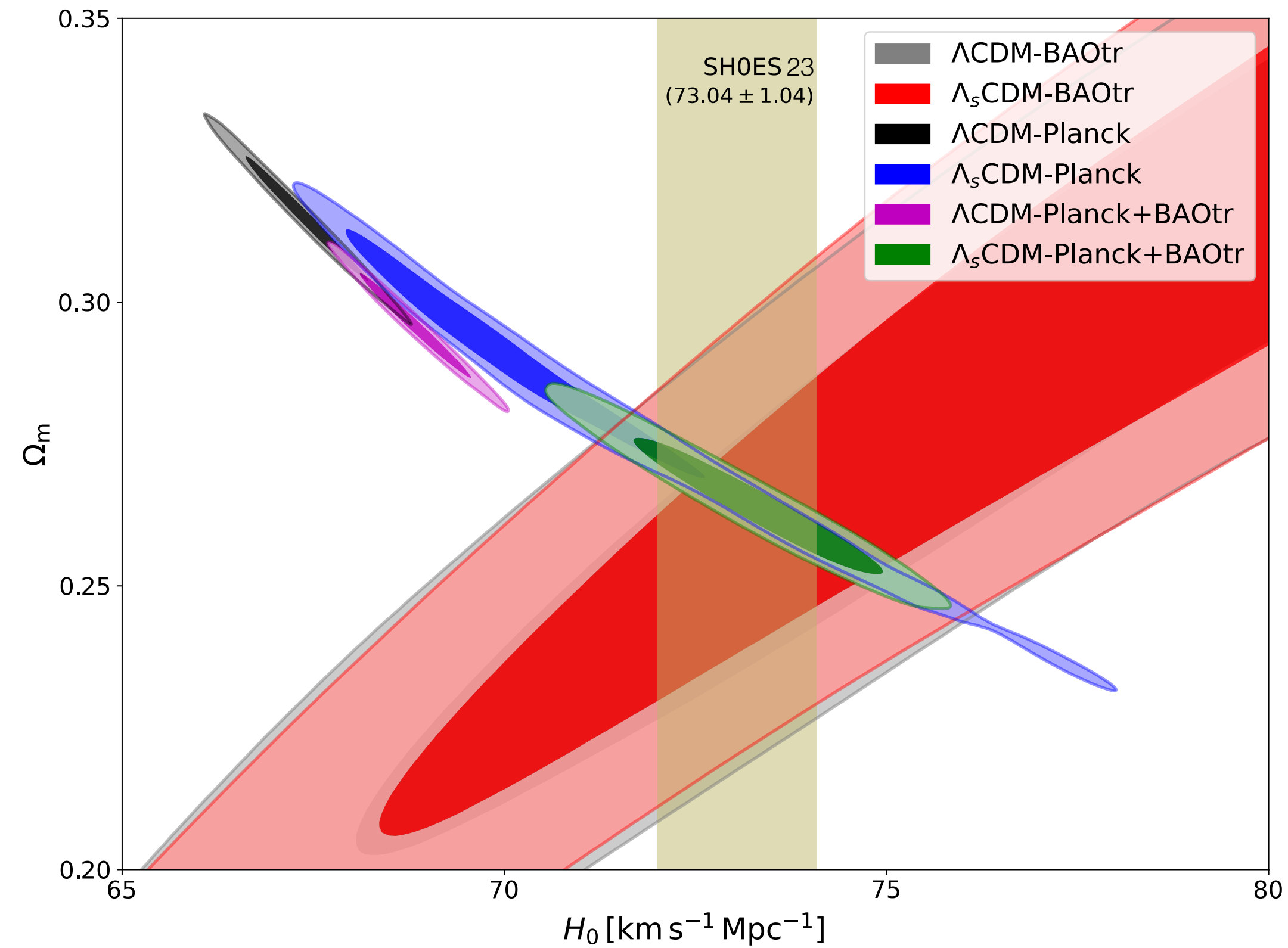
Angular scale of the sound horizon at last scattering	Comoving sound horizon at last scattering	Comoving angular diameter distance out to last scattering
$\theta_* = \frac{r_*}{D_M(z_*)}$	$r_* = \int_{z_*}^{\infty} \frac{c_{\text{s}}(z)}{H_{\Lambda\text{CDM}}(z)} \text{d}z$	$D_M(z_*) = c \int_0^{z_*} \frac{\text{d}z}{H(z)}$
$100\theta_* = 1.04110 \pm 0.00031 \text{ } (\Lambda\text{CDM PL18})$	$r_* = 144.43 \pm 0.26 \text{ Mpc } (\Lambda\text{CDM PL18})$	$D_{M(z_*)} = 13872.83 \pm 25.31 \text{ Mpc } (\Lambda\text{CDM PL18})$





Data set	CMB+Pan+ M_B		CMB+Pan+Ly- α + M_B		CMB+Pan+BAO+ M_B	
	Λ CDM	Λ_s CDM	Λ CDM	Λ_s CDM	Λ CDM	Λ_s CDM
$10^2\omega_b$	2.256 ± 0.015	2.248 ± 0.014	2.253 ± 0.013	$2.247^{+0.014}_{-0.013}$	2.255 ± 0.013	2.242 ± 0.014
ω_c	0.1181 ± 0.0011	0.1191 ± 0.0011	0.1183 ± 0.0008	0.1191 ± 0.0011	0.1181 ± 0.0009	$0.1200^{+0.0010}_{-0.0011}$
$100\theta_s$	1.04208 ± 0.00029	1.04197 ± 0.00031	1.04204 ± 0.00028	1.04196 ± 0.00028	$1.04207^{+0.00029}_{-0.00026}$	1.04186 ± 0.00028
$\ln(10^{10}A_s)$	$3.053^{+0.014}_{-0.017}$	3.039 ± 0.014	$3.052^{+0.013}_{-0.016}$	3.041 ± 0.015	$3.053^{+0.014}_{-0.016}$	3.041 ± 0.015
n_s	0.9701 ± 0.0040	$0.9687^{+0.0043}_{-0.0038}$	0.9697 ± 0.0035	0.9684 ± 0.0041	0.9702 ± 0.0035	0.9661 ± 0.0037
τ_{reio}	$0.0601^{+0.0072}_{-0.0085}$	0.0526 ± 0.0074	$0.0593^{+0.0064}_{-0.0079}$	0.0535 ± 0.0077	$0.0603^{+0.0070}_{-0.0078}$	0.0524 ± 0.0076
$z_{\dagger}$	—	$1.78^{+0.14}_{-0.18}$	—	$1.84^{+0.13}_{-0.21}$	—	2.36 ± 0.28
M_B [mag]	-19.399 ± 0.014	$-19.290^{+0.026}_{-0.029}$	-19.402 ± 0.011	-19.299 ± 0.028	-19.399 ± 0.011	$-19.366^{+0.013}_{-0.015}$
Ω_m	0.3028 ± 0.0068	0.2716 ± 0.0084	0.3043 ± 0.0050	$0.2743^{+0.0086}_{-0.0097}$	0.3030 ± 0.0051	0.2965 ± 0.0055
ω_m	0.1413 ± 0.0011	0.1422 ± 0.0010	0.1415 ± 0.0008	0.1422 ± 0.0011	0.1413 ± 0.0008	0.1431 ± 0.0010
H_0 [km/s/Mpc]	68.31 ± 0.52	$72.38^{+0.98}_{-1.10}$	68.19 ± 0.38	72.0 ± 1.1	68.29 ± 0.39	$69.48^{+0.48}_{-0.55}$
t_0 [Gyr]	13.76 ± 0.02	13.55 ± 0.05	13.76 ± 0.02	$13.56^{+0.04}_{-0.04}$	13.76 ± 0.02	13.67 ± 0.03
σ_8	0.8090 ± 0.0064	$0.8255^{+0.0072}_{-0.0081}$	$0.8091^{+0.0054}_{-0.0063}$	0.8243 ± 0.0076	$0.8092^{+0.0057}_{-0.0061}$	0.8176 ± 0.0063
S_8	0.813 ± 0.012	0.785 ± 0.012	0.815 ± 0.010	$0.788^{+0.012}_{-0.014}$	0.813 ± 0.010	0.813 ± 0.010
$-2\ln\mathcal{L}_{\text{max}}$	1913.28	1904.29	1918.68	1905.88	1919.85	1915.57
$\ln\mathcal{Z}$	-1947.83	-1940.06	-1954.17	-1941.85	-1955.02	-1951.79
$\Delta\ln\mathcal{Z}$	7.77	0	12.32	0	3.23	0





Here **BAOtr** dataset is the **2D BAO** dataset (less model independent) compiled in arXiv:2002.09293 (Nunes, Yadav, Jesus, Bernui, MNRAS 2020) and arXiv:2103.14121 (Carvalho, Bernui, Avila, Novaes, Nogueira-Cavalcante, A&A 2021)

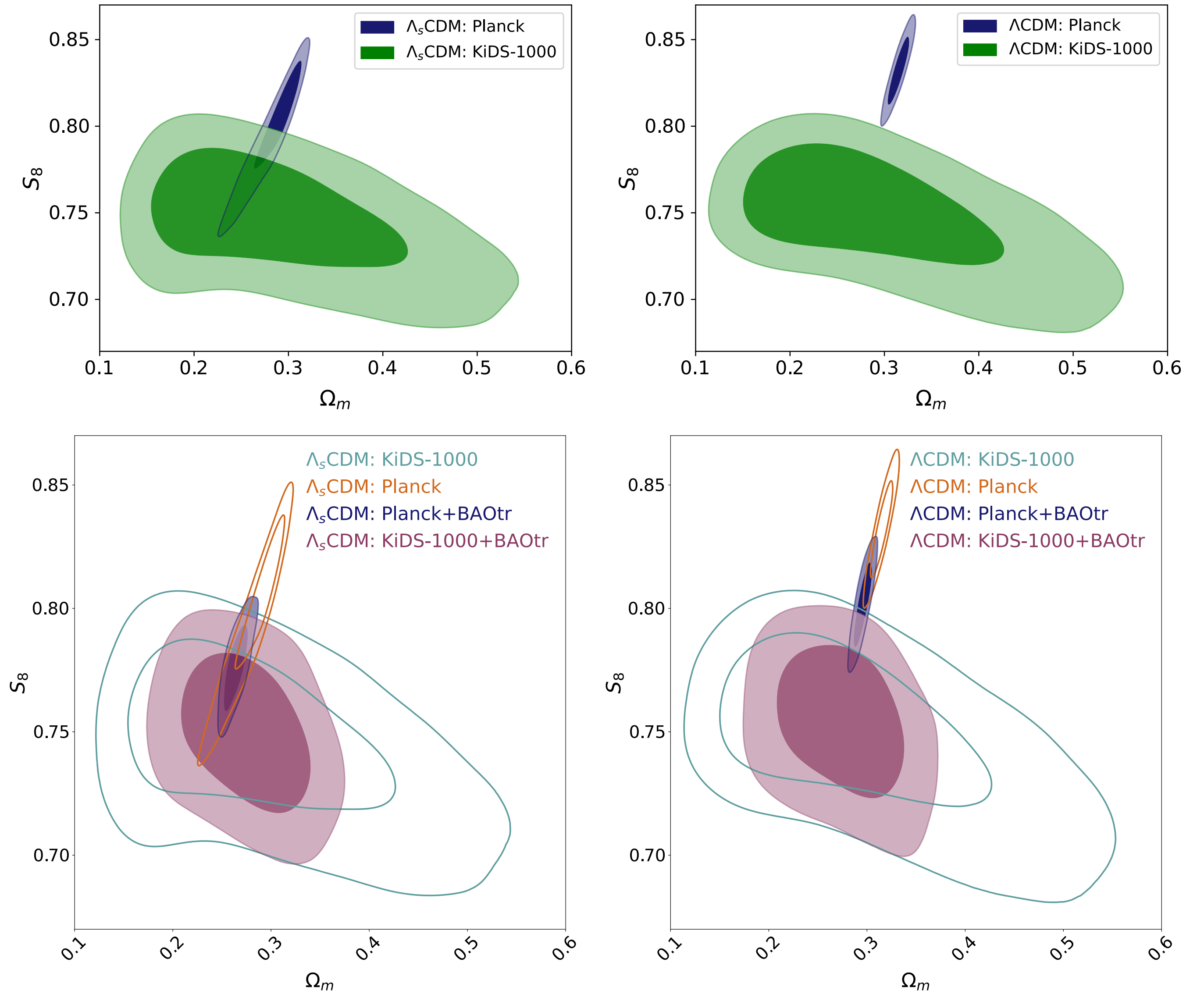


FIG. 1. 2D contours at 68%, and 95% CLs in the Ω_m - S_8 plane for the Λ_s CDM and Λ CDM models. $S_8 = 0.801^{+0.026}_{-0.016}$ (Λ_s CDM: Planck), $S_8 = 0.746^{+0.026}_{-0.021}$ (Λ_s CDM: KiDS), $S_8 = 0.832 \pm 0.012$ (Λ CDM: Planck), $S_8 = 0.749^{+0.027}_{-0.020}$ (Λ CDM: KiDS) at 68% CL.

work) CMB+BAOtr+PantheonPlus&SH0ES+KiDS-1000

CMB+BAOtr+PantheonPlus&SH0ES

CMB+BAOtr+PantheonPlus

CMB+BAOtr

CMB

CMB+Pantheon+BAO+ M_B

CMB+Pantheon+Ly- α + M_B

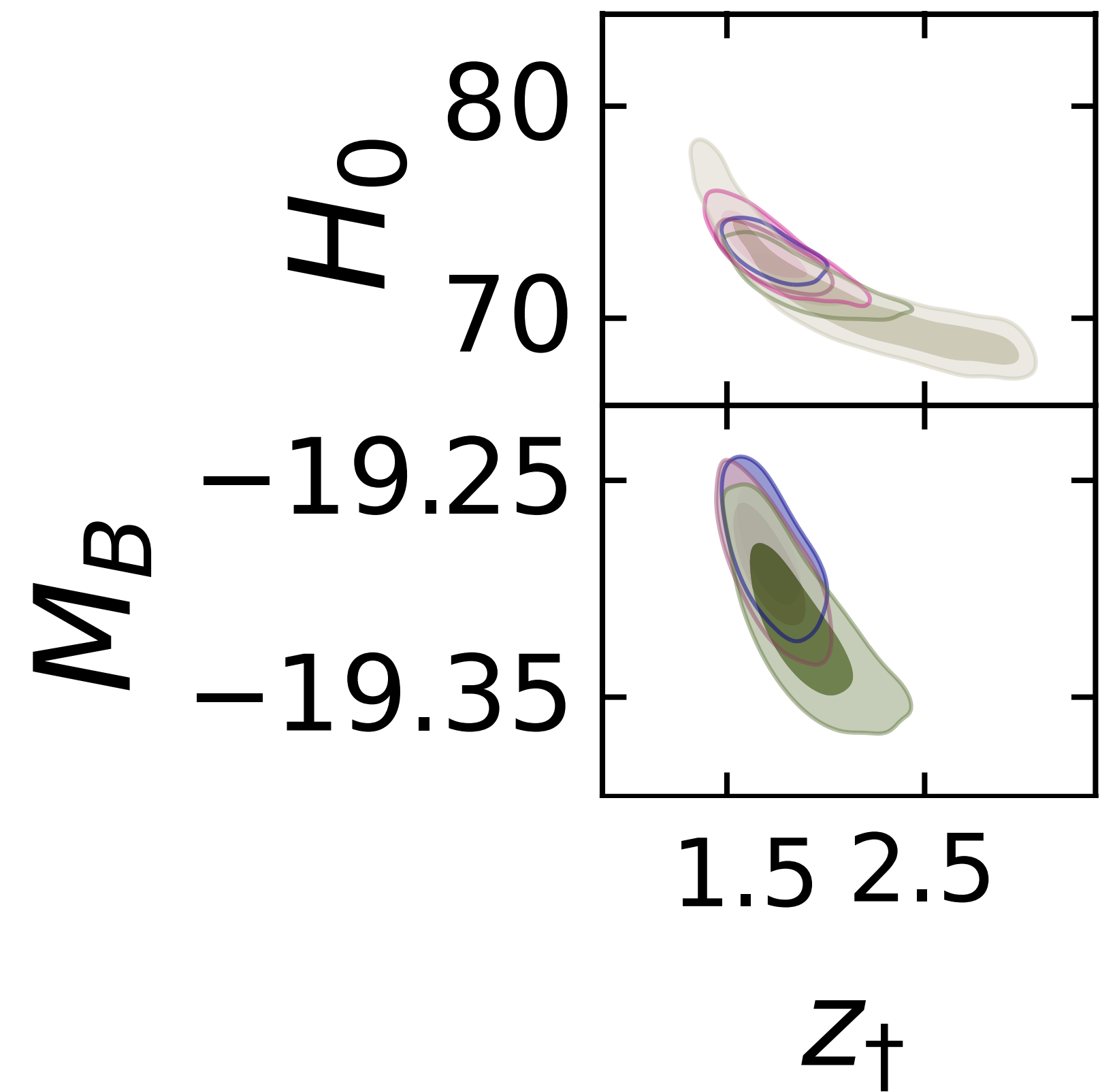
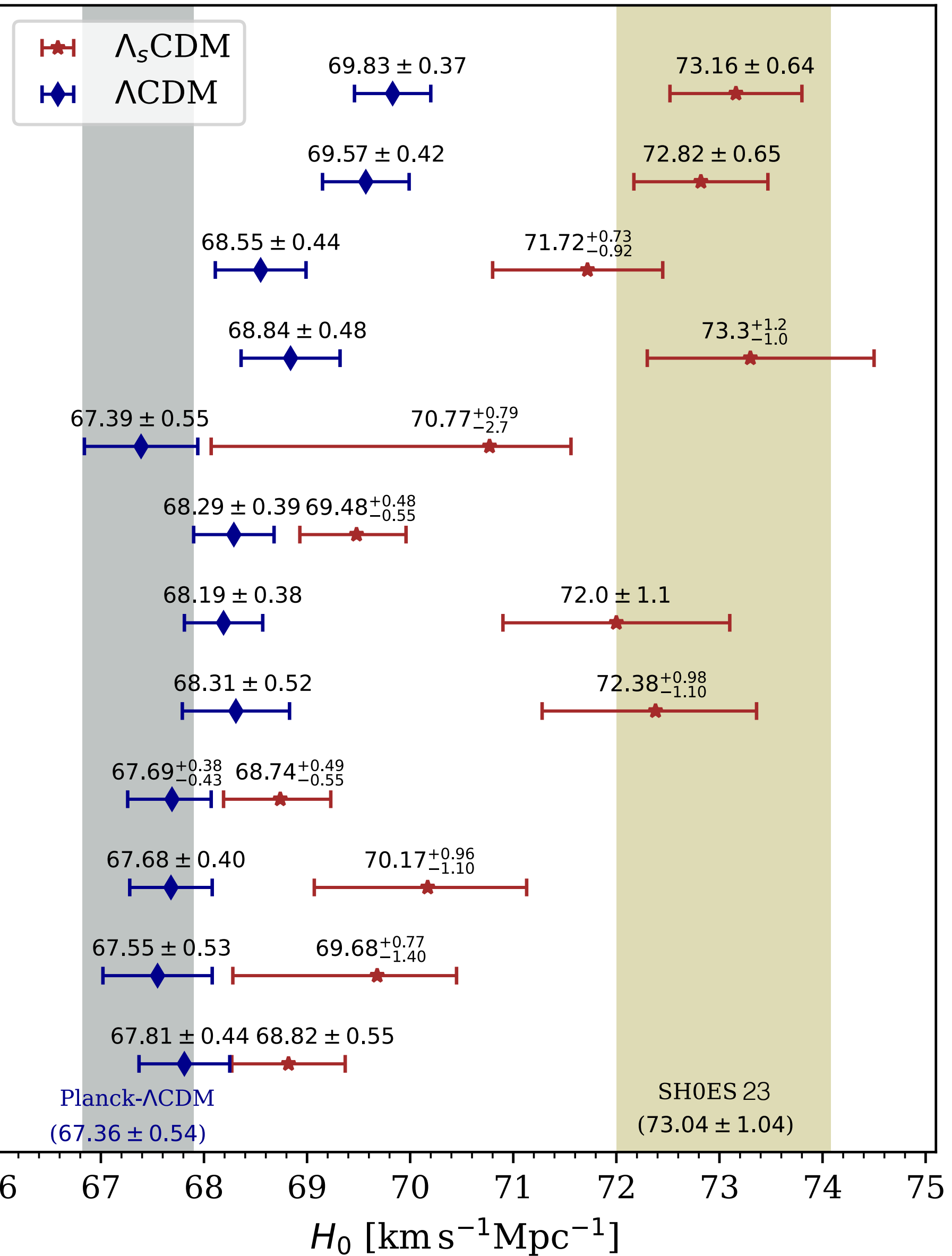
CMB+Pantheon+ M_B

CMB+Pantheon+BAO

CMB+Pantheon+Ly- α

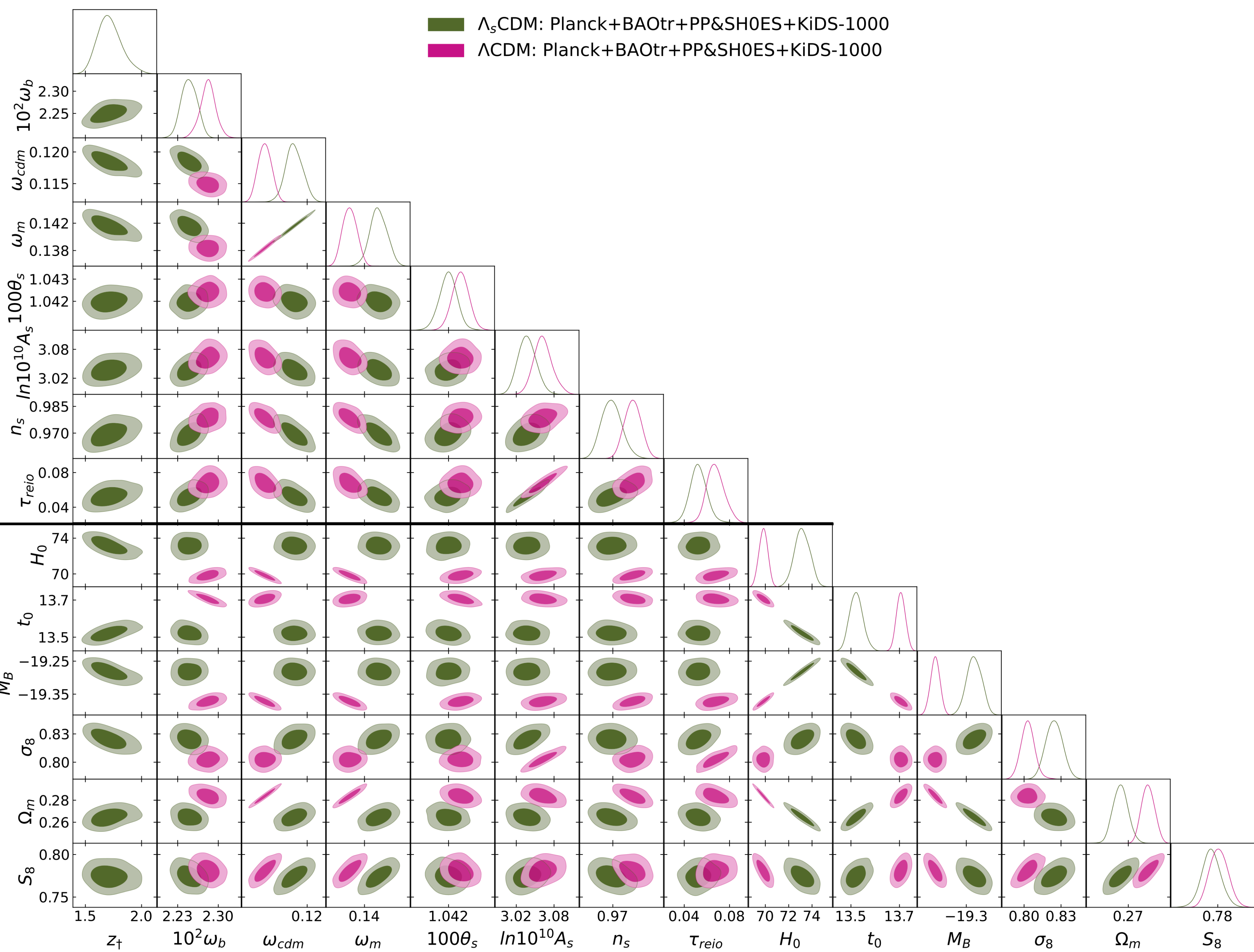
CMB+Pantheon

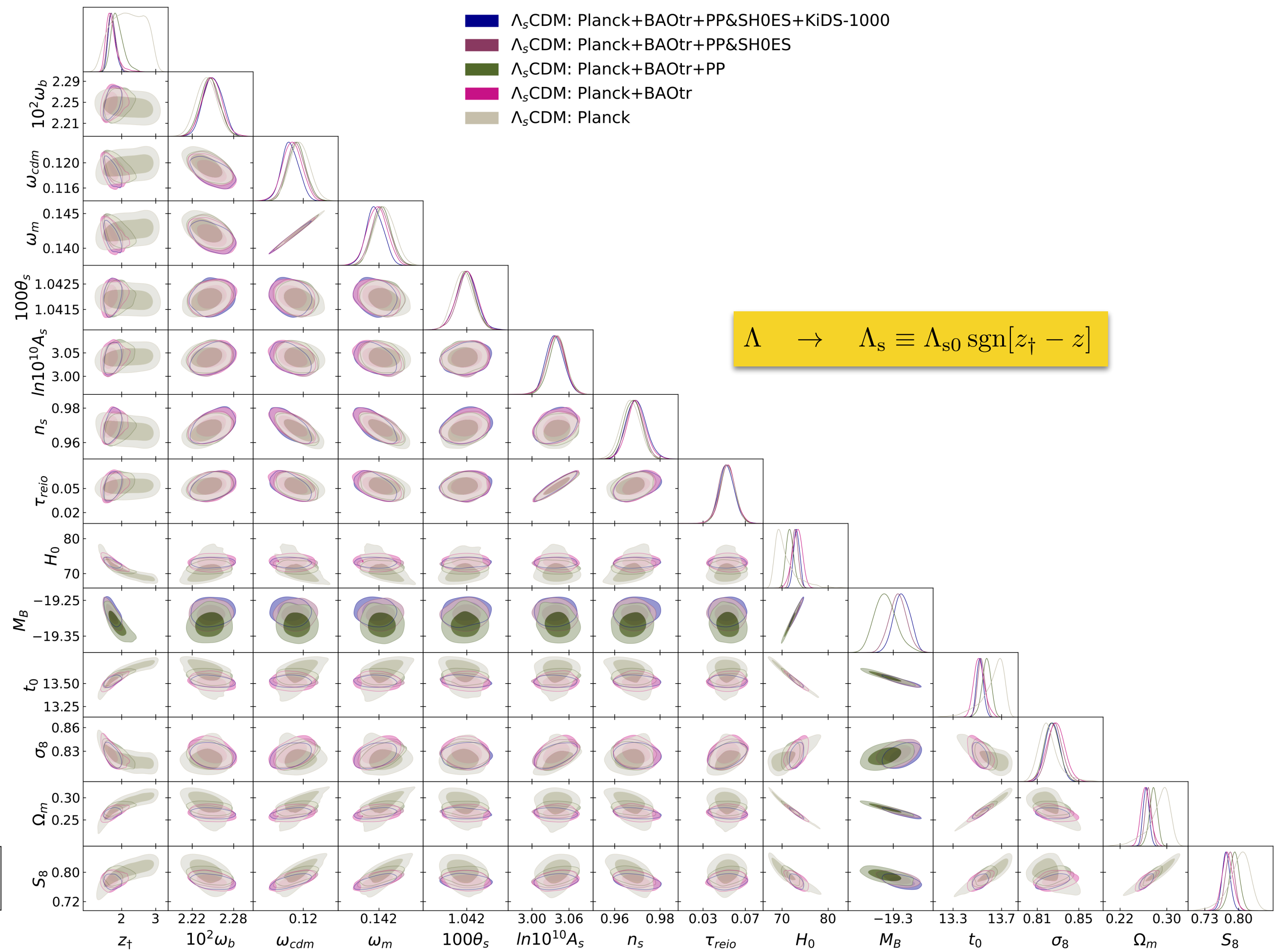
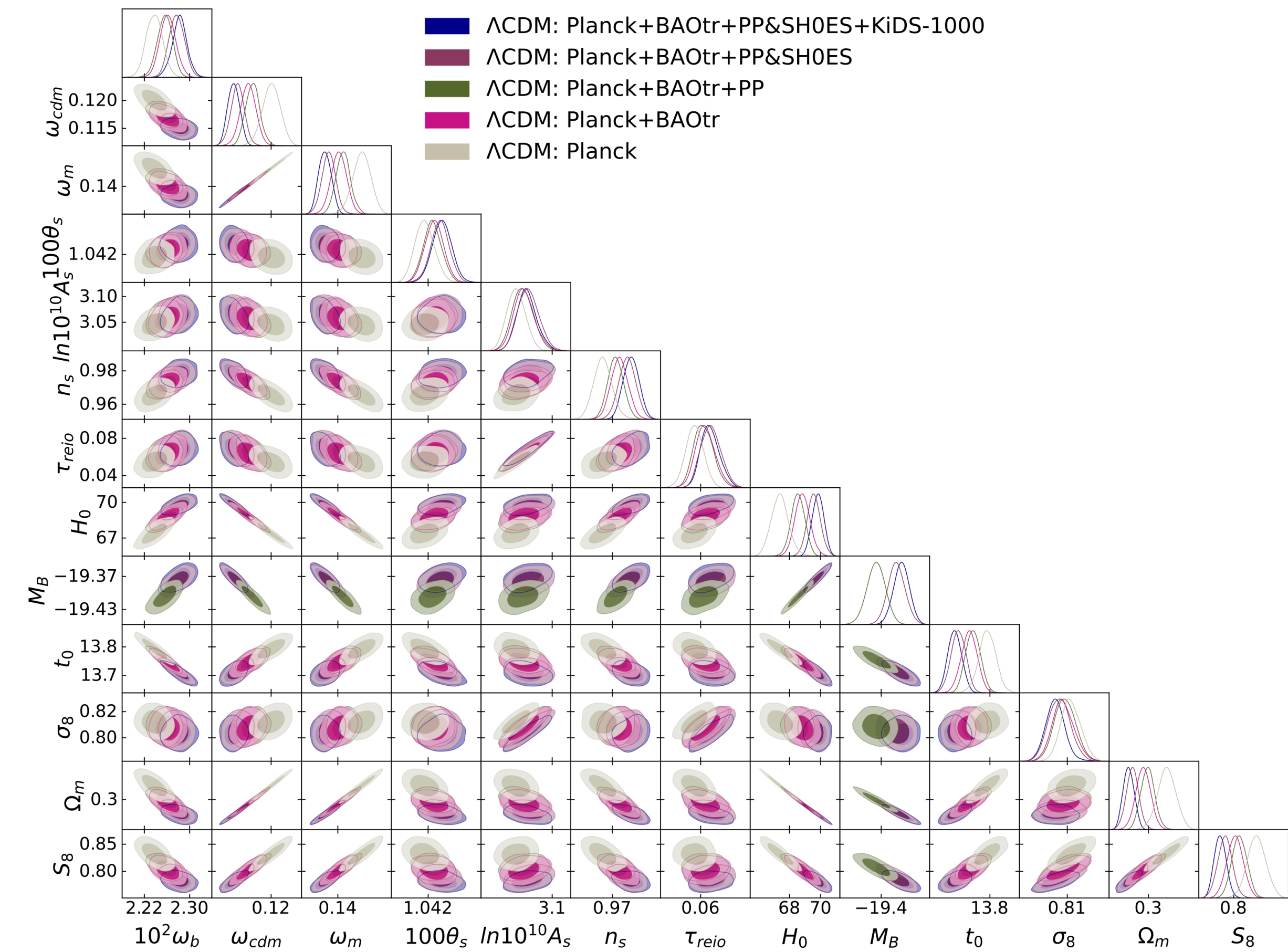
CMB+BAO



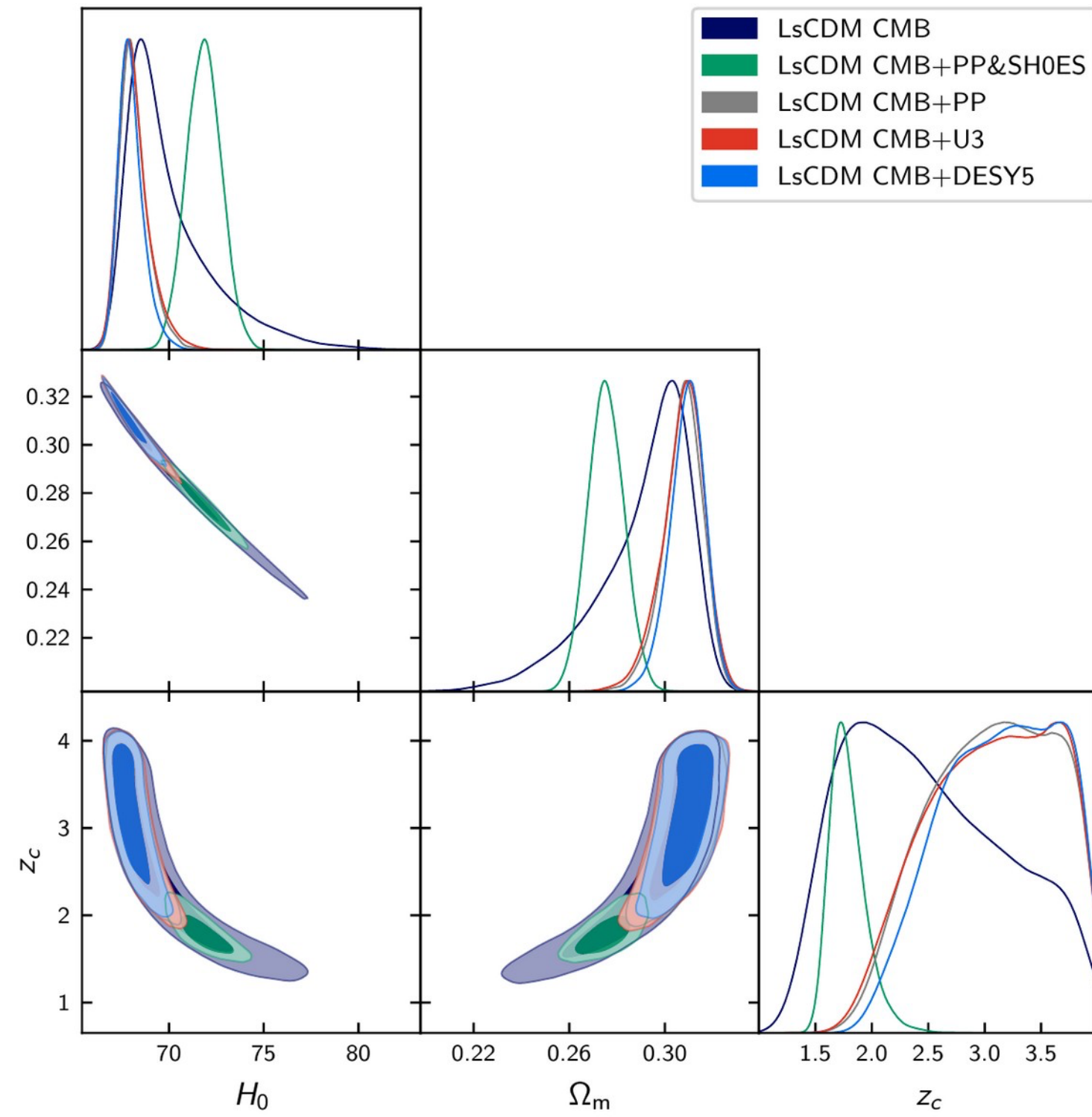
- Λ_5 CDM: Planck+BAOtr+PP&SH0ES+KiDS-1000
- Λ_5 CDM: Planck+BAOtr+PP&SH0ES
- Λ_5 CDM: Planck+BAOtr+PP
- Λ_5 CDM: Planck+BAOtr
- Λ_5 CDM: Planck

Common baseline free parameters

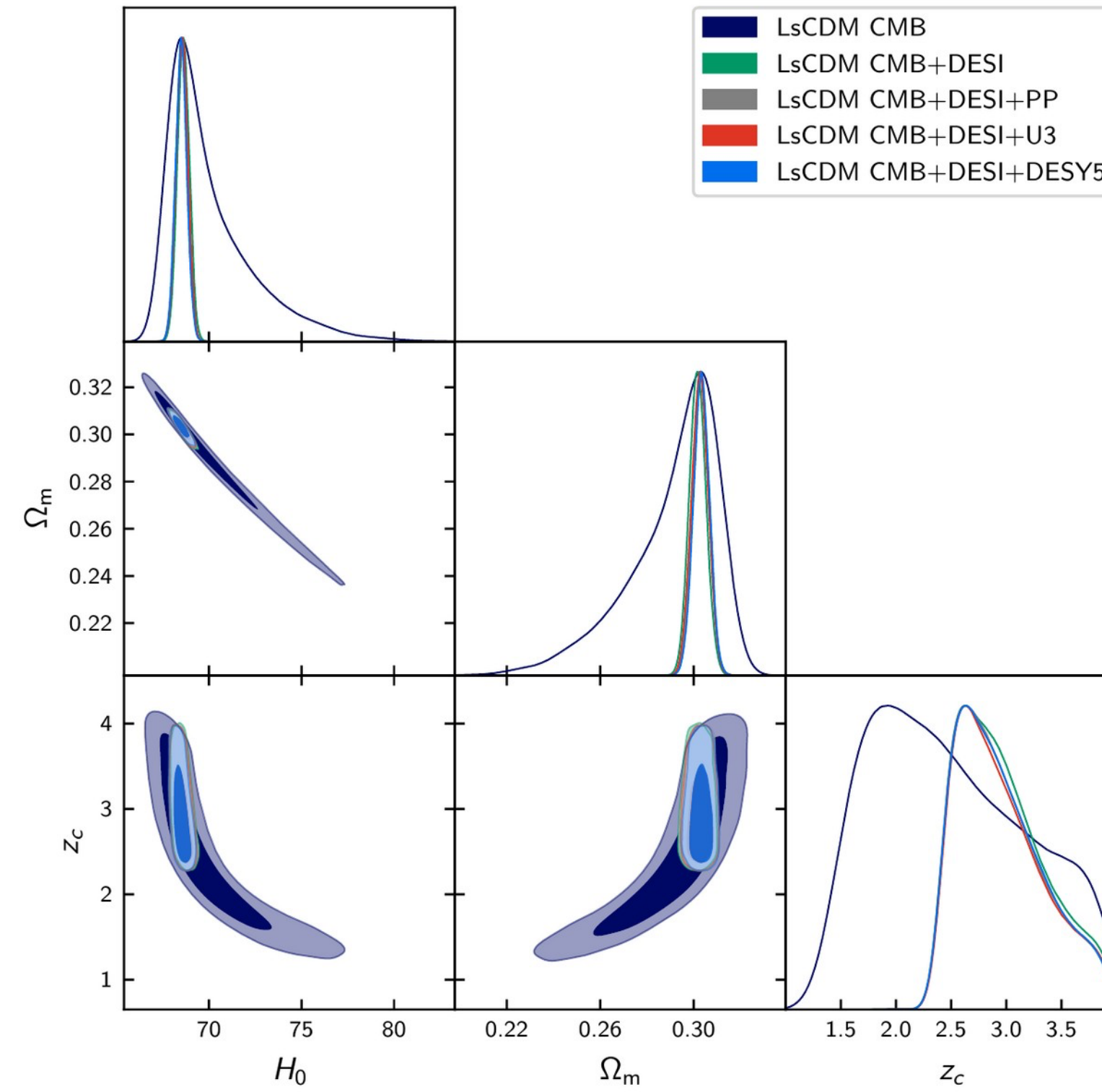




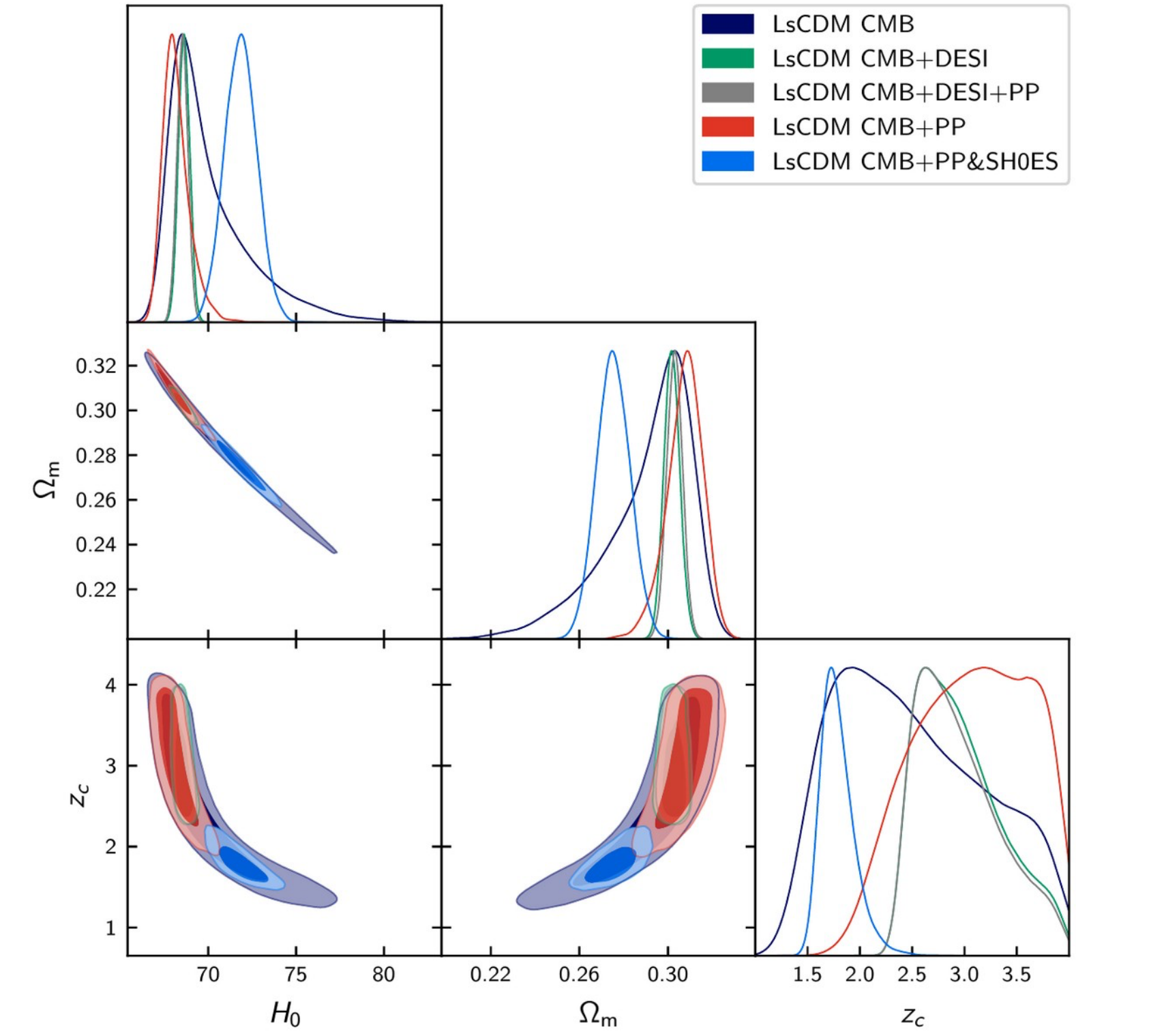
CMB + SNe / SH0ES



CMB + DESI DR2 + SNe



DESI DR2 versus SH0ES

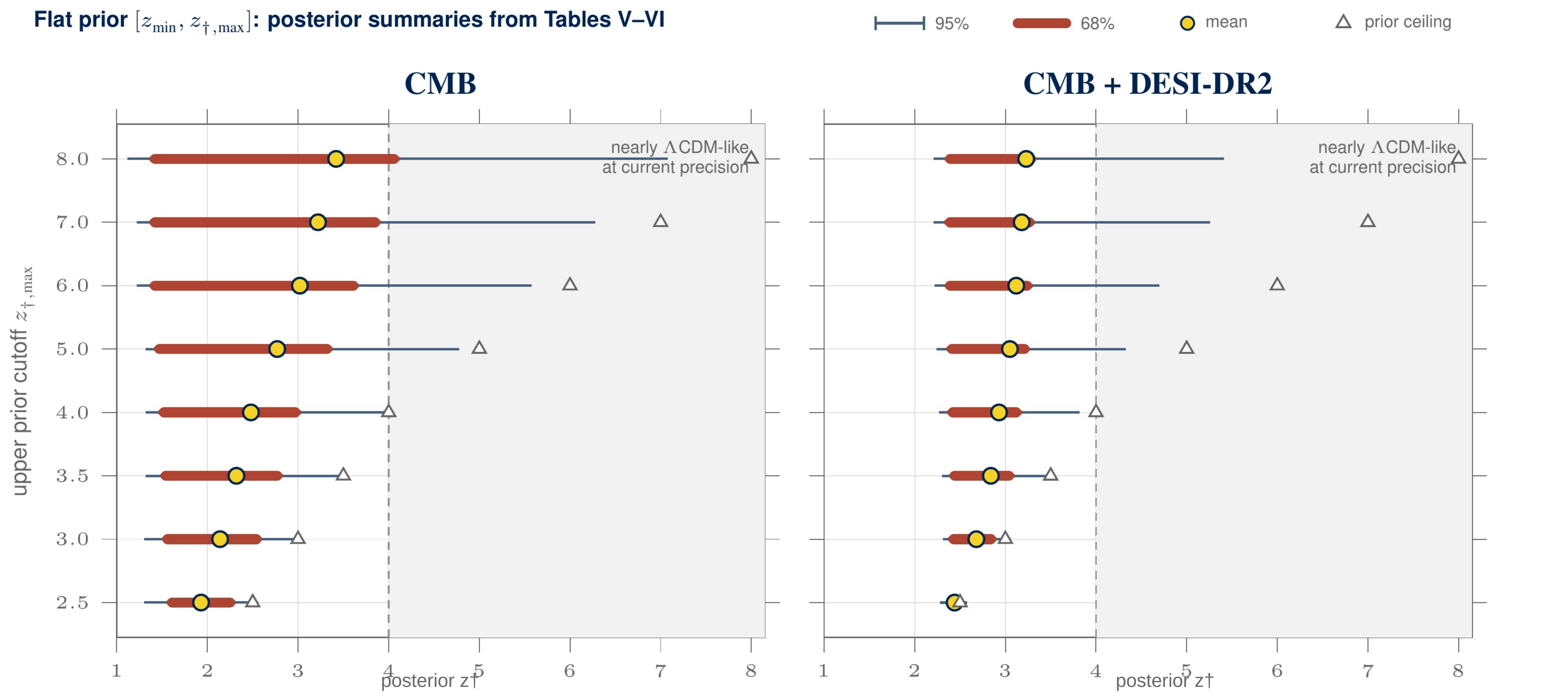
Plot notation: $z_c \equiv z_{\dagger}$.

DESI DR2 selects $z_{\dagger} \simeq 3$ and $H_0 \simeq 68.5$ km s⁻¹ Mpc⁻¹.

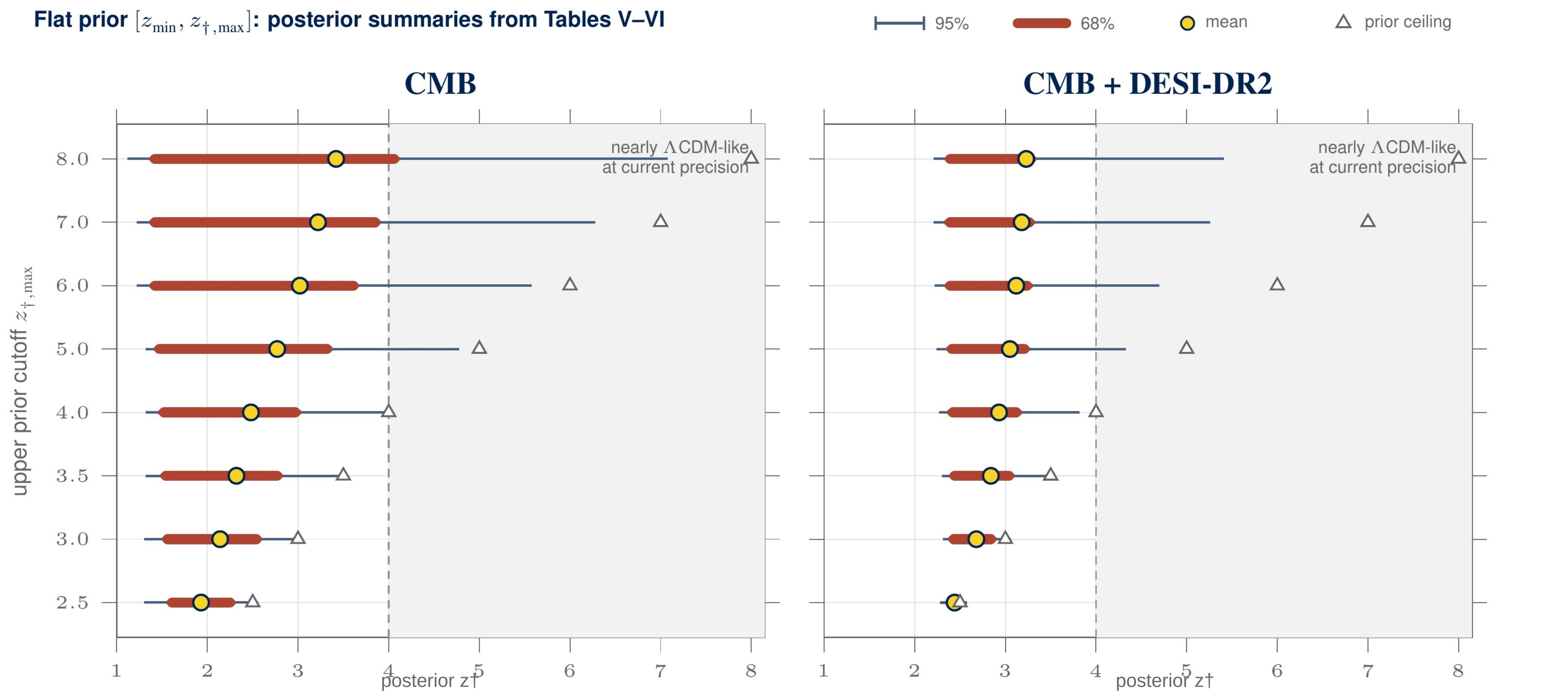
The finite-transition solution is stable across the supernova compilations; **SH0ES selects the lower- $z_{\dagger}$, higher- H_0 branch.**

Parameter	CMB	CMB+DESI-DR2	CMB+DESI-DR2+PP	CMB+DESI-DR2+Union3	CMB+DESI-DR2+DES-Dovekie
$\Omega_{\text{b}} h^2$	0.02221 ± 0.00014	0.02223 ± 0.00012	0.02221 ± 0.00012	0.02221 ± 0.00012	0.02220 ± 0.00012
	0.02218 ± 0.00013	0.02233 ± 0.00012	0.02231 ± 0.00012	0.02231 ± 0.00012	0.02231 ± 0.00012
$\Omega_{\text{c}} h^2$	0.1193 ± 0.0011	0.11919 ± 0.00079	0.11940 ± 0.00079	0.11935 ± 0.00079	0.11945 ± 0.00078
	0.1198 ± 0.0011	0.11760 ± 0.00062	0.11781 ± 0.00060	0.11776 ± 0.00062	0.11786 ± 0.00060
$100 \theta_{\text{MC}}$	1.04079 ± 0.00025	1.04081 ± 0.00024	1.04079 ± 0.00024	1.04080 ± 0.00024	1.04079 ± 0.00024
	1.04076 ± 0.00025	1.04101 ± 0.00024	1.04100 ± 0.00023	1.04098 ± 0.00024	1.04098 ± 0.00024
τ_{reio}	0.0510 ± 0.0073	0.0532 ± 0.0069	0.0524 ± 0.0069	0.0527 ± 0.0068	0.0523 ± 0.0069
	0.0526 ± 0.0071	$0.0597^{+0.0064}_{-0.0073}$	0.0590 ± 0.0069	$0.0590^{+0.0060}_{-0.0072}$	$0.0586^{+0.0064}_{-0.0073}$
n_{s}	0.9646 ± 0.0041	0.9649 ± 0.0035	0.9644 ± 0.0035	0.9644 ± 0.0035	0.9642 ± 0.0036
	0.9634 ± 0.0039	0.9685 ± 0.0034	0.9683 ± 0.0034	0.9684 ± 0.0033	0.9679 ± 0.0034
$\log(10^{10} A_{\text{s}})$	3.033 ± 0.014	3.038 ± 0.014	3.036 ± 0.014	3.037 ± 0.014	3.036 ± 0.014
	3.038 ± 0.014	3.048 ± 0.014	$3.048^{+0.013}_{-0.015}$	$3.047^{+0.012}_{-0.015}$	3.047 ± 0.014
$a_{\dagger}$	< 0.329	$0.255^{+0.034}_{-0.019}$	$0.256^{+0.033}_{-0.019}$	$0.256^{+0.033}_{-0.018}$	$0.256^{+0.033}_{-0.019}$
	—	—	—	—	—
H_0 [km/s/Mpc]	$69.99^{+0.87}_{-2.6}$	68.62 ± 0.32	68.53 ± 0.31	68.55 ± 0.31	68.50 ± 0.31
	67.21 ± 0.47	68.18 ± 0.28	68.09 ± 0.27	68.10 ± 0.28	68.06 ± 0.28
σ_8	$0.8152^{+0.0057}_{-0.010}$	0.8103 ± 0.0057	0.8105 ± 0.0057	0.8104 ± 0.0057	0.8106 ± 0.0057
	0.8079 ± 0.0055	0.8060 ± 0.0057	$0.8064^{+0.0053}_{-0.0059}$	$0.8062^{+0.0051}_{-0.0059}$	0.8061 ± 0.0056
S_8	$0.803^{+0.027}_{-0.014}$	0.8126 ± 0.0077	0.8144 ± 0.0076	0.8140 ± 0.0078	0.8150 ± 0.0076
	0.829 ± 0.011	0.8092 ± 0.0078	0.8114 ± 0.0076	0.8108 ± 0.0078	0.8115 ± 0.0077
Ω_{m}	$0.291^{+0.023}_{-0.0097}$	0.3017 ± 0.0036	0.3029 ± 0.0035	0.3026 ± 0.0036	0.3033 ± 0.0034
	0.3157 ± 0.0065	0.3024 ± 0.0036	0.3037 ± 0.0035	0.3035 ± 0.0036	0.3040 ± 0.0035
$\Omega_{\text{m}} h^2$	0.1422 ± 0.0010	0.14206 ± 0.00076	0.14225 ± 0.00075	0.14220 ± 0.00076	0.14229 ± 0.00075
	0.1426 ± 0.0010	0.14057 ± 0.00060	0.14077 ± 0.00059	0.14072 ± 0.00060	0.14081 ± 0.00059
r_d	147.46 ± 0.25	147.48 ± 0.21	147.45 ± 0.21	147.46 ± 0.21	147.44 ± 0.21
	147.39 ± 0.25	147.79 ± 0.19	147.75 ± 0.19	147.76 ± 0.19	147.74 ± 0.19
$z_{\dagger}$	$2.50^{+0.51}_{-0.97}$	$2.96^{+0.24}_{-0.53}$	$2.95^{+0.23}_{-0.52}$	$2.95^{+0.23}_{-0.52}$	$2.95^{+0.23}_{-0.52}$
χ^2_{min}	—	—	—	—	—
	10,978.054	10,988.182	12,393.466	11,016.094	12,622.136
$\Delta \chi^2_{\text{min}}$	10,980.632	10,997.728	12,402.844	11,025.218	12,632.452
	—2.578	—9.546	—9.378	—9.124	—10.316
ΔAIC	—0.578	—7.546	—7.378	—7.124	—8.316
$\Delta \ln \mathcal{Z}$	0.07	—3.28	—3.11	—3.11	—3.19

Parameter	SPA	SPA+DESI-DR2	SPA+DESI-DR2+PP	SPA+DESI-DR2+Union3	SPA+DESI-DR2+DES-Dovekie
$\Omega_{\rm b} \, h^2$	0.022446 ± 0.000096	0.022444 ± 0.000093	0.022438 ± 0.000092	0.022436 ± 0.000091	0.022436 ± 0.000091
	0.022425 ± 0.000096	0.022493 ± 0.000091	0.022488 ± 0.000091	0.022485 ± 0.000092	0.022485 ± 0.000093
$\Omega_{\rm c} \, h^2$	0.1193 ± 0.0010	$0.11933^{+0.00071}_{-0.00081}$	0.11947 ± 0.00077	0.11947 ± 0.00075	0.11947 ± 0.00075
	0.11980 ± 0.00099	0.11786 ± 0.00062	0.11803 ± 0.00061	0.11802 ± 0.00059	0.11807 ± 0.00060
$100 \, \theta_{\rm MC}$	1.04077 ± 0.00024	1.04077 ± 0.00023	1.04076 ± 0.00023	1.04077 ± 0.00023	1.04077 ± 0.00023
	1.04073 ± 0.00024	1.04092 ± 0.00022	1.04091 ± 0.00023	1.04092 ± 0.00023	$1.04091^{+0.00021}_{-0.00024}$
$\tau_{\rm reio}$	$0.0585^{+0.0054}_{-0.0062}$	0.0596 ± 0.0057	$0.0592^{+0.0051}_{-0.0060}$	$0.0590^{+0.0049}_{-0.0060}$	$0.0590^{+0.0049}_{-0.0060}$
	$0.0593^{+0.0055}_{-0.0061}$	$0.0650^{+0.0057}_{-0.0064}$	$0.0646^{+0.0058}_{-0.0066}$	$0.0647^{+0.0054}_{-0.0065}$	$0.0643^{+0.0054}_{-0.0064}$
$n_{\rm s}$	0.9711 ± 0.0035	0.9708 ± 0.0030	0.9705 ± 0.0031	0.9705 ± 0.0031	0.9705 ± 0.0031
	0.9697 ± 0.0034	0.9740 ± 0.0029	0.9736 ± 0.0030	0.9738 ± 0.0030	0.9734 ± 0.0029
$\log(10^{10} \, A_{\rm s})$	3.050 ± 0.011	3.055 ± 0.010	$3.0540^{+0.0097}_{-0.011}$	$3.0538^{+0.0095}_{-0.011}$	$3.0538^{+0.0095}_{-0.011}$
	3.056 ± 0.011	3.068 ± 0.010	$3.0667^{+0.0099}_{-0.011}$	$3.0667^{+0.0096}_{-0.012}$	$3.0659^{+0.0093}_{-0.011}$
$a_{\dagger}$	< 0.335	$0.252^{+0.032}_{-0.021}$	$0.252^{+0.032}_{-0.021}$	$0.252^{+0.032}_{-0.020}$	$0.252^{+0.032}_{-0.020}$
	—	—	—	—	—
H_0 [km/s/Mpc]	$70.49^{+0.94}_{-2.8}$	68.79 ± 0.31	68.72 ± 0.30	68.73 ± 0.31	68.73 ± 0.31
	67.48 ± 0.41	68.28 ± 0.25	68.22 ± 0.25	68.22 ± 0.24	68.20 ± 0.25
σ_8	$0.8261^{+0.0039}_{-0.0099}$	0.8206 ± 0.0040	0.8206 ± 0.0040	0.8205 ± 0.0040	0.8205 ± 0.0040
	0.8182 ± 0.0039	0.8177 ± 0.0041	0.8178 ± 0.0042	0.8178 ± 0.0042	$0.8176^{+0.0038}_{-0.0043}$
S_8	$0.808^{+0.027}_{-0.013}$	0.8220 ± 0.0063	0.8232 ± 0.0062	0.8230 ± 0.0062	0.8230 ± 0.0062
	0.8368 ± 0.0091	0.8210 ± 0.0065	0.8224 ± 0.0063	0.8223 ± 0.0062	0.8225 ± 0.0063
$\Omega_{\rm m}$	$0.288^{+0.024}_{-0.0098}$	0.3010 ± 0.0034	0.3019 ± 0.0033	0.3018 ± 0.0034	0.3018 ± 0.0034
	0.3138 ± 0.0058	0.3024 ± 0.0034	0.3034 ± 0.0034	0.3033 ± 0.0033	0.3036 ± 0.0034
$\Omega_{\rm m} \, h^2$	0.14237 ± 0.00099	0.14242 ± 0.00074	0.14256 ± 0.00075	0.14255 ± 0.00074	0.14255 ± 0.00074
	0.14287 ± 0.00097	0.14100 ± 0.00061	0.14116 ± 0.00061	0.14115 ± 0.00059	0.14120 ± 0.00059
r_d	147.21 ± 0.26	147.19 ± 0.20	147.16 ± 0.21	147.17 ± 0.20	147.17 ± 0.20
	147.09 ± 0.25	147.53 ± 0.18	147.49 ± 0.18	147.50 ± 0.18	147.48 ± 0.18
$z_{\dagger}$	$2.5^{+1.4}_{-1.1}$	$3.01^{+0.80}_{-0.63}$	$3.01^{+0.81}_{-0.63}$	$3.00^{+0.82}_{-0.63}$	$3.00^{+0.82}_{-0.63}$
$\chi^2_{\rm min}$	—	—	—	—	—
	993.82314	1005.1308	2410.8283	1033.1501	2640.1721
	994.39781	1011.523	2417.0608	1039.6012	2646.4171
$\Delta \chi^2_{\rm min}$	−0.5746	−6.392	−6.232	−6.45	−6.245
$\Delta {\rm AIC}$	1.425	−4.392	−4.232	−4.45	−4.245
$\Delta \ln \mathcal{Z}$	−0.4811	−3.2167	−3.2066	−3.2116	−3.1539



95% lower edge changes little: 1.20–1.40 (CMB), 2.29–2.39 (CMB+DESI DR2).
For $z_{\dagger} \gtrsim 4$, added prior volume is nearly Λ CDM-like. A flat $[1,10]$ prior places 67% of its mass above 4; the marginal comparison is therefore prior-volume dependent.



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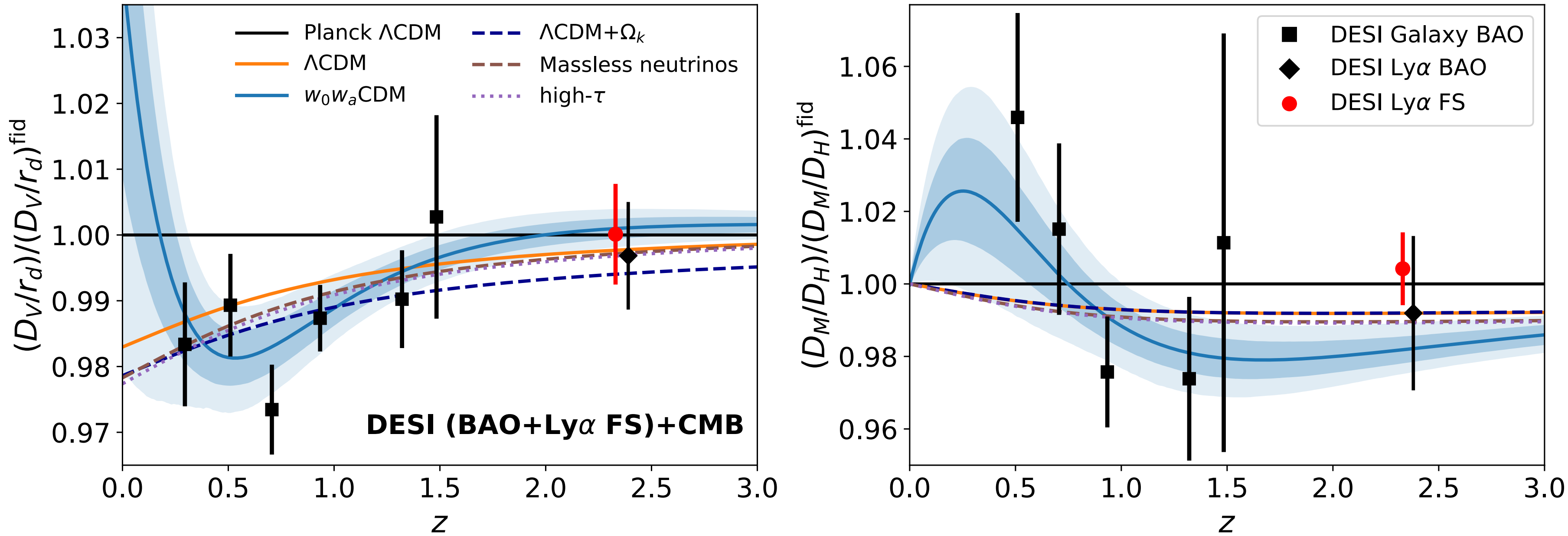


Fig. 13: joint CPL fit including Ly α full shape

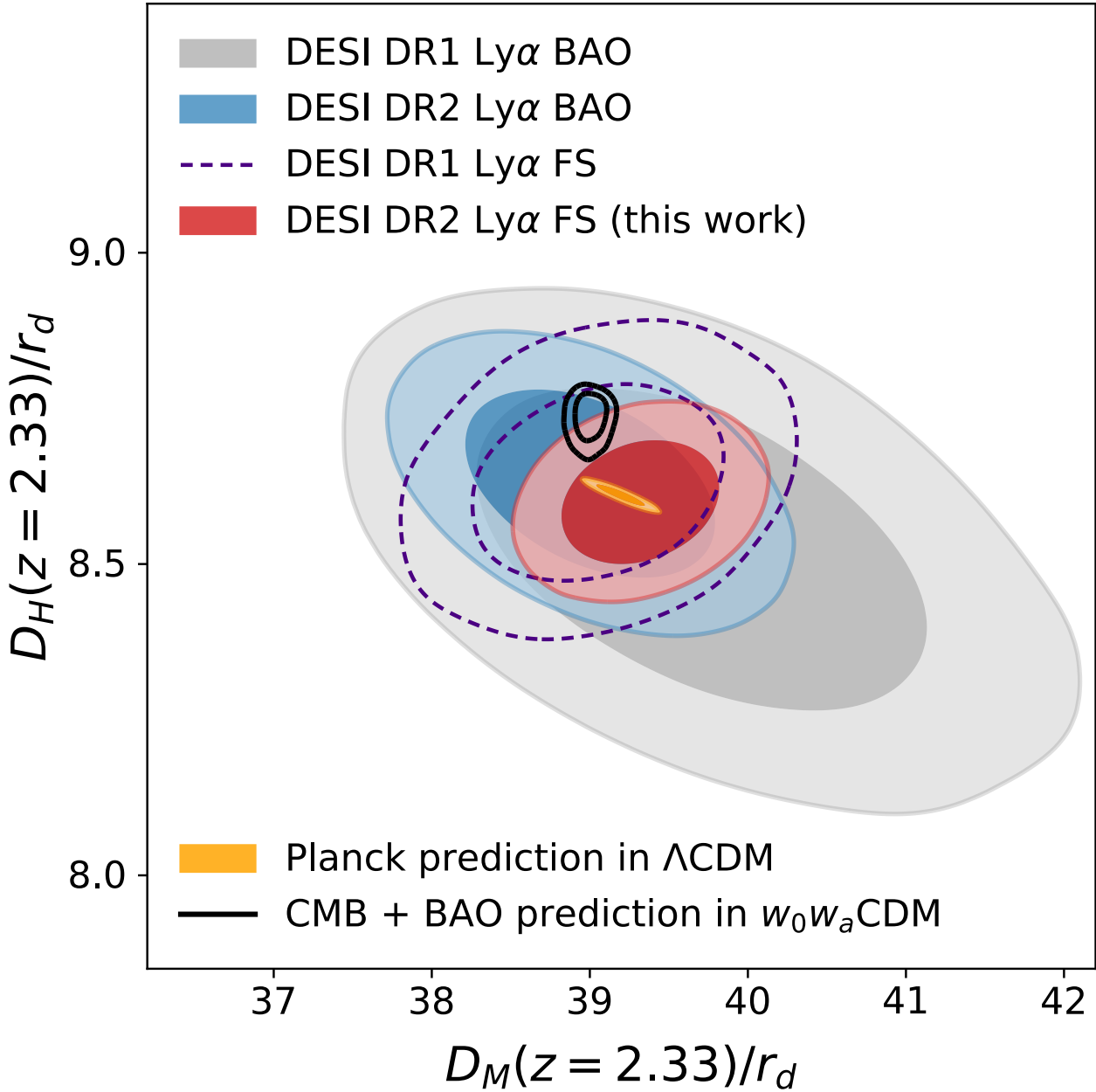


Fig. 7: pre-Ly α -FS CMB+BAO CPL prediction

$z_{\text{eff}} = 2.33$

$D_M/r_d = 39.32 \pm 0.33$

$D_H/r_d = 8.600 \pm 0.066$

$D_M/D_H = 4.572 \pm 0.046$
(1.0% precision)

WHAT DESI DEMONSTRATES

Planck Λ CDM: agreement within 0.5σ | CMB+BAO CPL prediction: $\approx 2\sigma$ away
DESI+CMB: CPL preference decreases $3.2\sigma \rightarrow 2.7\sigma$ after Ly α FS; $\Delta\chi^2_{\text{MAP}} = -9.8$
DESI+CMB+DES-Dovekie SNe: CPL remains preferred at 3.2σ ; $\Delta\chi^2_{\text{MAP}} = -12.9$

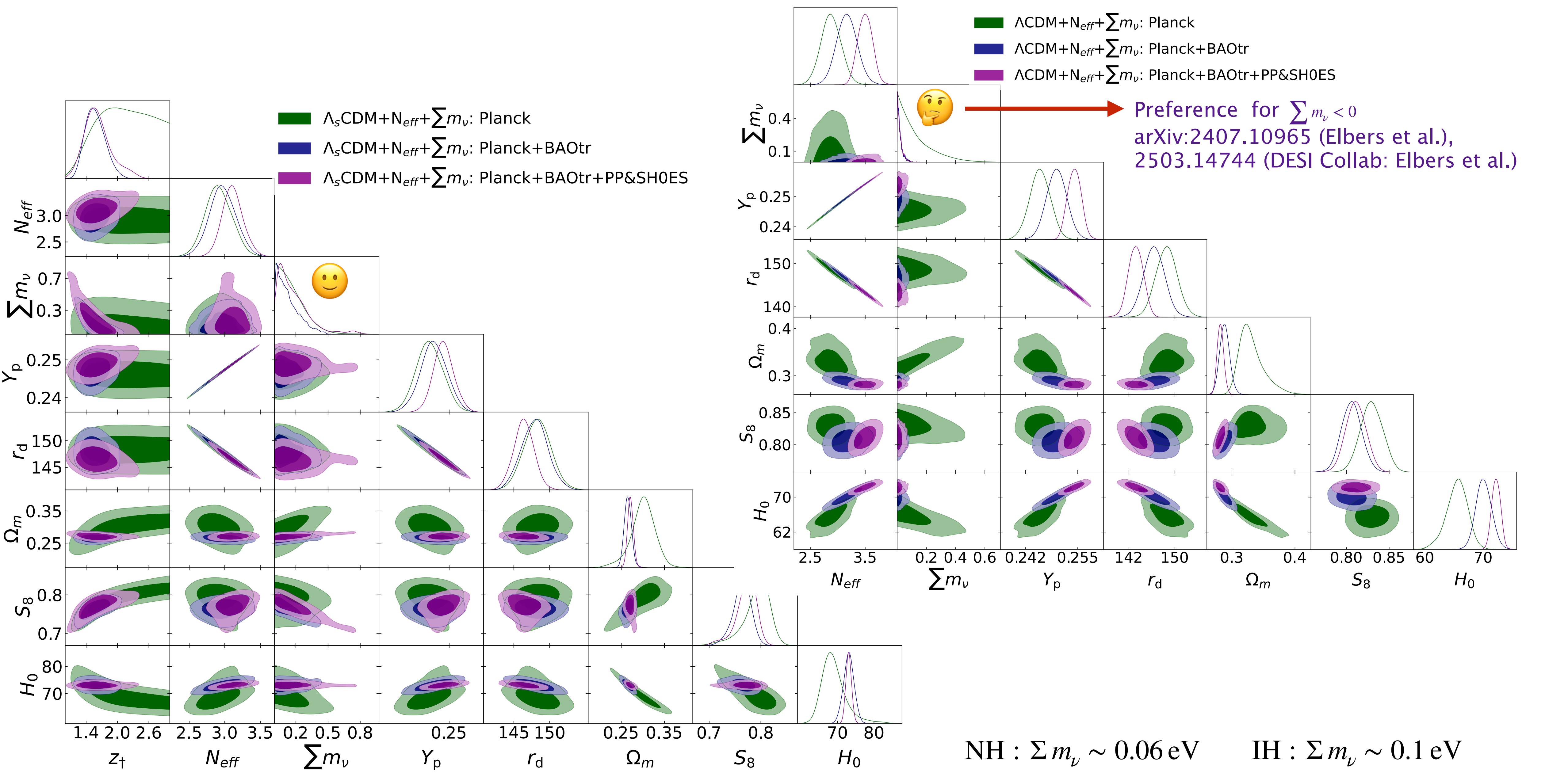
Quoted σ values are frequentist likelihood-ratio significances, not Bayesian evidence.

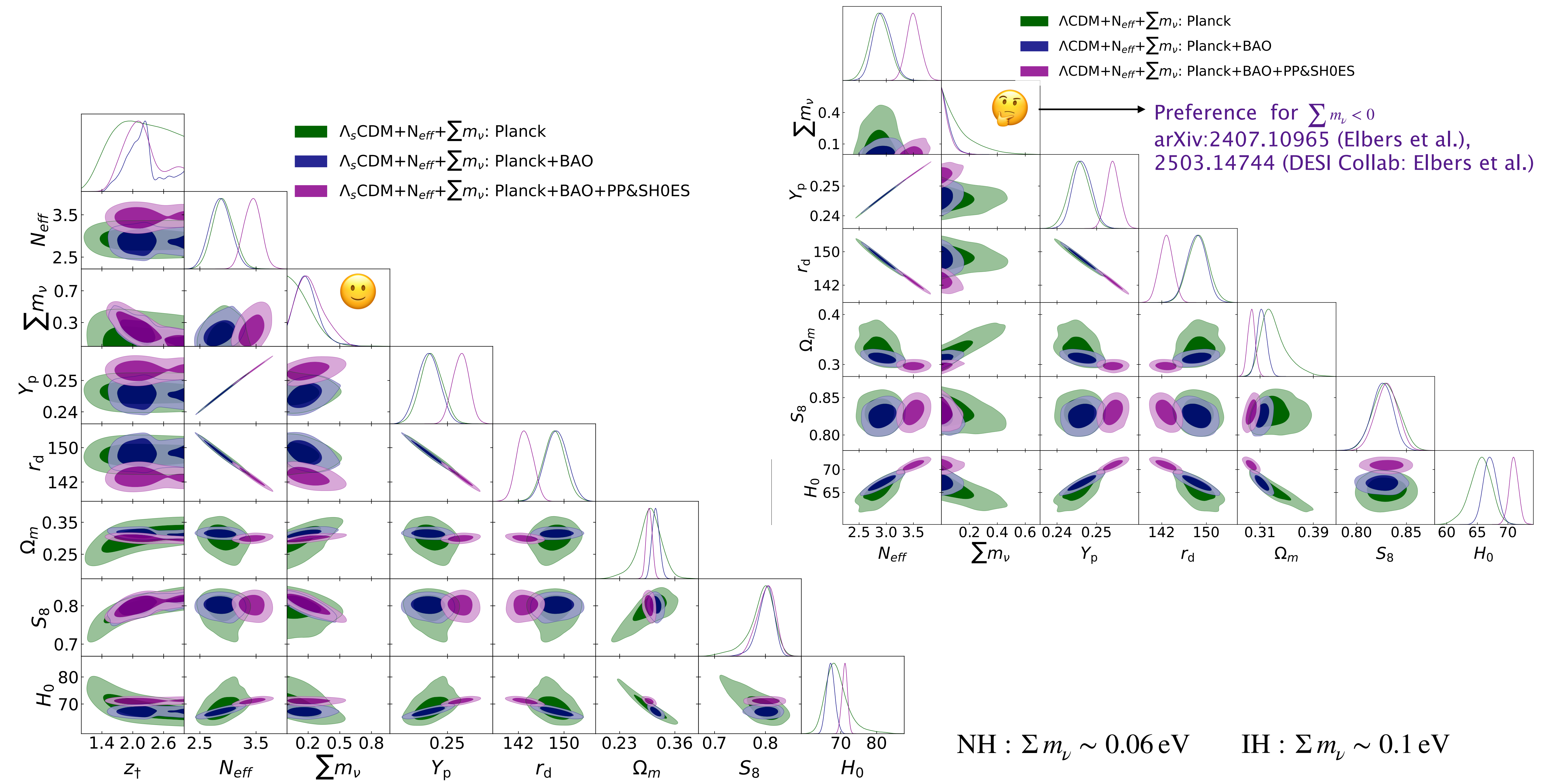
POSSIBLE RELEVANCE FOR Λ_s CDM - OUR INTERPRETATION

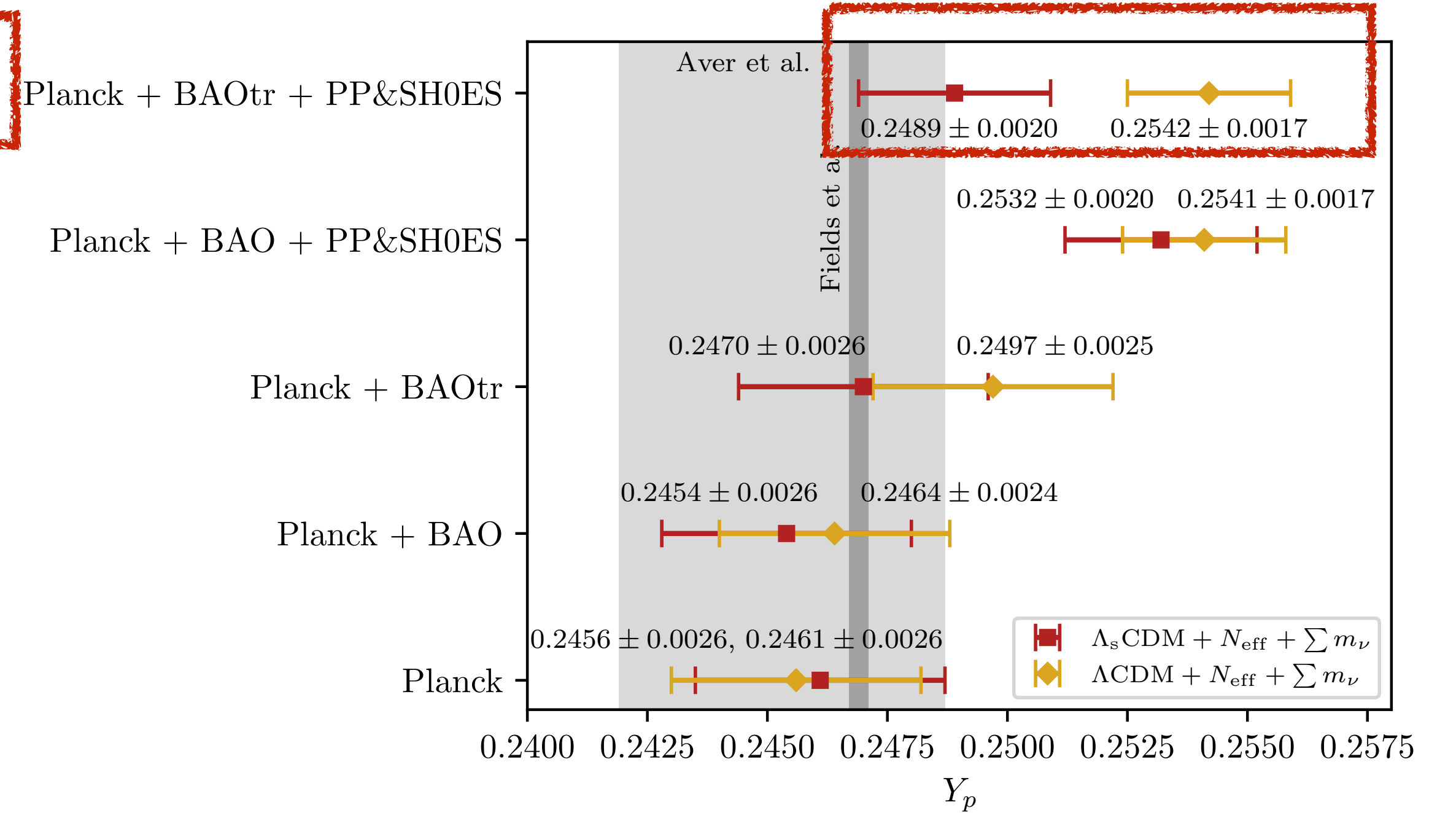
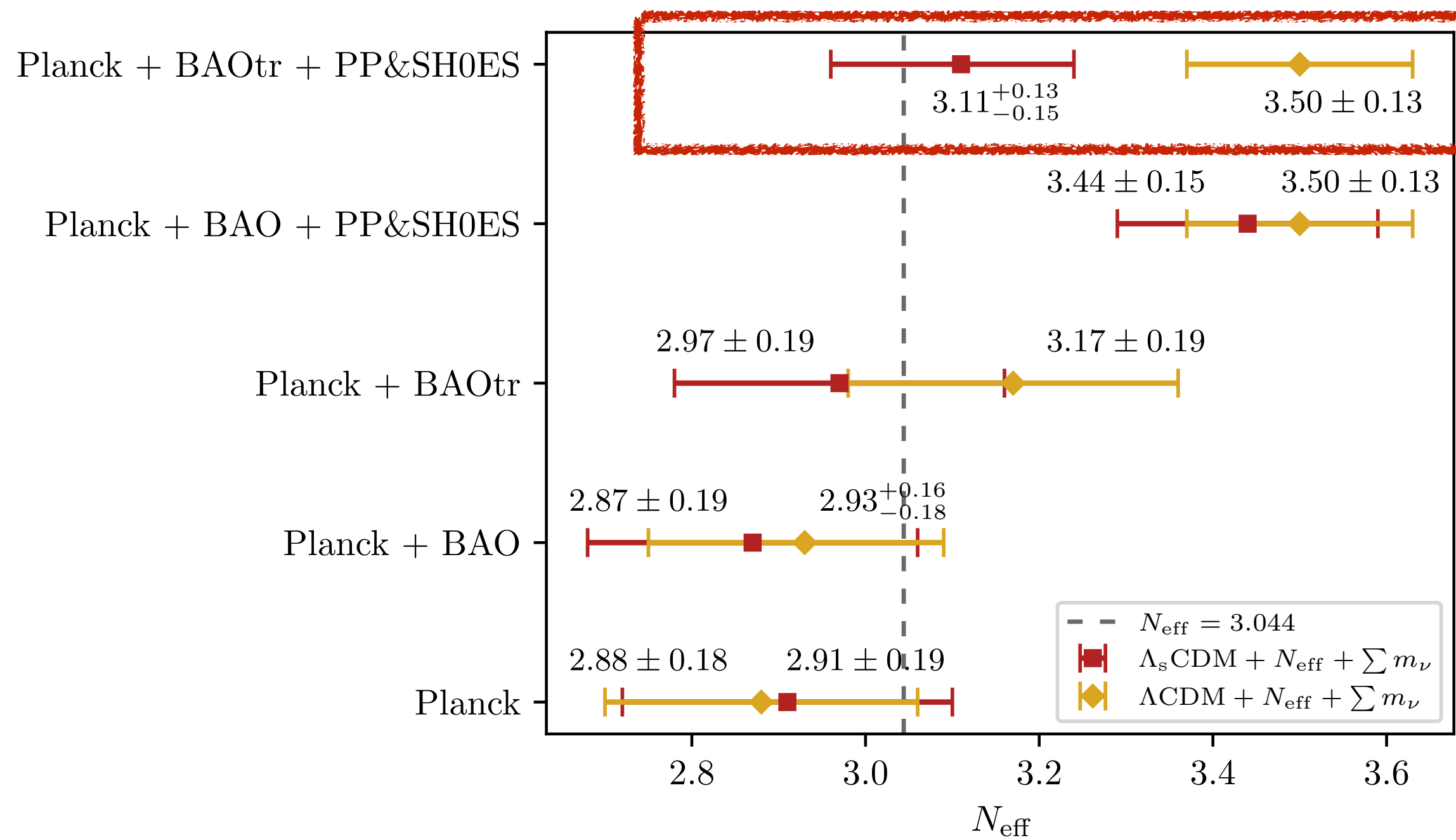
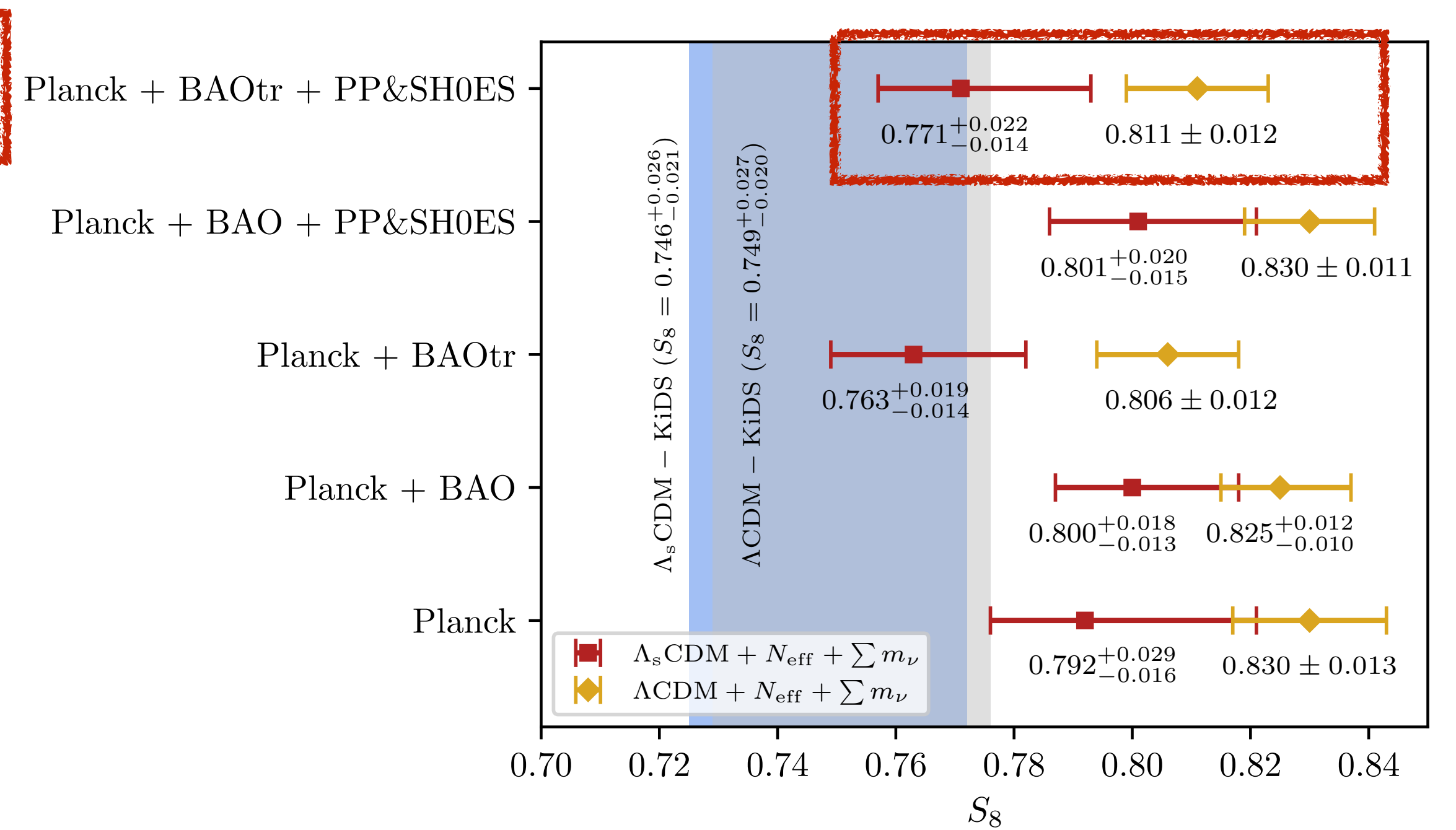
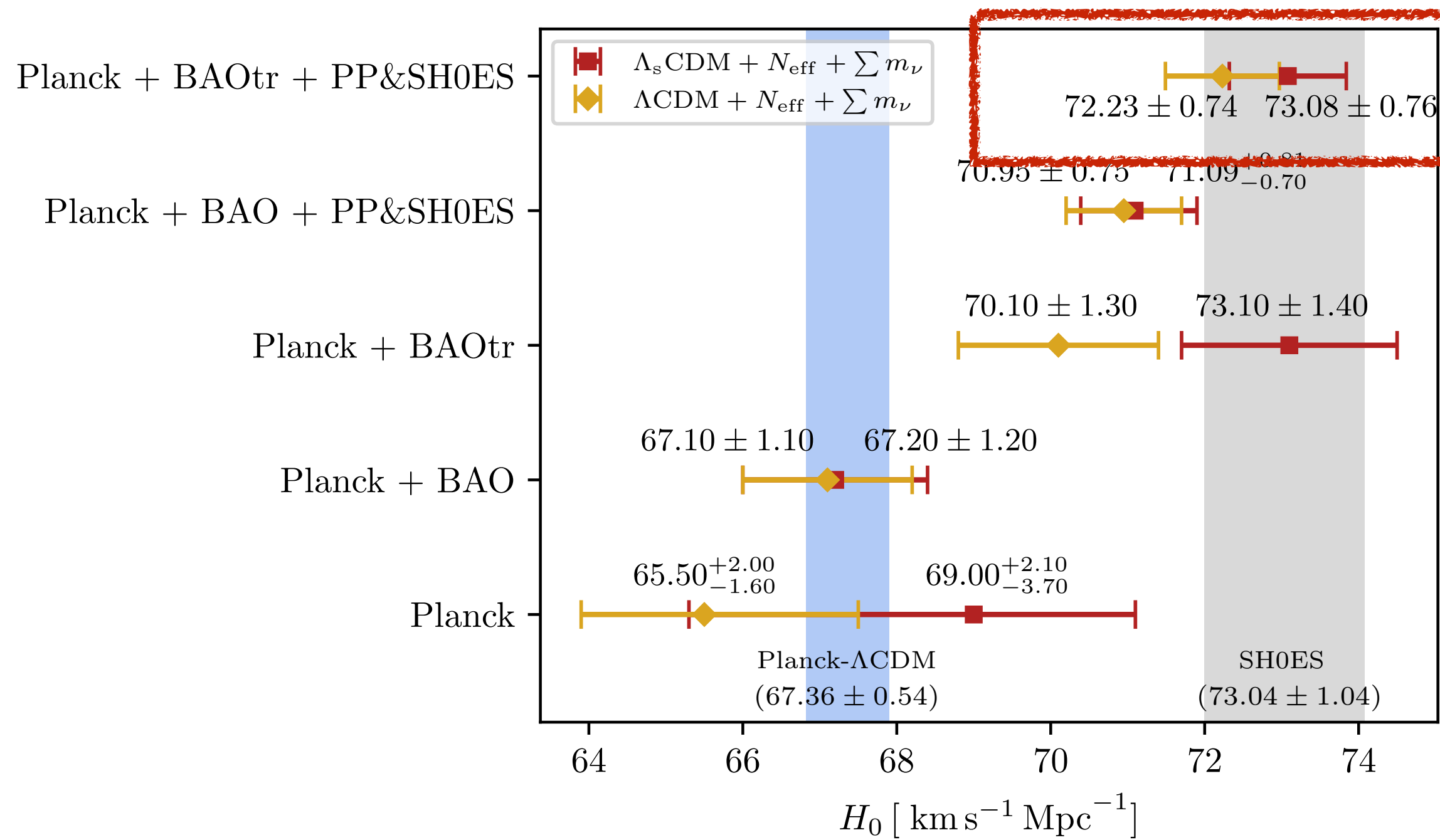
If $z_{\dagger} > 2.33$, the Ly α epoch lies on the positive- Λ , approximately Λ CDM-like branch. The new high-redshift anchor may therefore help distinguish smooth CPL evolution from a sign-switching history.

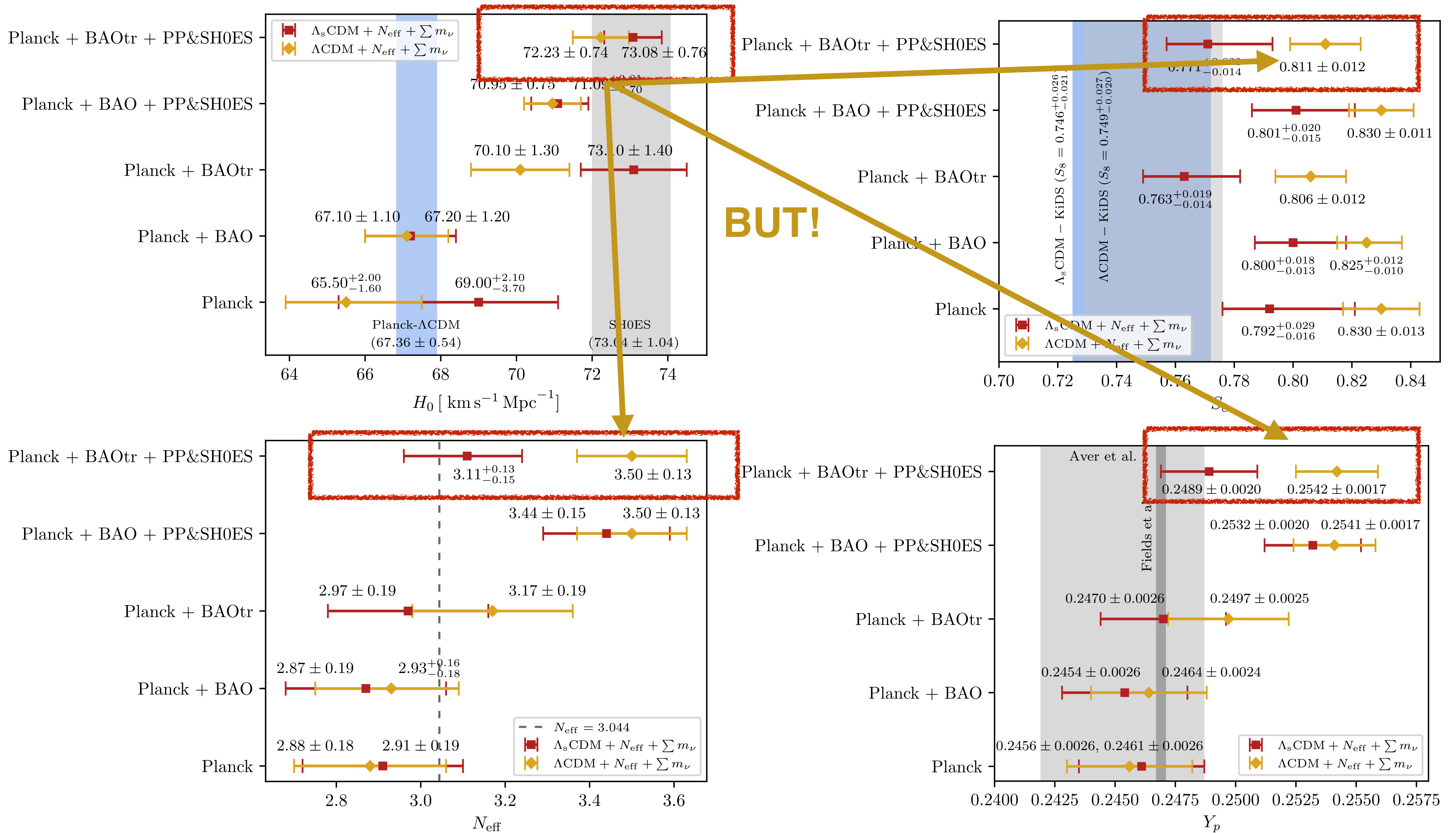
Next step: a common-likelihood fit is needed to assess the relative support for the two models.

$z_{\dagger} > 2.33 \Rightarrow \Lambda_s(z_{\text{eff}}) = +\Lambda_{s0}$



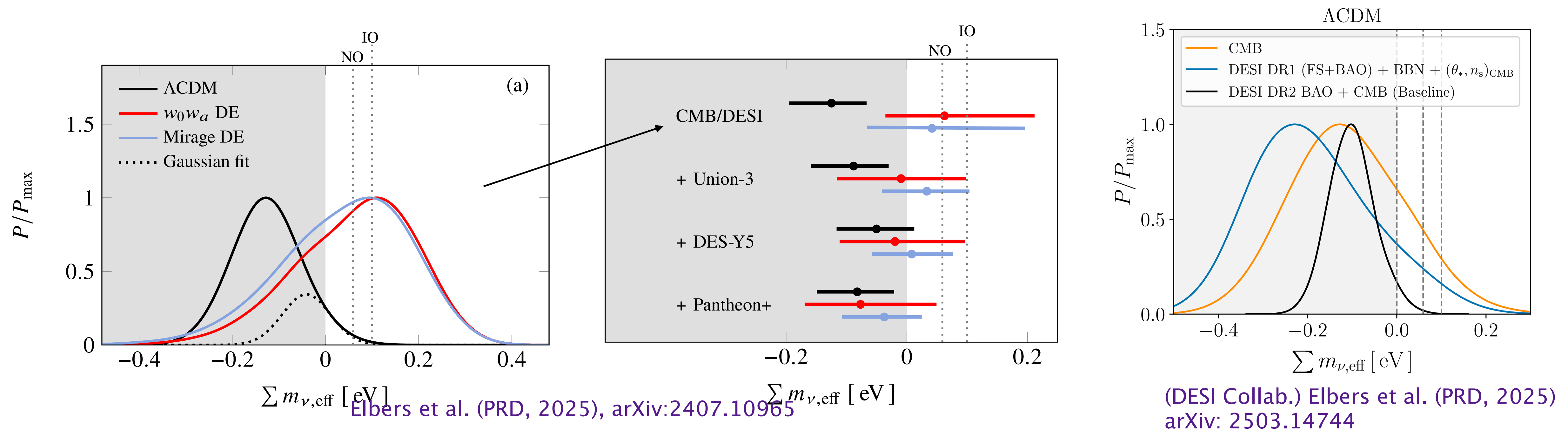


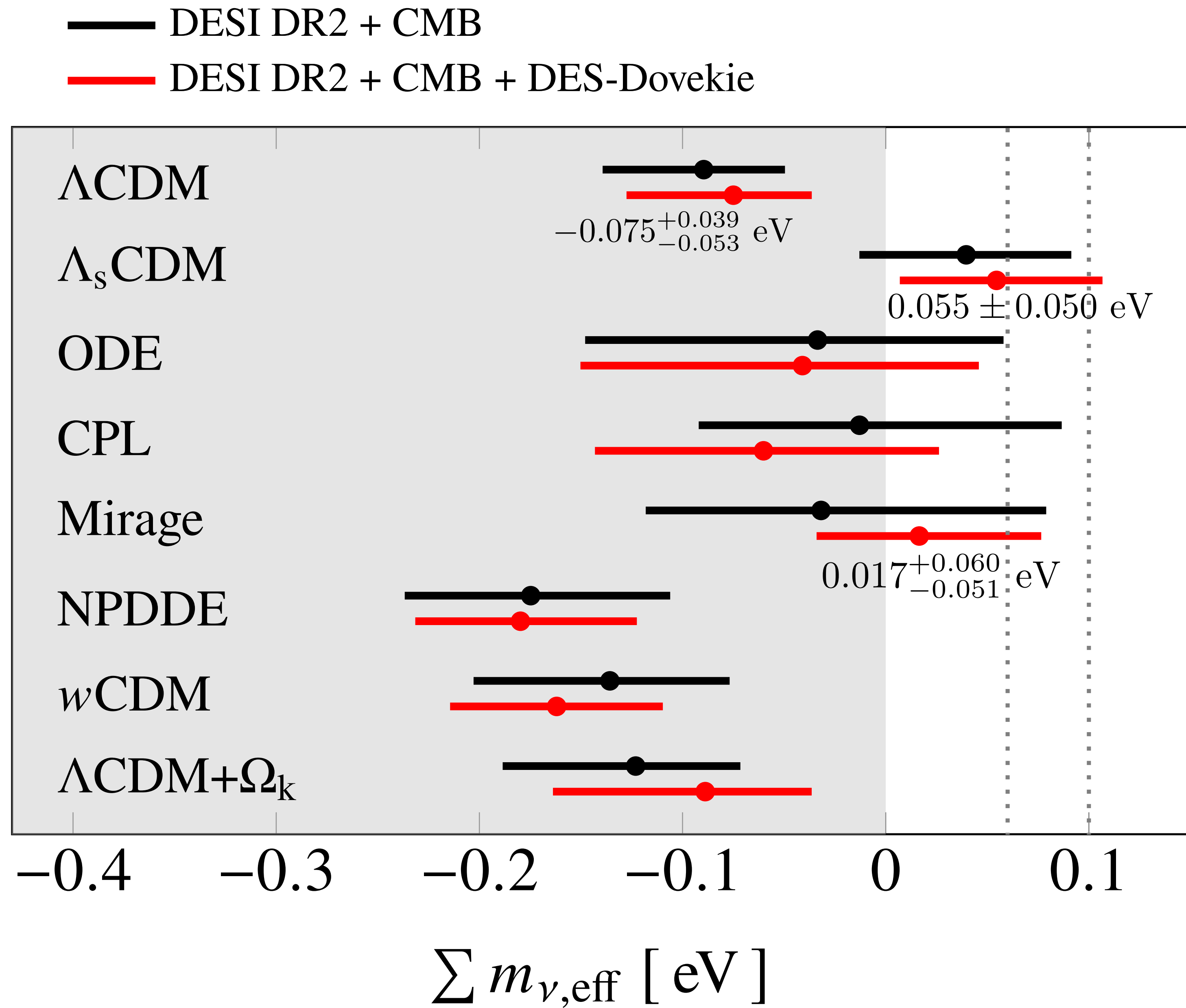
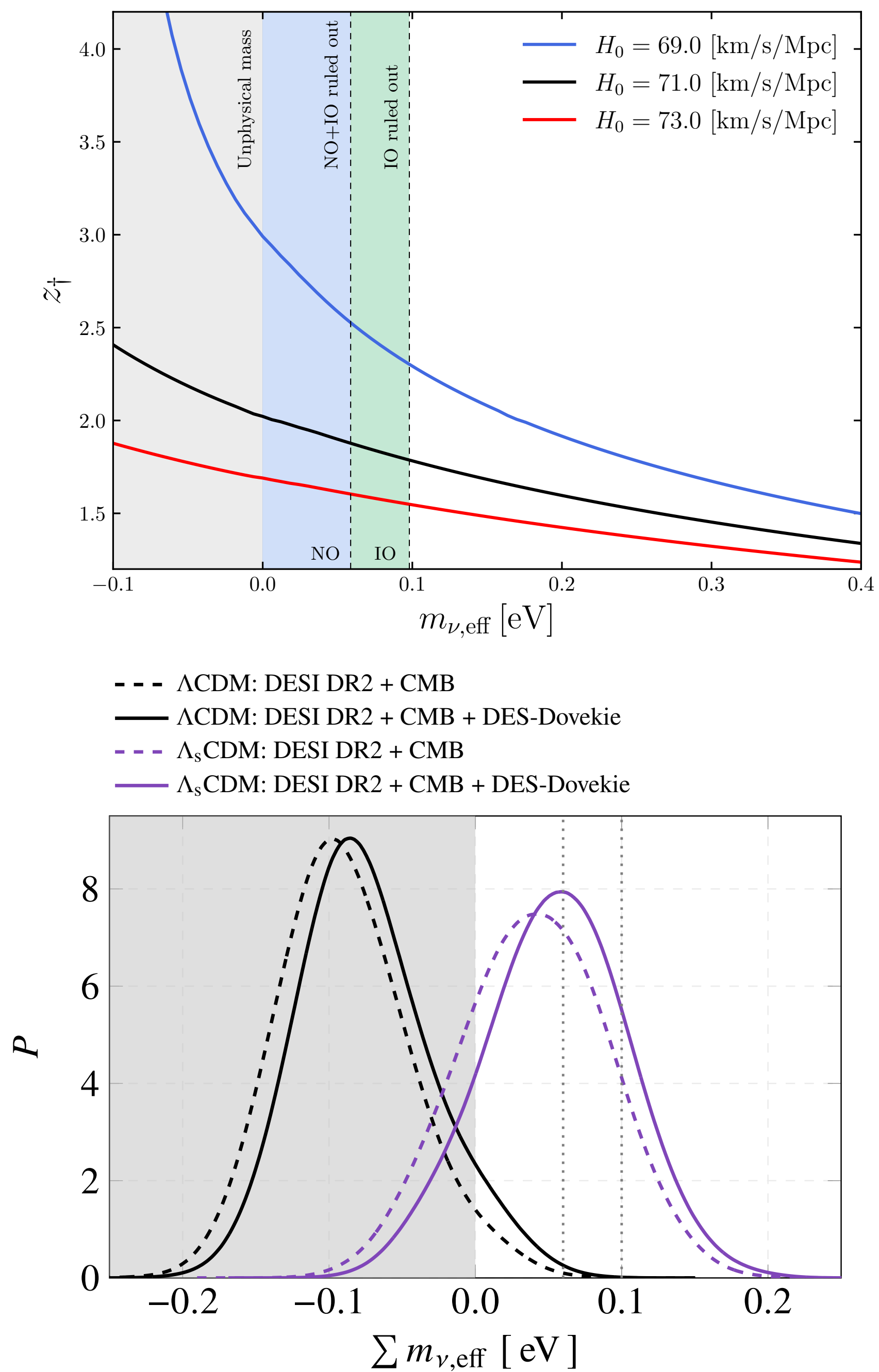




Elbers et al. (PRD, 2025), arXiv:2407.10965

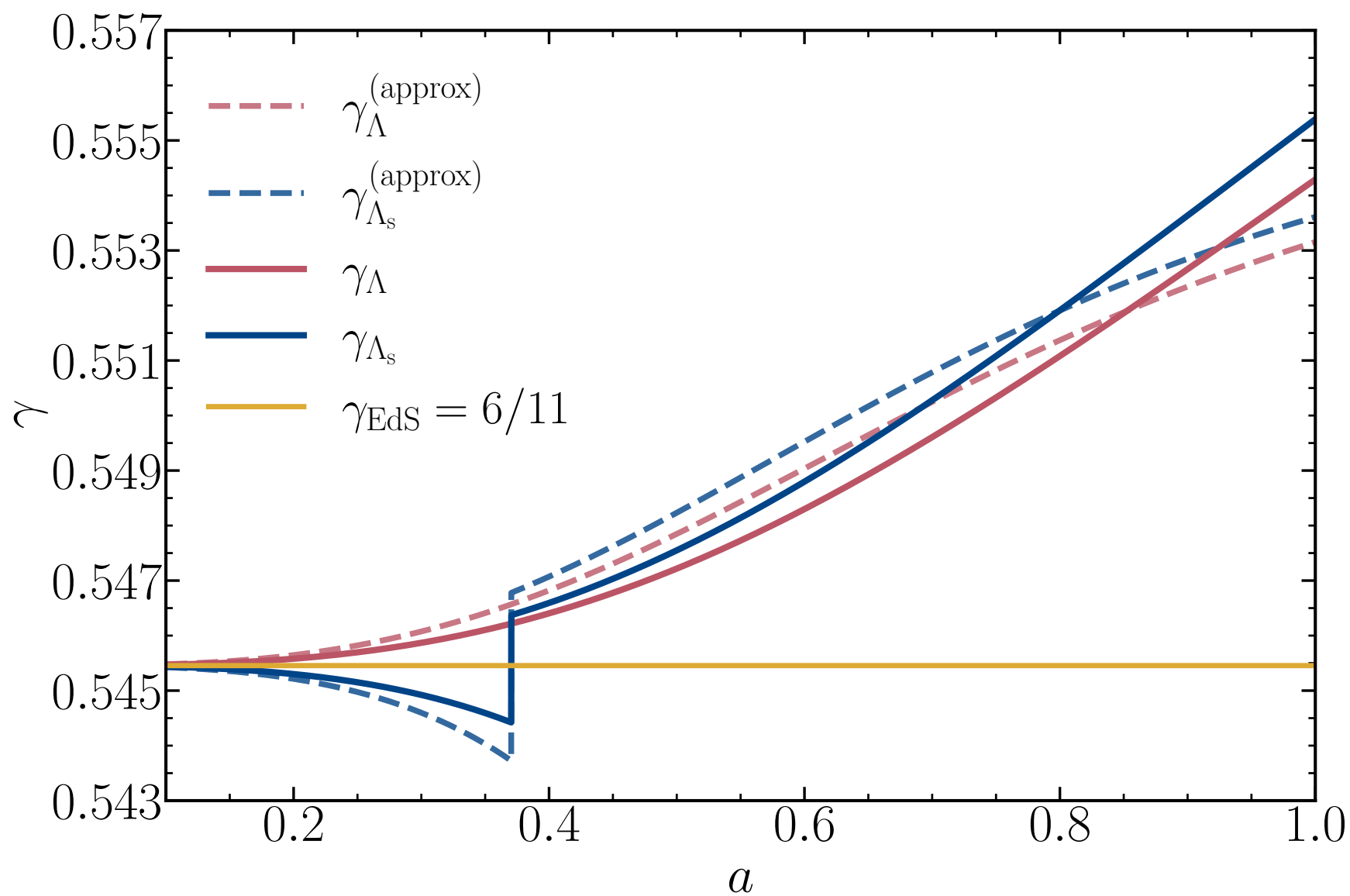
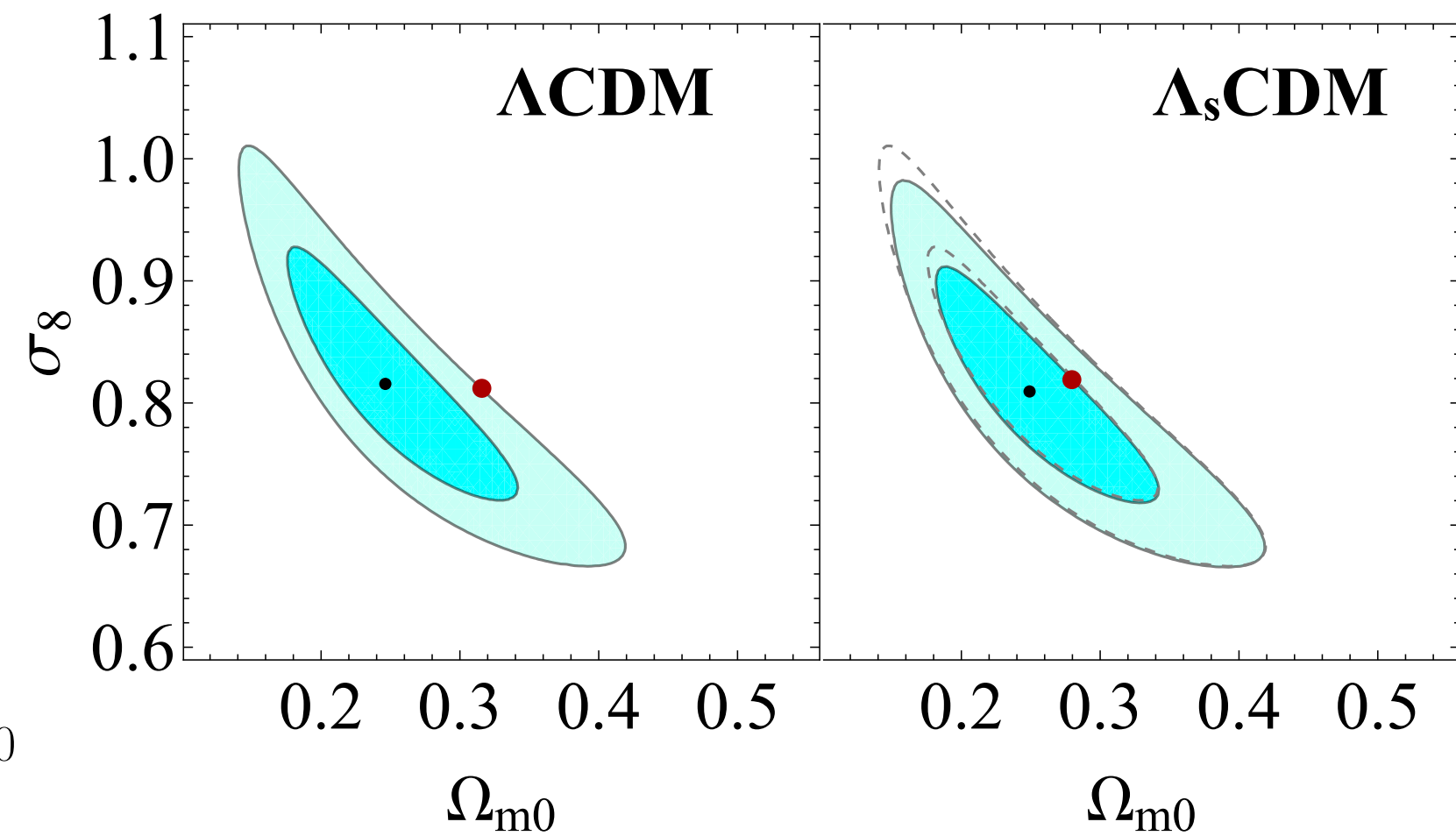
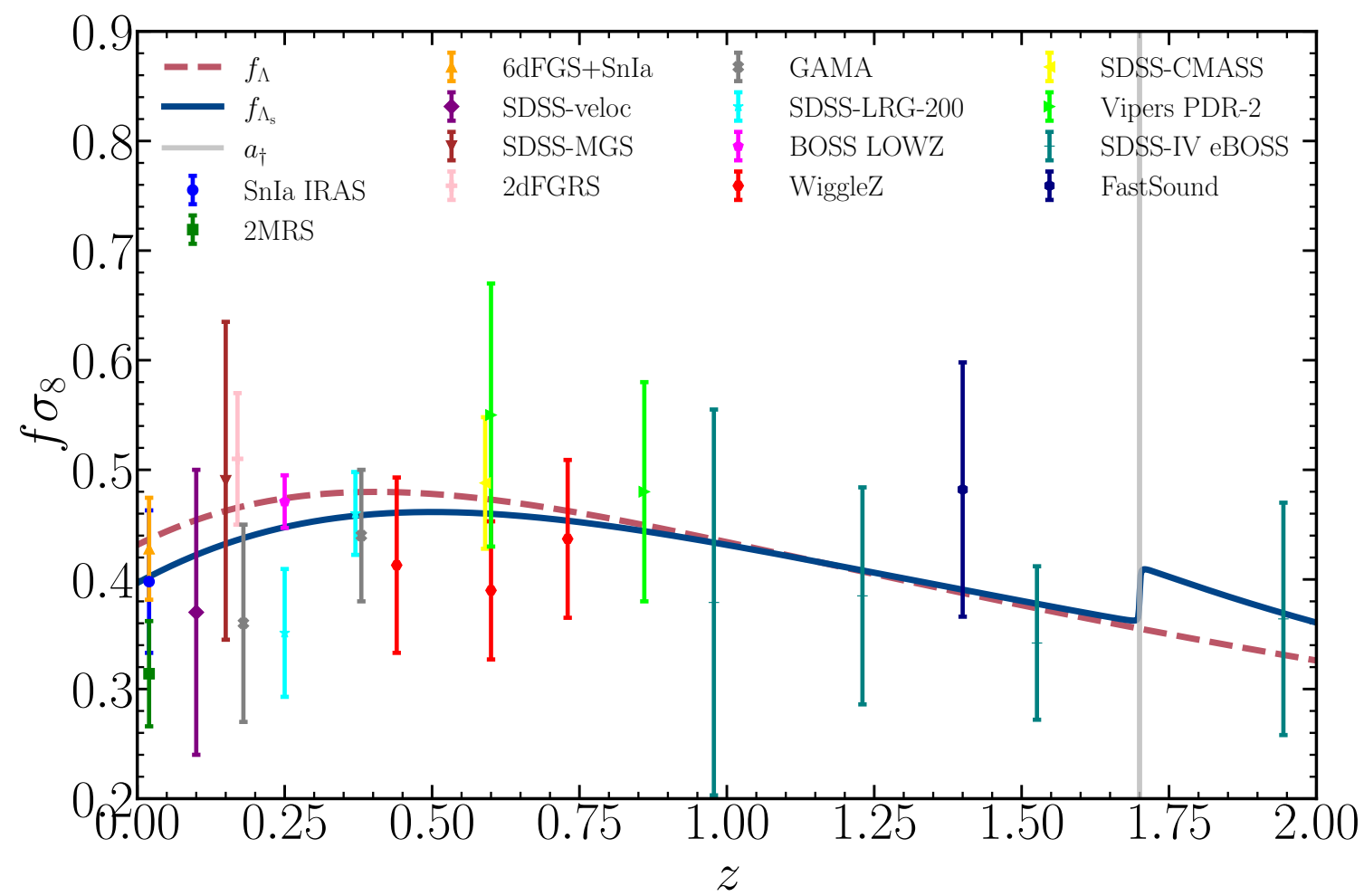
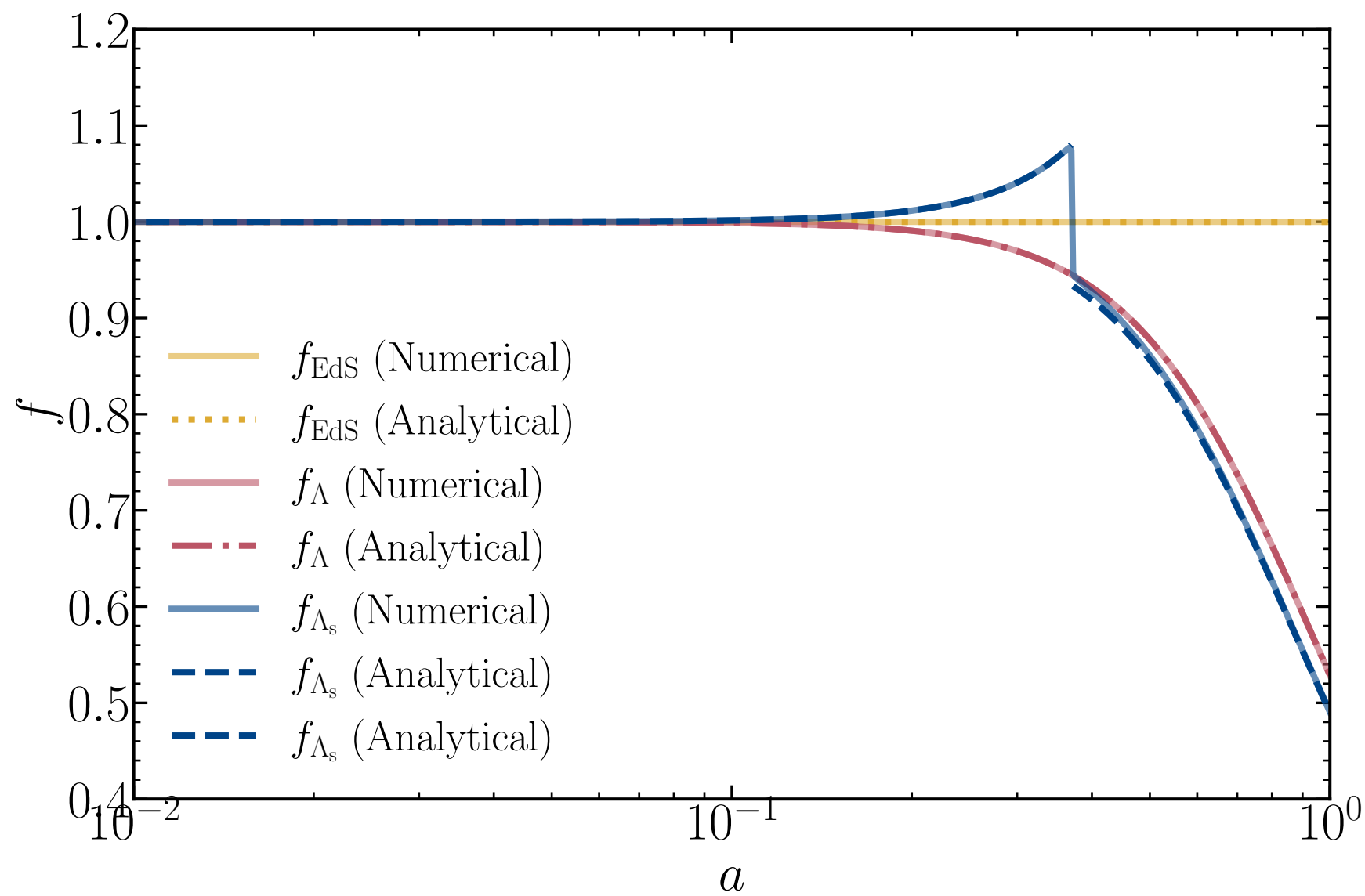
$$\sum m_\nu = m_3 + \sqrt{m_3^2 - \Delta m_{31}^2} + \sqrt{m_3^2 - \Delta m_{31}^2 + \Delta m_{21}^2} > 0.10 \text{ eV if } m_3 < m_1 < m_2 \text{ (the inverted ordering)}$$





Parameter	Λ CDM + fixed $\sum m_\nu$	Λ CDM	Λ_s CDM	ODE	CPL	Mirage	NPDDE	wCDM	Λ CDM+ Ω_k
$100\Omega_b h^2$	2.2482 ± 0.0094	2.2452 ± 0.0096	2.2414 ± 0.0095	2.2455 ± 0.0098	2.2458 ± 0.0098	2.2429 ± 0.0096	2.2493 ± 0.0096	2.2489 ± 0.0096	2.246 ± 0.010
$\Omega_{\text{cdm}} h^2$	0.11796 ± 0.00058	0.11889 ± 0.00065	0.11971 ± 0.00066	0.11894 ± 0.00077	0.11888 ± 0.00076	0.11931 ± 0.00066	0.11825 ± 0.00069	0.11833 ± 0.00068	0.1187 ± 0.0010
$100\theta_s$	1.04185 ± 0.00020	1.04176 ± 0.00020	1.04168 ± 0.00020	1.04175 ± 0.00020	1.04175 ± 0.00020	1.04172 ± 0.00020	1.04179 ± 0.00021	1.04179 ± 0.00020	1.04178 ± 0.00021
$\ln\left(10^{10} A_s\right)$	3.071 ± 0.012	$3.051^{+0.011}_{-0.013}$	$3.054^{+0.011}_{-0.013}$	$3.051^{+0.011}_{-0.013}$	$3.051^{+0.011}_{-0.012}$	$3.053^{+0.010}_{-0.013}$	$3.047^{+0.011}_{-0.012}$	$3.049^{+0.010}_{-0.013}$	$3.050^{+0.011}_{-0.014}$
n_s	0.9731 ± 0.0028	0.9712 ± 0.0029	0.9695 ± 0.0029	0.9711 ± 0.0030	0.9713 ± 0.0030	0.9703 ± 0.0028	0.9725 ± 0.0029	0.9725 ± 0.0029	0.9718 ± 0.0034
τ	$0.0703^{+0.0060}_{-0.0066}$	$0.0606^{+0.0056}_{-0.0069}$	$0.0608^{+0.0054}_{-0.0067}$	$0.0604^{+0.0055}_{-0.0066}$	$0.0603^{+0.0056}_{-0.0066}$	$0.0607^{+0.0054}_{-0.0066}$	$0.0596^{+0.0054}_{-0.0064}$	$0.0600^{+0.0055}_{-0.0066}$	$0.0602^{+0.0056}_{-0.0068}$
$\sum m_{\nu,\text{eff}} \text{ [eV]}$	0.06 (fixed)	$-0.075^{+0.039}_{-0.053}$	0.055 ± 0.050	$-0.041^{+0.087}_{-0.11}$	-0.060 ± 0.083	$0.017^{+0.060}_{-0.051}$	$-0.180^{+0.057}_{-0.052}$	-0.162 ± 0.054	$-0.089^{+0.052}_{-0.075}$
$z_{\dagger}$	—	—	> 2.40	—	—	—	—	—	—
α	—	—	—	$2.33^{+0.87}_{-1.3}$	—	—	—	—	—
β	—	—	—	$4.1^{+1.7}_{-3.1}$	—	—	—	—	—
a_{m}	—	—	—	> 0.895	—	—	—	—	—
w_0 or w	—	—	—	—	-0.830 ± 0.072	-0.822 ± 0.074	$-0.930^{+0.044}_{-0.040}$	-0.932 ± 0.028	—
w_a	—	—	—	—	-0.46 ± 0.30	-0.65 ± 0.27	$0.026^{+0.057}_{-0.13}$	—	—
Ω_k	—	—	—	—	—	—	—	—	-0.0005 ± 0.0015
$H_0 \text{ [km/s/Mpc]}$	68.16 ± 0.24	68.94 ± 0.37	68.88 ± 0.36	67.84 ± 0.59	67.31 ± 0.67	67.76 ± 0.59	67.61 ± 0.64	67.73 ± 0.61	68.92 ± 0.36
Ω_{m}	0.3037 ± 0.0033	0.2958 ± 0.0042	0.3009 ± 0.0042	0.3046 ± 0.0066	0.3107 ± 0.0072	0.3092 ± 0.0069	0.3037 ± 0.0059	0.3033 ± 0.0052	$0.2951^{+0.0047}_{-0.0052}$
σ_8	0.8170 ± 0.0045	0.8409 ± 0.0097	0.8222 ± 0.0094	0.829 ± 0.012	0.826 ± 0.011	0.825 ± 0.011	0.837 ± 0.010	0.8370 ± 0.0094	0.843 ± 0.011
S_8	0.8220 ± 0.0064	0.8349 ± 0.0079	0.8233 ± 0.0079	0.8353 ± 0.0087	0.8405 ± 0.0082	0.8373 ± 0.0078	0.8424 ± 0.0083	0.8414 ± 0.0083	0.8359 ± 0.0080
$r_{\text{d}} \text{ [Mpc]}$	147.53 ± 0.17	147.31 ± 0.18	147.14 ± 0.19	147.30 ± 0.20	147.31 ± 0.20	147.23 ± 0.18	147.45 ± 0.19	147.43 ± 0.19	147.36 ± 0.26
$\Delta\chi^2_{\text{MAP}}$	0.0	-8.104	-7.041	-15.00	-16.49	-14.85	-15.13	-14.62	-9.043
$\ln B$	0.0	-0.21	-2.14	-4.79	-2.06	-0.175	-3.44	-1.28	-5.18

DESI DR2 BAO combined with Type Ia supernovae from the DES-Dovekie compilation, CMB temperature and polarization data from Planck and ACT, and CMB lensing reconstruction from Planck, ACT, and SPT.

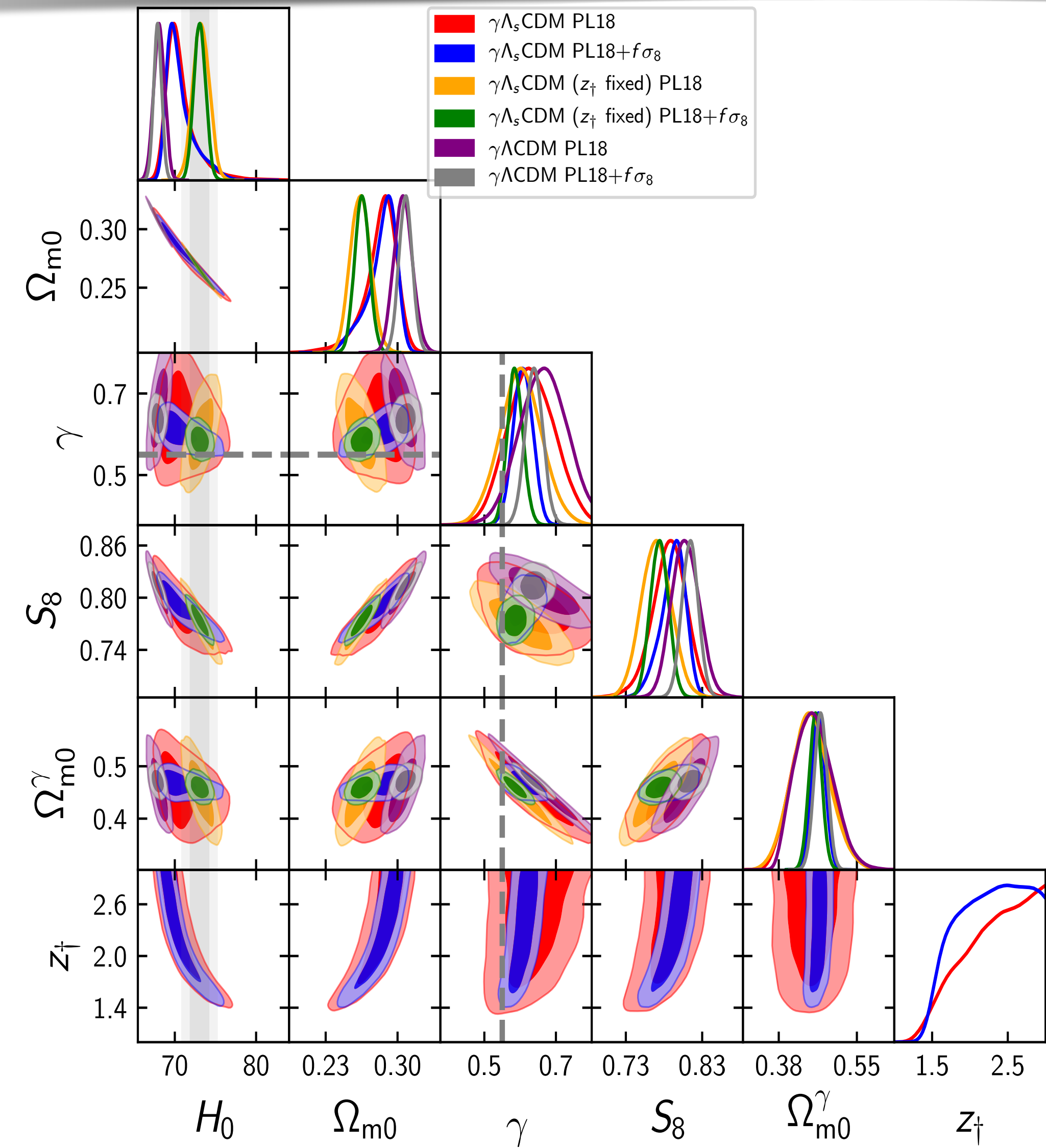


Dataset	$f\sigma_8$ (This Work)		Planck [71]	
	Λ_s CDM	Λ CDM	Λ_s CDM	Λ CDM
Ω_{m0}	0.249 ± 0.050	0.246 ± 0.050	$0.2860^{+0.0230}_{-0.0099}$ (0.2796)	0.3151 ± 0.0075 (0.3163)
σ_8	0.809 ± 0.060	0.816 ± 0.070	$0.8210^{+0.0064}_{-0.0110}$ (0.8191)	$0.8121^{+0.0055}_{-0.0061}$ (0.8136)
S_8	0.738 ± 0.089	0.739 ± 0.095	$0.8010^{+0.0260}_{-0.0160}$ (0.7910)	0.8320 ± 0.0130 (0.8350)
χ^2_{min}	12.04	12.36	2778.06	2780.52

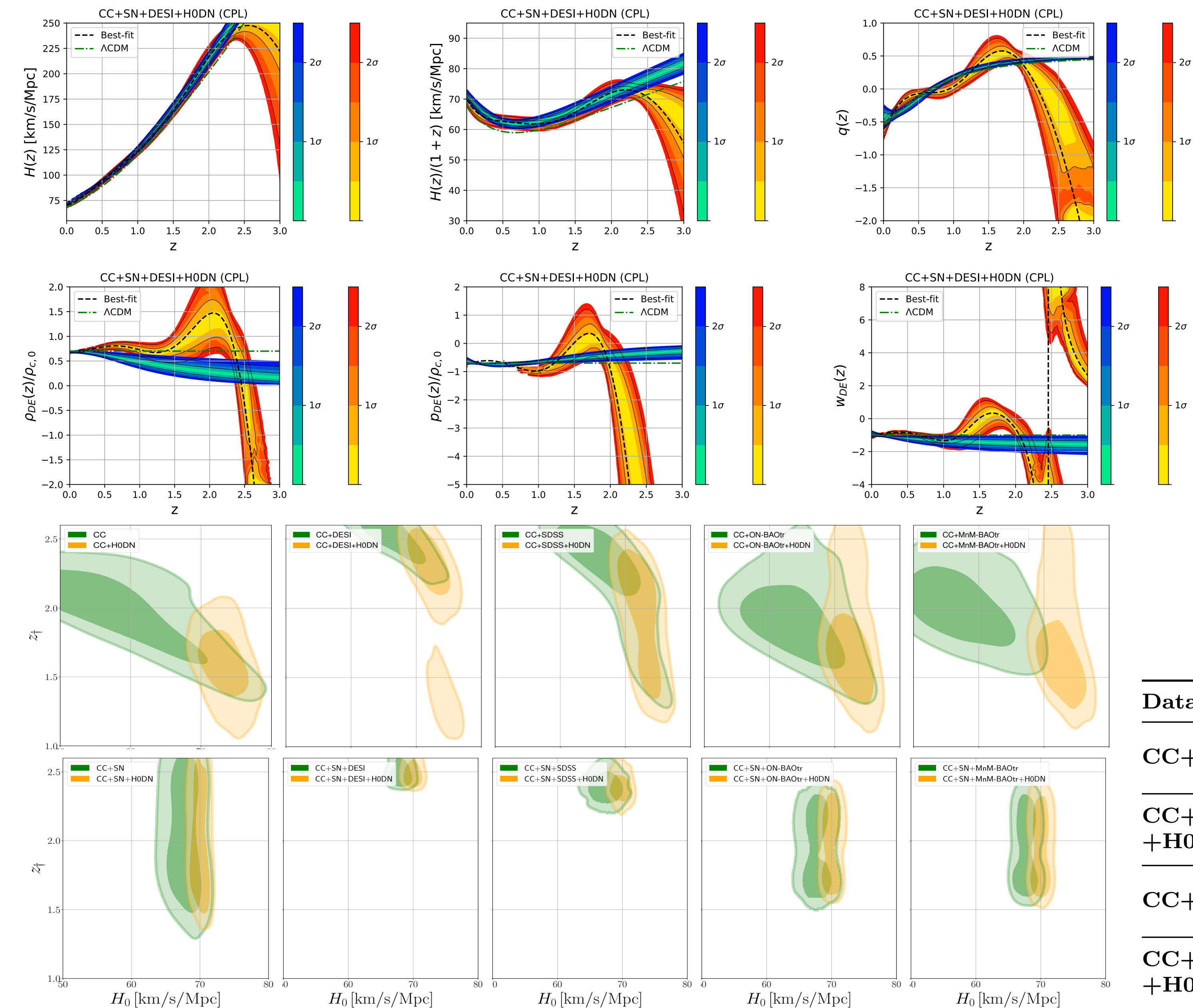
The growth index parameter γ yields almost the same value of ~ 0.55 in both Λ CDM and Λ_s CDM models!

Recently, it was shown that modeling the growth rate of perturbations with the growth index γ , current cosmological data favor values larger than the GR prediction ($\gamma = 0.55$) in flat Λ CDM. In particular, the combination of $f\sigma_8$ and Planck measurements yields $\gamma \simeq 0.64$, excluding $\gamma = 0.55$ at the **4.2 σ** level (Nguyen, Huterer & Wen, *PRL* 2023; arXiv:2302.01331).

Following a similar analysis, we have demonstrated that Λ_s CDM is capable of addressing this recently identified tension.



Model	Data	σ_γ	σ_{H_0}
$\gamma\Lambda_s$ CDM($z_\dagger$ free)	PL18	1.2 σ	1.3 σ
	PL18+ $f\sigma_8$	2.1 σ	1.4 σ
	PL18+ $f\sigma_8$ +PP	3.0 σ	3.2 σ
	PL18+ $f\sigma_8$ +PP&SH0ES	2.4 σ	1.9 σ
	PL18+ $f\sigma_8$ +PP+SDSS	3.1 σ	3.9 σ
	PL18+ $f\sigma_8$ +PP+DESI	3.0 σ	3.6 σ
	PL18+DESI	1.4 σ	3.3 σ
	PL18+DESI+PP&SH0ES	1.5 σ	3.2 σ
$\gamma\Lambda_s$ CDM($z_\dagger = 1.7$)	PL18	0.9 σ	0.1 σ
	PL18+ $f\sigma_8$	1.6 σ	0.0 σ
$\gamma\Lambda_s$ CDM($z_\dagger = 2.16$)	PL18+ $f\sigma_8$ +PP&SH0ES	2.2 σ	1.8 σ
$\gamma\Lambda$ CDM	PL18	1.8 σ	4.0 σ
	PL18+ $f\sigma_8$	3.6 σ	4.5 σ
	PL18+ $f\sigma_8$ +PP	3.8 σ	4.7 σ
	PL18+ $f\sigma_8$ +PP&SH0ES	3.6 σ	4.2 σ
	PL18+ $f\sigma_8$ +PP+SDSS	3.4 σ	4.7 σ
	PL18+ $f\sigma_8$ +PP+DESI	3.5 σ	4.4 σ
	PL18+DESI	2.6 σ	3.9 σ
	PL18+DESI+PP&SH0ES	2.8 σ	3.8 σ



Dataset	Parameter	Rec.	Λ CDM	w CDM	CPL
CC+DESI	H_0	65.96 ± 3.80	68.50 ± 0.53	67.0 ± 2.0	64.69 ± 2.12
	q_0	$-0.35^{+0.27}_{-0.31}$	-0.55 ± 0.01	-0.486 ± 0.077	$0.02^{+0.34}_{-0.22}$
	w_0	$-0.81^{+0.26}_{-0.30}$	-1	-0.939 ± 0.071	-0.48 ± 0.26
	w_a	—	—	—	$-1.66^{+0.87}_{-1.0}$
	$\Delta\chi^2_{\min,i}$	-3.47	—	-0.91	-3.42
	$\ln B_{\Lambda\text{CDM},i}$	10.76	—	2.01	1.27
CC+DESI+H0DN	H_0	72.41 ± 1.71	68.92 ± 0.42	72.14 ± 0.85	70.10 ± 1.09
	q_0	-0.61 ± 0.20	-0.55 ± 0.01	-0.709 ± 0.045	-0.46 ± 0.23
	w_0	-1.02 ± 0.19	-1	-1.143 ± 0.037	-0.92 ± 0.19
	w_a	—	—	—	$-0.45^{+0.76}_{-0.59}$
	$\Delta\chi^2_{\min,i}$	-6.88	—	-4.03	-4.87
	$\ln B_{\Lambda\text{CDM},i}$	10.59	—	2.14	1.54
CC+SN+DESI	H_0	68.05 ± 1.63	68.54 ± 0.52	66.7 ± 1.2	67.42 ± 1.16
	q_0	$-0.56^{+0.16}_{-0.22}$	-0.54 ± 0.01	-0.471 ± 0.037	-0.43 ± 0.07
	w_0	$-1.01^{+0.15}_{-0.21}$	-1	-0.926 ± 0.038	-0.89 ± 0.06
	w_a	—	—	—	$-0.30^{+0.34}_{-0.29}$
	$\Delta\chi^2_{\min,i}$	-6.61	—	-3.81	-4.11
	$\ln B_{\Lambda\text{CDM},i}$	12.23	—	1.08	2.11
CC+SN+DESI+H0DN	H_0	70.12 ± 1.31	68.92 ± 0.42	71.24 ± 0.63	71.24 ± 0.85
	q_0	$-0.57^{+0.11}_{-0.15}$	-0.548 ± 0.011	-0.598 ± 0.029	$-0.43^{+0.06}_{-0.07}$
	w_0	$-1.01^{+0.08}_{-0.20}$	-1	-1.060 ± 0.027	-0.91 ± 0.06
	w_a	—	—	—	$-0.46^{+0.36}_{-0.27}$
	$\Delta\chi^2_{\min,i}$	-7.24	—	-1.77	-3.11
	$\ln B_{\Lambda\text{CDM},i}$	12.12	—	2.92	2.11

Dataset	Parameter	Rec.	Λ CDM	w CDM	CPL
CC+DESI	$q(1.7)$	$0.589^{+0.090}_{-0.11}$	$0.340^{+0.0056}_{-0.0056}$	$0.324^{+0.020}_{-0.020}$	$0.350^{+0.024}_{-0.024}$
	$w_{\text{DE}}(1.7)$	$0.18^{+0.27}_{-0.34}$	-1	$-0.939^{+0.071}_{-0.071}$	$-1.53^{+0.32}_{-0.32}$
CC+DESI+H0DN	$q(1.7)$	$0.61^{+0.13}_{-0.11}$	$0.3422^{+0.0055}_{-0.0055}$	$0.374^{+0.009}_{-0.009}$	$0.375^{+0.011}_{-0.016}$
	$w_{\text{DE}}(1.7)$	$-0.81^{+2.61}_{-2.38}$	-1	$-1.143^{+0.037}_{-0.037}$	$-1.10^{+0.26}_{-0.21}$
CC+SN+DESI	$q(1.7)$	$0.565^{+0.078}_{-0.10}$	$0.3440^{+0.0049}_{-0.0049}$	$0.321^{+0.014}_{-0.013}$	$0.334^{+0.018}_{-0.019}$
	$w_{\text{DE}}(1.7)$	$0.08^{+0.36}_{-0.27}$	-1	$-0.926^{+0.038}_{-0.038}$	$-1.06^{+0.17}_{-0.16}$
CC+SN+DESI+H0DN	$q(1.7)$	$0.560^{+0.080}_{-0.10}$	$0.3457^{+0.0048}_{-0.0048}$	$0.362^{+0.008}_{-0.008}$	$0.391^{+0.015}_{-0.015}$
	$w_{\text{DE}}(1.7)$	$0.07^{+0.37}_{-0.31}$	-1	$-1.060^{+0.027}_{-0.027}$	$-1.48^{+0.18}_{-0.15}$

Model independent scalar field reconstruction

Double scalar field action (canonical+phantom)

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} R - \frac{1}{2} (\nabla Q)^2 - U_1(Q) + \frac{1}{2} (\nabla P)^2 - U_2(P) \right] + \mathcal{S}_m$$

Einstein field equations

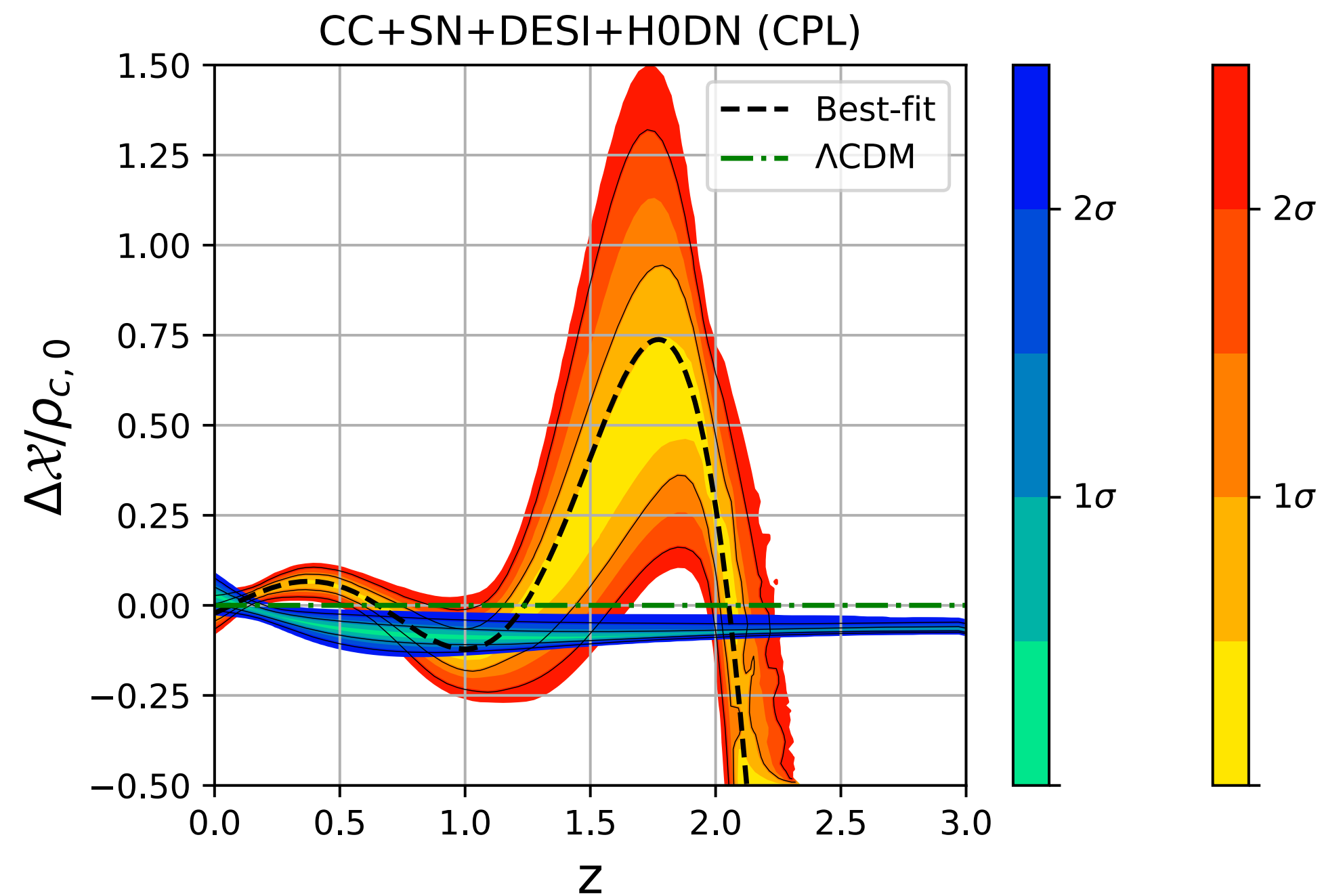
$$G_{\mu\nu} = \kappa^2 \left[\nabla_\mu Q \nabla_\nu Q - g_{\mu\nu} \left(\frac{1}{2} (\nabla Q)^2 + U_1 \right) - \nabla_\mu P \nabla_\nu P + g_{\mu\nu} \left(\frac{1}{2} (\nabla P)^2 - U_2 \right) + T_{\mu\nu}^{(m)} \right]$$

$$\square Q = U_{1,Q} \quad \square P = -U_{2,P}$$

Friedmann equations

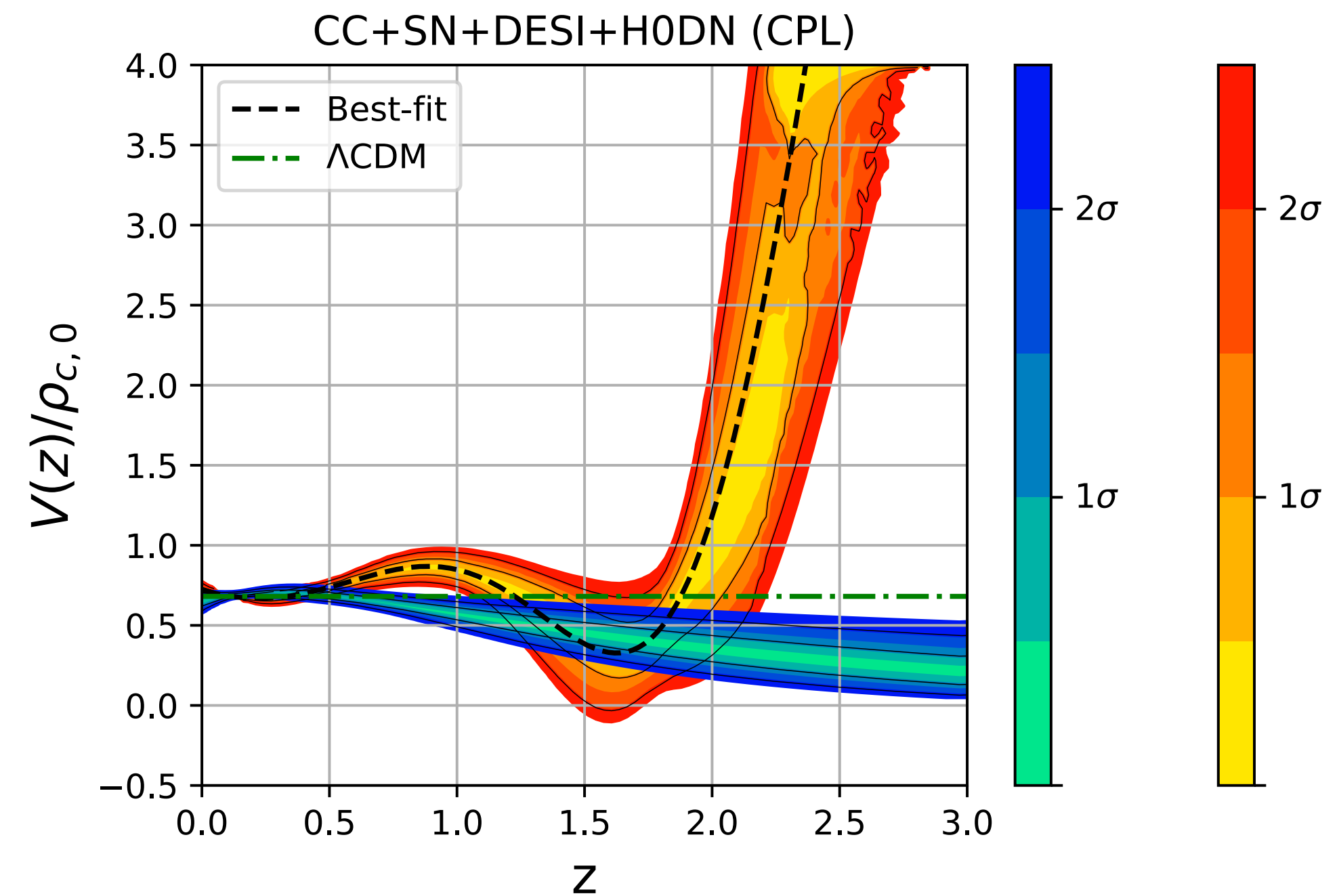
$$3H^2 = \kappa^2 \left[U_1(Q) + U_2(P) + \frac{1}{2} (\dot{Q}^2 - \dot{P}^2) + \rho_m \right]$$

$$3H^2 + 2\dot{H} = \kappa^2 \left[U_1(Q) + U_2(P) - \frac{1}{2} (\dot{Q}^2 - \dot{P}^2) - p_m \right]$$



$$\kappa^2 \Delta \mathcal{X} \equiv \frac{\kappa^2}{2} (\dot{Q}^2 - \dot{P}^2)$$

$$= -\dot{H} - \frac{\kappa^2}{2} (\rho_m + p_m)$$



$$\kappa^2 U \equiv \kappa^2 (U_1(Q) + U_2(P))$$

$$= 3H^2 + \dot{H} - \frac{\kappa^2}{2} (\rho_m - p_m)$$

- **Anti-de Sitter $\rightarrow$ de Sitter transition driven by Casimir forces and mitigating tensions in cosmological parameters** Anchordoqui, Antoniadis, Lüst, PLB (2024), arXiv:2312.12352
- **From infinite to infinitesimal: Using the Universe as a dataset to probe Casimir corrections to the vacuum energy from fields inhabiting the dark dimension** Anchordoqui, Antoniadis, Lüst, Noble, Soriano, PDU (2024), arXiv:2404.17334
- **Thin-wall vacuum decay in the presence of a compact dimension meets the H_0 and S_8 tensions** Anchordoqui, Antoniadis, Bielli, Chatrabhuti, Isono, JHEP (2025), arXiv:2410.18649
- **New insights on a sign-switching Λ** Soriano, Wohlberg, Anchordoqui, PDU (2025), arXiv:2502.19239

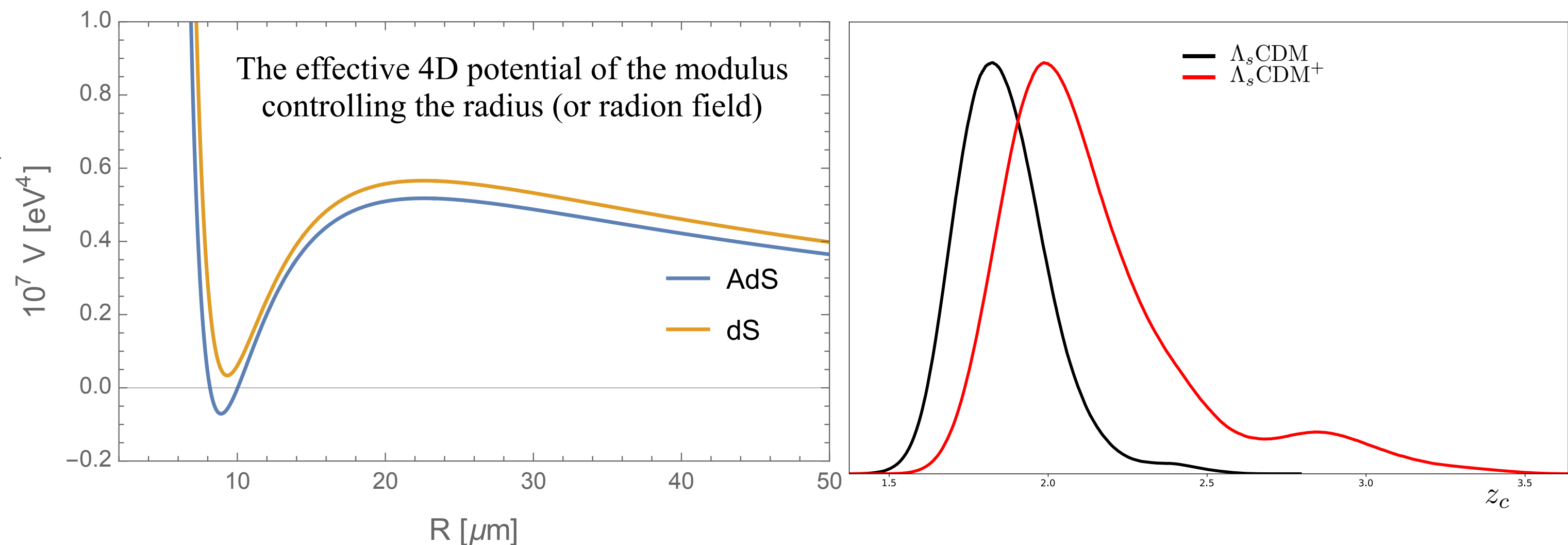
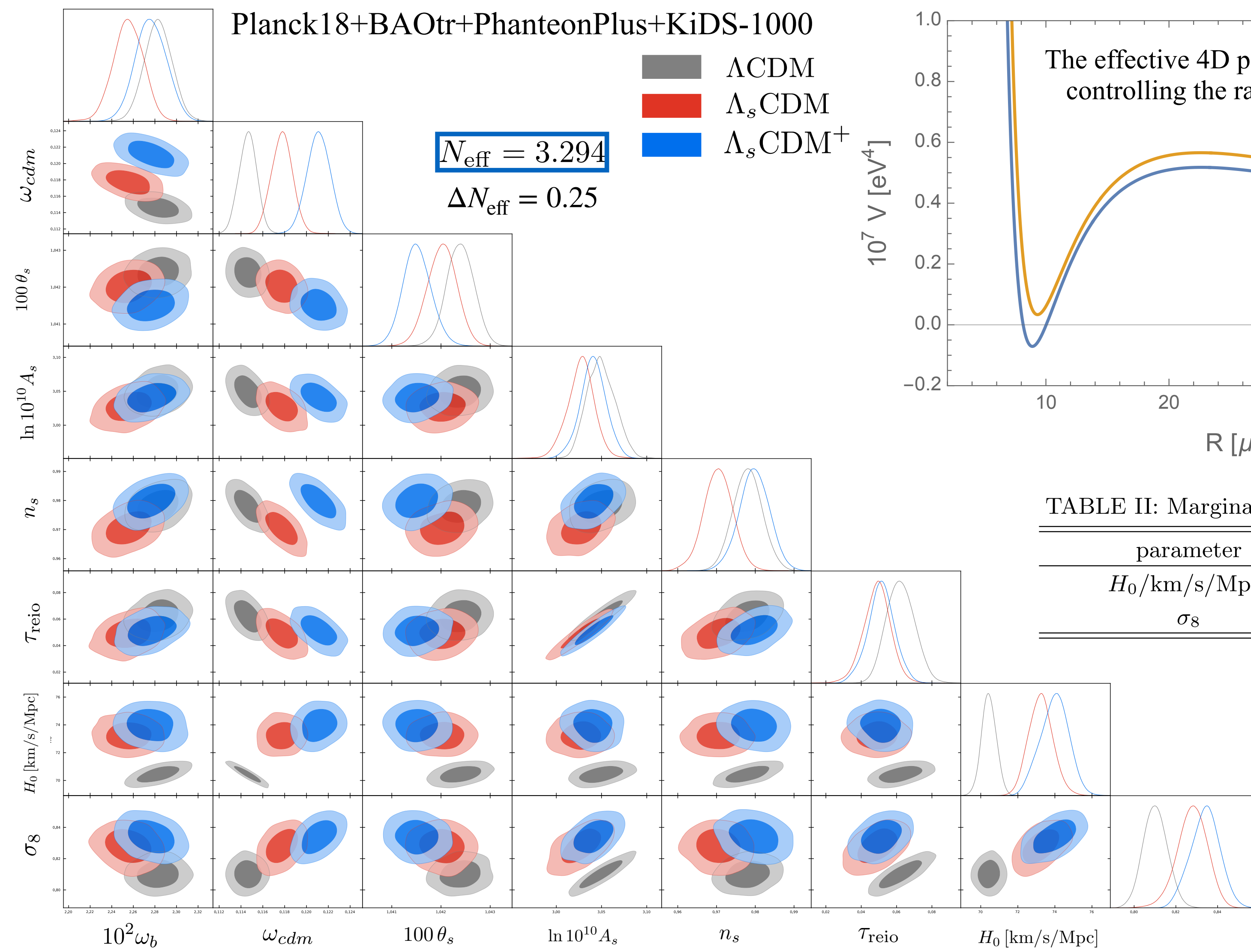


TABLE II: Marginalized constraints, mean values with 68% CL, on derived parameters of the models.

parameter	Λ CDM	Λ_s CDM	Λ_s CDM ⁺
H_0 /km/s/Mpc	70.44 ± 0.39	73.25 ± 0.65	74.04 ± 0.71
σ_8	0.8102 ± 0.0058	0.8284 ± 0.0070	0.8343 ± 0.0071

Despite the AdS swampland conjecture suggesting that AdS-dS transition seems unlikely given the AdS and dS vacua are infinitely distant from each other in moduli space, the Casimir energy of fields inhabiting the bulk can realize the AdS-dS transition conjectured in Λ_s CDM.

$$S = S_{\text{m}} + M_{\text{P}}^2 \int d^4x \, N \sqrt{\gamma} \left[\frac{1}{2} \left(R + K_{ij} K^{ij} - K^2 \right) \right.$$

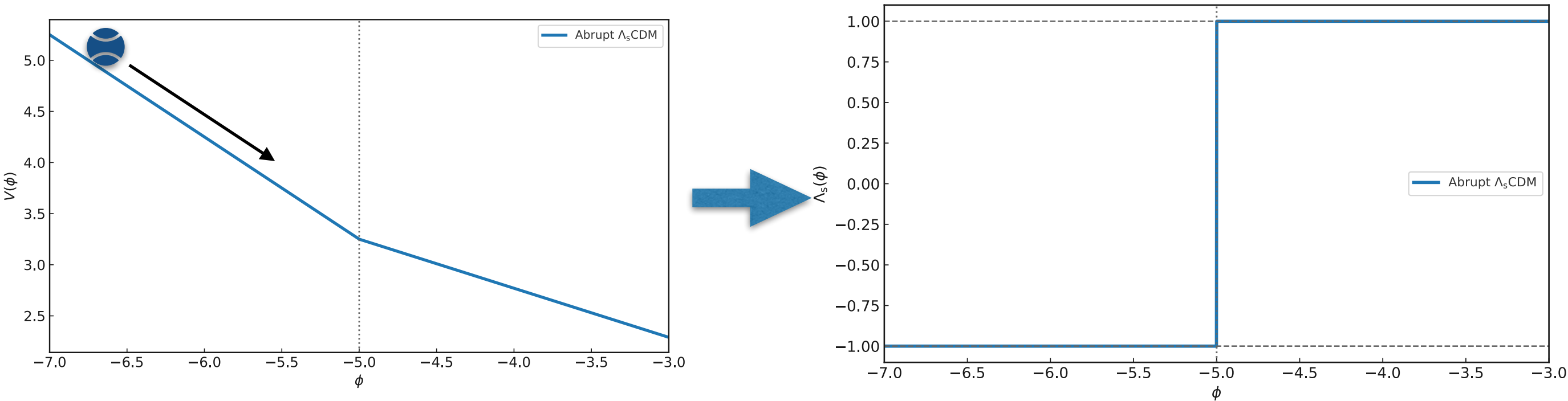
De Felice, Doll, Mukohyama, JCAP (2020), arXiv:2004.12549
De Felice, Mukohyama, Pookkillath, PLB (2021), arXiv:2009.08718

$$\left. - V(\phi) + \frac{\lambda_2}{N} \gamma^{ij} D_i D_j \phi - \frac{3\lambda^2}{4} - \lambda(K + \phi) \right],$$

$$3M_{\text{P}}^2 H^2 = \rho + \rho_{\phi}$$

$$\rho_{\phi} \equiv M_{\text{P}}^2 (V - \phi V_{,\phi}) + \frac{3}{4} M_{\text{P}}^2 V_{,\phi}^2$$

$$\rho_{\phi}/M_{\text{P}}^2 = \Lambda_{\text{eff}} \Rightarrow V(\phi) = \alpha\phi - \frac{3}{4}\alpha^2 + \Lambda_{\text{eff}}$$



Abrupt Λ_s VCDM model

$$V(\phi) = \begin{cases} \alpha_{\text{b}}(\phi - \phi_{\text{c}}) - \beta, & \text{for } \phi < \phi_{\text{c}} \\ \alpha_{\text{a}}(\phi - \phi_{\text{c}}) - \beta, & \text{for } \phi \geq \phi_{\text{c}} \end{cases} \rightarrow \Lambda_{\text{sft}} = \begin{cases} \Lambda_{\text{sft,b}} = \frac{3}{4}\alpha_{\text{b}}^2 - \alpha_{\text{b}}\phi_{\text{c}} - \beta, & \text{for } \phi < \phi_{\text{c}} \\ \Lambda_{\text{sft,a}} = \frac{3}{4}\alpha_{\text{a}}^2 - \alpha_{\text{a}}\phi_{\text{c}} - \beta, & \text{for } \phi \geq \phi_{\text{c}} \end{cases} \rightarrow \Lambda_{\text{sft,a}} = -\Lambda_{\text{sft,b}} = \Lambda_{\text{s0}} \rightarrow \Lambda_{\text{s}} = \Lambda_{\text{s0}} \text{sgn}[\phi_{\text{c}} - \phi]$$

$$\Delta\Lambda_{\text{sft}} = \frac{3}{2} \left[H_{\text{a}}(\phi_{\text{c}}) + H_{\text{b}}(\phi_{\text{c}}) \right] \Delta\alpha$$

$$\Delta H(\phi_{\text{c}}) = \frac{\Delta\alpha}{2}$$

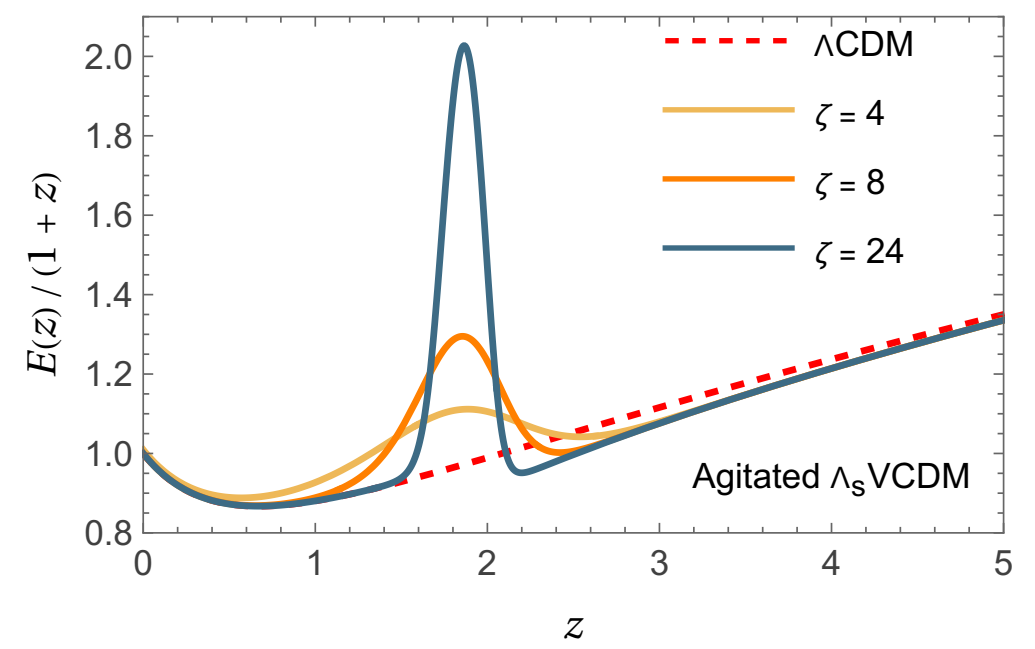
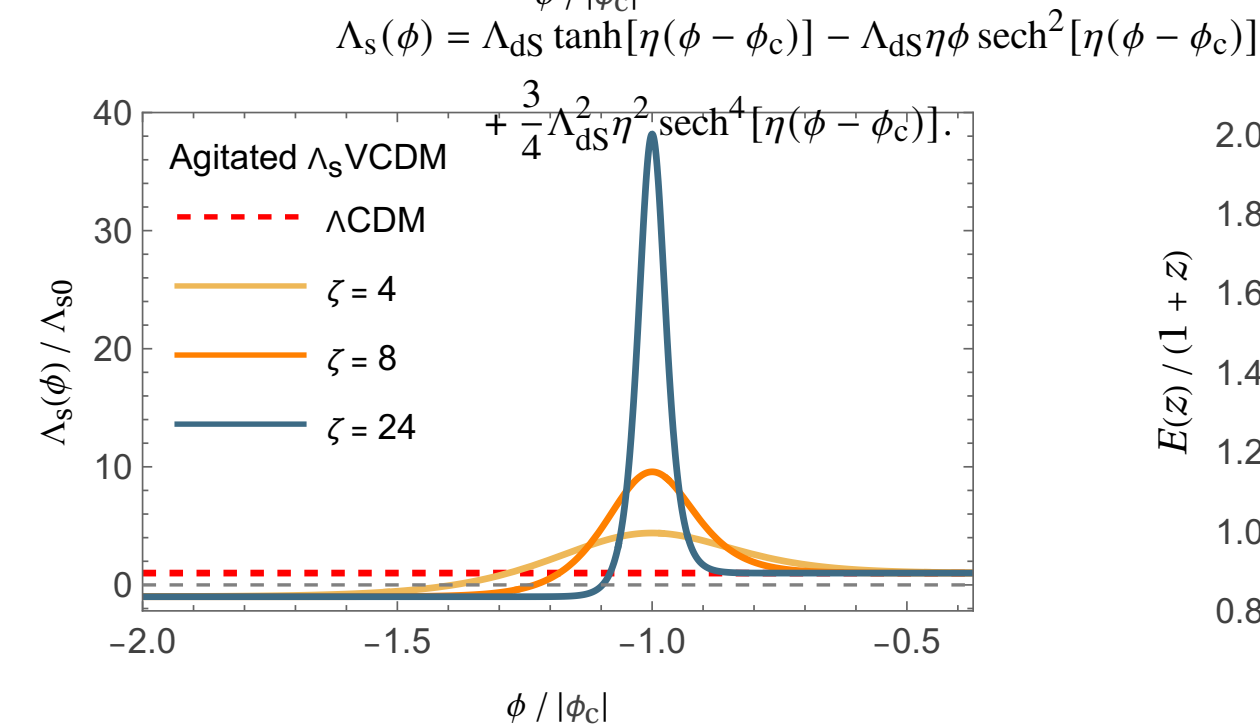
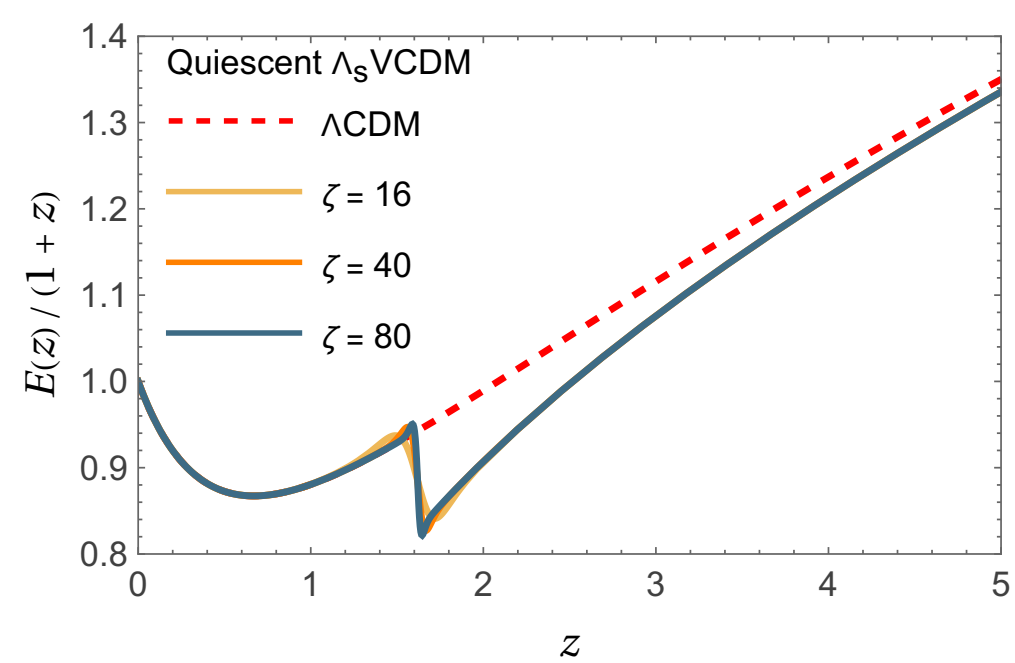
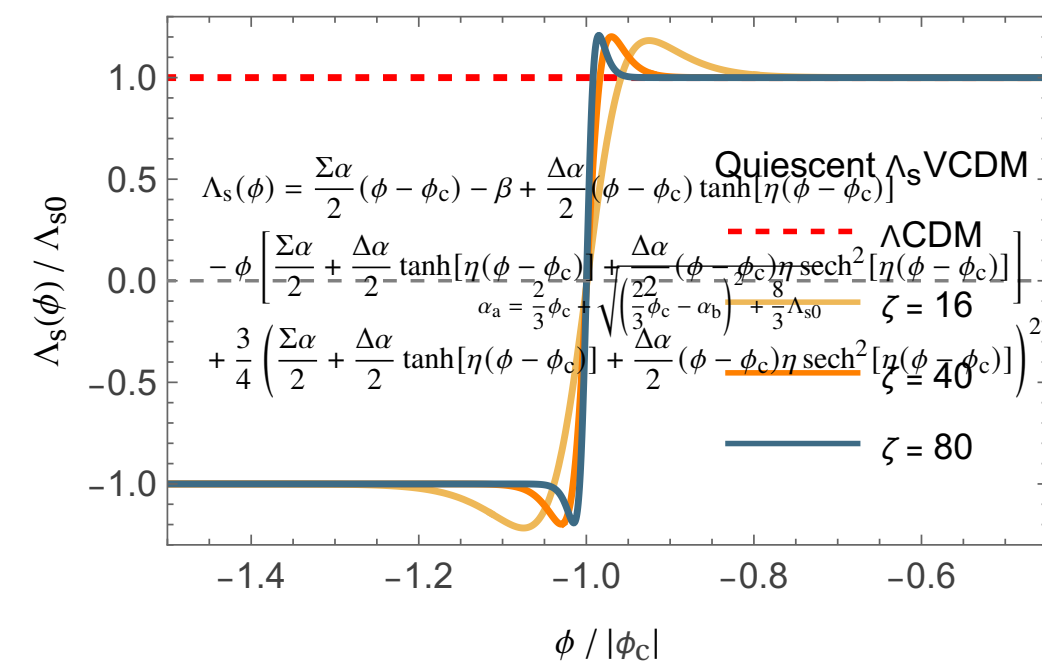
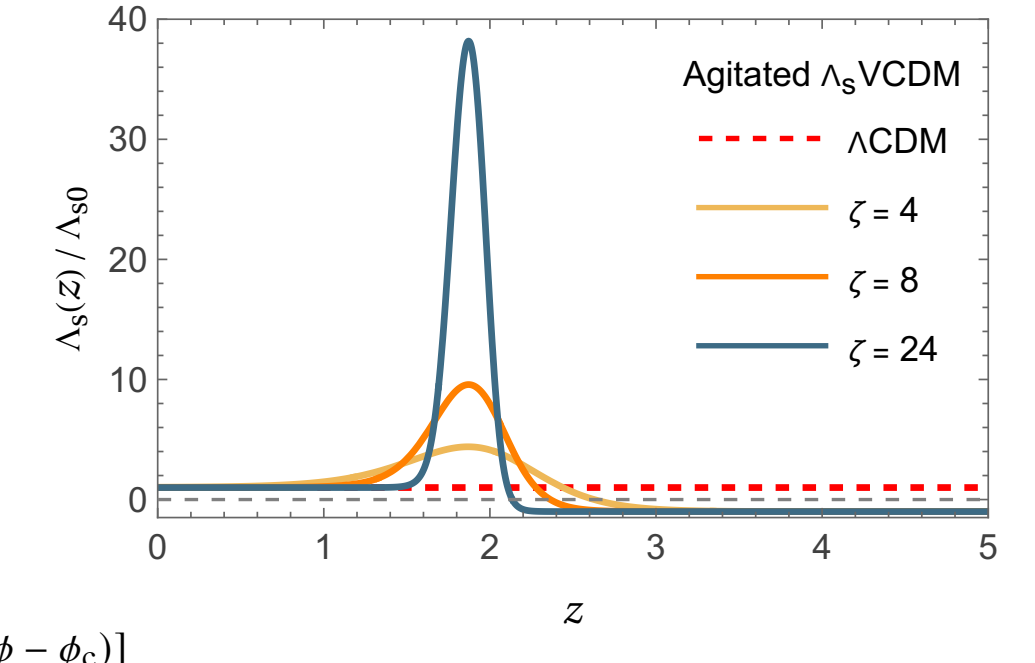
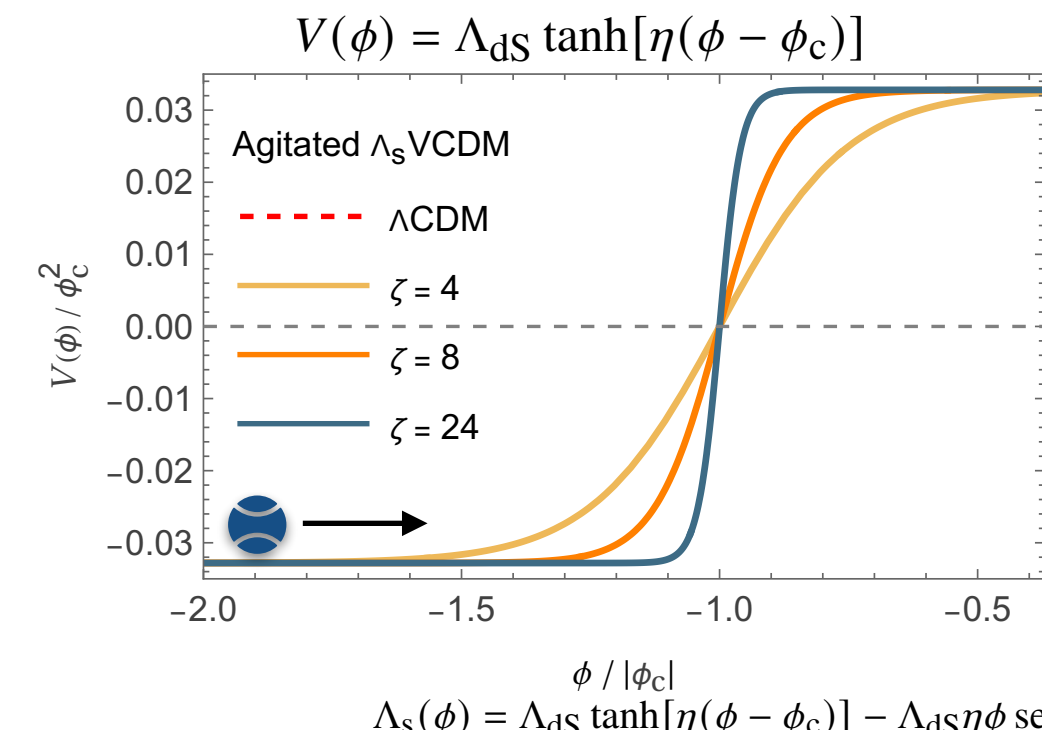
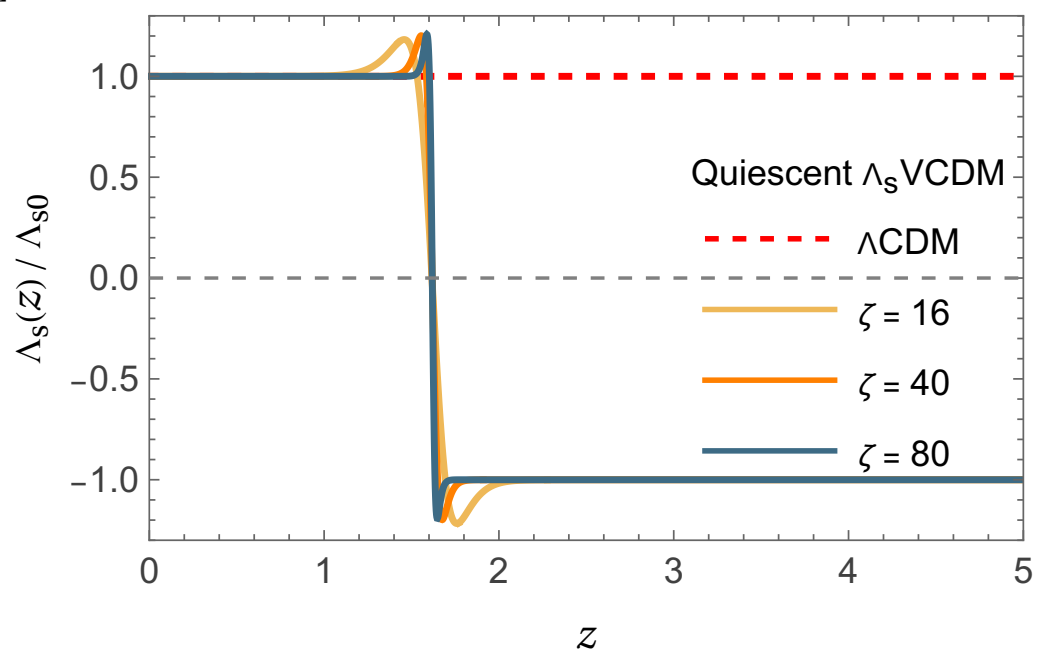
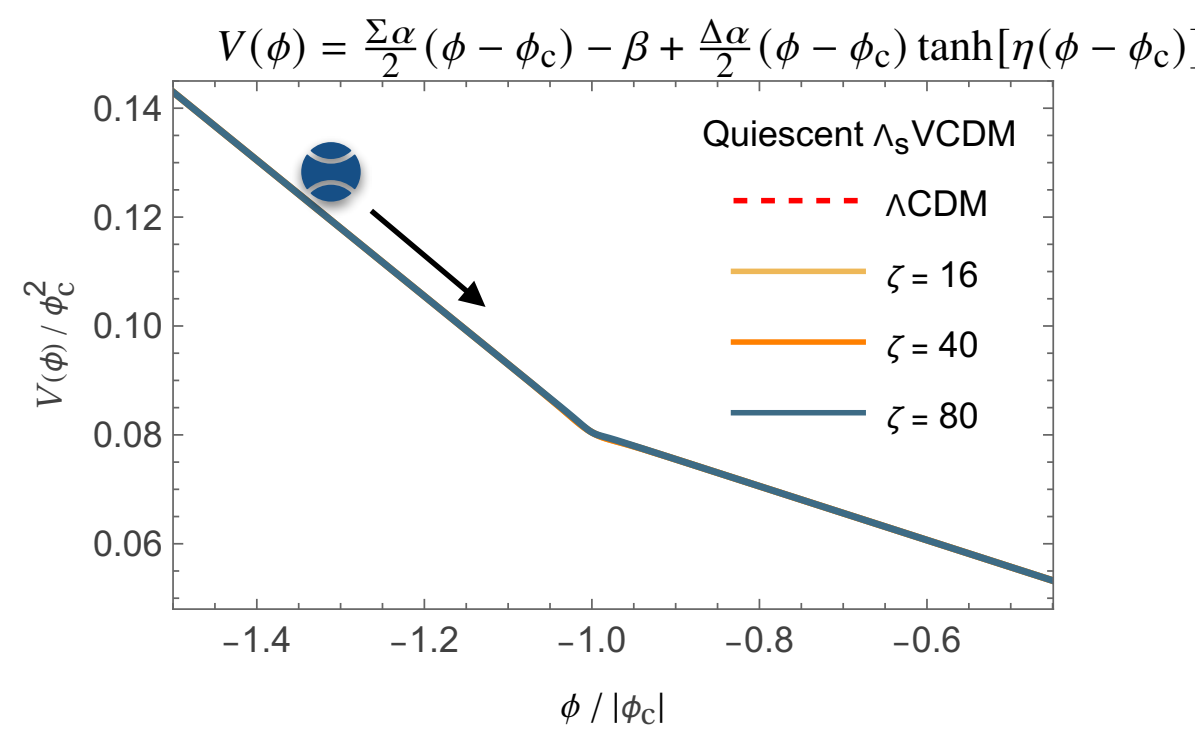
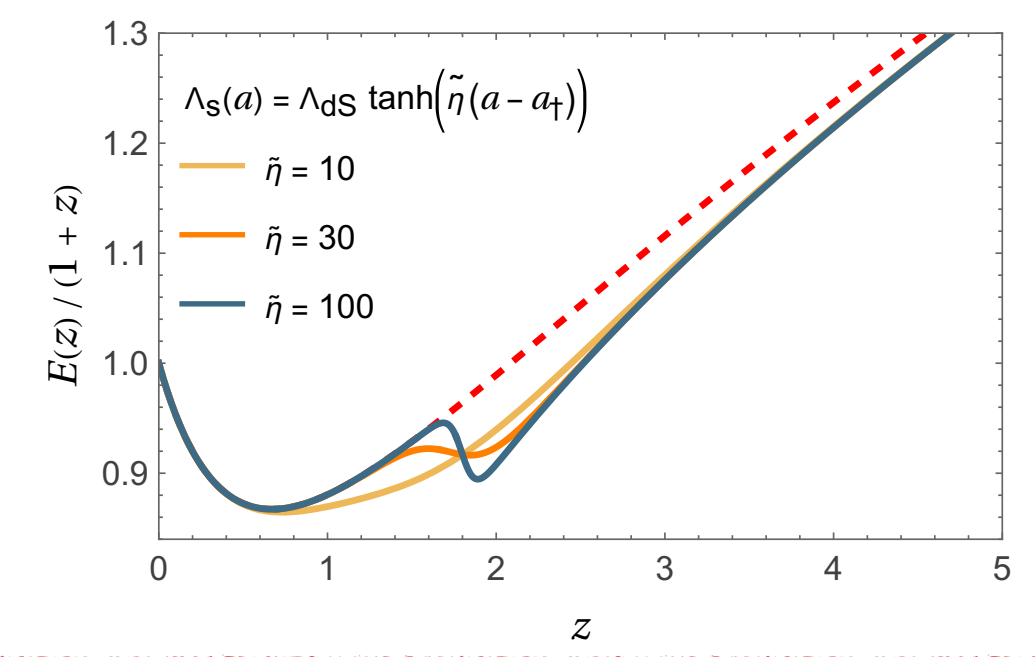
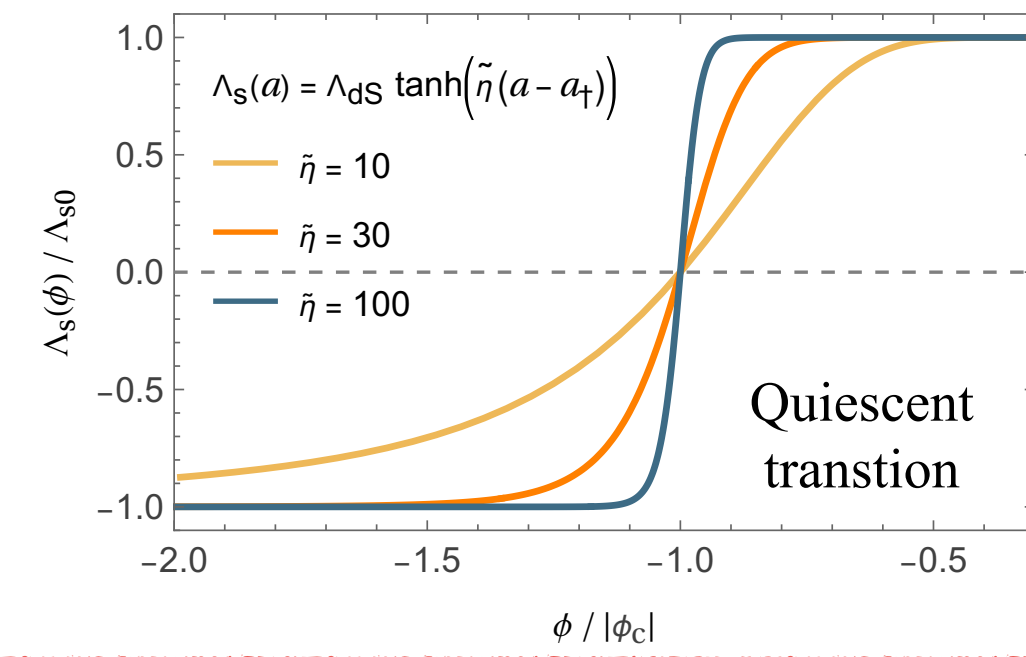
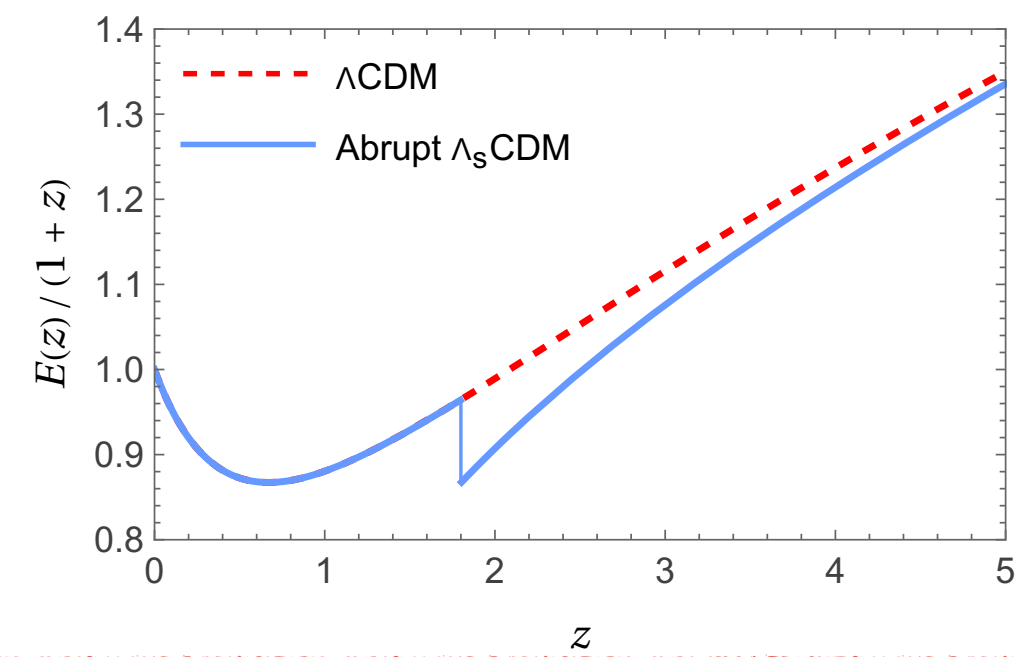
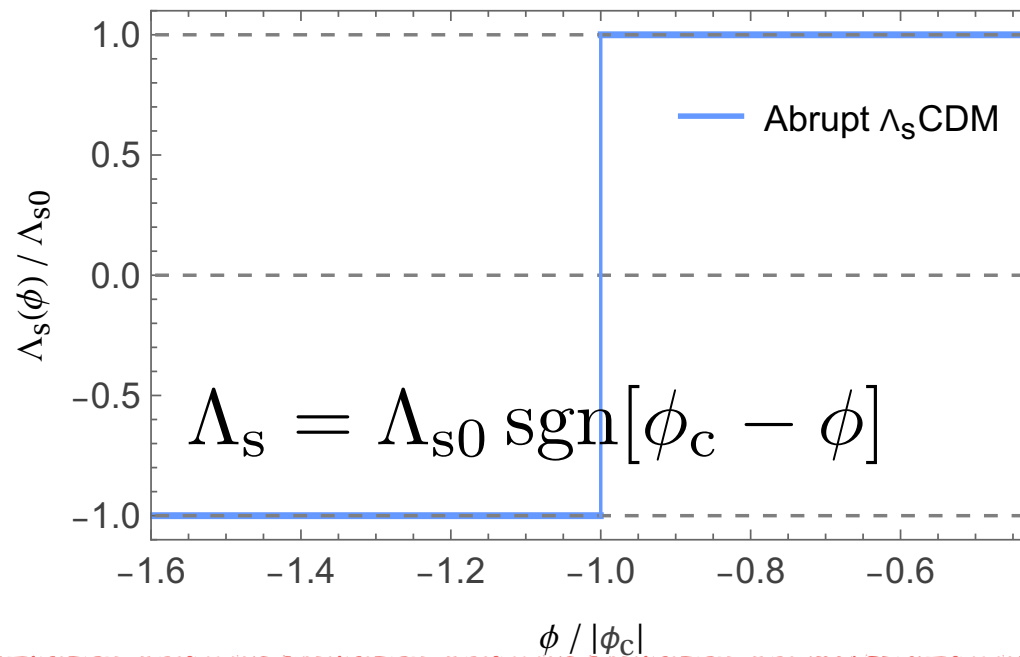
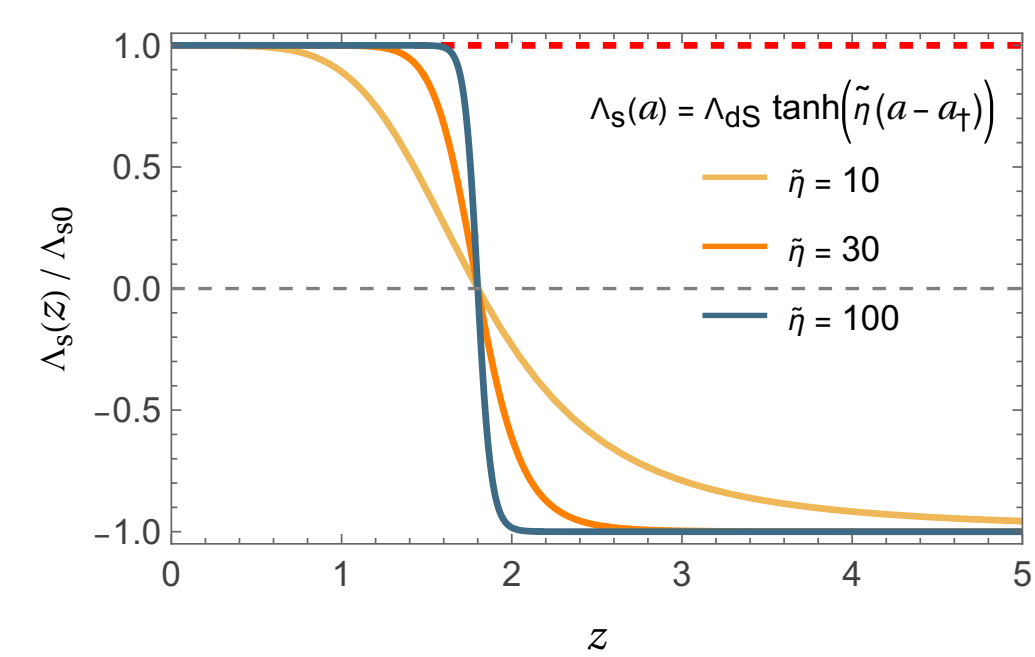
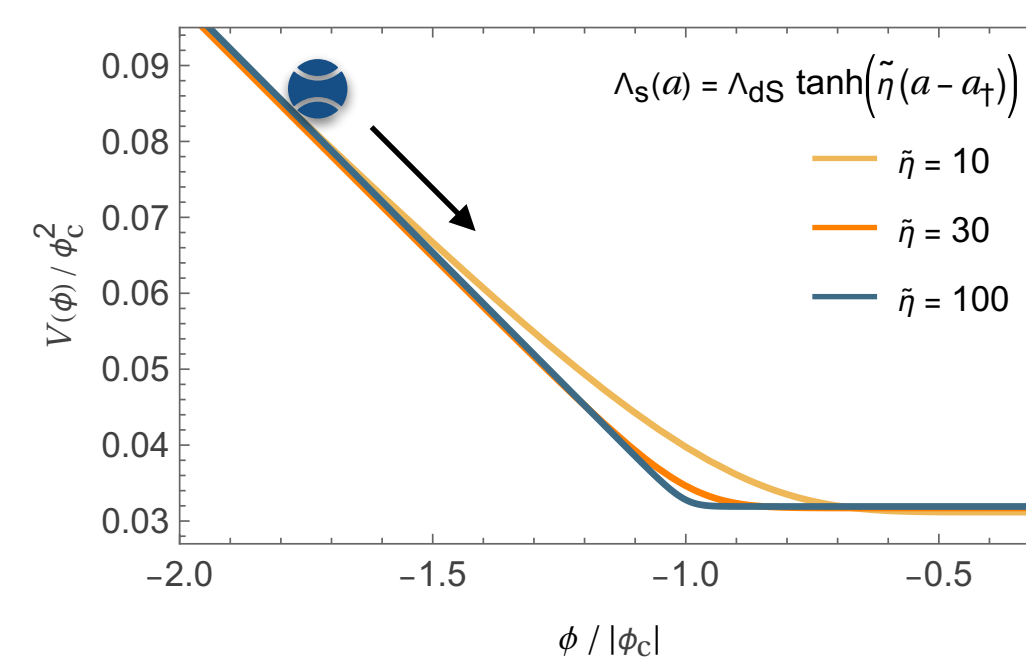
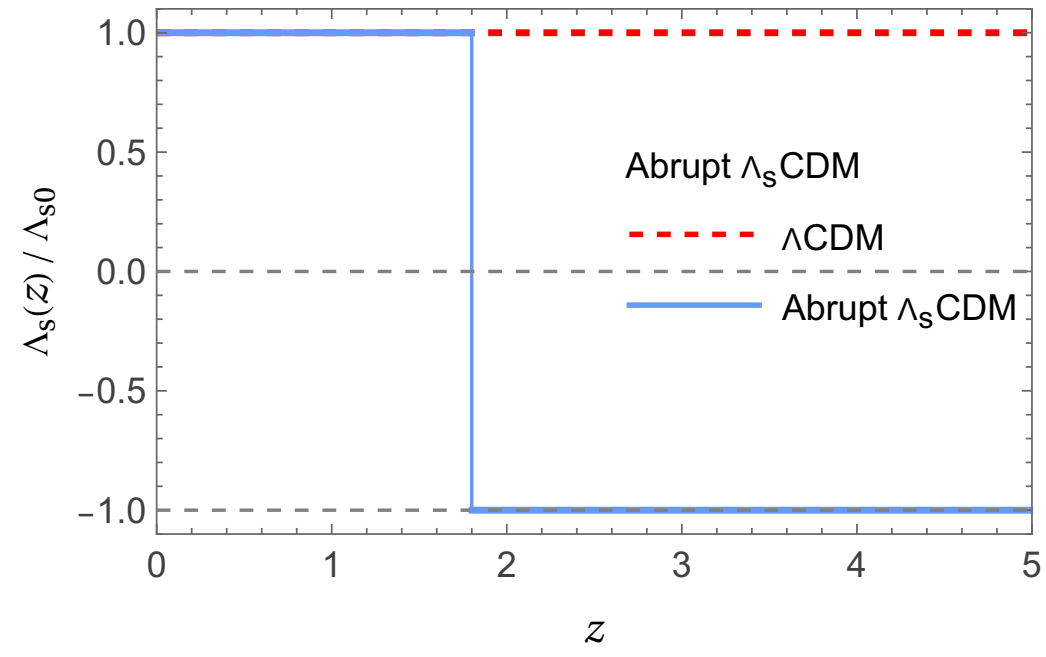
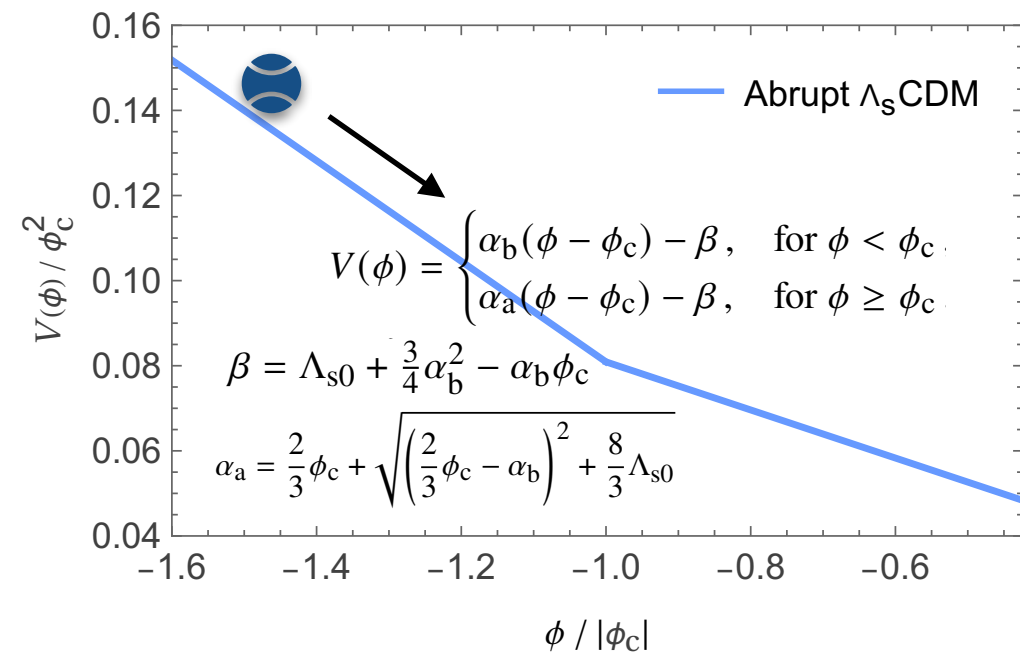
Smooth Λ_s VCDM model

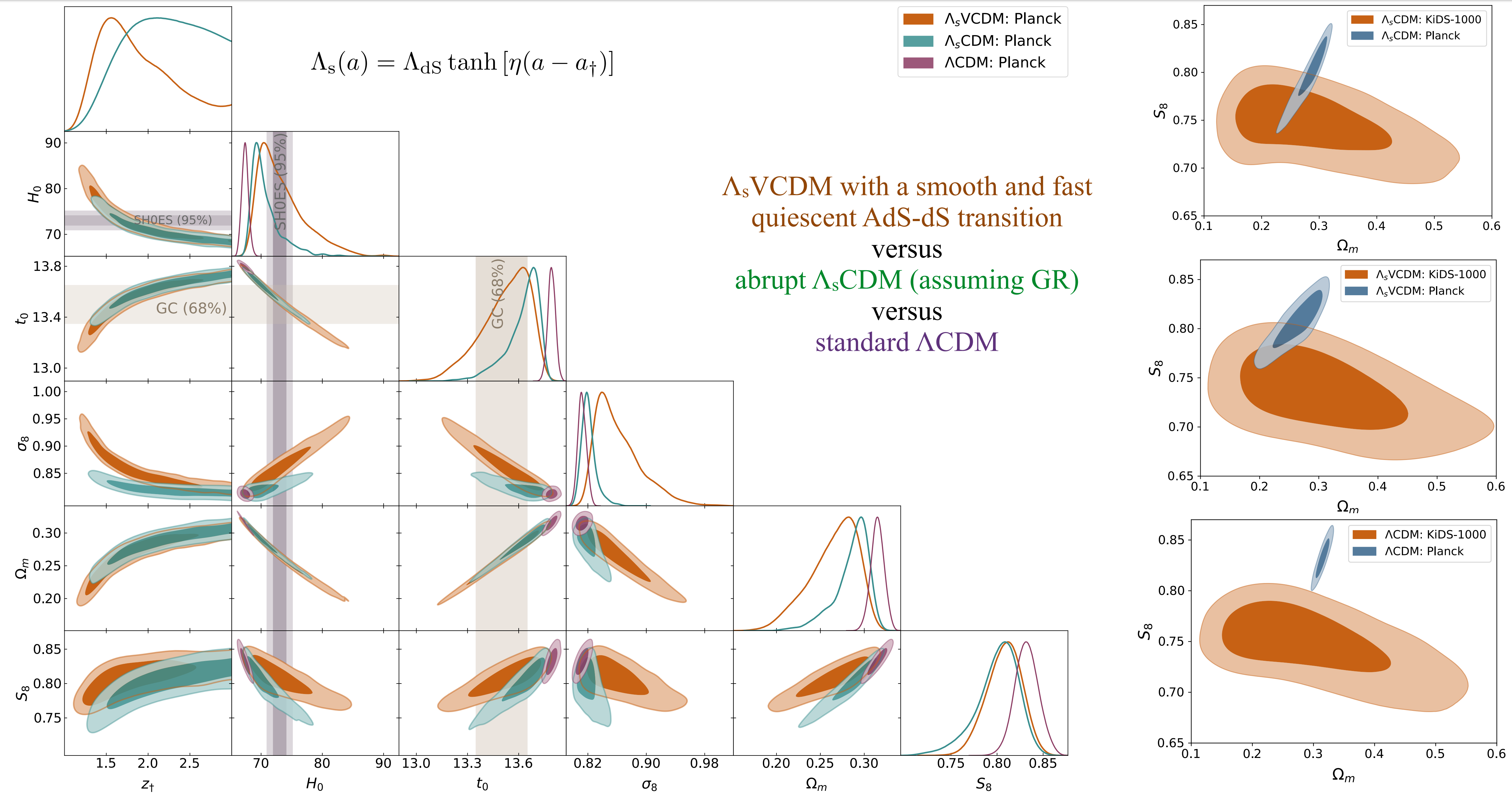
$$V = [\alpha_{\text{b}}(\phi - \phi_{\text{c}}) - \beta][1 - S(\phi)] + [\alpha_{\text{a}}(\phi - \phi_{\text{c}}) - \beta]S(\phi)$$

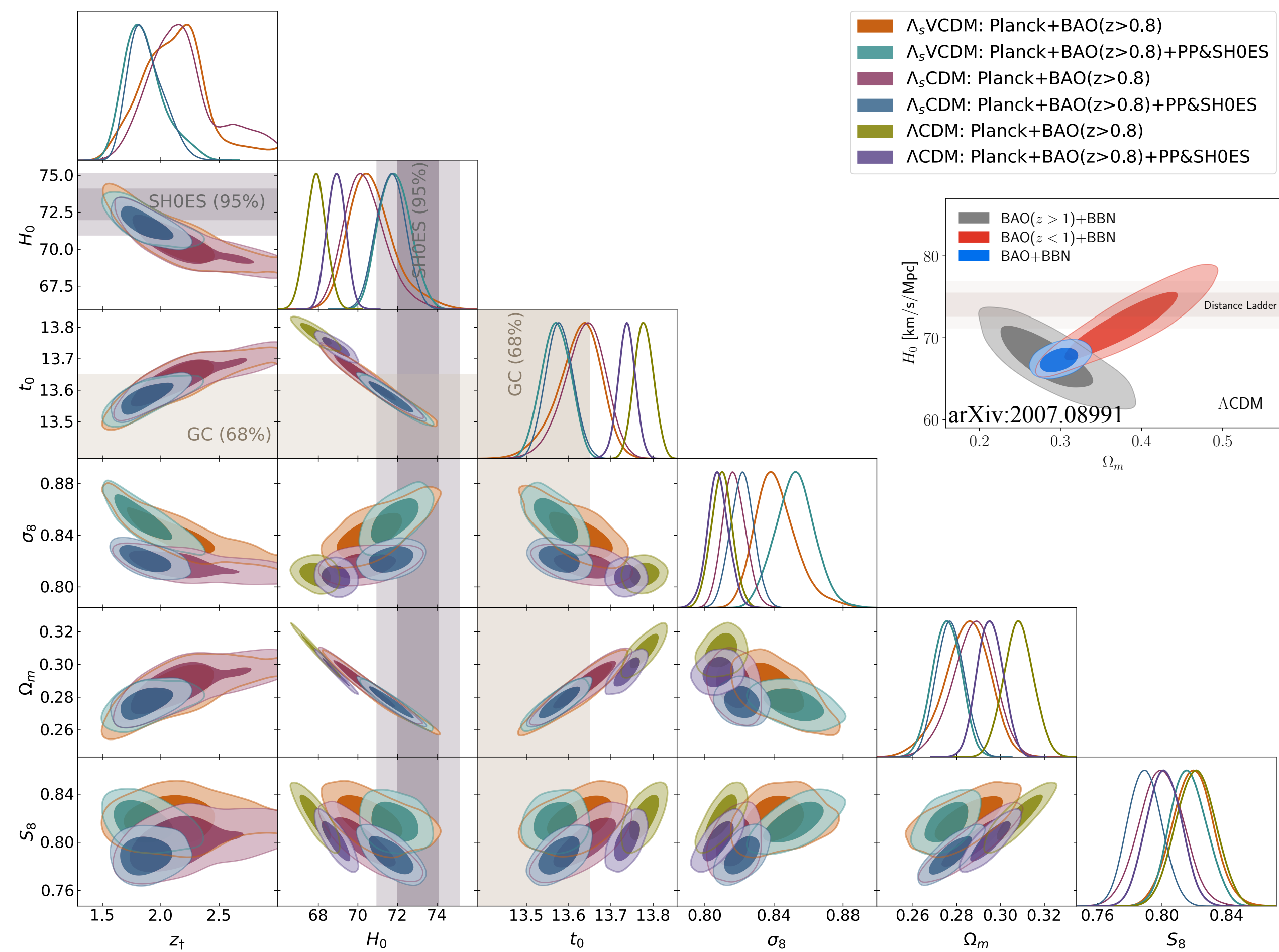
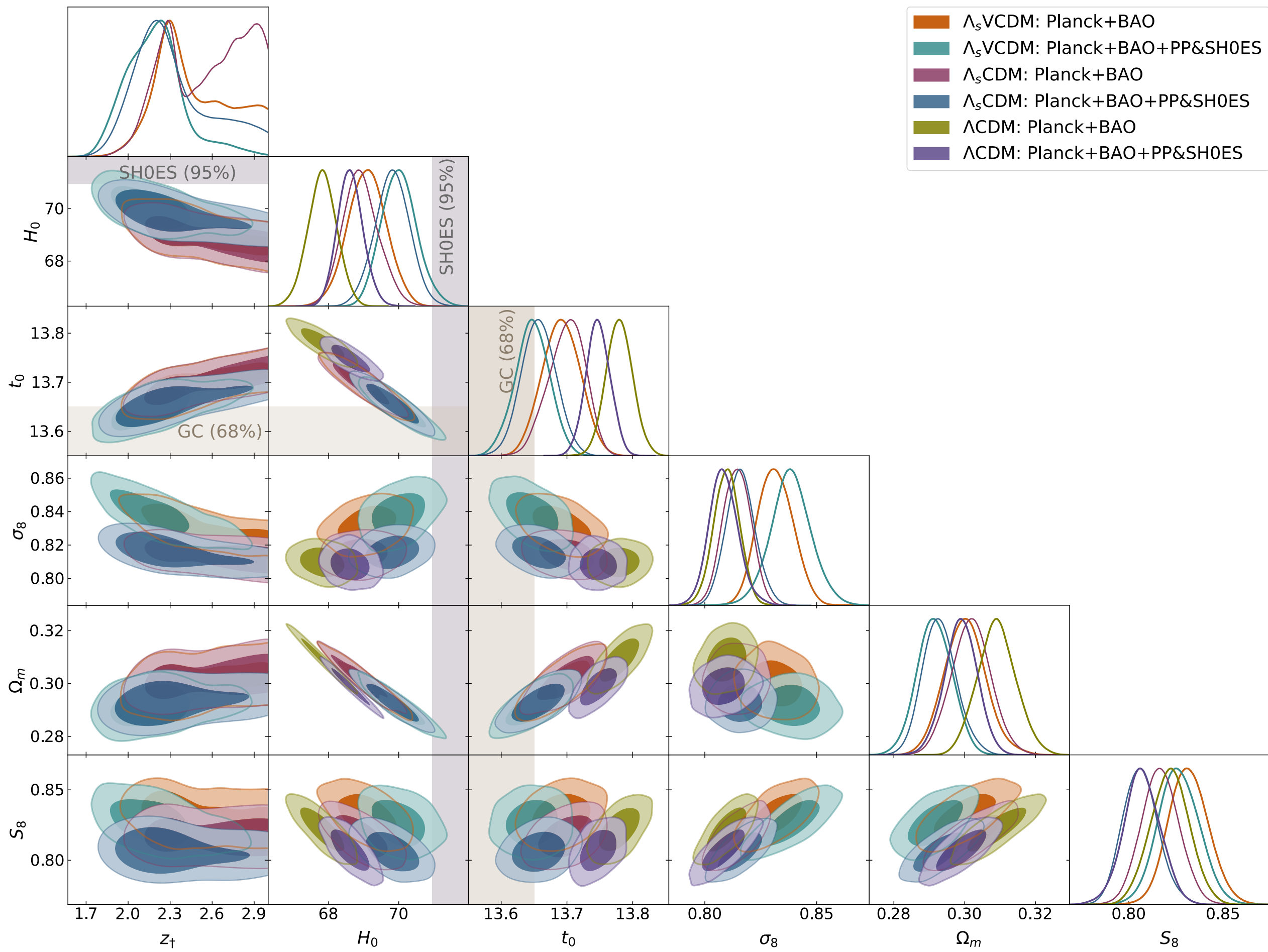
(Blended function)

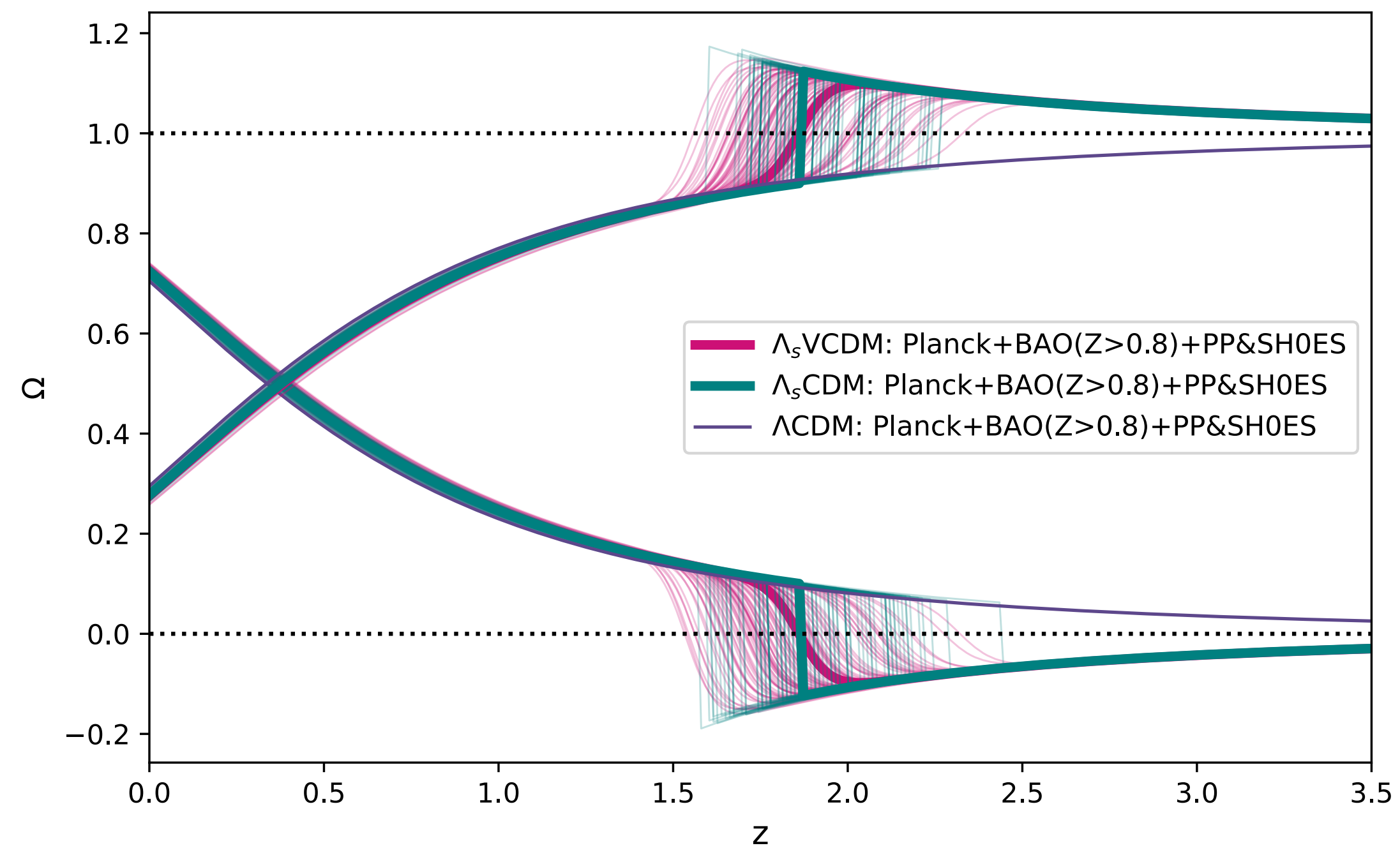
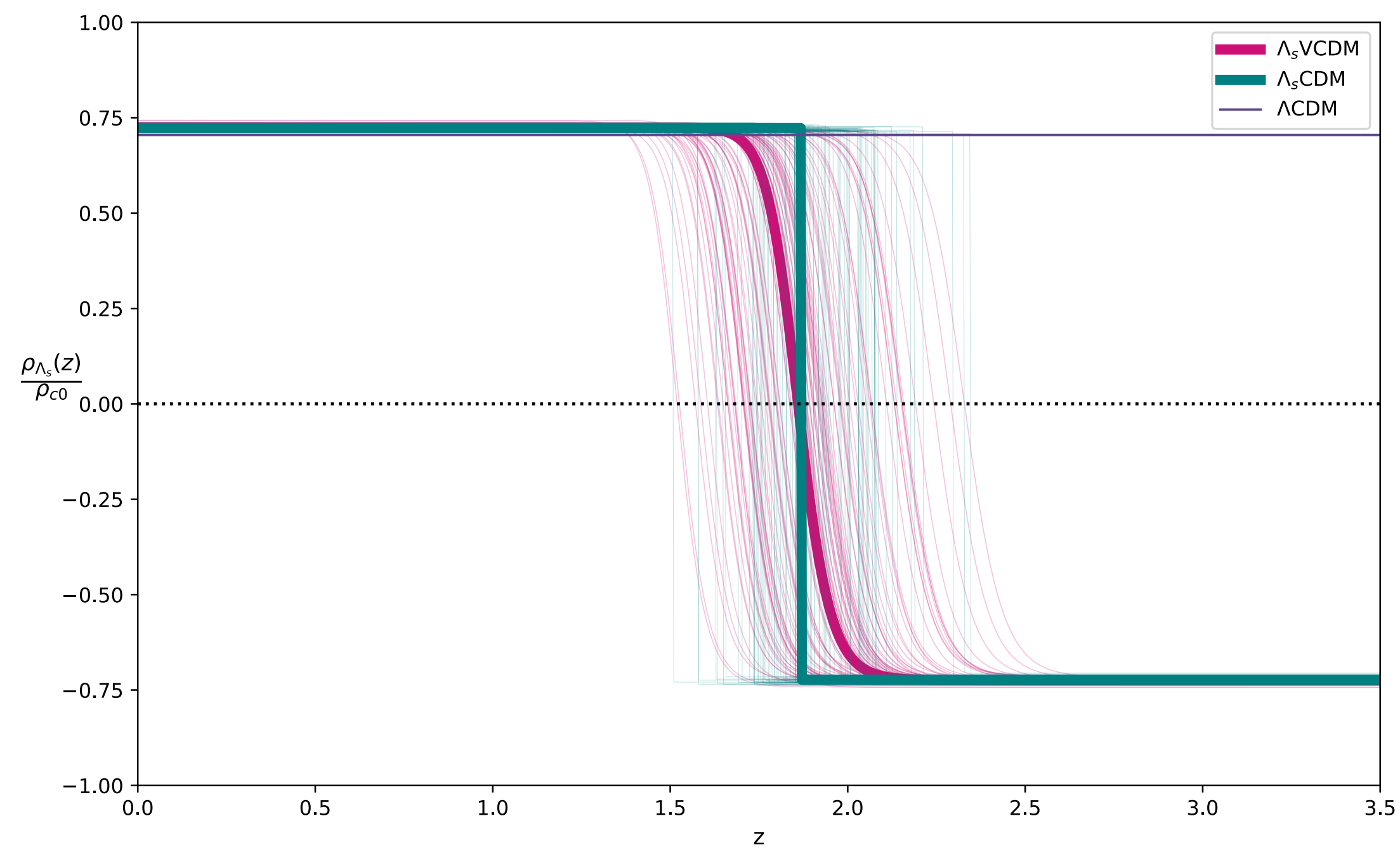
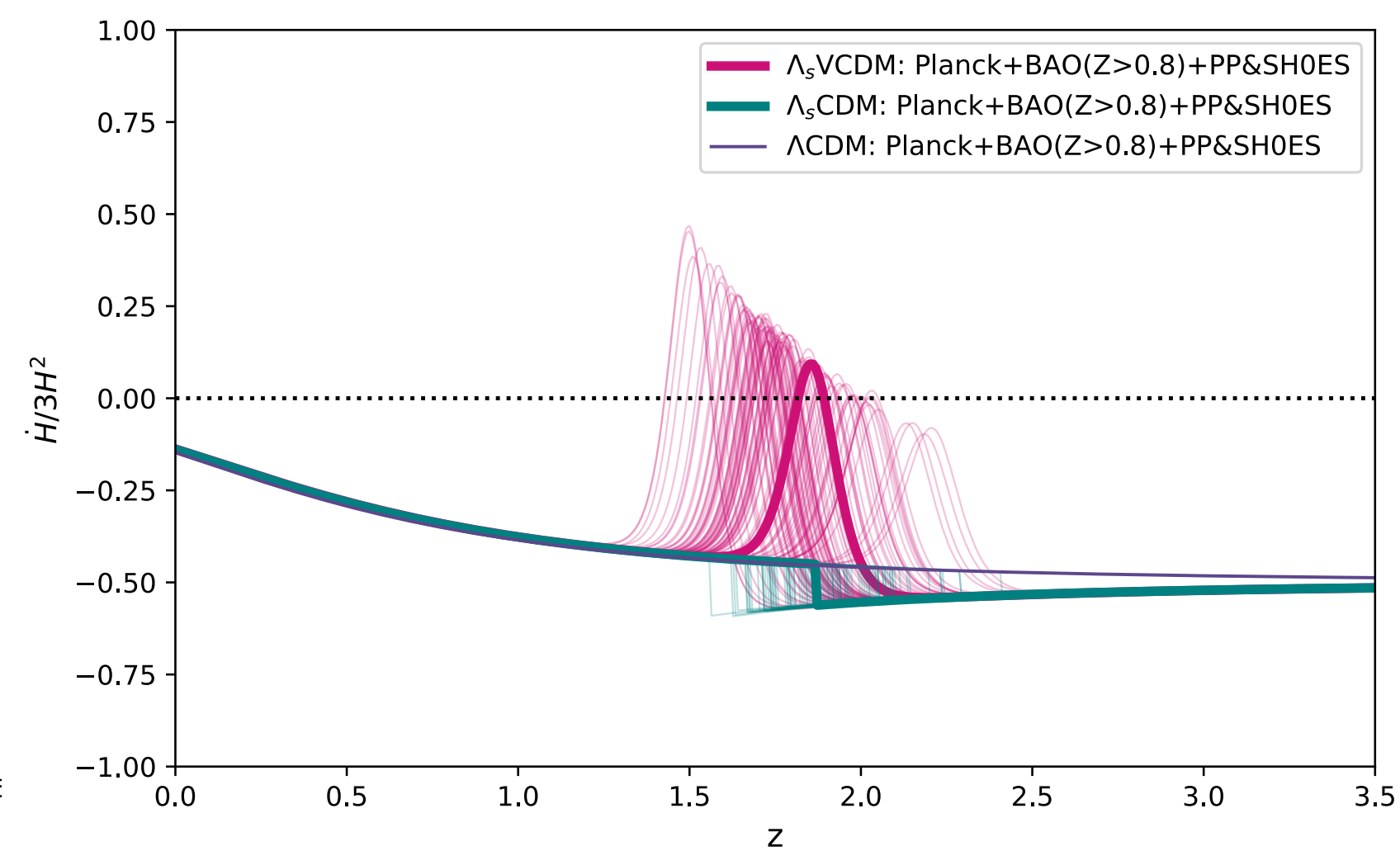
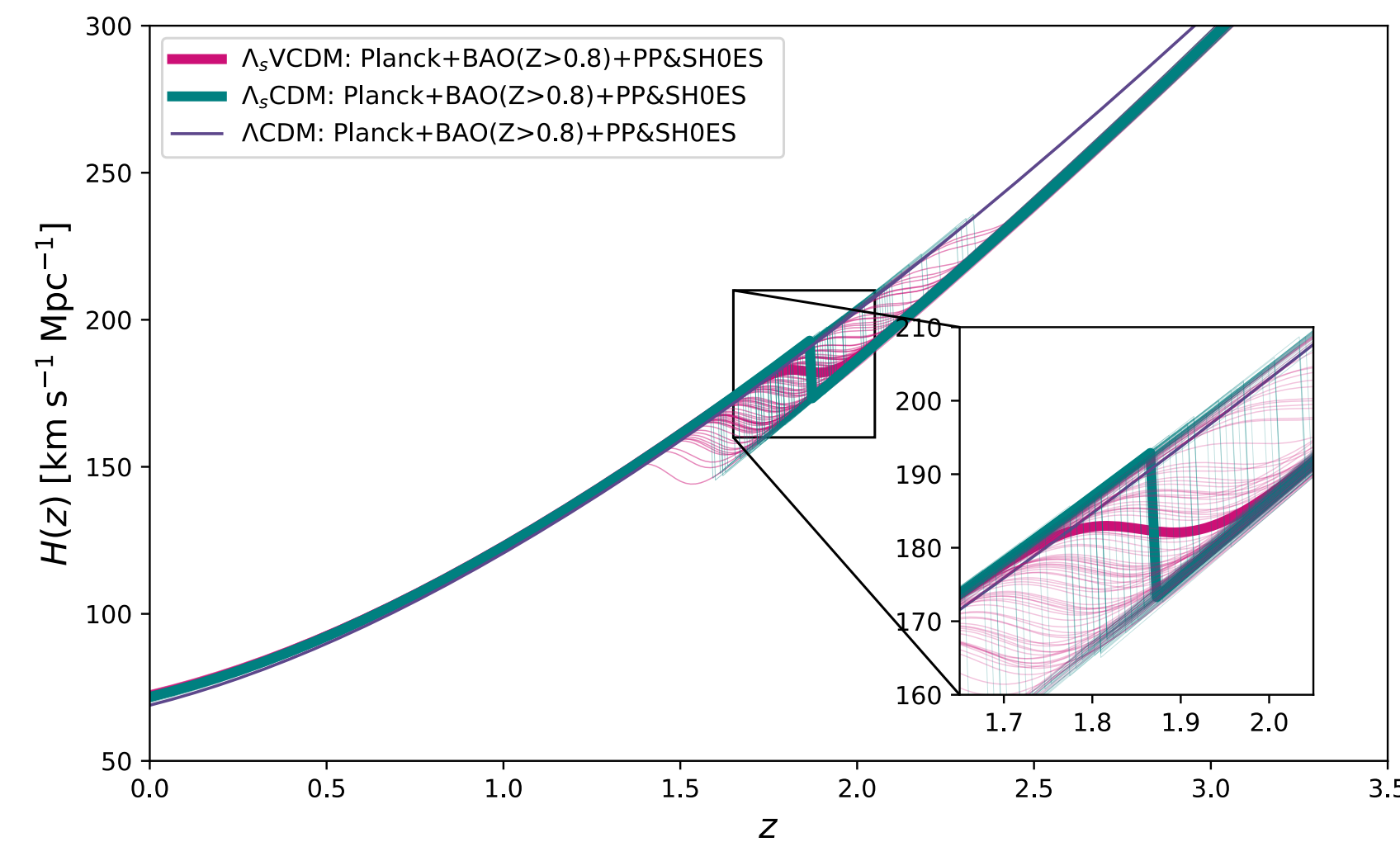
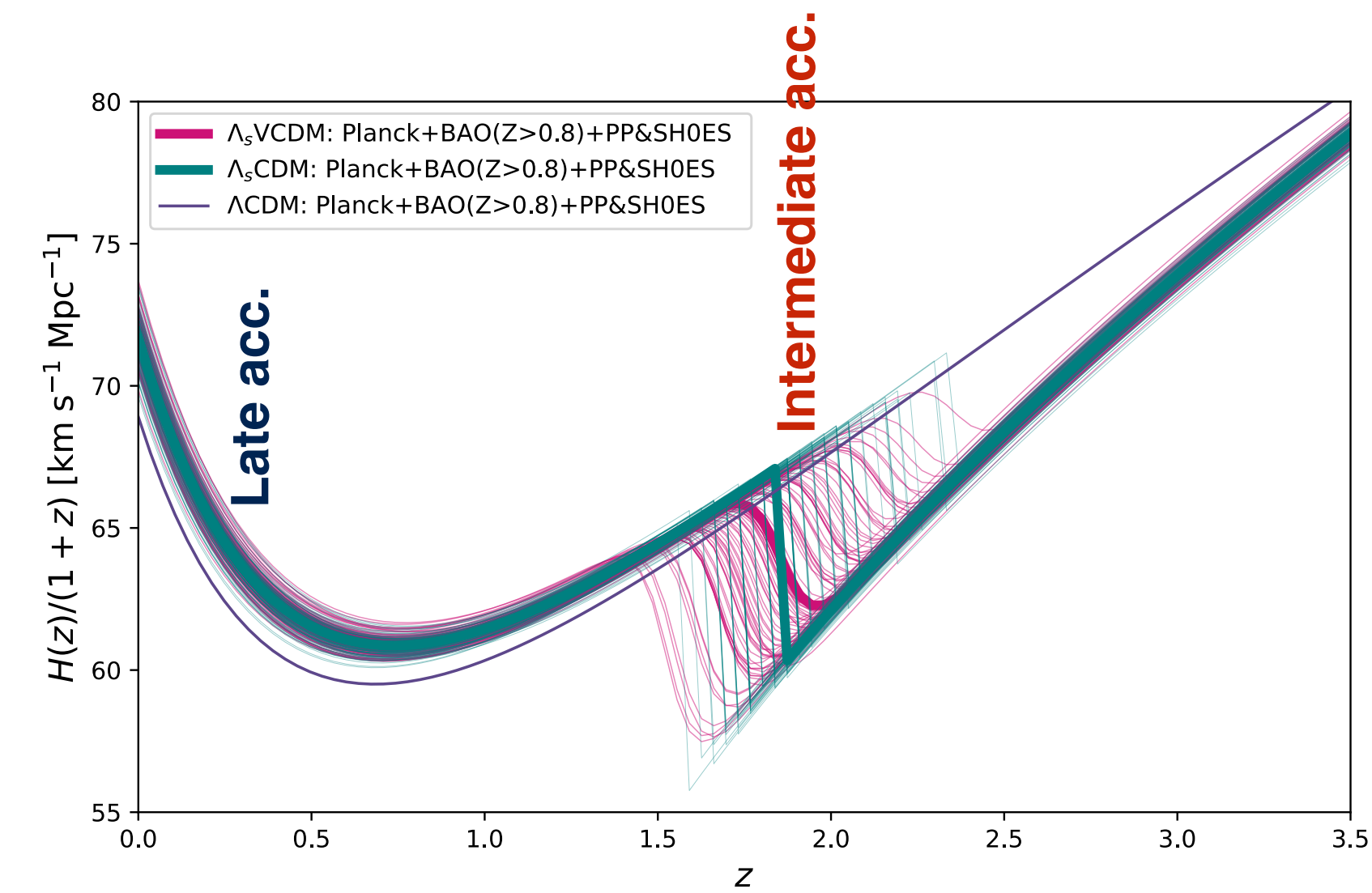
$$S(\phi) = \frac{1}{1 + e^{-2\eta(\phi - \phi_{\text{c}})}}$$

(Logistic interpolant function)









GR with a minimally interacting phantom scalar field with hyperbolic tangent potential

$$V(\tilde{\phi}) = \Lambda \tanh \left[\nu(\tilde{\phi} - \tilde{\phi}_c) \right], \quad \Lambda > 0$$

$$S_\phi = \int d^4x \sqrt{-g} \left[\frac{\xi}{2} g^{ik} \partial_i \phi \partial_k \phi - V(\phi) \right]$$

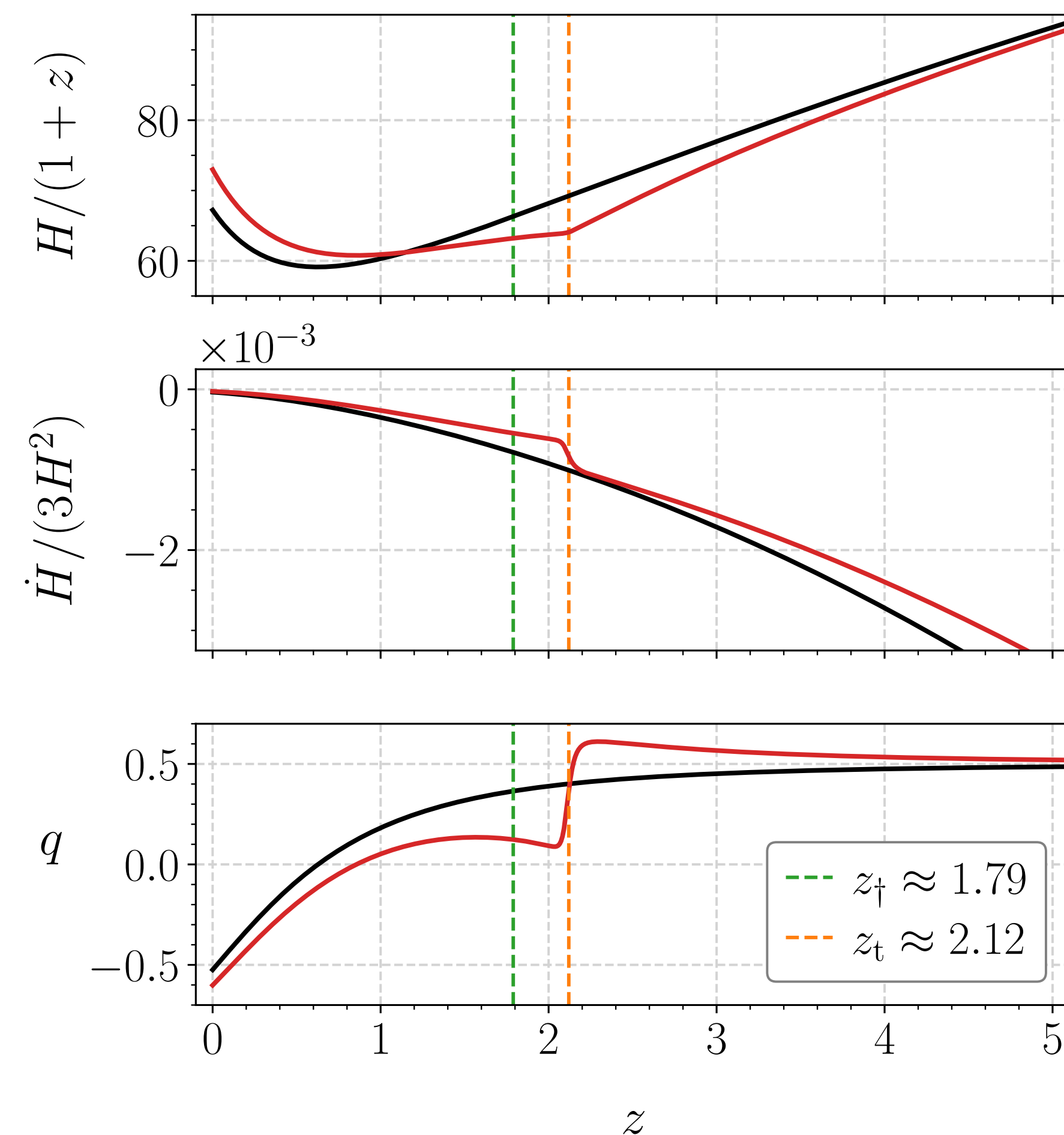
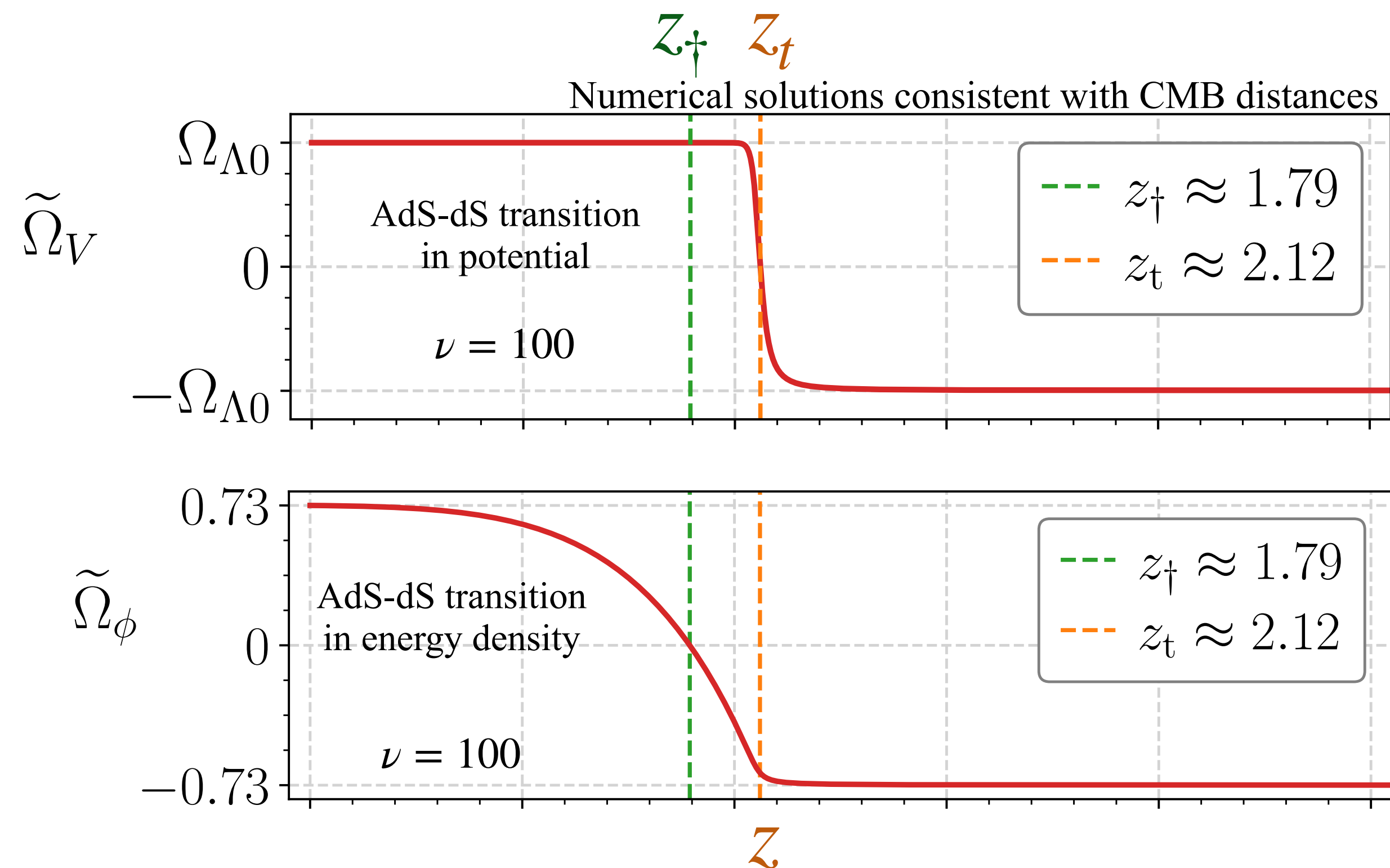
$\xi = +1$ (Quintessence)

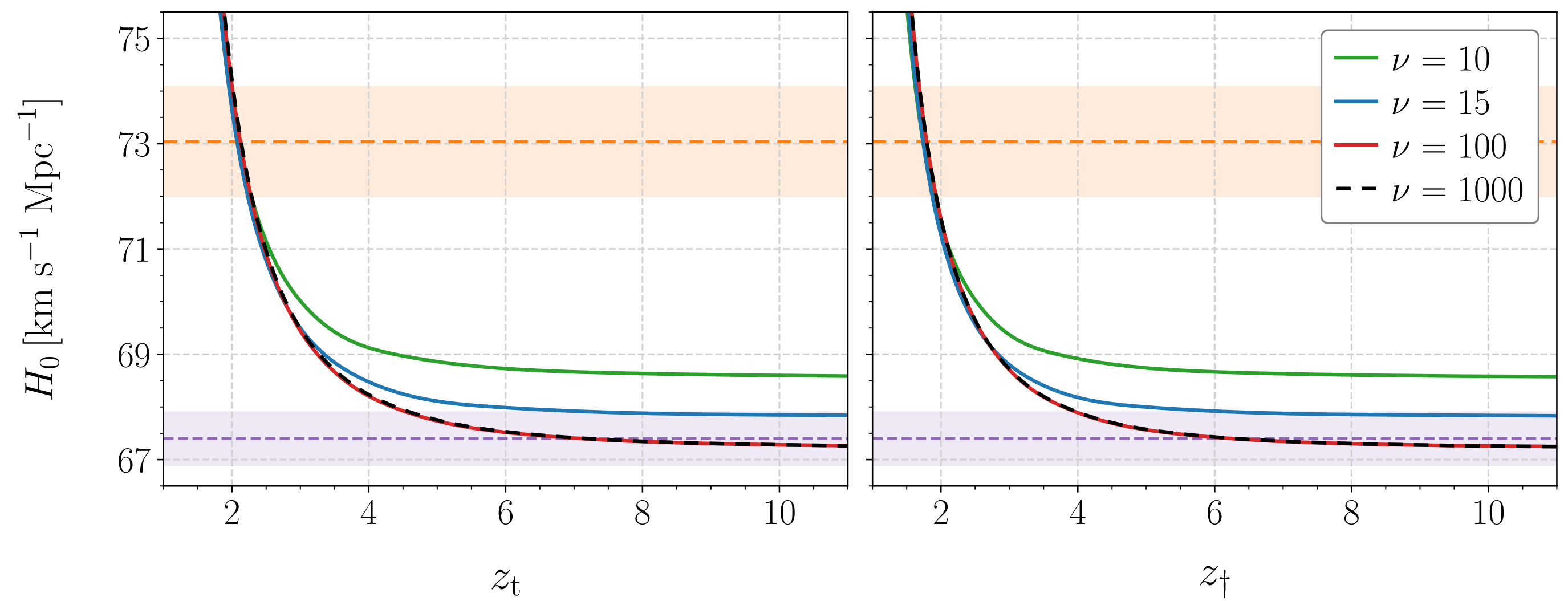
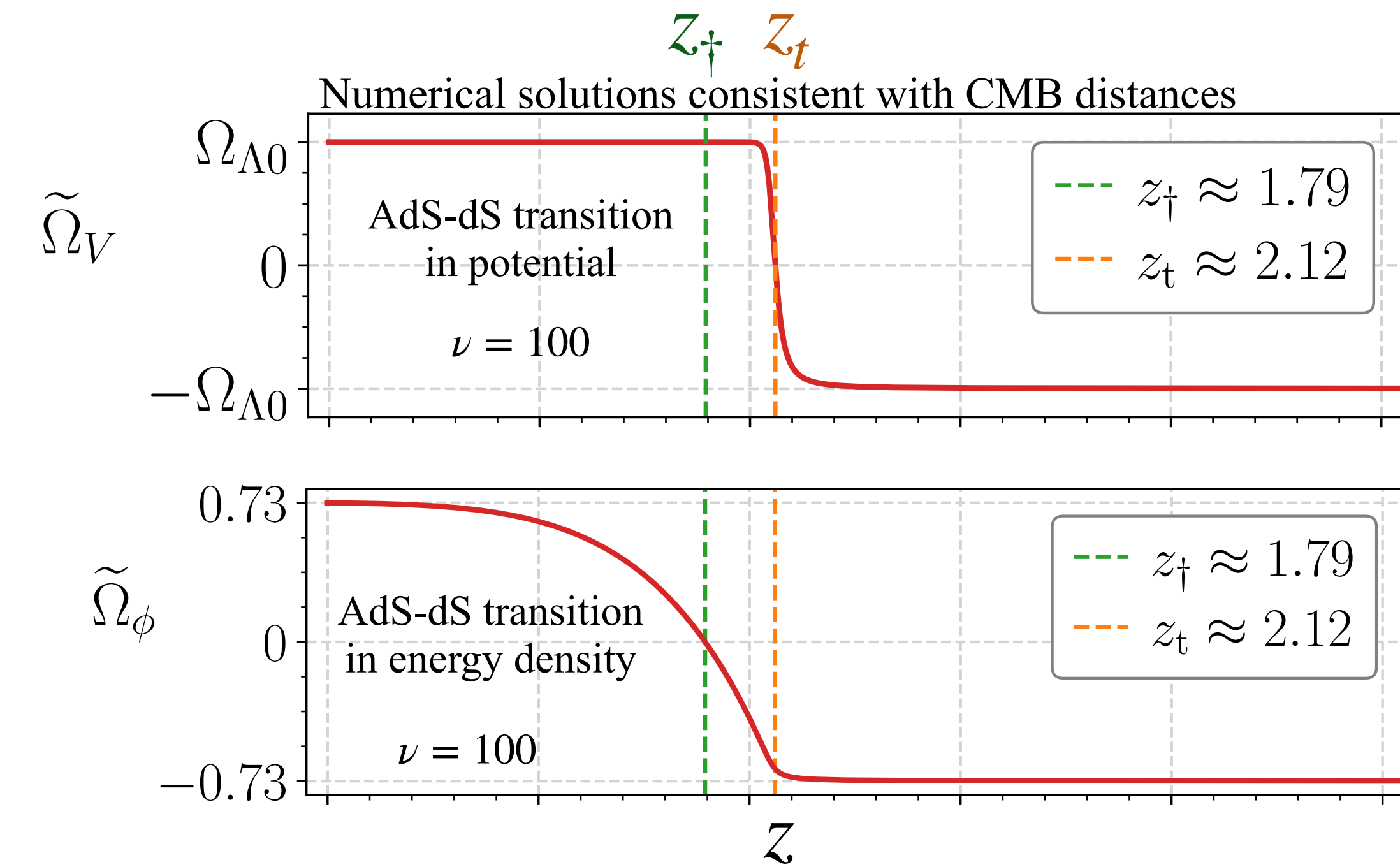
$\xi = -1$ (Phantom)

$$\varepsilon_\phi \equiv T_0^0(\phi) = \frac{\xi}{2c^2} (\dot{\phi})^2 + V(\phi)$$

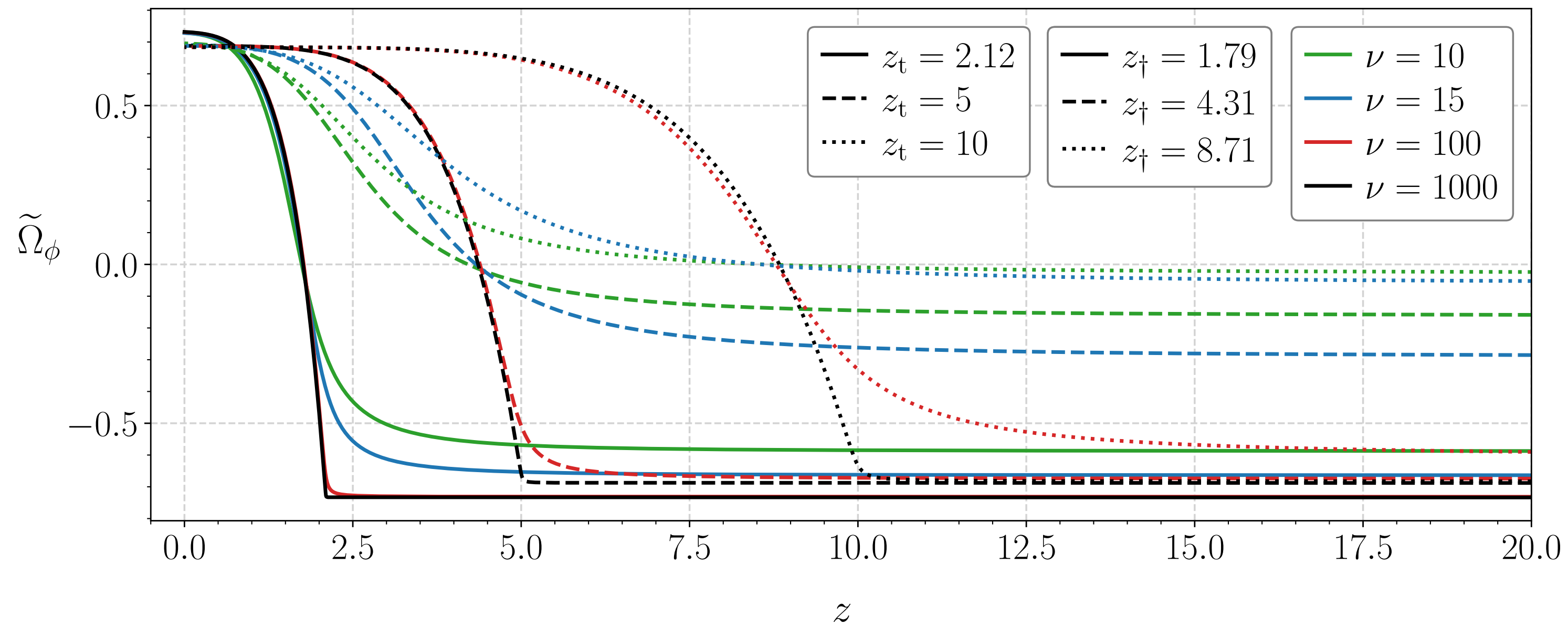
$$p_\phi \equiv -T_\alpha^\alpha(\phi) = \frac{\xi}{2c^2} (\dot{\phi})^2 - V(\phi)$$

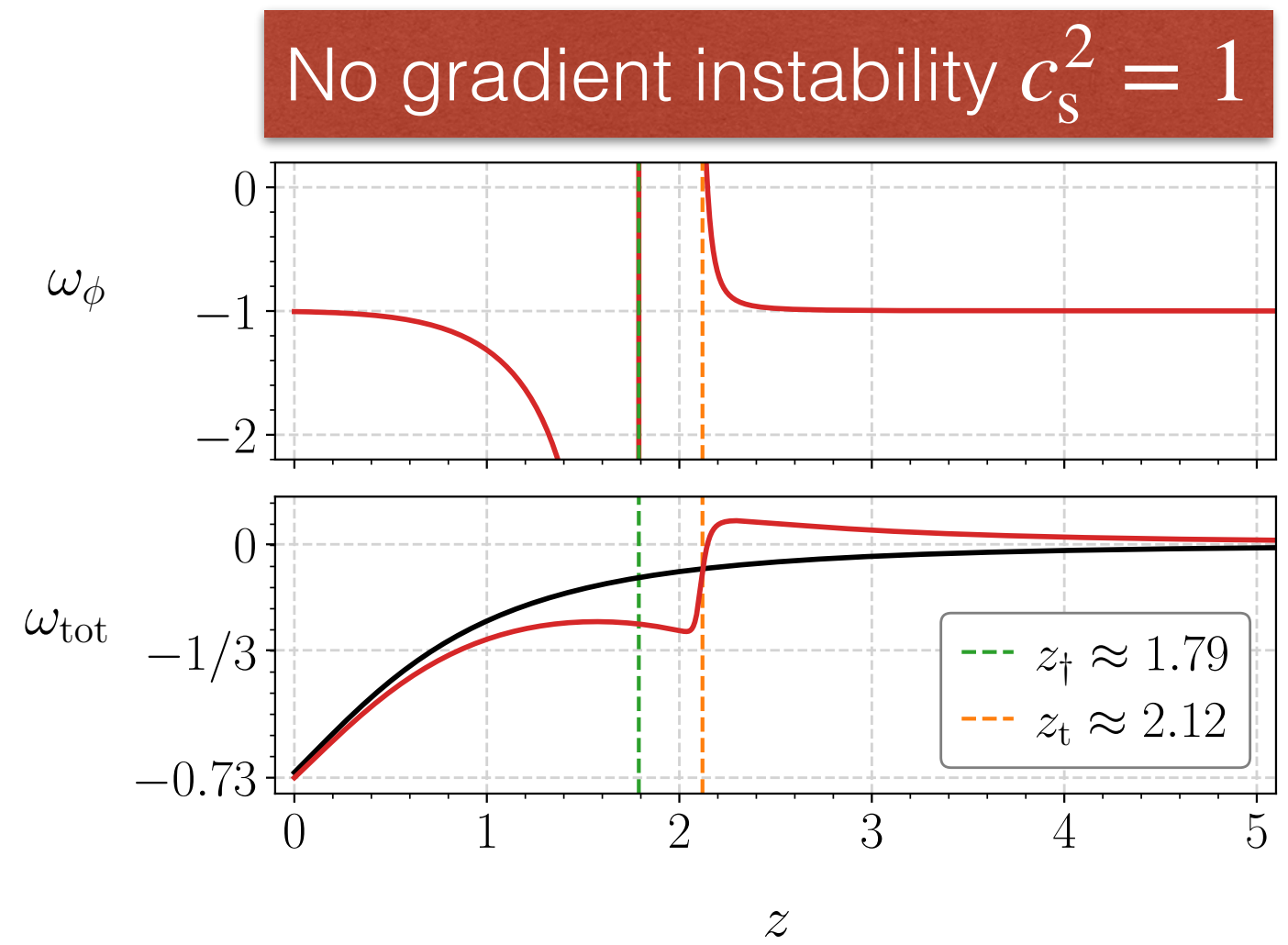
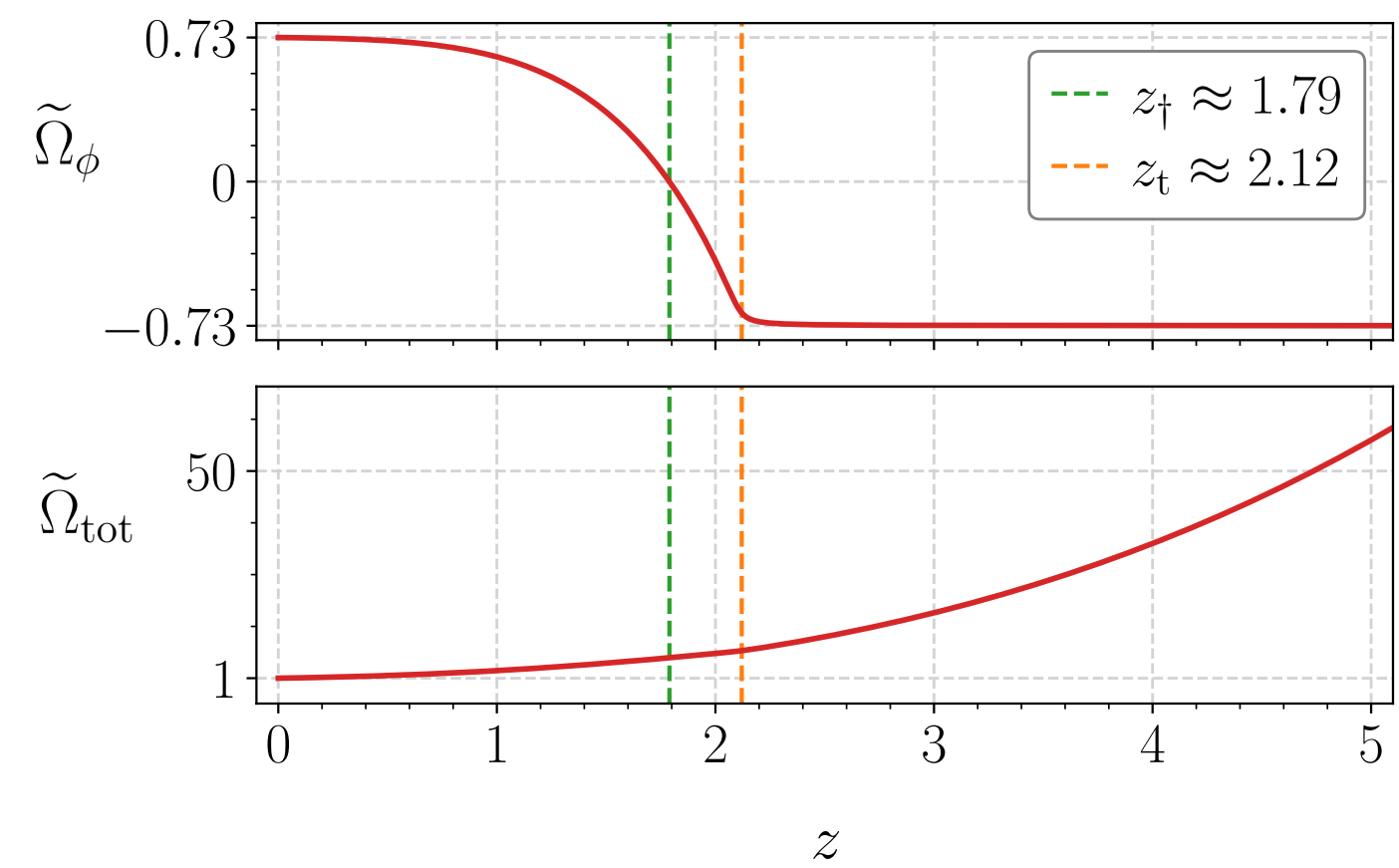
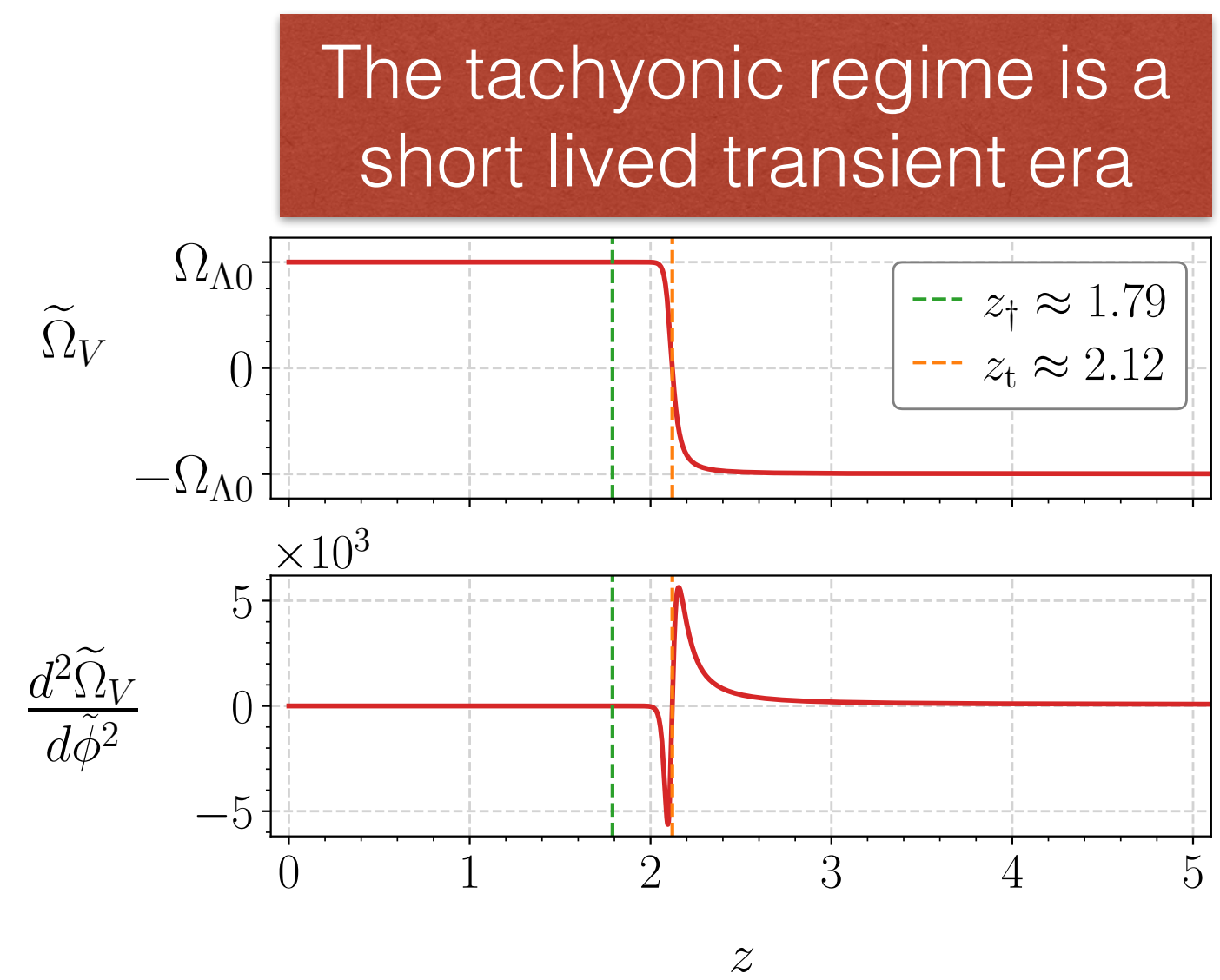
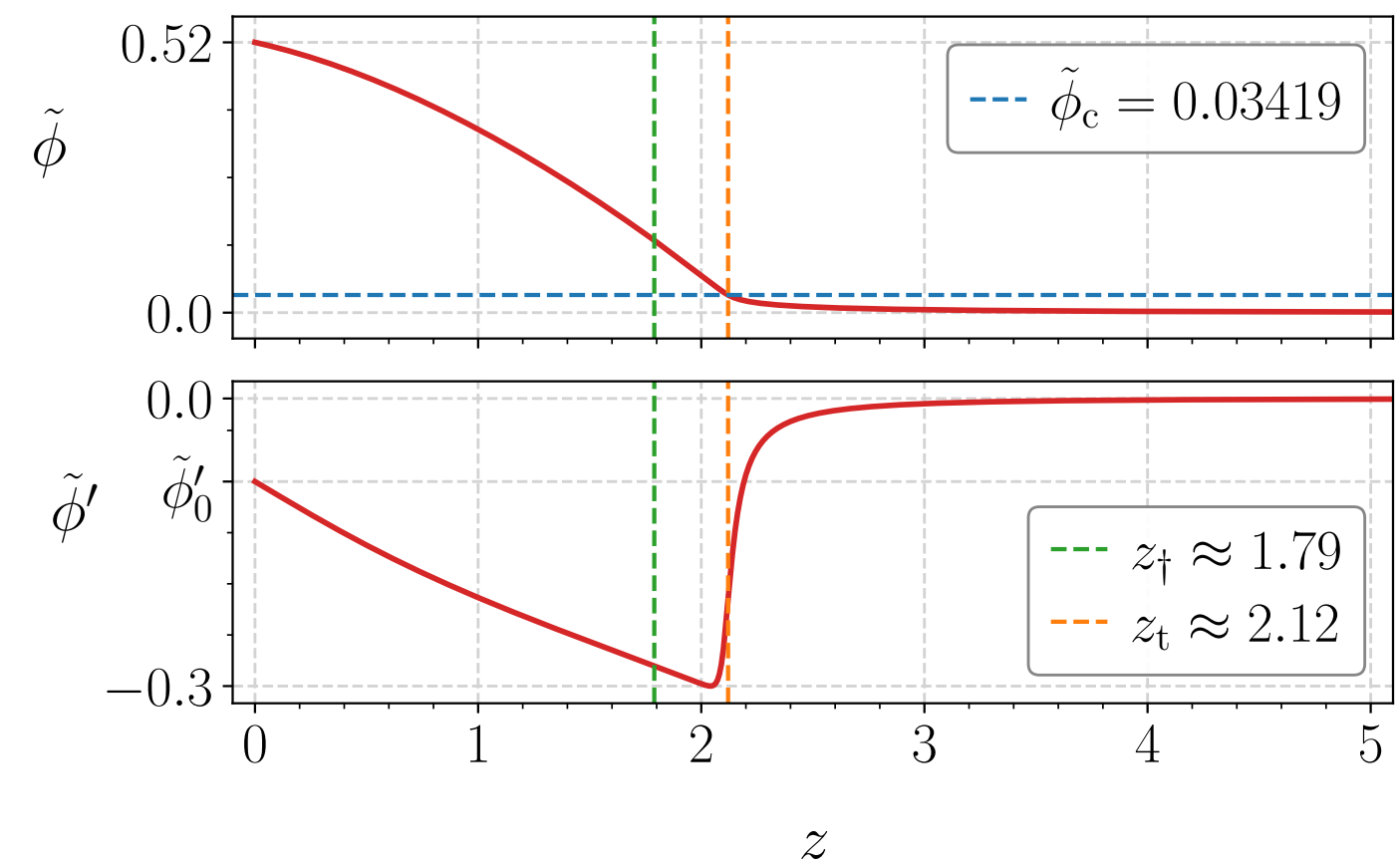
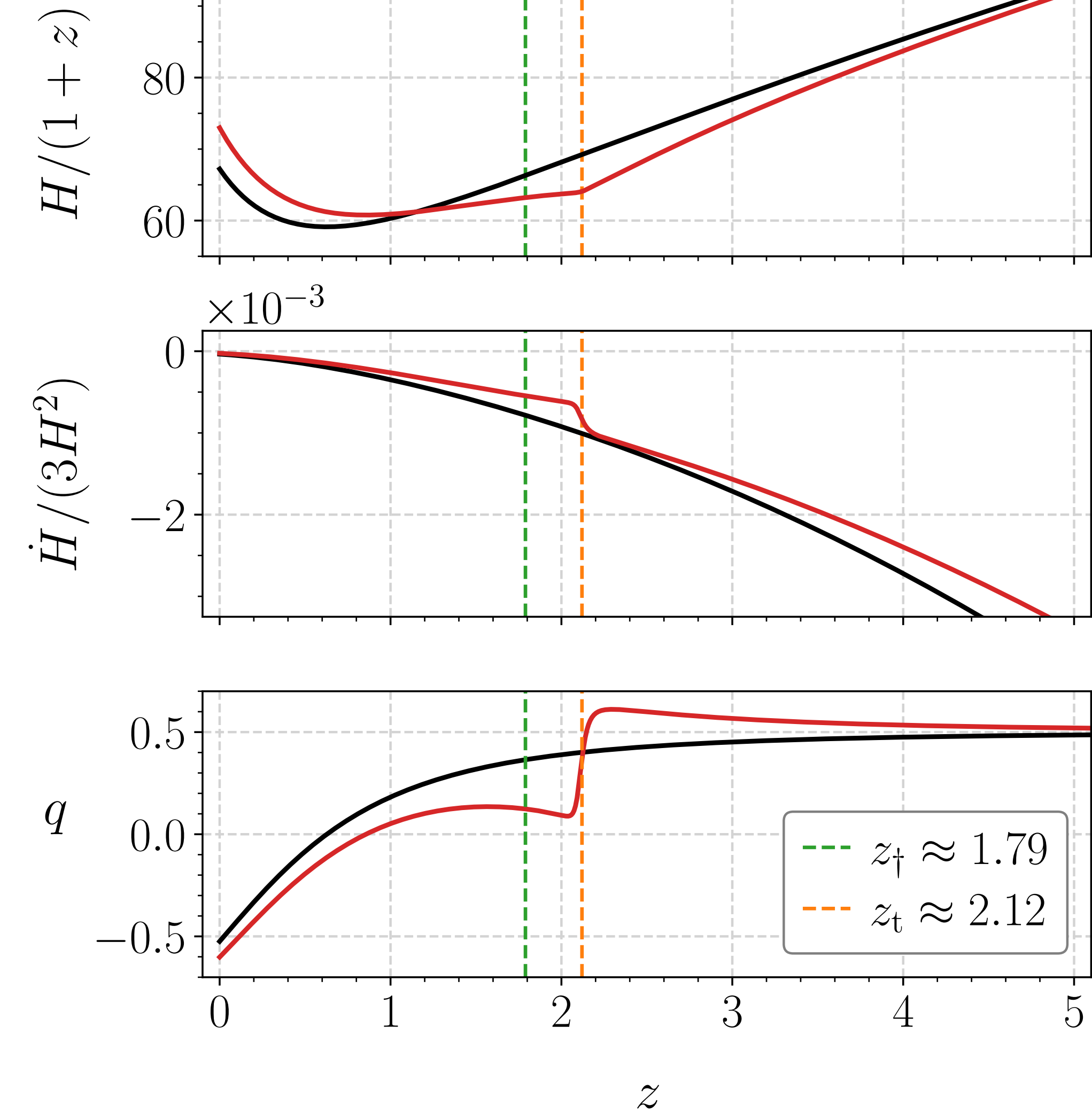
$$\ddot{\phi} + 3H\dot{\phi} + \xi c^2 \frac{dV}{d\phi} = 0$$





H_0	Ω_{m0}	ν	$\tilde{\phi}_c$	z_t	$z_\dagger$	Δz
75	0.2545	10	0.1269	1.871	1.545	0.326
73	0.2687		0.1106	2.104	1.744	0.360
71	0.2840		0.0861	2.547	2.122	0.425
69	0.3007		0.0314	4.382	3.707	0.675
68	-		-	-	-	-
75	0.2545	100	0.0356	1.919	1.609	0.310
73	0.2686		0.0342	2.127	1.796	0.331
71	0.2840		0.0321	2.476	2.109	0.367
69	0.3007		0.0287	3.251	2.807	0.444
68	0.3096		0.0250	4.347	3.791	0.556
67	-		-	-	-	-





$$\mathcal{S} = \int d^4x \, ||e|| \left[-\frac{1}{2\kappa} f(T) + \mathcal{L}_m \right]$$

$$f(T) = T e^{\beta T_0/T} \text{ with } T = 6H^2$$

Infrared teleparallel gravity

$$\left(\frac{H^2}{H_0^2} - 2\beta \right) e^{\beta H_0^2/H^2} = \Omega_{m0}(\beta) (1+z)^3$$

$$\Omega_{m0}(\beta) = (1 - 2\beta)e^\beta$$

$$\beta = \frac{1}{2} + W_k \left(-\frac{\Omega_{m0}}{2\sqrt{e}} \right) \text{ Lambert-}W \text{ function has two branches}$$

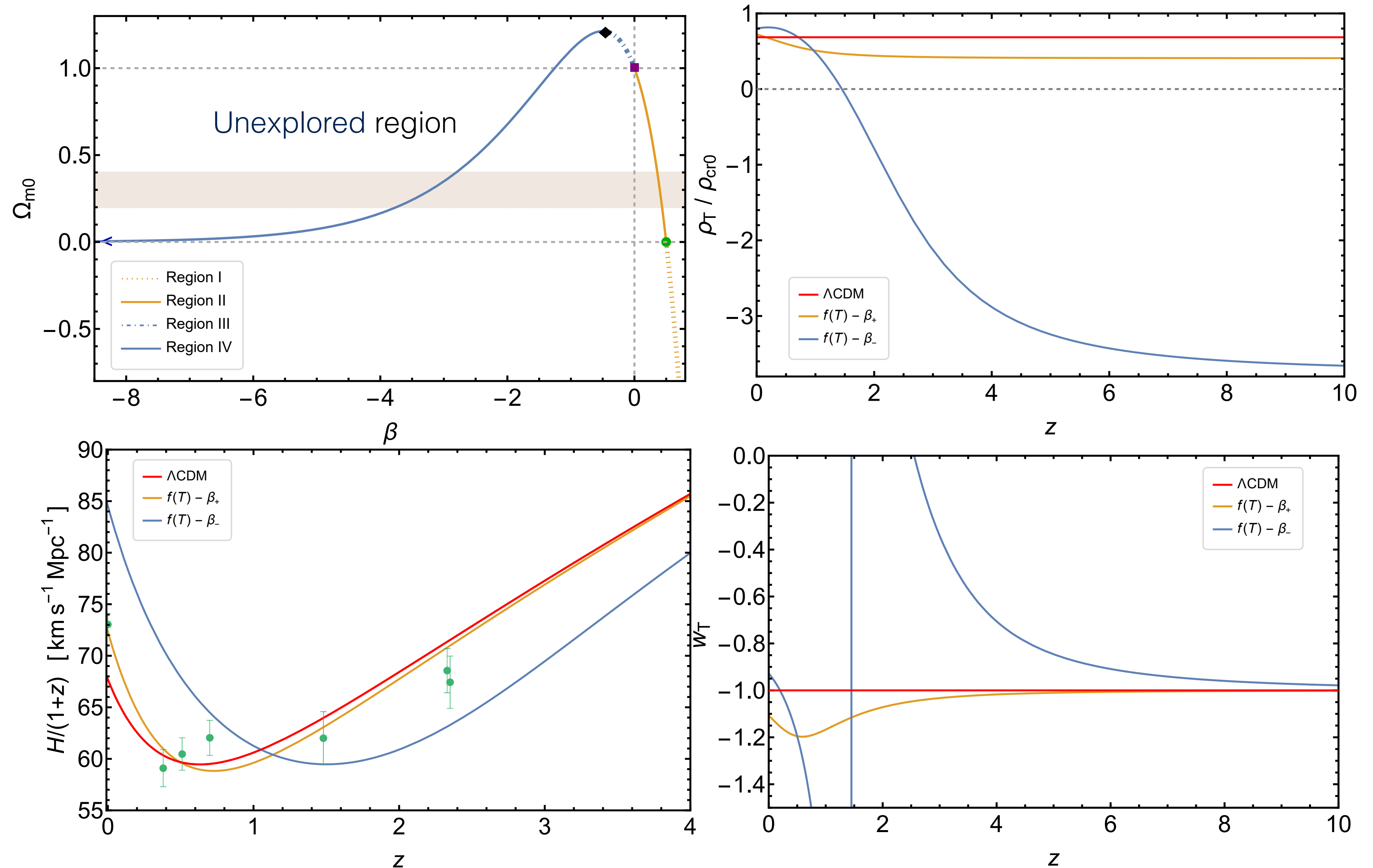
$$\rho_T(H) = \frac{3H^2}{\kappa} \left[1 - \left(1 - 2\beta H_0^2/H^2 \right) e^{\beta H_0^2/H^2} \right]$$

$$\ddot{\delta}_m + 2H\dot{\delta}_m - \frac{4\pi G_N}{f_T} \rho_m \delta_m = 0$$

$$f_T = e^{\beta H_0^2/H^2} (1 - \beta H_0^2/H^2)$$

$$\mathcal{G}_{\text{eff}} \equiv \frac{G_N}{f_T}$$

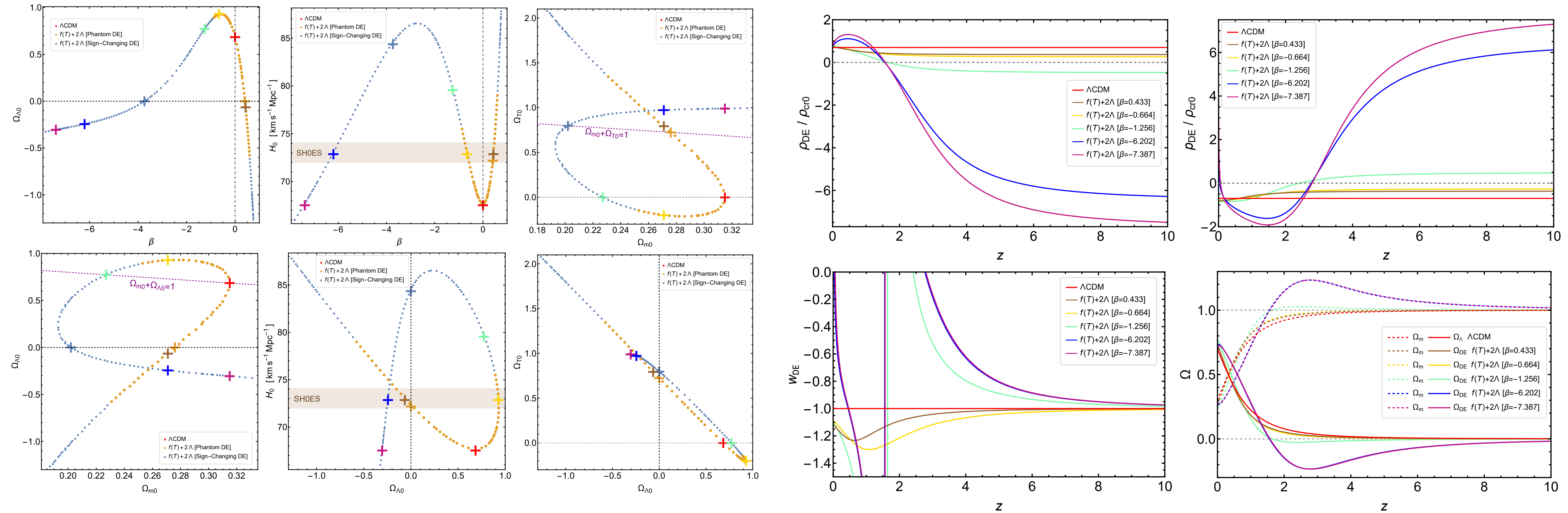
In fact $\beta < 0$ is safe!



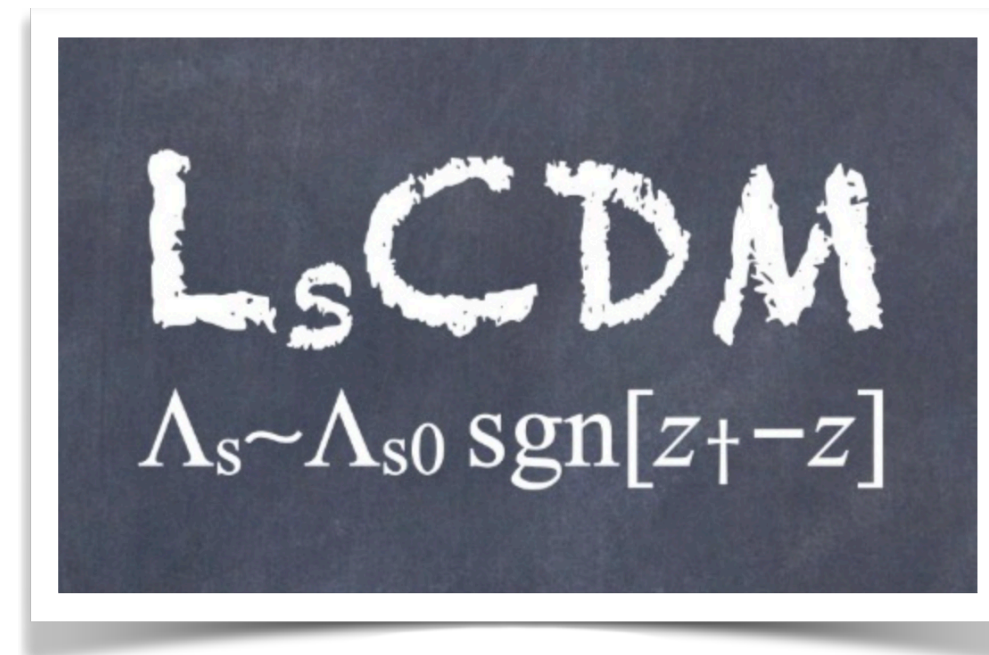
Plotted by ensuring consistency with Planck-CMB data

Infrared teleparallel gravity extended
with a cosmological constant

$$\mathcal{S} = \int d^4x \, ||e|| \left\{ -\frac{1}{2\kappa} [f(T) + 2\Lambda] + \mathcal{L}_m \right\} \quad \text{with} \quad f(T) = T e^{\beta T_0/T}$$



Conclusion



The introduction of a rapid (mirror-like) AdS-to-dS transition in the late universe into the current standard model of cosmology, the Λ_s CDM model, could represent a promising candidate for, or a guide towards, the development of a new concordance model of cosmology...

Thanks!

A flat $z_{\dagger} \in [1, 10]$ prior places 67% of its probability above $z_{\dagger} = 4$



The reported posterior barely contracts relative to the prior

$\mathcal{U}(1, 10) : \mu = 5.50, \sigma = 2.60$
reported: $z_{\dagger} = 6.0 \pm 2.4$

$$B_{[1,10]} \simeq \underbrace{\frac{1}{3} B_{\text{active}}}_{\text{finite-redshift switch}} + \underbrace{\frac{2}{3} \times 1}_{\Lambda\text{CDM-like tail}} \implies \text{evidence diluted toward } B = 1.$$

H_0 World Cup II reports:

$\Delta\chi^2 = -5.60$

$\Delta\text{AIC} = -3.60$

$\ln B = +0.92$

These numbers mildly favor Λ_s CDM; what fails is the H_0 shift for their particular CMB+BAO+SN setup.

Without a prior scan and profile $\Delta\chi^2(z_{\dagger})$, “weak support” is not a prior-robust verdict on the active Λ_s CDM regime.