

Constraining cosmology with transverse velocity and CMB cross-correlations

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GGI workshop: Exploring New Frontiers in Cosmology

Galileo Galilei Institute, Firenze

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Why peculiar velocities

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$$\mathbf{v}_{\mathbf{k}} = -i\frac{afH}{b_g}\frac{\mathbf{k}}{k^2}\delta_{\mathbf{k}}$$

Line-of-sight peculiar velocities

$$s = r + v_z/aH$$

Peculiar velocity surveys e.g.
CosmicFlow4

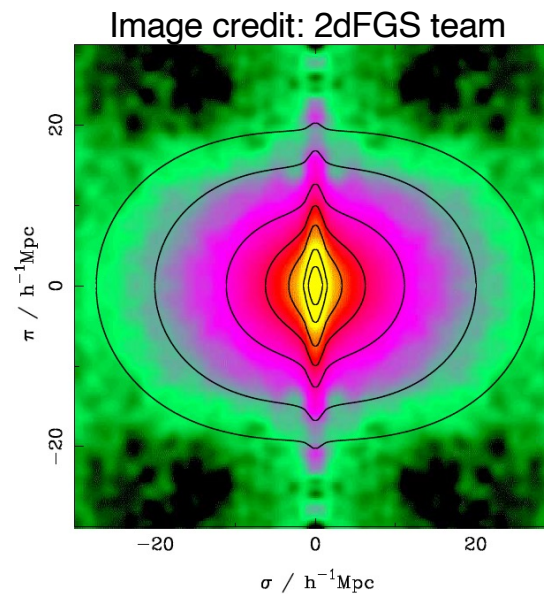
Distance indicators e.g.,
TF relation, standard candles

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$$1 + \xi^S(s_{\perp}, s_{\parallel}) = \int [1 + \xi(r)] \mathcal{P}(v_{\parallel}, \mathbf{r}) dv_{\parallel}$$

Redshift-space distortions

Line-of-sight peculiar velocities

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CosmicFlow4

Distance indicators e.g.,
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Image credit: 2dFGS team

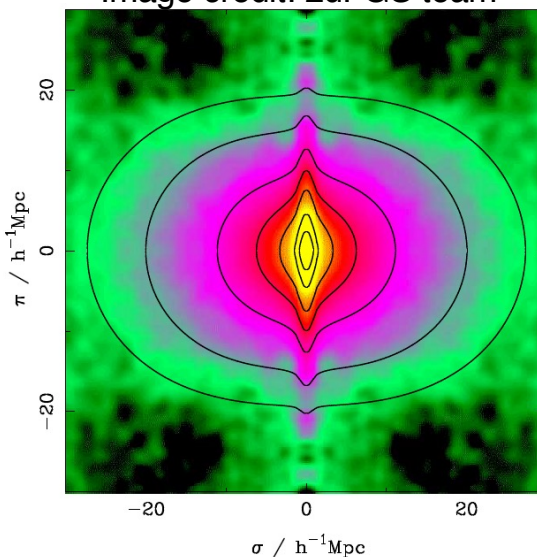
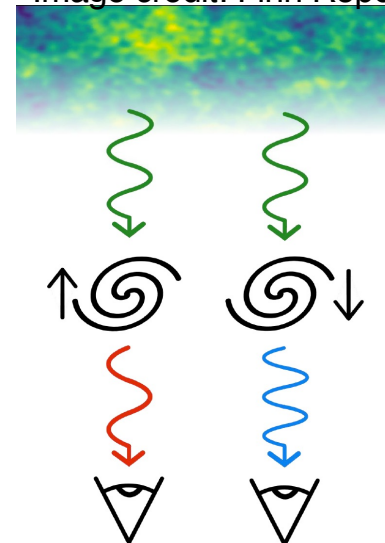


Image credit: Finn Roper



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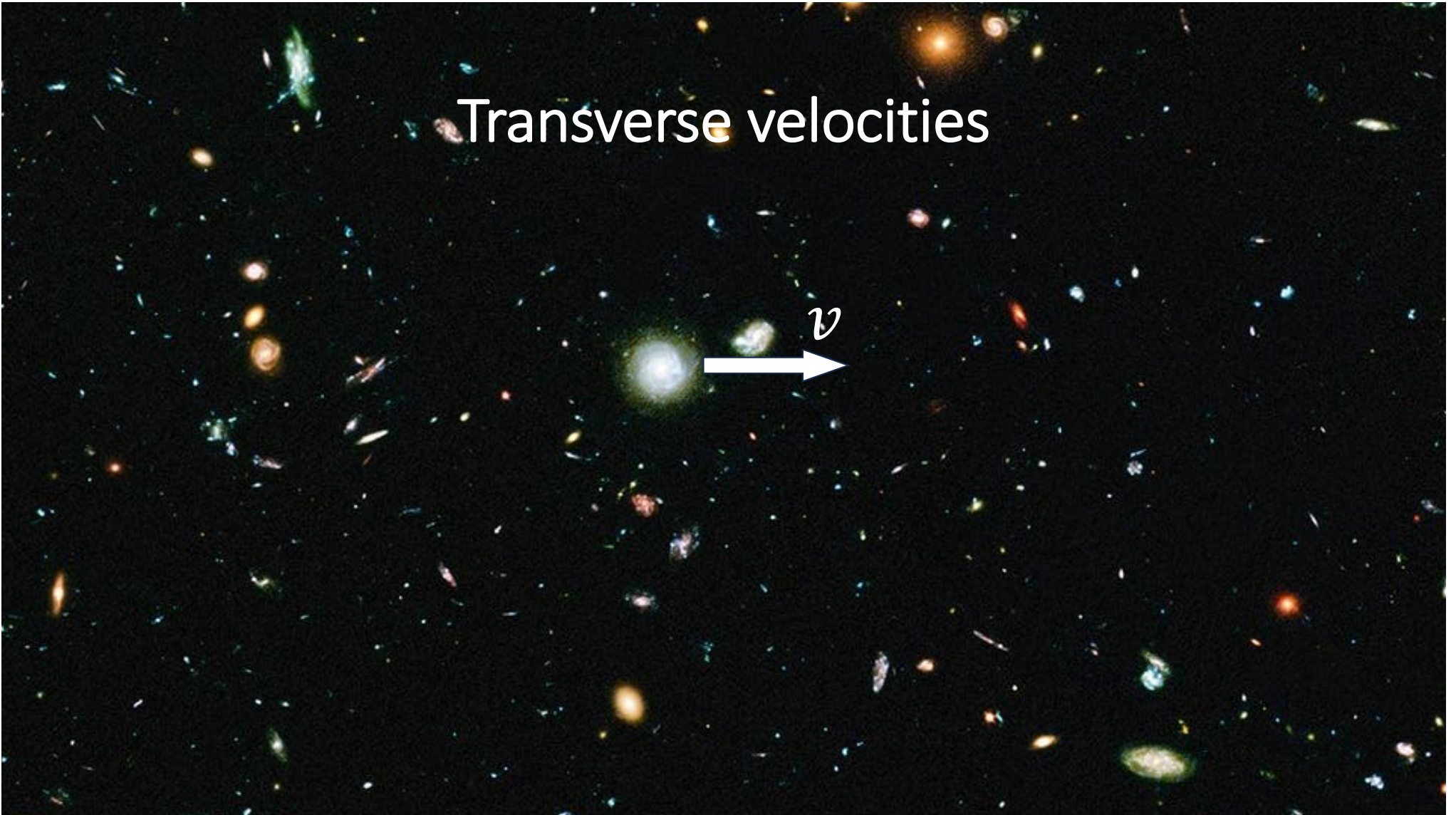
Redshift-space distortions

$$\frac{\Delta T_{\text{kSZ}}}{T_{\text{CMB}}} = -\sigma_T \int dl n_e(\mathbf{r}) \frac{\hat{\mathbf{r}} \cdot \mathbf{v}_e(\mathbf{r})}{c}$$

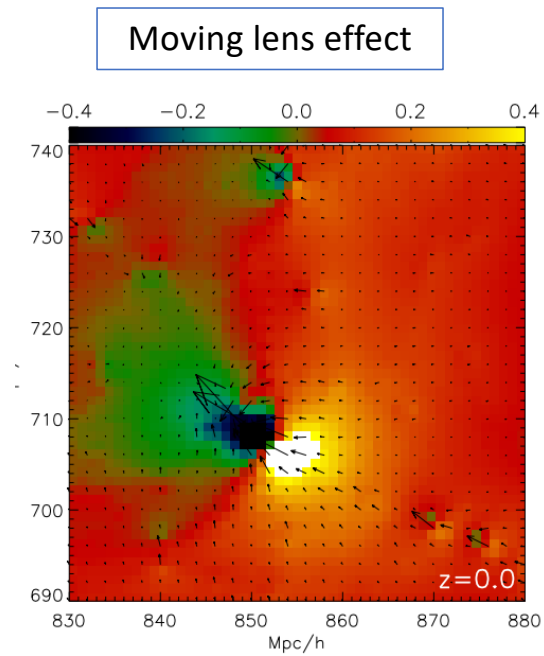
Kinetic Sunyaev Zeldovich effect

Edinburgh

Transverse velocities



Transverse velocities



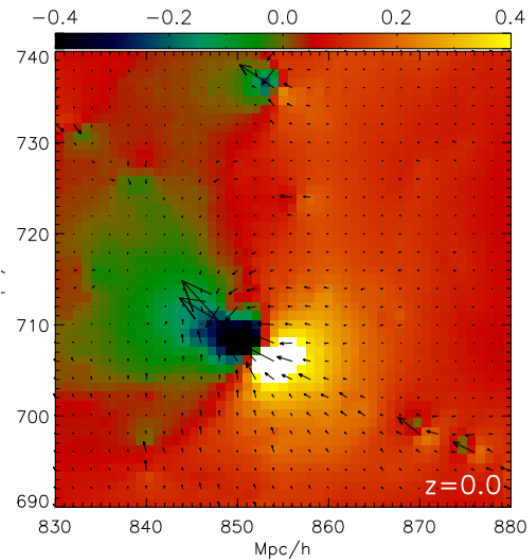
Birkinshaw & Gull 1983Natur.302..315B

Cai et al. 2010MNRAS.407..201C

Hotinli et al., 2022JCAP...10..026H & arXiv: 2605.18938

Transverse velocities

Moving lens effect



Polarised kinetic Sunyaev Zeldovich effect

$$T_0 = T_r [1 - \beta \mu_0 + \beta^2 (\mu_0^2 - 1/3) + \dots].$$

Sunyaev & Zeldovich, 1980MNRAS.190..413S

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Gon & Khatri, 2023JCAP...11..072G

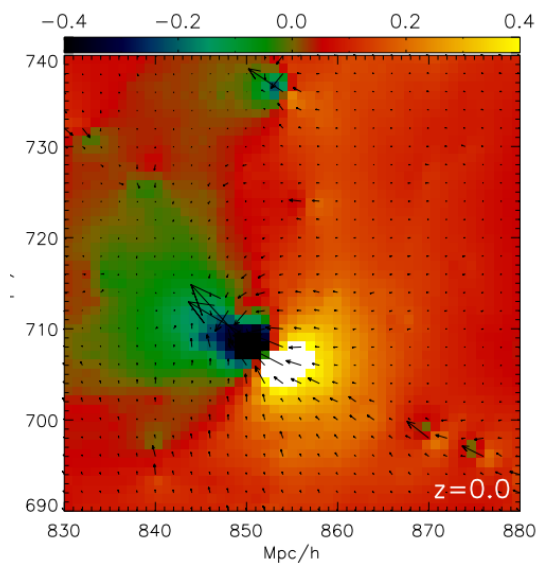
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Transverse Doppler redshift

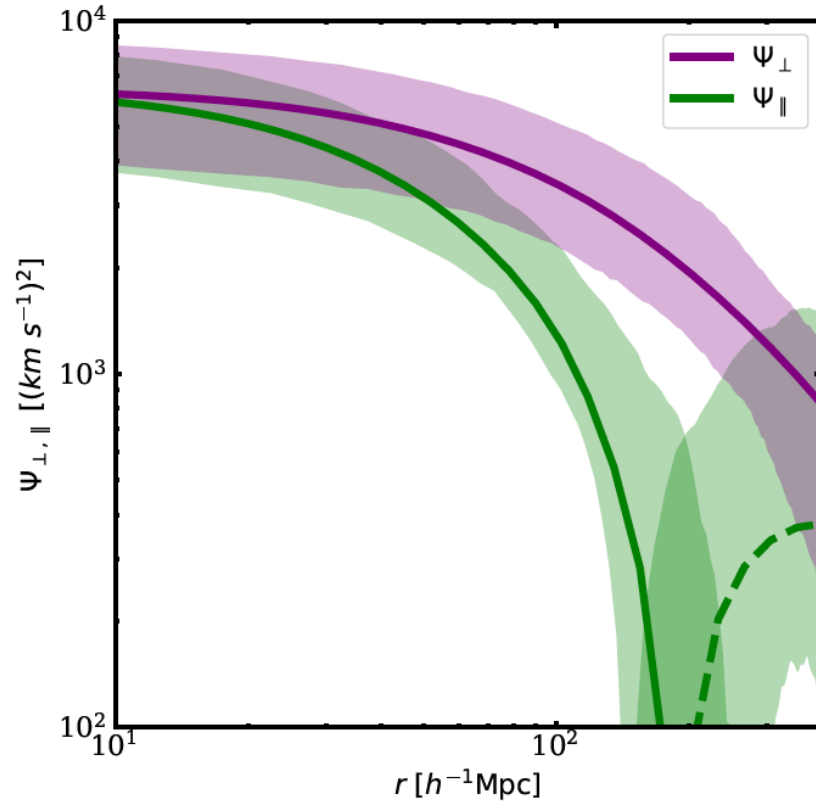
$$\propto v^2/c$$

Kaiser, 2013MNRAS.435.1278K

Zhao et al., 2013PhRvD..88d3013Z

Cai et al. 2017MNRAS.468.1981C

Correlation functions of velocities



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ON THE PATTERN OF PERTURBATIONS OF THE HUBBLE FLOW

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$$\langle v_i v'_j \rangle \equiv \Psi_{ij} = \Psi_{\perp}(r) \delta_{ij} + [\Psi_{\parallel}(r) - \Psi_{\perp}(r)] r_i r_j / r^2,$$

$$\Psi(r) = a^2 H(z)^2 f_g^2(z) \int \Delta^2(k, z) k^{-2} d \ln k K(kr),$$

$$K_{\perp}(x) = [\sin(x) - x \cos(x)]/x^3;$$

$$K_{\parallel}(x) = \sin(x)/x - 2[\sin(x) - x \cos(x)]/x^3.$$

Conditional velocity field



Conditional velocity field

$\mathbf{v}(\mathbf{0})$ along the x-axis, we have:

$$\langle \mathbf{v} \rangle = |\mathbf{v}(\mathbf{0})| \frac{1}{r^2} \begin{bmatrix} \psi_{\perp} y^2 + \psi_{\parallel} x^2 \\ (\psi_{\parallel} - \psi_{\perp}) xy \end{bmatrix}$$

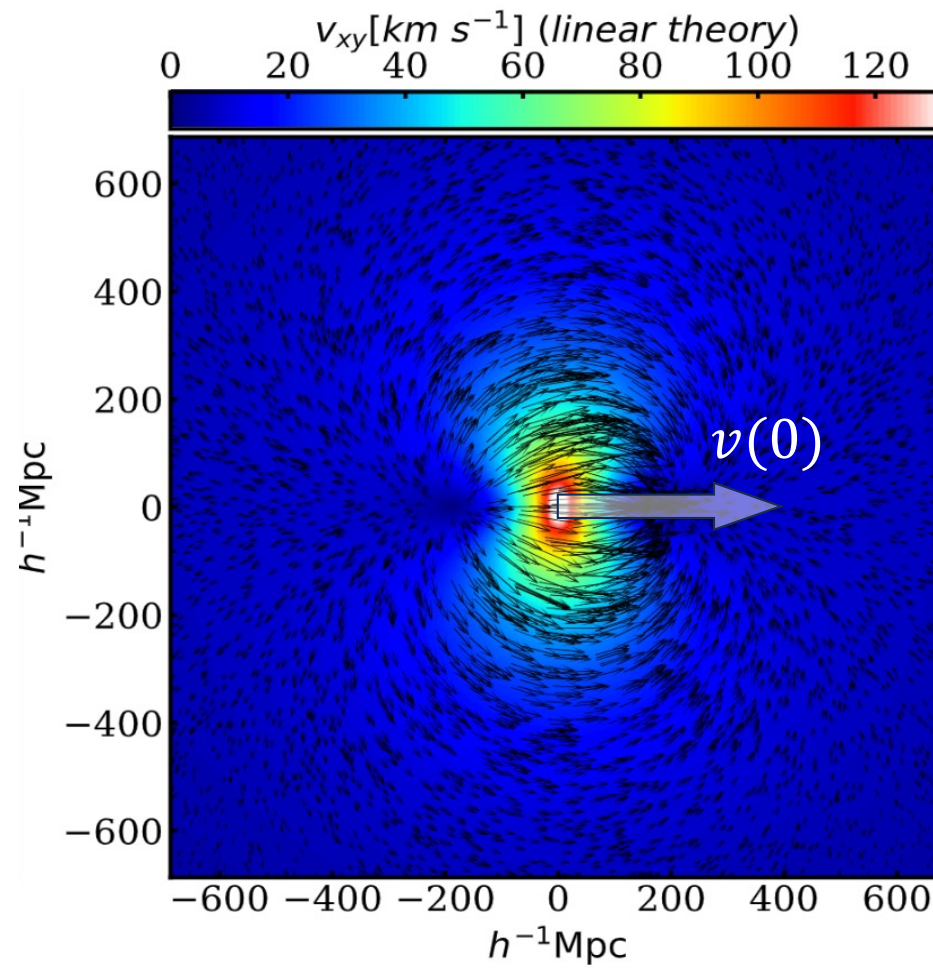
Conditional velocity field

$\mathbf{v}(\mathbf{0})$ along the x-axis, we have:

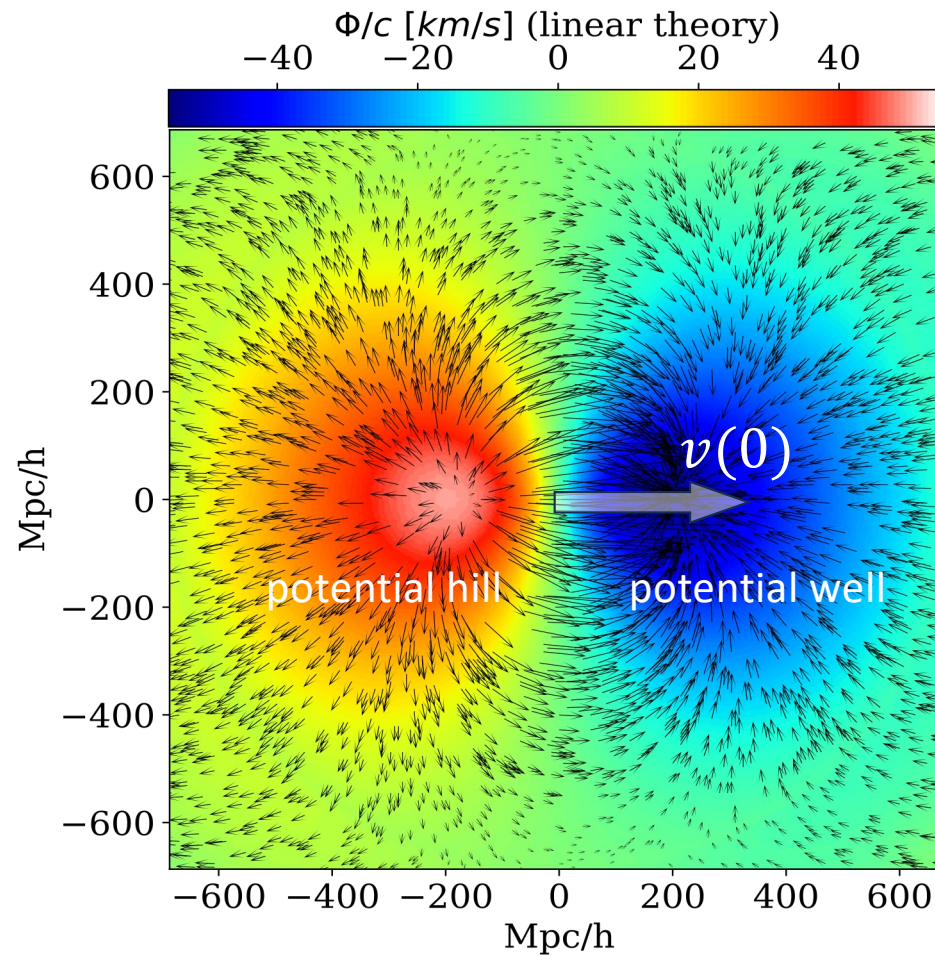
$$\langle \mathbf{v} \rangle = |\mathbf{v}(\mathbf{0})| \frac{1}{r^2} \begin{bmatrix} \psi_{\perp} y^2 + \psi_{\parallel} x^2 \\ (\psi_{\parallel} - \psi_{\perp}) xy \end{bmatrix}$$

Oriented stacked velocity field

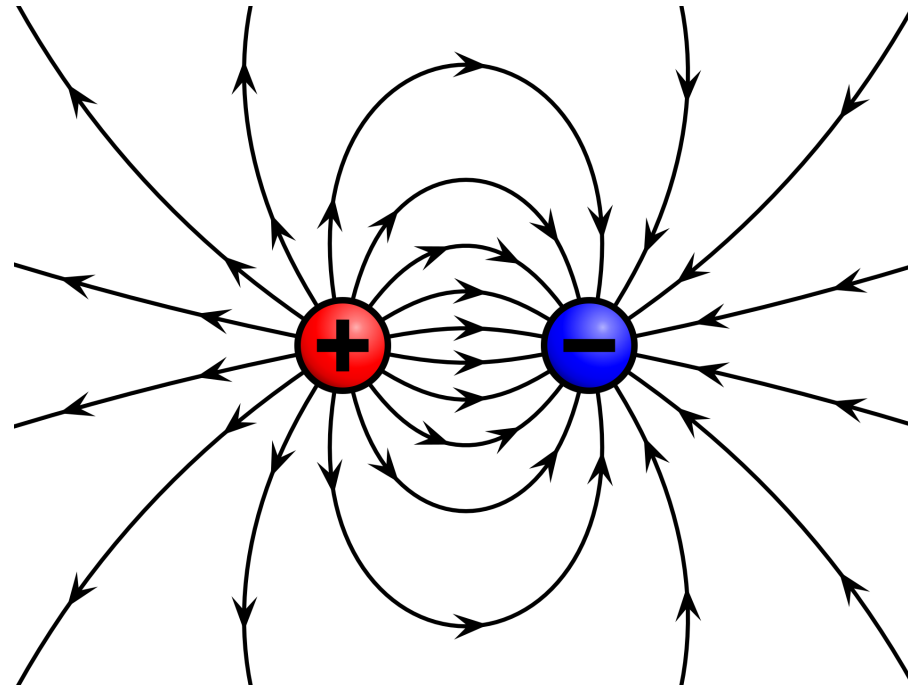
Velocity at the origin defines a preferred direction



Velocity points at the dipole of potential



Potential dipole resembles electric dipole



Conditional potential field

$v(0)$ along the x-axis, we have:

$$\langle \Phi | v \rangle \propto \cos\theta \mathcal{C}_{\Phi v}$$

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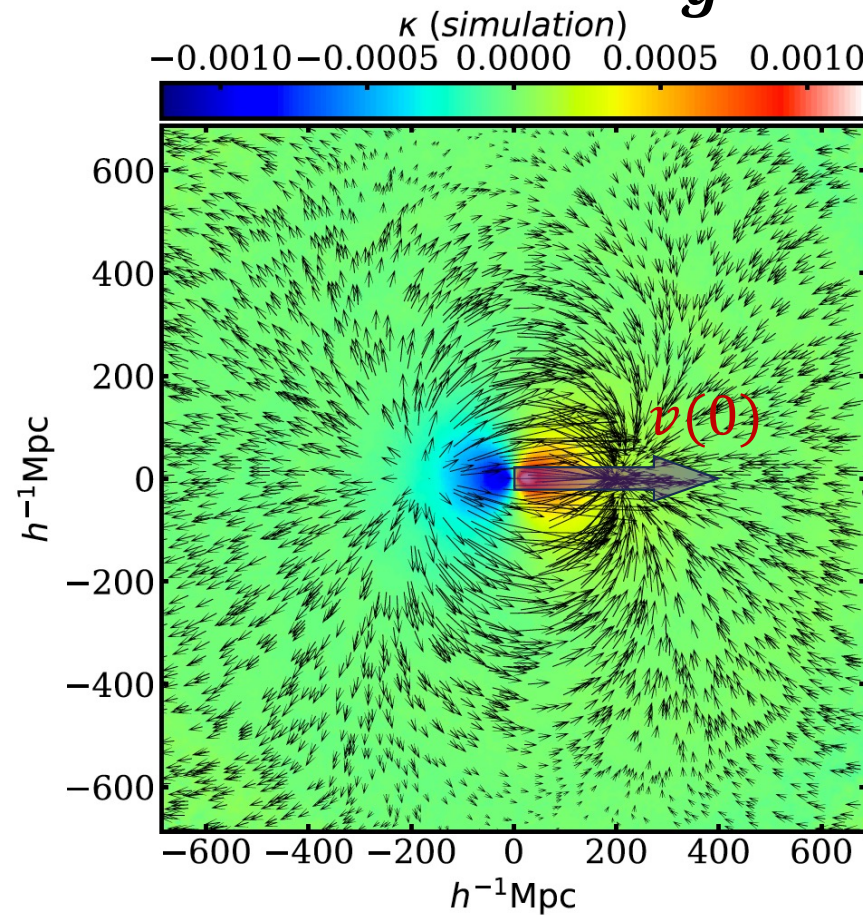
$$\langle \Phi | v \rangle \propto \cos\theta C_{\Phi v}$$

Oriented stacked potential field

$$\langle \delta_m | v \rangle \propto \cos\theta C_{\delta_m v}$$

...

v - κ correlation, v - δ_g correlation



Velocities X CMB

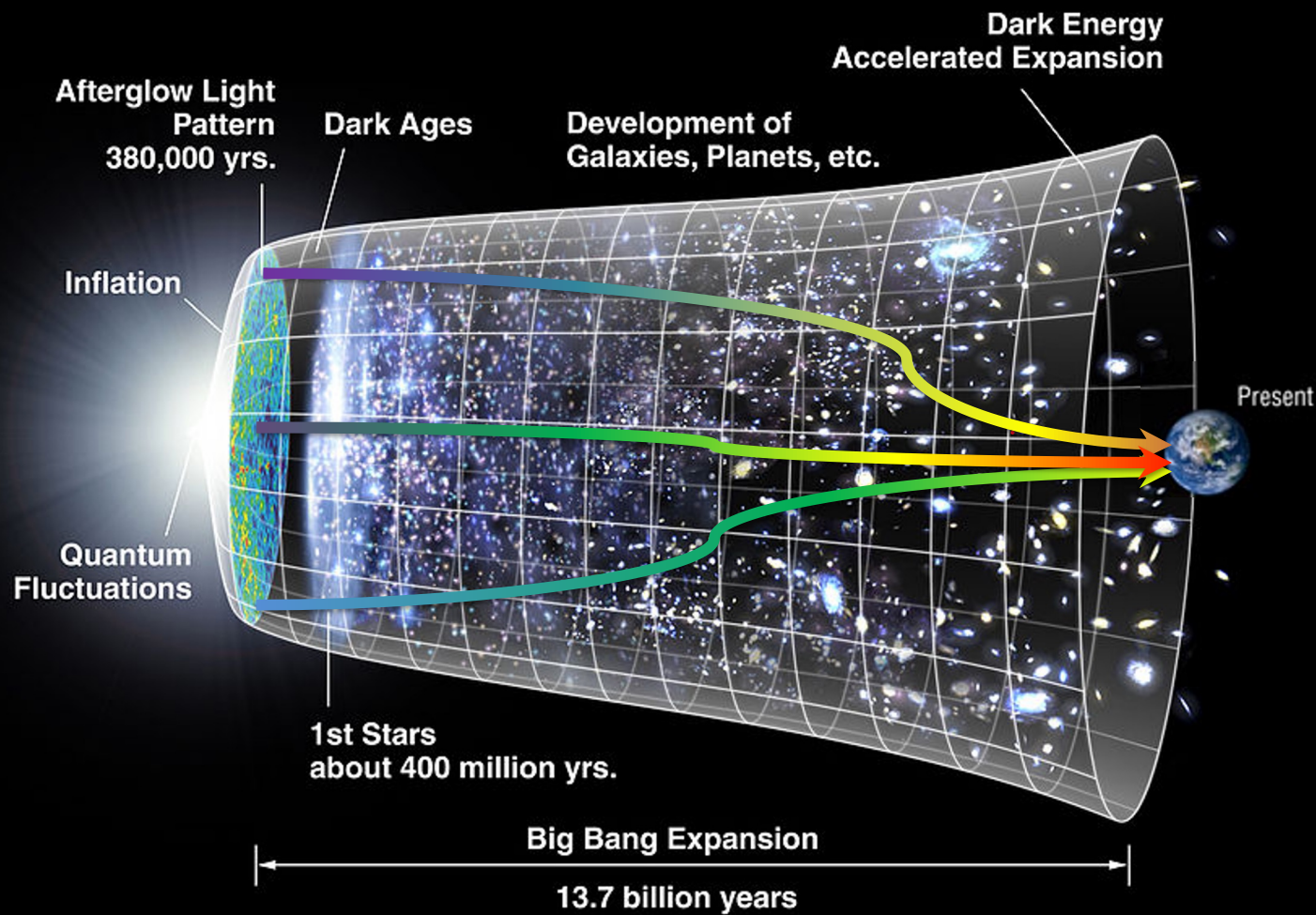


Image credit: NASA WMAP team

Galaxy field δ_g

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Velocity reconstruction

$$\mathbf{v}_{\mathbf{k}} = -i \frac{afH}{b_g} \frac{\mathbf{k}}{k^2} \delta_{\mathbf{k}},$$

Galaxy field δ_g

Velocity reconstruction

$$\mathbf{v}_{\mathbf{k}} = -i \frac{afH}{b_g} \frac{\mathbf{k}}{k^2} \delta_{\mathbf{k}},$$

Oriented stacking with $[T_{CMB}, \kappa, \delta_g]$ using $\mathbf{v}$

Traditional observables

Galaxy-X correlations:

- ISW monopole: δ_g - $ISW \langle \delta_g T \rangle$
- CMB lensing monopole: δ_g - κ
- Galaxy number density monopole: δ_g - δ_g

New observables

Velocity-X correlations:

- ISW dipole: $\boldsymbol{v-ISW} \quad \langle \boldsymbol{vT} \rangle$
- CMB lensing dipole: $\boldsymbol{v-\kappa}$
- Galaxy number density dipole: $\boldsymbol{v-\delta_g}$

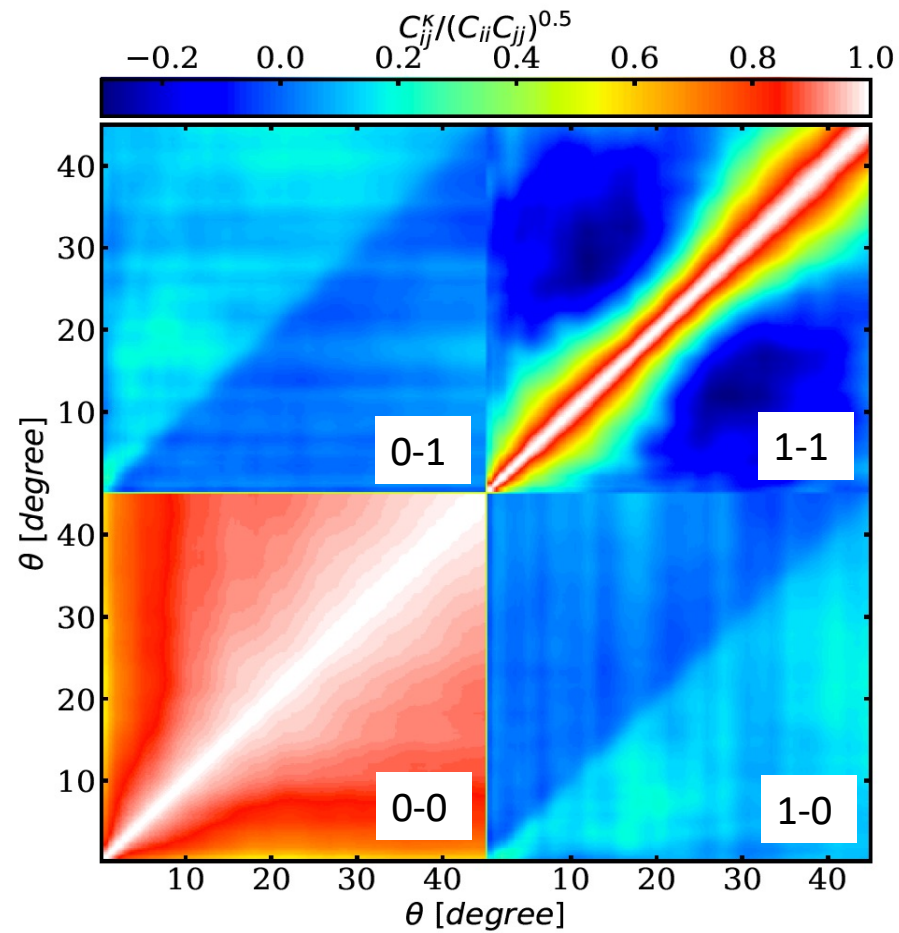
A signal independent of galaxy bias

$$\langle \delta_m | v \rangle \propto \cos\theta \, C_{\delta_m v} / \sigma_v$$

$$C_{\delta_m v} = aHf \int k^{-1} \Delta^2(k) j_1(kx) d \ln k$$

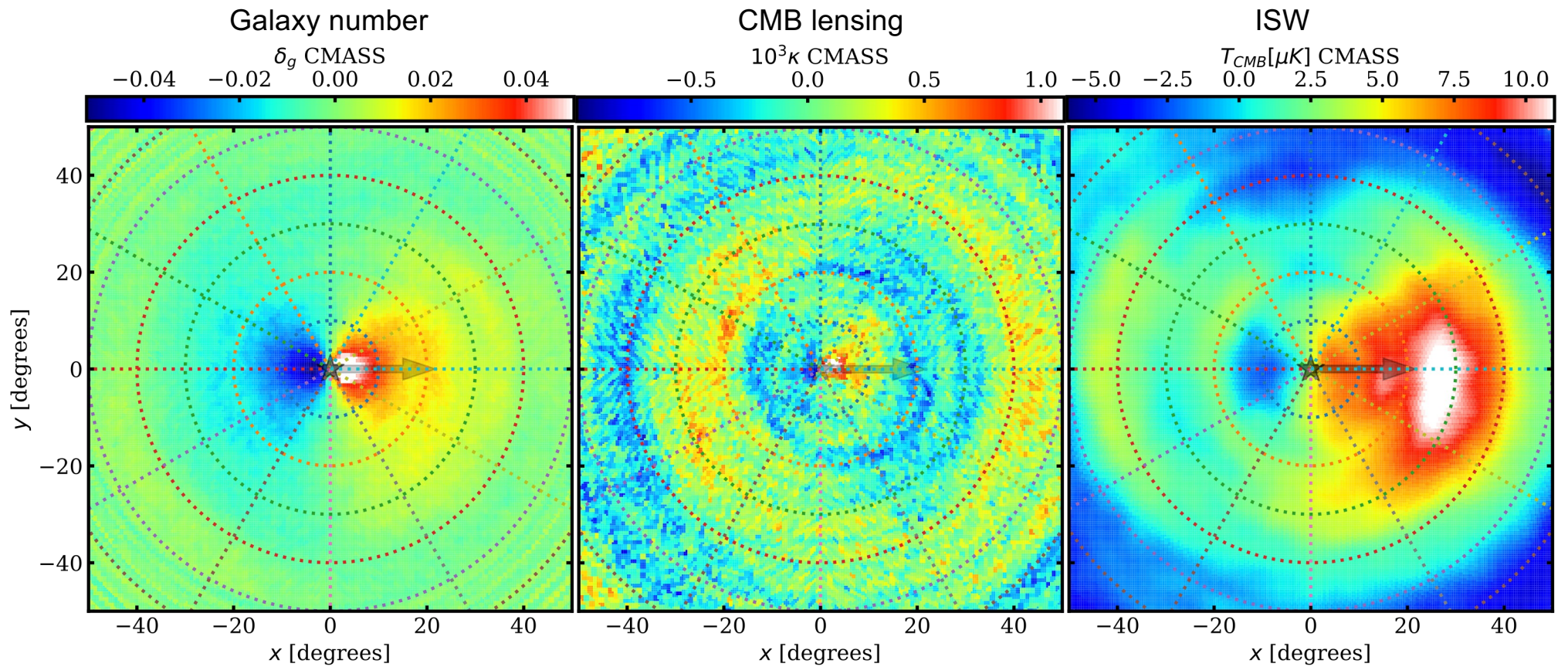
$$\sigma_v^2 = (aHf)^2 \int k^{-2} \frac{\Delta^2(k)}{3} d \ln k$$

Monopole and dipole are independent

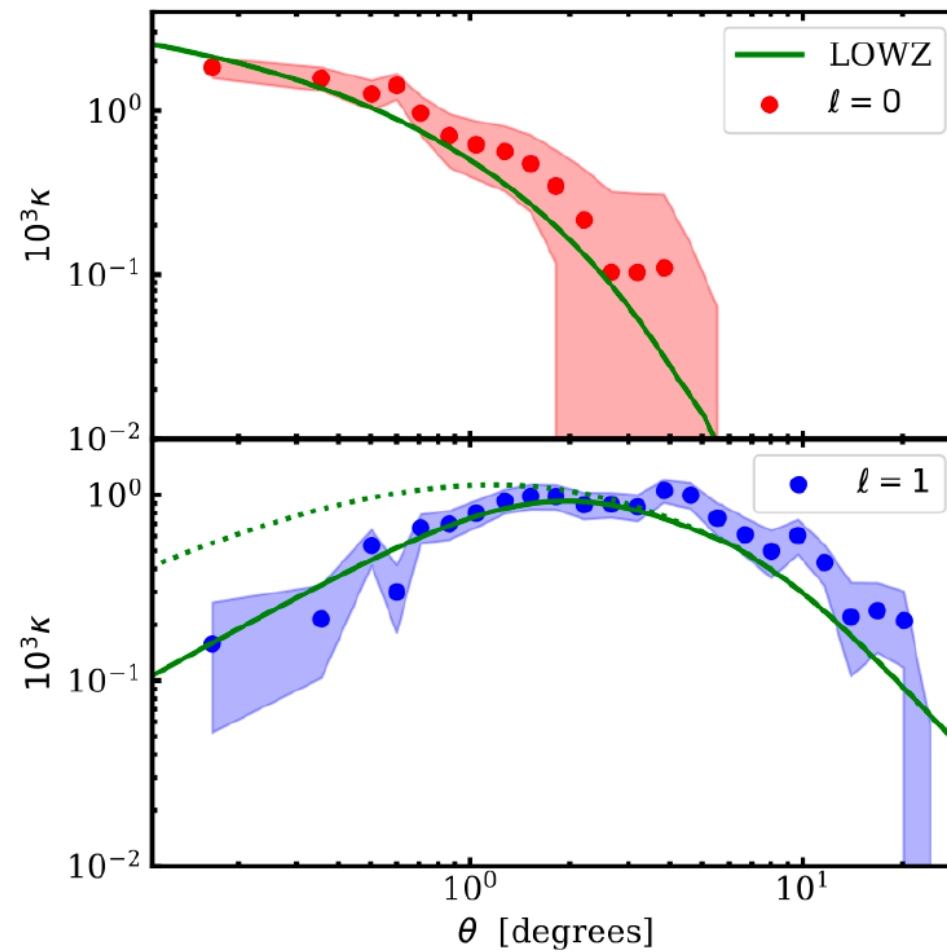
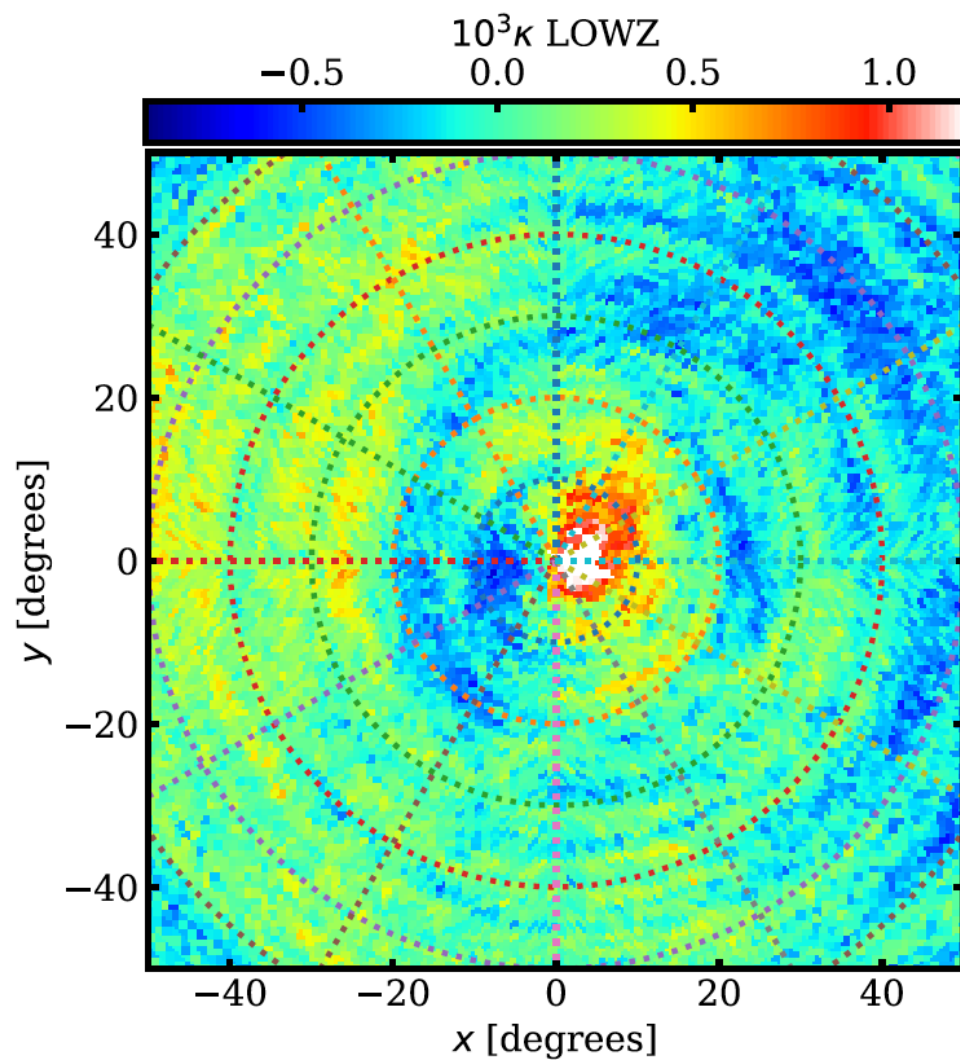


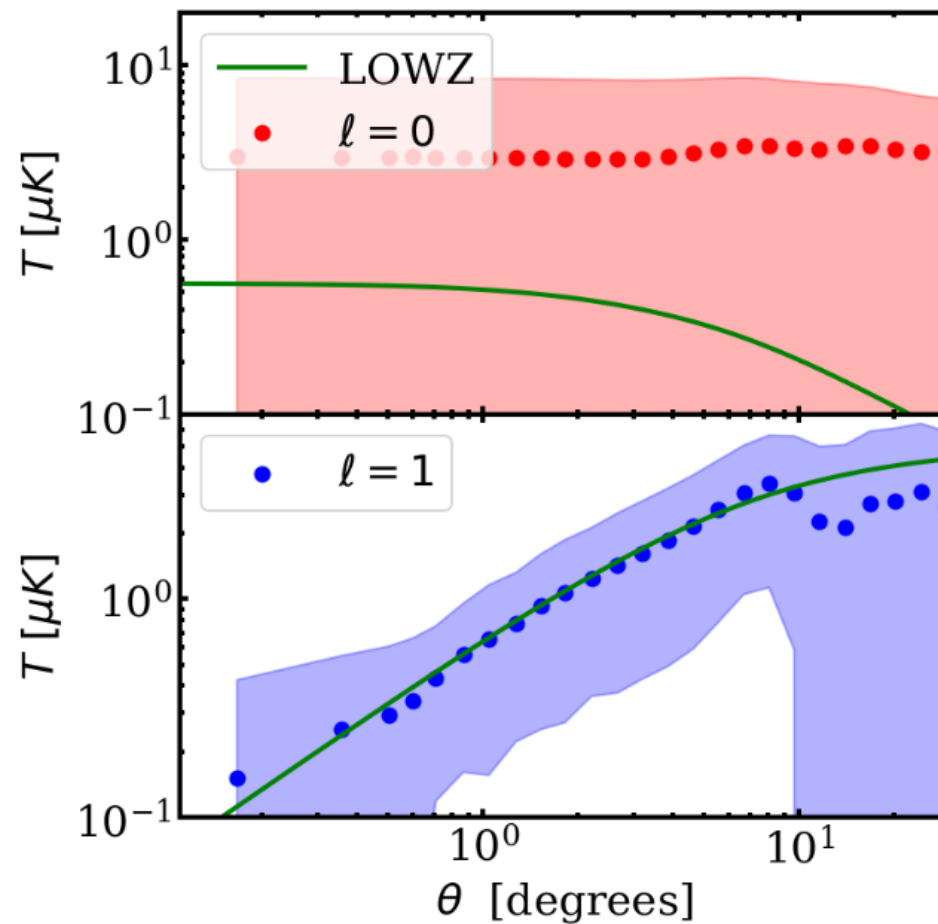
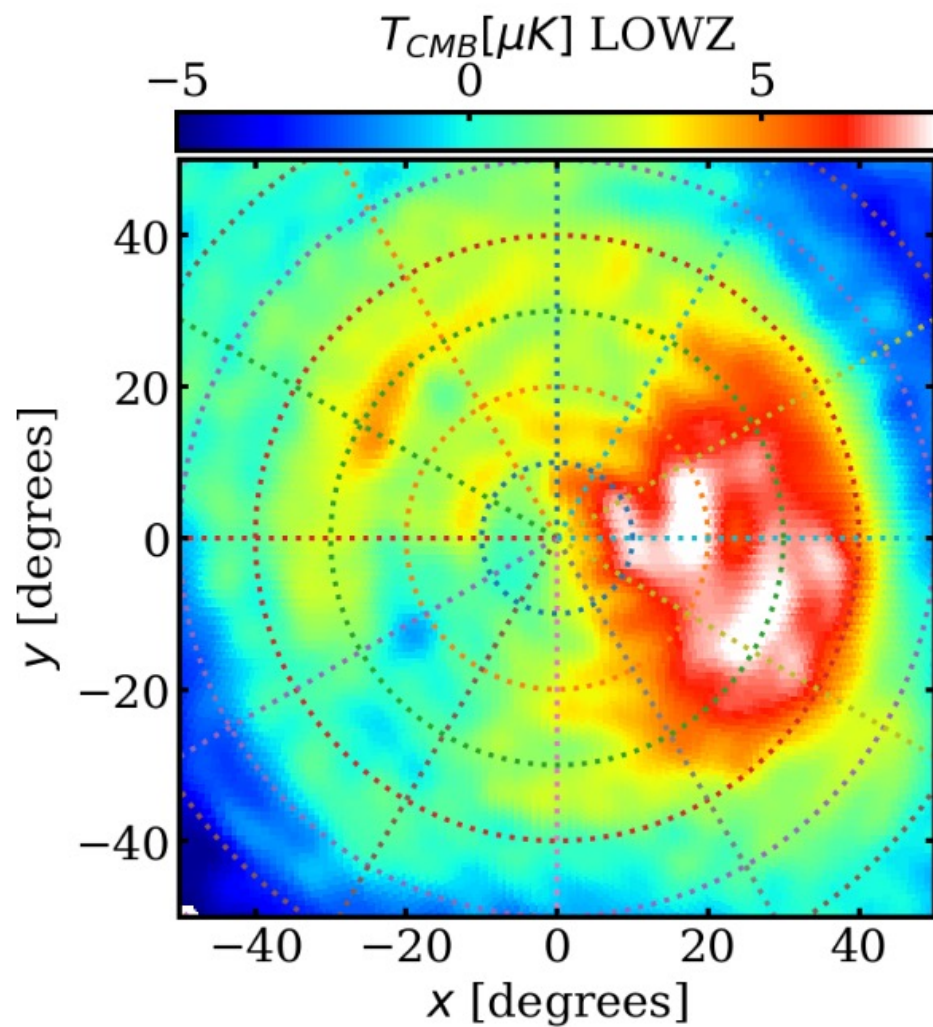
Detections with SDSS-BOSS x Planck

Oriented stacking

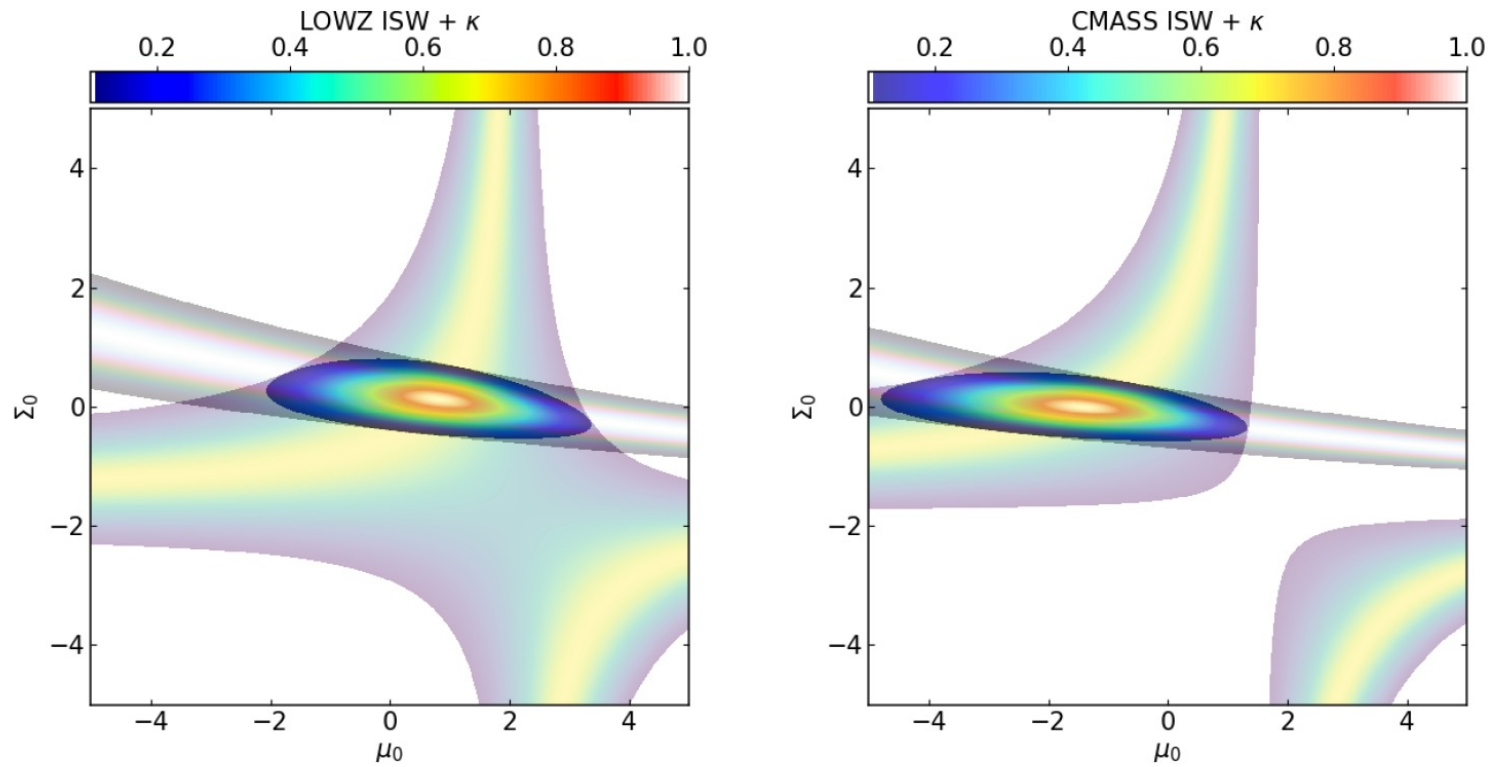


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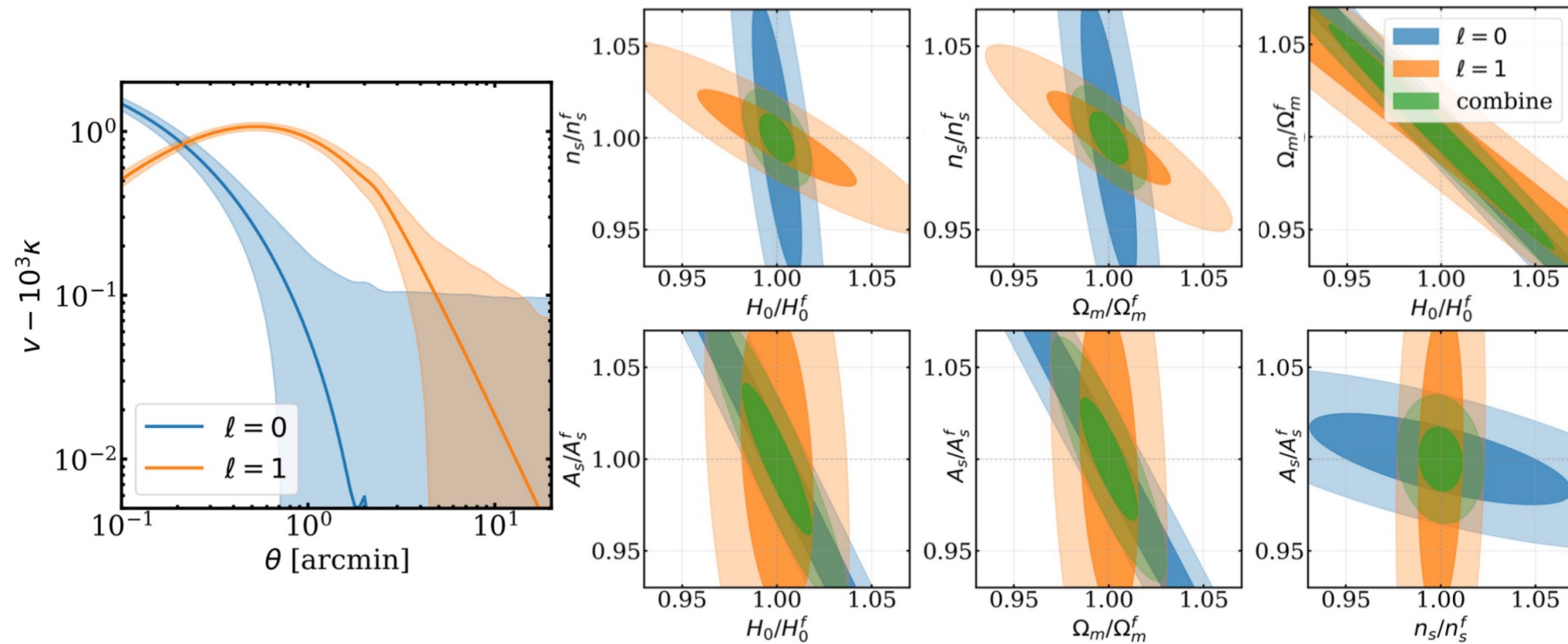


Constraints on modified gravity

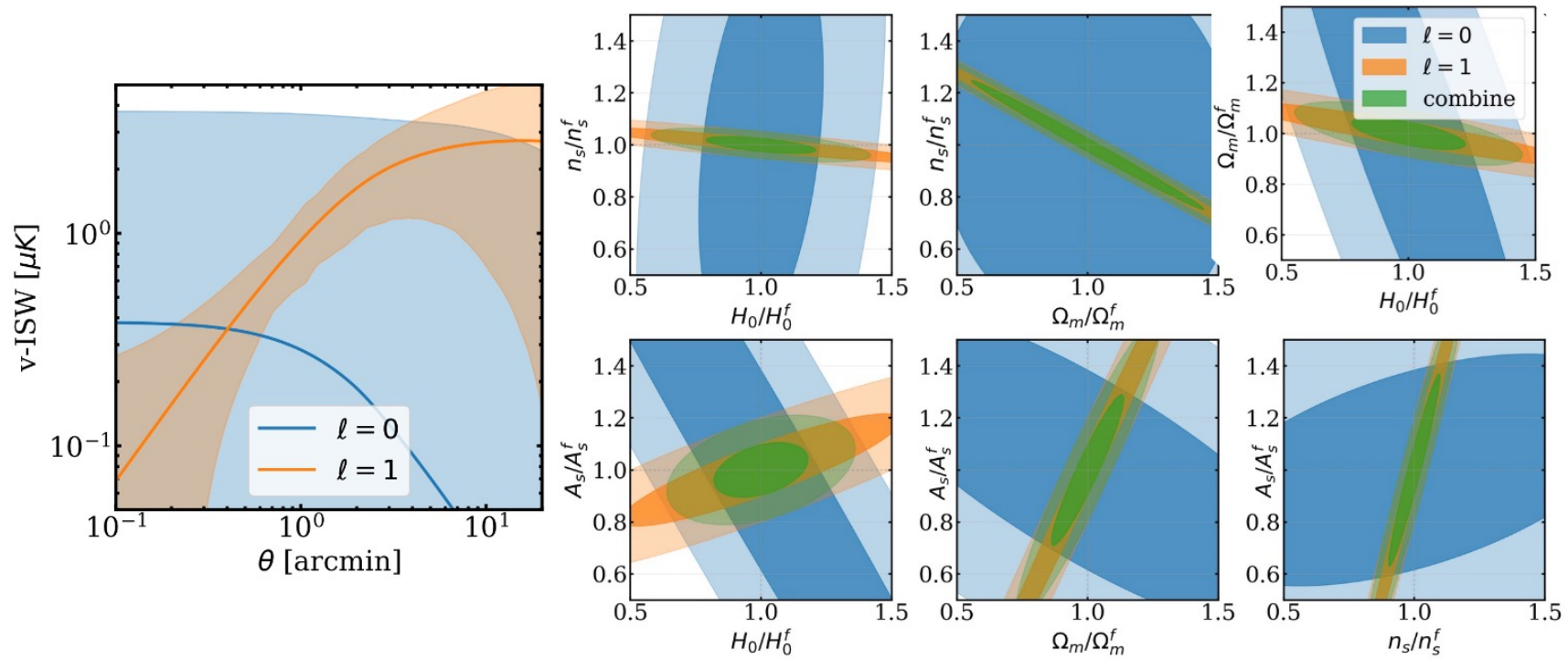


$$\Psi = (1 + \mu) \Psi_{\text{GR}}$$
$$\Psi + \Phi = (1 + \Sigma) [\Psi + \Phi]_{\text{GR}}$$

Expected constraints on cosmology



Expected constraints on cosmology



Summary

- Gpc-scale dipoles of velocities resemble electric dipoles
- Gravitational imprints on the CMB --- ISW & lensing
- Detections of both with galaxies x CMB
- Dipoles of cross-correlations are
 - independent from monopoles
 - free from galaxy bias
- Independent constraints on cosmology & gravity