

Isocurvature Induced Gravitational Waves at Pulsar Timing Arrays

Peizhi Du

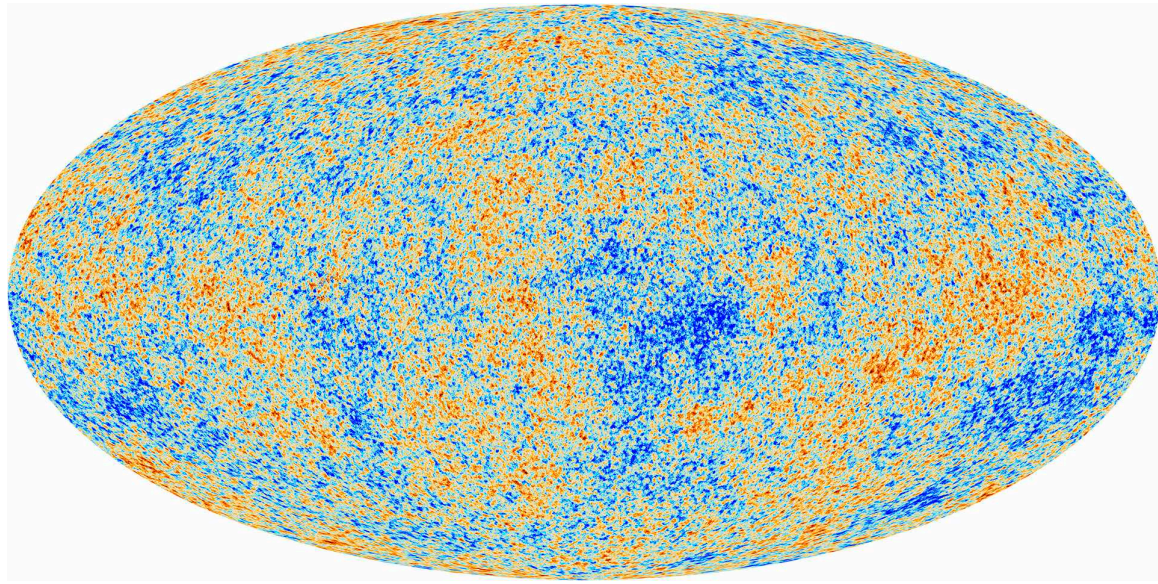
University of Science and Technology of China

GGI workshop: Exploring New Frontiers in Cosmology
July 22, 2026

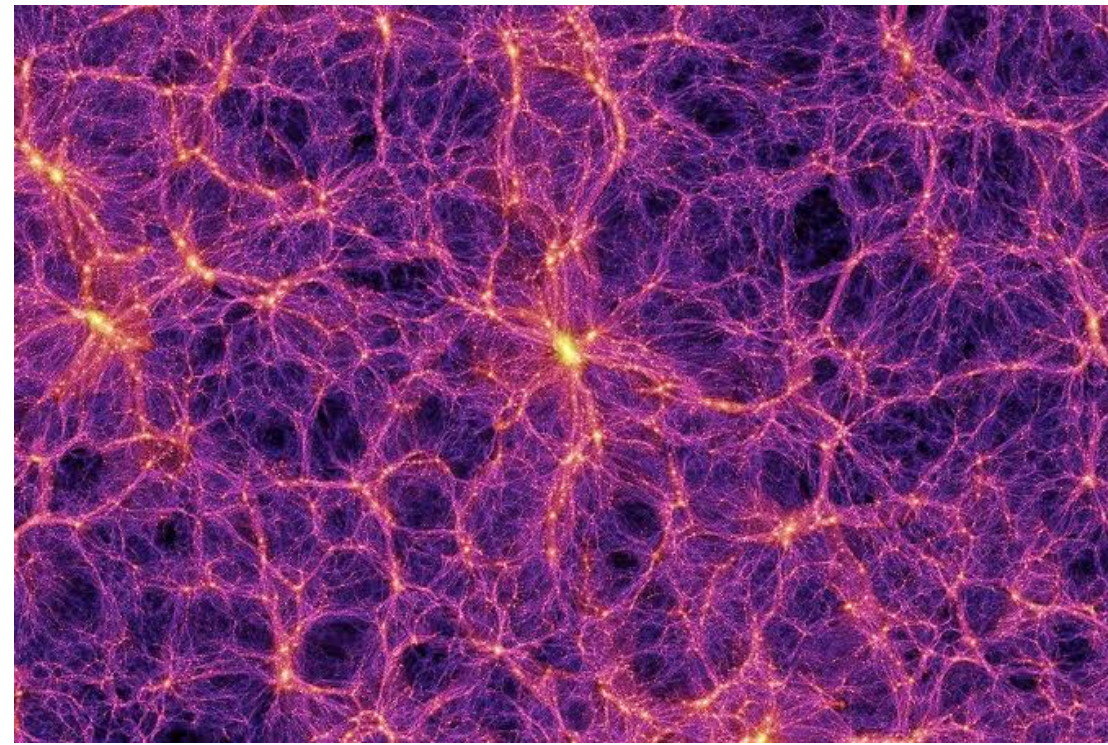
Based on works: Cai, PD, Zhong, *JCAP*, 2026 ; Buckley,PD,Fernandez,Weikert, *JCAP*, 2024&2025

Primordial Perturbations

- Our universe is mostly **homogeneous and isotropic**, but with small perturbations on large scales.



Cosmic Microwave Background

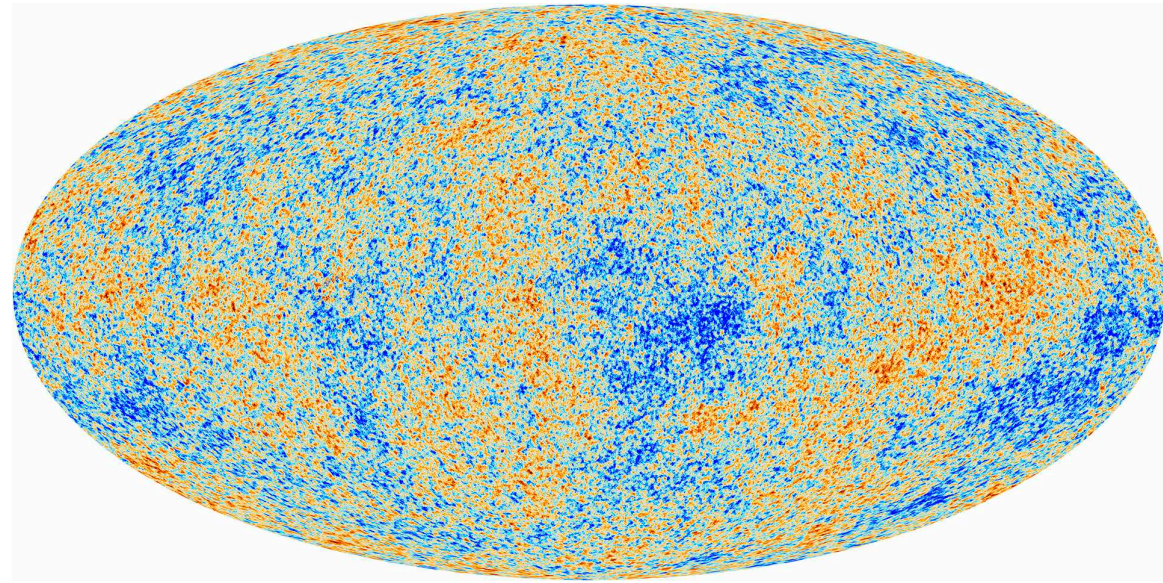


Large Scale Structure

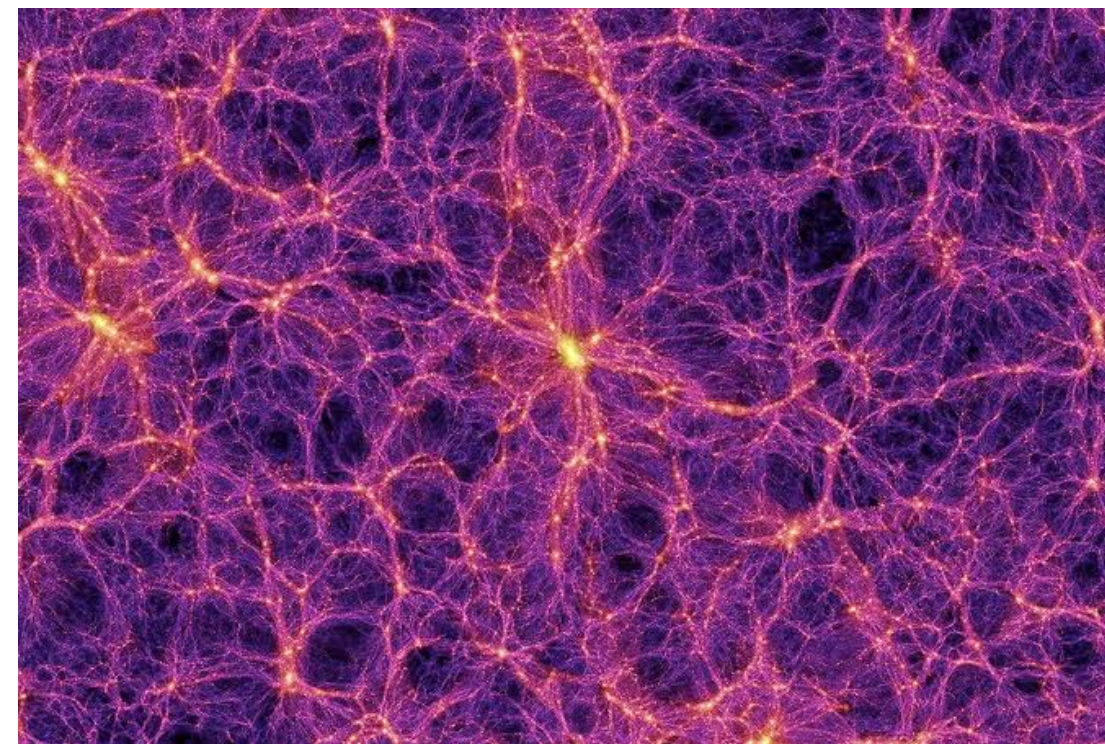
- Primordial density perturbation $\mathcal{O}(10^{-5})$

Primordial Perturbations

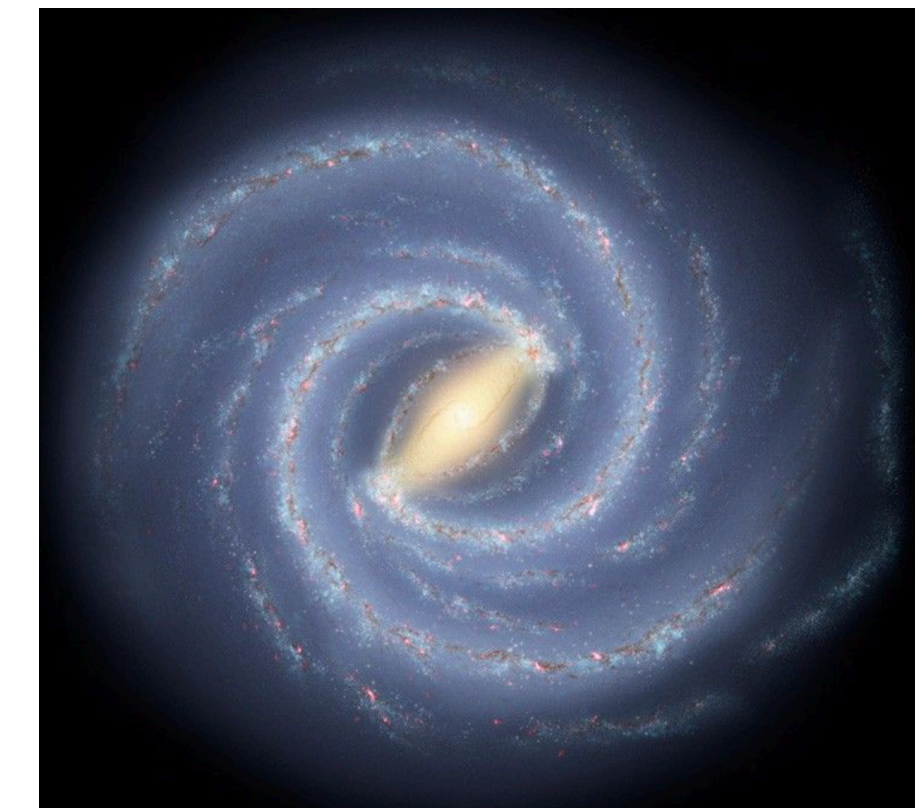
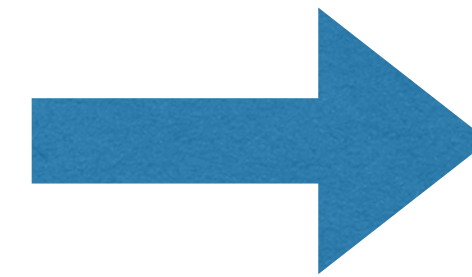
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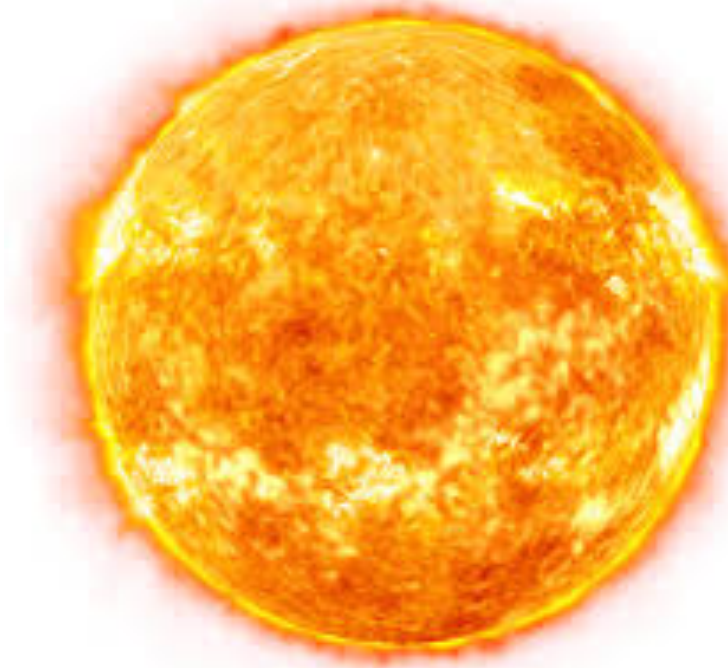
Cosmic Microwave Background



Large Scale Structure



Galaxies



Stars



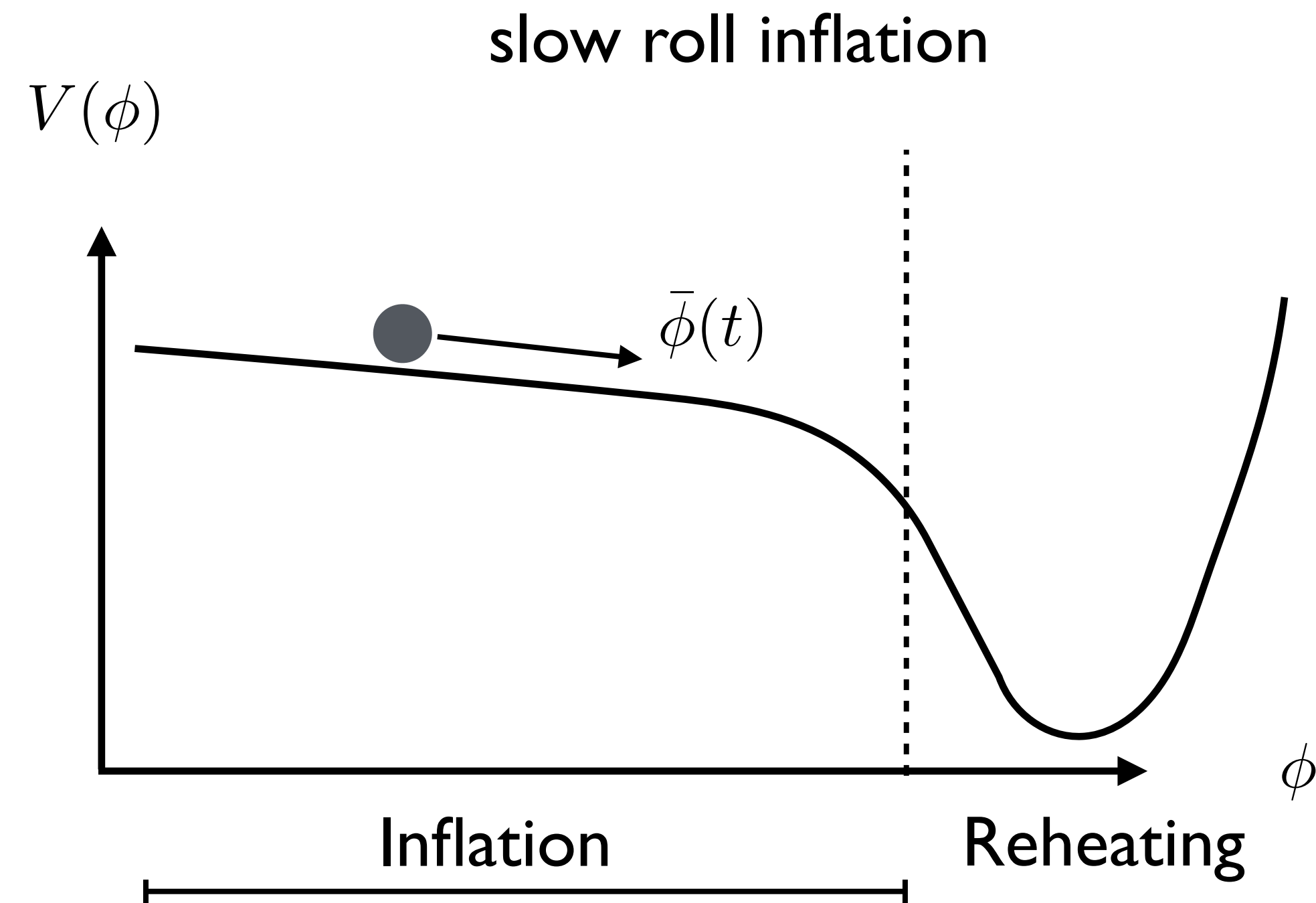
Planets

- Primordial density perturbation $\mathcal{O}(10^{-5})$

We are all coming from these small primordial perturbations!

Origin of Primordial Perturbations

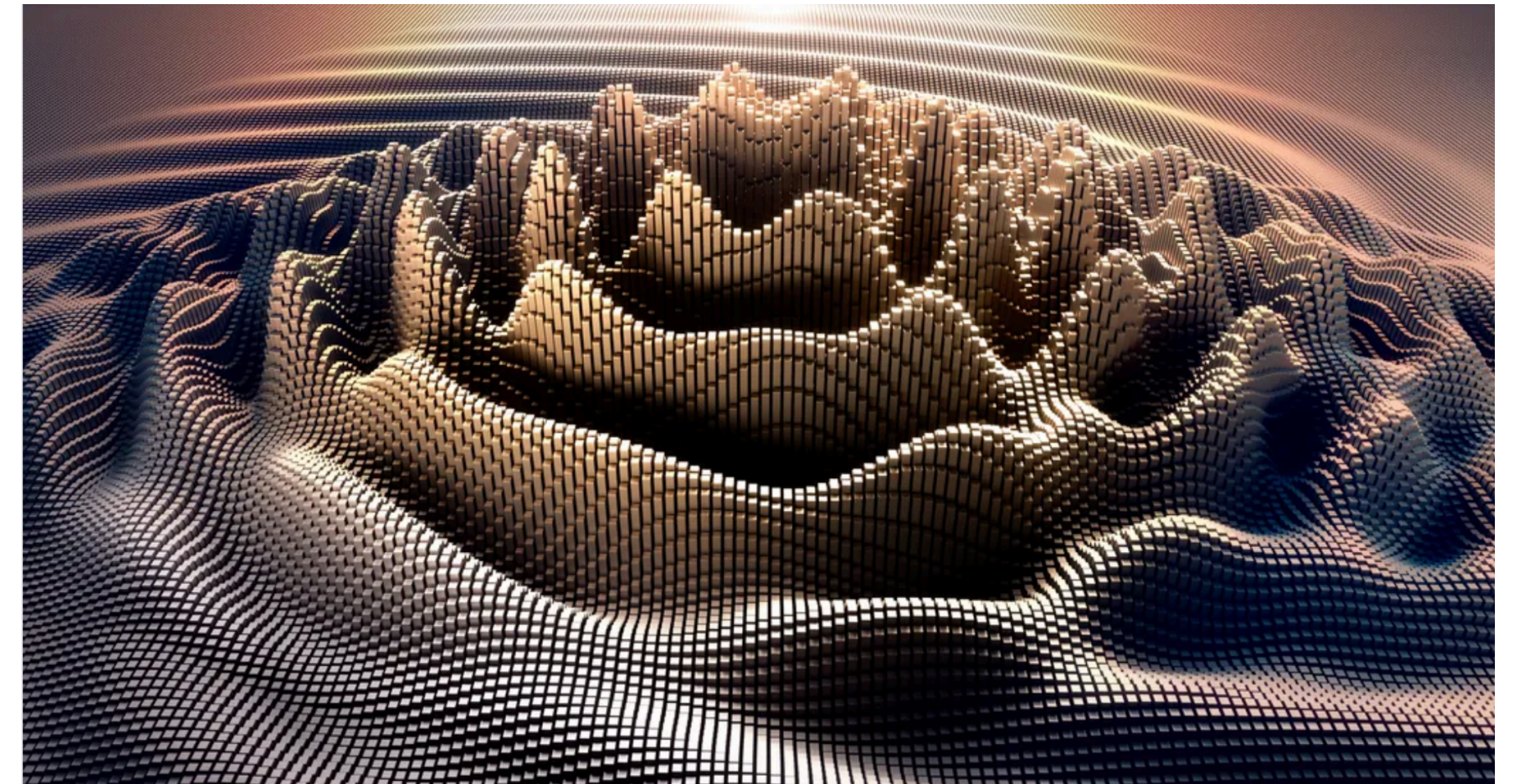
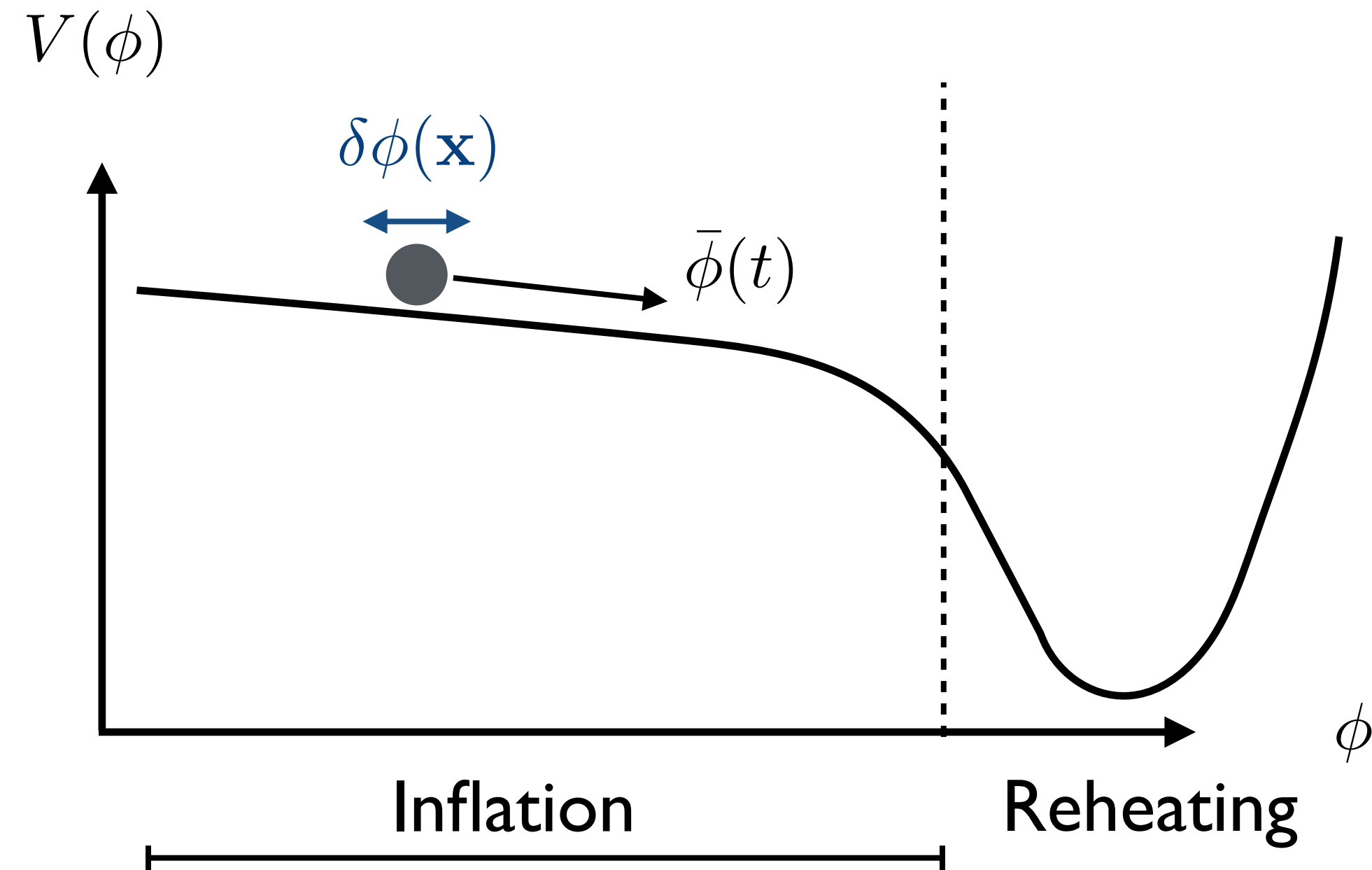
- Inflation:



Origin of Primordial Perturbations

- Inflation: **quantum fluctuations** of the inflaton generates primordial perturbations

slow roll inflation

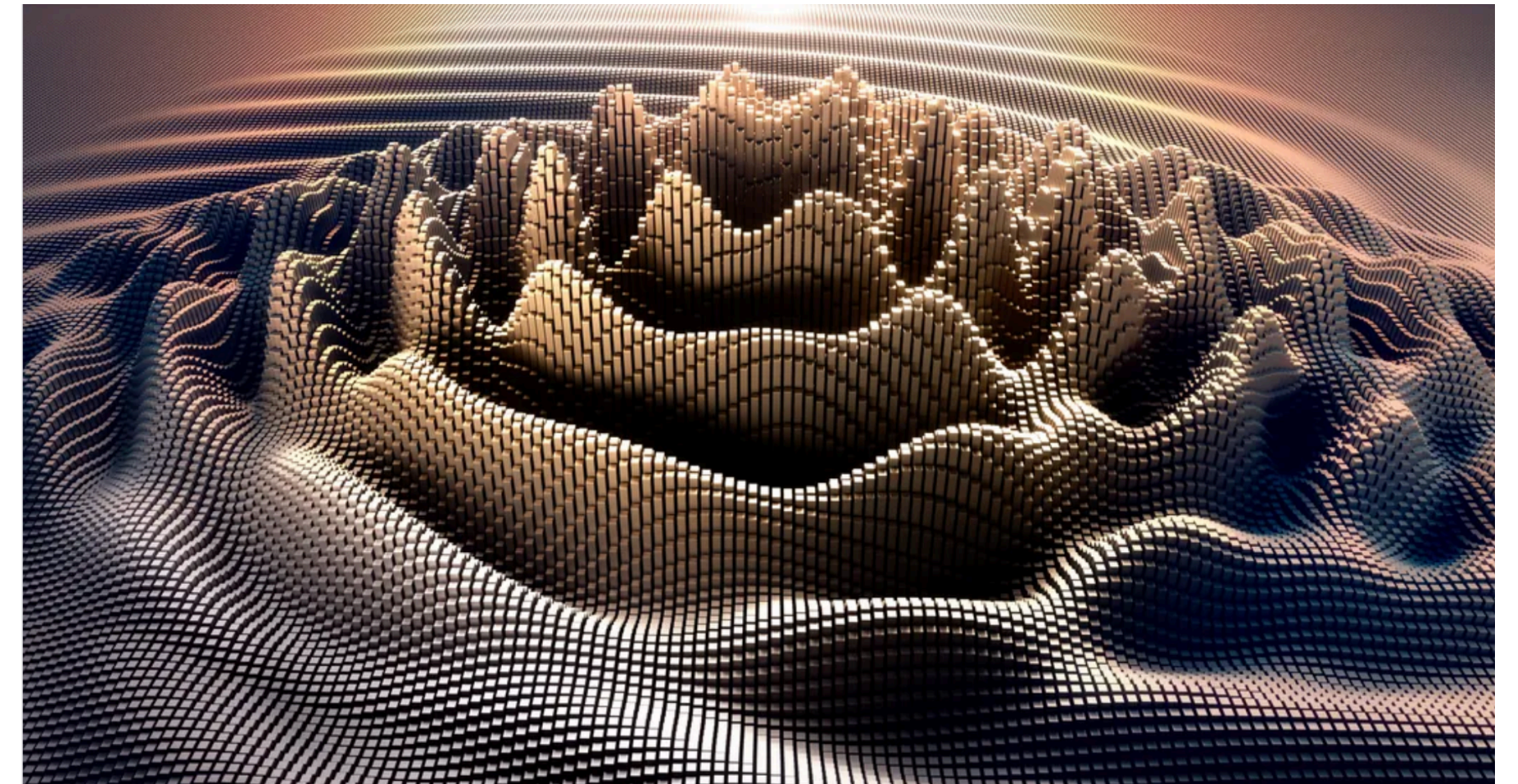
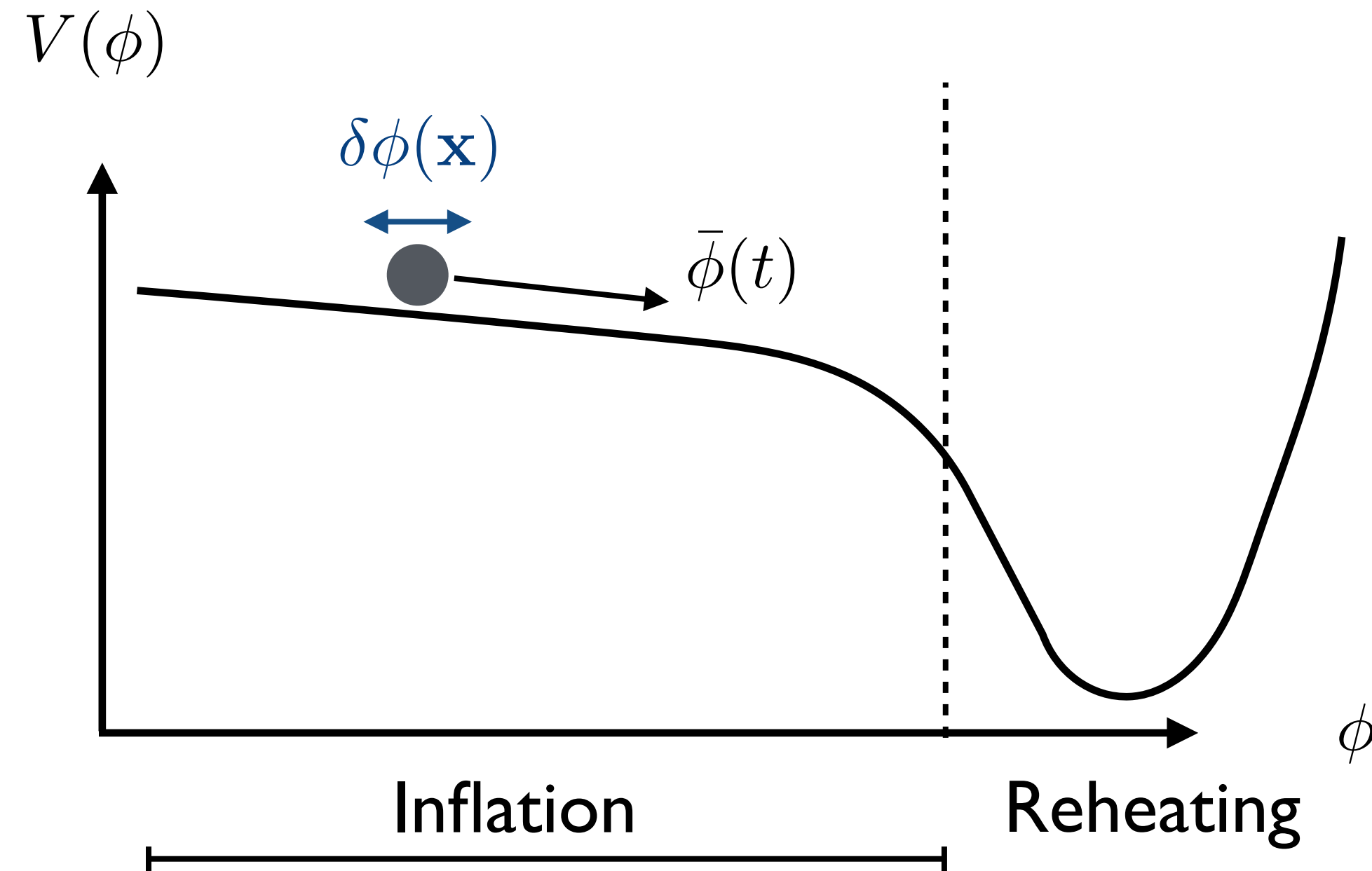


- $\delta\phi(\mathbf{x})$ sets **initial conditions** for metric and density perturbations ($\Psi, \Phi, \delta\rho/\rho$)

Origin of Primordial Perturbations

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- $\delta\phi(\mathbf{x})$ sets **initial conditions** for metric and density perturbations ($\Psi, \Phi, \delta\rho/\rho$)

Quantum fluctuations can be observed on **cosmological scales!**

More Options for Primordial Perturbations

Models/Dynamics

Initial conditions for metric and density perturbations

Single field inflation

Adiabatic

(Single source of perturbations)

More Options for Primordial Perturbations

Models/Dynamics

Initial conditions for metric and density perturbations

Single field inflation

Adiabatic

(Single source of perturbations)

Multi-field inflation

Axion/curvaton

Phase transitions

.....

Adiabatic + Isocurvature

(Multiple sources of perturbations)

Current data suggest adiabatic initial conditions, leaving constraints on isocurvature

Classification of Primordial Perturbations

- Perturbed FRW metric: $ds^2 = a^2(\tau) [-(1 + 2\Phi)d\tau^2 + (1 - 2\Psi)\delta_{ij}dx^i dx^j]$

Gauge-invariant perturbations

Curvature perturbation

$$\zeta = -\Psi - \mathcal{H}\delta\rho/\bar{\rho}'$$

Isocurvature perturbation

$$\mathcal{S}_{\alpha\beta} = 3(\zeta_\alpha - \zeta_\beta) = \frac{\delta_\alpha}{1 + w_\alpha} - \frac{\delta_\beta}{1 + w_\beta}$$

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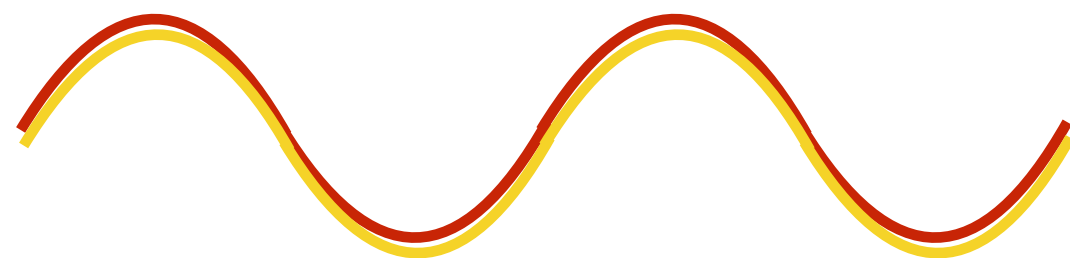
$$\mathcal{S}_{\alpha\beta} = 3(\zeta_\alpha - \zeta_\beta) = \frac{\delta_\alpha}{1 + w_\alpha} - \frac{\delta_\beta}{1 + w_\beta}$$

- Primordial perturbations determine **initial conditions** for $\Psi, \Phi, \delta_\alpha \equiv \delta\rho_\alpha/\bar{\rho}_\alpha \dots$

Adiabatic initial conditions

$$\zeta \neq 0, \mathcal{S}_{\alpha\beta} = 0$$

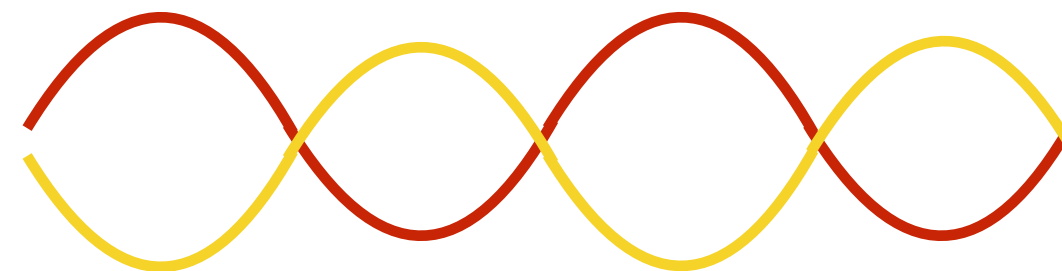
$$\delta_\gamma = \frac{4}{3}\delta_c$$



CDM Iso initial conditions

$$\zeta = 0, \mathcal{S}_{c\gamma} \neq 0$$

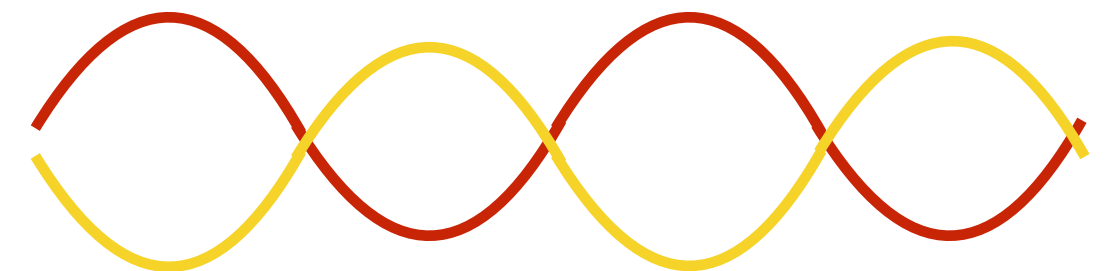
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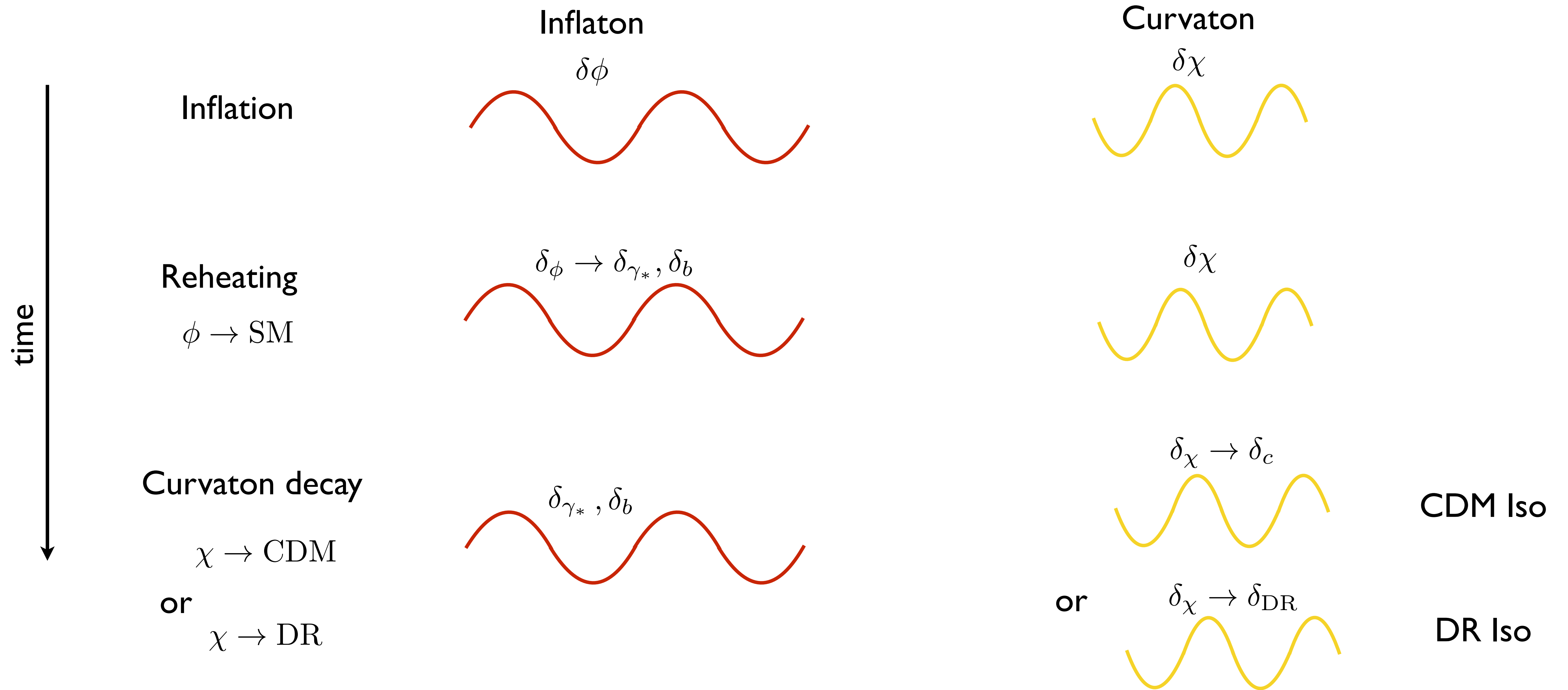
DR Iso initial conditions

$$\zeta = 0, \mathcal{S}_{\text{dr}\gamma} \neq 0$$

$$\delta_\gamma = \frac{4}{3}\delta_c \neq \delta_{\text{dr}}$$



Examples for Isocurvature Perturbations



Spatial distribution of Primordial Perturbations

$$X(\vec{k}, \tau) = \underbrace{c(\vec{k})}_{\text{Primordial spatial distribution}} \underbrace{X(k, \tau)}_{\text{Transfer function (time dependence)}}$$

Primordial spatial distribution

$$X \in \{\Phi, \Psi, \delta, \theta, \sigma \dots\}$$

- Power Spectrum encodes spatial distribution: $\langle c(\vec{k})c(\vec{k}') \rangle \equiv \frac{2\pi^2}{k^3} \underbrace{P(k)}_{\text{Power Spectrum}} (2\pi)^3 \delta^3(\vec{k} + \vec{k}'),$

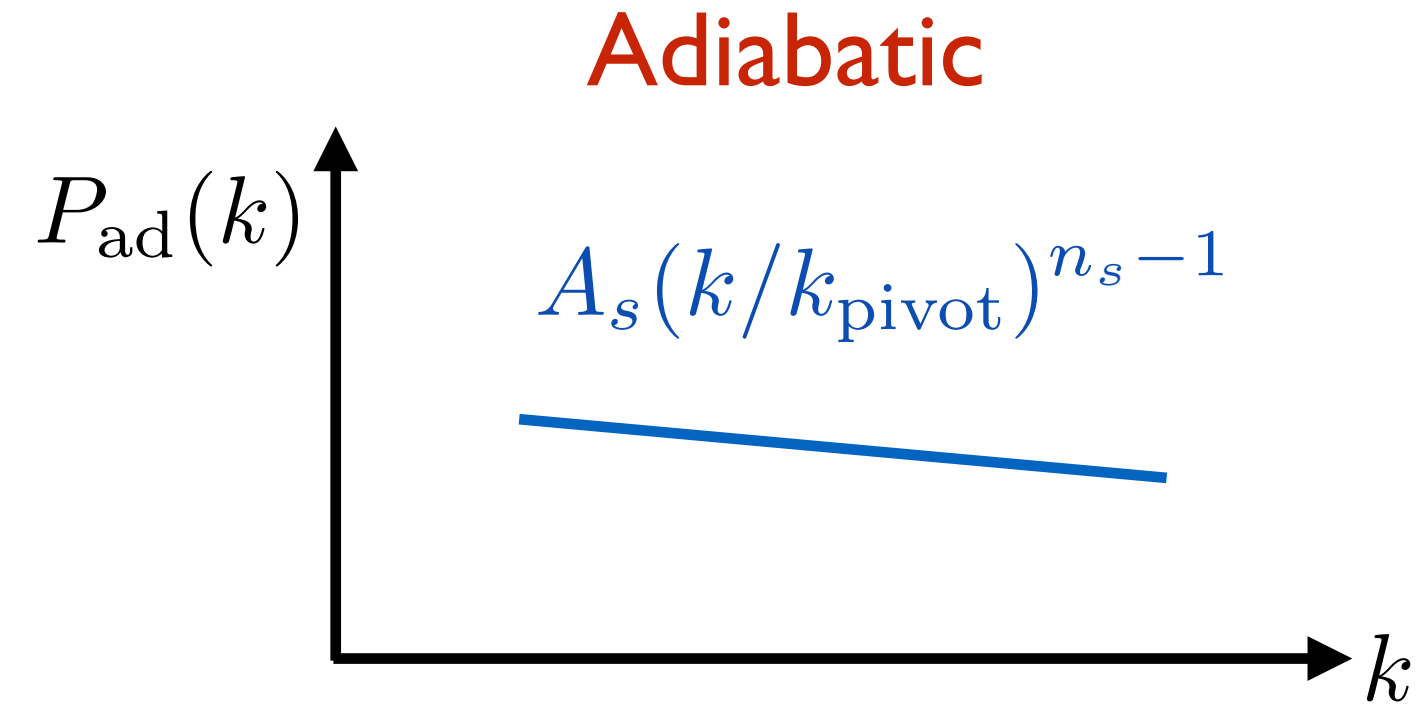
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$$A_s \approx 2 \times 10^{-9}$$

$$n_s \approx 0.96 \quad \text{Planck, 2018}$$

Slow roll inflation

Spatial distribution of Primordial Perturbations

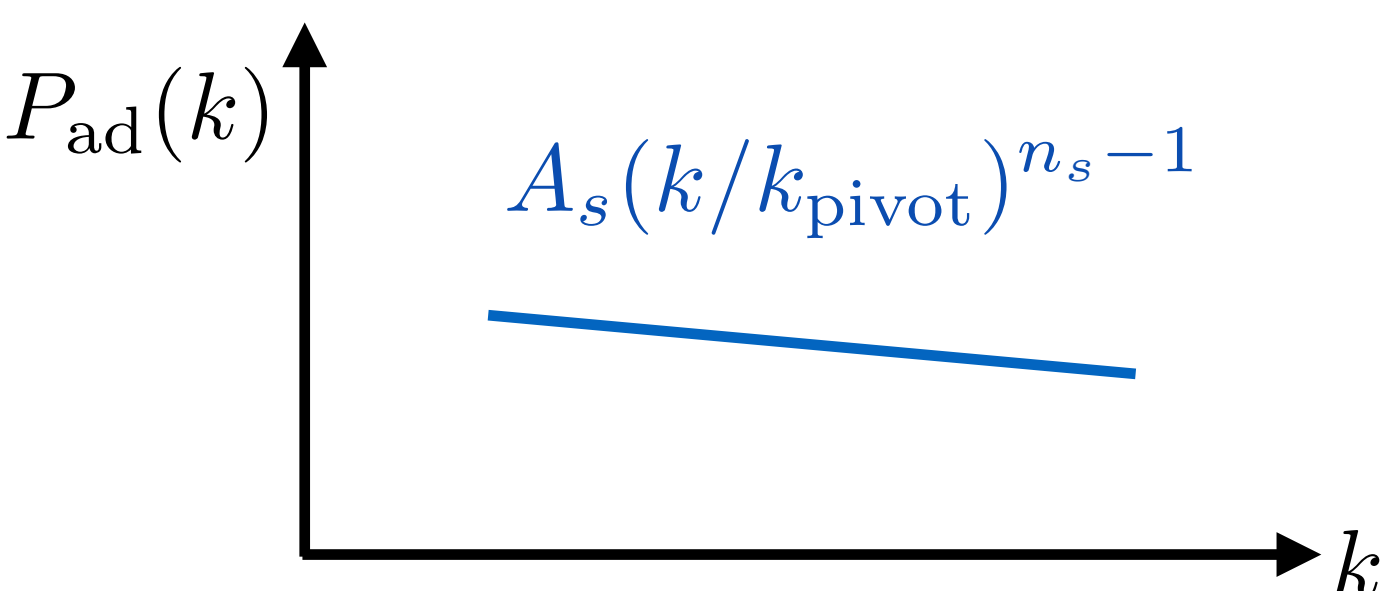
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Slow roll inflation

Isocurvature

Curvaton/Axion

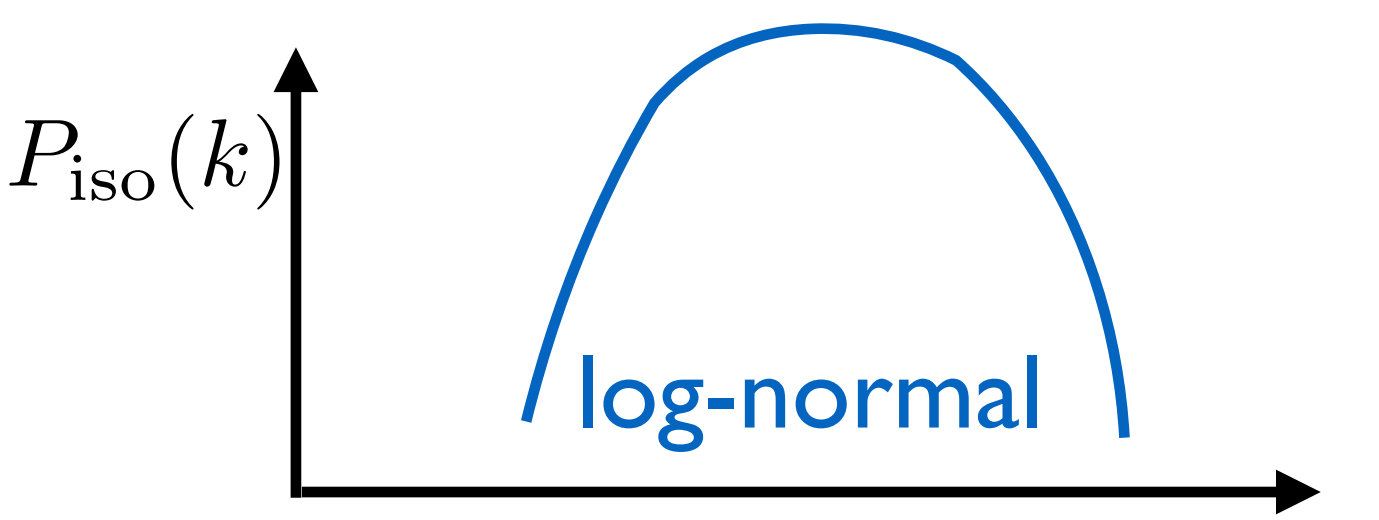
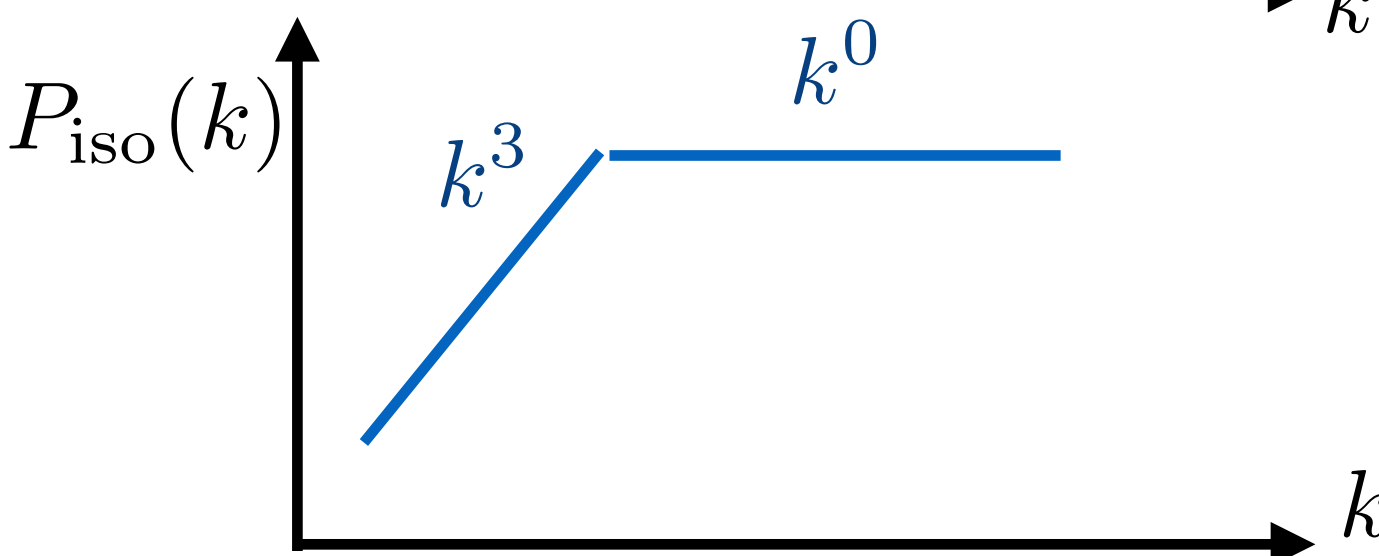
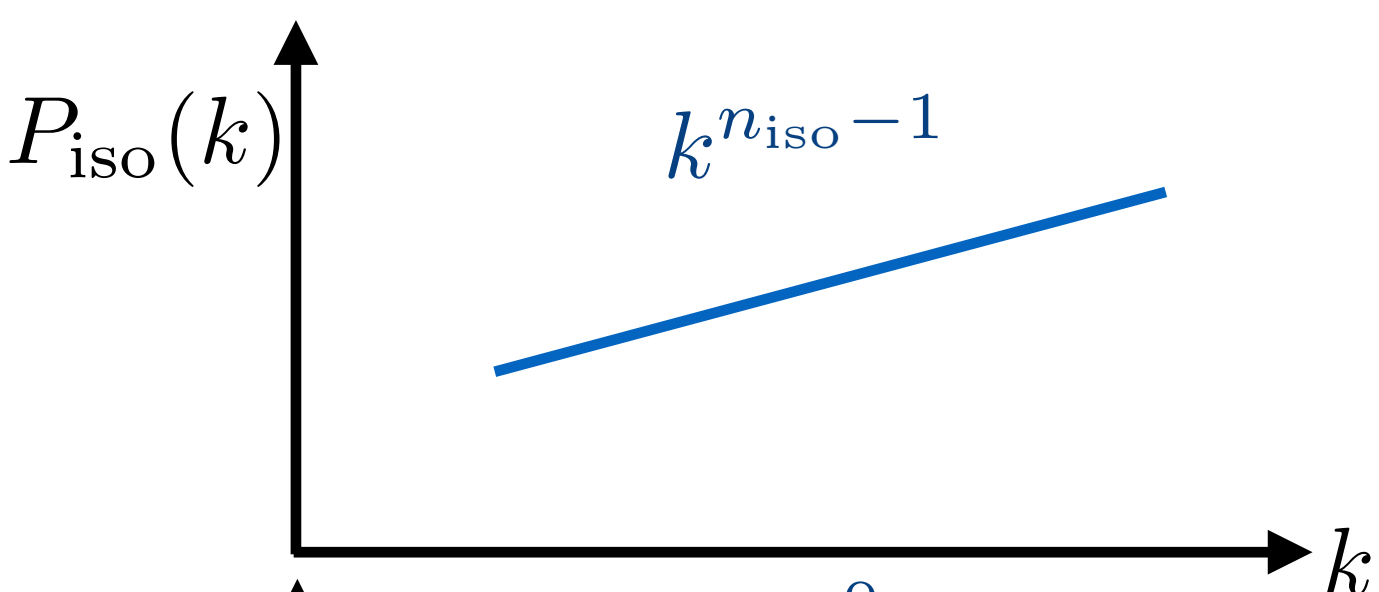
Mazumda, Rocher, *Phys.Rept.*, 2011
Marsh, *Phys.Rept.*, 2016

Phase transition during inflation

Barir, Geller, Sun, Volansky, *PRD*, 2023
Buckley,PD,Fernandez,Weikert, *JCAP*, 2024

Inflationary dynamics

Pi, Zhang, Huang, Sasaki, *JCAP*, 2018
Pi, Sasaki, *JCAP*, 2020



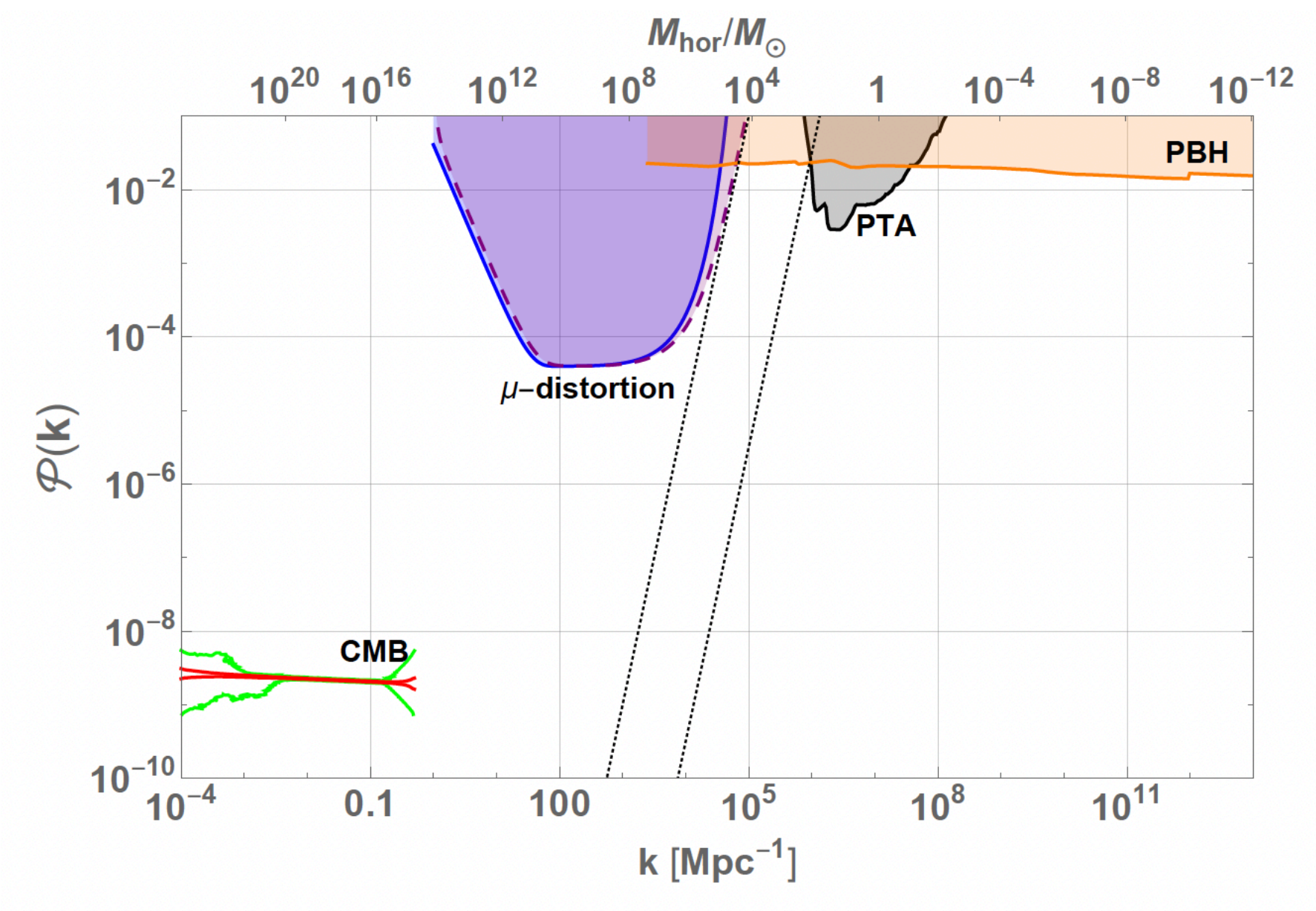
General Constraints on Primordial Perturbations

- General parametrization: $P(k) = A \delta(\ln k - \ln k_0)$ simple and useful

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Curvature Power Spectrum

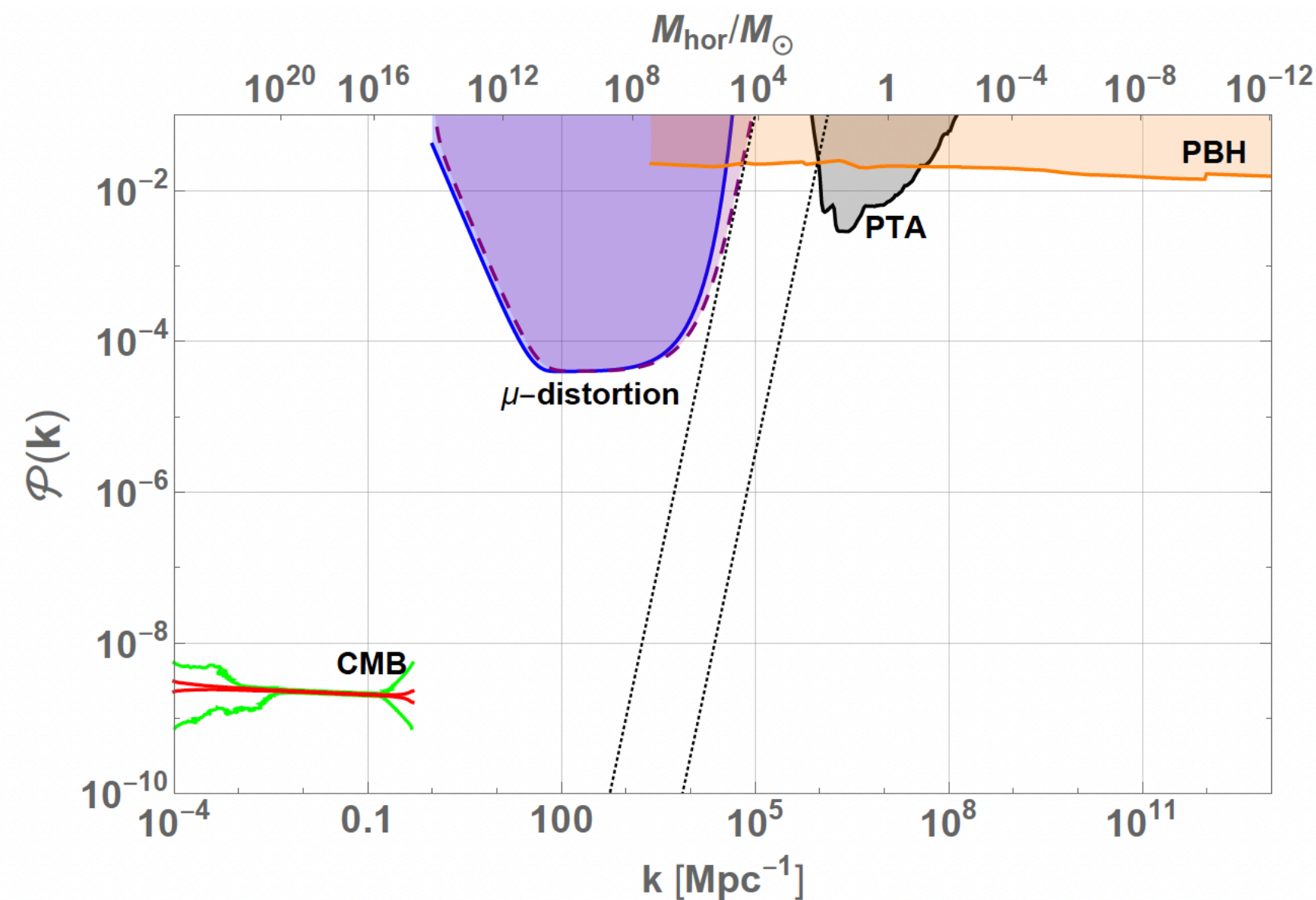


Byrnes, Cole, Patil, *JCAP*, 2019

General Constraints on Primordial Perturbations

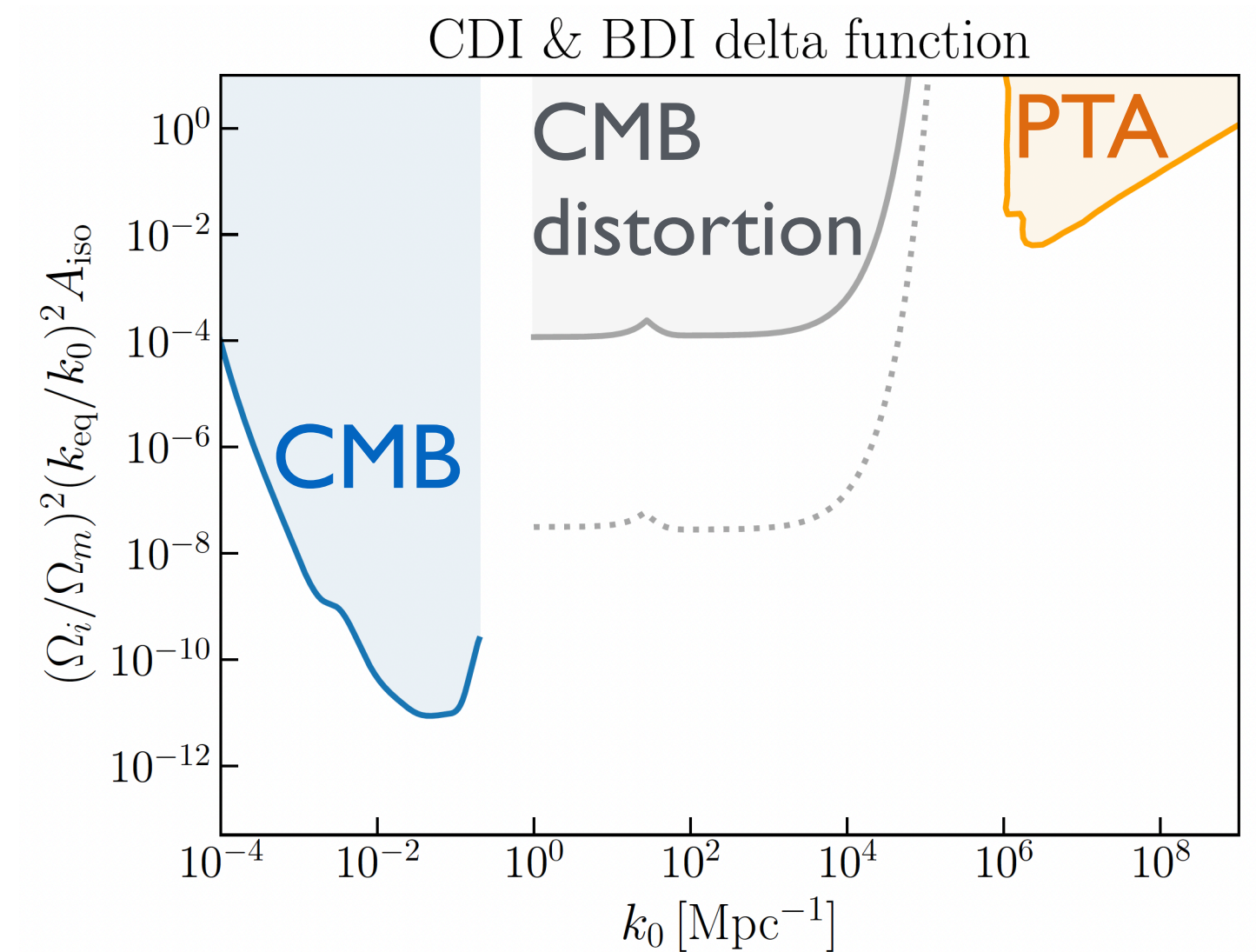
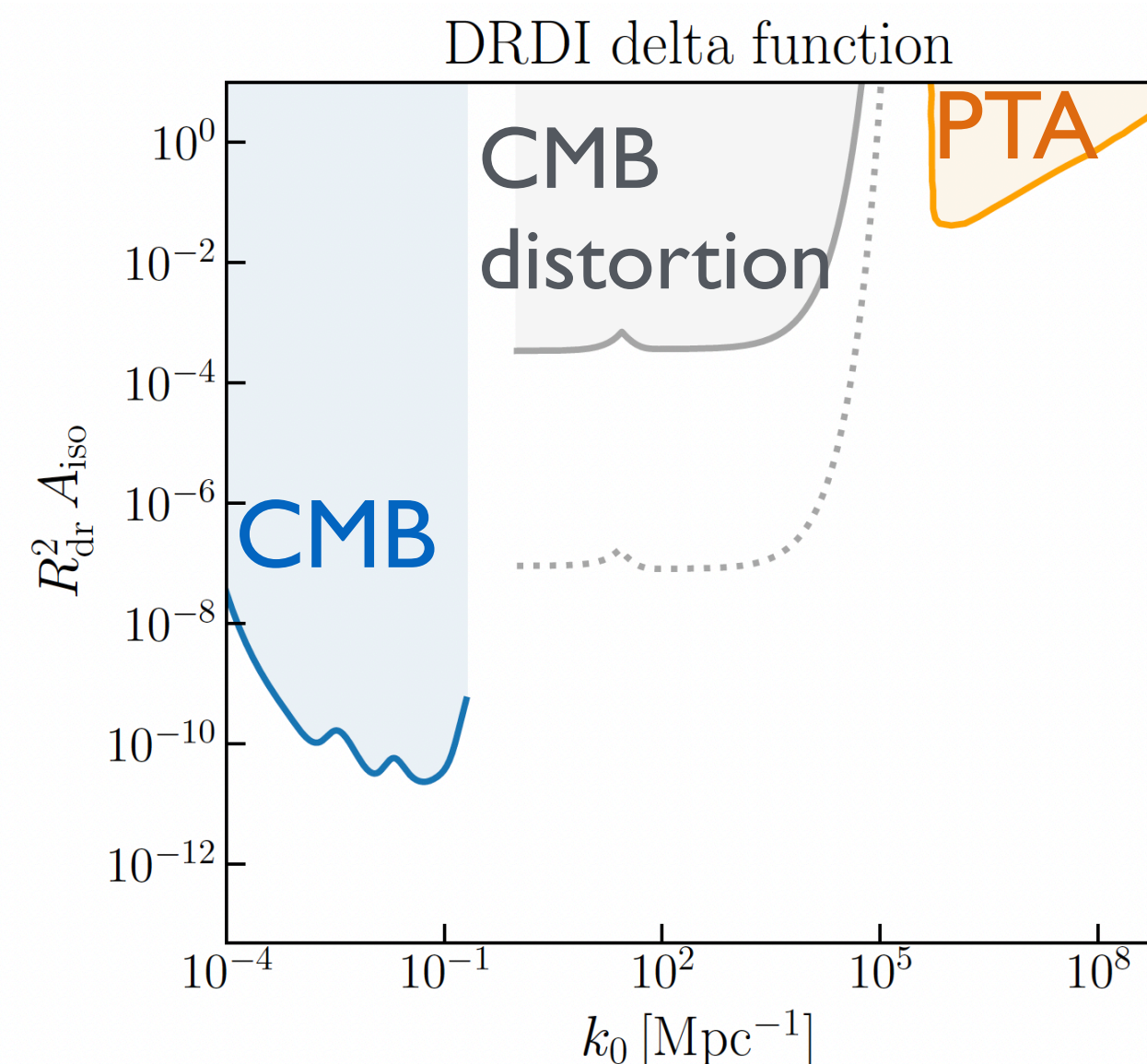
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Curvature Power Spectrum



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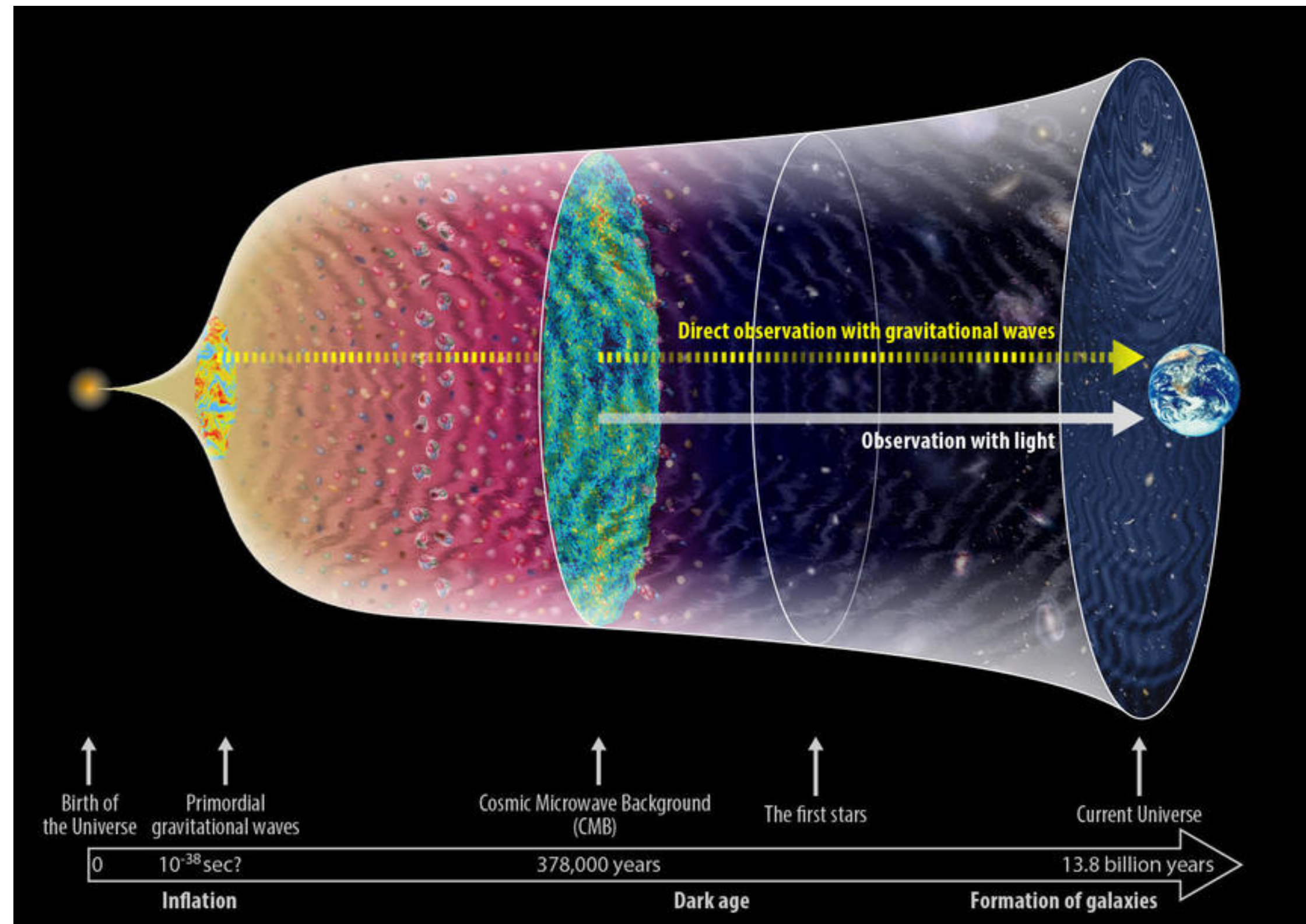
Isocurvature Power Spectrum



Buckley,PD,Fernandez,Weikert, *JCAP*, 2025

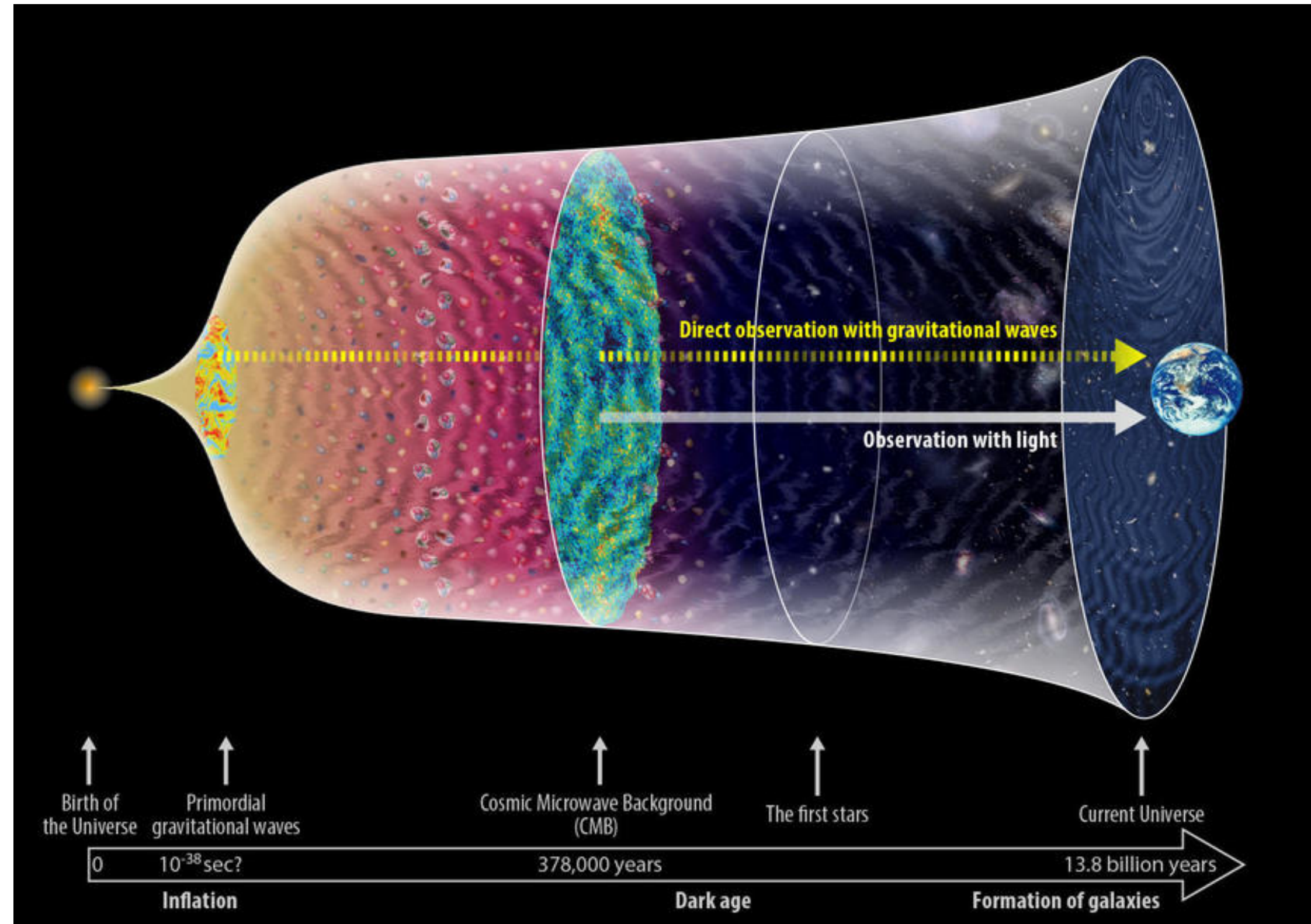
Cai, PD, Zhong, *JCAP*, 2026

Gravitational Waves

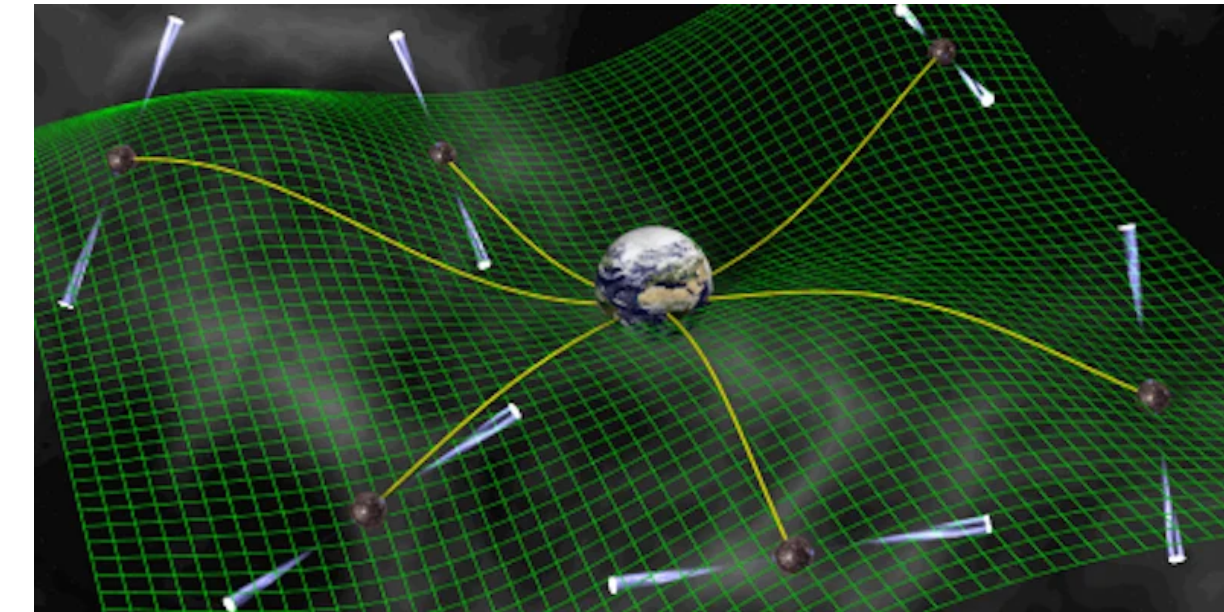


Powerful probes of physics in the early Universe

Gravitational Waves

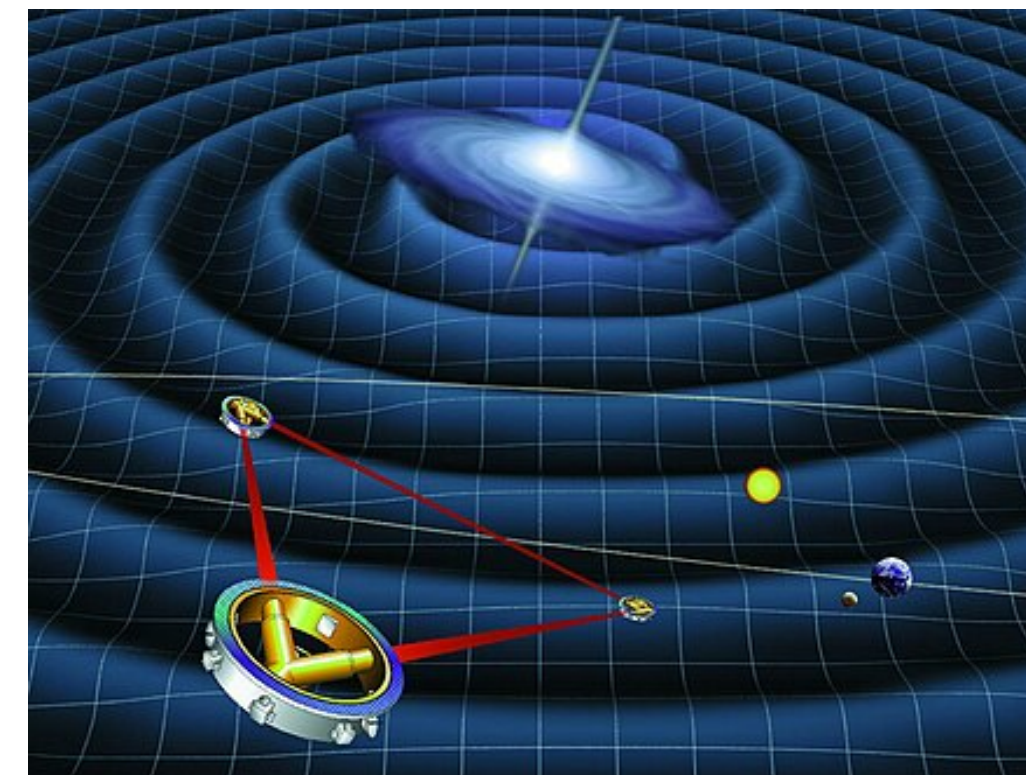


Powerful probes of physics in the early Universe



PTA

Ground Interferometers

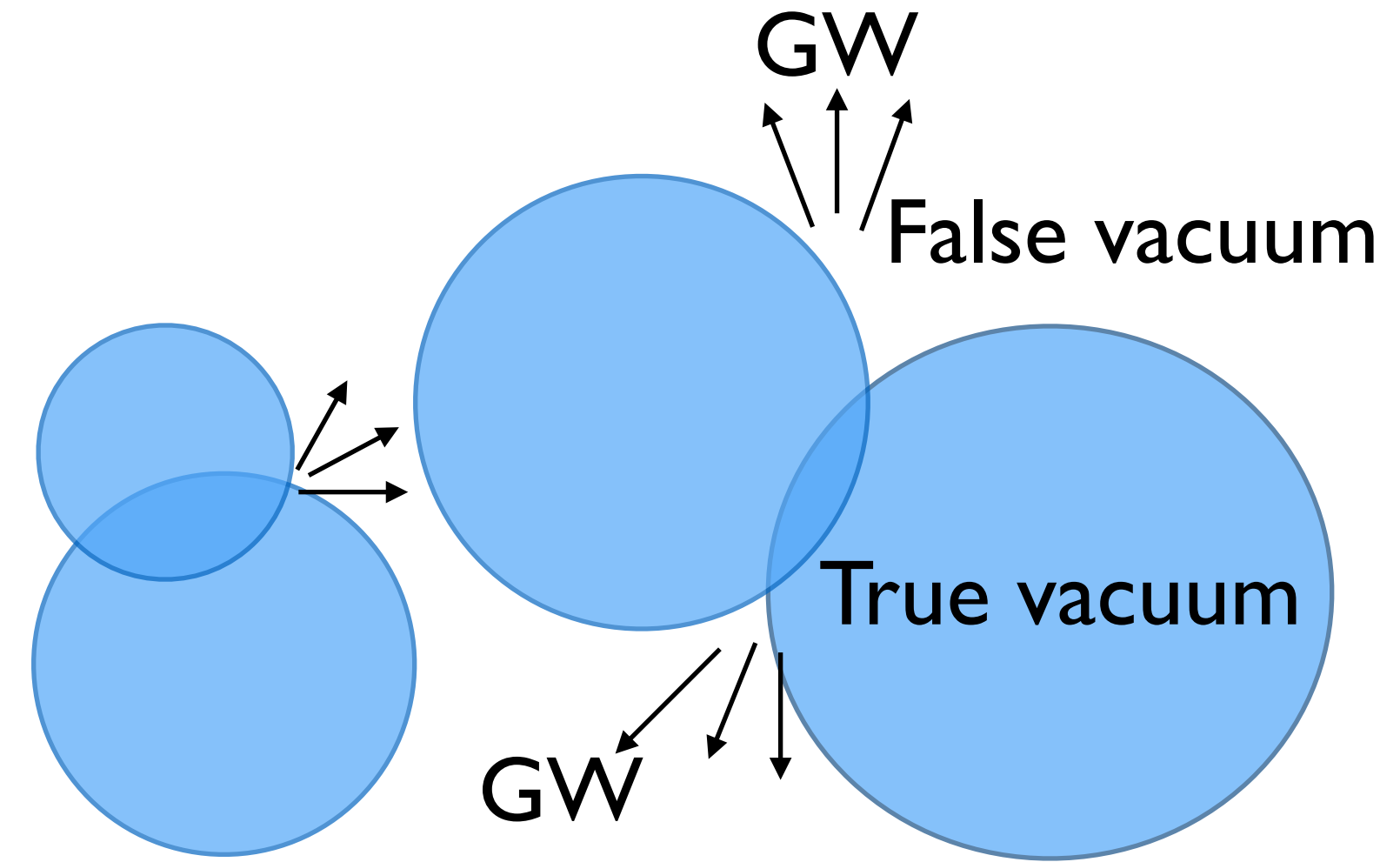


Space Interferometers

Exciting experimental probes

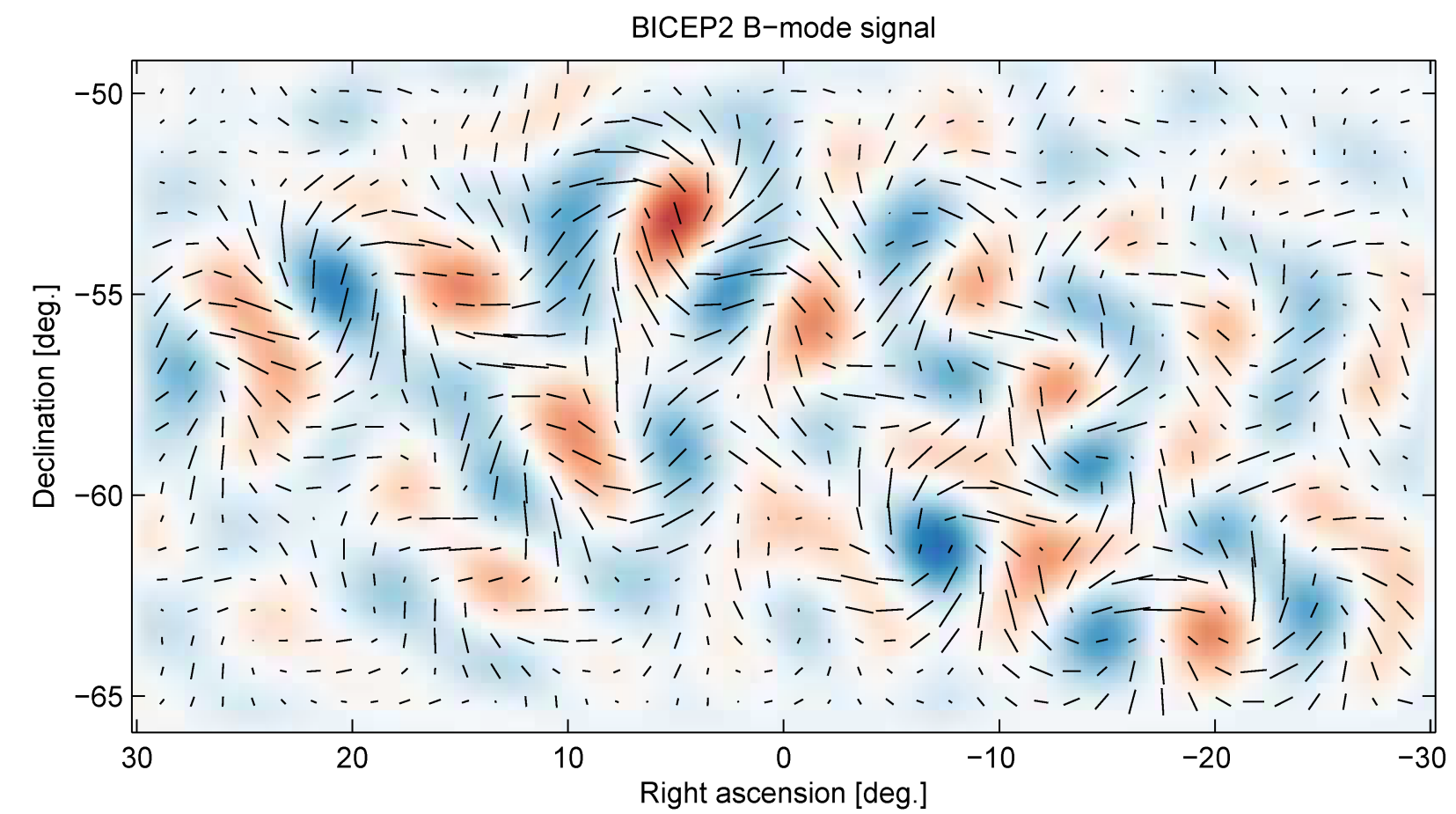
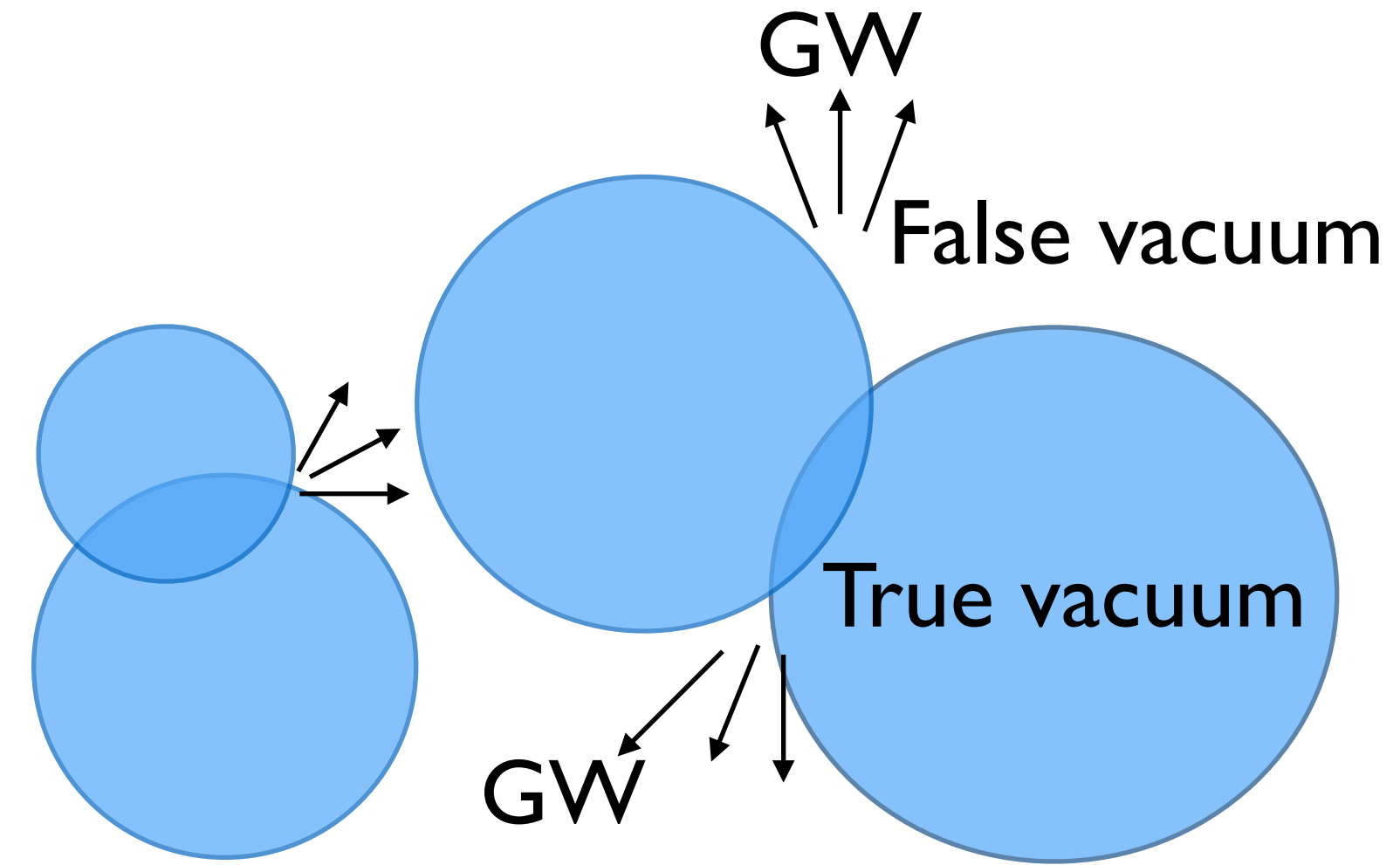
Sources of Gravitational Waves

- First order phase transitions



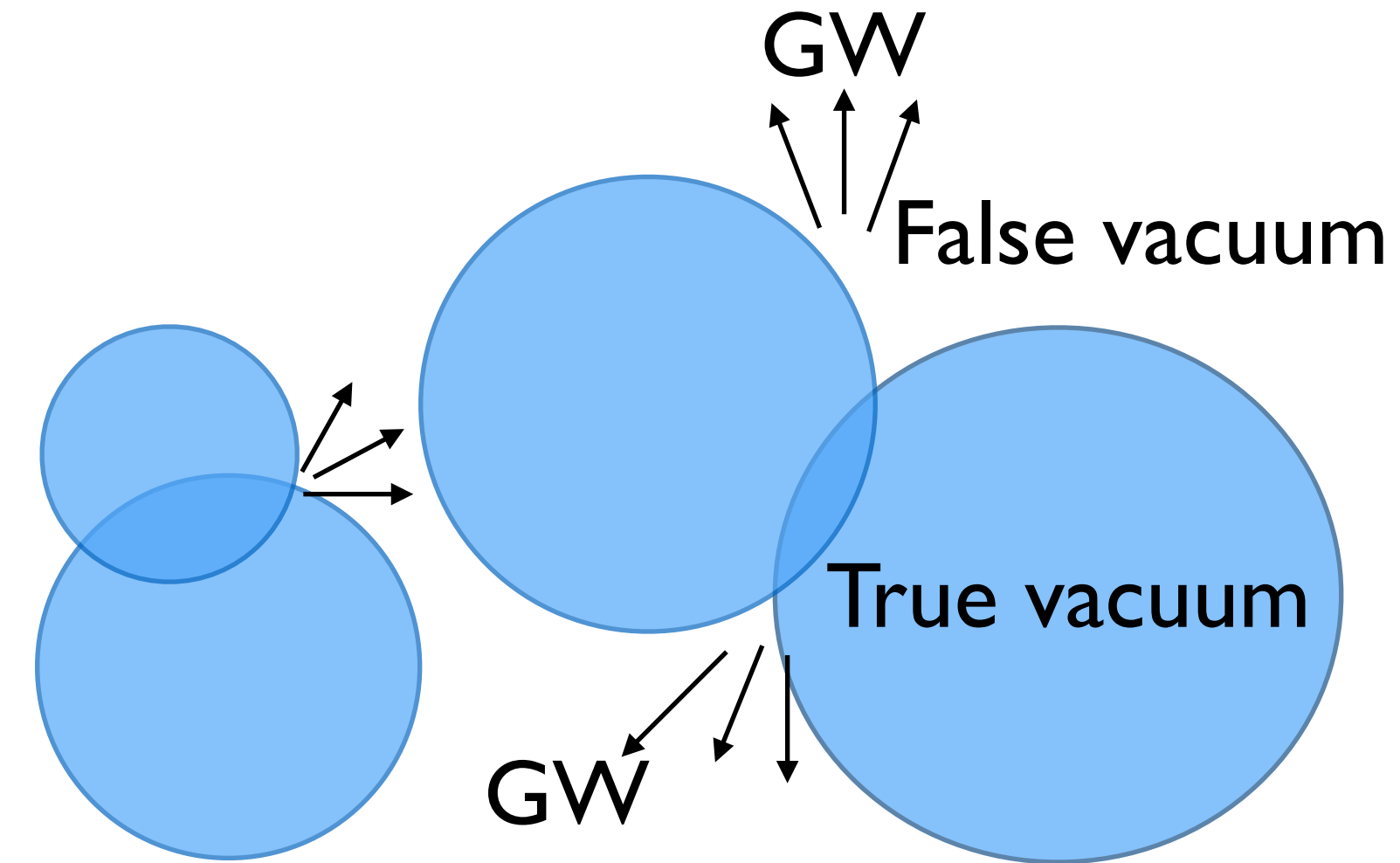
Sources of Gravitational Waves

- First order phase transitions
- Primordial tensor modes

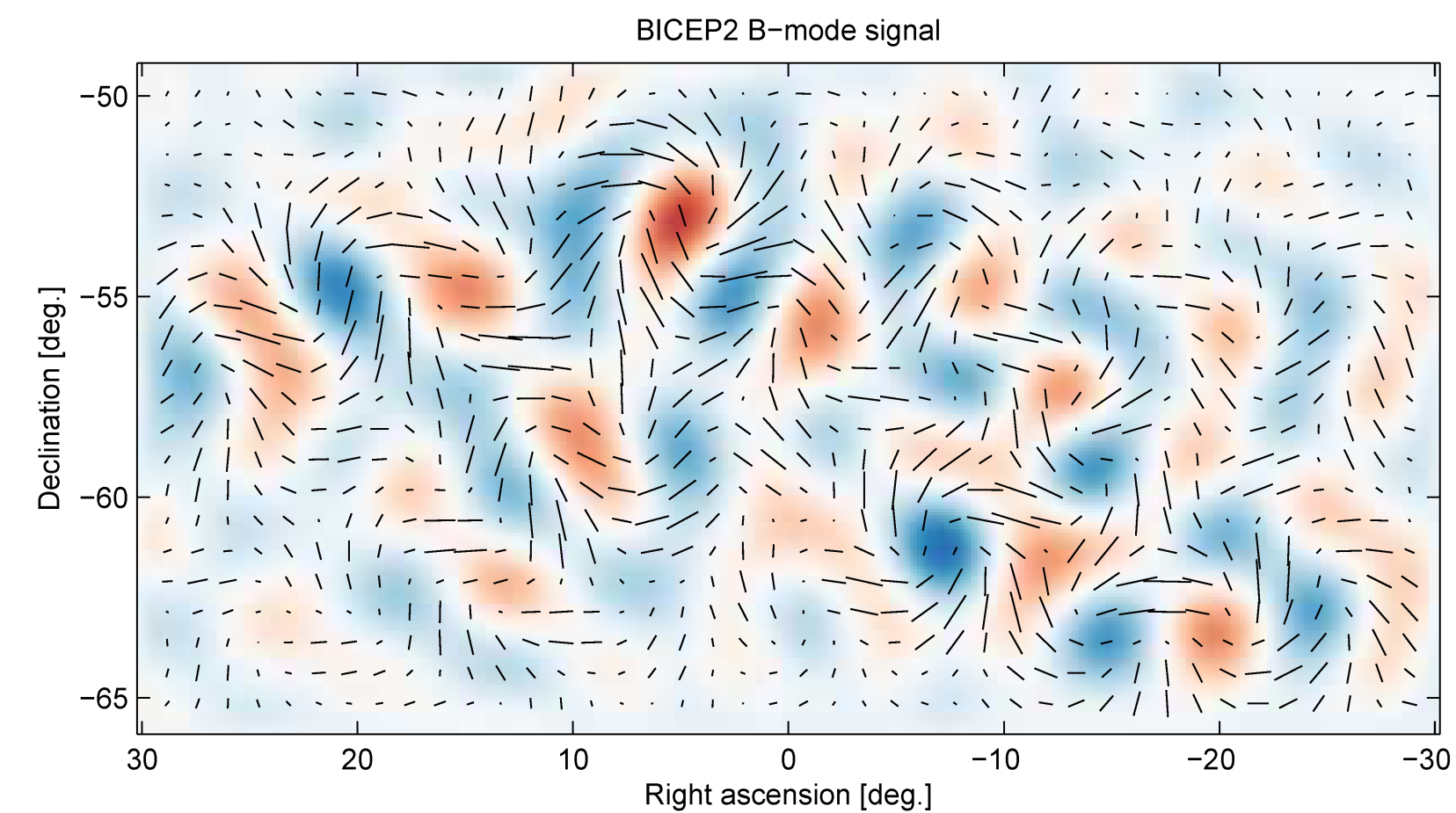


Sources of Gravitational Waves

- First order phase transitions



- Primordial tensor modes



- Scalar induced GWs

Baumann, Steinhardt, Takahashi, Ichiki, *PRD*, 2007
Domenech, *Universe*, 2021

$$\partial_i \Phi^{(1)} \partial_j \Phi^{(1)} \rightarrow h_{ij}^{(2)}$$

Scalar Induced GWs

$$ds^2 = a^2(\tau) \left[-(1 + 2\Phi)d\tau^2 + (\delta_{ij} - 2\Psi\delta_{ij} + h_{ij})dx^i dx^j \right]$$

- First-order perturbations: **no source term** for GWs
but a damping term if free-streaming radiation (neutrinos, DR) exists

Weinberg, *PRD*, 2004

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First-order Einstein's equation

$$\begin{aligned}
 G_{ij}^{(1)} &= 8\pi G T_{ij}^{(1)} \\
 G_{ij}^{(1)} &\supset h_{ij}^{(1)''} \\
 T_{ij}^{(1)} &\supset -\boxed{\bar{\rho}_{\text{fs}}} \int_{\tau_{\text{fs}}}^{\tau} d\tau' \frac{j_2(k(\tau - \tau'))}{(\tau - \tau')^2} \partial_{\tau} \underline{h_{ij}^{(1)}}
 \end{aligned}
 \quad \longrightarrow \quad
 \begin{aligned}
 \underline{h_{\vec{k},\lambda}''} + 2\mathcal{H}h_{\vec{k},\lambda}' + k^2 h_{\vec{k},\lambda} &= S_{\vec{k},\lambda} \\
 S_{\vec{k},\lambda} &= -24\boxed{R_{\text{fs}}}\mathcal{H}^2 \int_{\tau_{\text{fs}}}^{\tau} d\tau' \frac{j_2(k(\tau - \tau'))}{(\tau - \tau')^2} \partial_{\tau} \underline{h_{\vec{k},\lambda}} \\
 R_{\text{fs}} &\equiv \bar{\rho}_{\text{fs}}/\bar{\rho}_r
 \end{aligned}$$

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Weinberg, *PRD*, 2004

- Second-order perturbations:

$$G_{ij}^{(2)} = 8\pi G T_{ij}^{(2)}$$

$$G_{ij}^{(2)} \supset \underline{\partial_i \Phi^{(1)} \partial_j \Phi^{(1)}}, h_{ij}^{(2)''}$$

$$T_{ij}^{(2)} \supset \underline{\bar{\rho}_r \partial_i v^{(1)} \partial_j v^{(1)}}$$

Sources for GWs

$$h_{\vec{k},\lambda}'' + 2\mathcal{H}h_{\vec{k},\lambda}' + k^2 h_{\vec{k},\lambda} = S_{\vec{k},\lambda}$$

$$S_{\vec{k},\lambda} = 4 \int \frac{d^3 \vec{q}}{(2\pi)^3} e_{\vec{k},\lambda}^{ij} q_i q_j c(\vec{q}) c(\vec{k} - \vec{q}) \underline{f(\tau, q, |\vec{k} - \vec{q}|)}$$

- GW energy spectrum

$$\Omega_{\text{GW}} \sim \sum_{\lambda} \langle h_{\lambda} h_{\lambda} \rangle \propto A_{\text{ad,iso}}^2$$

Isocurvature Induced GWs

- Cases with no DR ($\Psi = \Phi$) **Studied in the literature** Domenech, *Universe*, 2021
Domenech, *AAPPS Bull.*, 2024

$$T_{\mu\nu}^{(2)} \approx \frac{4}{3} \bar{\rho}_r \partial_\mu v_{\gamma_*}^{(1)} \partial_\nu v_{\gamma_*}^{(1)}$$

$$f(x, u, v) = \Phi(u, x) \Phi(v, x) + \frac{1}{2} (\Phi(u, x) + x \partial_x \Phi(u, x)) (\Phi(v, x) + x \partial_x \Phi(v, x))$$

$$\Psi'/\mathcal{H}^2 + \Phi/\mathcal{H} = -2v$$

$$x \equiv k\tau \text{ , } u \equiv q/k \text{ , } v \equiv |\vec{k} - \vec{q}|/k$$

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$$x \equiv k\tau, \quad u \equiv q/k, \quad v \equiv |\vec{k} - \vec{q}|/k$$

- DR density iso (DRDI) ($R_{\text{dr}} \ll 1$) **Our new results** Cai, PD, Zhong, *JCAP*, 2026

Freestreaming DR

$$f(x, u, v) = \frac{1}{2} \Phi(u) \Phi(v) + \frac{1}{2} \Psi(u) \Psi(v)$$

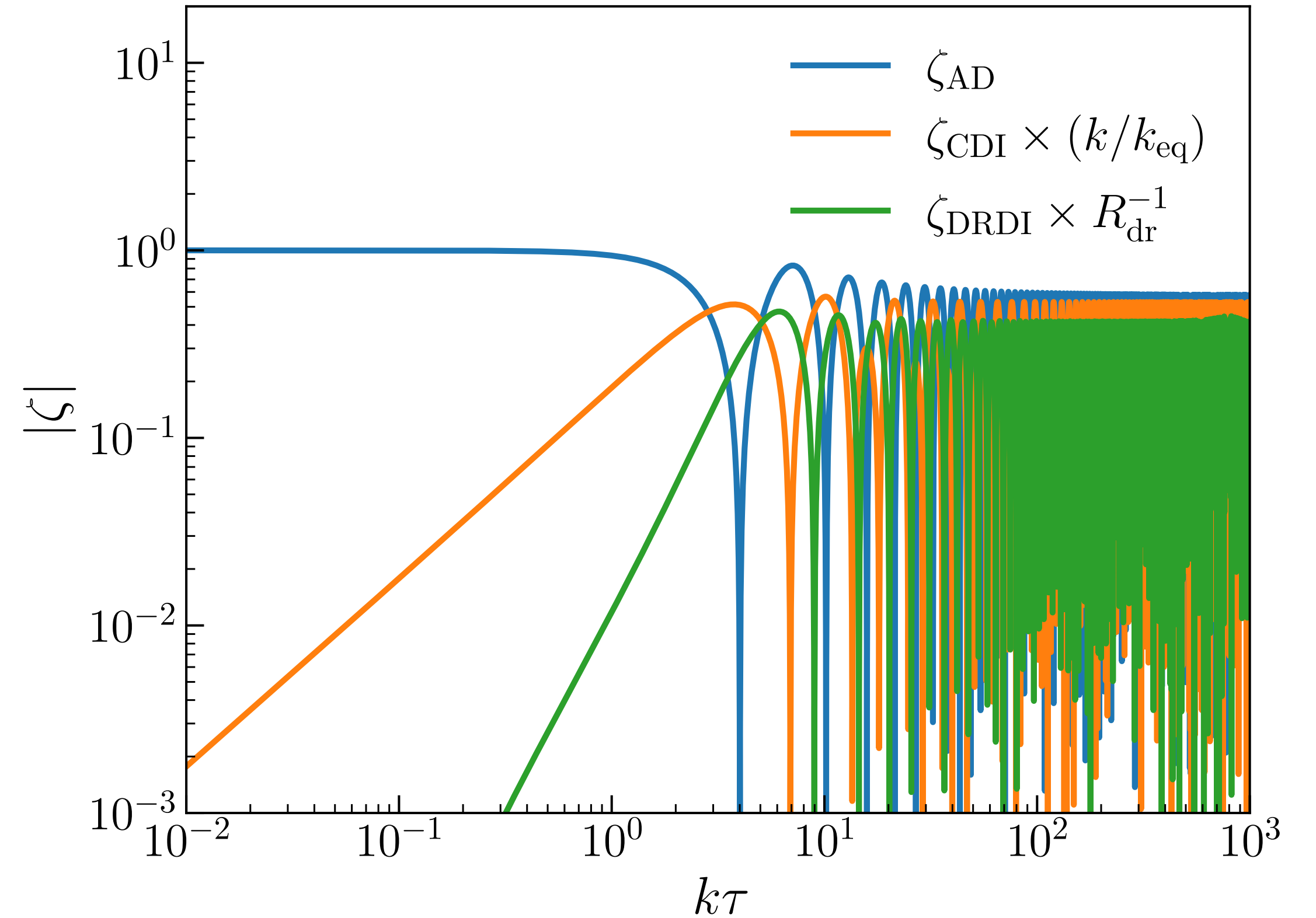
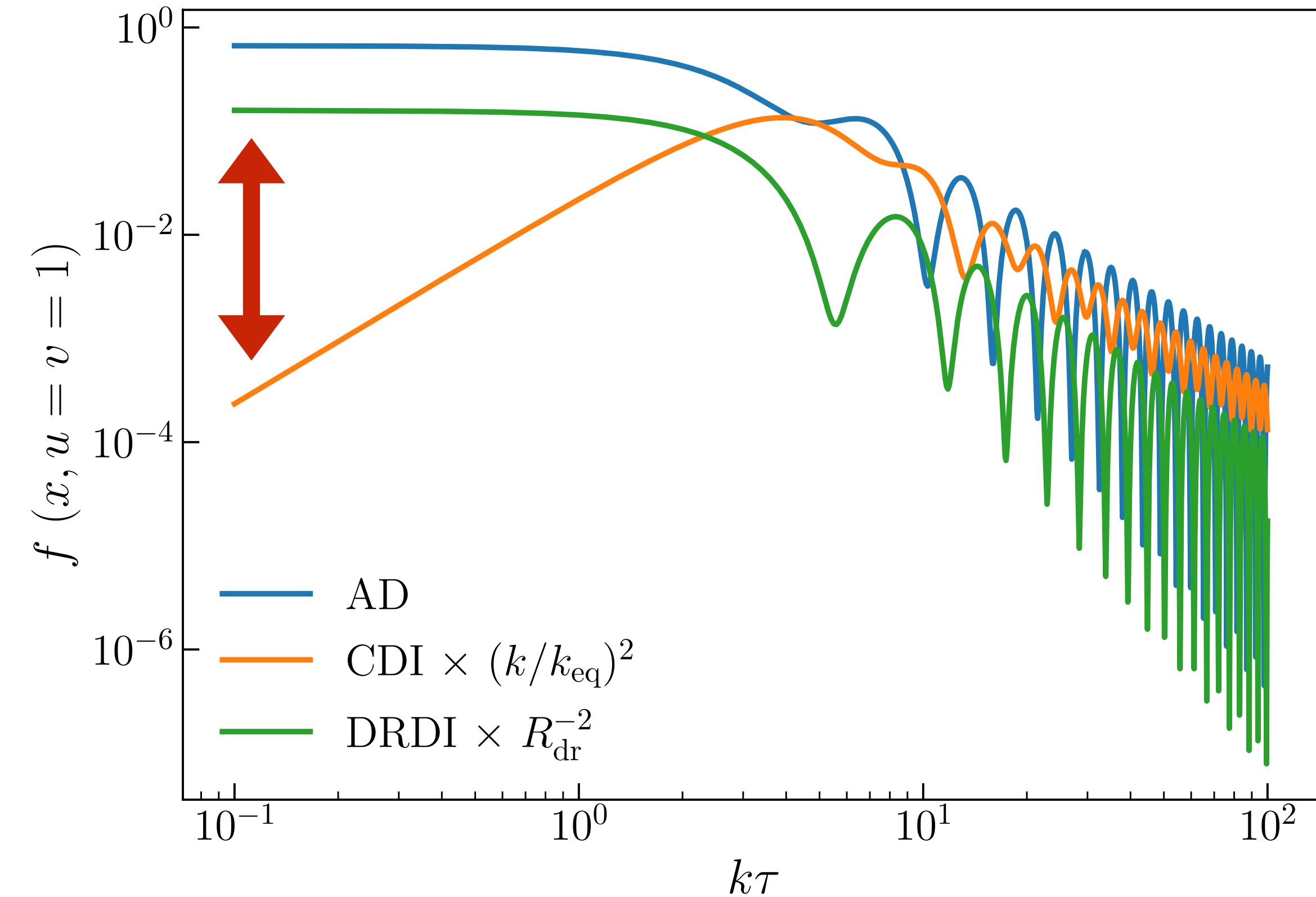
$$+ 2 \left(\frac{1}{2} (\Phi(u) + x \partial_x \Psi(u)) + R_{\text{dr}} \mathcal{H} v_{\text{dr}}(u) \right) \left(\frac{1}{2} (\Phi(v) + x \partial_x \Psi(v)) + R_{\text{dr}} \mathcal{H} v_{\text{dr}}(v) \right)$$

$$- \frac{16}{5x} R_{\text{dr}} \mathcal{H} \int_0^x d\tilde{x} v_{\text{dr}}(u) \Phi(v) \left[j_0(x - \tilde{x}) + \frac{10}{7} j_2(x - \tilde{x}) + \frac{3}{7} j_4(x - \tilde{x}) \right]$$

$$T_{\mu\nu}^{(2)} \approx \frac{4}{3} \bar{\rho}_r \partial_\mu v_{\gamma_*}^{(1)} \partial_\nu v_{\gamma_*}^{(1)} + \bar{\rho}_{\text{dr}} \Pi_{\text{dr} \mu\nu}^{(2)}$$

$$v_{\gamma_*} = -\frac{1}{2} (\Psi' / \mathcal{H}^2 + \Phi / \mathcal{H}) - R_{\text{dr}} v_{\text{dr}}$$

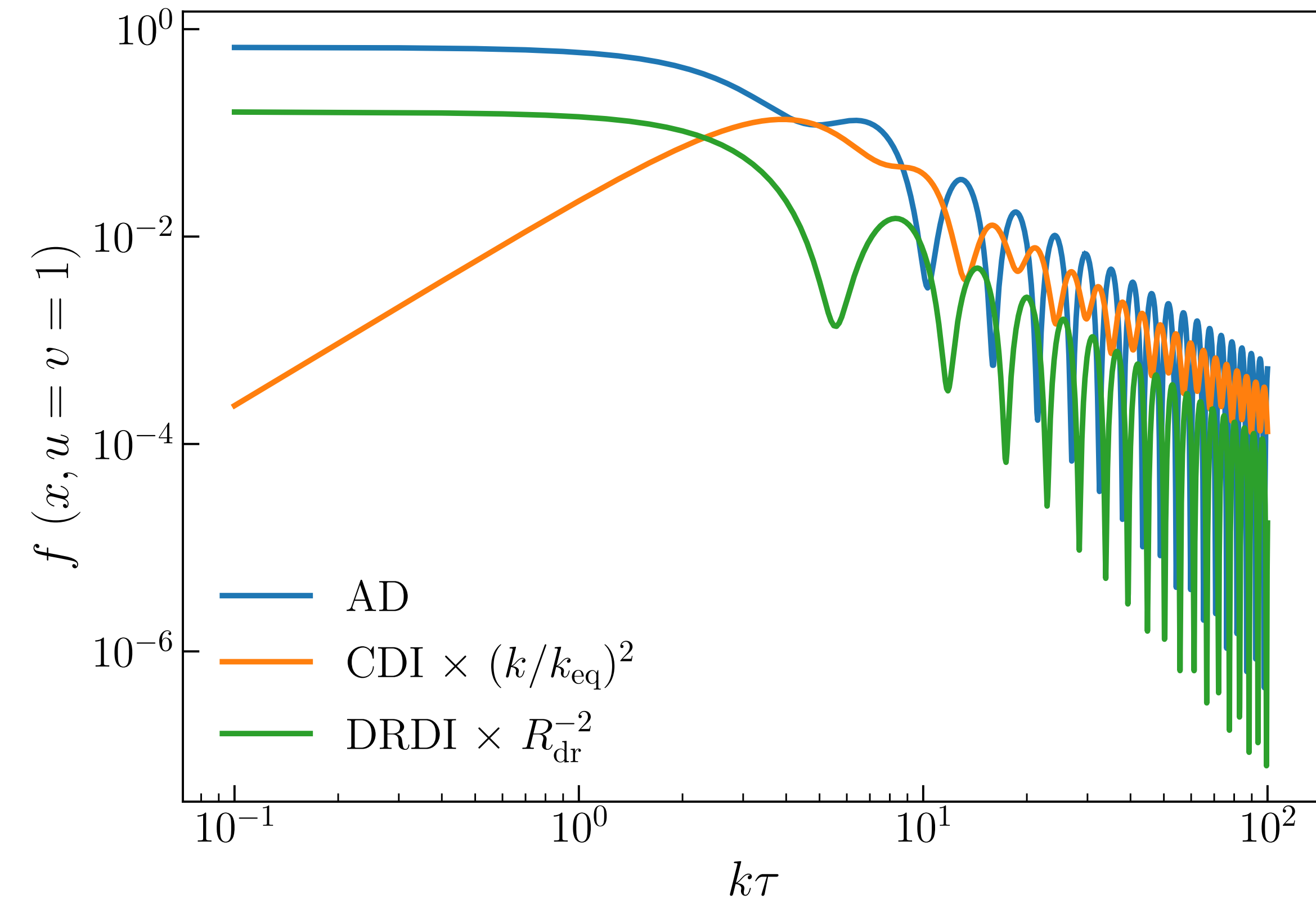
Isocurvature Induced GWs



- DRDI superhorizon source term is **not suppressed!** GW can be induced **without** curvature

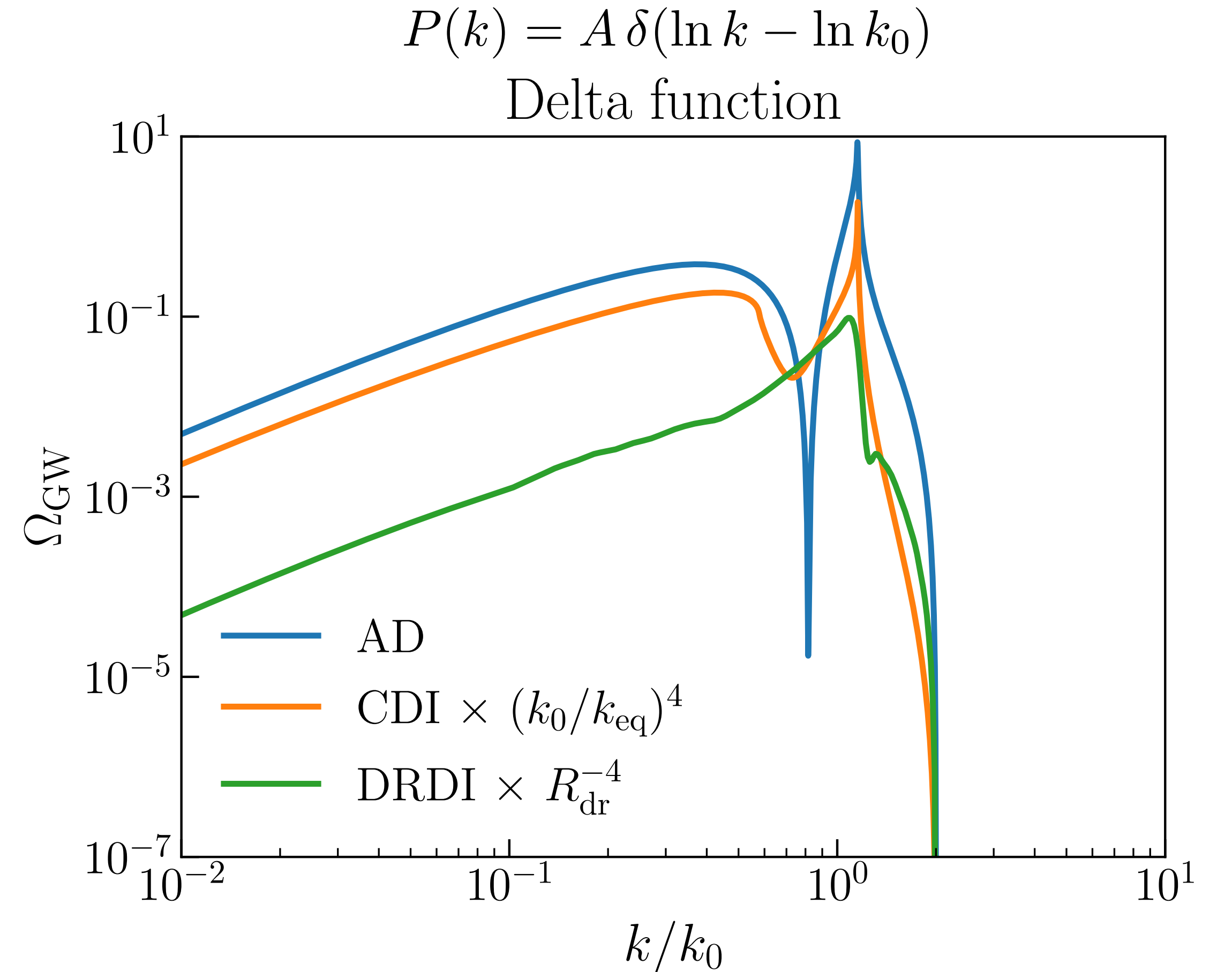
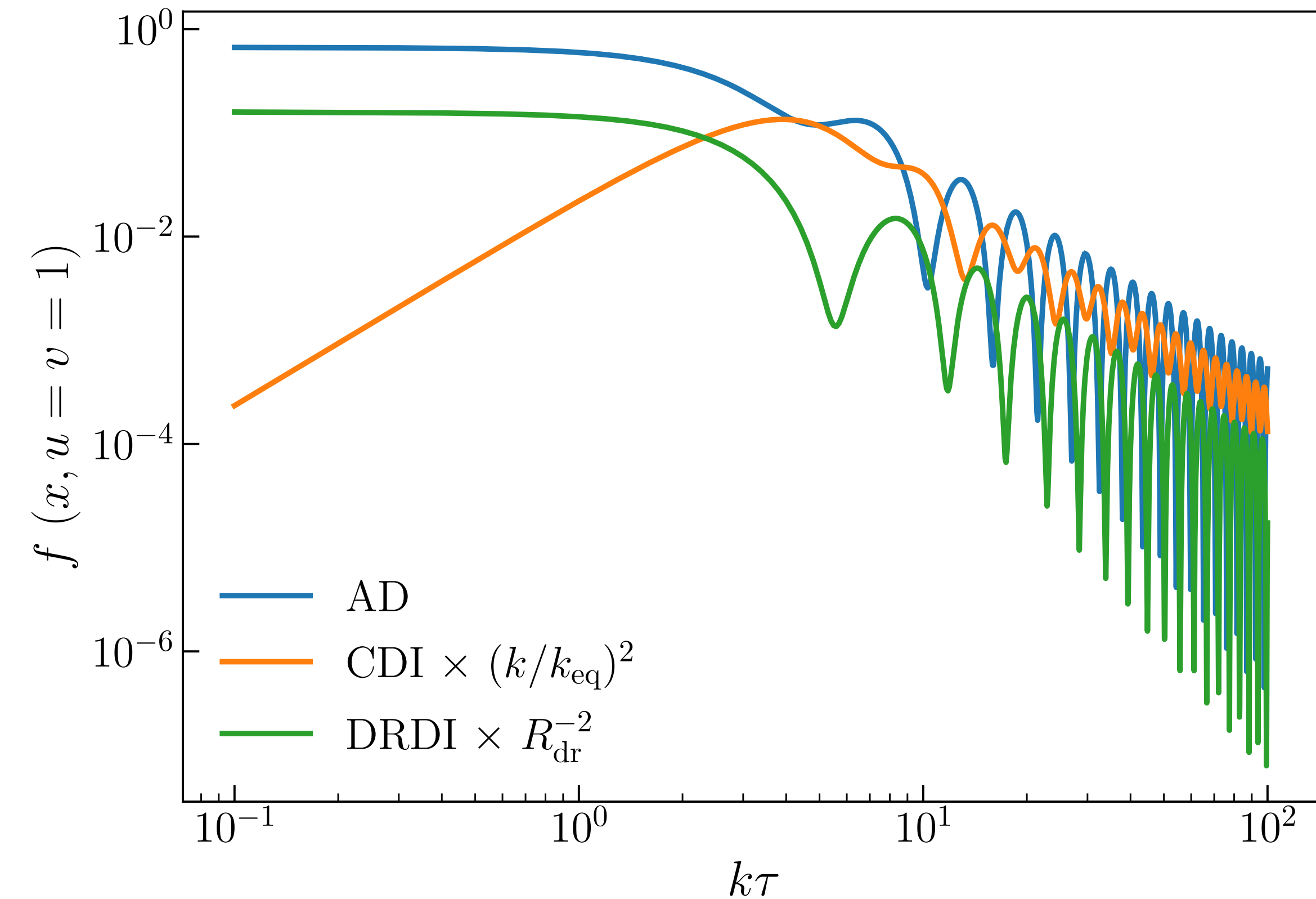
Cai, PD, Zhong, *JCAP*, 2026

Isocurvature Induced GWs



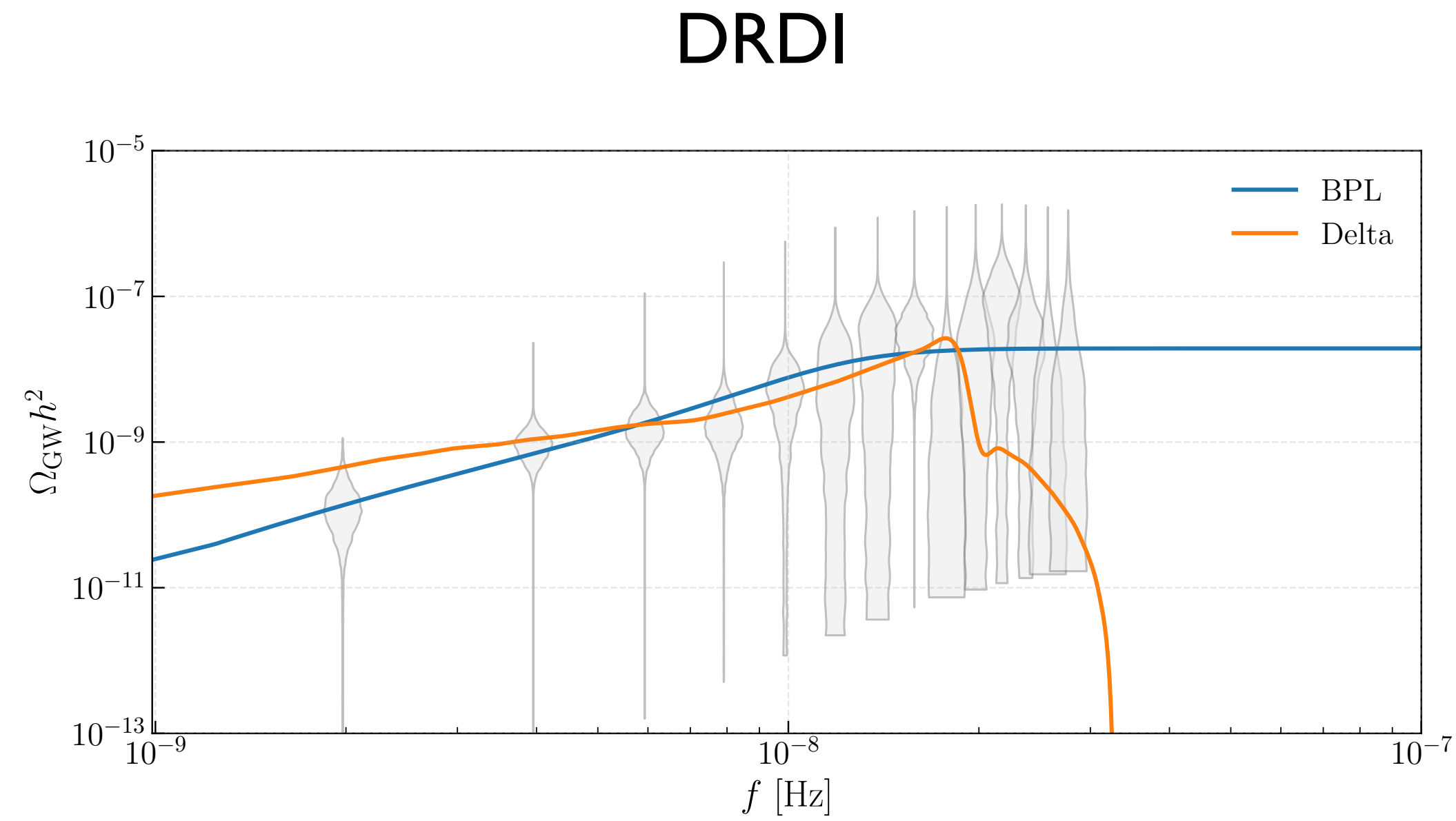
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- DRDI GWs can be **much larger** than CDI for large k

Isocurvature Induced GWs



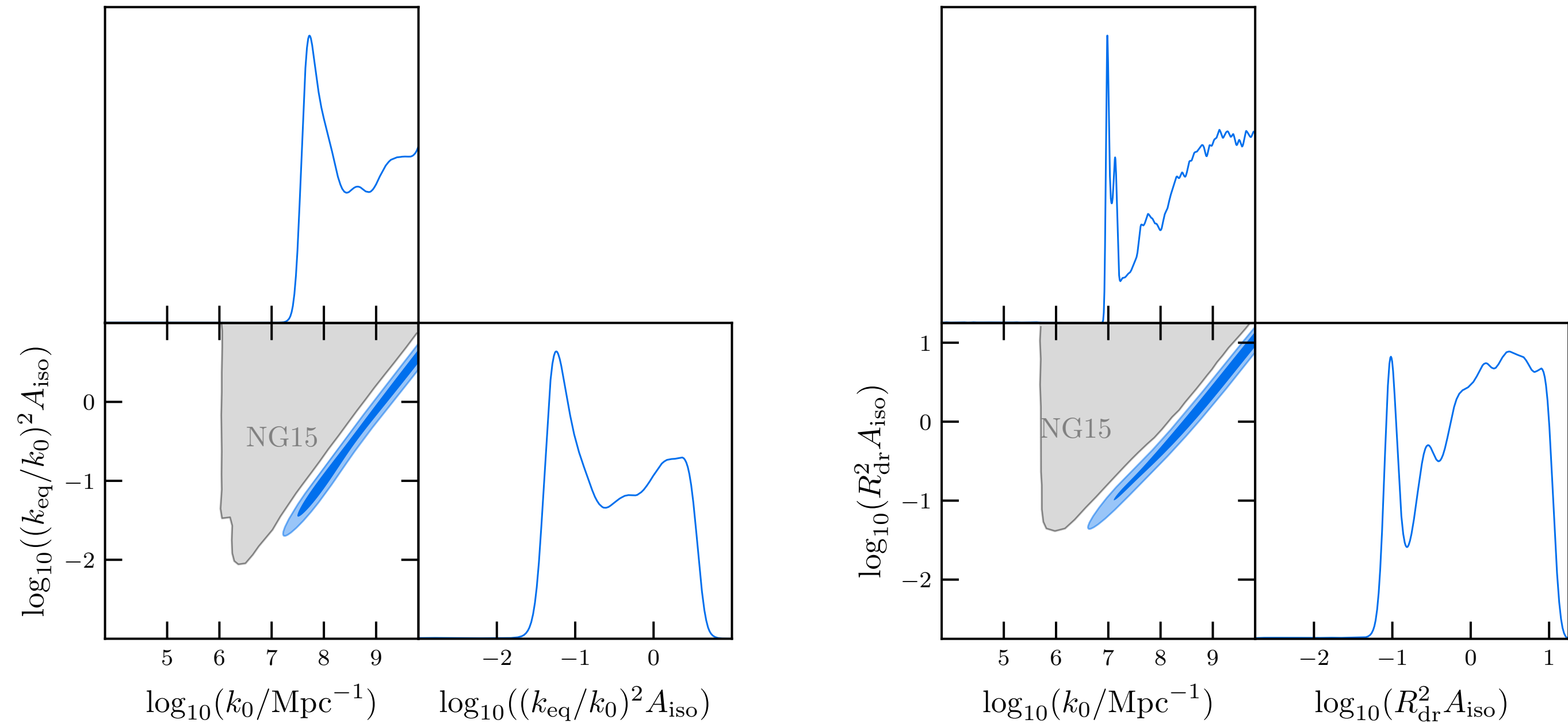
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Constraints from PTA



Data from NANOGrav 15 yr

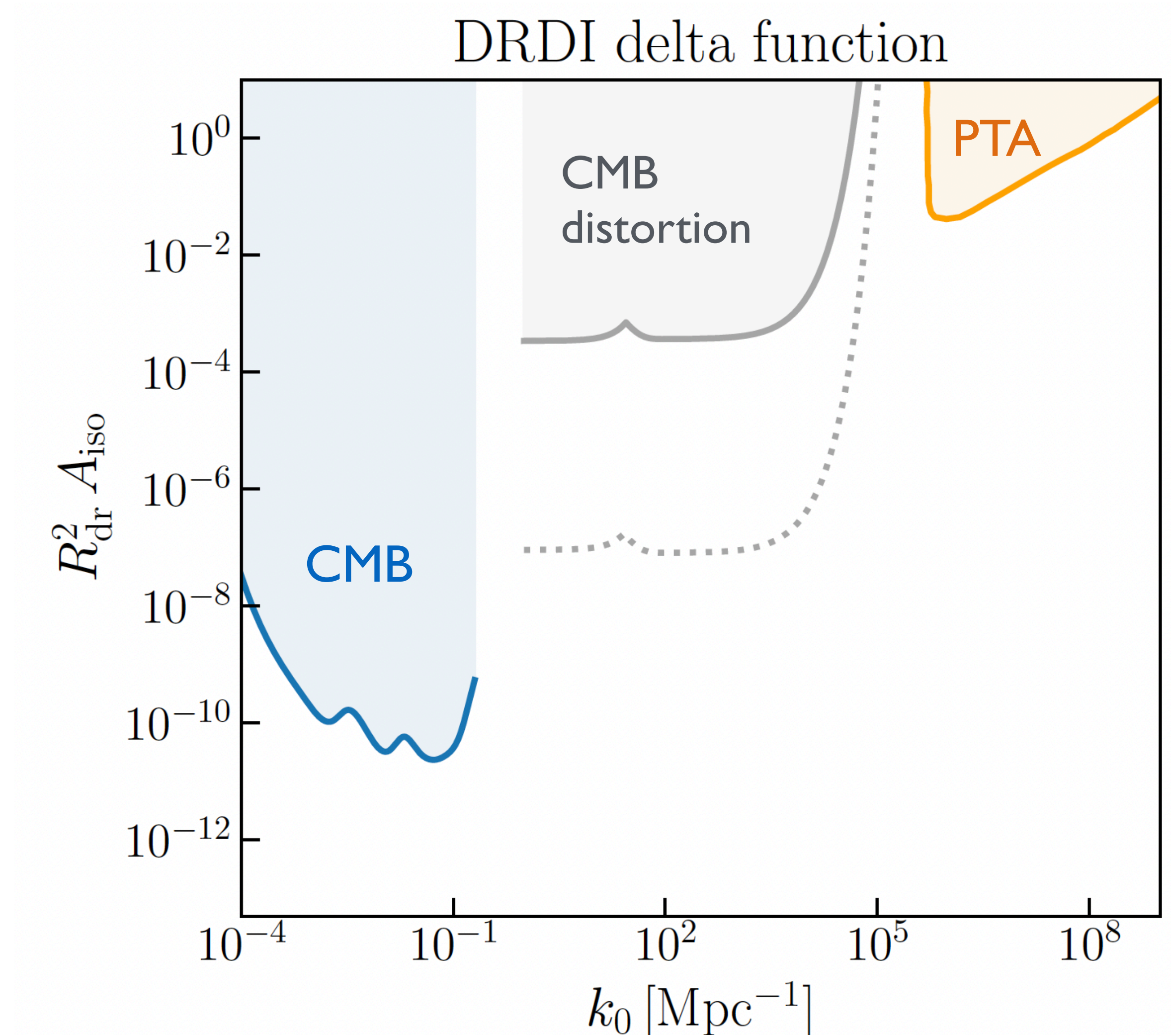
- Stringent constraints around 10^6 Mpc^{-1}



Data analysis with PTArcade

Cai, PD, Zhong, *JCAP*, 2026

Future Directions



- Primordial black holes
- GW probed at LISA/Taiji/Tianqin
- Mixture of different isocurvatures

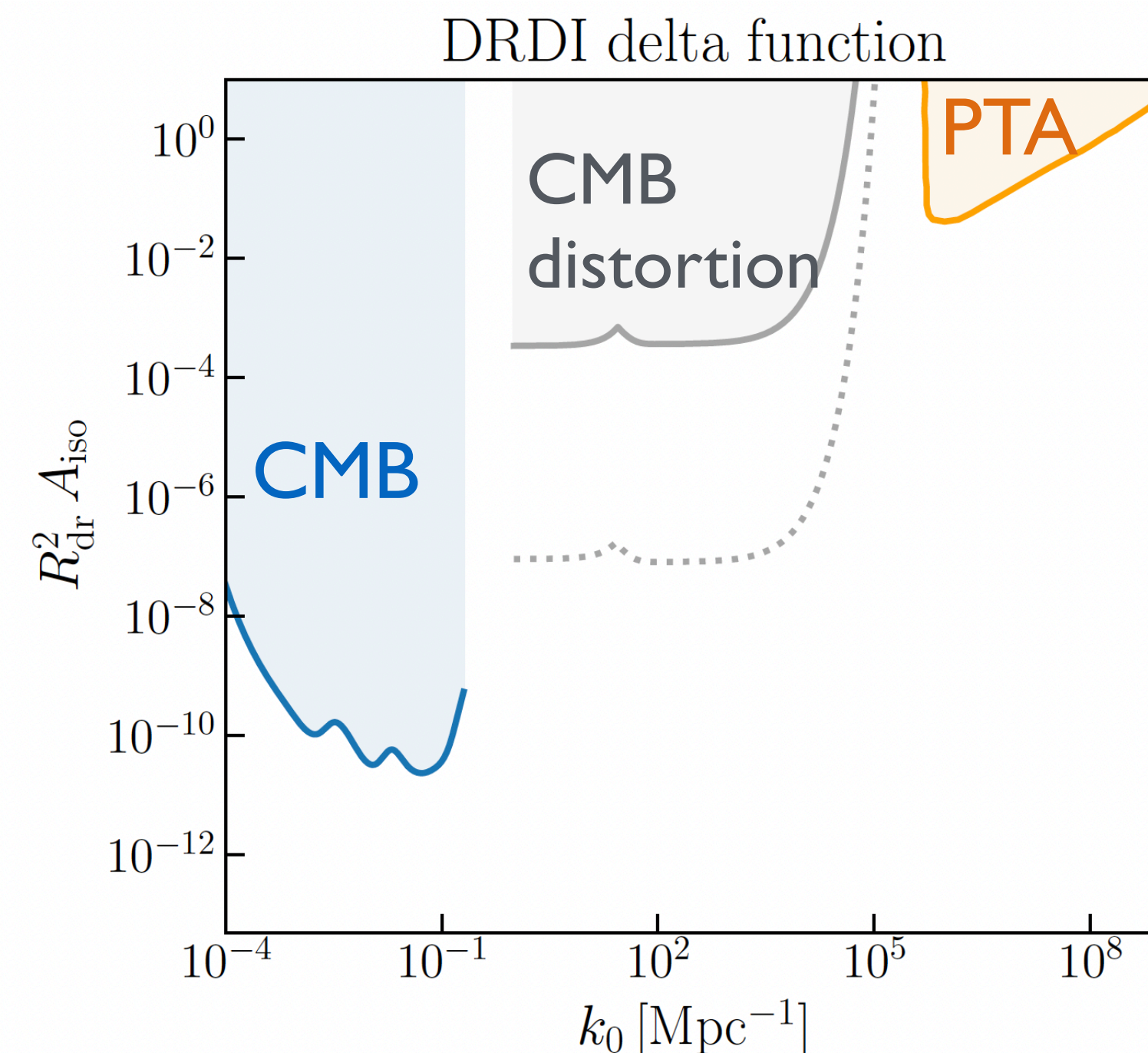
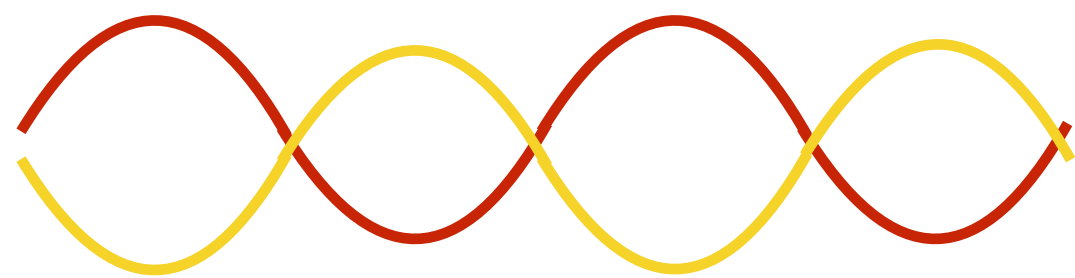
Conclusions

- Many well-motivated BSM models can generate isocurvature perturbations
- Isocurvature induced GWs have **distinct spectrum** compared to curvature induced GWs
- **Pulsar Timing Arrays** set limits on isocurvature power spectrum around **10^6 Mpc^{-1}**

DR Iso initial conditions

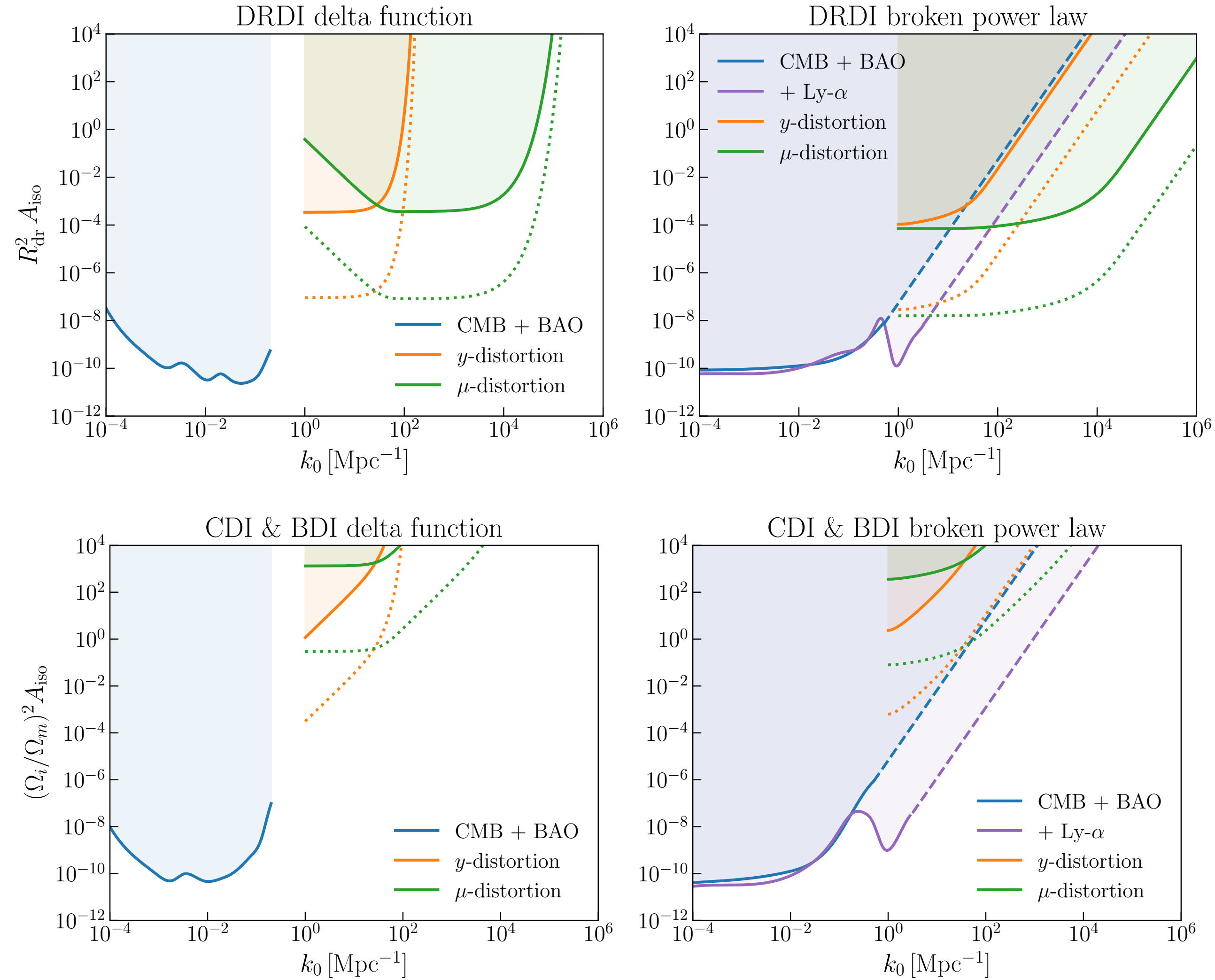
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Thank you!

General isocurvature constraints



Buckley,PD,Fernandez,Weikert, 2025