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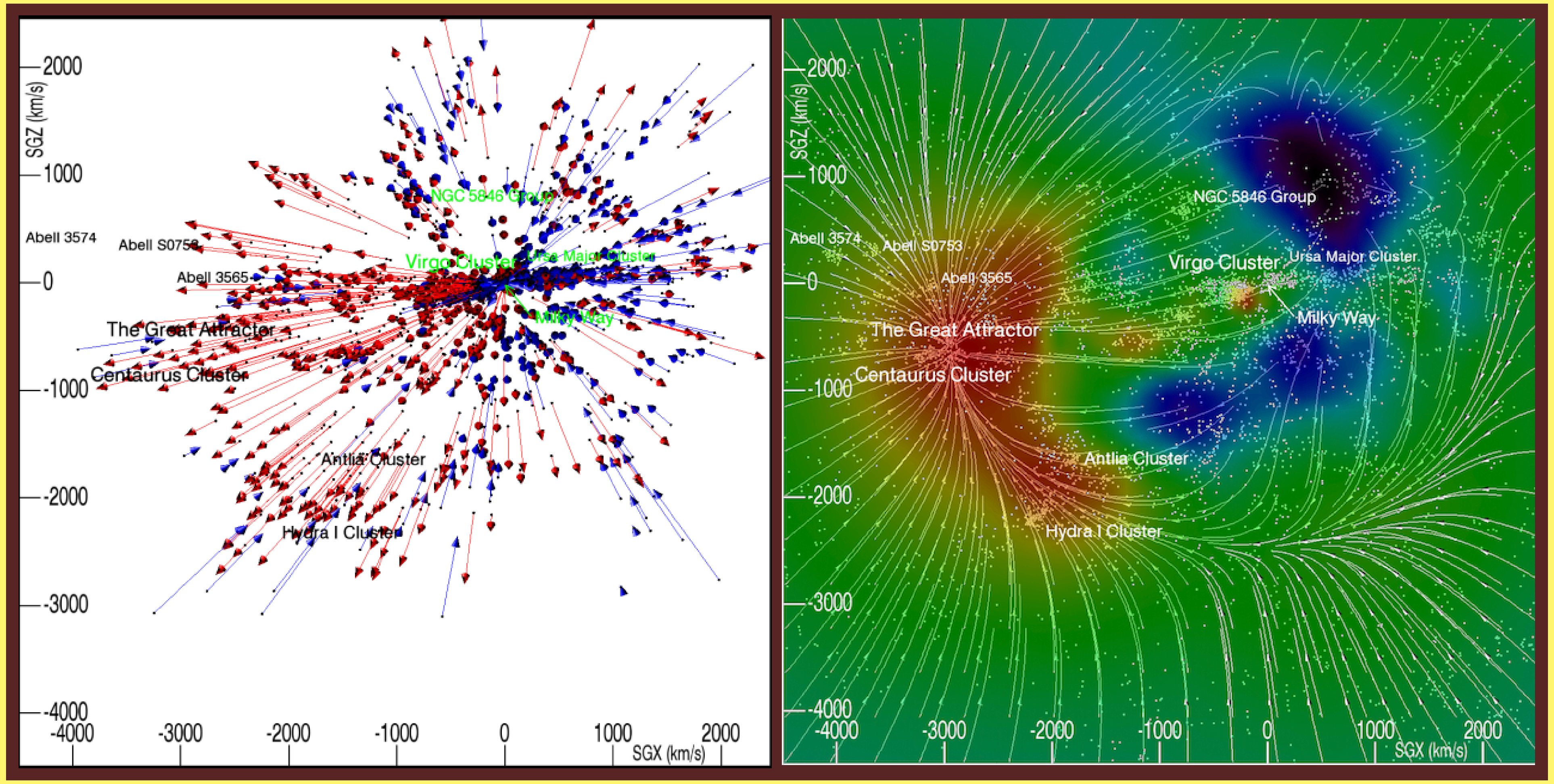
QUANTIFYING ANISOTROPIES IN THE LOCAL EXPANSION RATE WITH CF4

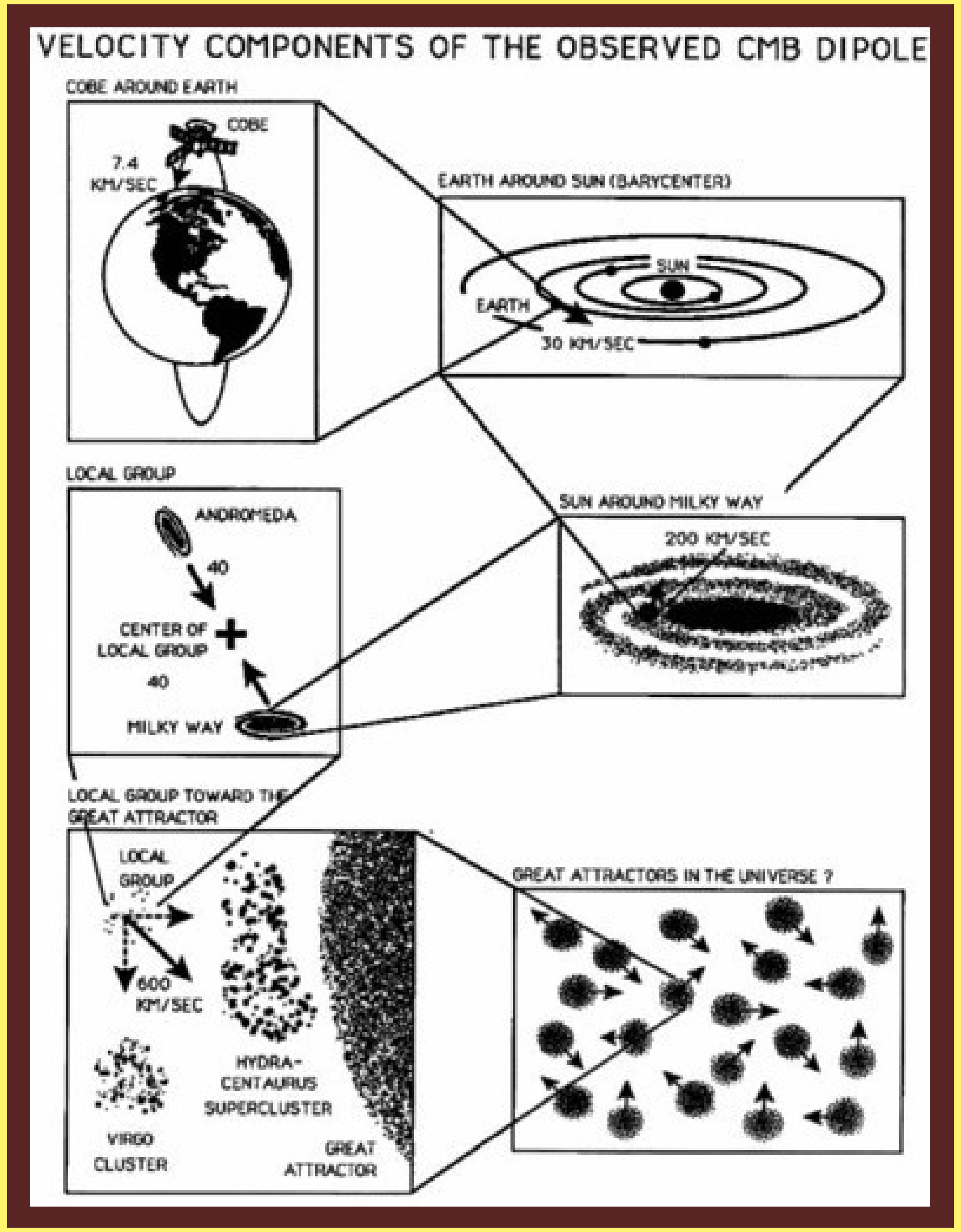
arXiv: 2312.09875 – JCAP09(2024)070
arXiv: 2408.04333 – JCAP02(2025)076
arXiv: 2510.02510 – JCAP03(2026)084

THE COSMOLOGICAL PRINCIPLE

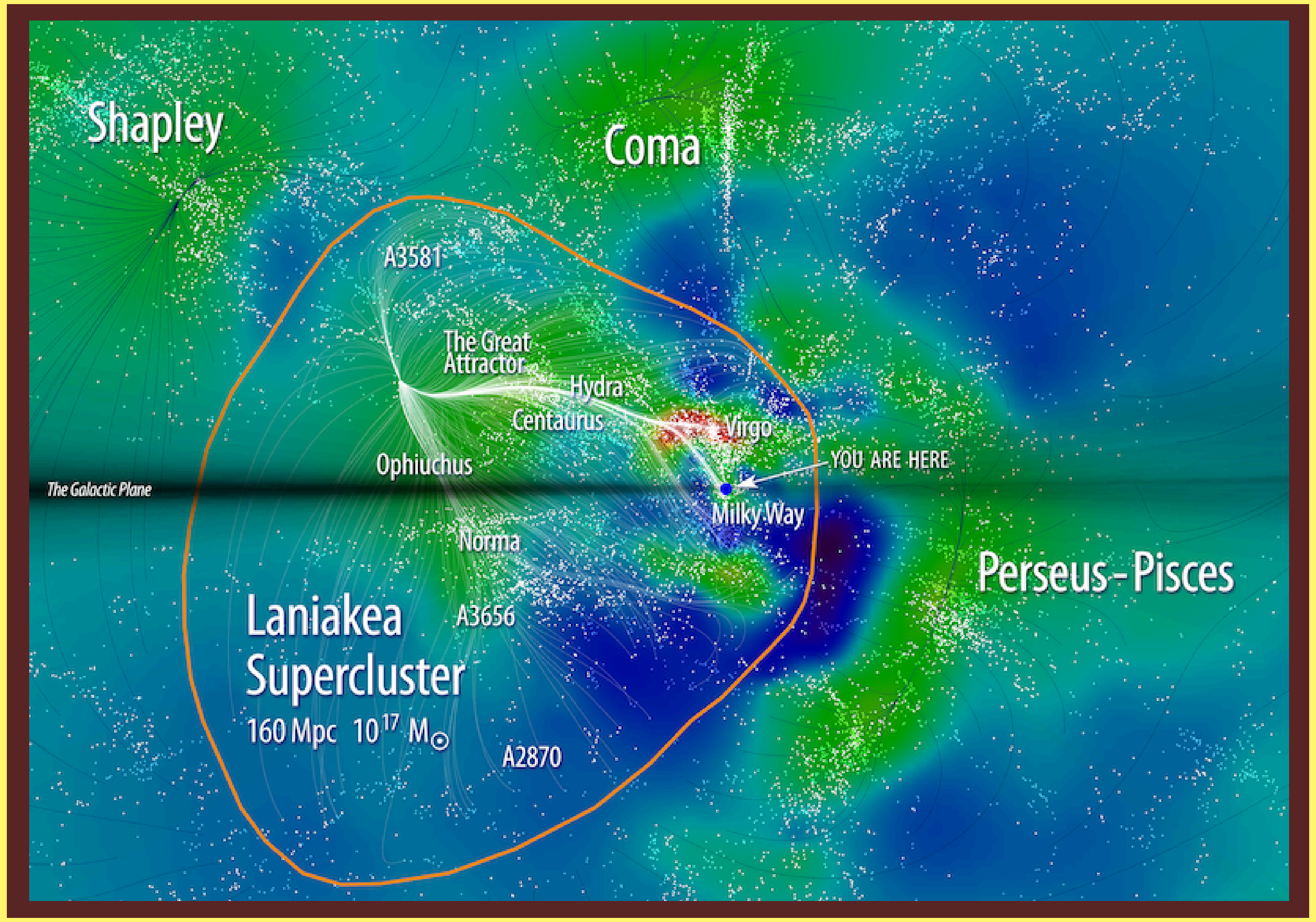
Locally, the cosmological principle doesn’t apply



Hélène M. Courtois et al 2013 AJ 146 69

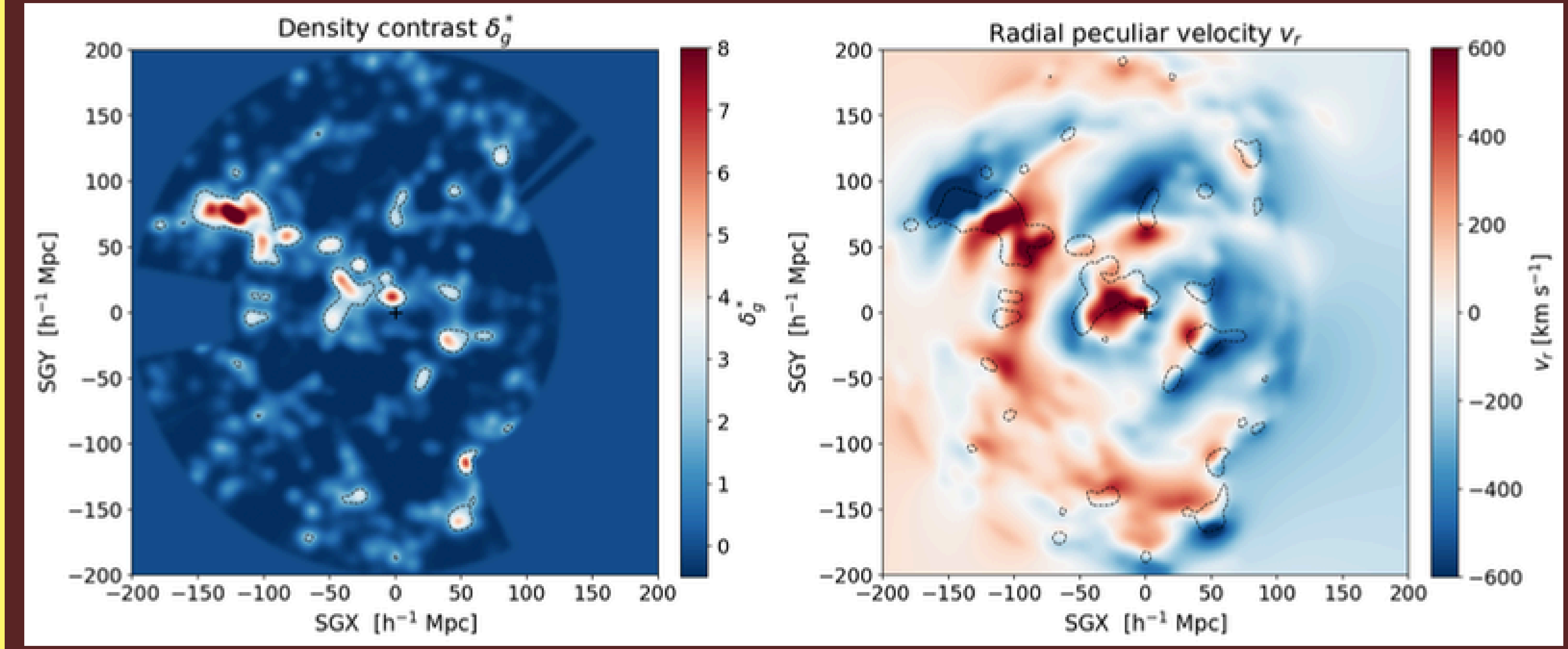


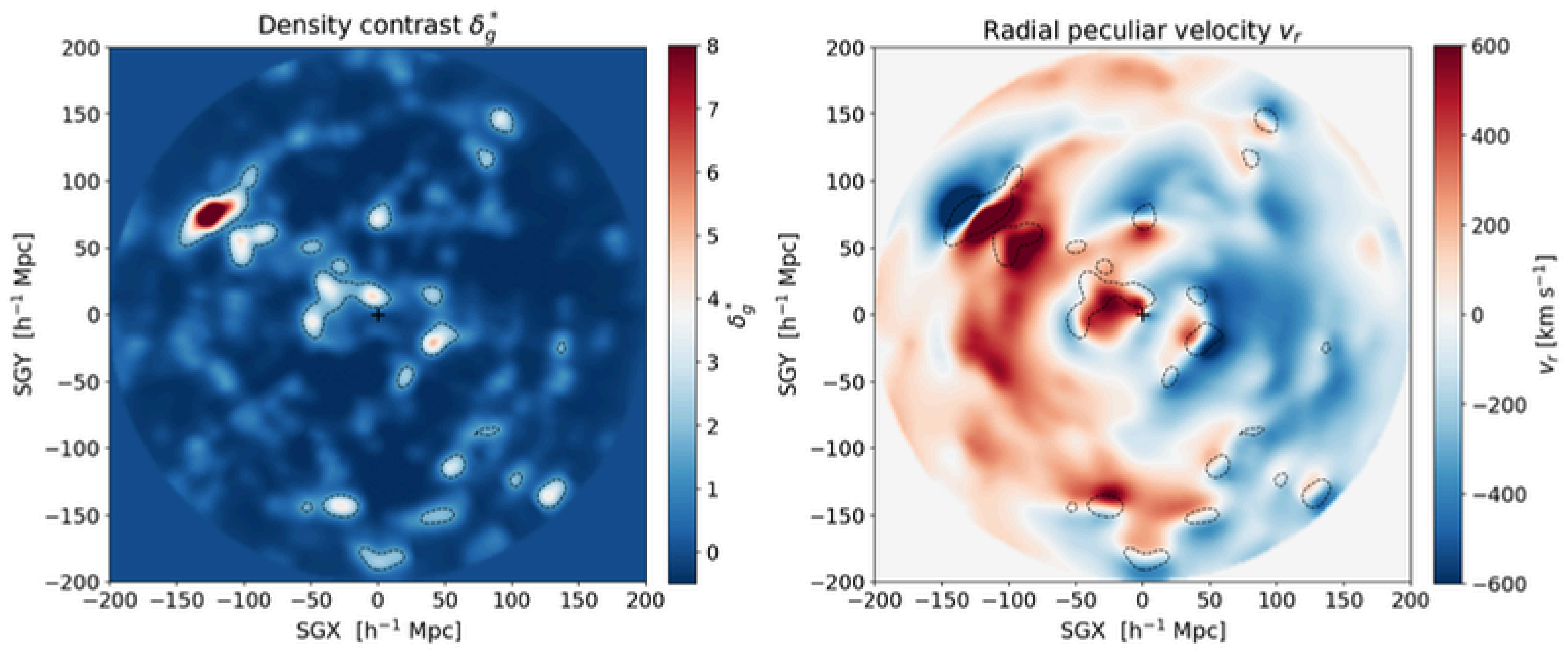
George F. Smoot Rev. Mod. Phys. 79, 1349 (2007)



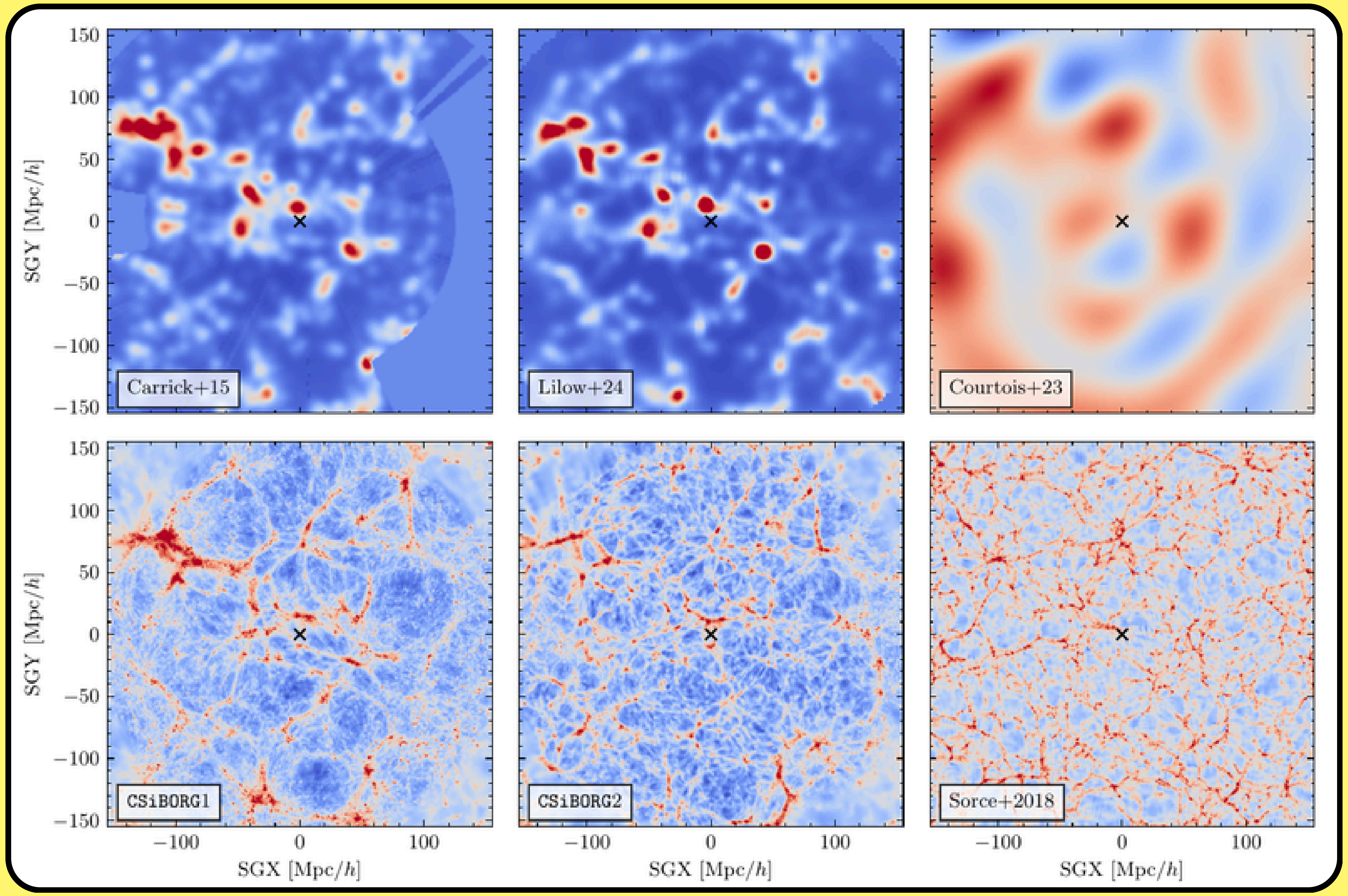
Tully et al. 2014
Nature, vol 513,
7516 – 71

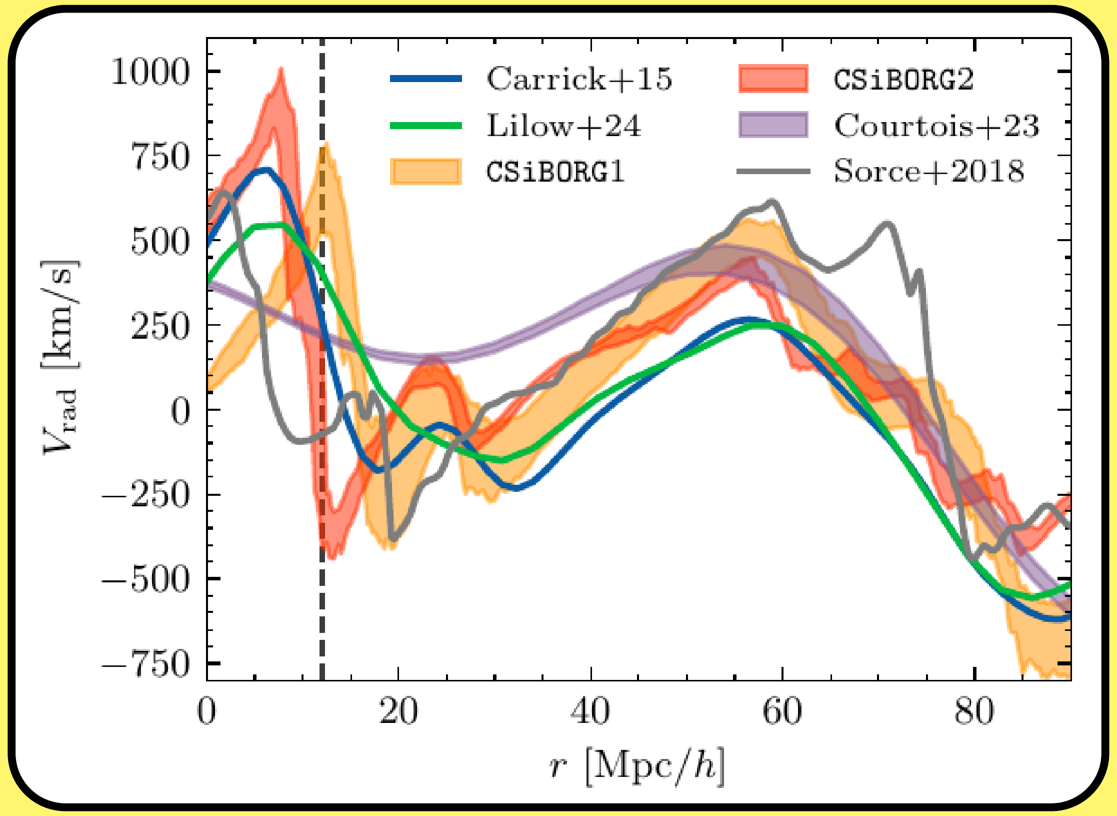
We correct for
perturbations
by taking into account
peculiar velocities...





Carrick et. al 2015 (right) and Lilow & Nusser (below) peculiar velocity surveys with high density constract regions highlighted and compared to peculiar velocities values



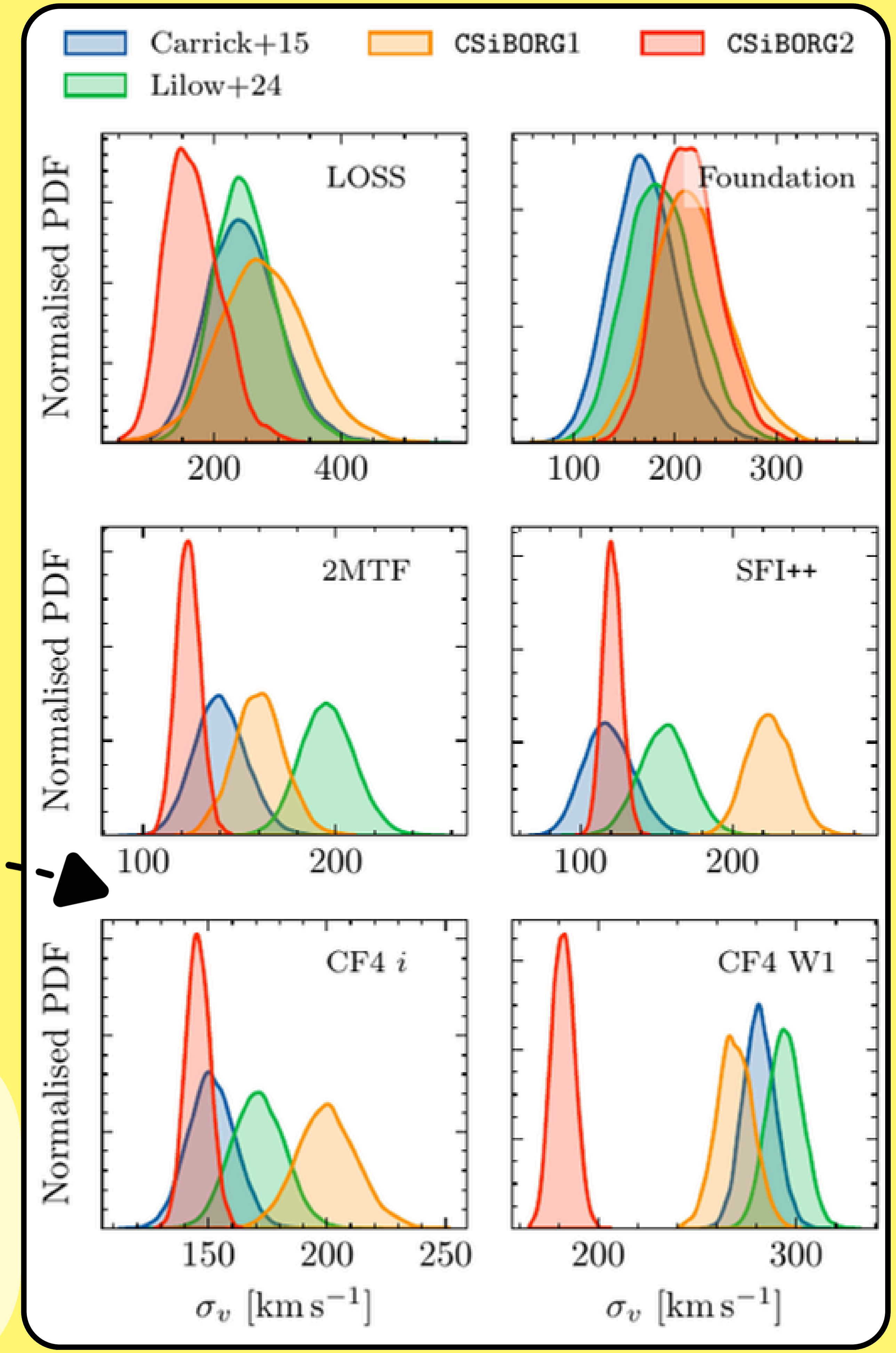


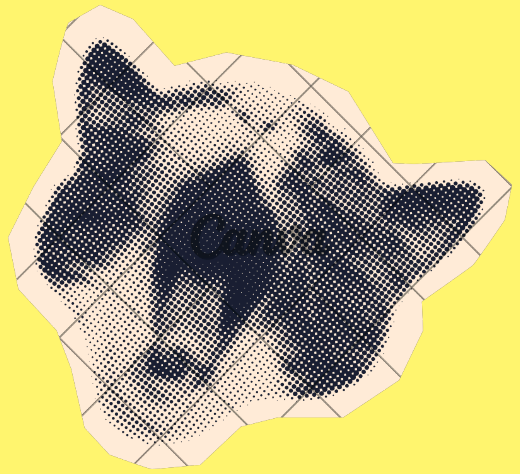
The Velocity Field Olympics

Richard Stiskalek et. al

Mon Not R Astron Soc (2025)

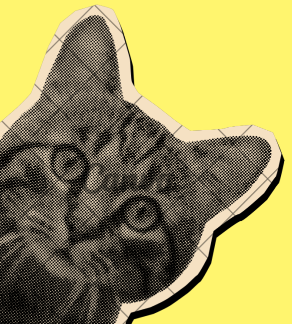
A Gaussian uncertainty between the field's prediction and the "observed" value.






if not peculiar velocities

then what?

TWO DIFFERENT THEORETICAL APPROACHES

$$1 + z = \frac{a(t)}{a(t_0)} = z(t)$$

$$H(t) = \frac{\dot{a}(t)}{a(t)}, \qquad q(t) = -\frac{a\,\ddot{a}(t)}{\dot{a}^2(t)} = -\left(1 + \frac{\dot{H}}{H^2}\right)$$

$$d_L(z) = \frac{z}{H_0}\Big[1 + \frac{1}{2}(1 - q_0)z + \frac{1}{6}\left(3q_0^2 + q_0 - 1 + \Omega_{K0} - j_0\right)z^2 + \frac{1}{24}\left(2 - q_0(2 + 15q_0 + 15q_0^2) + 5j_0(1 + 2q_0) - 2\Omega_{K0}(1 + 3q_0) + s_0\right)z^3\Big] + \mathcal{O}(z^5)\,.$$

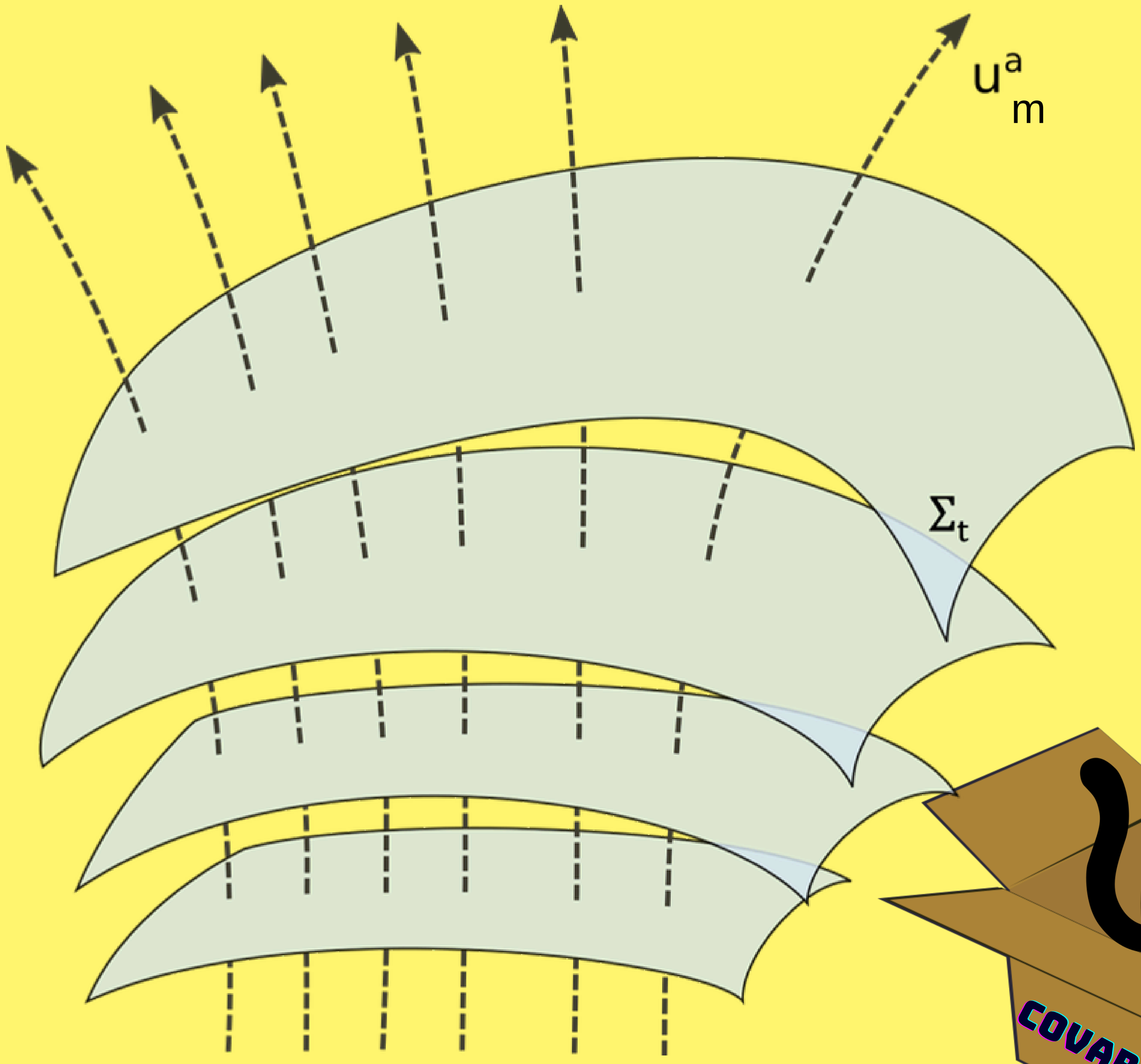
- Cannot account for possible spatial anisotropies
- Doesn't allow directional dependence
- Cannot handle the influence of local effects into data

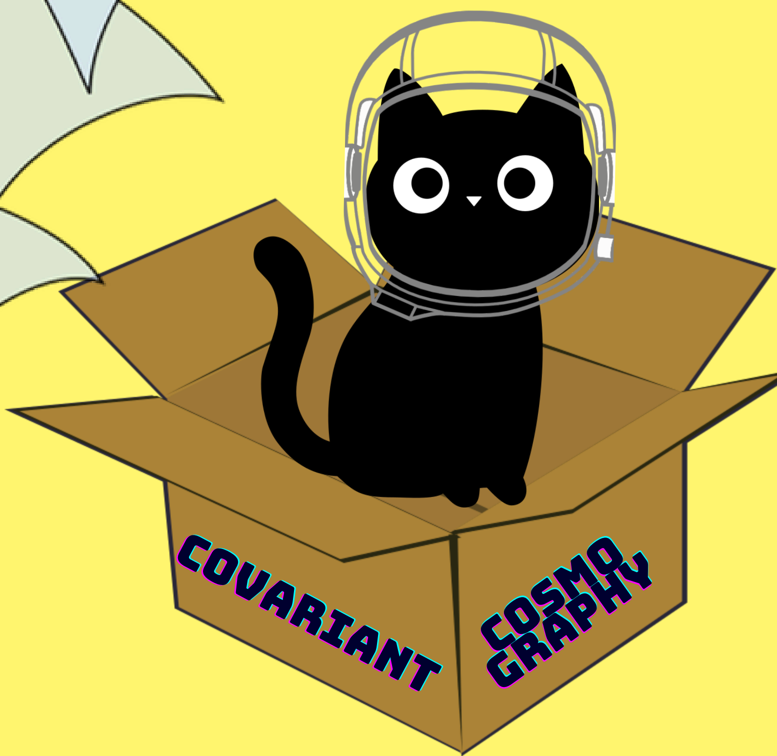
COVARIANT COSMOGRAPHY

- Define a congruence of matter observers with four-velocity u^a_m .
- Realize a 3+1 split, where the metric on each surface is given by:

$$h_{ab} = g_{ab} + u_a u_b\ ,$$

where h_{ab} is also known as the projection operator.



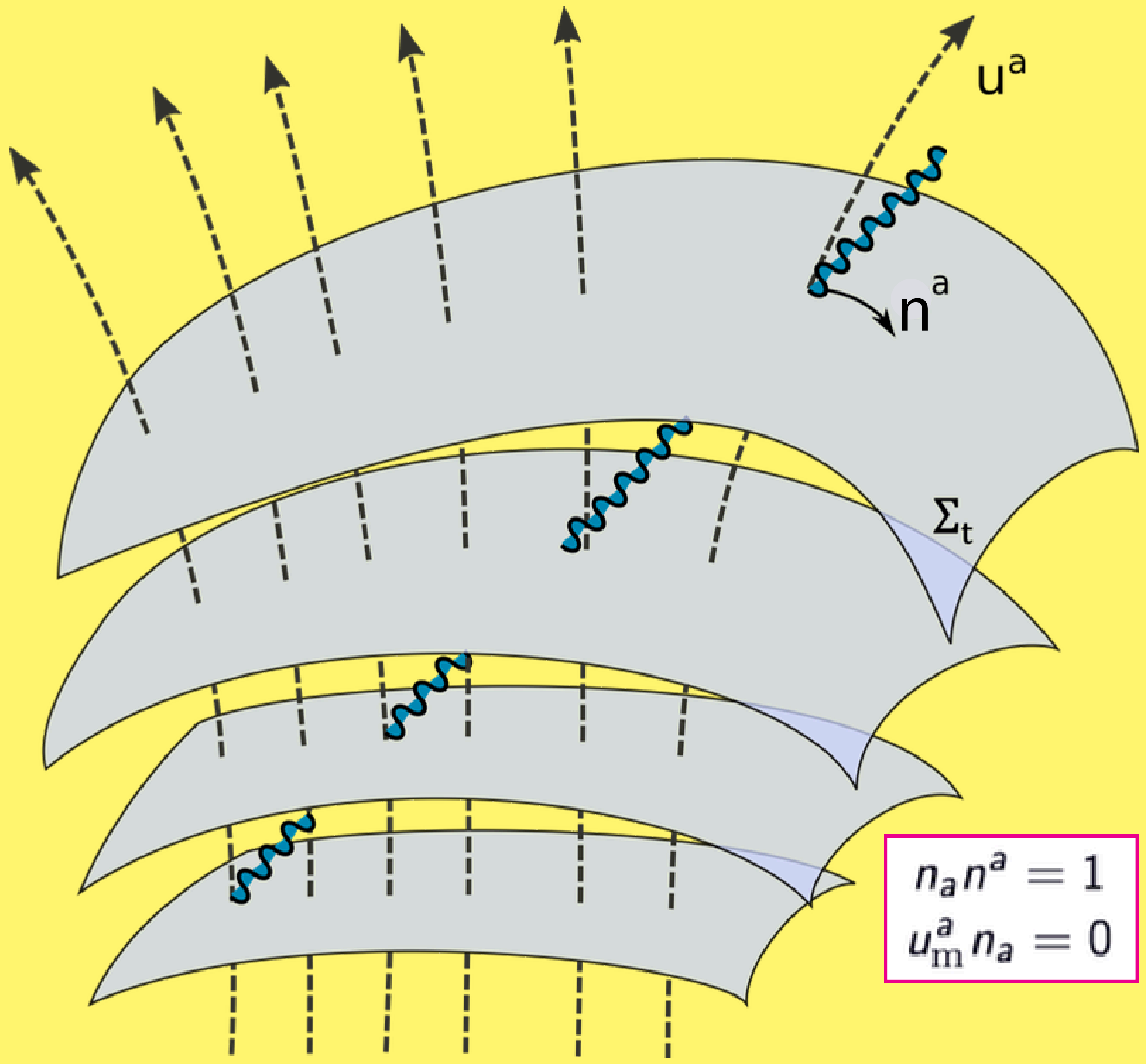


HOW MATTER MOVES

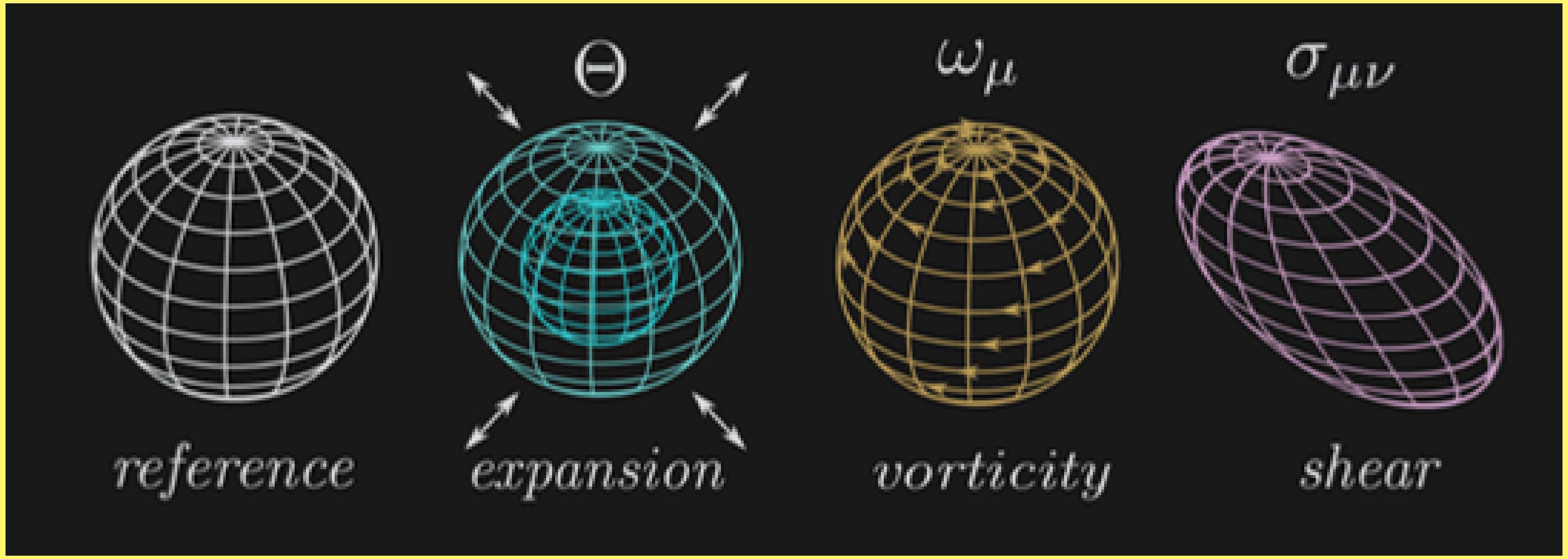
How the matter congruence varies along all 4 space-time directions can be used to infer the dynamical properties of the universe:

$$\nabla^b u^a_{\rm m} = \frac{1}{3}\Theta_{\rm m} h^{ab}_{\rm m} + \sigma^{ab}_{\rm m} \equiv \Theta^{ab}_{\rm m} \,,$$

where $\Theta_{\rm m}$ and $\sigma^{ab}_{\rm m}$ are the *expansion* and *shear*, respectively, while $\Theta^{ab}_{\rm m}$ represents the **expansion tensor**.
Note we are assuming a vorticity free geodesic fluid.



Rezzolla & Zanotti (2013)



COVARIANT COSMOGRAPHY

$$d_L(z,\mathbf{n}) = \frac{z}{\mathbb{H}_o}\Big[1 + \frac{1}{2}(1-\mathbb{Q}_o)z + \frac{1}{6}\left(3\mathbb{Q}_o^2 + \mathbb{Q}_o - 1 + \mathbb{R}_o - \mathbb{J}_o\right)z^2$$
$$+ \frac{1}{24}\left(2 - \mathbb{Q}_o\left(2 + 15\mathbb{Q}_o + 15\mathbb{Q}_o^2\right) + 5\mathbb{J}_o(1+2\mathbb{Q}_o) - 2\mathbb{R}_o(1+3\mathbb{Q}_o) + \mathbb{S}_o\right)z^3\Big] + \mathcal{O}(z^5)\,,$$

Cosmographic parameters are line-of-sight dependent:

$$\mathbb{H}_o = \mathbb{H}_o(\mathbf{n})$$
$$\mathbb{Q}_o = \mathbb{Q}_o(\mathbf{n})$$
$$\mathbb{J}_o = \mathbb{J}_o(\mathbf{n})$$
$$\mathbb{S}_o = \mathbb{S}_o(\mathbf{n})$$
$$\mathbb{R}_o = \mathbb{R}_o(\mathbf{n})$$

The Hubble parameter is given by:

$$\mathbb{H} = K_a K_b \Theta_{\mathrm{m}}^{ab} = \frac{\Theta_{\mathrm{m}}}{3} + \sigma_{\mathrm{m}}^{ab} n_a n_b\,,$$

and, in the presence of a boost:

$$\langle\tilde{\mathbb{H}}\rangle \mathrel{\hat{=}} \gamma^2\left[\left(1+\frac{v^2}{3}\right)\langle\mathbb{H}\rangle + \sigma_{\mathrm{m}}^{ab}\, v_a v_b\right]\,,$$
$$\tilde{\mathbb{H}}^a \mathrel{\hat{=}} -2\gamma\left(\langle\mathbb{H}\rangle\, v^a + \sigma_{\mathrm{m}}^{ab}\, v_b\right)\,,$$
$$\tilde{\mathbb{H}}^{ab} \mathrel{\hat{=}} \sigma_{\mathrm{m}}^{ab} + \langle\mathbb{H}\rangle\, v^{\langle a} v^{b\rangle}\,.$$

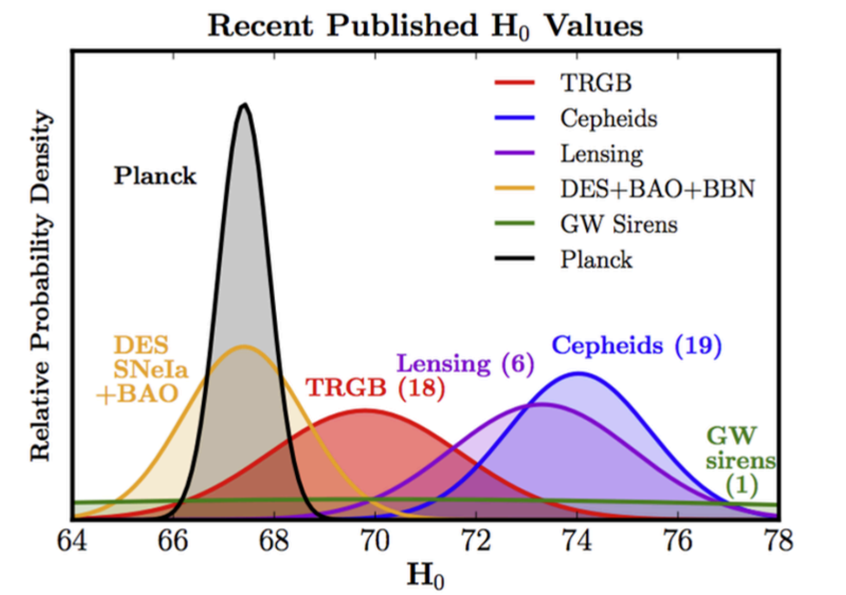
[Kristian and R.K. Sachs (1966), Maartens et. al 2024
Clarkson 1999, Umeh 2013, Heinesen 2020]

MEASURING THE ANISOTROPIC RATE OF EXPANSION FIELD

$$\eta(z,n) = \log\left[\frac{z}{d_L(z,n)}\right] - \mathcal{M}(z) \equiv \log\left(\frac{z}{d_L(z,n)}\right) - \frac{1}{4\pi}\int_S \log\left(\frac{z}{d_L(z,n)}\right) d\Omega$$

directly from data!

⋆ η is independent of distance calibration, since calibration affects only the monopole

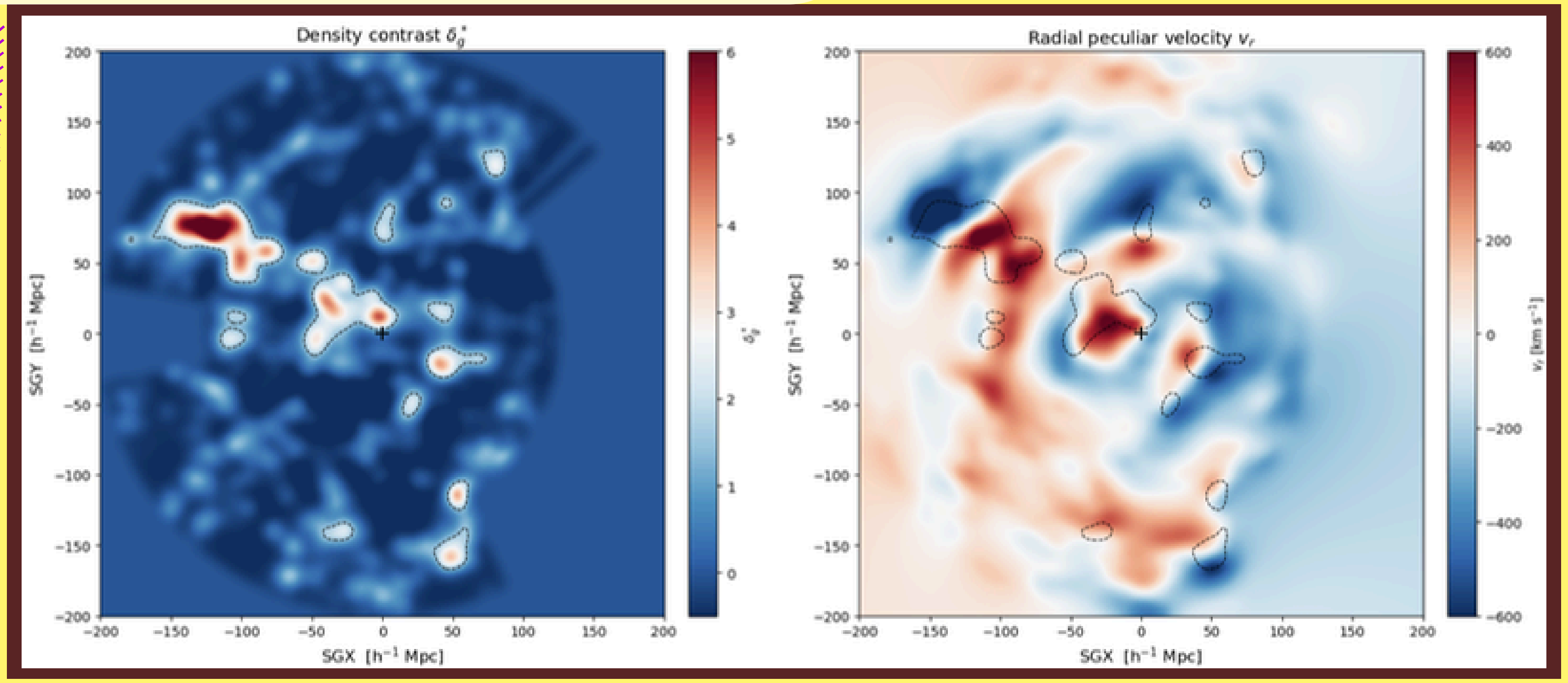


⋆ The monopole is subtracted, so ⟨η⟩=0

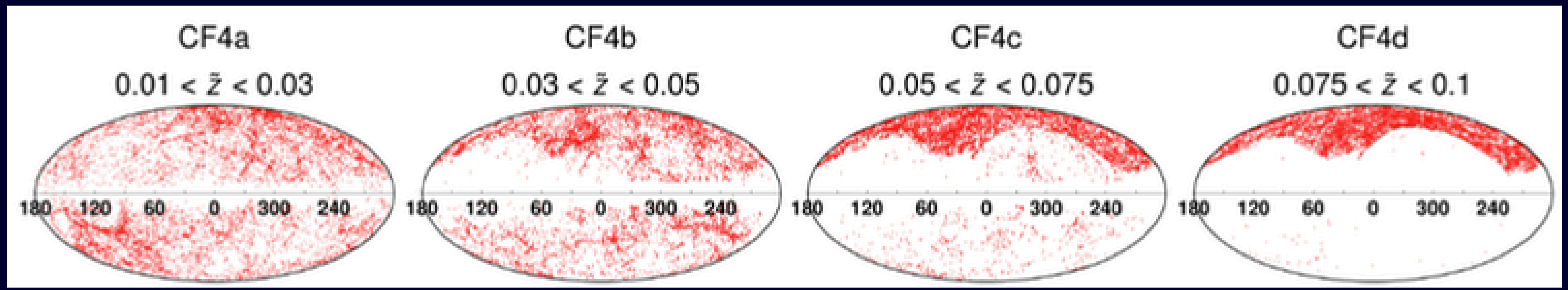
⋆ η is only sensitive to anisotropies in dL. So η = 0 in FLRW.

★ NO NEED for peculiar velocities!

OUR POLICY:
we don't correct for what is not a problem

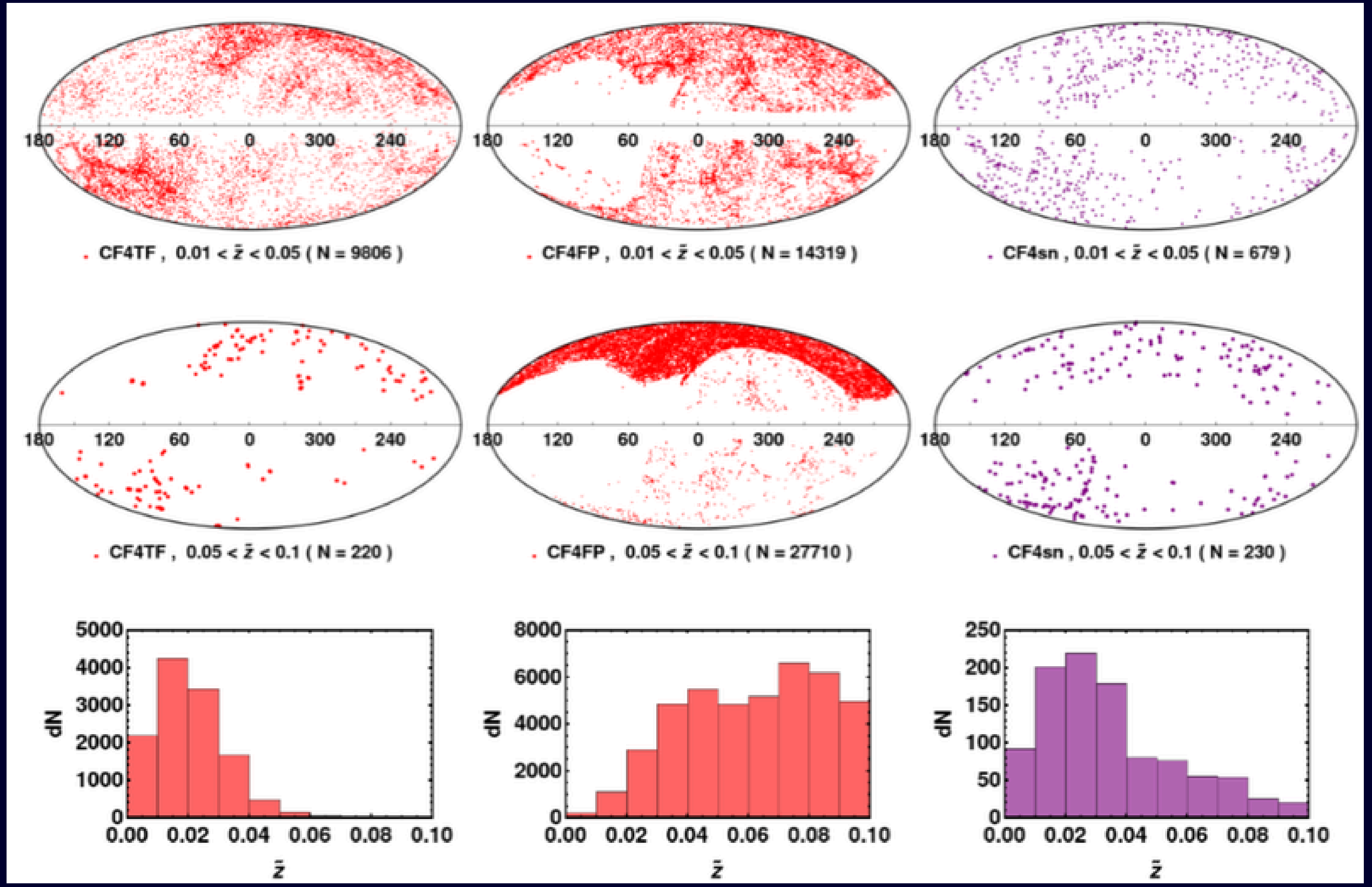


COSMIC FLOWS 4

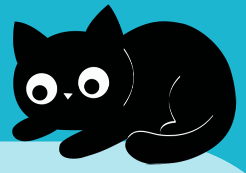


55.874 nearby galaxies (z ≲ 0.1)

CF4 TF CF4 FP CF4 SN



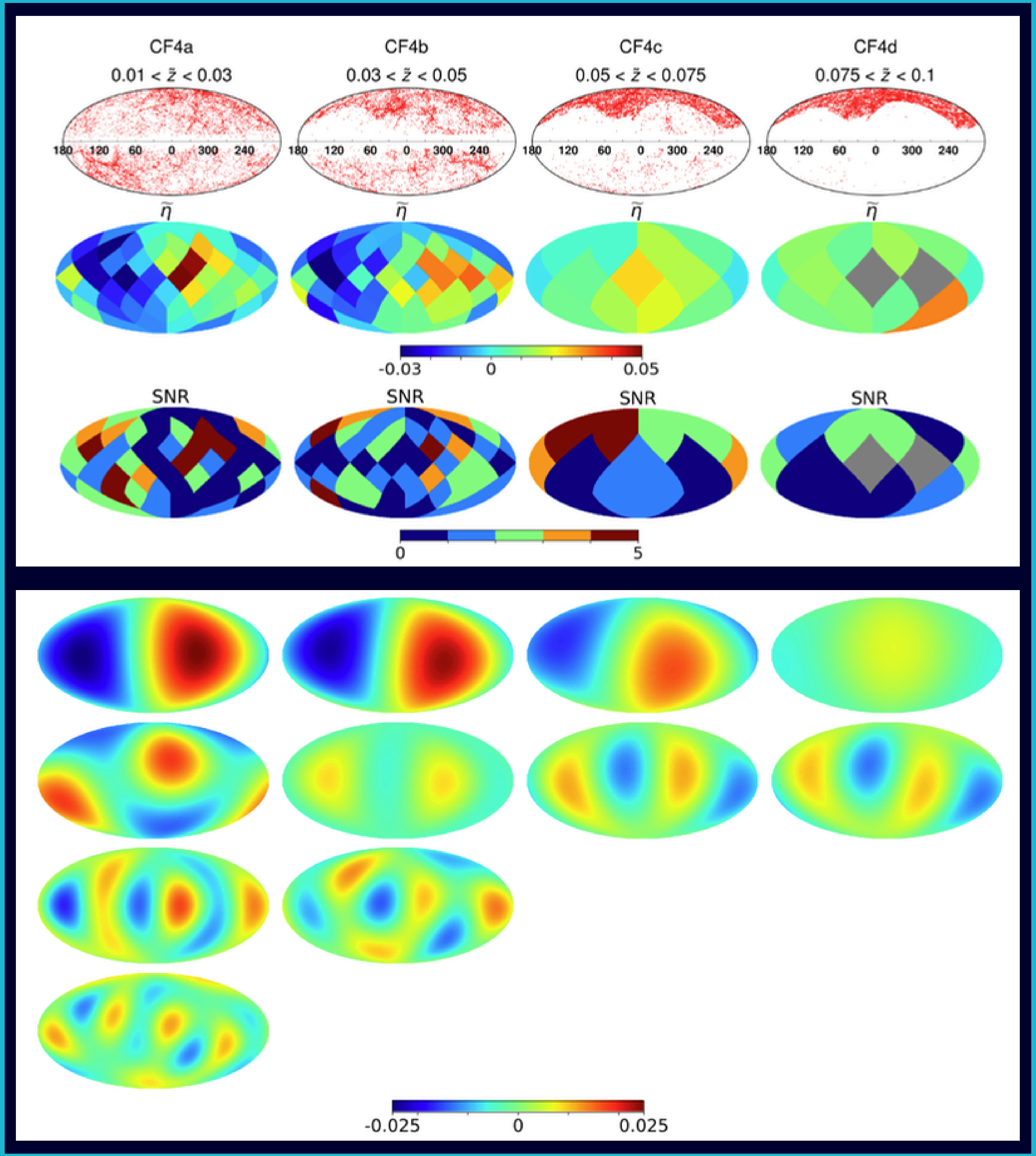
Redshift-independent distance from complementary methods:



- ⋆ 42.221 via Fundamental Plane (FP) relation.
- ⋆ 12.221 via Tully–Fisher (TF) relation
- ⋆ ~1.000 Type Ia supernova (mainly Pantheon+ and SH0ES)

A FINITE NUMBER OF DEGREES OF FREEDOM

$2\ell + 1$ per multipole



1. measure η:

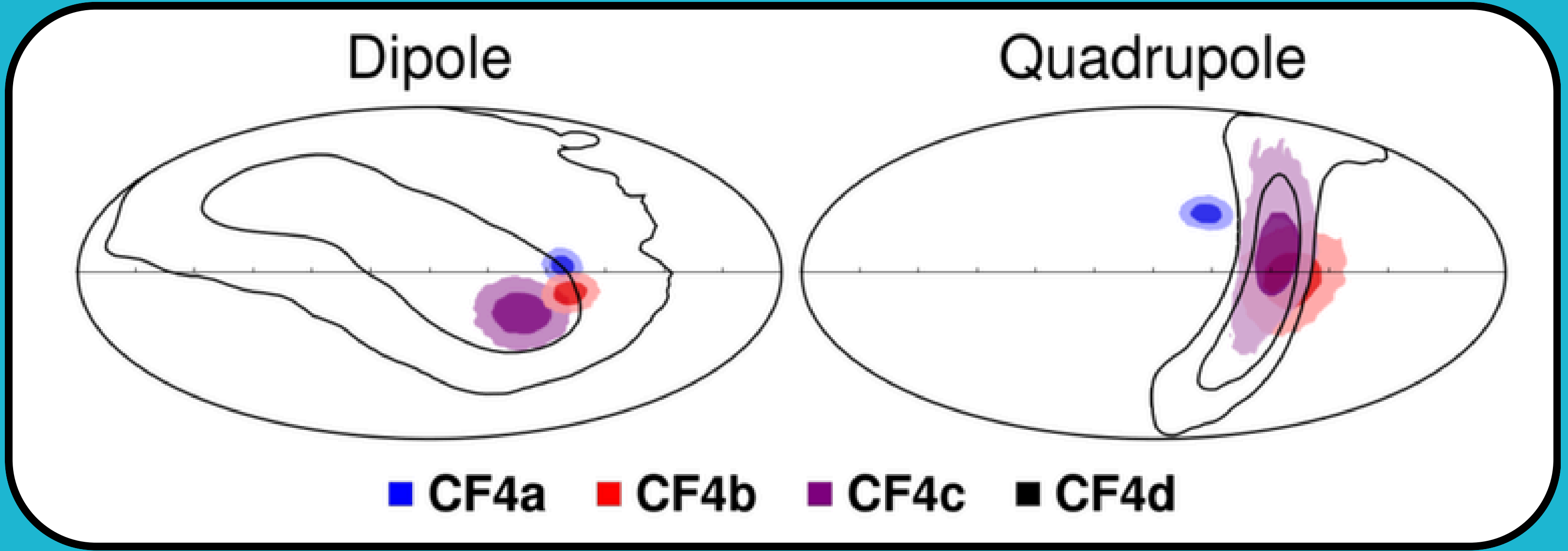
$$\eta(z, \boldsymbol{n}) \equiv \log\left(\frac{z}{d_L(z, \boldsymbol{n})}\right) - \frac{1}{4\pi}\int_S \log\left(\frac{z}{d_L(z, \boldsymbol{n})}\right) d\Omega$$

2. spherical harmonic decomposition:

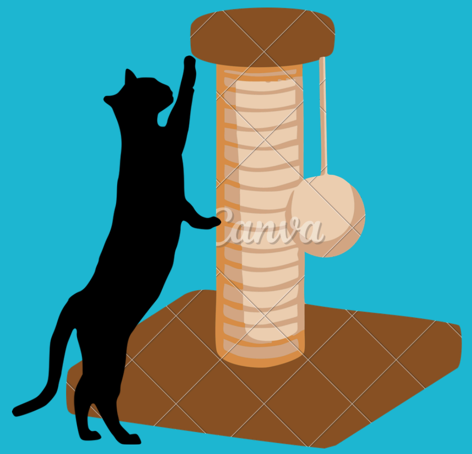
$$\tilde{\eta}(z, \boldsymbol{n}) = \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} \tilde{\eta}_{\ell m}(z)\, Y_{\ell m}(\boldsymbol{n}),$$

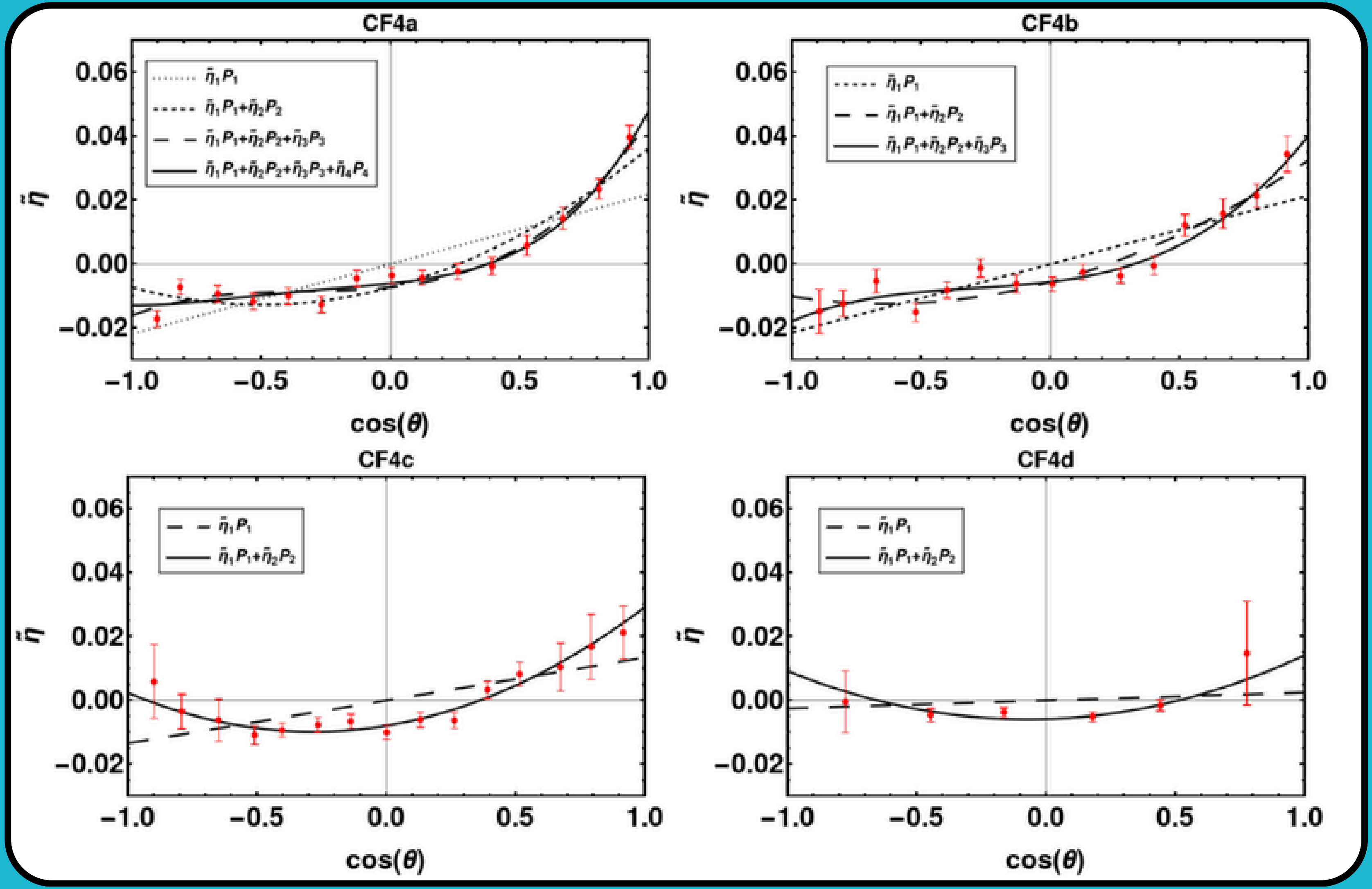
- ⋆ strong quadrupole
- ⋆ indication for axisymmetry
- ⋆ persistency along all redshift bins

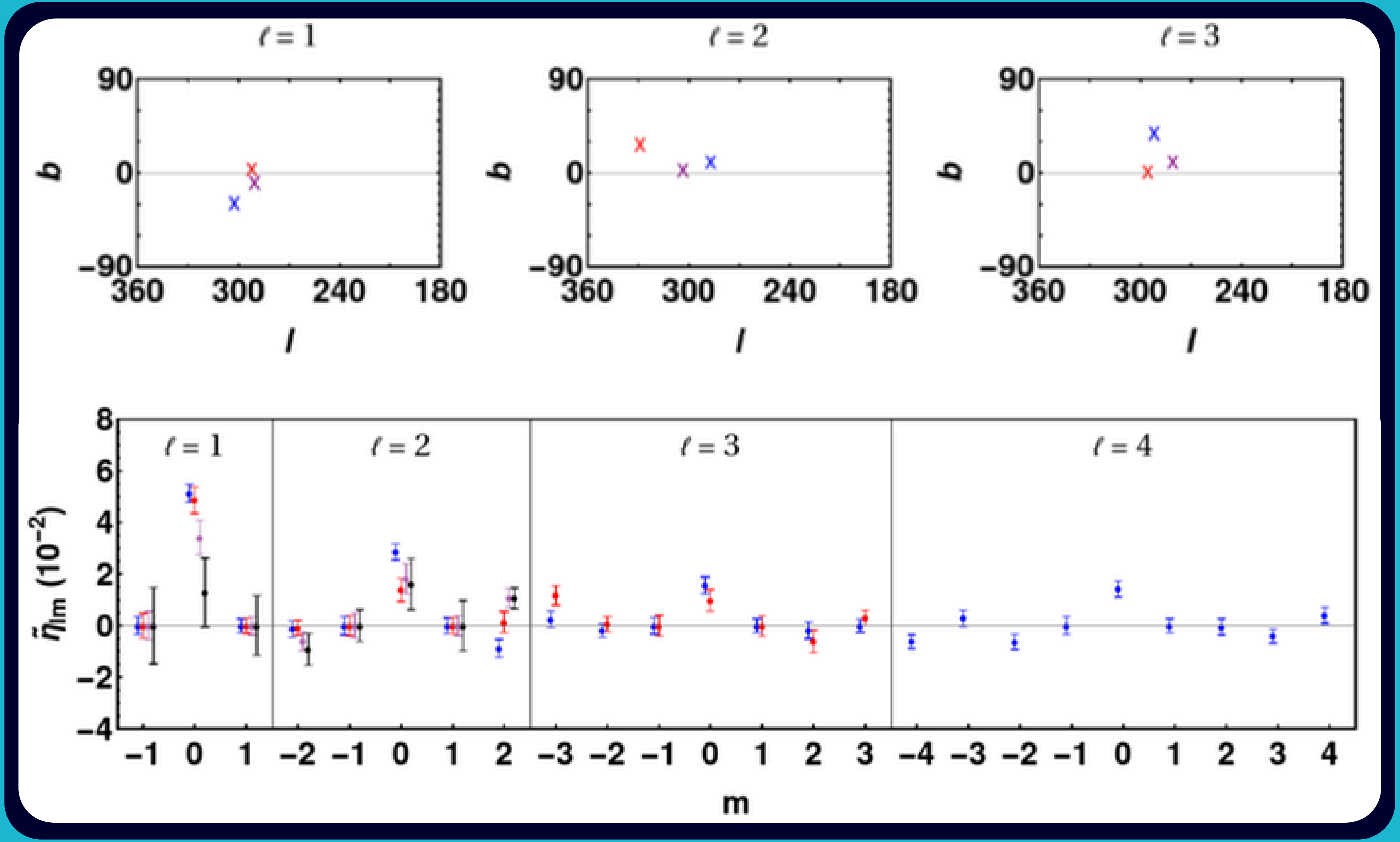
INVESTIGATING THE PRESENCE OF AXIAL SYMMETRY

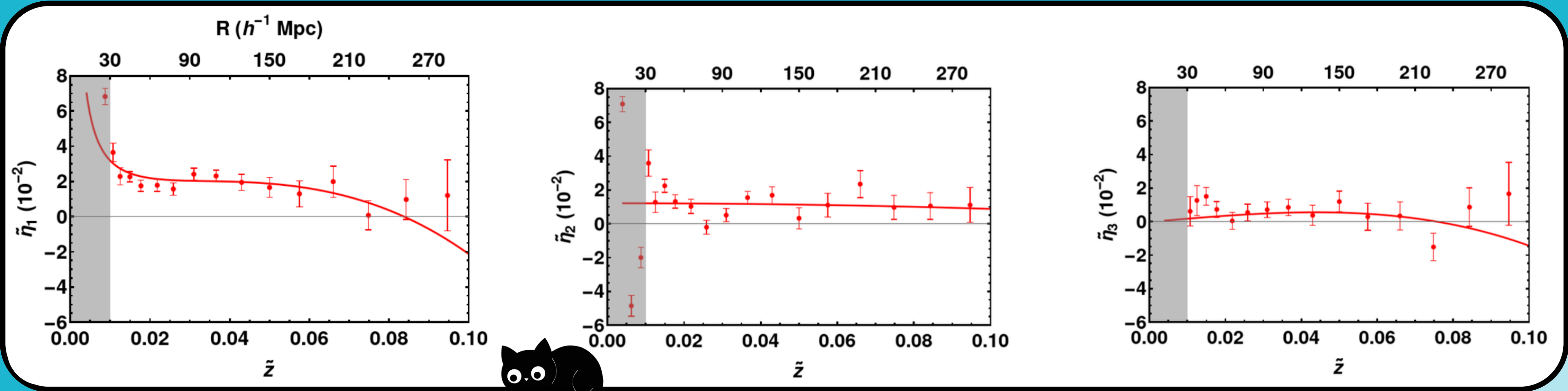


$$\tilde{\eta}(z, \cos\theta) = \sum_{\ell=1}^{\ell_{\max}} \tilde{\eta}_\ell(z) P_\ell(\cos\theta),$$

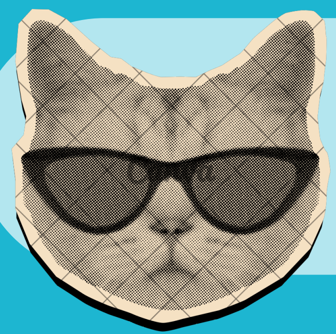








AXISYMMETRIC ETA EVOLUTION



CONNECTING WITH PERTURBED LCDM

$$\eta(z,n) = \log\left[\frac{z}{d_L(z,n)}\right] - \mathcal{M}(z)$$

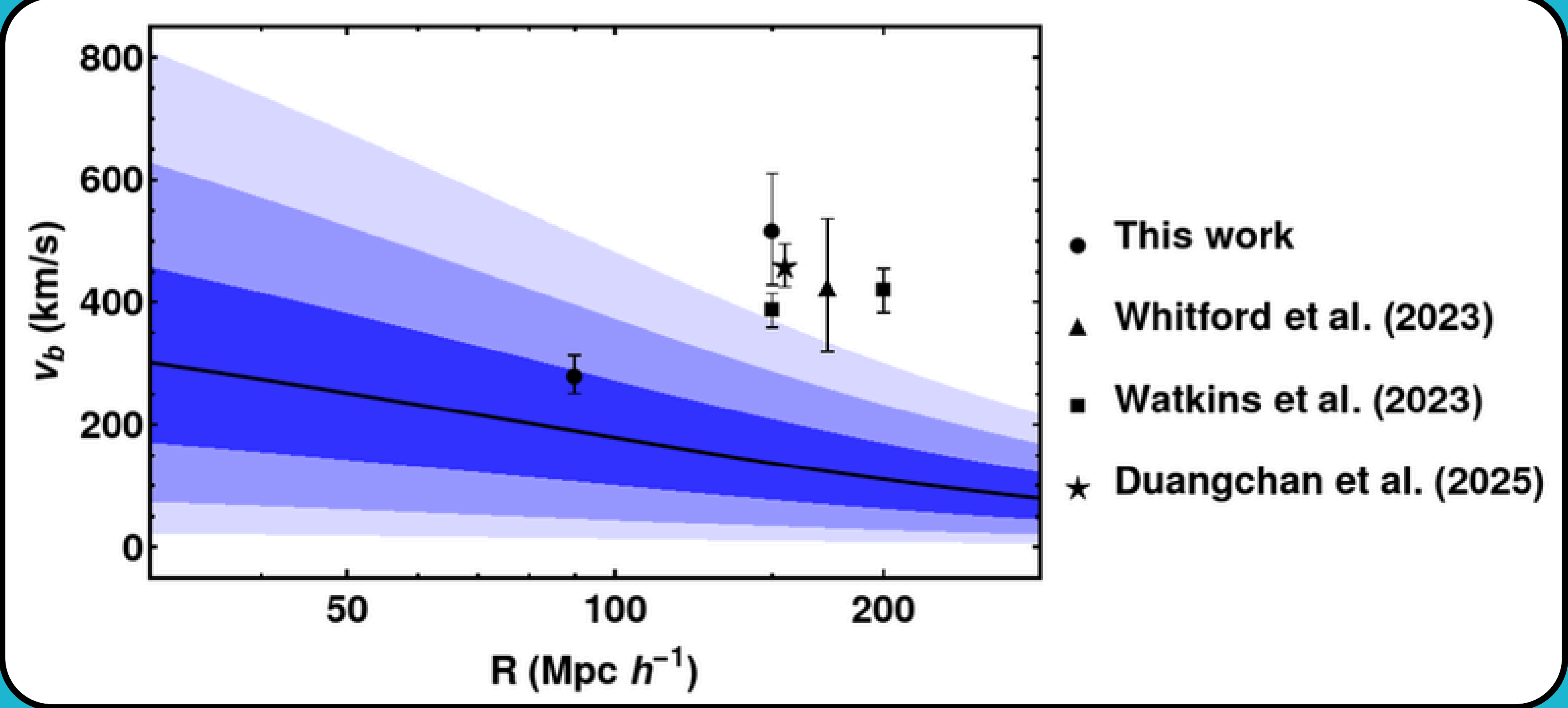
$$\tilde{\eta} = \log\left[\frac{z_c + v(1+z_c)}{d_L(1+2v)}\right] - \tilde{\mathcal{M}} \approx \log H_0 - \tilde{\mathcal{M}} + \frac{\boldsymbol{v}\cdot\boldsymbol{n}}{\tilde{z}\ln 10}\,.$$

$$\tilde{\eta}_{\ell m} = \frac{(\boldsymbol{v}\cdot\boldsymbol{n})_{\ell m}}{\tilde{z}\ln 10}$$

The average of the dipole of eta over a spherical volume of radius R is:

$$v_b = \frac{\ln 10}{V(R)}\int_0^{\tilde{z}(R)} \tilde{\eta}_1\ \tilde{z}\,\mathrm{d}V \qquad \text{with}\ \ z(R) \approx H_0 R$$

RECOVERING THE BULK FLOW DOES NOT REQUIRE ASSUMING ANY VALUE OF H_0



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ROBUSTNESS

★ 10.000 standard LCDM mock realizations to obtain the p-values;

★ Followed by mocks of LCDM augmented by a 400km/s bulk flow;

★ Division into subsamples: spiral galaxies, early-type galaxies, and SNIa;

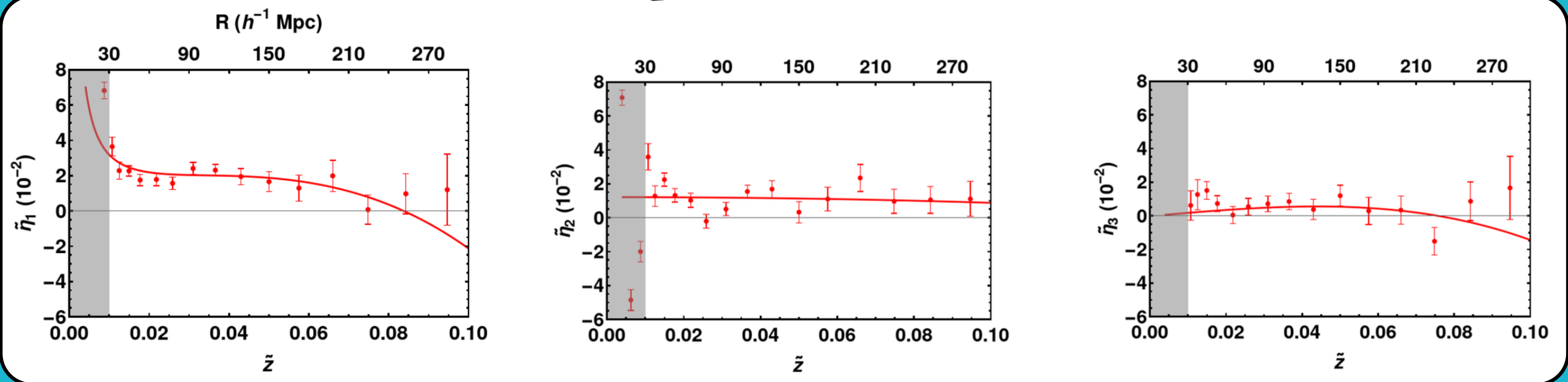
★ Investigated the effect of the ZoA and anisotropic source distribution.

Sample	*p*-value (%)					
	Uniform ΛCDM				ΛCDM + bulk (400 km/s)	
	Dipole	Quadrupole	Octupole	Joint	Quadrupole	Octupole
CF4a	0	0	0.06	0	0	6.89
CF4b	0	7.7	0.3	0	6.62	0.42
CF4c	0.02	0.02	-	0	0.02	-
CF4d	63.46	1.44	-	2.55	1.43	-
CF4TF [0.01,0.05]	0	0	-	0	0	-
CF4FP [0.01,0.05]	0	38.13	0.02	0	24.36	0.36
CF4FP [0.05,0.1]	2.2	0	-	0	0	-
CF4TF+CF4SN [0.05,0.1]	6.21	54.24	-	34.71	59.48	-

BEST-FIT COVARIANT COSMOGRAPHIC PARAMETERS

Sample	CF4	CF4TF	CF4FP	Pantheon+
$\mathbb{H}_2/\mathbb{H}_0$ (10^{-2})	2.8 ± 0.4	2.3 ± 0.7	2.6 ± 0.9	0.0 ± 1.3
$\mathbb{Q}_1$	2.0 ± 0.4	1.3 ± 0.7	1.5 ± 0.7	0.6 ± 1.0
$\mathbb{Q}_3$	0.9 ± 0.3	1.2 ± 0.7	0.5 ± 0.5	0.3 ± 0.9
$\mathbb{J}_2$	5 ± 10	3 ± 47	-8 ± 17	-6 ± 42
$\mathbb{S}_1 + 4\mathbb{Q}_1(2\mathbb{J}_0 - \mathbb{R}_0)$	3745 ± 1012	1959 ± 2984	1177 ± 1683	2373 ± 2893
$\mathbb{S}_3 + 4\mathbb{Q}_3(2\mathbb{J}_0 - \mathbb{R}_0)$	1901 ± 947	3604 ± 5062	-508 ± 1507	224 ± 2721
v_o (km/s)	188 ± 22	204 ± 35	328 ± 63	267 ± 80
χ^2_ν (ν)	0.891 (52793)	1.017 (10017)	0.836 (42016)	0.955 (619)

AXISYMMETRIC ETA EVOLUTION



COVARIANT COSMOGRAPHY AND ETA CONNECTION

$$d_L(z,\mathbf{n}) = \frac{z}{\mathbb{H}_o}\Big[1+\frac{1}{2}(1-\mathbb{Q}_o)z+\frac{1}{6}\big(3\mathbb{Q}_o^2+\mathbb{Q}_o-1+\mathbb{R}_o-\mathbb{J}_o\big)z^2$$
$$+\frac{1}{24}\big(2-\mathbb{Q}_o\big(2+15\mathbb{Q}_o+15\mathbb{Q}_o^2\big)+5\mathbb{J}_o(1+2\mathbb{Q}_o)-2\mathbb{R}_o(1+3\mathbb{Q}_o)+\mathbb{S}_o\big)z^3\Big]+\mathcal{O}(z^5)\,,$$

$$\eta(z,n) = \log\left[\frac{z}{d_L(z,n)}\right]-\mathcal{M}(z)$$

$$\eta(z,\boldsymbol{n}) = -\mathcal{M}(z)+\log\mathbb{H}_o(\boldsymbol{n})-\frac{1-\mathbb{Q}_o(\boldsymbol{n})}{2\ln 10}\,z+\frac{7-\mathbb{Q}_o(\boldsymbol{n})\big[10+9\mathbb{Q}_o(\boldsymbol{n})\big]+4\big[\mathbb{J}_o(\boldsymbol{n})-\mathbb{R}_o(\boldsymbol{n})\big]}{24\ln 10}\,z^2$$
$$+\frac{1}{24\ln 10}\Big\{-5\mathbb{J}_o(\boldsymbol{n})\big[2\mathbb{Q}_o(\boldsymbol{n})+1\big]+2\big[\mathbb{J}_o(\boldsymbol{n})-\mathbb{R}_o(\boldsymbol{n})\big]\big[\mathbb{Q}_o(\boldsymbol{n})-1\big]$$
$$+\mathbb{Q}_o(\boldsymbol{n})\Big[9+2\mathbb{Q}_o(\boldsymbol{n})\big[5\mathbb{Q}_o(\boldsymbol{n})+8\big]+6\mathbb{R}_o(\boldsymbol{n})\Big]+2\mathbb{R}_o(\boldsymbol{n})-5-\mathbb{S}_o(\boldsymbol{n})\Big\}z^3+\mathcal{O}(z^4)$$

Kalbouneh, Marinoni and Maartens, JCAP09(2024)069

$$\eta_1(z) \approx \frac{\mathbb{Q}_1}{2\ln 10} z - \frac{(9\mathbb{Q}_0+5)\mathbb{Q}_1 z^2}{12\ln 10} - \frac{1}{27720\ln 10}\Big\{-33\mathbb{Q}_1\Big[280\mathbb{J}_0+112\mathbb{J}_2 - 5\big(14\mathbb{Q}_0(15\mathbb{Q}_0+16)+46\mathbb{Q}_3^2+28\mathbb{R}_0+63\big)\Big] + 8\mathbb{Q}_3\big(-297\mathbb{J}_2-220\mathbb{J}_4+135\mathbb{Q}_3^2\big) + 6930\mathbb{Q}_1^3+5940\mathbb{Q}_1^2\mathbb{Q}_3-1155\mathbb{S}_1\Big\}z^3,$$

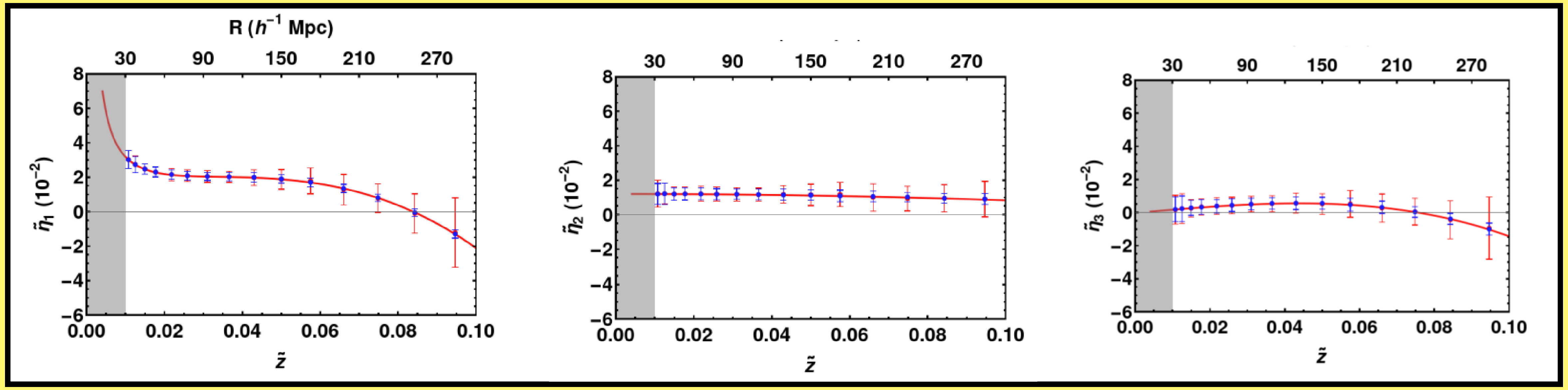
$$\eta_2(z) \approx \frac{\mathbb{H}_2}{\mathbb{H}_0\ln 10} + \frac{14\mathbb{J}_2-3(\mathbb{Q}_1+\mathbb{Q}_3)(7\mathbb{Q}_1+2\mathbb{Q}_3)}{84\ln 10}z^2 + \frac{1}{504\ln 10}\Big[-21\mathbb{J}_2(8\mathbb{Q}_0+7) + 60\mathbb{Q}_0(\mathbb{Q}_1+\mathbb{Q}_3)(7\mathbb{Q}_1+2\mathbb{Q}_3)+224\mathbb{Q}_1^2+4\mathbb{Q}_3(81\mathbb{Q}_1+20\mathbb{Q}_3)\Big]z^3,$$

Expansion rate fluctuation field and the dominant multipoles of the covariant cosmographic parameters in the axial symmetric configuration

$$\eta_3(z) \approx \frac{\mathbb{Q}_3}{2\ln 10} z - \frac{(9\mathbb{Q}_0+7)\mathbb{Q}_3 z^2}{12\ln 10} + \frac{1}{154440\ln 10}\Big[-6435\big(\mathbb{Q}_3(8\mathbb{J}_0-6\mathbb{Q}_0(5\mathbb{Q}_0+6)-13)+\mathbb{S}_3\big) - 3432\mathbb{J}_2(9\mathbb{Q}_1+4\mathbb{Q}_3)-1040\mathbb{J}_4(22\mathbb{Q}_1+9\mathbb{Q}_3) + 30\big(858\mathbb{Q}_1^3+3289\mathbb{Q}_1^2\mathbb{Q}_3+1404\mathbb{Q}_1\mathbb{Q}_3^2+723\mathbb{Q}_3^3+858\mathbb{Q}_3\mathbb{R}_0\big)\Big]z^3,$$

$$\eta_4(z) \approx \frac{154\mathbb{J}_4-9\mathbb{Q}_3(44\mathbb{Q}_1+9\mathbb{Q}_3)}{924\ln 10}z^2 + \frac{\big[12\mathbb{Q}_3(44(5\mathbb{Q}_0+3)\mathbb{Q}_1+15(3\mathbb{Q}_0+2)\mathbb{Q}_3)-77\mathbb{J}_4(8\mathbb{Q}_0+7)\big]}{1848\ln 10}z^3,$$

$$\eta_5(z) \approx \frac{\big\{60\mathbb{Q}_3\big[5\big(26\mathbb{Q}_1^2+27\mathbb{Q}_1\mathbb{Q}_3+6\mathbb{Q}_3^2\big)-52\mathbb{J}_2\big]-40\mathbb{J}_4(91\mathbb{Q}_1+36\mathbb{Q}_3)-819\mathbb{S}_5\big\}}{19656\ln 10}z^3,$$


R (h^{-1} Mpc)
30
90
150
210
270
$\tilde{\eta}_1$ (10^{-2})
$\tilde{\eta}_2$ (10^{-2})
$\tilde{\eta}_3$ (10^{-2})
0.00
0.02
0.04
0.06
0.08
0.10
$\tilde{z}$