Defocusing dark energy: Raychaudhuri diagnostics beyond $w < -1/3$ and the phantom divide

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GGI workshop: Exploring New Frontiers in Cosmology
Galileo Galilei Institute, Florence — 21 July 2026

[arXiv:2607.18008] — *on arXiv today*

Why sign-aware diagnostics, and why now

- DESI-era BAO+CMB+SN combinations revive evolving dark energy, usually phrased through $w_{\rm de}(z)$ (CPL); inferred dynamics is sensitive to dataset choices and to the functional prior — and standard EoS parametrizations are sign-preserving by construction: density-level structure gets smeared into smoother $w_{\rm de}(z)$.
- Independent threads point at sign-changing effective DE densities:
  - negative-Λ / $\rho_{\rm de} < 0$ hints at high z; $\Lambda_{\rm s}$CDM-type transitions easing H_0 (& M_B), S_8, γ, with $\sum m_\nu$ back in the physical region;
  - model-agnostic reconstructions: inferred density zero around $z \sim 1.5$–2.5, with localized intermediate-redshift structure.

The problem

These histories enter exactly the regime where $w_{\rm de} = p_{\rm de}/\rho_{\rm de}$ ceases to be a regular variable: the inequalities reverse for $\rho_{\rm de} < 0$ and the ratio is undefined at $\rho_{\rm de} = 0$.
Which variables should organize the description?

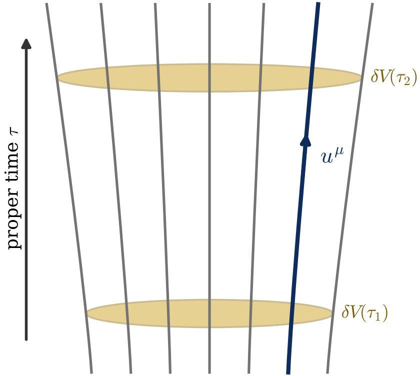
[$\Lambda_{\rm s}$CDM: Akarsu+ '20–'25 · reconstructions: Akarsu+ '26; Gupta Choudhury+ '26 · CPL with sign switch: Gökçen+ '26]

The object under study: a timelike congruence

- A congruence: a family of timelike worldlines filling spacetime — exactly one through each event — with unit tangent u^μ. For cosmology: the comoving observers.
- Its expansion scalar is the fractional growth rate of a comoving volume element:

$$\Theta \equiv \nabla_\mu u^\mu = \frac{1}{\delta V}\frac{\mathrm{d}\,\delta V}{\mathrm{d}\tau}, \qquad \Theta_{\mathrm{FLRW}} = 3H.$$

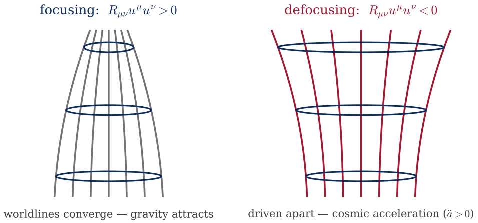
- Focusing / defocusing = whether neighboring worldlines are driven together or apart — a statement about *geometry*, before any matter content is specified.



The Raychaudhuri equation, term by term

$$\frac{\mathrm{d}\Theta}{\mathrm{d}\tau} = \underbrace{-\tfrac{1}{3}\Theta^2}_{\substack{\text{expansion}\\ \text{self-interaction}}} \quad \underbrace{-\,\sigma_{\mu\nu}\sigma^{\mu\nu}}_{\substack{\text{shear:}\\ \text{always focuses}}} \quad \underbrace{+\,\omega_{\mu\nu}\omega^{\mu\nu}}_{\substack{\text{rotation:}\\ \text{resists focusing}}} \quad \underbrace{-\,R_{\mu\nu}u^\mu u^\nu}_{\substack{\text{gravity: matter}\\ \text{enters only here}}} \quad \underbrace{+\,\nabla_\mu A^\mu}_{\substack{\text{non-geodesic}\\ \text{(forces)}}}$$

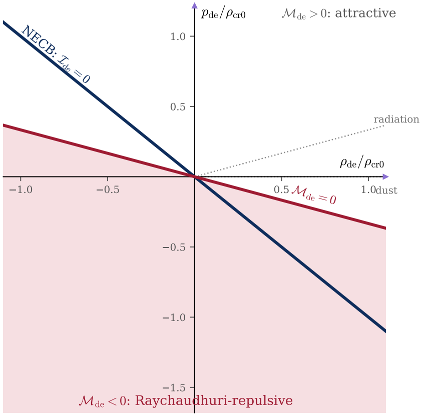
- A purely geometric identity: no EoS parameter, no stress–energy tensor appears.
- FLRW comoving congruence: $\sigma = \omega = A^\mu = 0$, $\Theta = 3H \;\Rightarrow\; \ddot{a}/a = -\frac{1}{3} R_{\mu\nu}u^\mu u^\nu$.
- Acceleration = defocusing, $R_{\mu\nu}u^\mu u^\nu < 0$ — it does *not* require $\dot{\Theta} > 0$: in flat FLRW $\dot{\Theta} = 3\dot{H} = -12\pi G\,\mathcal{I}_{\rm tot}$, so $\dot{\Theta} < 0$ whenever the total NEC holds; acceleration needs only $\mathcal{M}_{\rm tot} < 0$. GR maps it to matter: $R_{\mu\nu}u^\mu u^\nu = 4\pi G \sum_i \mathcal{M}_i$, $R_{\mu\nu}k^\mu k^\nu = 8\pi G\,\mathcal{E}^2 \sum_i \mathcal{I}_i$, and $\rho_i' = \frac{3}{1+z}\mathcal{I}_i$. The equations select $\mathcal{M} = \rho + 3p$, $\mathcal{I} = \rho + p$; $w = p/\rho$ is a derived, branch-dependent ratio.



Convergence conditions — and why $(\mathcal{I}, \mathcal{M})$ deserve their names

- Convergence conditions: gravity focuses timelike congruences iff $R_{\mu\nu} u^\mu u^\nu \geq 0$ (timelike CC), null ones iff $R_{\mu\nu} k^\mu k^\nu \geq 0$ (null CC). The field equations translate: TCC $\iff \mathcal{M}_{\rm tot} \geq 0$; NCC $\iff \mathcal{I}_{\rm tot} \geq 0$ (the total NEC).
- Active gravitational mass density $\mathcal{M}_i = \rho_i + 3p_i$: the Newtonian limit of Raychaudhuri is $\nabla^2\Phi = 4\pi G \sum_i(\rho_i + 3p_i)$ — pressure gravitates, and $\mathcal{M}$ is precisely *what attracts*.
- Inertial mass density $\mathcal{I}_i = \rho_i + p_i$: the relativistic Euler equation, $(\rho + p)\, a^\mu = -\nabla_\perp^\mu p$, makes $\rho + p$ the inertia a pressure gradient must push; and continuity transports exactly it: $\dot{\rho}_i = -\Theta\, \mathcal{I}_i$, i.e. $\rho_i' = \frac{3}{1+z}\, \mathcal{I}_i$.

One picture: the $(\rho_{\rm de}, p_{\rm de})$ plane



- The field equations draw half-planes: repulsive $\mathcal{M}_{\rm de} < 0$; phantom-like $\mathcal{I}_{\rm de} < 0$. Regular everywhere — including $\rho_{\rm de} = 0$. Λ lives *on* the NECB: $T^{(\Lambda)}_{\mu\nu} = -\rho_\Lambda g_{\mu\nu} \Rightarrow \mathcal{I}_\Lambda \equiv 0$; $w_\Lambda = -1$ is only its ratio form.

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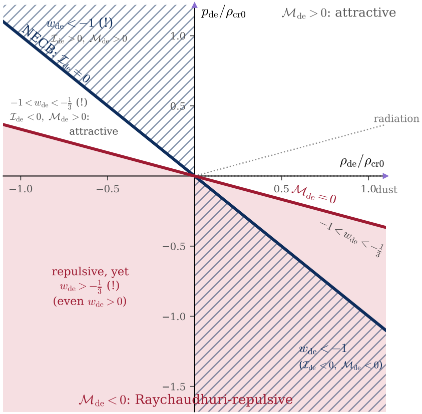

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- Ratio language draws bowties: $\{w_{\rm de} < -1\}$ is a union of two *opposite* sectors; on $\rho_{\rm de} < 0$ even $w_{\rm de} > 0$ can be repulsive, and $\mathcal{M}_{\rm de} < 0 \Rightarrow \mathcal{I}_{\rm de} < 0$ there.

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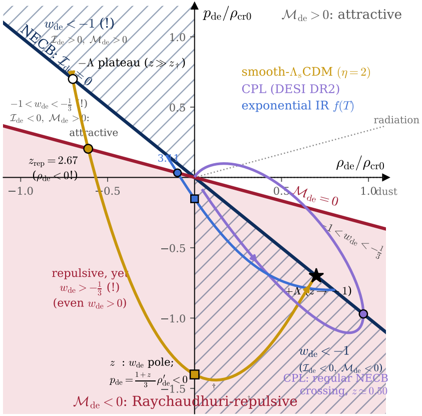

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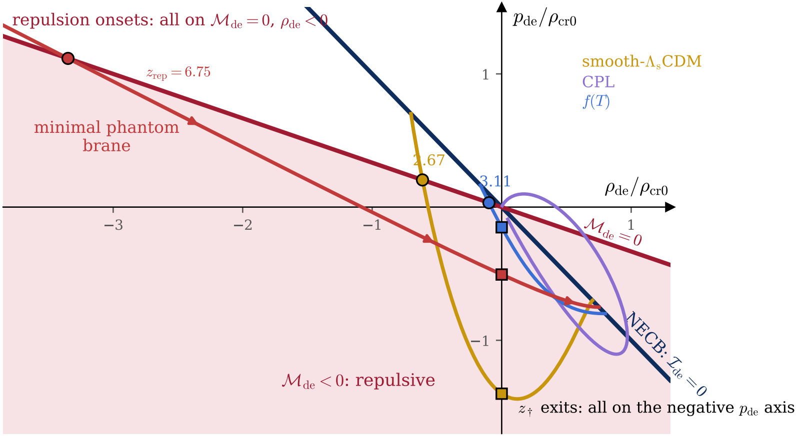
- Trajectories: smooth-$\Lambda_{\rm s}$CDM enters on the NECB at the $-\Lambda$ plateau, crosses $\mathcal{M}_{\rm de} = 0$ at $z_{\rm rep}$ (*while* $\rho_{\rm de} < 0$), exits through the $p_{\rm de}$ axis *below* the origin at $z_\dagger$, lands on $+\Lambda$. CPL instead crosses the NECB regularly.

One picture: the $(\rho_{\rm de}, p_{\rm de})$ plane



Zoomed out: the brane traces the same route at a very different scale. All three $z_\dagger$ exits pierce the negative $p_{\rm de}$ axis; all three repulsion onsets sit on $\mathcal{M}_{\rm de} = 0$ with $\rho_{\rm de} < 0$.

The branch dictionary

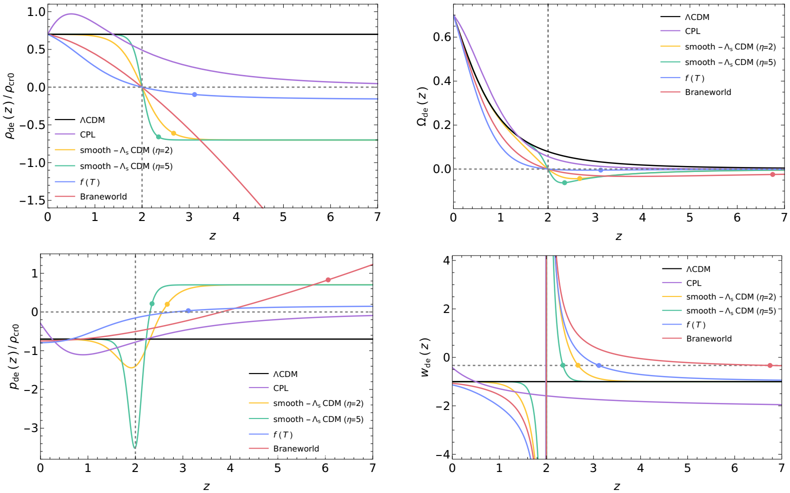
$$\mathcal{M}_{\rm de} < 0 \iff \begin{cases} w_{\rm de} < -\frac{1}{3}, & \rho_{\rm de} > 0, \\ w_{\rm de} > -\frac{1}{3}, & \rho_{\rm de} < 0, \end{cases} \qquad \mathcal{I}_{\rm de} = p_{\rm de}, \ \ \mathcal{M}_{\rm de} = 3p_{\rm de} \ \text{ at } \rho_{\rm de} = 0.$$

	$\mathcal{I}_{\rm de} > 0$	$\mathcal{I}_{\rm de} < 0$
$\rho_{\rm de} > 0$	$w_{\rm de} > -1$	$w_{\rm de} < -1$
$\rho_{\rm de} < 0$	$w_{\rm de} < -1$	$w_{\rm de} > -1$

The line $w_{\rm de} = -1$ represents $\mathcal{I}_{\rm de} = 0$ only on a nonzero-density branch, with the conventional side assignment valid only for $\rho_{\rm de} > 0$.

Two frequent questions: (i) the Friedmann constraint bounds only the *total* source, $\rho_{\rm tot} \geq 3K/(8\pi Ga^2)$ — no positivity condition follows for any individual sector; (ii) the opposite local assignment near a crossing ($\mathcal{I}_{\rm de} > 0$, i.e. $w_{\rm de} < -1$ on the $\rho_{\rm de} < 0$ side) is impossible for *any* continuous zero — realizing it requires a sign flip *without* a zero (jump- or pole-like, hence singular).

The sector histories



$\rho_{\rm de}$ switches sign; $\Omega_{\rm de}$ flips with it; $p_{\rm de}$ stays finite; only the ratio $w_{\rm de}$ diverges at $z_\dagger$ (dashed).

Exact local structure at a density zero

Separately conserved sector; smooth negative $\to$ positive crossing of finite odd order n at $z_\dagger$.

- $\mathcal{I}_{\rm de}, \mathcal{M}_{\rm de} < 0$ in a punctured neighborhood of $z_\dagger$; at the zero, ≤ 0 (strict for a simple zero), with $p_{\rm de}(z_\dagger) = \frac{1+z_\dagger}{3}\,\rho'_{\rm de}(z_\dagger)$ — the slope at the zero fixes the pressure there: finite and negative.
- $w_{\rm de}$ develops a kinematic and universal pole at $z_\dagger$, fixed by $(n, z_\dagger)$ alone — no amplitude, no model details.
- $w_{\rm de} = -1$ is *never attained* near $z_\dagger$: the apparent quintessence $\to$ phantom transition never occurs; the two sides of the divide connect only through the pole.

[zero $\leftrightarrow$ pole for conserved sectors: Özülker '22]

Sector repulsion precedes the switch: $z_{\rm rep} > z_\dagger$

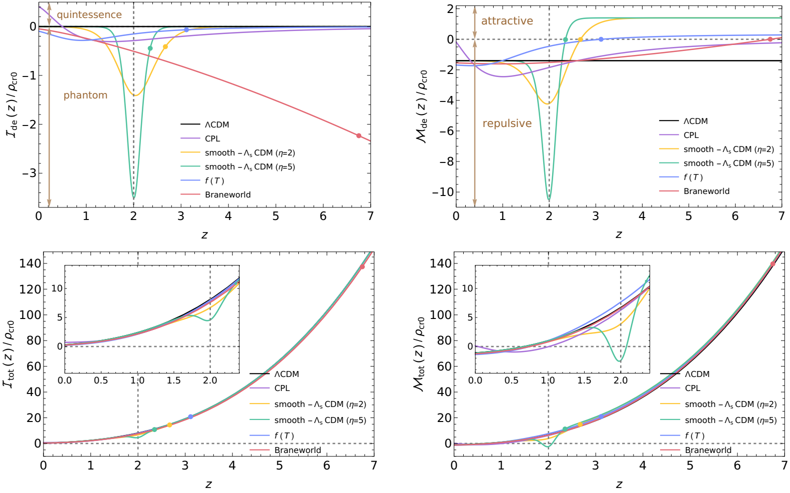
Theorem (continuity)

If $\mathcal{M}_{\rm de} > 0$ at some $z_{\rm hi} > z_\dagger$, there is *at least one* $z_{\rm rep} \in (z_\dagger, z_{\rm hi})$ with $\mathcal{M}_{\rm de}(z_{\rm rep}) = 0$: a finite interval where $\rho_{\rm de} < 0$ *and* $\mathcal{M}_{\rm de} < 0$.

$w_{\rm de} < -1/3$ misdiagnoses this interval as non-repulsive.

Smooth-$\Lambda_{\rm s}$CDM profile $\rho_{\Lambda_{\rm s}} \propto \tanh[\eta(z_\dagger - z)]$: the onset is *unique*; for $z_\dagger = 2$: $z_{\rm rep} = 2.67$ (η=2), 2.35 (η=5) — and 3.11 for $f(T)$, 6.75 for the brane: same pattern, widely different separations.

The Raychaudhuri diagnostics



Top: sign-switchers keep $\mathcal{I}_{\rm de} < 0$ (CPL crosses the NECB regularly); circles mark $z_{\rm rep}$. Bottom: $\mathcal{I}_{\rm tot} > 0$ throughout — no total NEC violation; $q < 0 \iff \mathcal{M}_{\rm tot} < 0$, and only the $\eta = 5$ dip re-enters.

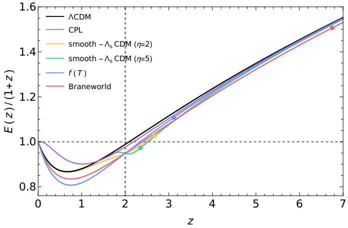
When does the Universe accelerate *at* the crossing?

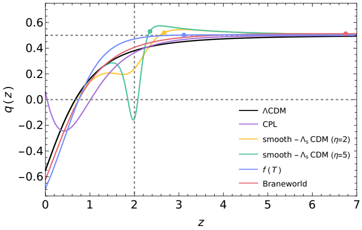
At $\rho_{\rm de}(z_\dagger) = 0$ (flat, matter era):
$q(z_\dagger) < 0 \iff \rho'_{\rm de}(z_\dagger) < -\rho_{\rm m0}(1+z_\dagger)^2$, and
the total NEC bounds the slope from below:

$$\boxed{-3\rho_{\rm m0}(1+z_\dagger)^2 \leq \rho'_{\rm de}(z_\dagger) < -\rho_{\rm m0}(1+z_\dagger)^2}$$

acceleration at the crossing without total NEC violation.

$z_\dagger = 2$, $\Omega_{\rm m0} = 0.3$: the dip must pass $\mathcal{M}_{\rm de}(z_\dagger) < -8.1\,\rho_{\rm cr0}$ — η=2 gives -4.2 (fails), η=5 gives -10.5 (succeeds); window $3.9 \lesssim \eta \lesssim 11.6$.





The kinematics: transient $q < 0$ episode around $z_\dagger$ only for the sharper transition.

Which branch powers which epoch — and how many $q = 0$ crossings?

- $\mathcal{I}_{\rm de}$ drives the history: $\rho'_{\rm de} = \frac{3}{1+z}\mathcal{I}_{\rm de}$ — a negative → positive crossing *requires* $\mathcal{I}_{\rm de} < 0$ there; sign-switchers keep $\mathcal{I}_{\rm de} < 0$ throughout (phantom-like).
- $\mathcal{M}_{\rm de}$ drives repulsion, competing with matter in $\mathcal{M}_{\rm tot} = \rho_{\rm m} + \mathcal{M}_{\rm de}$: on $\rho_{\rm de} < 0$ the localized dip must beat $\rho_{\rm m} \propto (1+z)^3 \Rightarrow$ at most a transient intermediate window; on $\rho_{\rm de} > 0$, $\mathcal{M}_{\rm de} < 0$ persists while matter dilutes $\Rightarrow$ the late-time epoch follows automatically.
- Counting (single impulse in $\mathcal{M}_{\rm de}$; $\mathcal{M}_{\rm tot}$ C^1, at most two stationary points): Rolle $\Rightarrow$ exactly one or three sign-changing zeros of q; η=5 realizes $z_{\rm rep} > z_{\rm acc}^{\rm int,b} > z_\dagger > z_{\rm acc}^{\rm int,e} > z_{\rm acc}^{\rm late}$: $2.35 > 2.08 > 2 > 1.85 > 0.67$.

Density sign change $\neq$ sector repulsion $\neq$ cosmic acceleration $\neq$ total NEC violation.

Scalar fields: the kinetic sign — not the ratio — decides "phantom"

$\mathcal{L}_\phi = \epsilon X - V(\phi)$, $X = \dot\phi^2/2 \geq 0$: $\quad \mathcal{I}_\phi = \epsilon\,\dot\phi^2$, $\quad \mathcal{M}_\phi = 2\big(\epsilon\,\dot\phi^2 - V\big)$.

- EoS labels mislead: canonicality fixes $\operatorname{sgn}\mathcal{I}_\phi = \epsilon$, *not* $\operatorname{sgn}\rho_\phi$. A canonical field ($\epsilon = +1$) with V sufficiently negative has $\rho_\phi < 0$, $p_\phi > 0$: $w_\phi < -1$ with perfectly healthy kinetics.
- Negative energy density is not forbidden: no known physical principle prevents a canonical scalar from reaching $\rho_\phi < 0$.
- The crossing is what costs: $\dot\rho_\phi = -\Theta\,\mathcal{I}_\phi$, so climbing from $\rho_\phi < 0$ to $\rho_\phi > 0$ in an expanding universe requires $\mathcal{I}_\phi < 0$, i.e. $\epsilon = -1$: a phantom field climbing its potential. The minimal realization is a genuine ghost — kinematic $\neq$ microphysical.
- Fixed ϵ forbids an NECB crossing (touched only at $\dot\phi = 0$) — the single-field no-go. Stable embedding used in practice: $\Lambda_{\rm s}$VCDM (type-II minimally modified gravity; no extra propagating scalar mode).

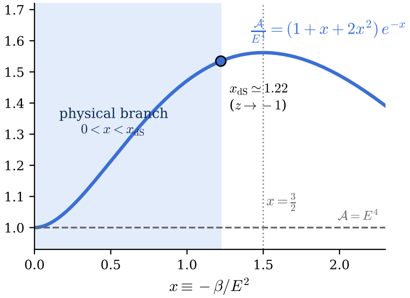
Modified gravity does it too, I: exponential infrared $f(T)$

$S = \frac{1}{2\kappa^2}\int d^4x\,|e|\,f(T) + S_{\rm m}$, $f(T) = T\,e^{\beta T_0/T} + 2\Lambda$ $(T = 6H^2)$, negative-β branch; GR-like split gives

$$\rho_{\rm tp}(E) = \rho_{\rm cr0}\big[E^2 - (E^2 - 2\beta)e^{\beta/E^2} + \Omega_{\Lambda 0}\big], \qquad {\rm sgn}\,\rho'_{\rm tp} = {\rm sgn}\big[E^4 - \mathcal{A}(E)\big].$$

- Whole-branch result: with $x \equiv -\beta/E^2$, $\mathcal{A}/E^4 = (1 + x + 2x^2)e^{-x}$ rises monotonically from 1 on $0 < x < x_{\rm dS} \simeq 1.22 < \frac{3}{2} \Rightarrow E^4 < \mathcal{A} \Rightarrow \mathcal{I}_{\rm tp} < 0$ for all $z > -1$.
- $z_\dagger = 2$, $\Omega_{\rm m0} = 0.3 \Rightarrow \beta \simeq -0.977$, $\Omega_{\Lambda 0} \simeq 0.81$; high-$z$: $\rho_{\rm tp} \to \rho_{\rm cr0}(\beta + \Omega_{\Lambda 0}) < 0$, attractive $(\mathcal{M}_{\rm tp} > 0) \Rightarrow z_{\rm rep} = 3.11$; *no* intermediate window.
- Caveat: a recent perturbation-level analysis disfavors the *minimal* $\Lambda = 0$ branch; the Λ-extended branch here is not automatically subject — still, kinematic regularity $\neq$ viability.

[Awad+ '18; Hashim+ '21; Akarsu+ '25 · caveat: Hashim+ '26]



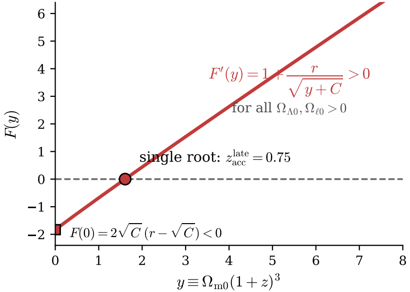
The whole-branch proof in one curve: $\mathcal{A}/E^4$ rises from unity and the physical branch never leaves $(0, \frac{3}{2})$.

Modified gravity does it too, II: the minimal phantom brane

Normal branch of the induced-gravity (DGP) braneworld, flat bulk
$[S = M^3\int_{\rm bulk}\sqrt{-g}\,R + m^2\int_{\rm brane}\sqrt{-h}\,R - 2\sigma\int_{\rm brane}\sqrt{-h} + S_{\rm m};\quad \ell = 2m^2/M^3,\ \Omega_{\ell 0} = 1/\ell^2 H_0^2]$:

$$E(z) = \sqrt{\Sigma(z)} - \sqrt{\Omega_{\ell 0}},\quad \Sigma = \Omega_{\rm m0}(1+z)^3 + \Omega_{\Lambda 0} + \Omega_{\ell 0},\quad E(0) = 1 \Rightarrow \Omega_{\Lambda 0} = 1 - \Omega_{\rm m0} + 2\sqrt{\Omega_{\ell 0}}.$$

- The sign switch is automatic — for every parameter choice: $\rho_{\rm bw}(0) = (1-\Omega_{\rm m0})\rho_{\rm cr0} > 0$ while $\rho_{\rm bw} \simeq -2\rho_{\rm cr0}\sqrt{\Omega_{\ell 0}\Omega_{\rm m0}}\,(1+z)^{3/2} < 0$ at high z.
- $z_{\rm rep}$ from an exact quadratic: $z_{\rm rep} = 6.75$ for $\Omega_{\ell 0} = 0.036$, $\Omega_{\Lambda 0} = 1.08$ ($z_\dagger = 2$).
- $q = 0$ crossing unique, analytically, for all parameters; acceleration at the switch would need $6\Omega_{\ell 0}/(\Omega_{\Lambda 0} + 2\Omega_{\ell 0}) > 1$, i.e. $\Omega_{\ell 0} \gtrsim 0.54$ vs the adopted $\Omega_{\ell 0} = 0.036$ (LHS $\simeq 0.19$): repulsive while $\rho_{\rm de} < 0$, yet uniqueness forbids a separate acceleration episode for *every* parameter choice.



Uniqueness in one curve: F starts negative and increases for every parameter choice.

[Sahni–Shtanov '02–'03; Alam+ '06, '17; Bag+ '21]

Six histories, one pattern

Model	$z_\dagger$	$z_{\rm rep}$	NEC character	Accelerating $q < 0$
ΛCDM	—	—	$\mathcal{I}_\Lambda \equiv 0$	$-1 < z < 0.67$
CPL (DESI DR2)	—	—	regular NECB crossing at $z \simeq 0.50$	$0.04 < z < 1.01$
Smooth-$\Lambda_{\rm s}$CDM (η=2)	2	2.67	$\mathcal{I}_{\Lambda_{\rm s}} < 0$ for all $z > -1$	$-1 < z < 0.68$
Smooth-$\Lambda_{\rm s}$CDM (η=5)	2	2.35	$\mathcal{I}_{\Lambda_{\rm s}} < 0$ for all $z > -1$	$-1 < z < 0.67$; $1.85 < z < 2.08$
Exponential infrared $f(T)$	2	3.11	$\mathcal{I}_{\rm tp} < 0$ for all $z > -1$	$-1 < z < 0.75$
Minimal phantom brane	2	6.75	$\mathcal{I}_{\rm bw} < 0$ for all $z > -1$	$-1 < z < 0.75$

Same qualitative pattern — density zero, phantom-like NEC character, $z_{\rm rep} > z_\dagger$ — with widely different separations; only the sufficiently sharp $\eta = 5$ profile opens an intermediate acceleration window.

In practice: what is split-independent, and is it regular?

- Kinematic, from $H(z)$ alone (flat GR): $\mathcal{M}_{\rm tot} = 2q\,\rho_{\rm cr}$, $\quad \mathcal{I}_{\rm tot} = \dfrac{(1+z)HH'}{4\pi G}$ — no sector split needed; report these first.
- Sector-level quantities follow by subtracting matter+radiation: quote $z_\dagger$, $z_{\rm rep}$ jointly with the adopted $\Omega_{\rm m0}$ ($|\Delta z_\dagger| \simeq 0.19$ (η=2), 0.08 (η=5) per $\Delta\Omega_{\rm m0} = 0.01$).
- Linear theory is ratio-safe: evolve unnormalized $(\delta\rho_i, \delta p_i,\ Q_i = \mathcal{I}_i\theta_i,\ \Pi_i)$ — no division by ρ_i or $\mathcal{I}_i$ anywhere.
- Stability and closure belong to the completion: stable embedding in practice is $\Lambda_{\rm s}$VCDM (no new propagating scalar mode); departure concentrated near/above the horizon $\Rightarrow$ late-time ISW $\times$ LSS as the natural discriminator.

Take home: geometry first, sectors second, ratios last

- Density sign change, sector repulsion, cosmic acceleration, total NEC violation: four distinct notions — the Raychaudhuri pair $(\mathcal{I}_{\rm de}, \mathcal{M}_{\rm de})$ keeps them apart; $w_{\rm de}$ cannot.
- Exact results at a smooth crossing: $\mathcal{I}_{\rm de}, \mathcal{M}_{\rm de} \le 0$; universal pole residue $n(1+z_\dagger)/3$; $z_{\rm rep} > z_\dagger$; acceleration-with-NEC window; one-or-three $q=0$ transitions.
- In practice: track $(\rho_{\rm de}, p_{\rm de}, \mathcal{I}_{\rm de}, \mathcal{M}_{\rm de})$; use parametrizations and priors that admit both density branches (at the density, pressure, or $\mathcal{I}$ level — $\mathcal{M}$-level as a natural next step); read $w_{\rm de}$ poles as kinematic signatures; take the NECB, not the PDL, as the regular boundary; quote $z_\dagger, z_{\rm rep}$ with $\Omega_{\rm m0}$.

The field equations already single out these combinations;
we propose only that inference follow them.

[arXiv:2607.18008]