Halo Abundance in Decaying Dark Matter Models

Thomas Montandon

Decaying Dark Matter Halo Abundance from a Revised Spherical Collapse Model

TM, Vivian Poulin, Oliver Hahn, Jozef Bucko and Aurel Schneider

arXiv: 2607.19244

UNIVERSITÉ DE MONTPELLIER

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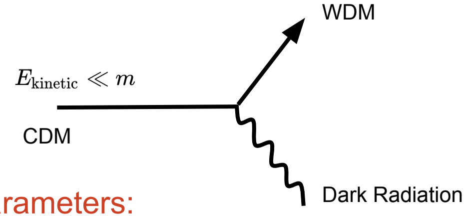
Decaying Dark Matter

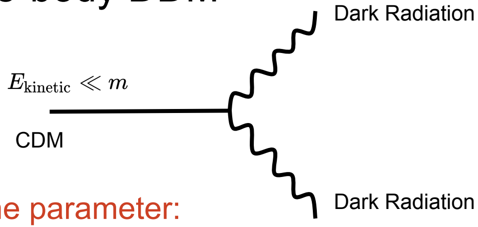
Two-body DDM



Two parameters:

- Life time Γ^{-1}
- Velocity kick $\varepsilon = \frac{1}{2}\left(1 - \frac{m_{\text{wdm}}}{m_{\text{cdm}}}\right) \simeq \frac{v_k}{c}$

One-body DDM



One parameter:

- Life time

Γ^{-1}

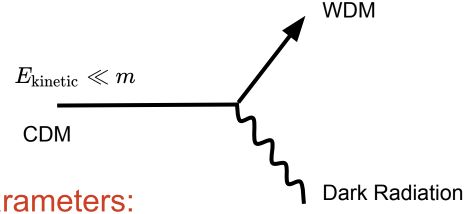
(I ignore here the fraction of unstable particles)

In particle physics:

- T. Hambye: *On the stability of particle dark matter* (1012.4587)
- M. Drewes et al. : *A White Paper on keV Sterile Neutrino Dark Matter* (1602.04816)
- V. Berezinsky et al. : *Cosmological signatures of supersymmetry with spontaneously broken R parity*
- L. Covi: *Axinos as Cold Dark Matter* (9905212)
- H.-B. Kim and J. E. Kim: *Late decaying axino as CDM and its lifetime bound* (0108101)
- C.-H. Chou and K.-W. Ng: *Decaying Superheavy Dark Matter and Subgalactic Structure of the Universe* (0306437)
- And many others…

Decaying Dark Matter

Two-body DDM



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- Life time Γ^{-1}
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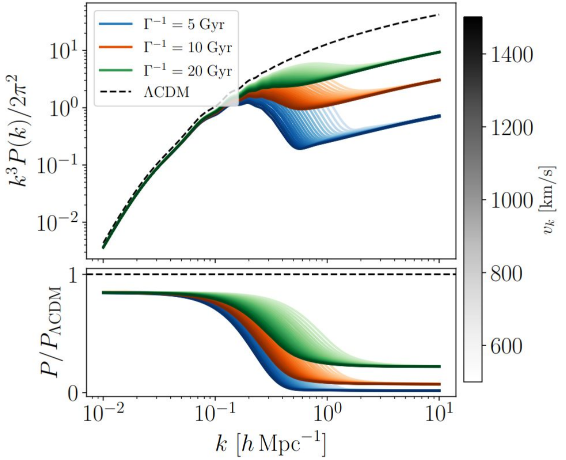
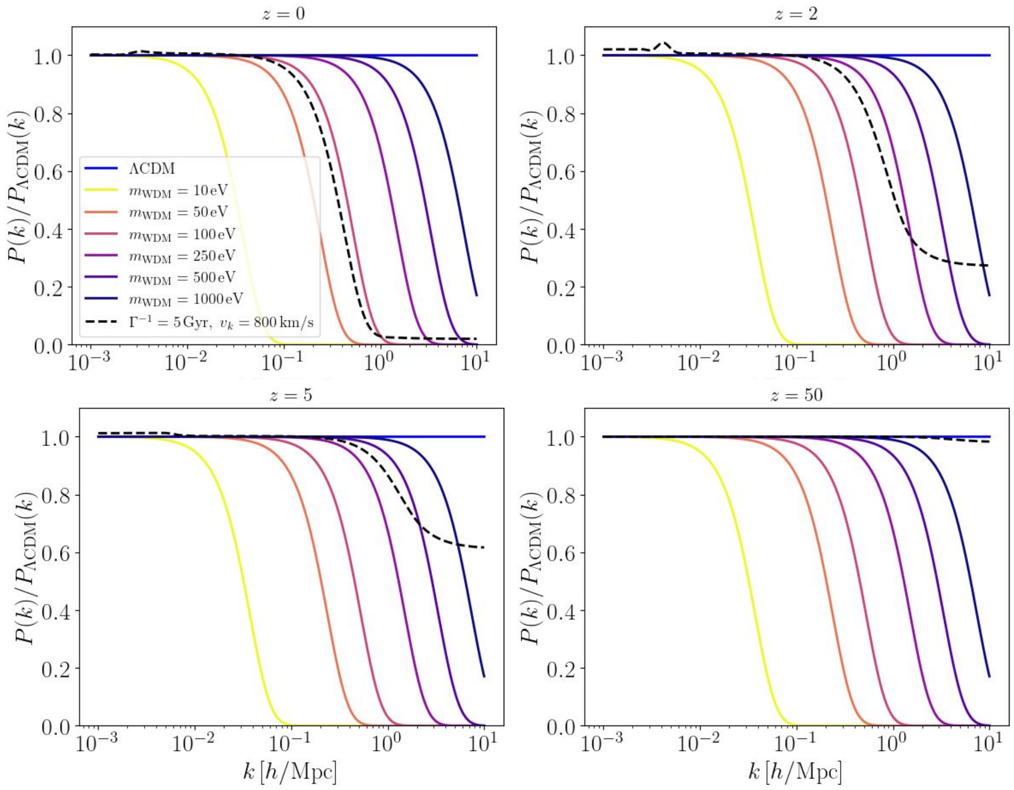
Einstein-Boltzmann solver:
class_decays



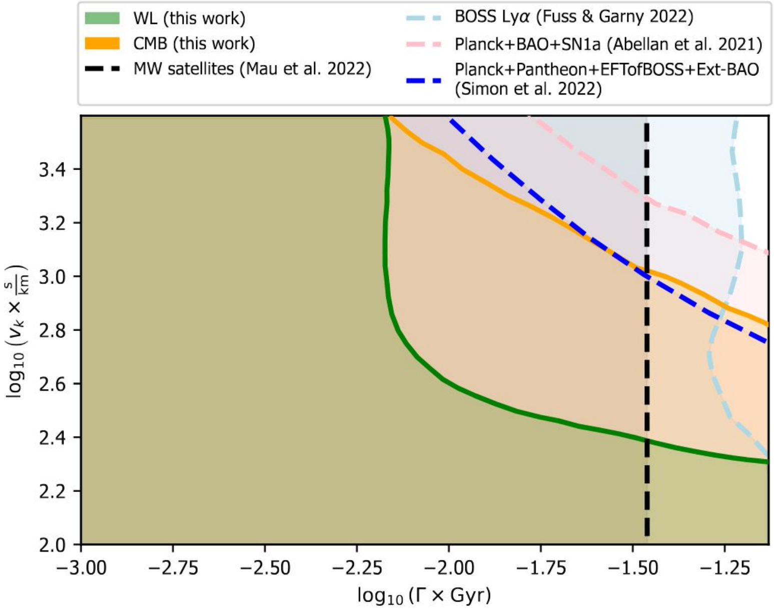


Phenomenology of DDM in Cosmology

- Mei-Yu Wang and Andrew R. Zentner (1201.2426)
- Guillermo F. Abellan, Riccardo Murgia and Vivian Poulin (2102.12498)
- G. F. Abellan, R. Murgia, V. Poulin, and J. Lavalle (2008.09615)

Decaying Dark Matter: Constraints

J.Bucko, A.Schneider et al. (2307.03222)



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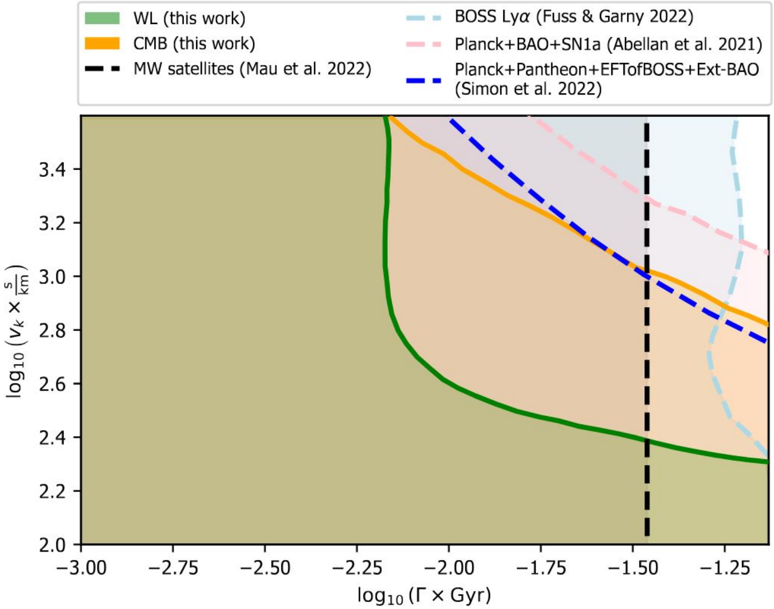
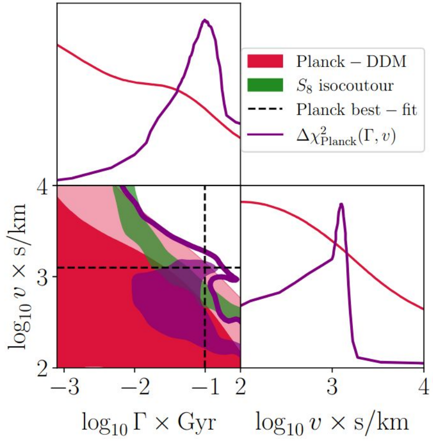
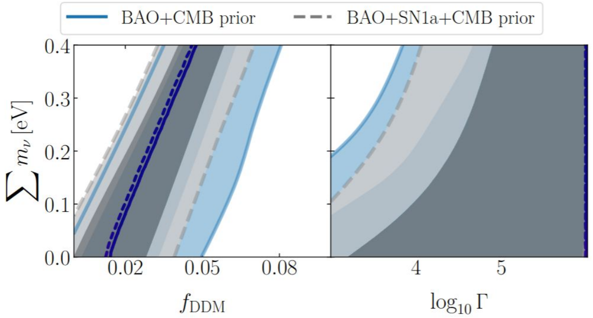


TM, E. M. Teixeira, A. Poudou and V. Poulin (2505.20193)



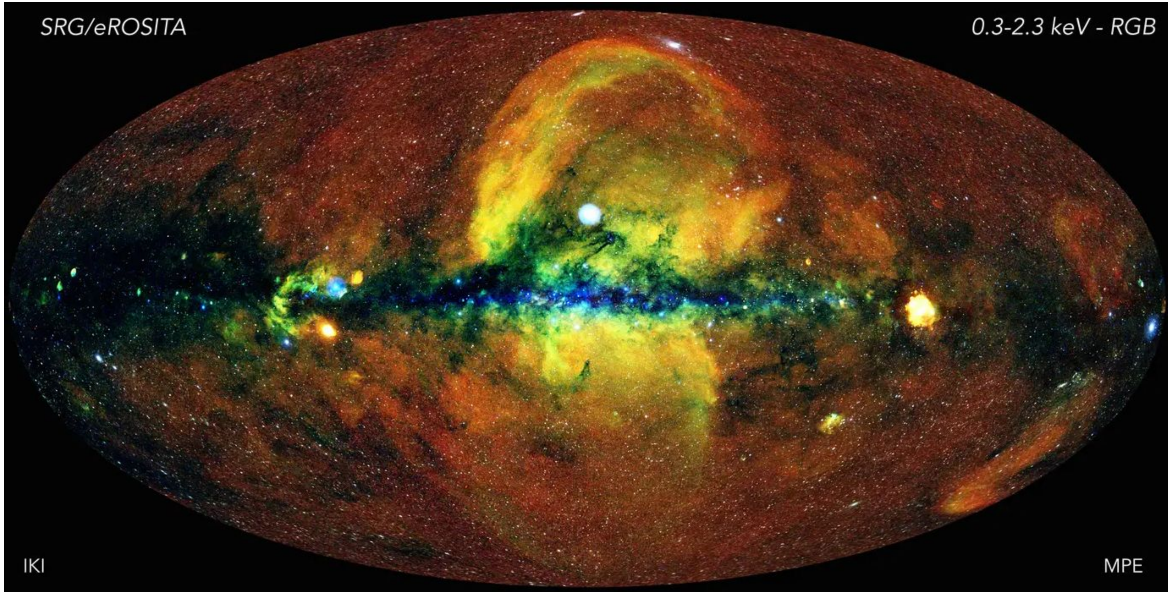
TM, V. Poulin, T. Rink, T. Schwetz (2603.03284)



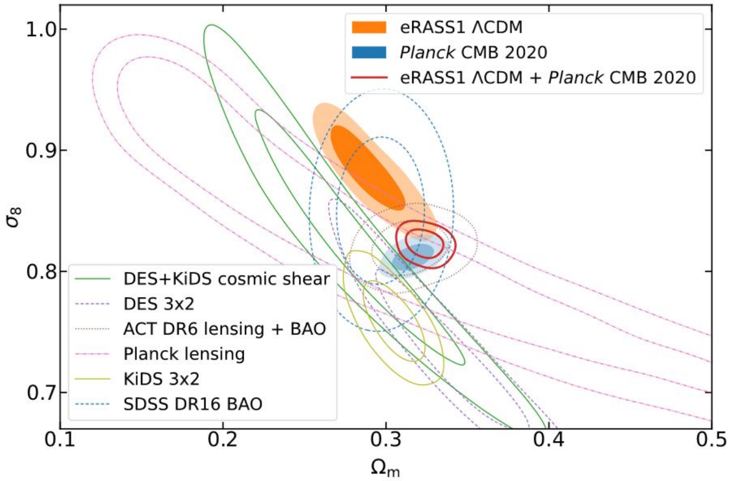
Decaying Dark Matter: Constraints

Halo Mass Function Measurements:

- Planck-detected galaxy clusters (I. Zubeldia and A. Challinor 2005.14607)
- SPT collaboration + DES and HST data (S. Bocquet et al 2401.02075)
- Planck + SPT cluster catalogs (L. Salvati et al 2112.03606)
- Planck and Chandra data (G. Aymerich 2402.04006)
- DES Y1 and SPT (M. Costanzi et al 2010.13800)
- HSC weak lensing data (T. Sunayama 2309.13025)



eROSITA All-Sky-Survey
(V.Ghirardini 2402.08458)

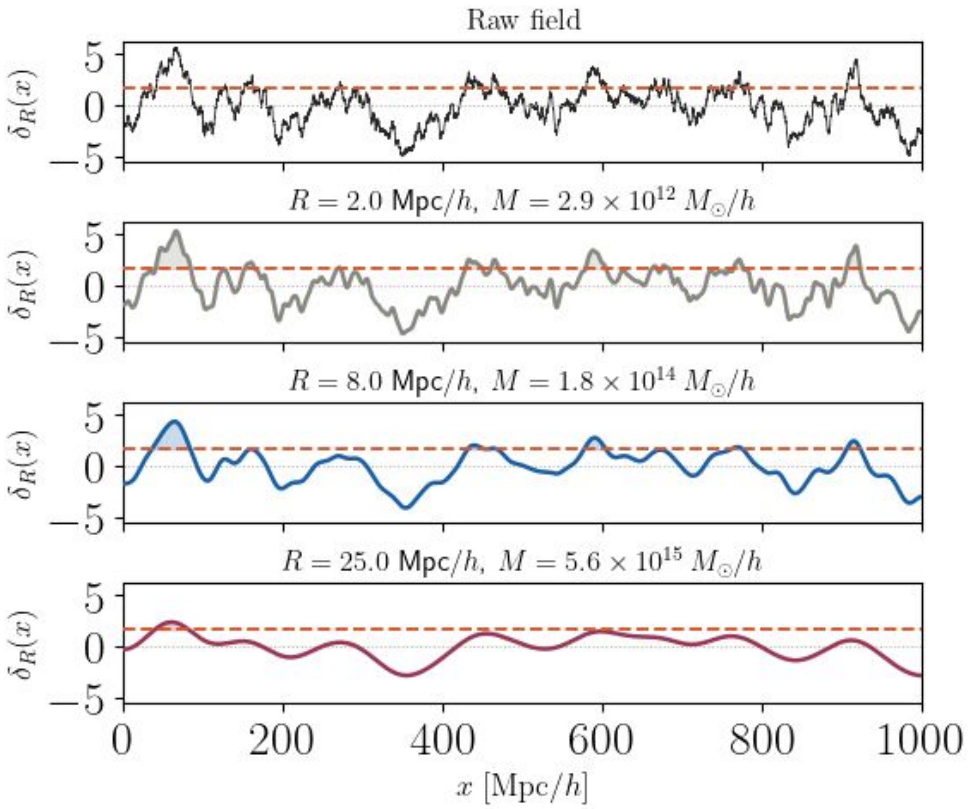


We need an HMF prediction for Decaying Dark Matter models

Halo Mass Function:
Press and Schechter

$$\frac{dn_h}{d\ln M} = \frac{\bar{\rho}}{M} f(\nu_c) \left| \frac{d\ln\sigma}{d\ln M} \right|$$

$$\sigma^2(R) = \int_0^\infty \frac{k^2\,dk}{2\pi^2}\, W^2(kR)\, P_m(k)$$



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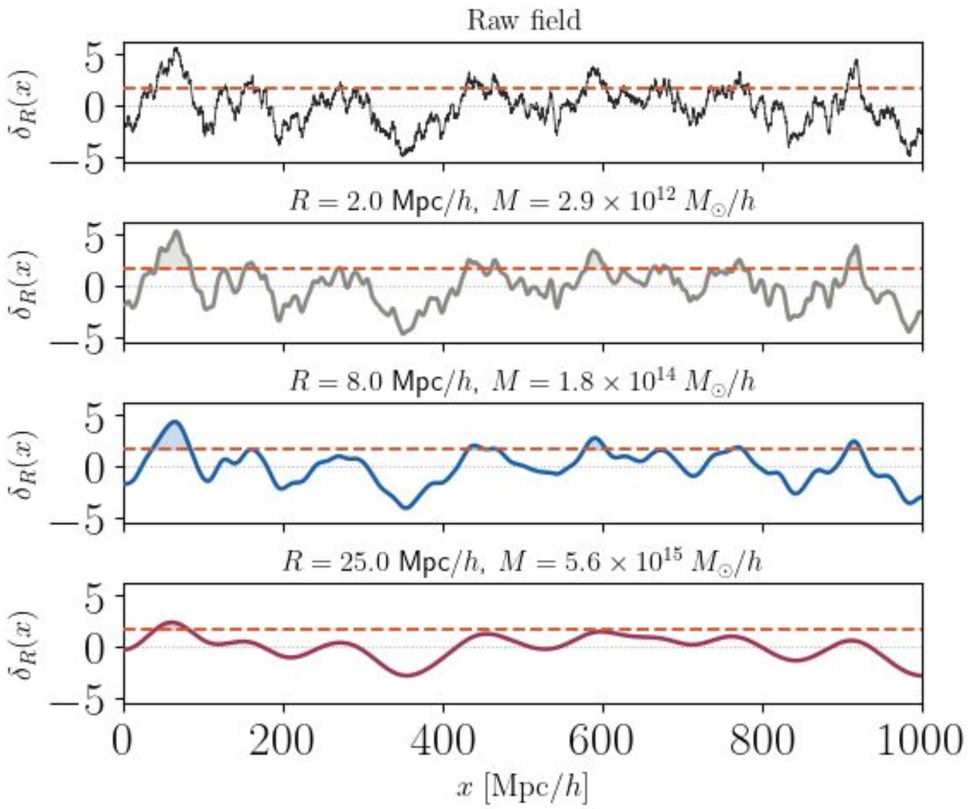

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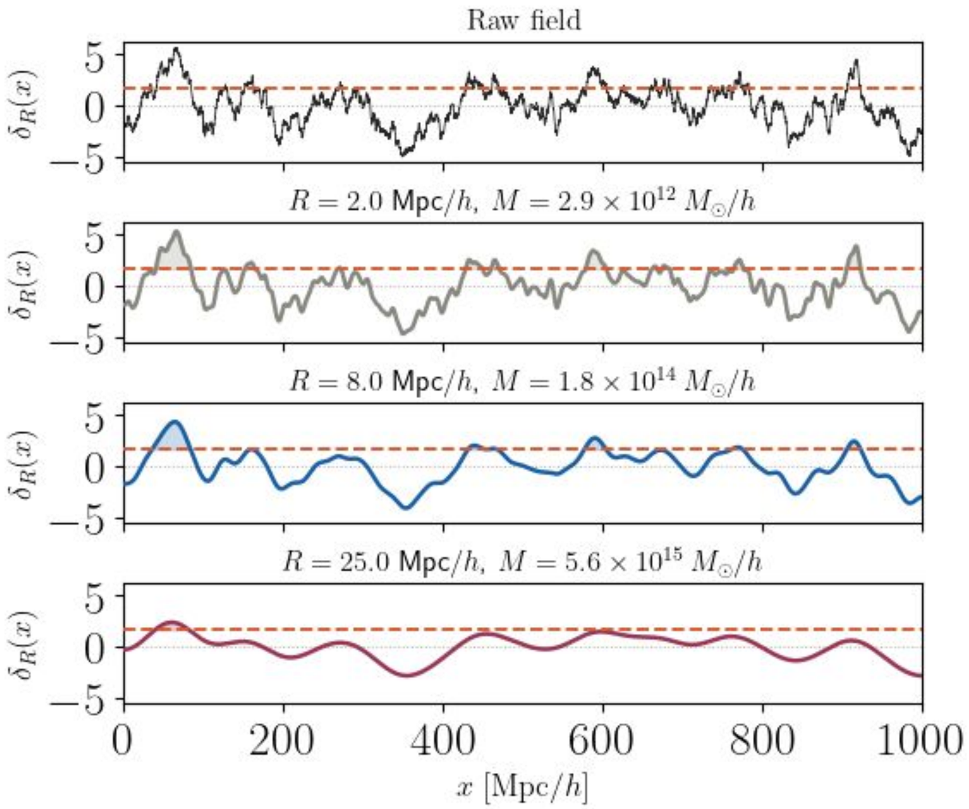
$$M = \frac{4\pi}{3} R^3 \rho_m$$

$$\nu_c = \boxed{\frac{\delta_c}{\sigma} \simeq \frac{1.686}{\sigma}}$$

Spherical collapse model



Spherical Collapse model: Einstein-De Sitter

$$\boxed{\delta_c \simeq 1.686}$$

Initial conditions

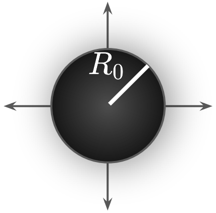
δ_0

M_0

t_0

time

$$\dot{R}(t_0) = H(t_0)R(t_0)\left(1 - \frac{\delta_0}{3} - \frac{2\delta_0^2}{21}\right)$$



Spherical Collapse model: Einstein-De Sitter

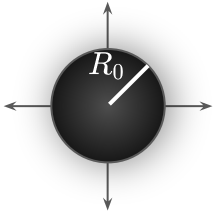
$$\boxed{\delta_c \simeq 1.686}$$

Initial conditions: δ_0, M_0, t_0

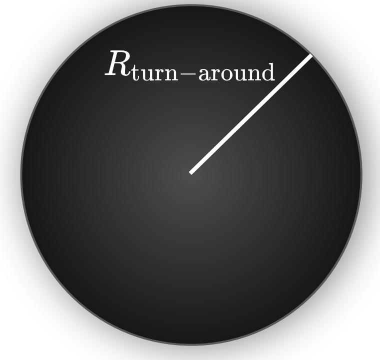
$t_{\mathrm{turn-around}}$

time

$$\dot{R}(t_0) = H(t_0)R(t_0)\left(1 - \frac{\delta_0}{3} - \frac{2\delta_0^2}{21}\right)$$



$$\dot{R}(t_{\mathrm{ta}}) = 0$$



Spherical Collapse model: Einstein-De Sitter

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δ_0

M_0

t_0

$t_{\mathrm{turn-around}}$

t_{collapse}

time

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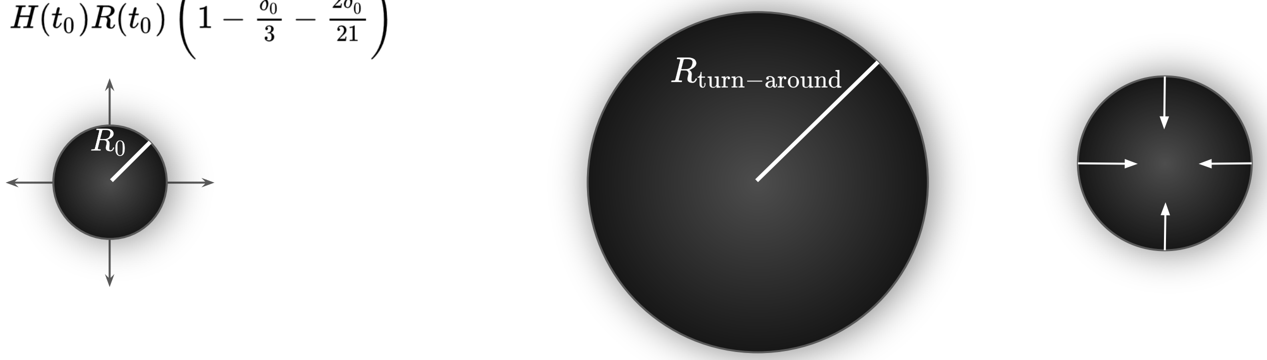
$$\dot{R}(t_{\mathrm{ta}}) = 0$$

$$R(t_{\mathrm{collapse}}) = 0$$



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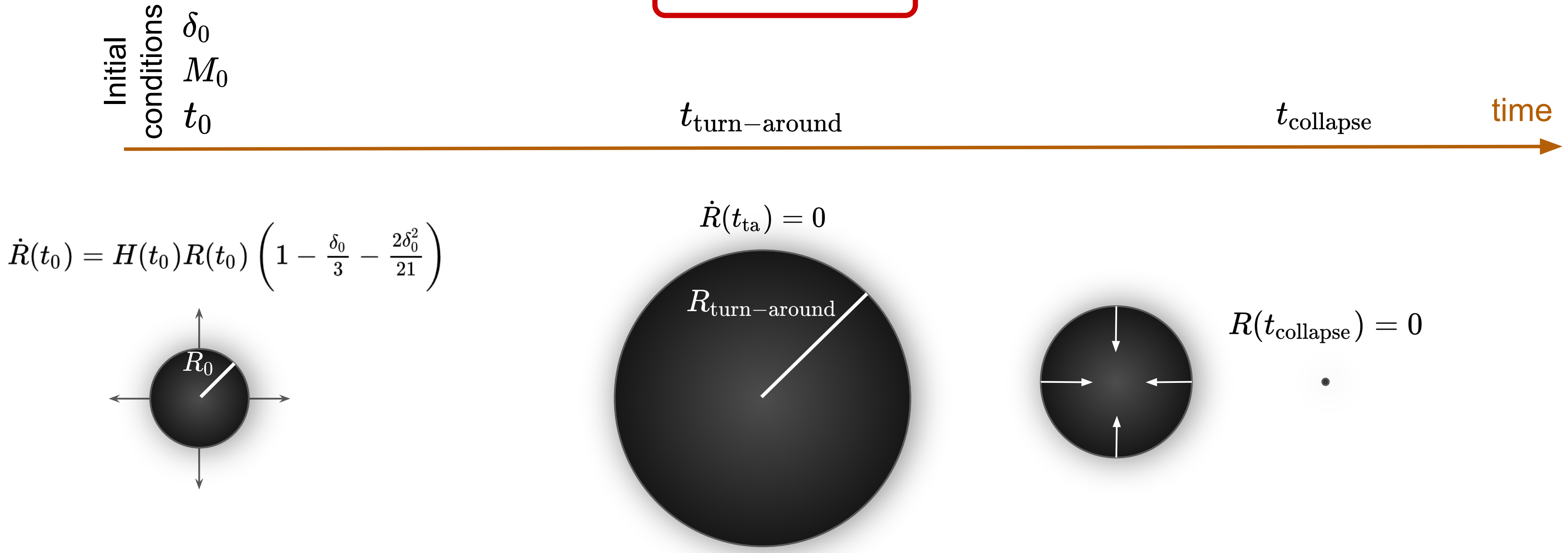


EOM:

$$\ddot{R} = -\frac{GM(<R)}{R^2}$$

Solution:

$$R(\theta) = \frac{R_{\mathrm{ta}}}{2}(1 - \cos\theta)\,,$$

$$t(\theta) = \frac{t_{\mathrm{ta}}}{\pi}(\theta - \sin\theta)$$

Linear Extrapolation:

$$\delta^{(1)}(t) = \frac{3}{20}\left(\frac{6\pi t}{t_{\mathrm{ta}}}\right)^{2/3}$$

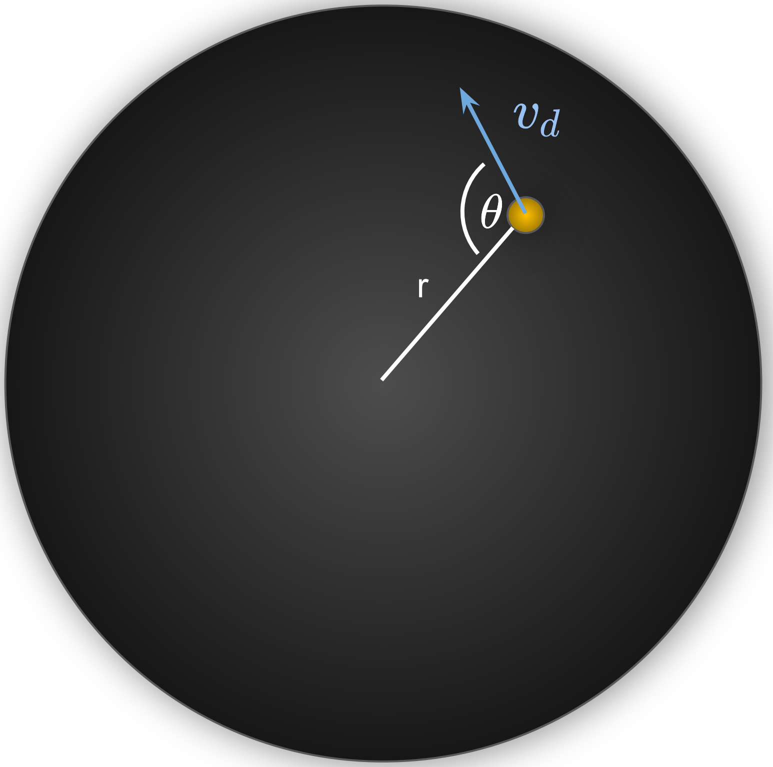
$$\delta_c = \delta^{(1)}(t_{\mathrm{collapse}}) = \frac{3}{5}\left(\frac{3\pi}{2}\right)^{2/3} \simeq 1.686$$

Spherical Collapse model: Decaying Dark Matter

$$\boxed{\delta_c = ?}$$

$$\ddot{R} = -\frac{GM_{\text{grav}}}{R^2}$$

δ_0	Initial conditions
$M_p = M_0$	
$M_d = 0$	t_0



Model inspired by E. O. Nadler and A. J. Benson (2501.12636)

Spherical Collapse model: Decaying Dark Matter

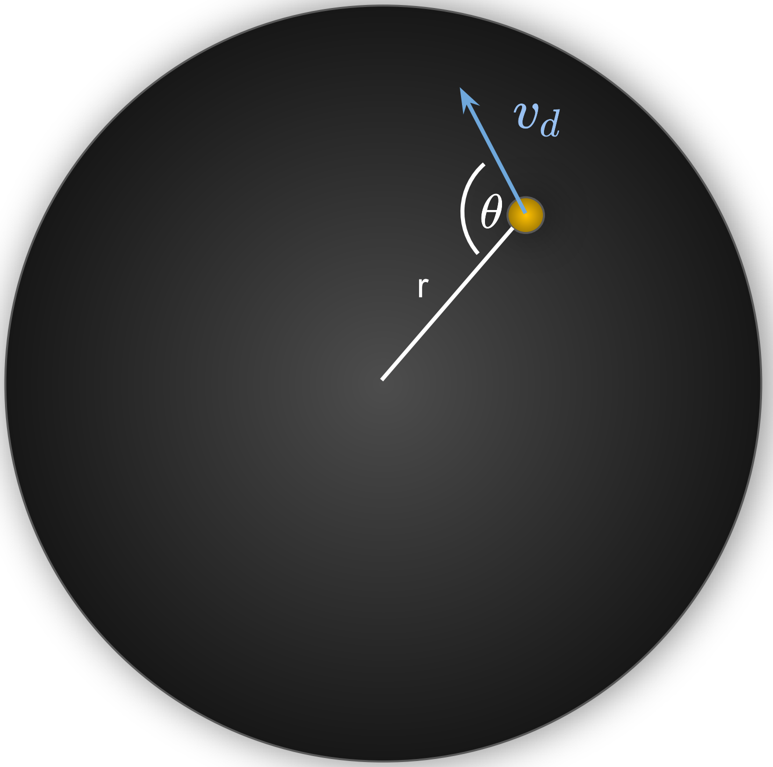
$$\boxed{\delta_c = ?}$$

$$\ddot{R} = -\frac{GM_{\rm grav}}{R^2}$$

$$\dot{M}_p = -\Gamma M_p(t)$$

$$\dot{M}_{\rm d} = f_{\rm bound}(t)\sqrt{1-2\epsilon}\,\Gamma M_p(t)$$

δ_0 $M_p = M_0$ $M_d = 0$	Initial conditions t_0



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$$\dot{M}_p = -\Gamma M_p(t)$$

$$\dot{M}_{\rm d} = f_{\rm bound}(t)\sqrt{1-2\epsilon}\,\Gamma M_p(t)$$

$$\cos\theta < \frac{-2\Phi(r) - v_p^2 - v_k^2}{2 v_p v_k} = C(r)$$

$$P(r) = \begin{cases} 1 & C_(r) \geq 1 \\ \frac{1+C_(r)}{2} & -1 \leq C_(r) < 1 \\ 0 & C_(r) < -1 \end{cases}$$

$$f_{\rm bound} = \frac{3}{R^3}\int_0^R dr\, r^2\, P(r)$$

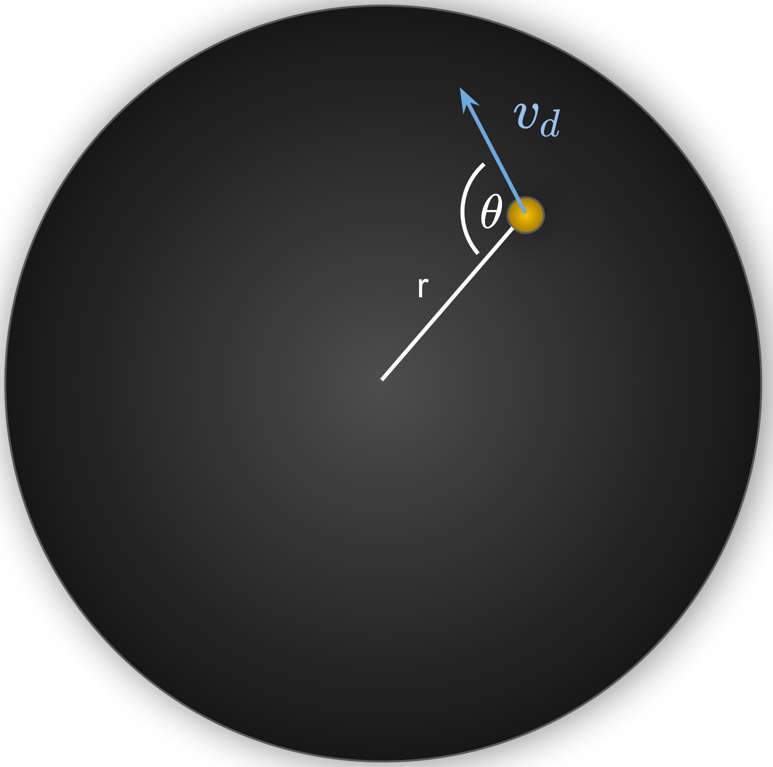
δ_0

$M_p = M_0$

$M_d = 0$

Initial conditions

t_0



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$$f_{\rm bound} = \frac{3}{R^3}\int_0^R dr\, r^2\, P(r)$$

$$M_{\rm grav} = M_p + \left[\frac{f_{\rm in}}{f_{\rm bound}} + \frac{M_d}{M_d + M_p}\left(1 - \frac{f_{\rm in}}{f_{\rm bound}}\right)\right] M_d$$

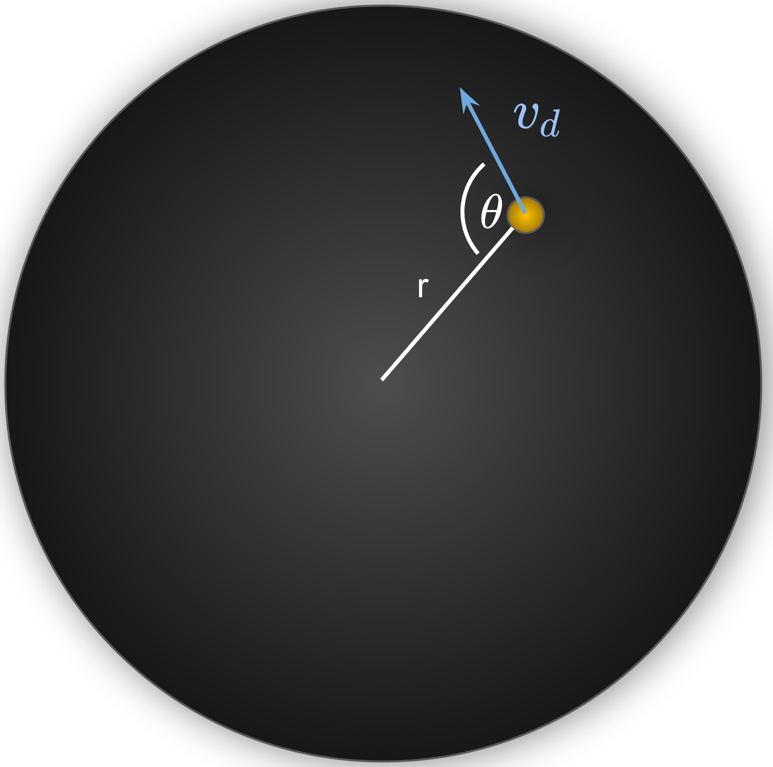
Initial conditions

δ_0

$M_p = M_0$

$M_d = 0$

t_0



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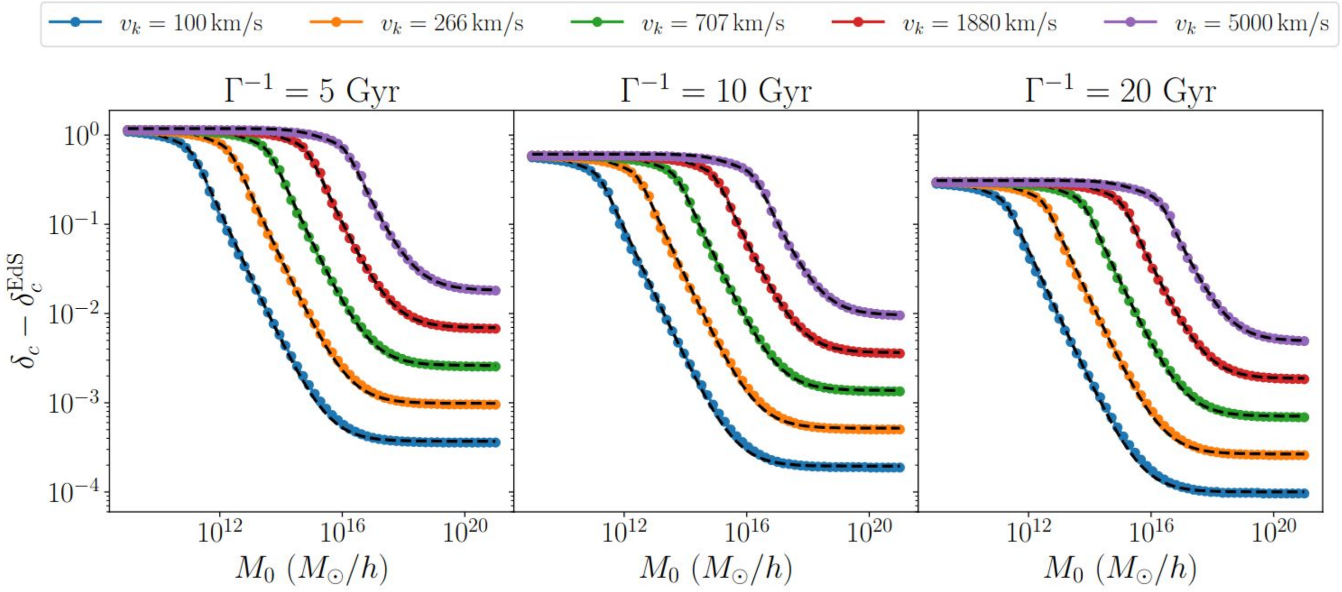
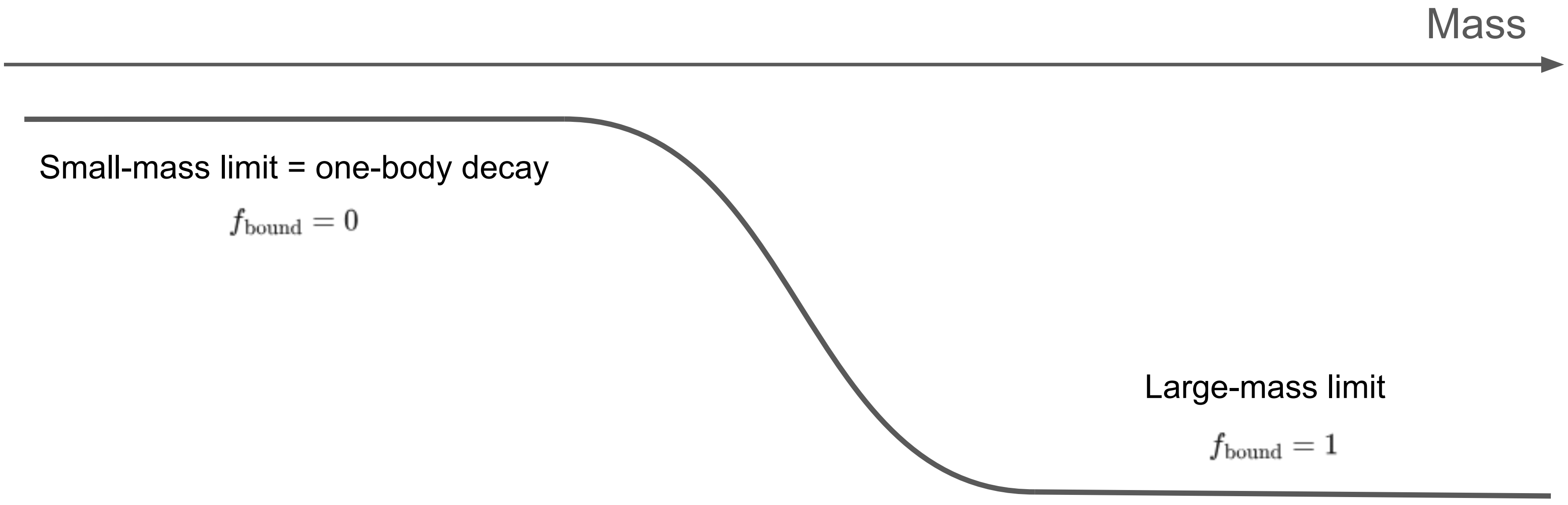
Spherical Collapse model: Decaying Dark Matter



EdS: $\delta_c \simeq 1.686$ $\longrightarrow$ DDM: $\delta_c(M_0, \Gamma, \epsilon)$

Spherical Collapse model: Decaying Dark Matter



Spherical Collapse model: Decaying Dark Matter

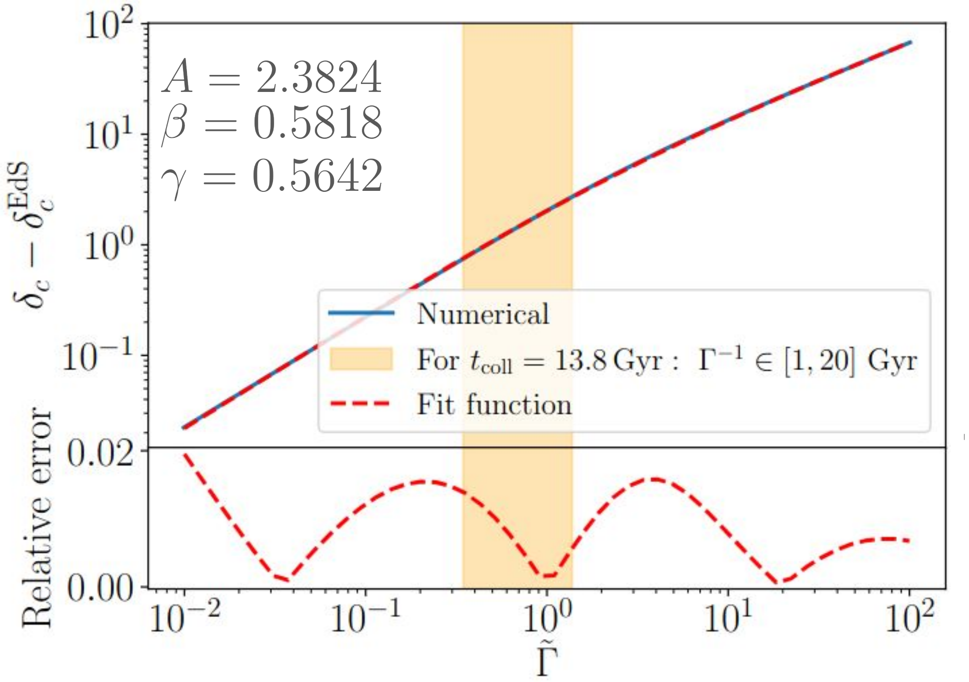
Mass

Small-mass limit = one-body decay

$$f_{\text{bound}} = 0$$

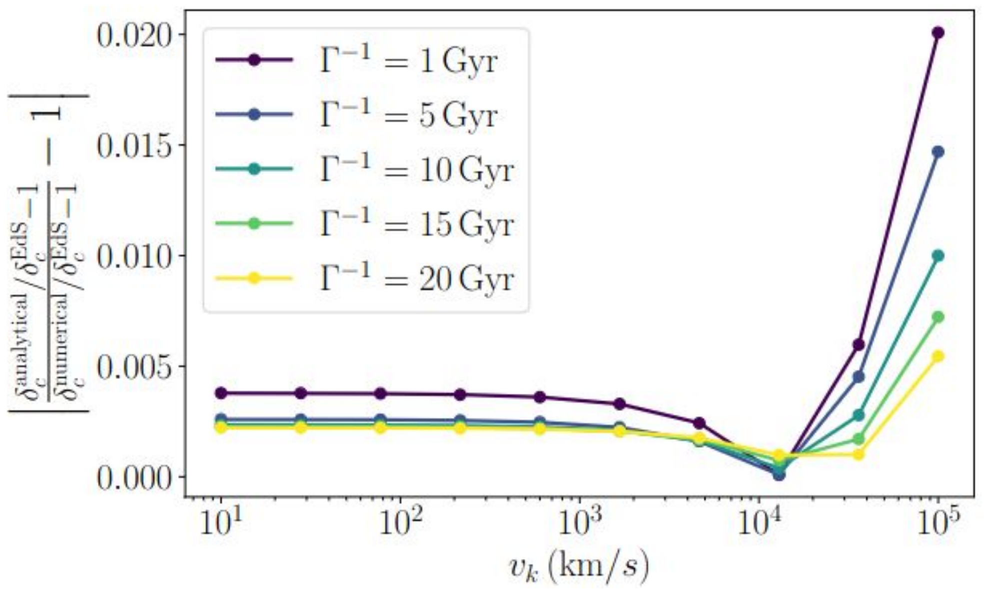
$$\tilde{R}'' = -\frac{\pi^2}{8\tilde{R}^2} e^{-\tilde{\Gamma}\tilde{t}}$$

$$\delta_c^{\text{small}}(\tilde{\Gamma}) = \delta_{\text{c}}^{\text{EdS}} + A\tilde{\Gamma}^{\beta} \ln(1+\tilde{\Gamma})^{1-\gamma}$$



Spherical Collapse model:
Decaying Dark Matter

Mass



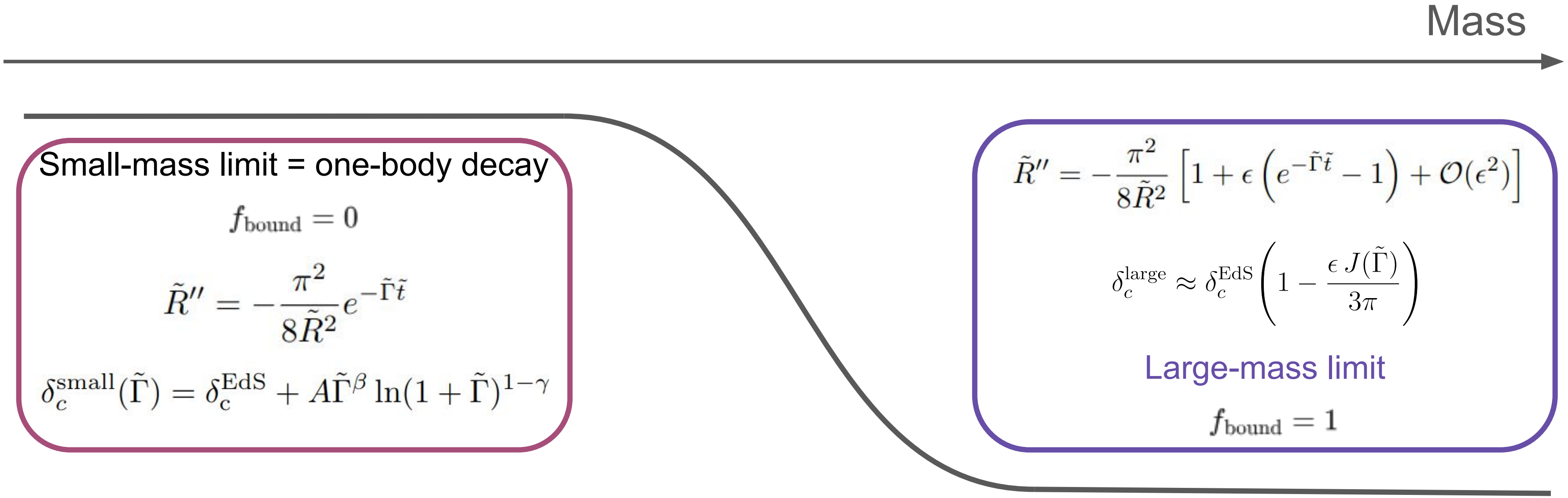
$$\tilde{R}'' = -\frac{\pi^2}{8\tilde{R}^2}\left[1 + \epsilon\left(e^{-\tilde{\Gamma}\tilde{t}} - 1\right) + \mathcal{O}(\epsilon^2)\right]$$

$$\delta_c^{\rm large} \approx \delta_c^{\rm EdS}\left(1 - \frac{\epsilon\, J(\tilde{\Gamma})}{3\pi}\right)$$

Large-mass limit

$$f_{\rm bound} = 1$$

Spherical Collapse model: Decaying Dark Matter



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$$\delta_c^{\text{large}} \approx \delta_c^{\text{EdS}}\left(1 - \frac{\epsilon\, J(\tilde{\Gamma})}{3\pi}\right)$$

Large-mass limit

$$f_{\text{bound}} = 1$$

Intermediate mass

$$\delta_c^{\text{fit}}(M) = \delta_c^{\text{large}} + \frac{\delta_c^{\text{small}} - \delta_c^{\text{large}}}{\left[\left(1+\frac{M}{M_1}\right)\left(1+\left(\frac{M}{M_2}\right)^4\right)\right]^{\nu}}$$

$$\nu \approx 0.15$$

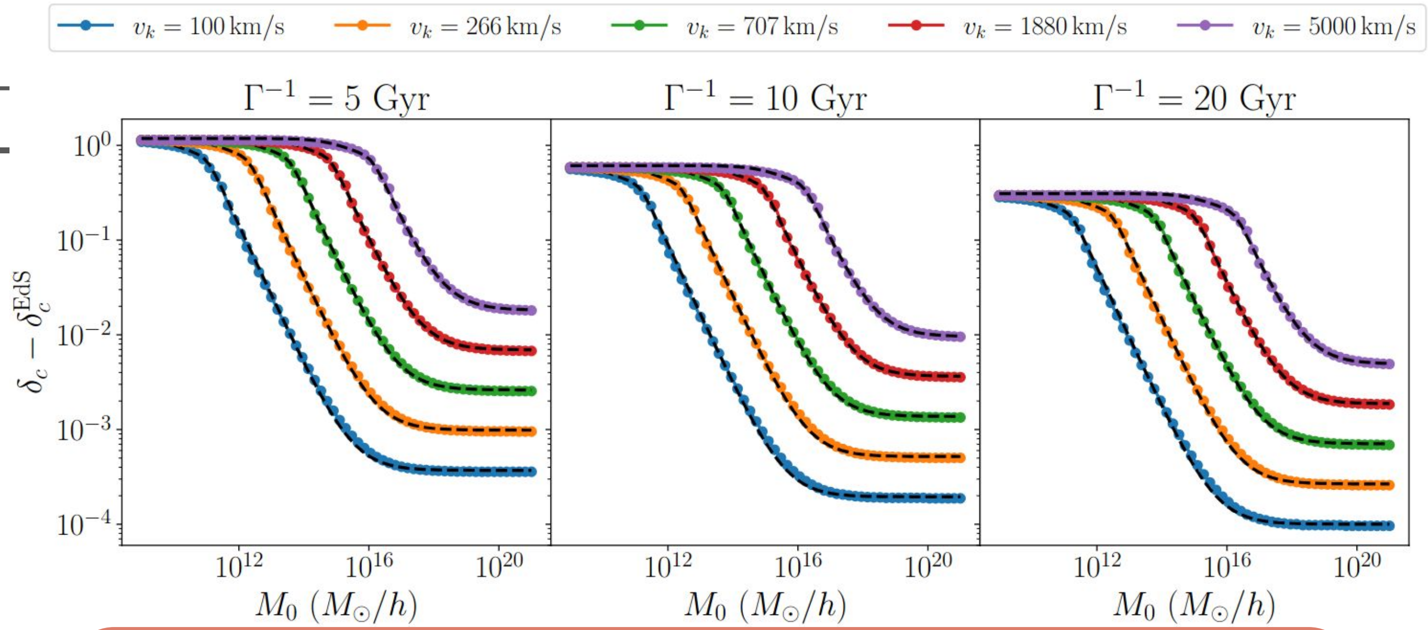
$$\frac{M_2}{M_1} \approx 24$$

$$v_k = \omega_{\text{ta}} R_{\text{ta}}$$

$$\downarrow$$

$$M_1 = B v_k^3 \tilde{\Gamma}^{-0.5} t_{\text{ta}}$$

Spherical Collapse model: Decaying Dark Matter



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$$M_1 = B v_k^3 \tilde{\Gamma}^{-0.5} t_{\mathrm{ta}}$$

Results: two-body DDM

$$\frac{dn}{d\ln M_{\rm coll}} = \frac{\rho_c \Omega_m}{M_0} f\left(\nu_c\right) \left|\frac{d\ln\sigma}{d\ln M_0}\right| \frac{d\ln M_0}{d\ln M_{\rm coll}}$$

$$\sigma^2(R) = \int_0^\infty \frac{k^2\,dk}{2\pi^2}\, W_{\rm TH}^2(kR)\, P_m^{\Lambda\rm CDM}(k)$$

$$M = \frac{4\pi}{3} R^3 \rho_m$$

$$\nu_c = \frac{\delta_c(M_0)}{\sigma}$$

$\mathrm{top-hat\ filter}$

$\Lambda\mathrm{CDM\ power\ spectrum}$

All DDM physics contained in the critical density!

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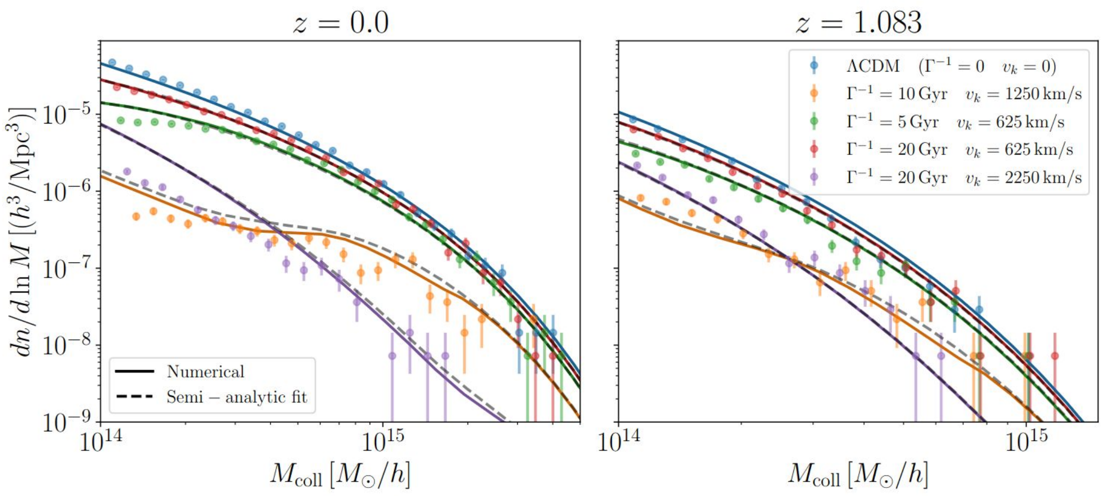

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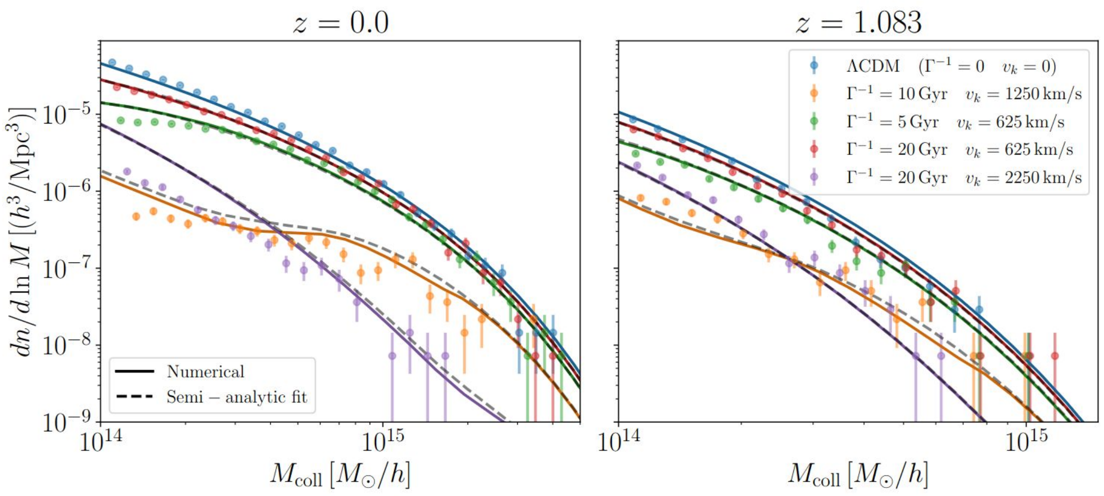
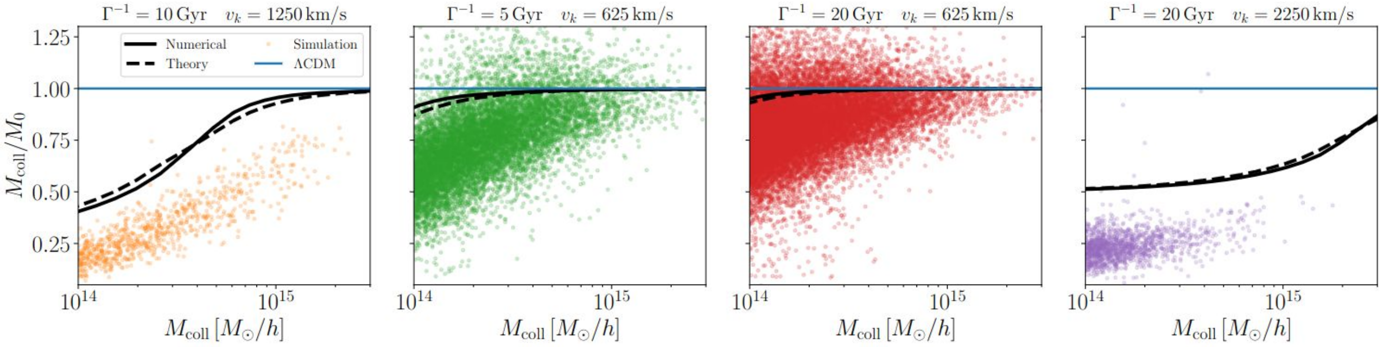


$$M_{\rm coll} \overset{?}{=} M_{200m}$$



Conclusion: A Revised Spherical Collapse Model

Two-body DDM

$$\frac{dn}{d\ln M_{\rm coll}} = \frac{\rho_c \Omega_m}{M_0} f(\nu_c) \left|\frac{d\ln\sigma}{d\ln M_0}\right| \frac{d\ln M_0}{d\ln M_{\rm coll}}$$

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$$M = \frac{4\pi}{3} R^3 \rho_m$$

$$\nu_c = \frac{\delta_c(M_0)}{\sigma}$$

- LCDM Linear Power Spectrum
- Top-Hat Window Function
- Modified Spherical Collapse

$\xrightarrow{\text{Small mass limit}}$ $\mathrm{One-body\ DDM}$

$\xrightarrow{\text{Large mass limit}}$ $\mathrm{Analytical\ Solution}$

$\longrightarrow$ Semi-analytical fitting formula

Funded by the European Union. Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or ERCEA. Neither the European Union nor the ERCEA can be held responsible for them

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Press and Schechter

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$$\sigma^2(R) = \int_0^\infty \frac{k^2\, dk}{2\pi^2} W^2(kR)\, P_m(k)$$

$$M = \frac{4\pi}{3} R^3 \rho_m$$

$$\nu_c = \boxed{\frac{\delta_c}{\sigma} \simeq \frac{1.686}{\sigma}}$$

First try: inject the DDM linear power spectrum

Halo Mass Function:
Press and Schechter

$$\frac{dn_h}{d\ln M} = \frac{\bar{\rho}}{M} f(\nu_c) \left| \frac{d\ln\sigma}{d\ln M} \right|$$

$$\sigma^2(R) = \int_0^\infty \frac{k^2\,dk}{2\pi^2} W^2(kR)\, P_m(k)$$

$$M = \frac{4\pi}{3} R^3 \rho_m$$

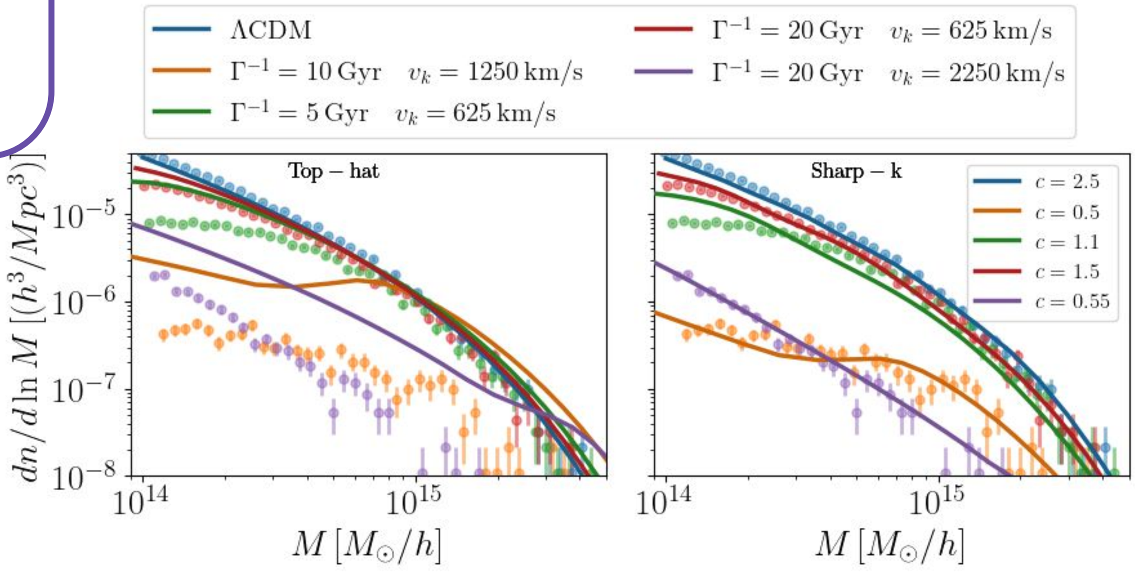
$$\nu_c = \boxed{\frac{\delta_c}{\sigma} \simeq \frac{1.686}{\sigma}}$$

First try: inject the DDM linear power spectrum

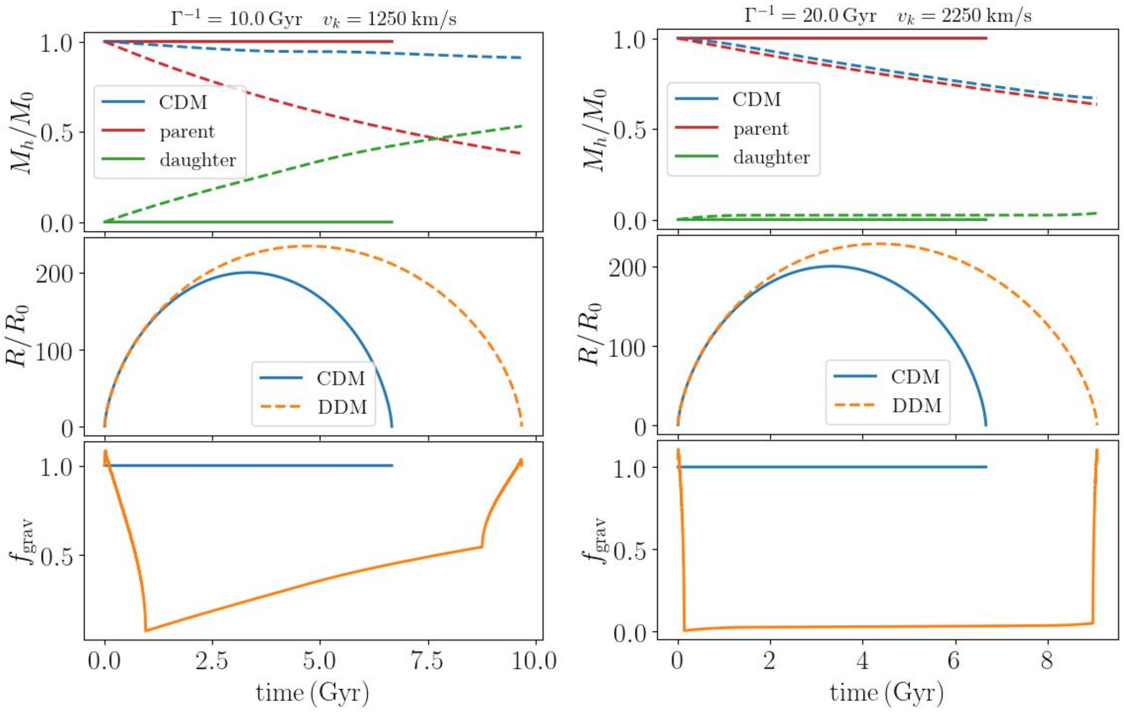
A.Schneider (1303.0839 and 1412.2133)

$$W_{\rm TH}(kR) = \frac{3}{(kR)^3}[\sin(kR) - kR\cos(kR)]$$

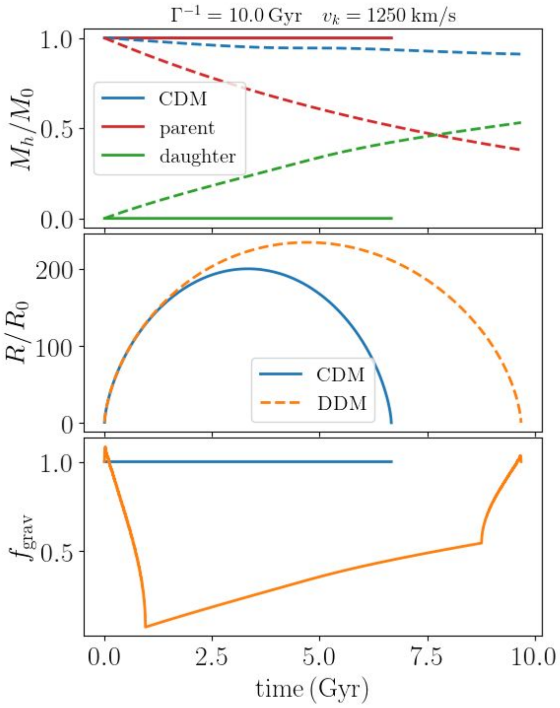
$$W_{\rm SK}(kR) = \Theta(1-kR)$$
$$M = \frac{4\pi}{3}(cR)^3 \rho_m$$

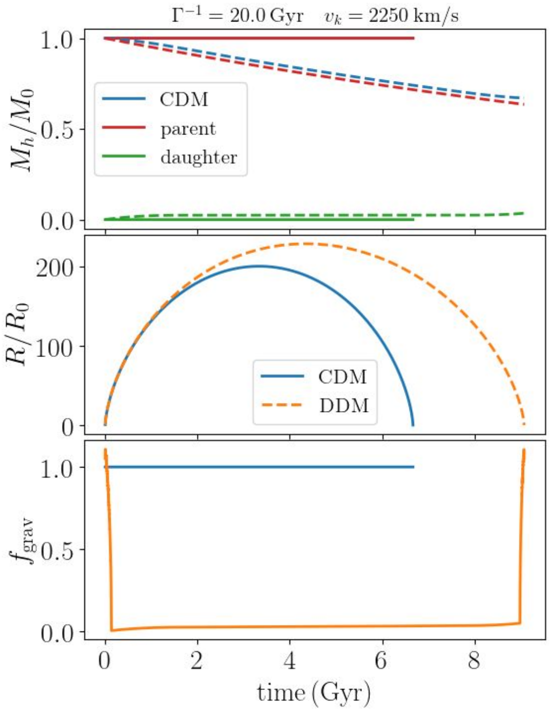


Spherical Collapse model: Decaying Dark Matter

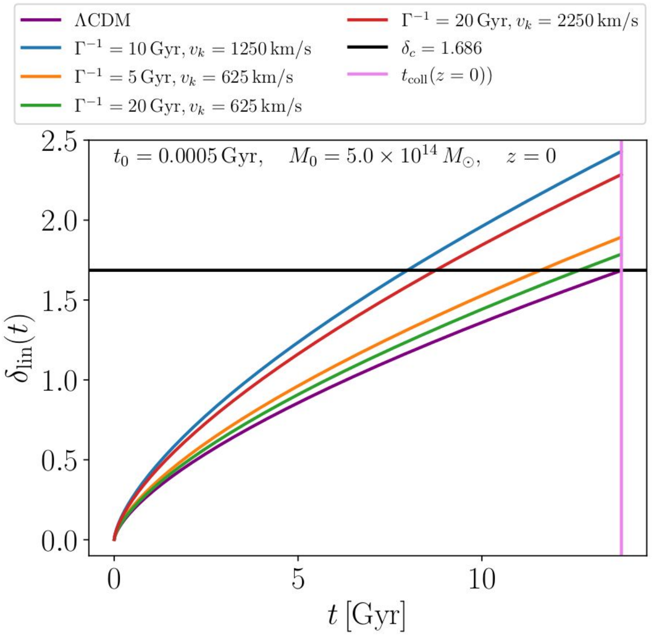


Spherical Collapse model: Decaying Dark Matter





Getting the critical density at z=0:



EdS: $\delta_c \simeq 1.686$ $\longrightarrow$ DDM: $\delta_c(M_0, \Gamma, \epsilon)$