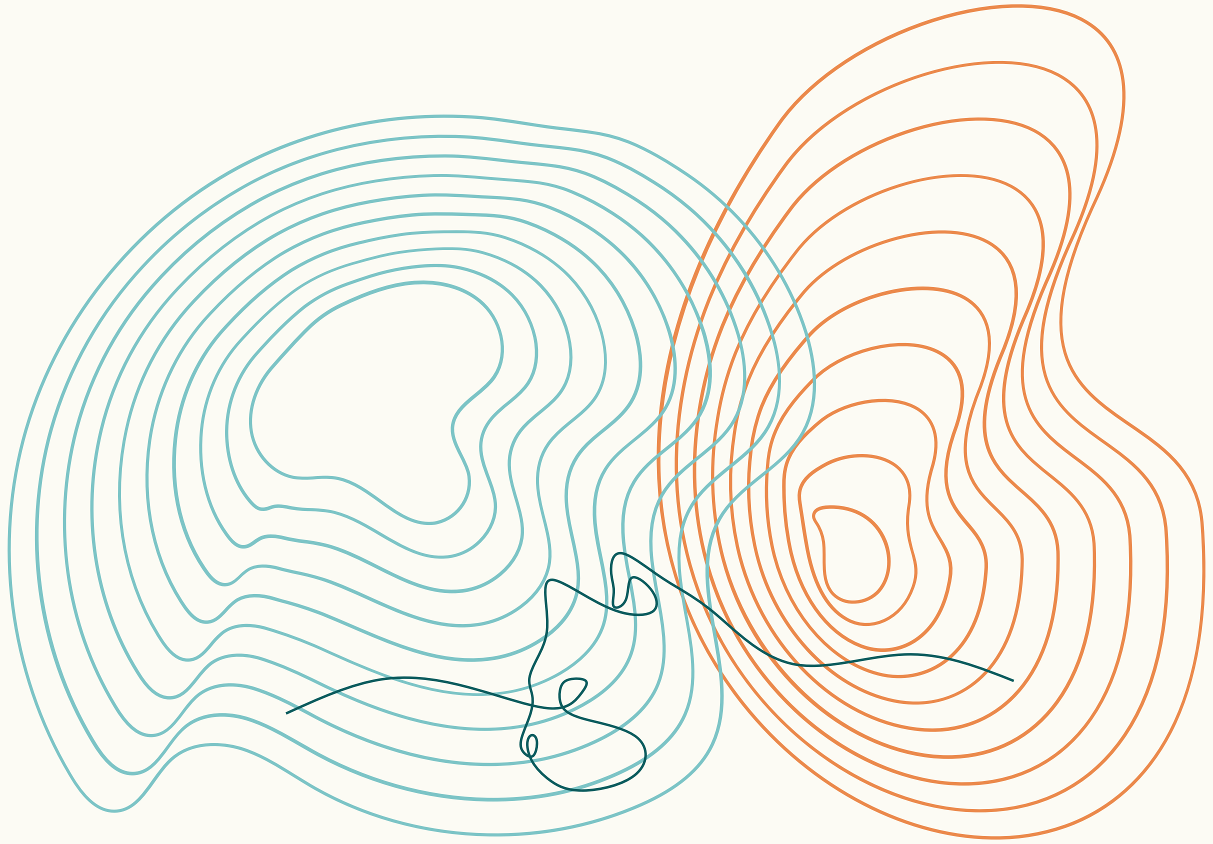
A SUPERPOSITION QUANTUM UNIVERSE AND ITS PERTURBATIONS

GGI workshop: Exploring New Frontiers in Cosmology
Week 3
22 July 2026



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LEVERHULME TRUST

IN THE GRAND SCHEME...

- Homogeneous isotropic spacetime described by Friedmann-Lemaître-Robertson-Walker (FLRW) metric

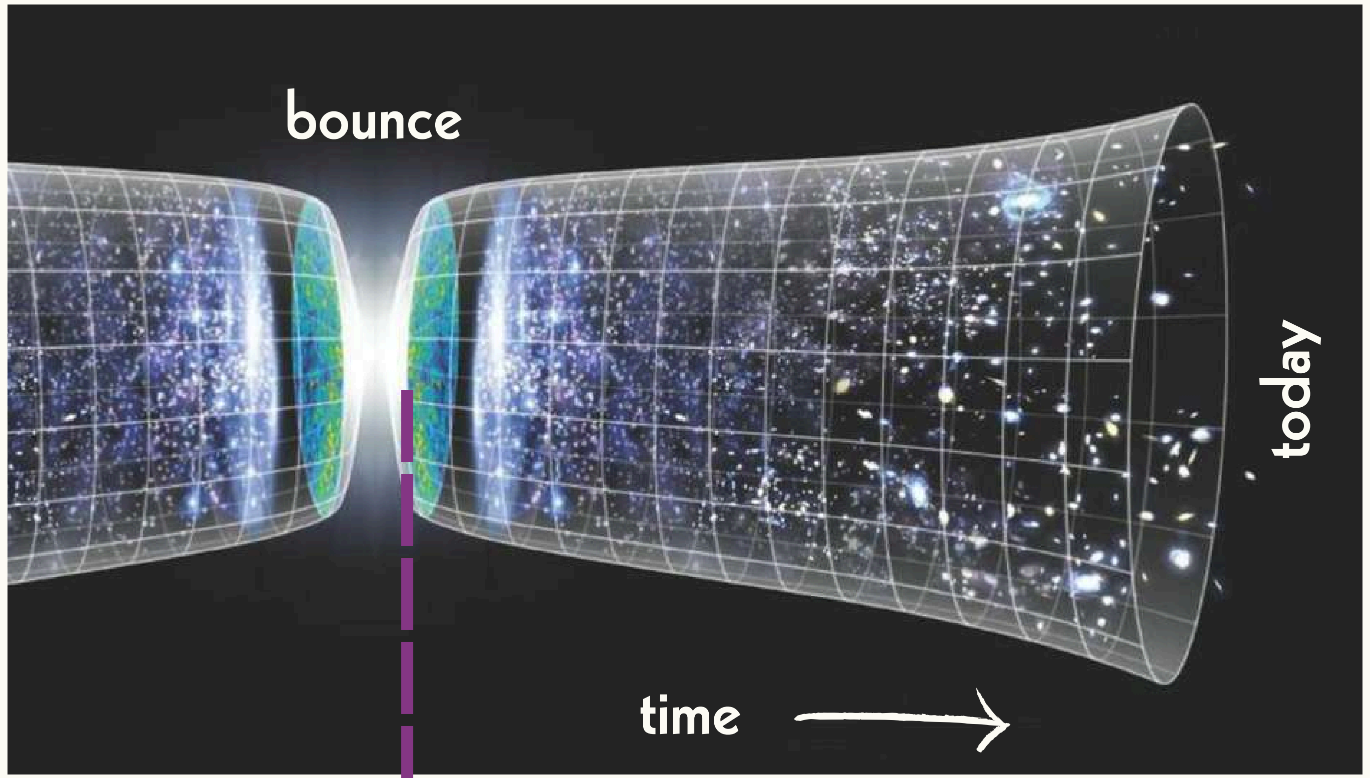
$$\mathrm{d}s^2 = -N^2(t)\mathrm{d}t^2 + a^2(t)\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j$$

  - Evolution of the universe's geometry is encoded in the **scale factor** a

$\downarrow$

GR: singularity at $a \to 0$

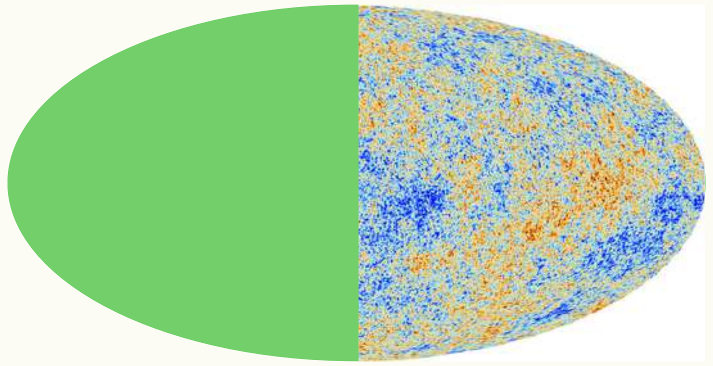
- **Quantum treatment of the background dynamics**
  - Avoid the singularity → **bounce**



[NASA/WMAP]

Primordial power spectrum → CMB (Planck, ACT, SPT) →

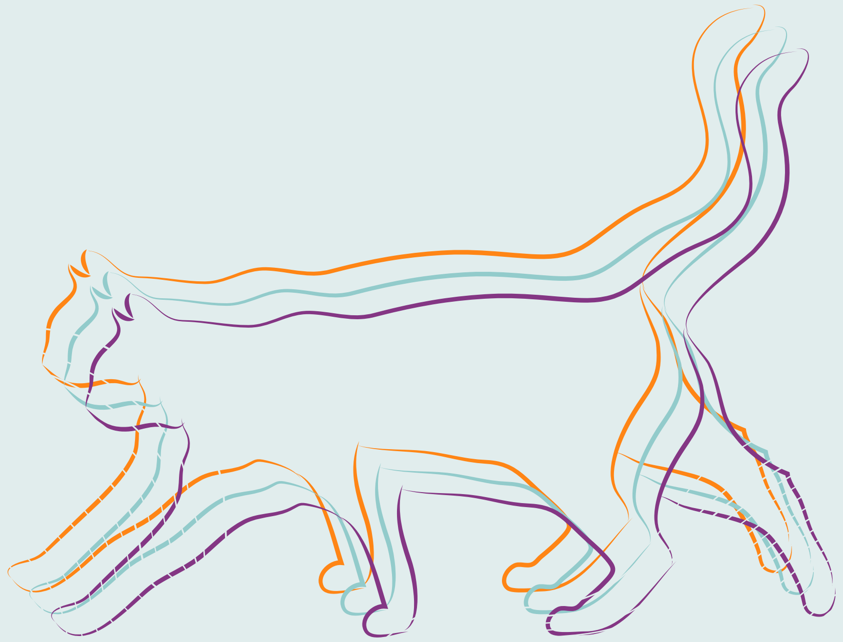
T ~ 2.7 K



[ESA/Planck]

Here: imprints of the quantum bounce in cosmological perturbations

OUTLINE



- **Introduction**

  Quantum cosmology, classical system and its quantisation

- **Bouncing universe**

  Superposition states

- **Perturbations over bounces**

  Primordial tensor power spectrum

- **Conclusion**

  Outlook: embedding in cosmology

QUANTUM COSMOLOGY

- **Minisuperspace models**: Wheeler-DeWitt quantisation on the reduced phase space of general relativity to obtain a quantum description of the universe $(\hat{\mathcal{H}}_{\mathrm{FLRW}} + \hat{\mathcal{H}}_{\mathrm{matter}})\Psi(a,\phi) = 0$ [DeWitt ('67)]

  approximation to a full theory of quantum gravity

- **Problem of time** [Isham, ('93)]
  - GR is a fully constrained system → no external time parameter
  - Evolution happens w.r.t. an internal degree of freedom serving as a clock (here: perfect fluid)

[Małkiewicz, Peter, ('19)]
[Gielen, Menéndez-Pidal, ('20, `21)]
[de Cabo Martin, Małkiewicz, Peter, ('22)]
[Bergeron, Dapor, Gazeau, Małkiewicz, ('14)]
....

- **Ambiguities**
  - Quantisation: choice of clock degree of freedom, canonical variables, quantisation scheme
  - Extraction of an effective evolution of the scale factor
  - (Semiclassical) state of the universe

  different phenomenology (e.g. singularity resolution)

CLASSICAL BACKGROUND

- Geometry: FLRW Hamiltonian

lapse

$$\mathcal{H}_{\mathrm{ADM}} \to \mathcal{H}_{\mathrm{FLRW}} = -\frac{\kappa N}{12\mathcal{V}_0 a} p_a^2$$

$\kappa = 8\pi G$

momentum conjugate to scale factor $\{a, p_a\} = 1$

spatial section volume

scale factor

- Matter: perfect fluid $\mathcal{H}_{\mathrm{fluid}} = \gamma(w)\frac{N}{a^{3w}}\, p_\phi^{1+w}$

[Schutz, ('70) ('71)]

  - Perfect fluid as matter clock fixes the lapse $N = -a^{3w}$

equation of state parameter

- Canonical transformations to convenient variables

$(a, p_a) \to (q, p)$ with $p \propto a^{\frac{3}{2}(1-w)} H$ $q \propto a^{\frac{3}{2}(1-w)}$ and $p_\phi \to p_\tau = -\gamma p_\phi^{1+w}$

scale factor expansion rate of the universe

Hubble rate

matter clock

$$\boxed{\text{Total Hamiltonian:} \quad \mathcal{H} = \mathcal{H}_{\mathrm{FLRW}} + \mathcal{H}_{\mathrm{fluid}} \propto p^2 + p_\tau}$$

QUANTISATION OF THE BACKROUND

Total classical Hamiltonian: $\mathcal{H} = \mathcal{H}_{\mathrm{FLRW}} + \mathcal{H}_{\mathrm{fluid}} \propto p^2 + p_T$

scale factor matter clock

[Małkiewicz, Peter, ('19)] [Gielen, Menéndez-Pidal, ('20, `21)] [de Cabo Martin, Małkiewicz, Peter, ('22)] [Bergeron, Dapor, Gazeau, Małkiewicz, ('14)] ....

- Quantisation map based on the affine group introduces repulsive potential that leads to a bounce

Essential assumption: non-negative scale factor

constant $\geq \frac{3}{4}$

time evolution

$$\mathcal{H} \longrightarrow \boxed{\hat{\mathcal{H}}\Psi = -\partial_x^2\Psi + \frac{K}{x^2}\Psi - \mathrm{i}\partial_\tau\Psi}$$

introduces a bounce

- Schrödinger equation in perfect fluid clock variable $\hat{\mathcal{H}}\psi = 0 \longrightarrow \hat{\mathcal{H}}_{\mathrm{FLRW}}\psi - \mathrm{i}\partial_\tau\psi = 0$

BOUNCING UNIVERSE

... in a superposition

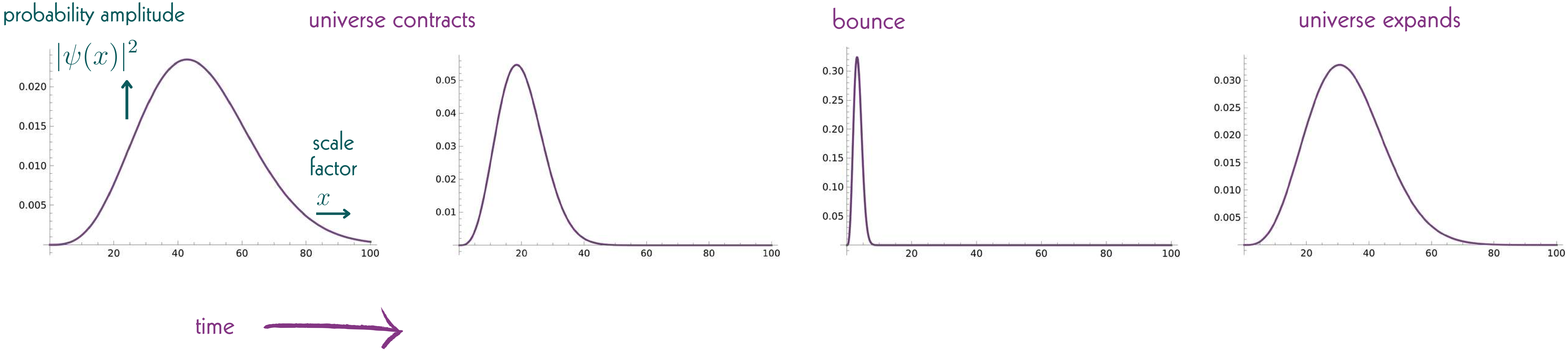
STATE CHOICE FOR A SEMICLASSICAL BOUNCE

[Bergeron, Gazeau, Małkiewicz, Peter ('23)]

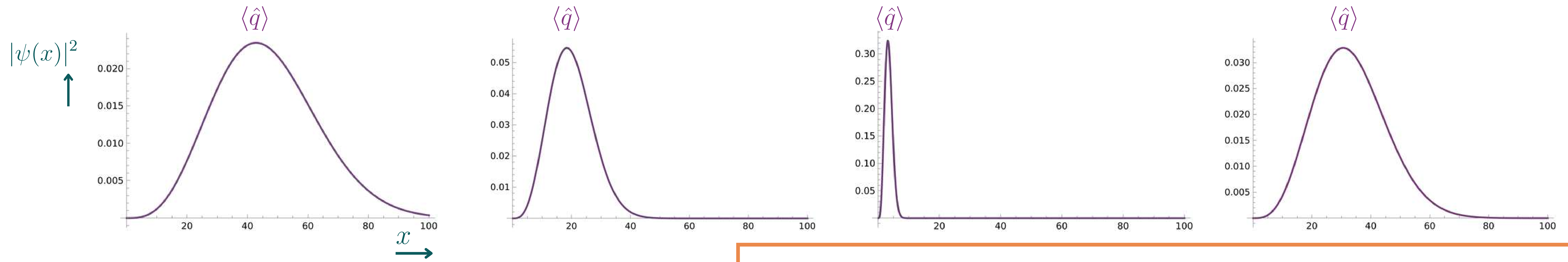
- Choose semiclassical state $|\psi\rangle = e^{-\mathrm{i}\phi(\tau)}|q(\tau), p(\tau)\rangle$
  - Follows dynamics generated by the semiclassical Hamiltonian $\mathcal{H}_{\text{sem}} = p^2 + \frac{\xi^2}{q^2}$

$$q(\tau) = q_B\sqrt{1+\omega^2(\tau-\tau_B)^2} \qquad p(\tau) = \frac{1}{2}\dot{q}(\tau) = \frac{q_B\,\omega^2(\tau-\tau_B)}{2\sqrt{1+\omega^2(\tau-\tau_B)^2}}$$

Wave function evolution



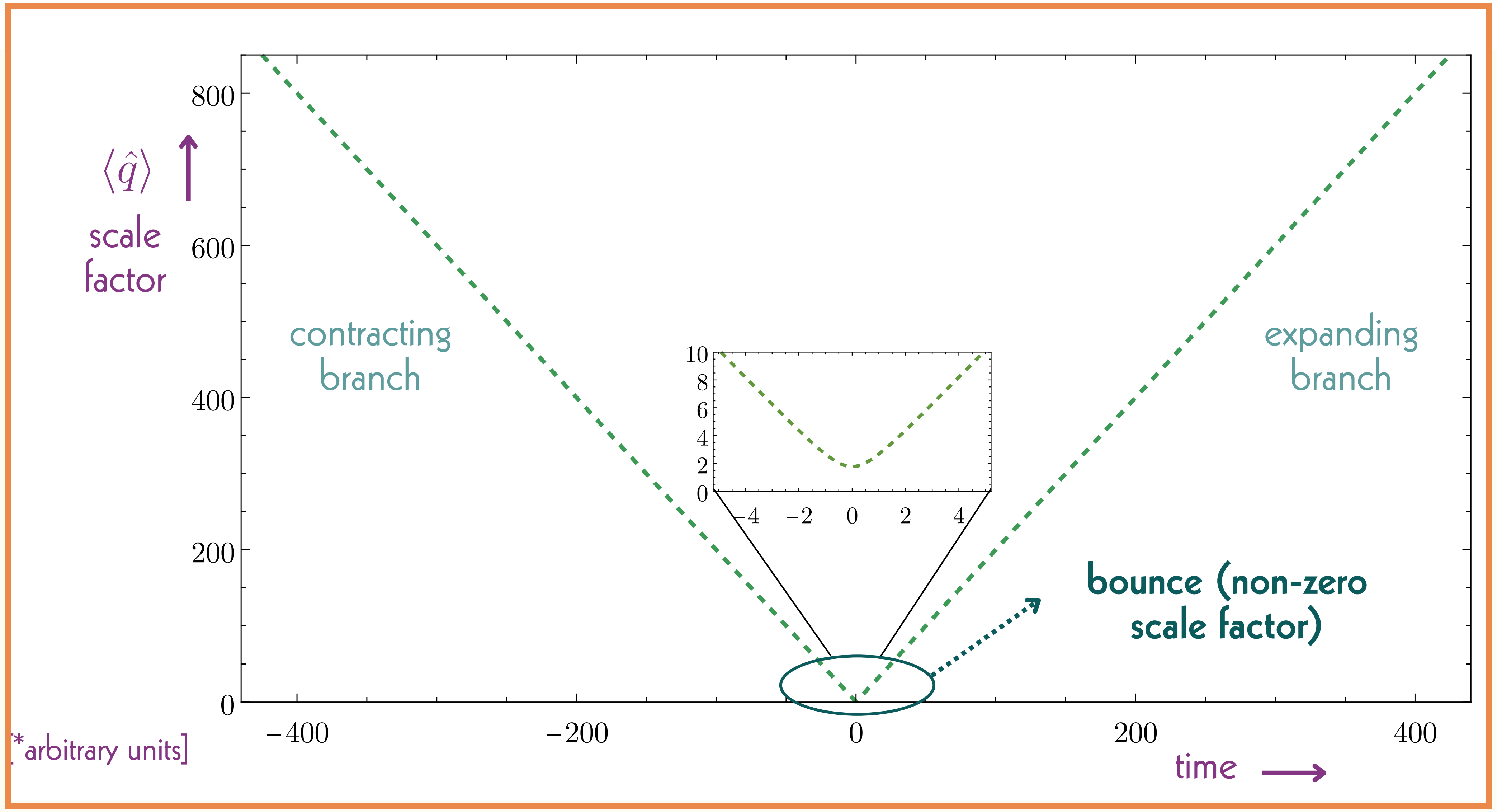
EFFECTIVE SCALE FACTOR EVOLUTION



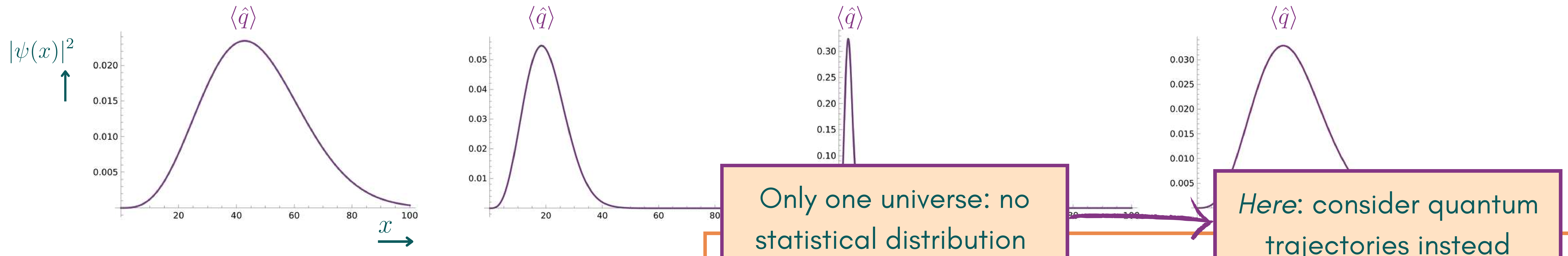
- Operator related to the scale factor

$$q \propto a^{\frac{3}{2}(1-w)}$$

- *Copenhagen viewpoint*: effective scale factory from operator expectation value in semiclassical state



EFFECTIVE SCALE FACTOR EVOLUTION



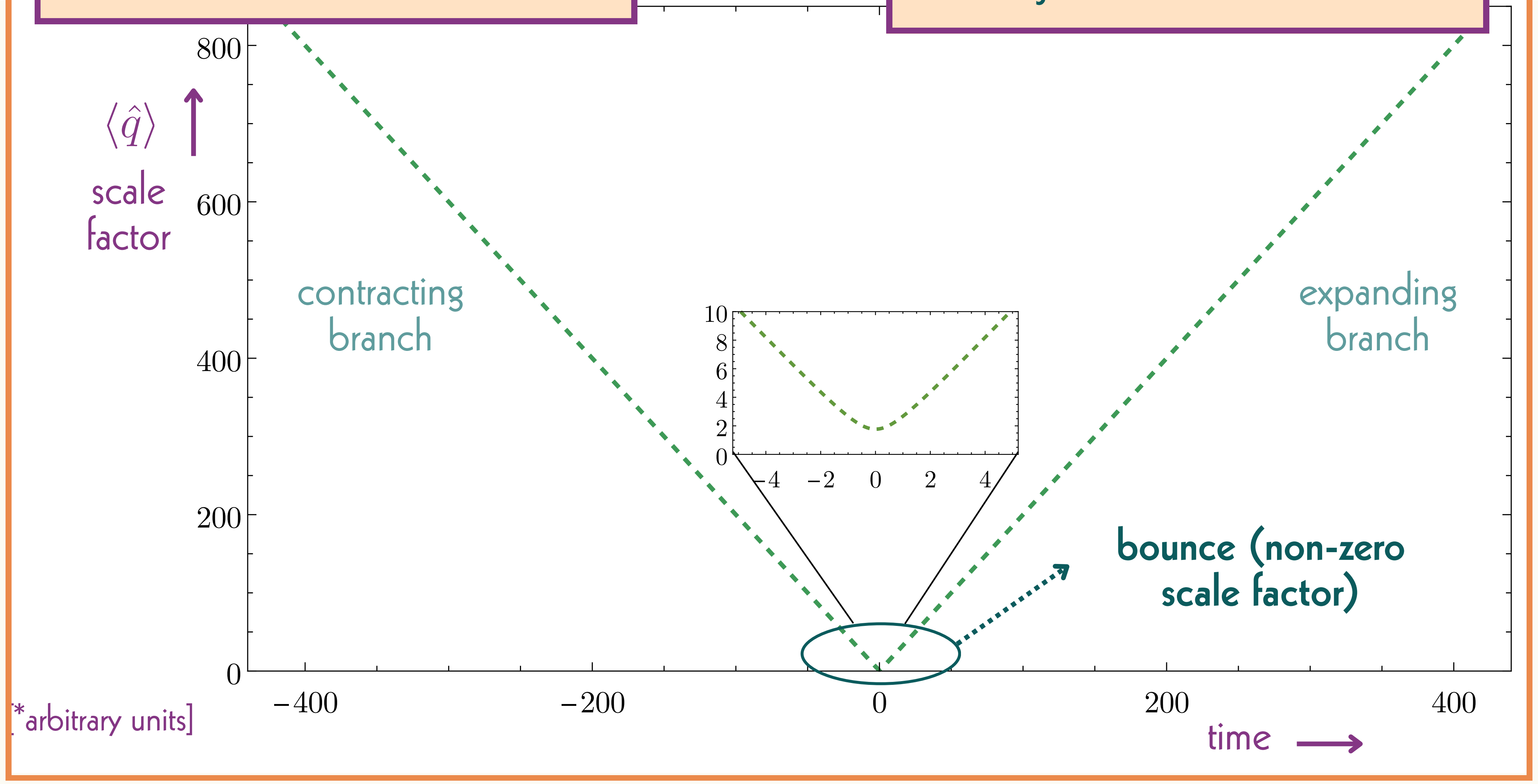
Only one universe: no statistical distribution

Here: consider quantum trajectories instead

- Operator related to the scale factor

$$q \propto a^{\frac{3}{2}(1-w)}$$

- *Copenhagen viewpoint*: effective scale factory from operator expectation value in semiclassical state



THE TRAJECTORY APPROACH

[de Broglie ('27)] [Bohm ('52)] [Holland ('93)]

In a nutshell: The quantum system follows a trajectory, but our knowledge of system properties at any given moment is limited

- Physical system = wave + point particle

$$\psi(x,t) = R(x,t)e^{\mathrm{i}S(x,t)} \longrightarrow x(t)$$

trajectory

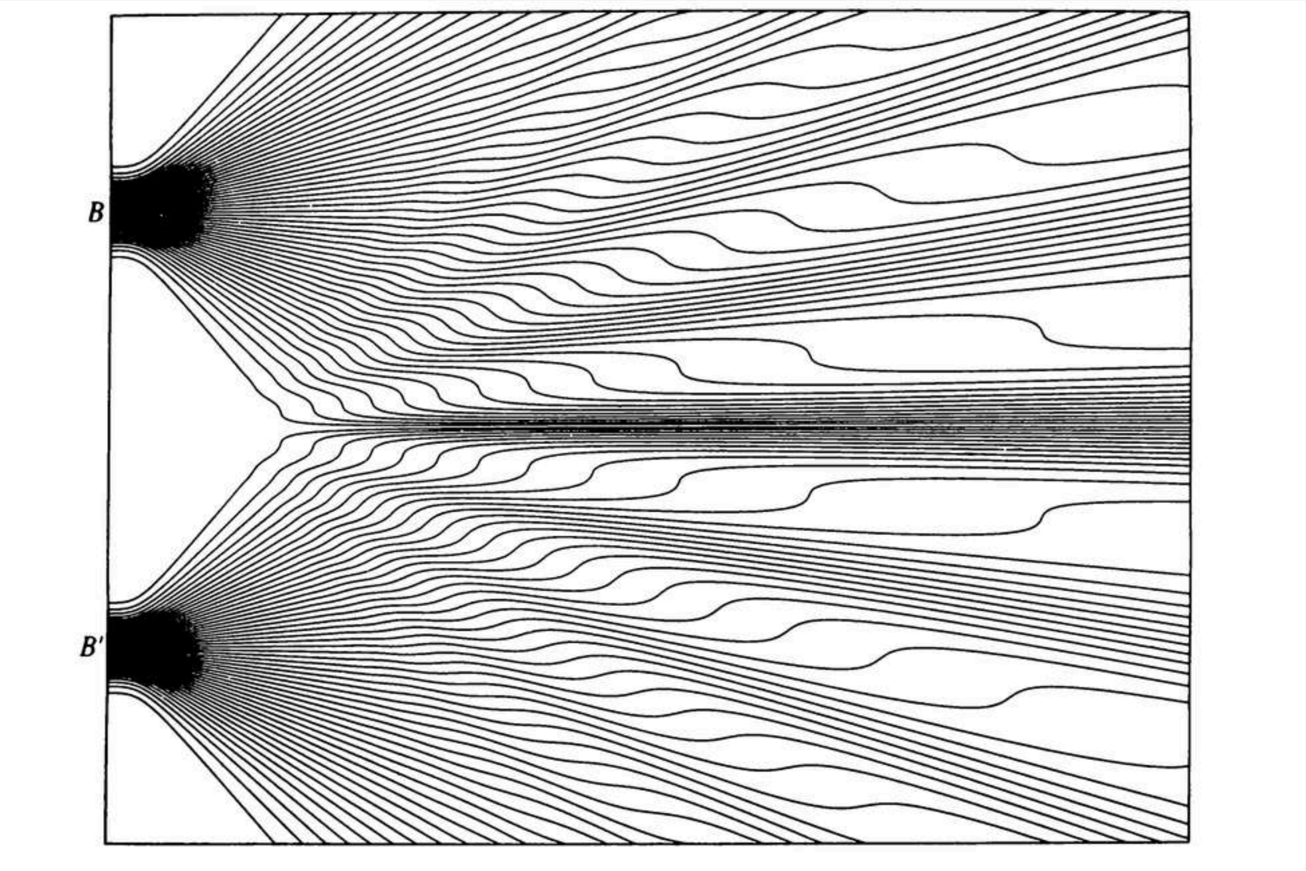
- Wave function obeys the Schrödinger equation $\mathrm{i}\partial_t\psi = H\psi$
- Particle motion from the phase of the wave function

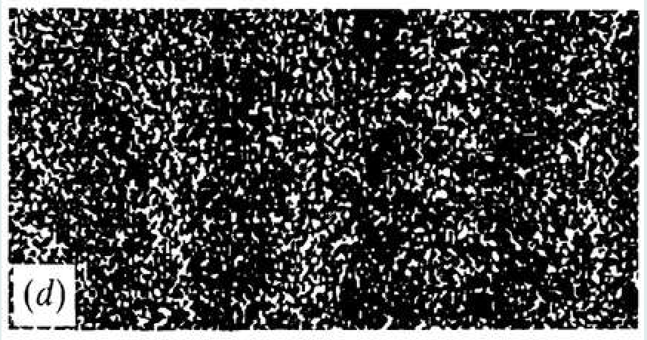
$$\dot{x} = \frac{1}{m}\nabla S(x,t)$$

  - Different initial conditions $x(t_0)$ give an ensemble of particles
- Probability of the particle to lie in an interval $x + \mathrm{d}x$ is determined by the wave function amplitude $R^2(x,t)\mathrm{d}x$

Example: trajectories for the double slit experiment

[Image: Philippidis et al., ('82)]





[Image: Tonomura et al., ('89)]

BOUNCING TRAJECTORIES

- Trajectories from semiclassical states follow semiclassical solutions

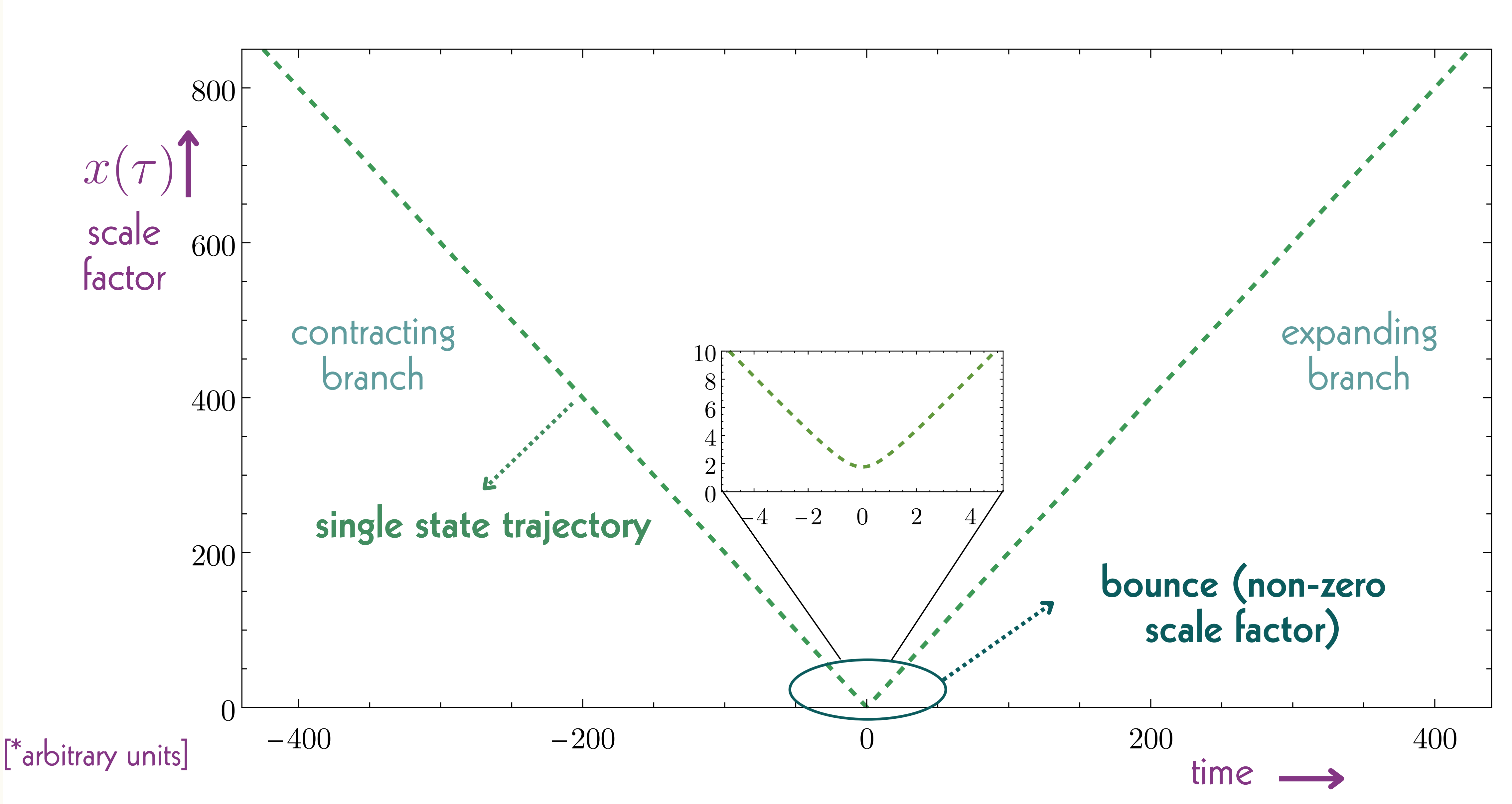
trajectory gives the scale factor: $x(\tau) \propto a^{\frac{3}{2}(1-w)}$

evolution governed by the wave function: $\frac{\mathrm{d}x}{\mathrm{d}\tau} = -\mathrm{i}\partial_x \ln \frac{\psi}{\psi^\star}$

with

$$|\psi\rangle = e^{-\mathrm{i}\phi(\tau)}|q(\tau), p(\tau)\rangle$$

$$\hat{\mathcal{H}}_{\mathrm{FLRW}}\psi - \mathrm{i}\partial_\tau\psi = 0$$



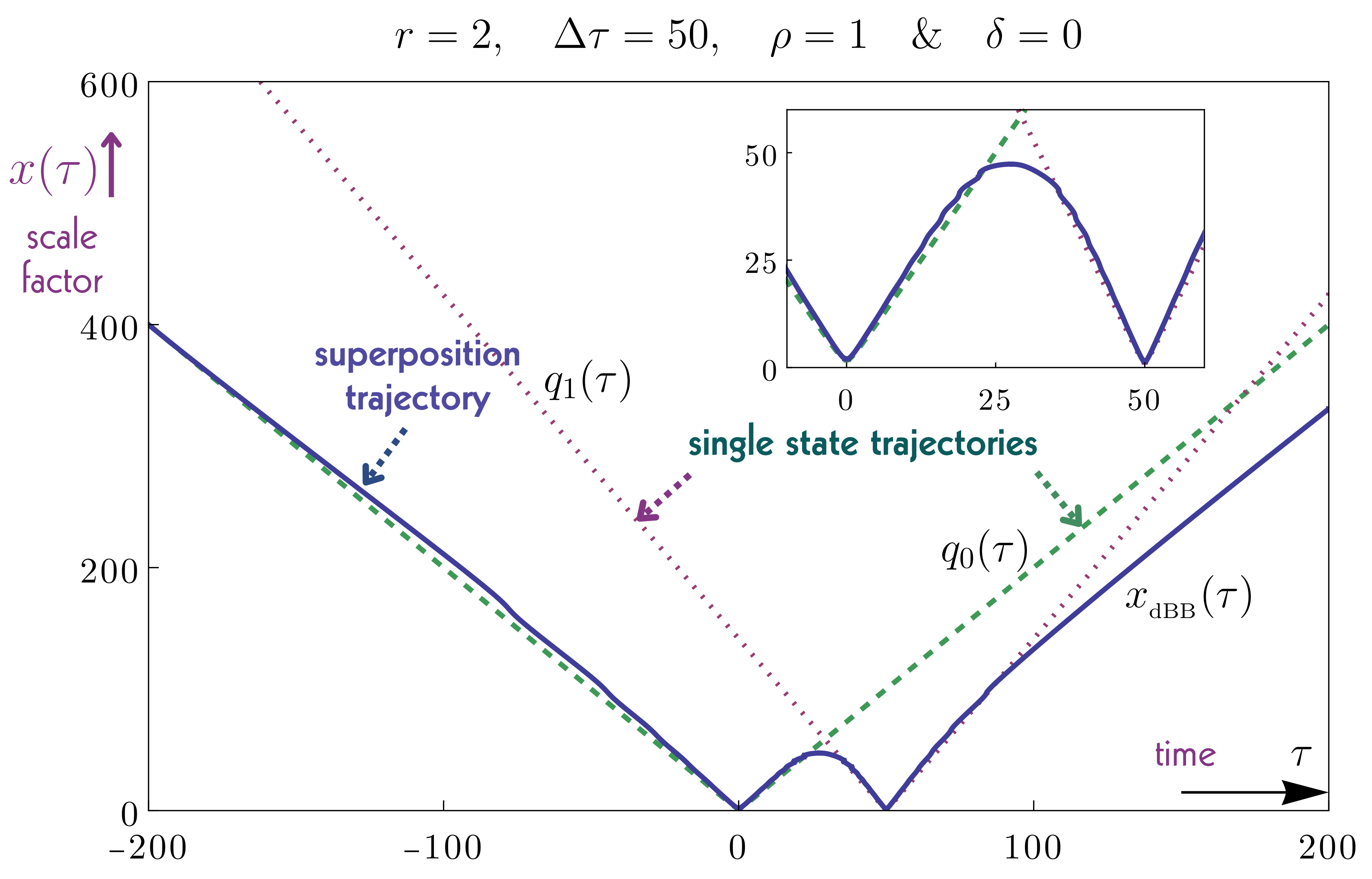
- Concrete value for the effective scale factor from trajectories at all times
- Classical dynamics away from bounce

BIVERSE TRAJECTORY

- Go beyond highly fine-tuned single state universe: **quantum superposition state**
  - Two states as sufficiently complex, but traceable example
- Biverse: $\Psi = \mathcal{N}(\psi_0 + \rho e^{i\delta}\psi_1)$

$$\boxed{\frac{\mathrm{d}x}{\mathrm{d}\tau} = -i\partial_x \ln\frac{\Psi}{\Psi^*} \quad \text{with} \quad x \propto a^{\frac{3}{2}(1-w)}}$$

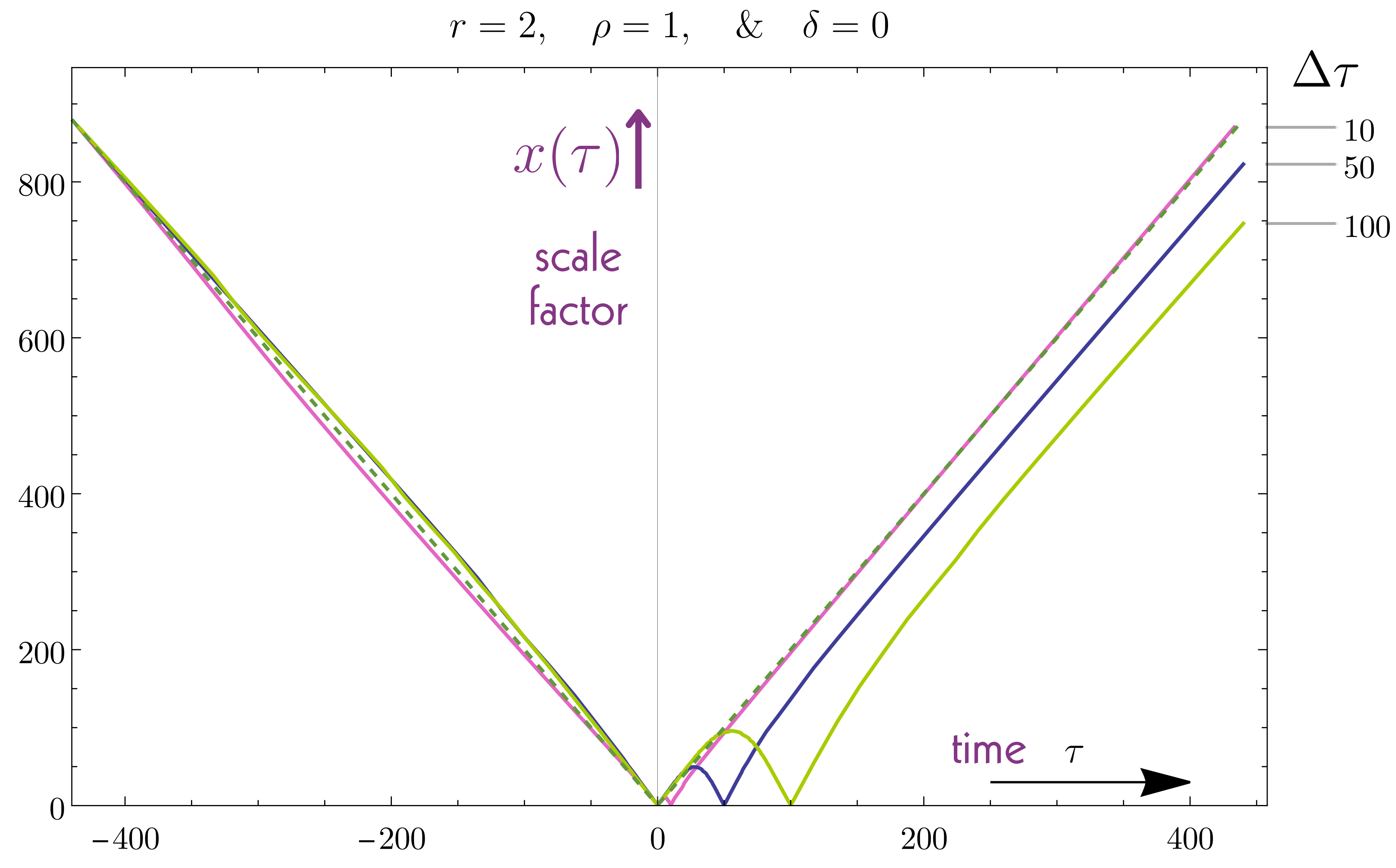
- Start on semiclassical trajectory
- Parameters:
  - r ◦ ratio of late time momenta of semiclassical solutions
  - $\Delta\tau$ ◦ difference in bounce times
  - $\rho,\ \delta$ ◦ contribution of second wave function

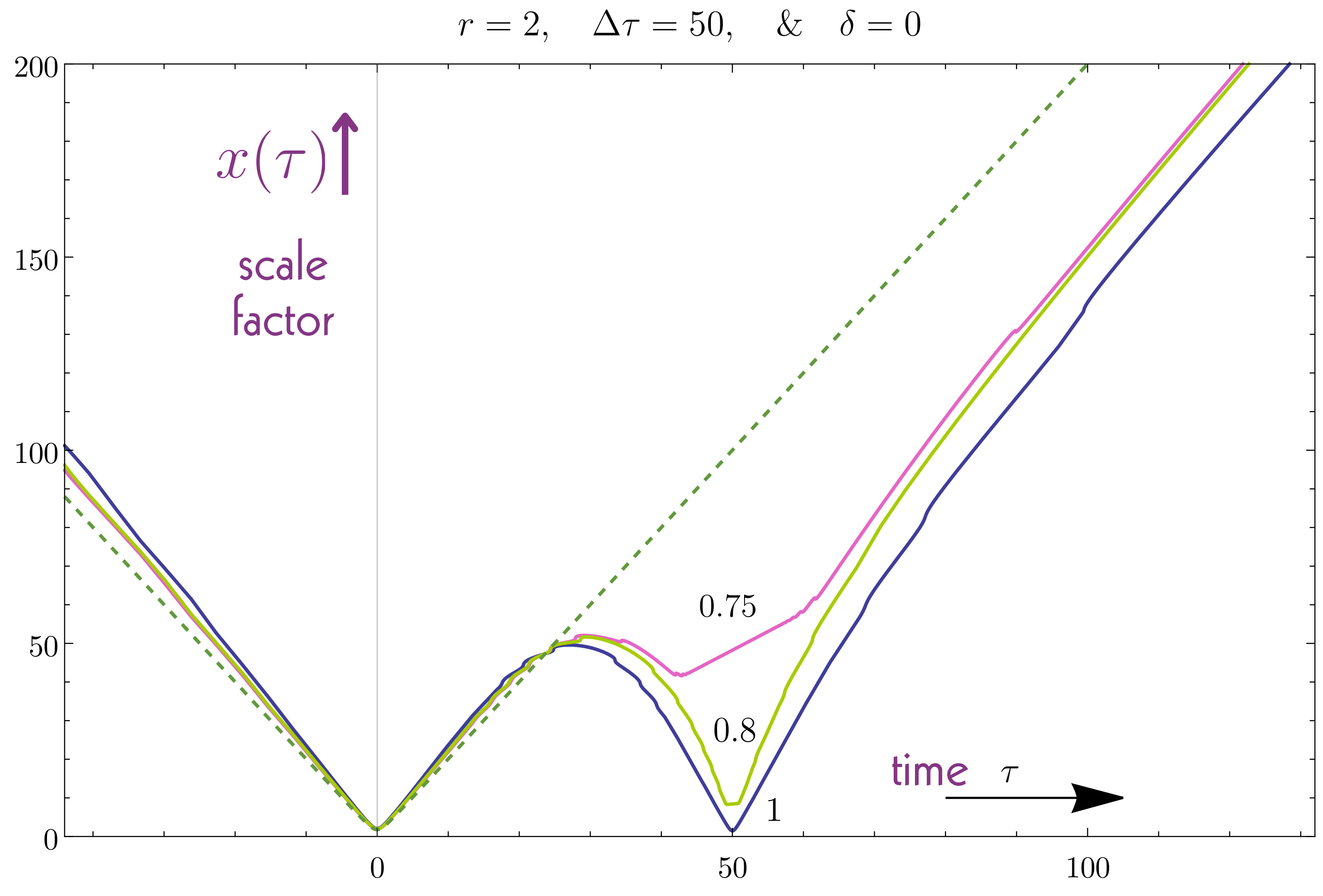


BIVERSE TRAJECTORIES: PARAMETERS

- Vary time differences between the bounces $\Delta\tau$

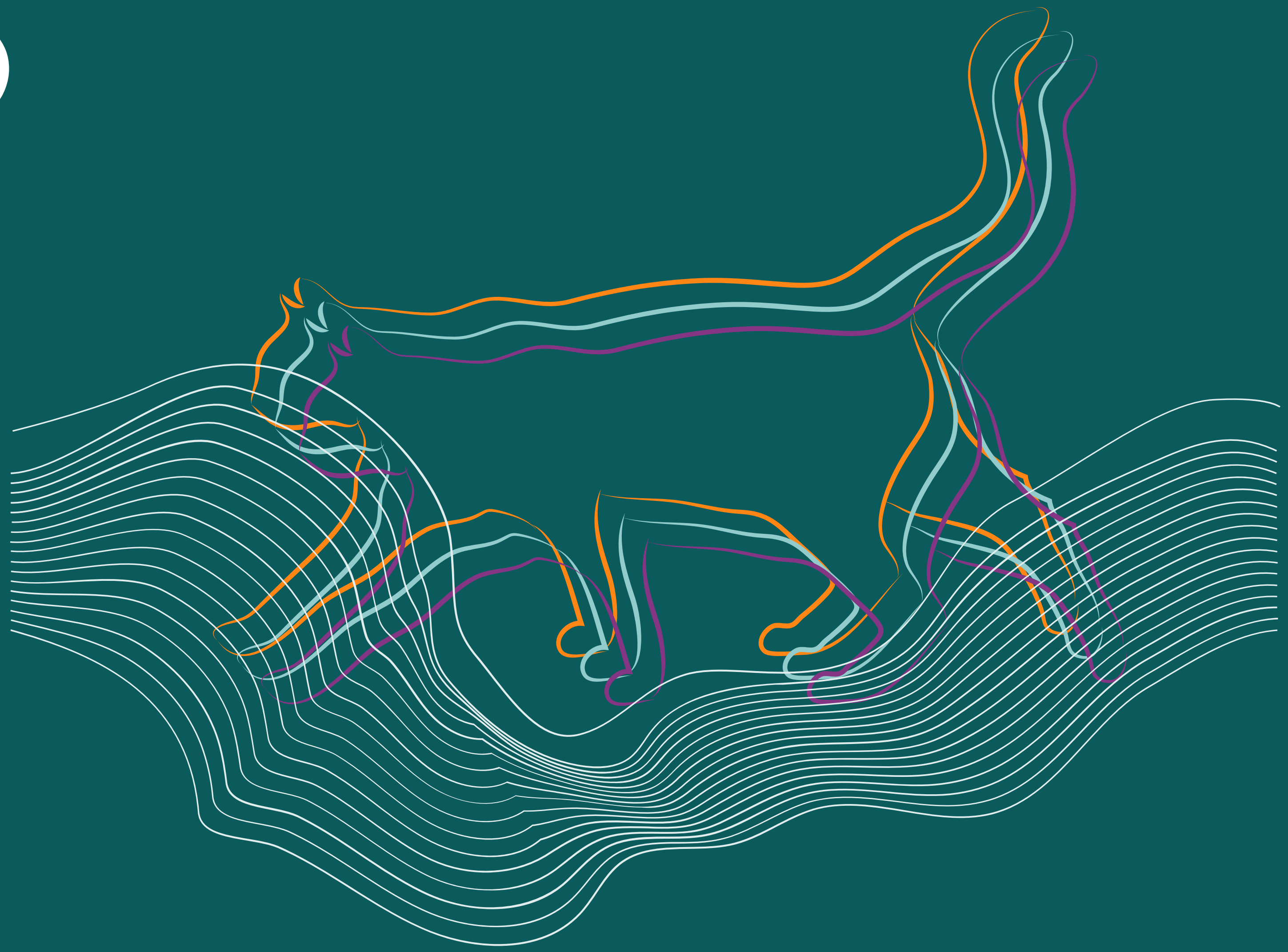
- Vary relative contribution ρ of the second wave function $\Psi = \mathcal{N}(\psi_0 + \rho e^{i\delta}\psi_1)$



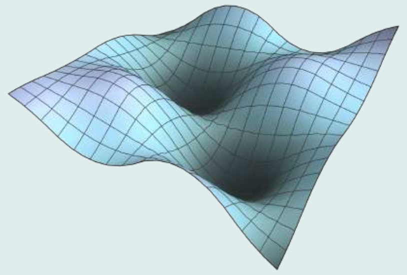


PERTURBATIONS

and the impact of biverse trajectories



FIRST APPROACH TO PERTURBATIONS



- Consider **tensor** perturbations in a flat FLRW spacetime with a radiation fluid (set $w = 1/3$)

$$\mathrm{d}s^2 = a^2(\eta)\left(-\mathrm{d}\eta^2 + [\delta_{ij} + h_{ij}(\eta, \vec{x})]\,\mathrm{d}x^i\mathrm{d}x^j\right)$$

[Peter, Pinho, Pinto-Neto, ('06)] [Peter, Pinho, Pinto-Neto, ('05)]

- Second order linear perturbative Hamiltonian for tensor modes: sum of the two polarisations and Fourier modes

$$\mathcal{H}_{\rm FLRW} \to \mathcal{H}_{\rm FLRW} + \mathcal{H}^{(2)}$$

$$\mathcal{H}^{(2)} = \sum_{\vec{k}} \left(\mathcal{H}^{(2)}_{\vec{k},+} + \mathcal{H}^{(2)}_{\vec{k},\times}\right) \quad \text{with} \quad \mathcal{H}^{(2)}_{\vec{k},\lambda} = \pi^{(\lambda)}_{\vec{k}}\pi^{(\lambda)}_{-\vec{k}} + \left(k^2 - \frac{a''}{a}\right)\mu^{(\lambda)}_{\vec{k}}\mu^{(\lambda)}_{-\vec{k}}$$

$\lambda = +, \times$

conjugate momentum of $\mu^{(\lambda)}_{\vec{k}}$

$h_{ij} \propto \mu_{ij}/a$

- Wave function Ansatz: $\Psi(x, \{\mu_k\}) = \psi_{\rm bg}(x)\psi_{\rm pert}(x, \{\mu_k\})$

Background:	Perturbations:
$\mathcal{H}_{\rm FLRW}\psi_{\rm bg}(x) = \mathrm{i}\partial_\tau\psi_{\rm bg}(x)$	$\mathcal{H}^{(2)}\psi_{\rm pert}(x(\tau), \{\mu_k\}) = \mathrm{i}\partial_\tau\psi_{\rm pert}(x(\tau), \{\mu_k\})$ background dependent trajectory extracted from the background

PERTURBATIVE DYNAMICS

- Dynamics of perturbation modes

$$\mu_k'' + \left(k^2 - \frac{a''}{a}\right)\mu_k = 0$$

$h_{ij} \propto \mu_{ij}/a$ $\quad V_{\rm eff}$ modified by trajectories

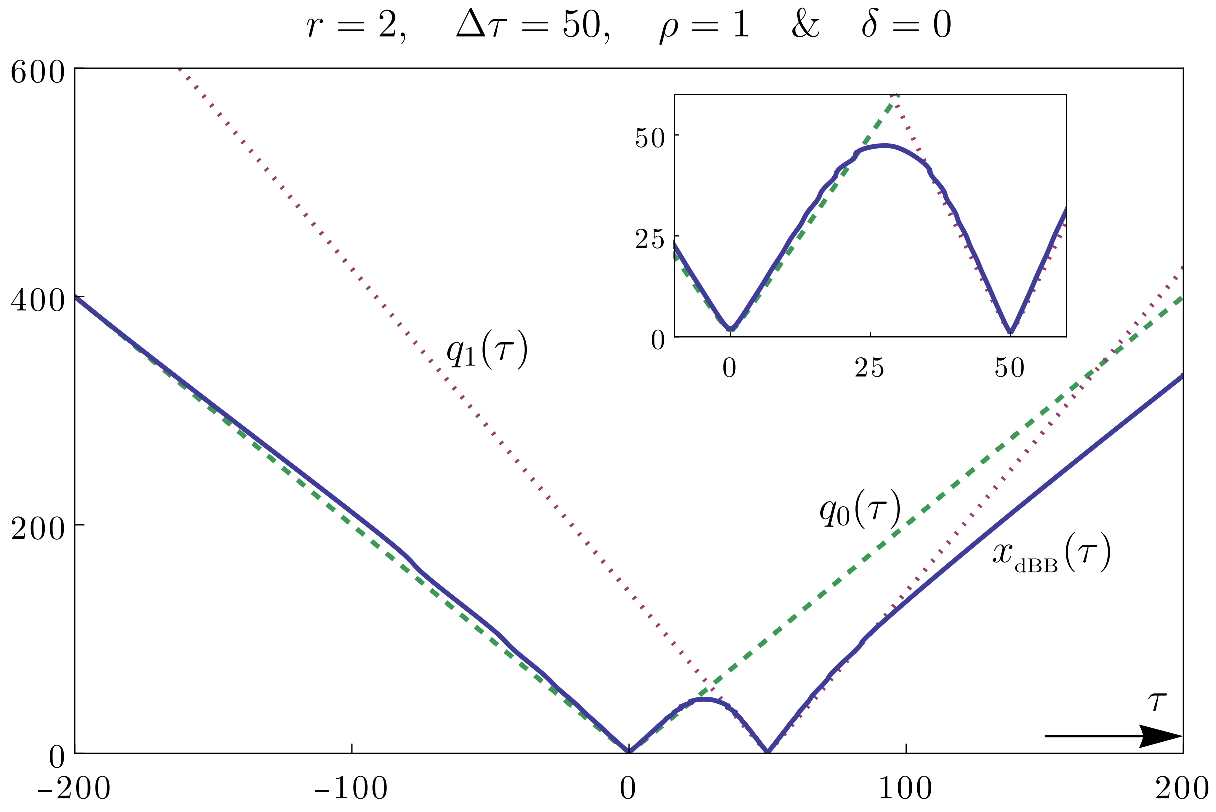
With $\hat{\mu}_{\vec{k}}(\eta) = \mu_k(\eta)\hat{a}_{\vec{k}} + \mu_k^*(\eta)\hat{a}^\dagger_{-\vec{k}}$

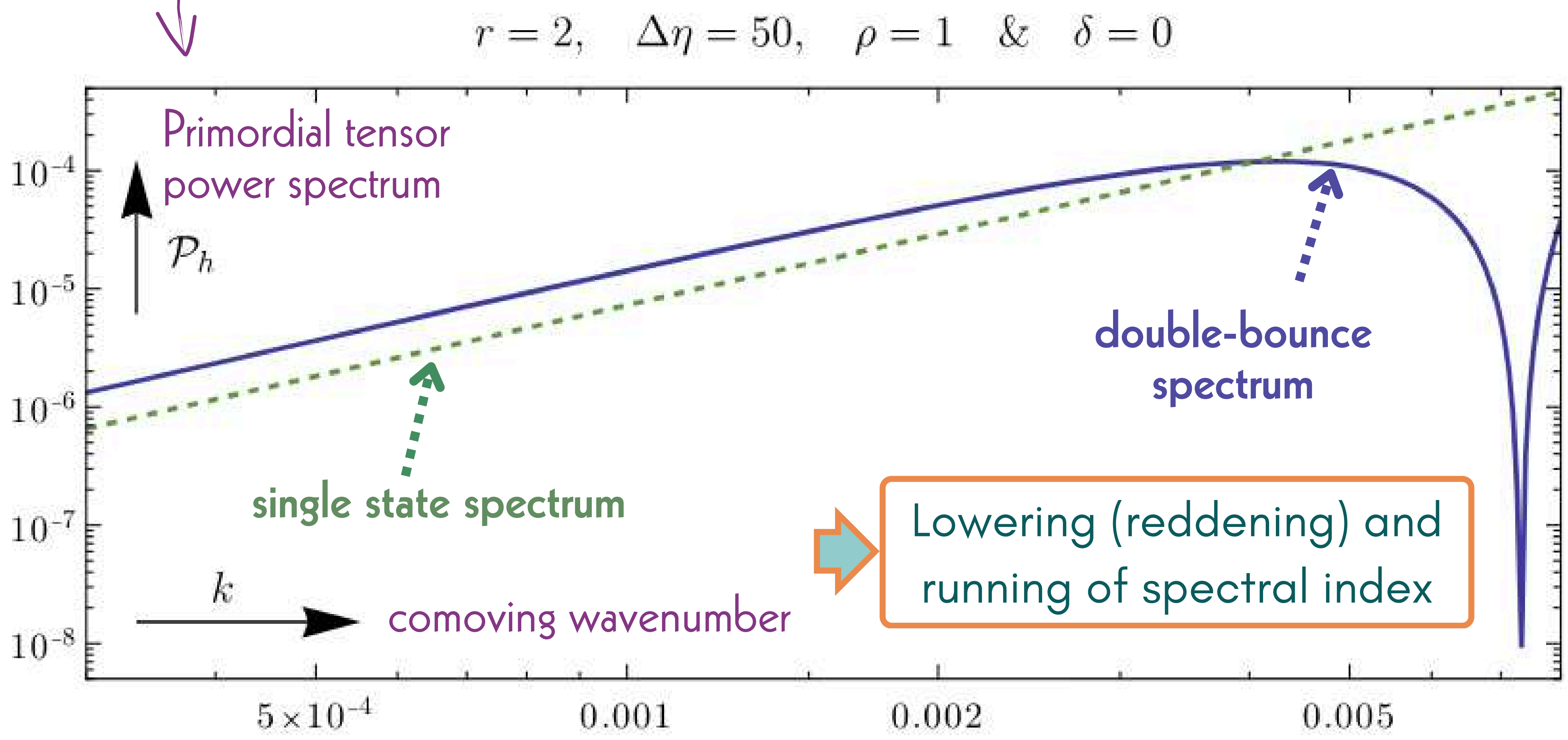
creation and annihilation operators

- Initial conditions: "Bunch-Davies" vacuum

$$\mu_k(\eta) = \frac{1}{\sqrt{2k}} e^{-ik(\eta-\eta_i)}$$

- Power spectrum $\mathcal{P}_h \propto k^3 \left|\frac{\mu_k(\eta)}{a(\eta)}\right|^2_{\eta=\eta^*}$



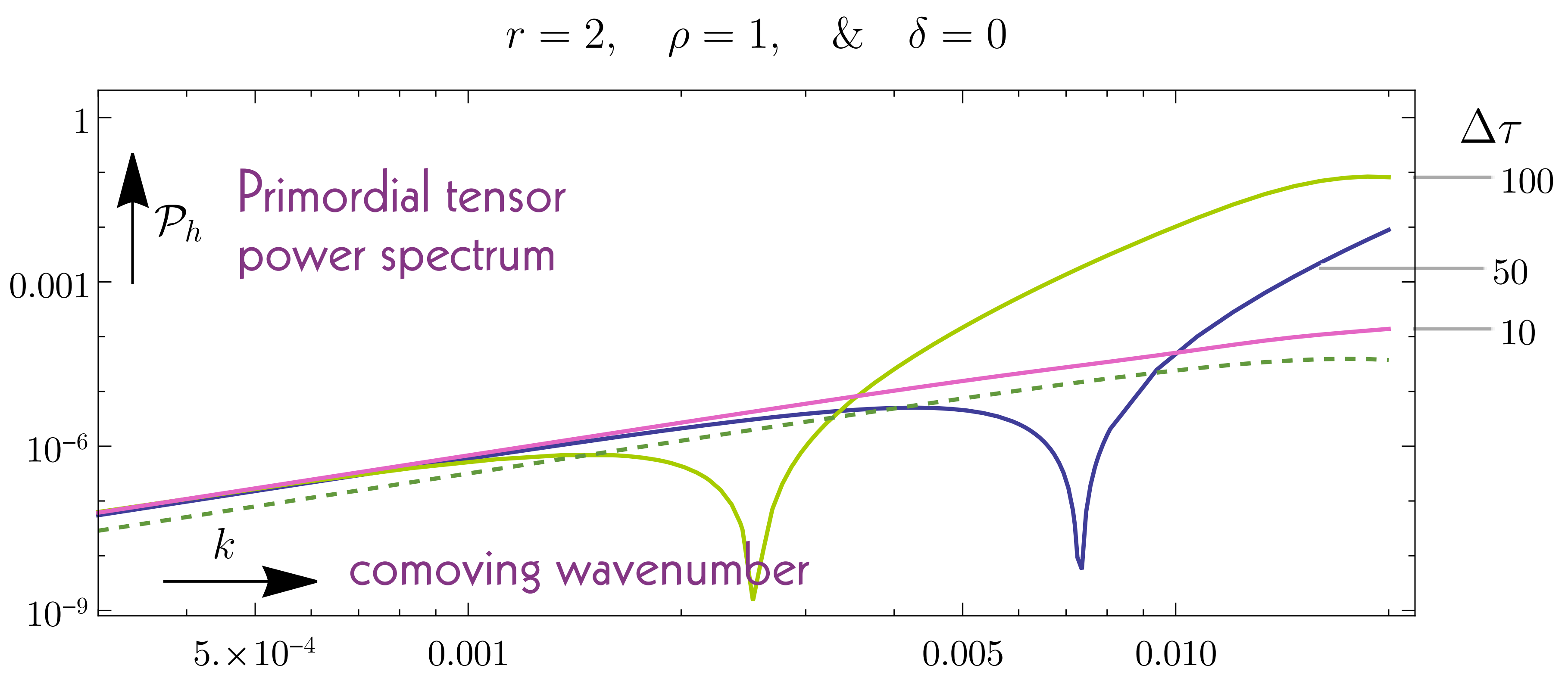


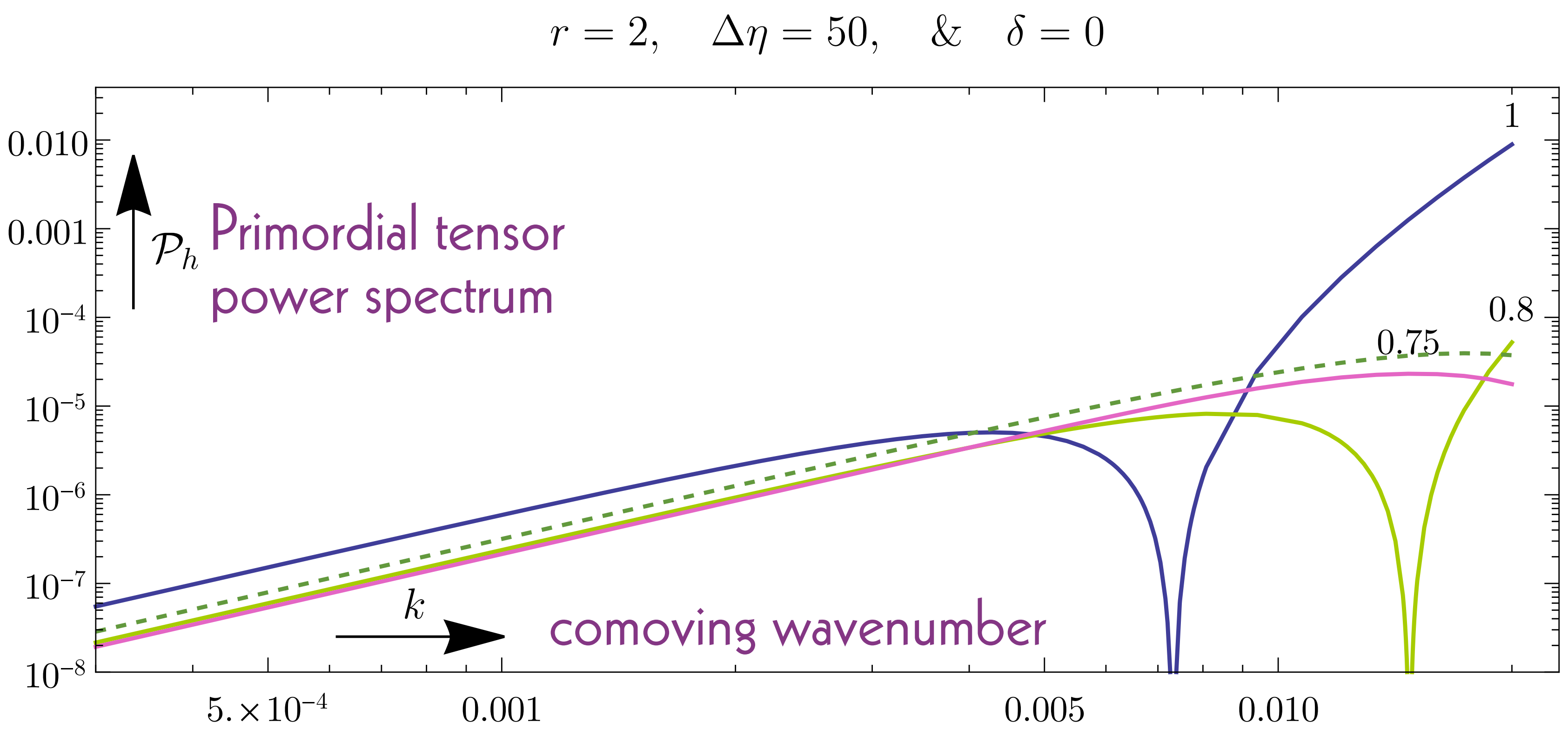
PARAMETER DEPENDENCE OF SPECTRA

- Changing time difference between bounces

  $\rightarrow$ feature at different scale

- Changing the relative contribution ρ of the second semiclassical state $\Psi = \mathcal{N}(\psi_0 + \rho e^{i\delta}\psi_1)$





EMBEDDING THE BIVERSE IN COSMOLOGY

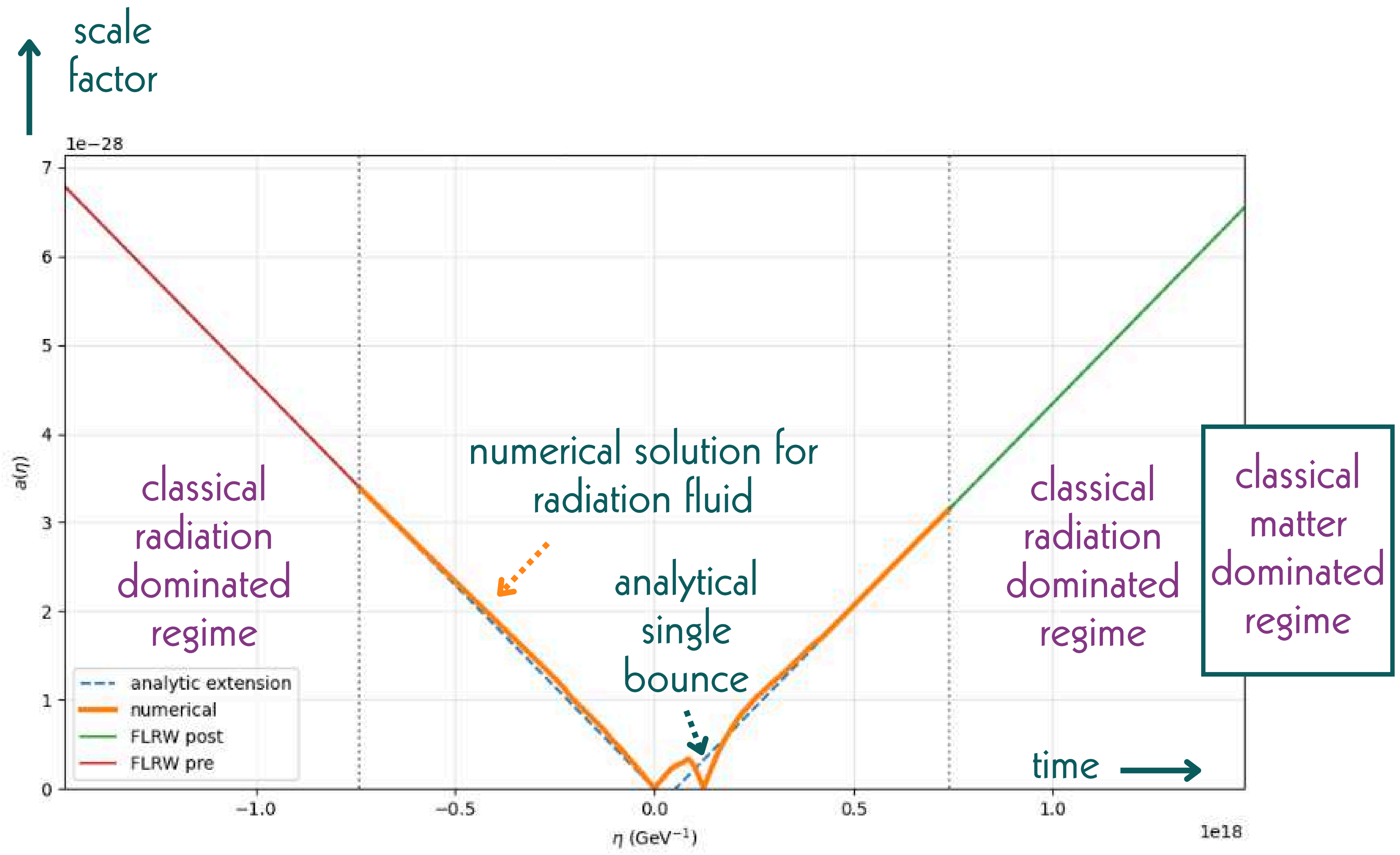
work in progress

- Late time biverse trajectory mimics classical evolution
- Embed numerical solution in single bounce model described by an effective Friedmann equation

$$\frac{H^2}{H_0^2} = \underbrace{\Omega_r a^{-4}}_{\text{radiation}} + \underbrace{\Omega_m a^{-3}}_{\text{matter}} + \underbrace{\Omega_\Lambda}_{\text{cosm. const.}} \underbrace{- \Omega_Q a^{-6} \left(\frac{a}{a-\lambda_Q}\right)^2}_{\text{effective quantum contribution}}$$

- Single bounce two-fluid model consistent with scale invariant primordial power spectrum

[Vitenti, Pinto-Neto, Peter, Demétrio, (`26)]



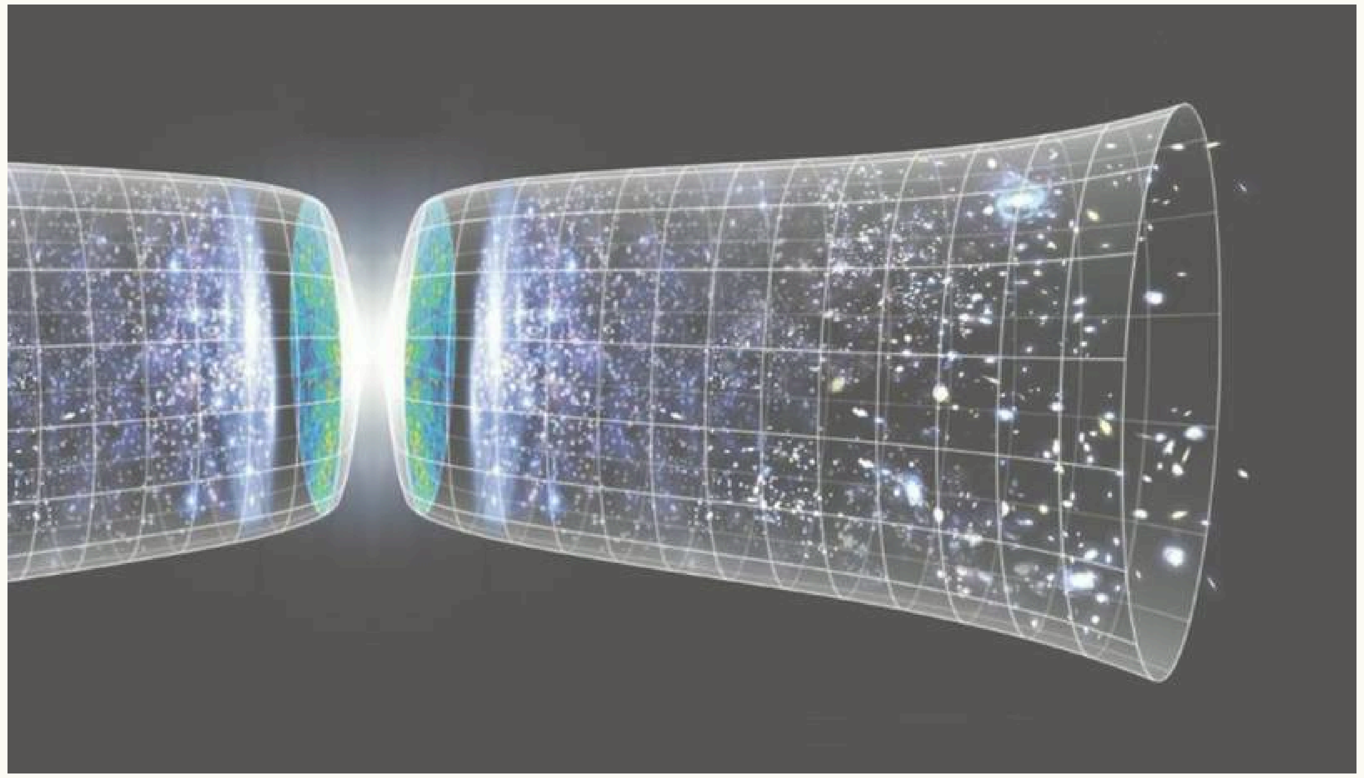
[Image: Guillaume Desaché]

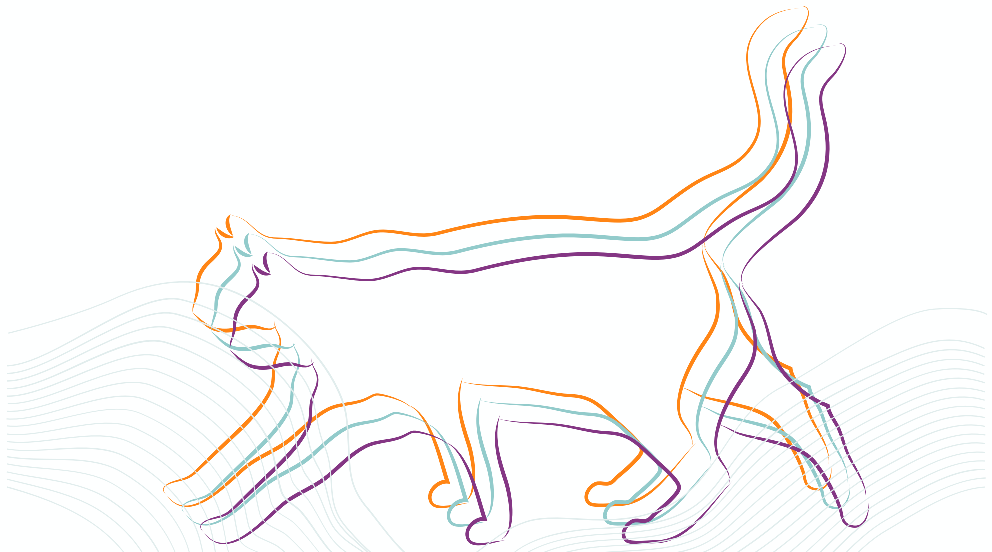
CONCLUSION

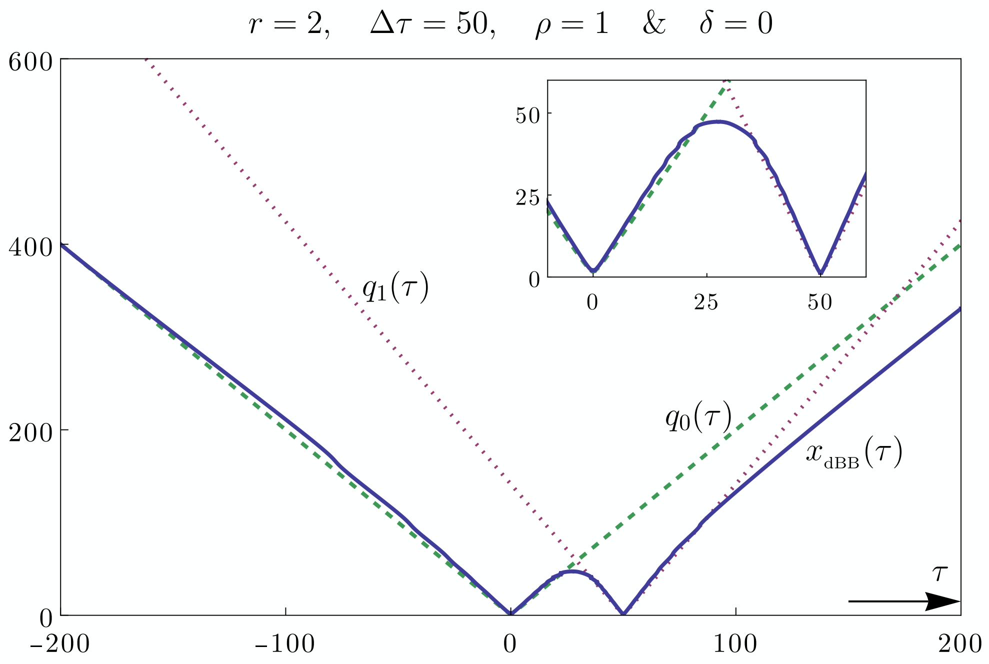
... and next steps

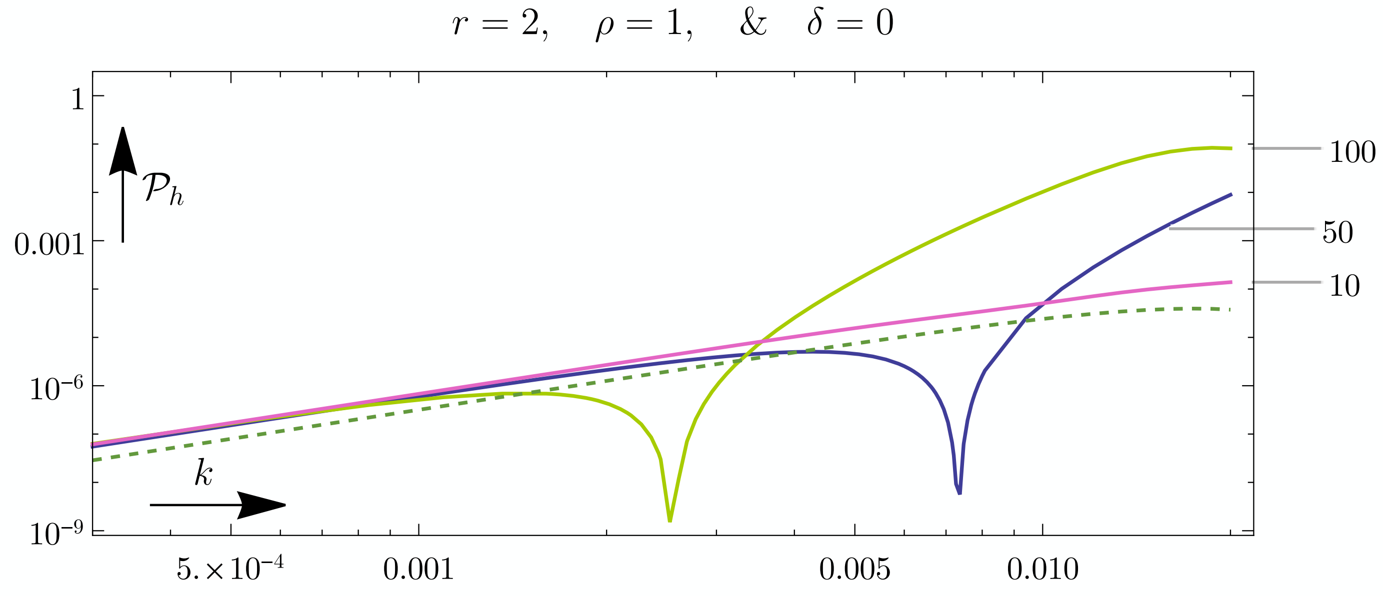
CONCLUSION

- Affinely quantise an FLRW spacetime to obtain a universe in which the big bang is replaced by a bounce
- **Trajectories** for a universe in a **superposition** introduce distinct features in the scale factor evolution
- These features affect the evolution of **perturbations** and thereby the **tensor power spectrum**



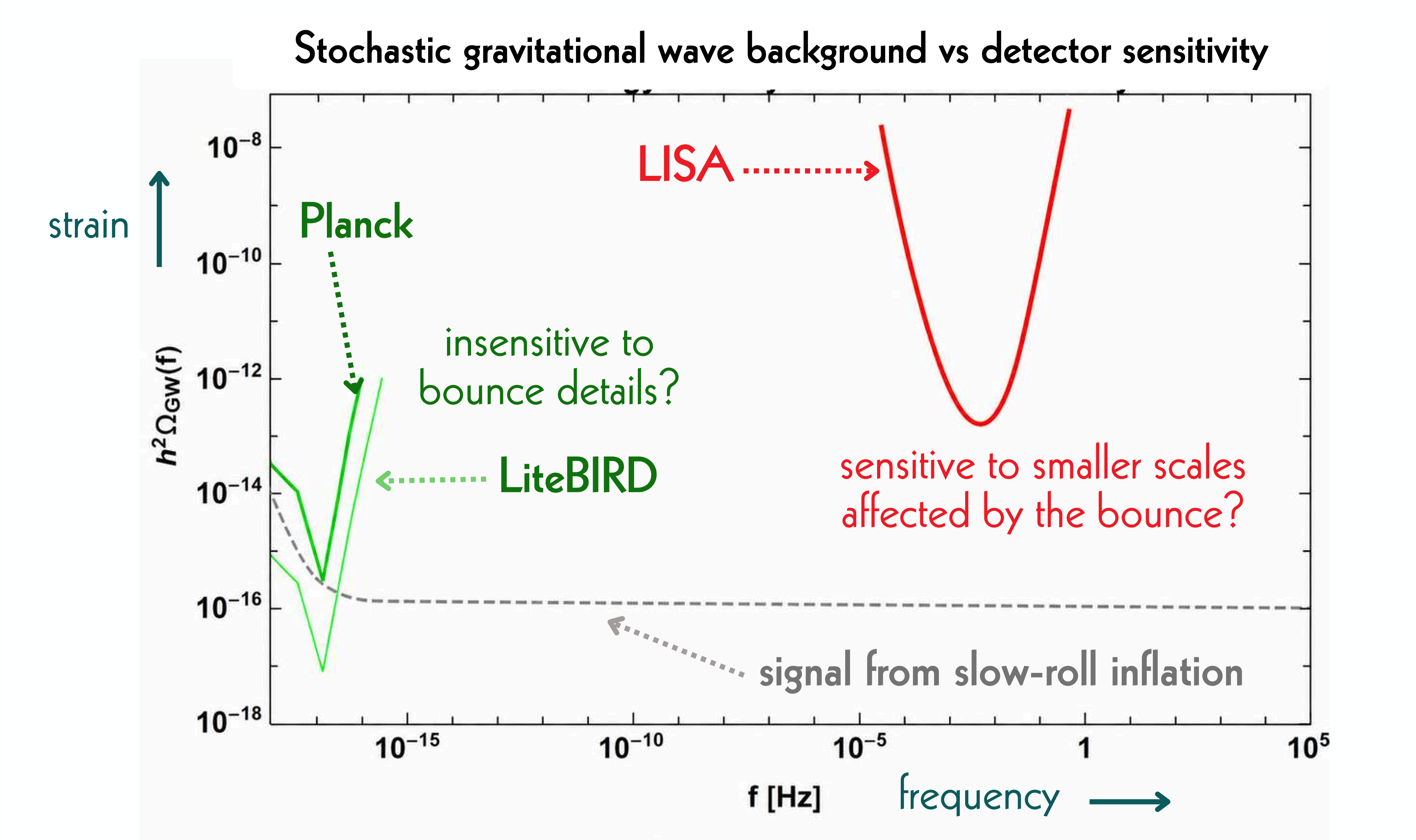






OUTLOOK

- Develop theoretical treatment of scalar perturbations: equations of motion for amended background evolution
- Constrain the parameter space from current CMB data
- Consider imprints in future data for early universe gravitational waves

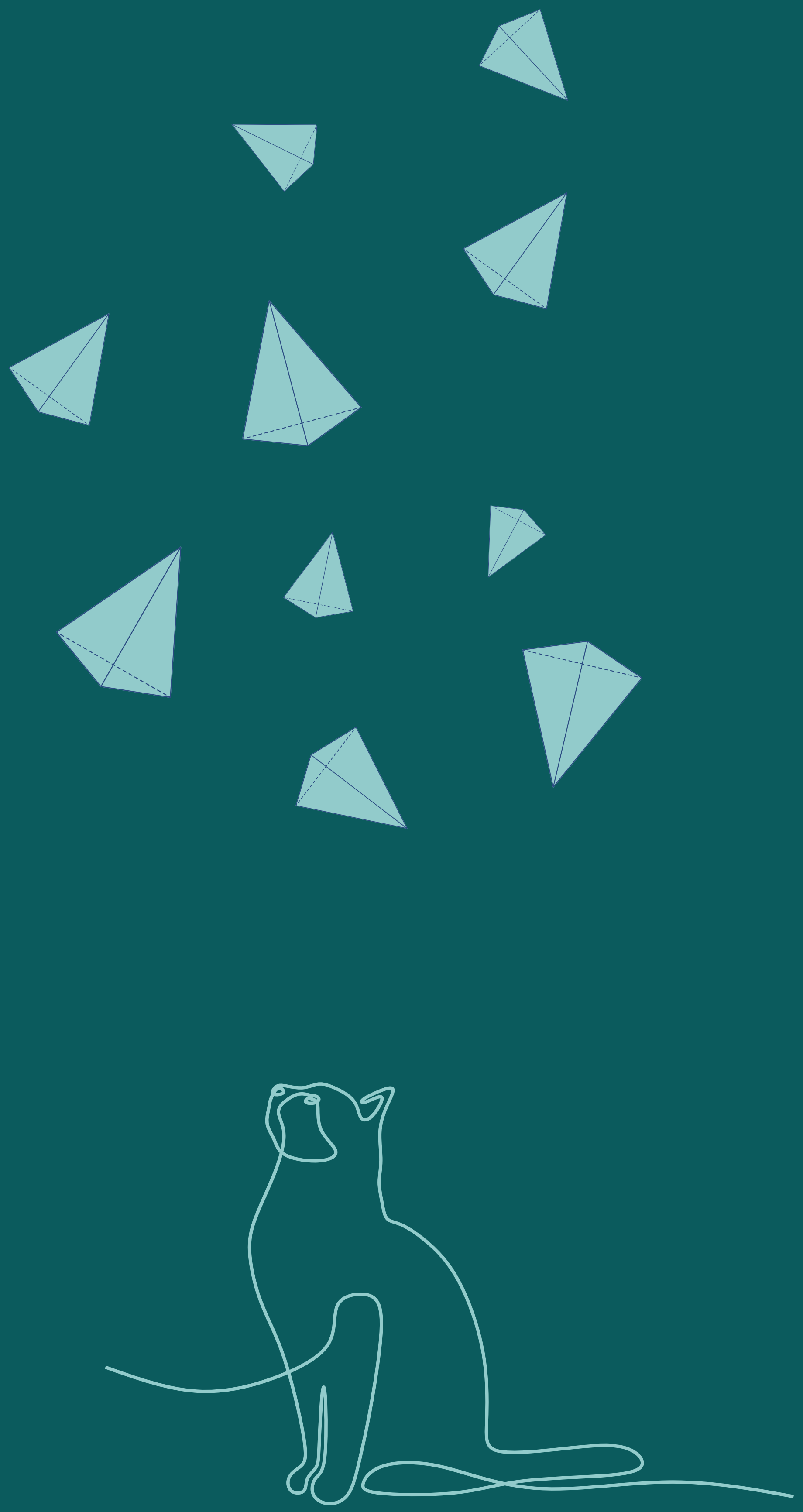


[Adapted from Auclair et. at. (`23)]

THANK YOU!

Questions...?

BACKUP



ADM HAMILTONIAN FOR GENERAL RELATIVITY

- Foliation of spacetime: spatial hypersurfaces with spatial metric and time direction $M = \Sigma \times \mathbb{R}$
- Time evolution: shift and lapse give translations parallel and orthogonal to hypersurfaces

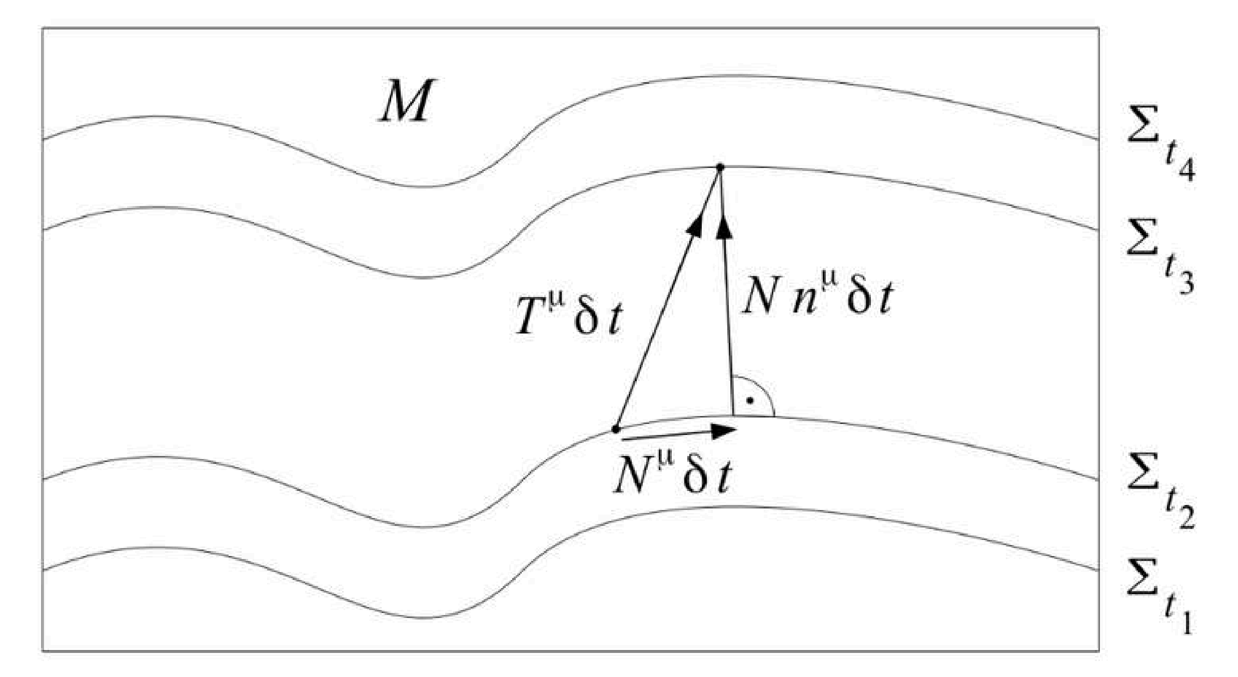
lapse

$$g_{\mu\nu} = \begin{pmatrix} -N^2 + N^a N_a & N_a \\ N_a & q_{ab} \end{pmatrix}$$

shift

spatial metric

[Image: Bodendorfer, ('16)]



- Hamiltonian: sum of Hamiltonian and diffeomorphism constraint
  - Lapse and shift give gauge choice
  - Equal time Poisson brackets

$$\{q_{kl}(x), \pi^{ab}(y)\} = \delta^a_{(k}\delta^b_{l)}\delta^{(3)}(x,y)$$

conjugate momentum of q_{ab}

spatial Ricci scalar

$$\mathcal{H}_{\mathrm{ADM}} = \int_\Sigma \mathrm{d}^3x \left[\underbrace{N_a(\nabla_b \pi^{ab})}_{\mathcal{H}^a_{\mathrm{ADM}}} + N \underbrace{\left(\frac{2\kappa}{\sqrt{q}}(\pi^{ab}\pi_{ab} - \frac{1}{2}\pi^2) - \frac{\sqrt{q}}{2\kappa}{}^{(3)}R \right)}_{\mathcal{H}_{\mathrm{ADM,S}}} \right]$$

$\kappa = 8\pi G$

- "GR is a fully constrained system"

$$\mathcal{H}_{\mathrm{GR}} = N(\underbrace{\mathcal{H}_{\mathrm{ADM,S}} + \mathcal{H}_{\mathrm{matter,S}}}_{\approx 0}) + N_a(\underbrace{\mathcal{H}^a_{\mathrm{ADM}} + \mathcal{H}^a_{\mathrm{matter}}}_{\approx 0})$$

WHEELER-DEWITT QUANTISATION

- Wave functional $\Psi[q_{ab}(x), \Phi(x)]$ with constraints $\hat{\mathcal{H}}^a\Psi = 0$ and $\hat{\mathcal{H}}_{\mathrm{S}}\Psi = 0$
- Issues:
  - Definition of Hilbert space – inner product?
  - Problem of time: evolution happens w.r.t. an internal degree of freedom serving as a clock

[DeWitt ('67)]

[Isham, ('93)]

- **Minisuperspace models**: Wheeler-DeWitt quantisation on the reduced phase space of general relativity to obtain a quantum description of the universe
  - Quantise phase space of FLRW spacetime $\mathrm{d}s^2 = -N^2(t)\mathrm{d}t^2 + a^2(t)\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j$
    - $q_{ab} \to a\ ,\quad \pi^{ab} \to p_a\ ,\quad \{a, p_a\} = 1$

lapse

$\kappa = 8\pi G$

$$\mathcal{H}_{\mathrm{ADM}} \to \mathcal{H}_{\mathrm{FLRW}} = -\frac{\kappa N}{12\mathcal{V}_0 a}p_a^2$$

momentum conjugate to scale factor

scale factor

spatial section volume

approximation to a full theory of quantum gravity

QUANTISATION

- Quantisation based on the Weyl-Heisenberg group $x \in \mathbb{R}\,,\ \ p \in \mathbb{R}$
  - $U(q,p)\psi(x) = e^{\mathrm{i}p(x-q/2)}\psi(x-q)$

- Here: $x \geq 0\,,\ \ p \in \mathbb{R}$ $\longrightarrow$ use affine group $U(q,p)\psi(x) = \dfrac{e^{\mathrm{i}px}}{\sqrt{q}}\psi\left(\dfrac{x}{q}\right)$

- Quantisation map $\hat{A}_f = \mathcal{N}\int_{\mathbb{R}\times\mathbb{R}^+} \mathrm{d}p\mathrm{d}q\,|q,p\rangle\, f(p,q)\,\langle q,p|$ where $|q,p\rangle = U(q,p)|\psi_0\rangle$

  fiducial state

  phase space function

  coherent state

- Normalisation condition:

$$\langle x|\hat{A}_1(\psi_0)|\psi\rangle = \mathcal{N}\int_{\mathbb{R}\times\mathbb{R}^+}\mathrm{d}q\mathrm{d}p\langle x|q,p\rangle\int_{\mathbb{R}^+}\mathrm{d}y\langle q,p|y\rangle\langle y|\psi\rangle = \left(2\pi\mathcal{N}\int_0^\infty \frac{\mathrm{d}z}{z^2}\psi_0^2(z)\right)\psi(x) \stackrel{!}{=} \psi(x)$$

- Action of the position and momentum operator:
  - $\hat{q}\,\psi = x\,\psi$
  - $\hat{p}\,\psi = -\mathrm{i}\partial_x\psi$

  imposing on the fiducial state $\int_0^\infty \frac{\mathrm{d}z}{z^2}\psi_0^2(z) = \int_0^\infty \frac{\mathrm{d}z}{z^3}\psi_0^2(z)$

[Klauder, ('99)]

[Bergeron, Dapor, Gazeau, Małkiewicz, ('14)]

QUANTUM HAMILTONIAN

- Here: $x \geq 0\,, \quad p \in \mathbb{R}$ $\longrightarrow$ use affine group $\quad U(q,p)\psi(x) = \dfrac{e^{ipx}}{\sqrt{q}}\psi\left(\dfrac{x}{q}\right)$
- Quantisation map $\hat{A}_f = \mathcal{N}\displaystyle\int_{\mathbb{R}\times\mathbb{R}^+} \mathrm{d}p\mathrm{d}q\,|q,p\rangle\, f(p,q)\,\langle q,p|$ where $\underbrace{|q,p\rangle}_{\text{coherent state}} = U(q,p)\underbrace{|\psi_0\rangle}_{\text{fiducial state}}$

- Applying the quantisation map to the FLRW Hamiltonian $\mathcal{H}_{\mathrm{FLRW}} \propto p^2$ leads to a repulsive potential

$$\boxed{\hat{A}_{p^2}\,\psi = -\partial_x^2\,\psi + \frac{\overbrace{K}^{\text{constant}}}{x^2}\psi}$$

- Total Hamiltonian

$$\mathcal{H} \propto p^2 + p_\tau \quad \longrightarrow \quad \boxed{\hat{\mathcal{H}} \propto \hat{p}^2 + \underbrace{\frac{K}{\hat{q}^2}}_{\text{introduces a bounce}} + \overbrace{\hat{p}_\tau}^{\text{time evolution}}}$$

with

$$\begin{aligned} \hat{p}\,\psi &= -\mathrm{i}\partial_x\psi \\ \hat{q}\,\psi &= x\,\psi \\ \hat{p}_\tau\psi &= -\mathrm{i}\partial_\tau\psi \end{aligned}$$

- Schrödinger equation $\hat{\mathcal{H}}\psi = \hat{\mathcal{H}}_{\mathrm{FLRW}}\psi - \mathrm{i}\partial_\tau\psi = 0$

[Klauder, ('99)] [Bergeron, Dapor, Gazeau, Małkiewicz, ('14)]

THE CHOICE OF STATE

[Bergeron, Gazeau, Małkiewicz, Peter ('23)]

- Use semiclassical state $|\psi\rangle = e^{-\mathrm{i}\phi(\tau)}|q(\tau), p(\tau)\rangle$
  - Satisfies the Schrödinger equation $\hat{\mathcal{H}}_{\mathrm{FLRW}}\psi - \mathrm{i}\partial_\tau\psi = 0$
  - Follows dynamics generated by the semiclassical Hamiltonian $\mathcal{H}_{\mathrm{sem}} = p^2 + \dfrac{K}{q^2}$

$$q(\tau) = q_B\sqrt{1+\omega^2(\tau-\tau_B)^2} \qquad p(\tau) = \frac{1}{2}\dot{q}(\tau) = \frac{q_B\,\omega^2(\tau-\tau_B)}{2\sqrt{1+\omega^2(\tau-\tau_B)^2}}$$

Bounce trajectory

  - Specific choice of state $\langle x|q(\tau), p(\tau)\rangle = \dfrac{1}{\sqrt{q(\tau)}}\exp\left(\mathrm{i}\dfrac{p(\tau)}{2q(\tau)}x^2\right)\Phi_n\left(\dfrac{x}{q(\tau)}\right)$

Note: use different representation

$$V(q,p)\psi(x) = \frac{1}{\sqrt{q}}\exp\left(\mathrm{i}\frac{p}{2q}x^2\right)\psi\left(\frac{x}{q}\right)$$

- Full wave function: $\psi_a(x) = \langle x|\psi_a\rangle$ with $x \propto a^{\frac{3}{2}(1-w)}$

$n \in \mathbb{N}$
$\nu^2 \geq 1$

$$\psi_a(x) = \sqrt{\frac{2\,n_a!}{\Gamma(\nu+n_a+1)}}\left(\frac{\xi_{\nu,n_a}-\mathrm{i}q_a p_a}{\xi_{\nu,n_a}+\mathrm{i}q_a p_a}\right)^{\frac{1}{2}(2n_a+\nu+1)}\xi_{\nu,n_a}^{\frac{\nu+1}{2}}\frac{x^{\nu+1/2}}{q_a^{\nu+1}}L^{\nu}_{n_a}\left(\xi_{\nu,n_a}\frac{x^2}{q_a^2}\right)\exp\left(-\frac{1}{2}(\xi_{\nu,n_a}-\mathrm{i}\,q_a p_a)\frac{x^2}{q_a^2}\right)$$

Phase: $e^{-\mathrm{i}\phi_a(\tau)} = \left(\dfrac{\xi_{\nu,n_a}-\mathrm{i}q_a p_a}{\xi_{\nu,n_a}+\mathrm{i}q_a p_a}\right)^{\frac{1}{2}(2n_a+\nu+1)}$

Laguerre polynomial

$$\xi_{\nu,n} = \left(\frac{n!}{\Gamma(n+\nu+1)}\int_0^\infty y^{\nu+\frac{1}{2}}\left(L_n^{(\nu)}(y)\right)^2 e^{-y}\mathrm{d}y\right)^2$$

STATE CHOICE FOR A SEMICLASSICAL BOUNCE

- Choose semiclassical state

$$|\psi\rangle = e^{-\mathrm{i}\phi(\tau)}|q(\tau), p(\tau)\rangle$$

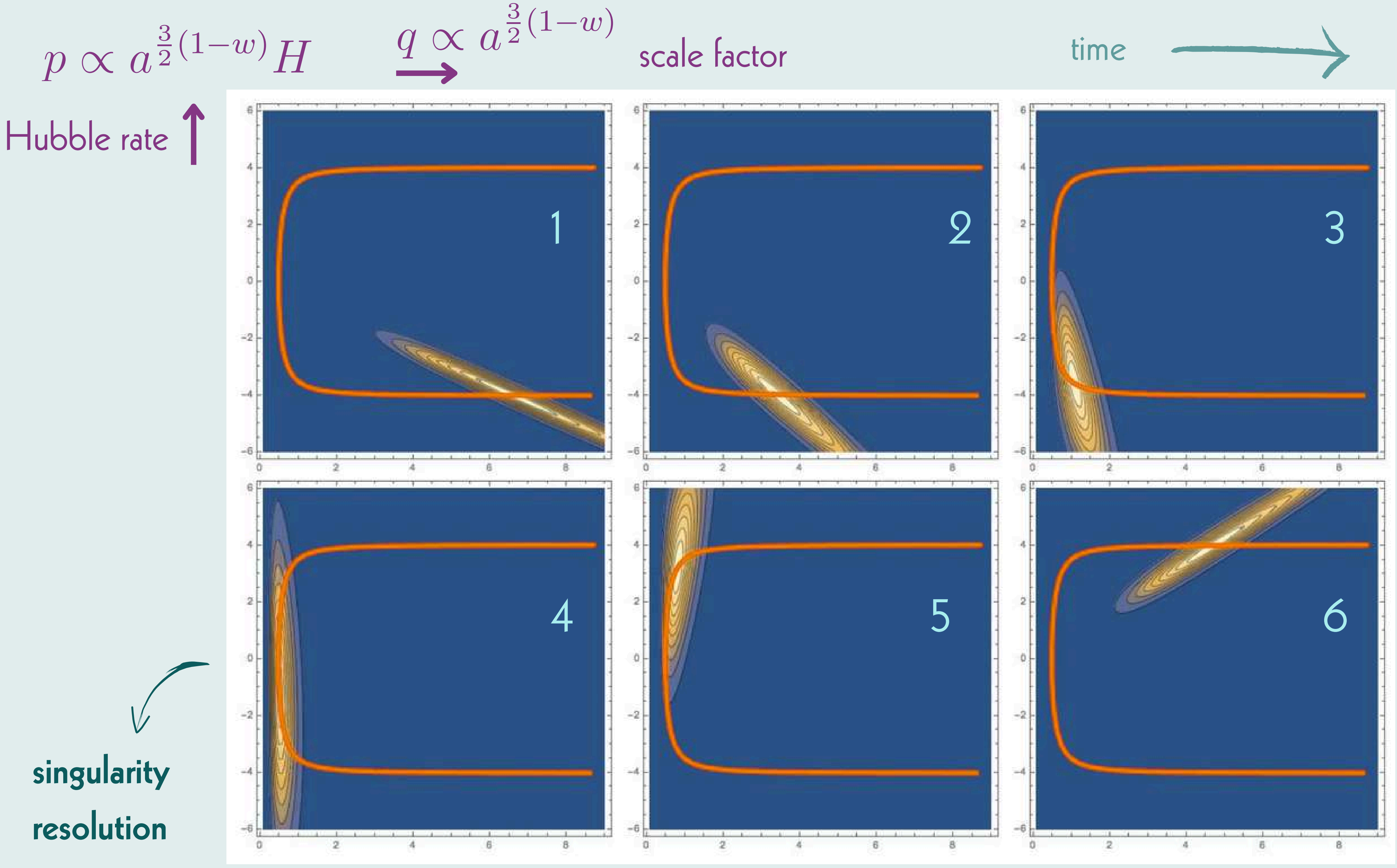
  - Follows dynamics generated by the semiclassical Hamiltonian $\mathcal{H}_{\mathrm{sem}} = p^2 + \dfrac{\xi^2}{q^2}$

$$q(\tau) = q_B\sqrt{1+\omega^2(\tau-\tau_B)^2}$$

$$p(\tau) = \frac{1}{2}\dot{q}(\tau) = \frac{q_B\,\omega^2(\tau-\tau_B)}{2\sqrt{1+\omega^2(\tau-\tau_B)^2}}$$

[Bergeron, Gazeau, Małkiewicz, Peter ('23)]

- Peak of the semiclassical state follows the semiclassical trajectories $q(\tau)$, $p(\tau)$ $\rightarrow$ bouncing solutions



[Images: Bergeron, Gazeau, Małkiewicz, Peter ('23)]

EFFECTIVE SCALE FACTOR

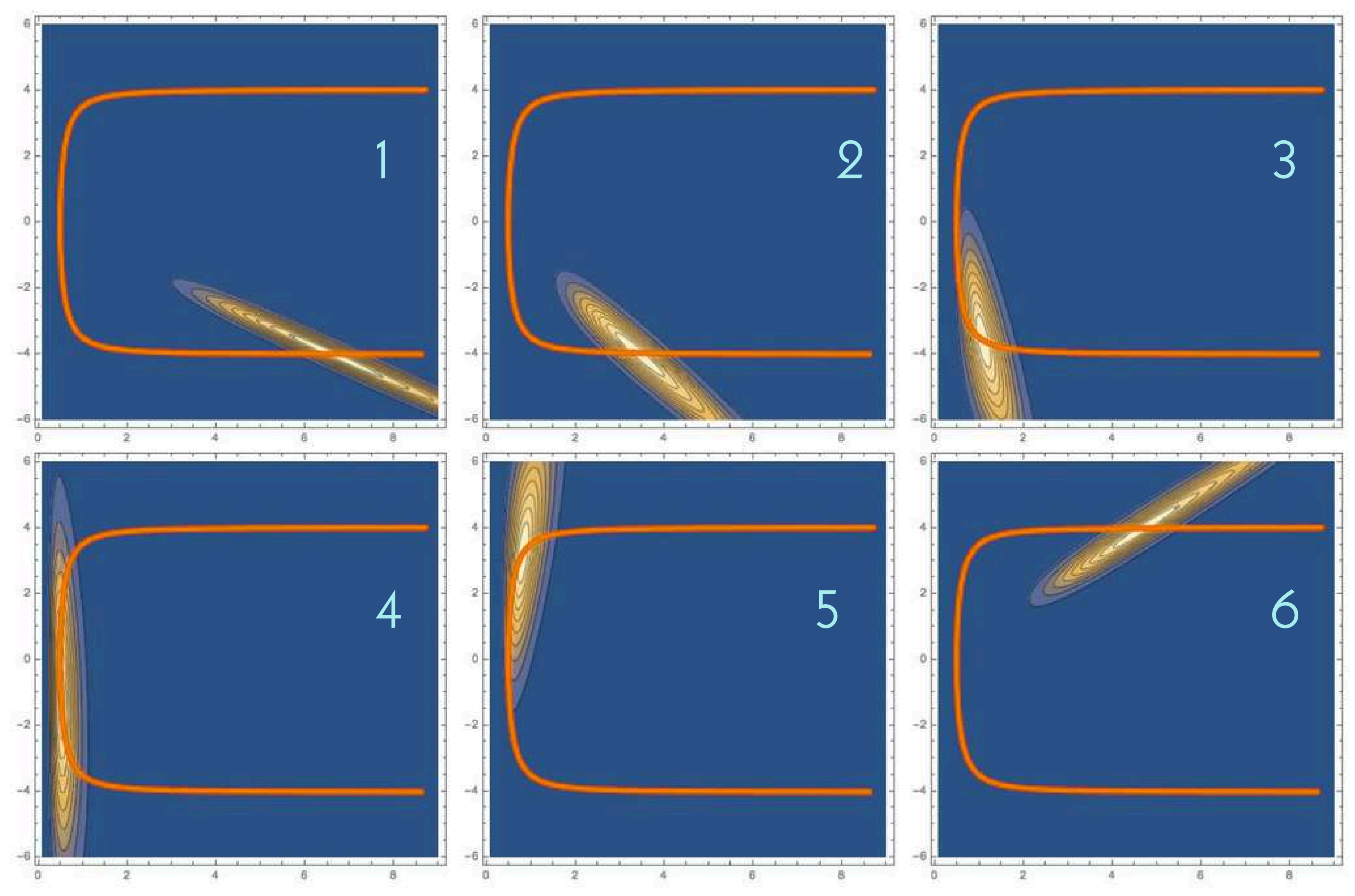
[Bergeron, Gazeau, Małkiewicz, Peter ('23)]

- Peak of the semiclassical state follows the semiclassical trajectories $q(\tau)$, $p(\tau)$

$p \propto a^{\frac{3}{2}(1-w)} H$ Hubble rate

$q \propto a^{\frac{3}{2}(1-w)}$ scale factor

time



singularity resolution

[Images: Bergeron, Gazeau, Małkiewicz, Peter ('23)]

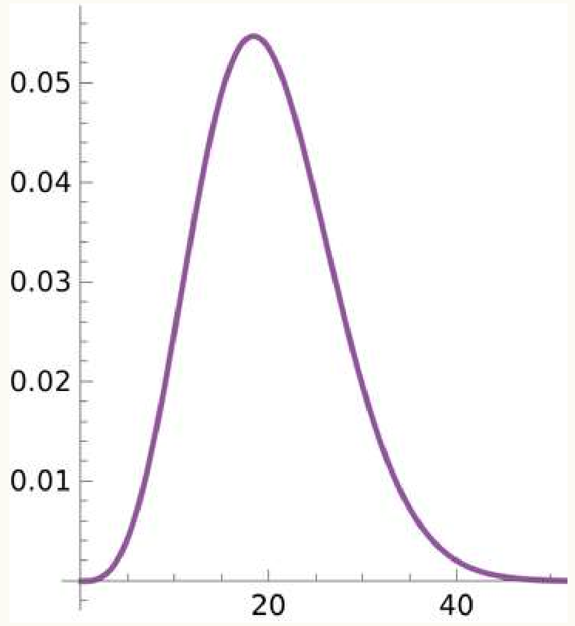
- Operator related to the scale factor

$$q \propto a^{\frac{3}{2}(1-w)}$$

- *Copenhagen viewpoint*: effective scale factory from operator expectation value in a highly peaked semiclassical state $\langle\psi|\hat{q}|\psi\rangle \propto a^{\frac{3}{2}(1-w)}$

$\langle\hat{q}\rangle$

$|\psi(x)|^2$



x

Only one universe: no statistical distribution

Here: consider quantum trajectories instead

QUANTUM CORRECTED SCALE FACTOR

- Quantum corrected scale factor has the same dynamics as a semiclassical trajectory → recover GR dynamics at late times

  $$x(\tau) \to x_0\,\omega\,\tau \Rightarrow a \propto t^{2/(3(1+w))} \quad \text{with} \quad x \propto a^{\frac{3}{2}(1-w)} \quad \text{and} \quad N\mathrm{d}\tau = \mathrm{d}t$$

  - Initial condition related to the expansion rate of the universe $\dot{x}(\tau) \to x_0\,\omega$

- Quantum potential $Q(x,\tau) = \frac{2\xi_\nu(\nu+1)}{q^2} - \frac{\xi_\nu^2 x^2}{q^4} - \frac{\nu^2 - \frac{1}{4}}{x^2}$ gives $\ddot{x} = -2\frac{\partial}{\partial x}\left(\frac{\nu^2-\frac{1}{4}}{x^2} + Q(x,\tau)\right) = \frac{4\xi_\nu^2}{q(\tau)^4}x$

- Hubble rate from trajectories
  - FLRW with perfect fluid for $\partial_x S = \text{const.}$

  $$H^2 = \left(\frac{\dot{a}}{Na}\right)^2 = \frac{4}{9(1-w)^2}\frac{\dot{x}^2}{N^2x^2} \propto \frac{(\partial_x S)^2}{a^{3(1+w)}} \propto \frac{(\partial_a S)^2}{a^4}$$

- Acceleration
  - FLRW for $\ddot{x} \to 0$

  $$\dot{H} = \frac{2}{3(1-w)}\frac{\ddot{x}}{Nx} - \frac{3}{2}(1+w)NH^2$$

BOUNCING TRAJECTORIES

- Trajectories from semiclassical states

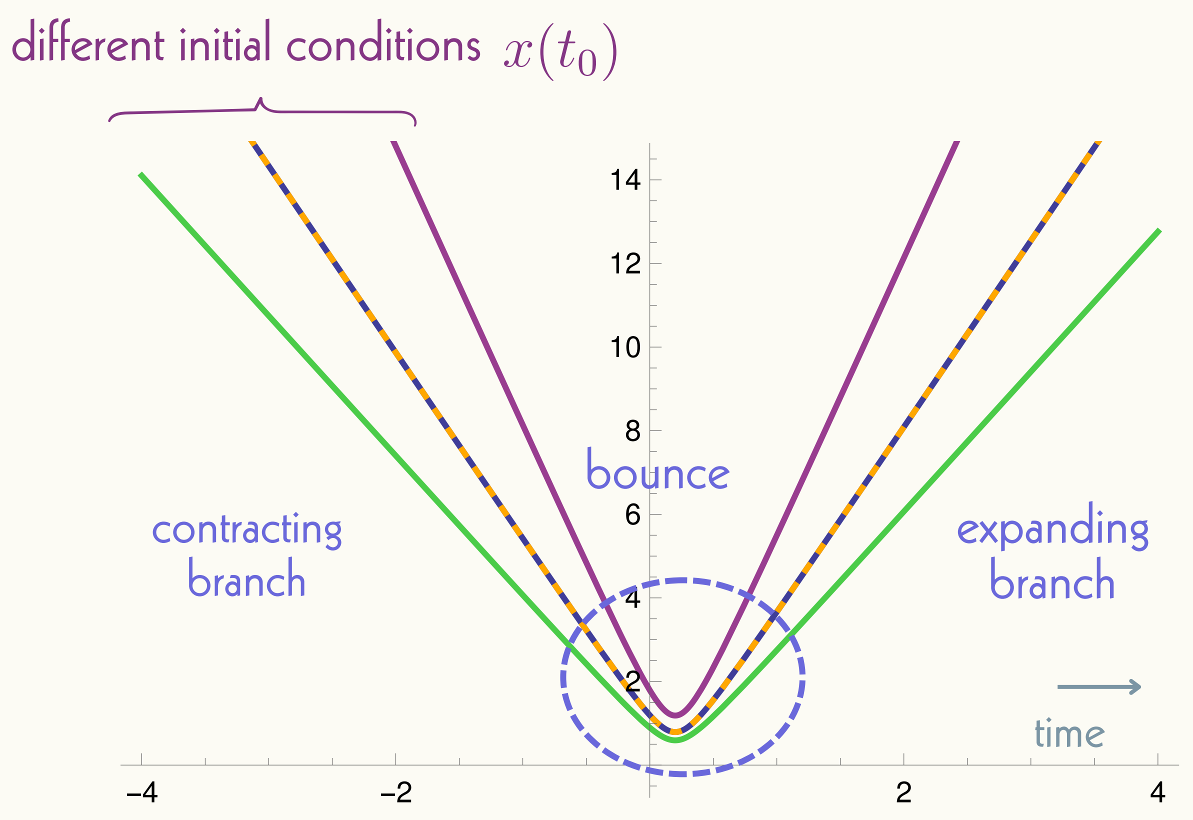
trajectory gives the scale factor: $x(\tau) \propto a^{\frac{3}{2}(1-w)}$

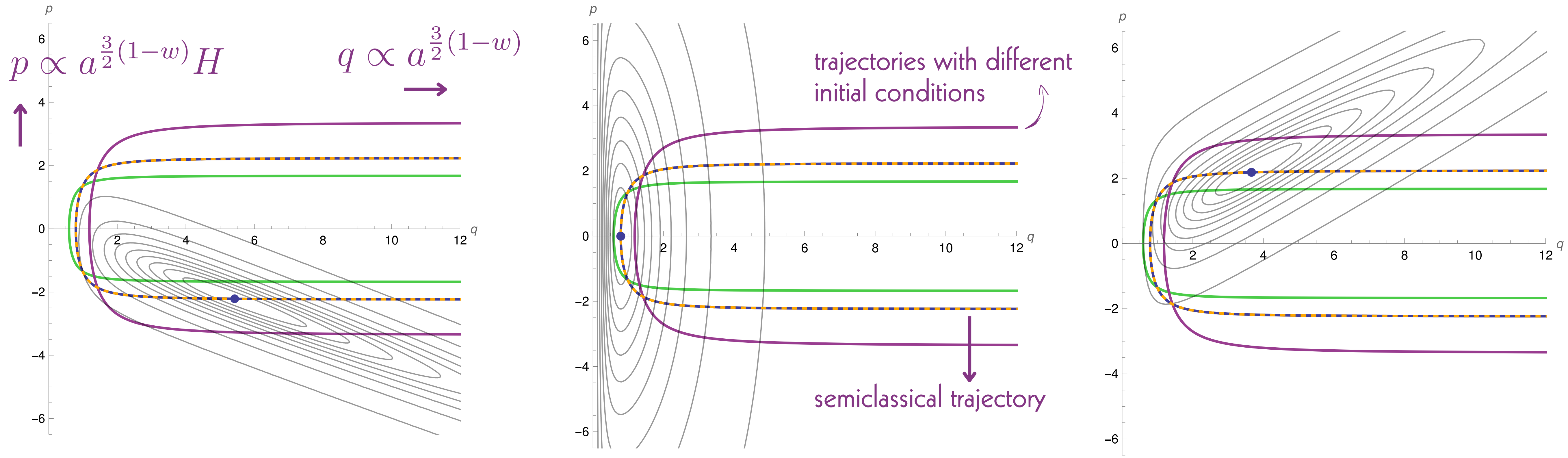
evolution governed by the wave function: $\frac{\mathrm{d}x}{\mathrm{d}\tau} = -\mathrm{i}\partial_x \ln \frac{\psi}{\psi^\star}$

with

$$|\psi\rangle = e^{-\mathrm{i}\phi(\tau)}|q(\tau), p(\tau)\rangle$$

$$\hat{\mathcal{H}}_{\mathrm{FLRW}}\psi - \mathrm{i}\partial_\tau \psi = 0$$



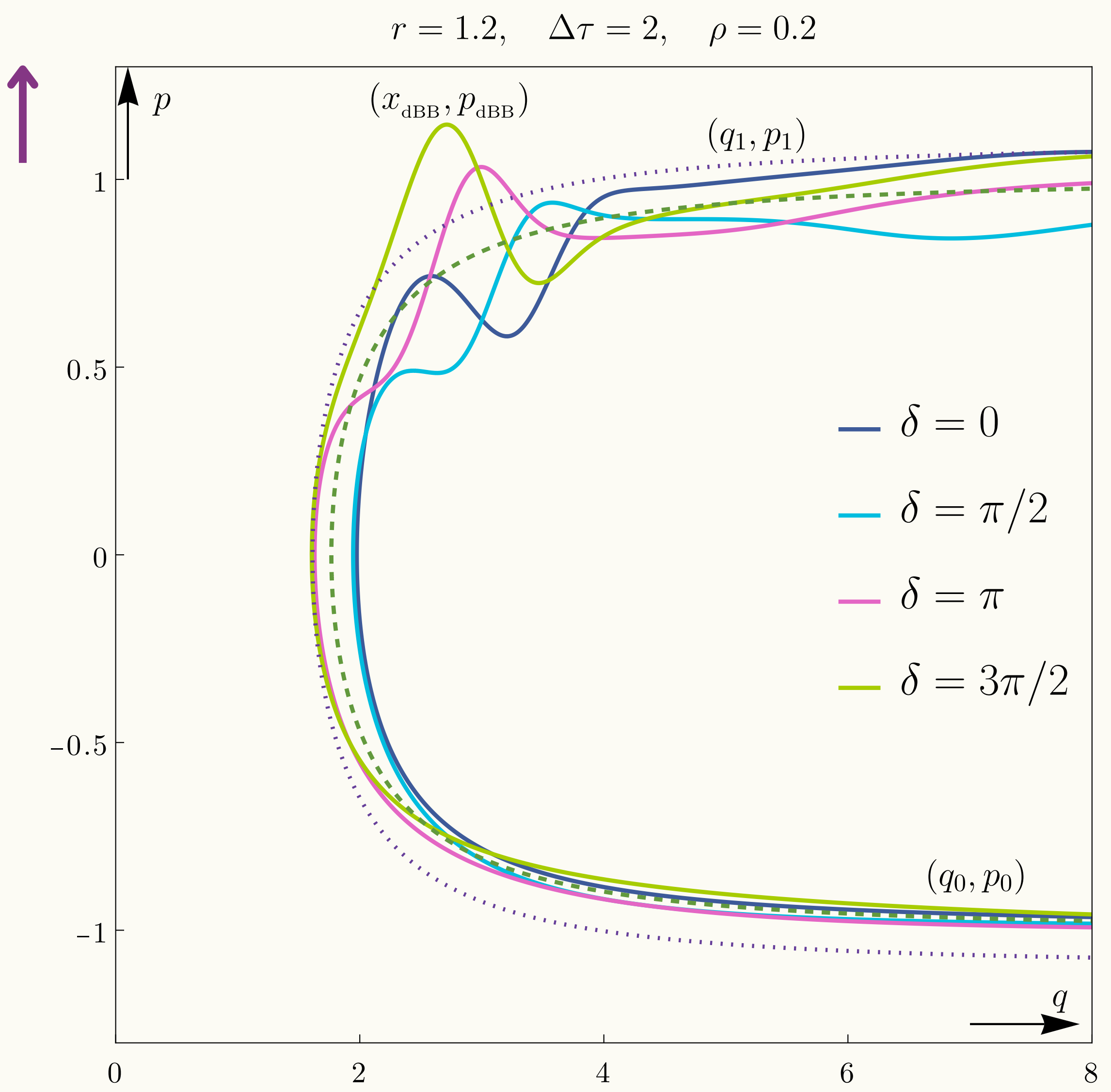


- Concrete value for the effective scale factor from trajectories at all times
- Classical dynamics away from bounce

UNIVERSE IN A SUPERPOSITION

- Phase space portraits and wave functions for the biverse $\Psi = \mathcal{N}(\psi_0 + \rho e^{\mathrm{i}\delta}\psi_1)$

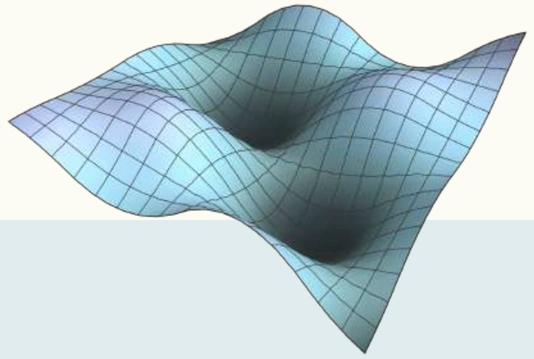
$p \propto a^{\frac{3}{2}(1-w)} H \uparrow$



$\longrightarrow \; q \propto a^{\frac{3}{2}(1-w)}$

- Trajectories highly dependent on initial conditions, but generally exhibit features that differ from single state trajectories

FIRST APPROACH TO PERTURBATIONS



- Consider tensor perturbations in a flat FLRW spacetime with a radiation fluid (set $w = 1/3$)

$$\mathrm{d}s^2 = a^2(\eta)\left(-\mathrm{d}\eta^2 + [\delta_{ij} + h_{ij}(\eta,\vec{x})]\,\mathrm{d}x^i\mathrm{d}x^j\right)$$

- Describe perturbations in Fourier space, encoded by $\mu_{\vec{k}}$

$$h_{ij}(\vec{x},\eta) \propto \sum_{\vec{k},\lambda} \varepsilon_{ij}^{(\lambda)} \frac{\mu_{\lambda,\vec{k}}(\eta)}{a(\eta)} e^{\mathrm{i}\vec{k}\cdot\vec{x}}$$

$\lambda = +, \times$

[Micheli, Peter, ('23)]

- Full Hamiltonian $\mathcal{H}_{\mathrm{FLRW}} = \mathcal{H}^{(0)} + \mathcal{H}^{(2)}$ with

$$\mathcal{H}^{(2)} = \sum_{\vec{k},\lambda} \frac{1}{2}\left(\pi_{\lambda,\vec{k}}\pi_{\lambda,-\vec{k}} + 2\frac{p}{q}(\pi_{\lambda,\vec{k}}\mu_{\lambda,-\vec{k}} + \mu_{\lambda,\vec{k}}\pi_{\lambda,-\vec{k}}) + k^2\mu_{\lambda,\vec{k}}\mu_{\lambda,-\vec{k}}\right)$$

conjugate momentum of $\mu_{\vec{k}}^{(\lambda)}$

background quantities

- Born-Oppenheimer approximation to solve perturbative dynamics $\Psi(a,\{\mu_{\vec{k}}\}) = \psi_{\mathrm{bg}}(a)\psi_{\mathrm{pert}}(a(\eta),\{\mu_{\vec{k}}\})$

Background:	Perturbations:
$\mathcal{H}^{(0)}\psi_{\mathrm{bg}}(a) = \mathrm{i}\partial_\eta\psi_{\mathrm{bg}}(a)$	$\mathcal{H}^{(2)}\psi_{\mathrm{pert}}(a(\eta),\{\mu_{\vec{k}}\}) = \mathrm{i}\partial_\eta\psi_{\mathrm{pert}}(a(\eta),\{\mu_{\vec{k}}\})$
defines a background trajectory $a(\eta)$	insert background trajectory; trajectory extracted from the background

- Single state: same background from quantum trajectories and expectation value
- Biverse: use quantum trajectory of superposition background

PERTURBATIONS OVER BIVERSE TRAJECTORIES

[Peter, Pinho, Pinto-Neto, ('05)]
[Peter, Pinho, Pinto-Neto, ('06)]
[Micheli, Peter, ('23)]

- Canonical quantisation of perturbation modes with $\hat{\mu}_{\vec{k}}(\eta) = \mu_k(\eta)\hat{a}_{\vec{k}} + \mu_k^*(\eta)\hat{a}^\dagger_{-\vec{k}}$

creation and annihilation operators

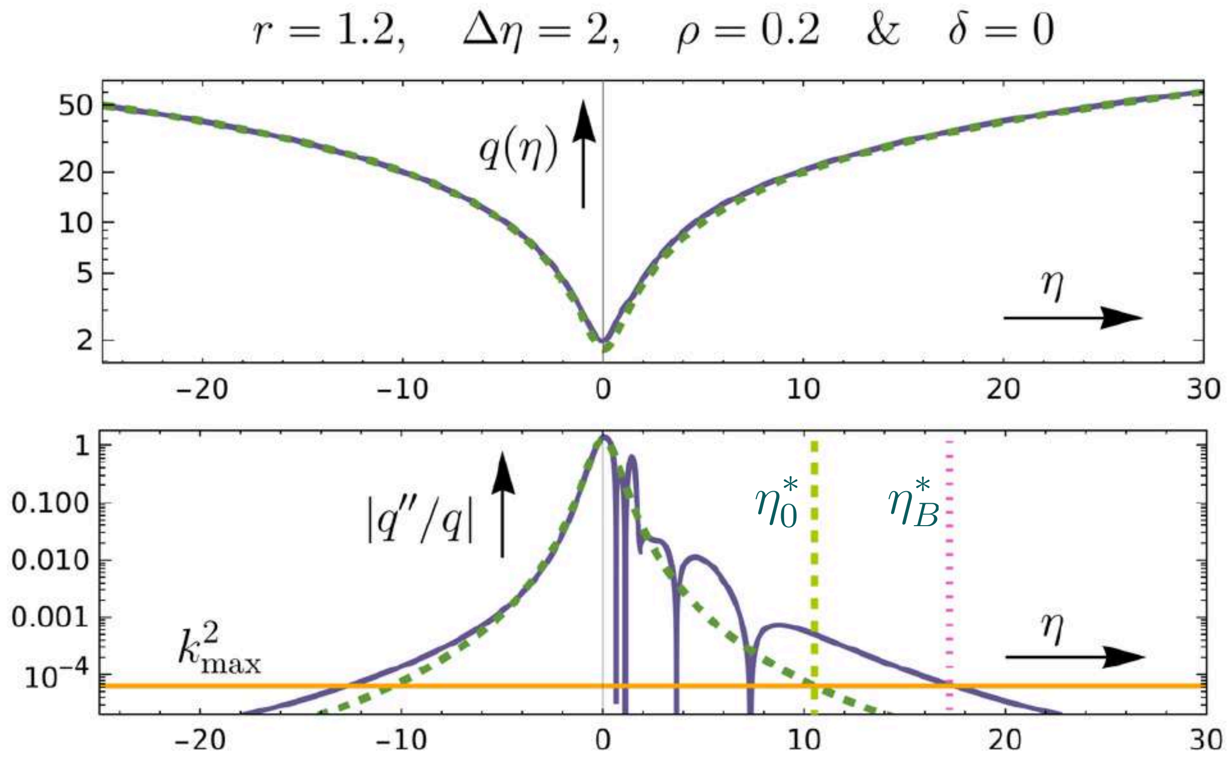
Dynamics:

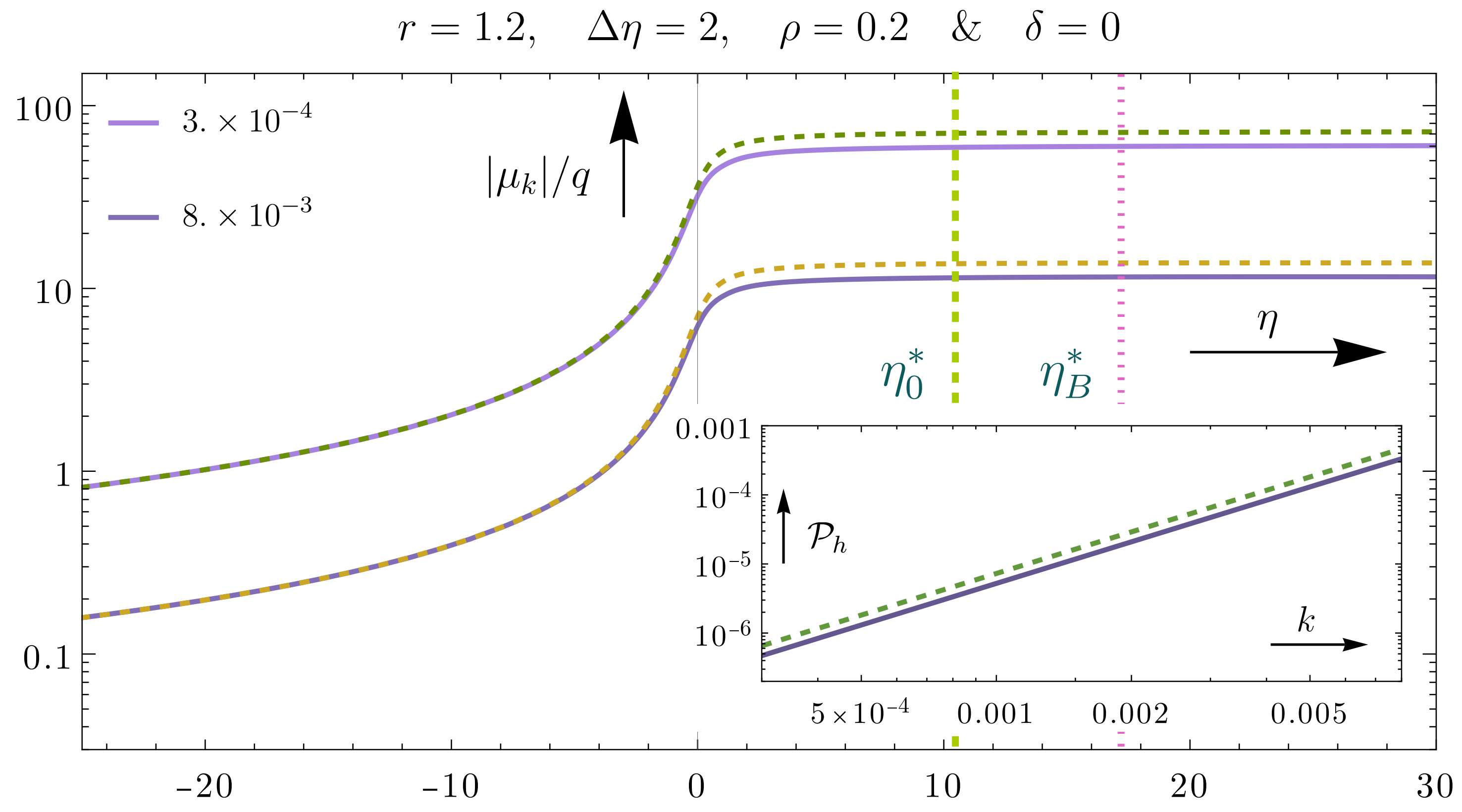
$$\mu_k'' + \left(k^2 - \frac{a''}{a}\right)\mu_k = 0$$

V_{eff} modified by trajectories

- Range of Fourier modes: $k \in (8\times 10^{-4}, 3\times 10^{-3})$
- Initial conditions: "Bunch-Davies" vacuum $\mu_k(\eta) = \frac{1}{\sqrt{2k}}e^{-ik(\eta-\eta_i)}$

$q(\eta) = a(\eta)$





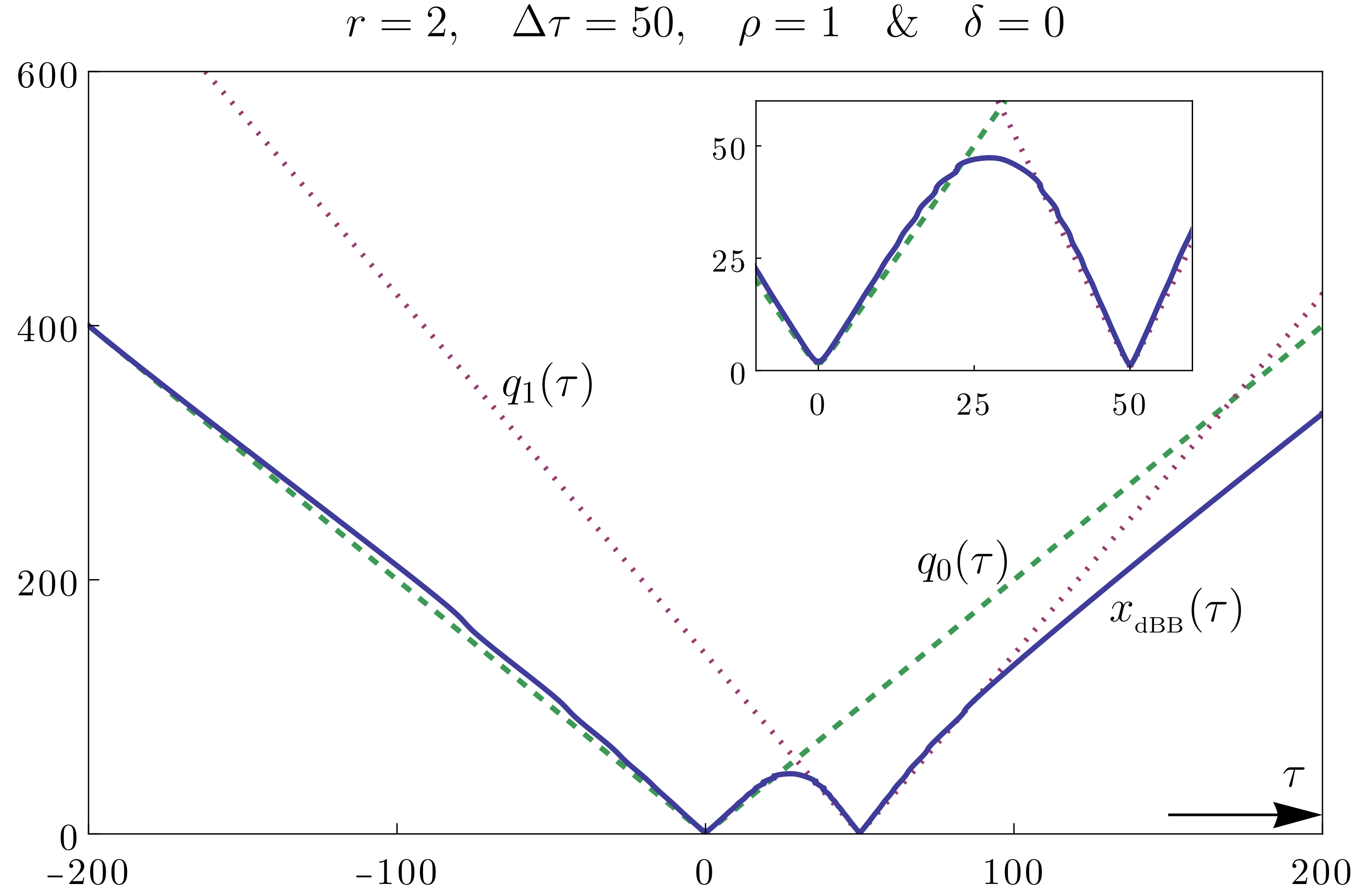
- Power spectrum $\mathcal{P}_h(k) \propto k^3 \left|\frac{\mu_k}{a}\right|^2_{\eta=\eta^*}$ taken at time when $V_{\text{eff}}(\eta^*) = k^2_{\text{max}}$

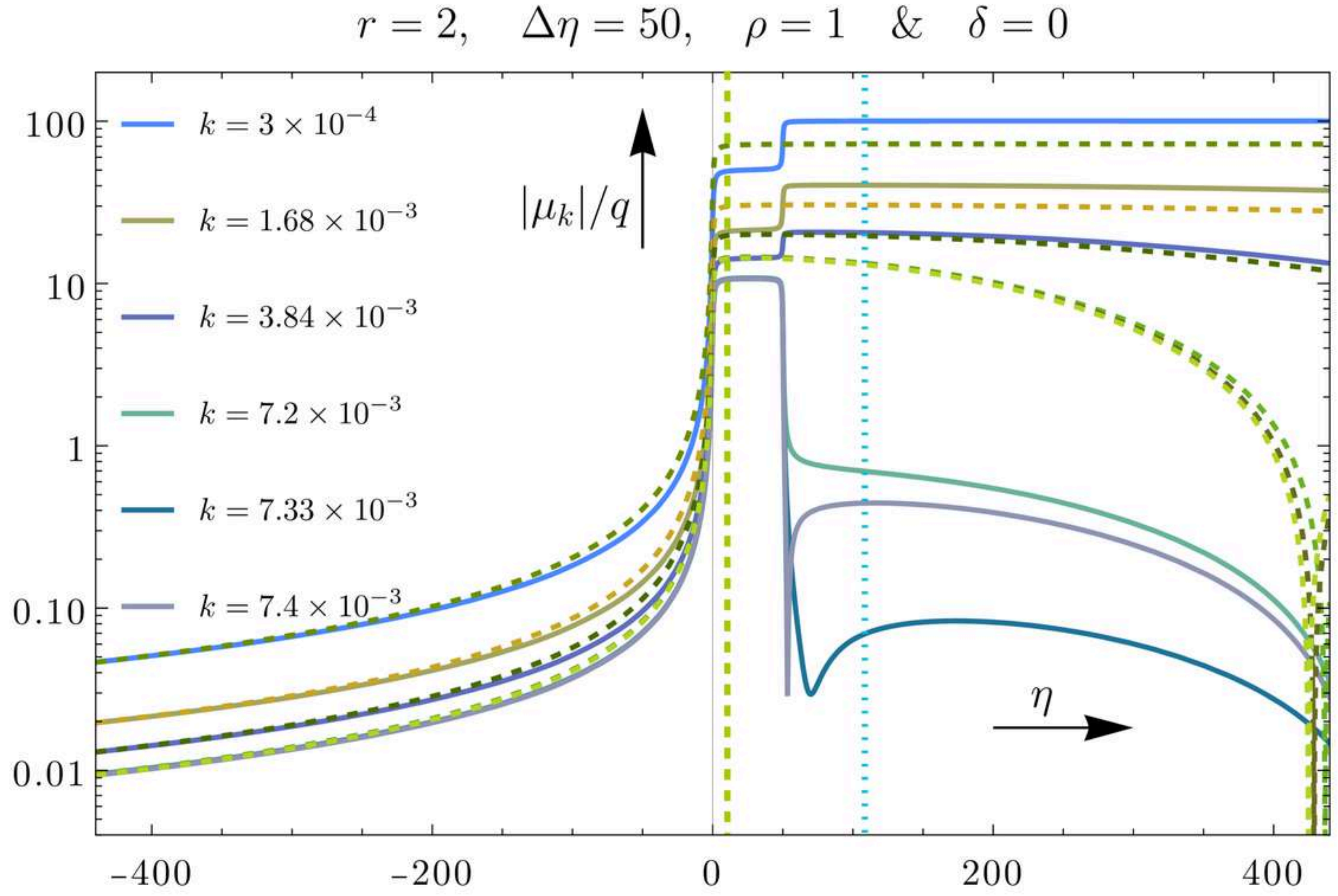
PERTURBATIONS OVER BIVERSE TRAJECTORIES

- Dynamics of perturbation modes $\mu_k'' + \left(k^2 - \frac{a''}{a}\right)\mu_k = 0$

V_{eff} modified by trajectories

- Range of Fourier modes: $k \in (8 \times 10^{-4}, 3 \times 10^{-3})$





- In potential dominated regime: $\mu_k(\eta) \approx a(\eta)\left(\frac{\mu_{k,\mathrm{i}}}{a_\mathrm{i}} + (\mu'_{k,\mathrm{i}}a_\mathrm{i} - \mu_{k,\mathrm{i}}a'_\mathrm{i})\int_{\eta_\mathrm{i}}^{\eta}\frac{d\tilde{\eta}}{a^2}\right)$

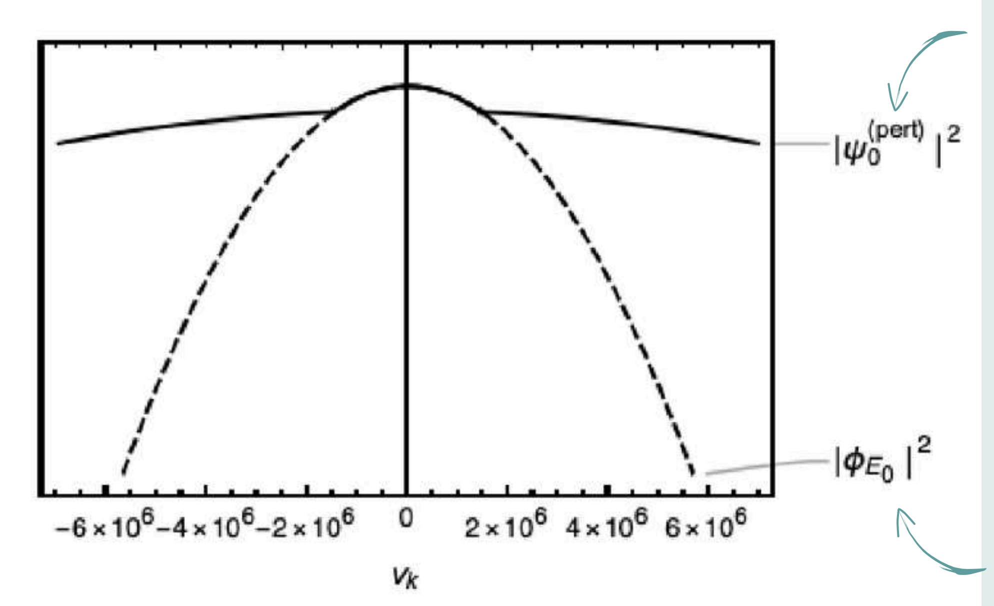
for single state: $\int_{\eta_\mathrm{i}}^{\eta}\frac{d\tilde{\eta}}{a^2} \propto \arctan((\eta - \eta_B)\omega)$

PERTURBATIONS OVER A COPENHAGEN BIVERSE

- State in which each universe has its own perturbations $\Psi = \psi_{\mathrm{bg},0}\psi_{\mathrm{pert},0} + \psi_{\mathrm{bg},1}\psi_{\mathrm{pert},1}$

Background:	Perturbations over each backgound:
$\mathcal{H}_{\mathrm{FLRW}}\psi_{\mathrm{bg}}(x) = \mathrm{i}\partial_\tau\psi_{\mathrm{bg}}(x)$	$\mathrm{i}\partial_\tau\psi_{\mathrm{pert},n} = \sum_{lm}\langle\psi_{\mathrm{bg},n}\|\psi_{\mathrm{bg},l}\rangle^{-1}\langle\psi_{\mathrm{bg},l}\|\mathcal{H}^{(2)}\|\psi_{\mathrm{bg},m}\rangle\psi_{\mathrm{pert},m}$ effect of "virtual" states

- Influence of superposition on perturbations from $\langle\psi_{\mathrm{bg},l}|\mathcal{H}^{(2)}|\psi_{\mathrm{bg},m}\rangle$
- Imprint: non-Gaussian profile in the perturbation modes



total perturbations

single state $\Psi = \Psi_0$

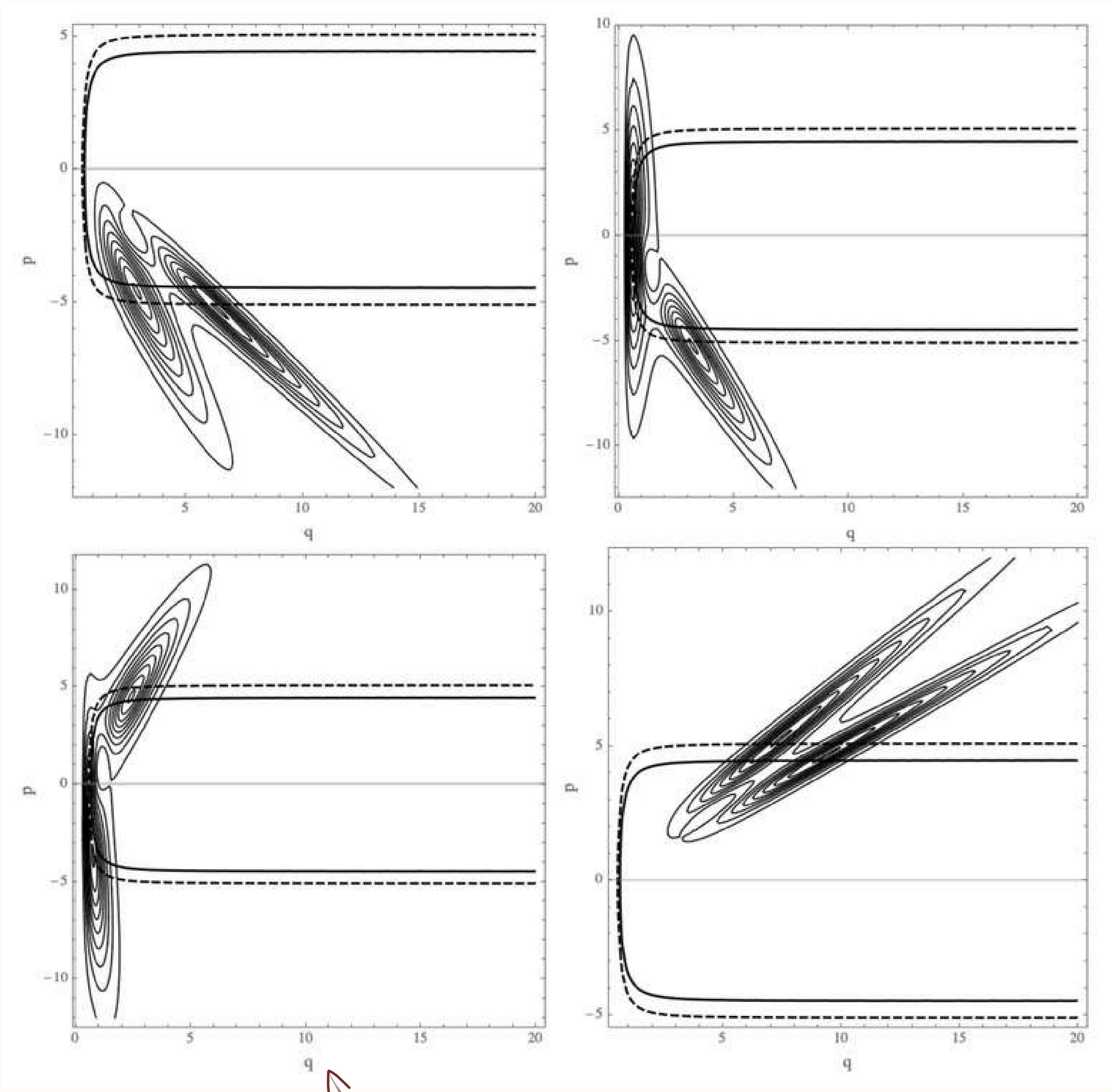
perturbation

PERTURBATIONS OVER A COPENHAGEN BIVERSE

[Bergeron, Małkiewicz, Peter, ('24), arXiv: 2405.09307]

- Biverse $\Psi = \mathcal{N}(\psi_0 + \alpha\,\psi_1)$
- Our universe: projection on a single wave function

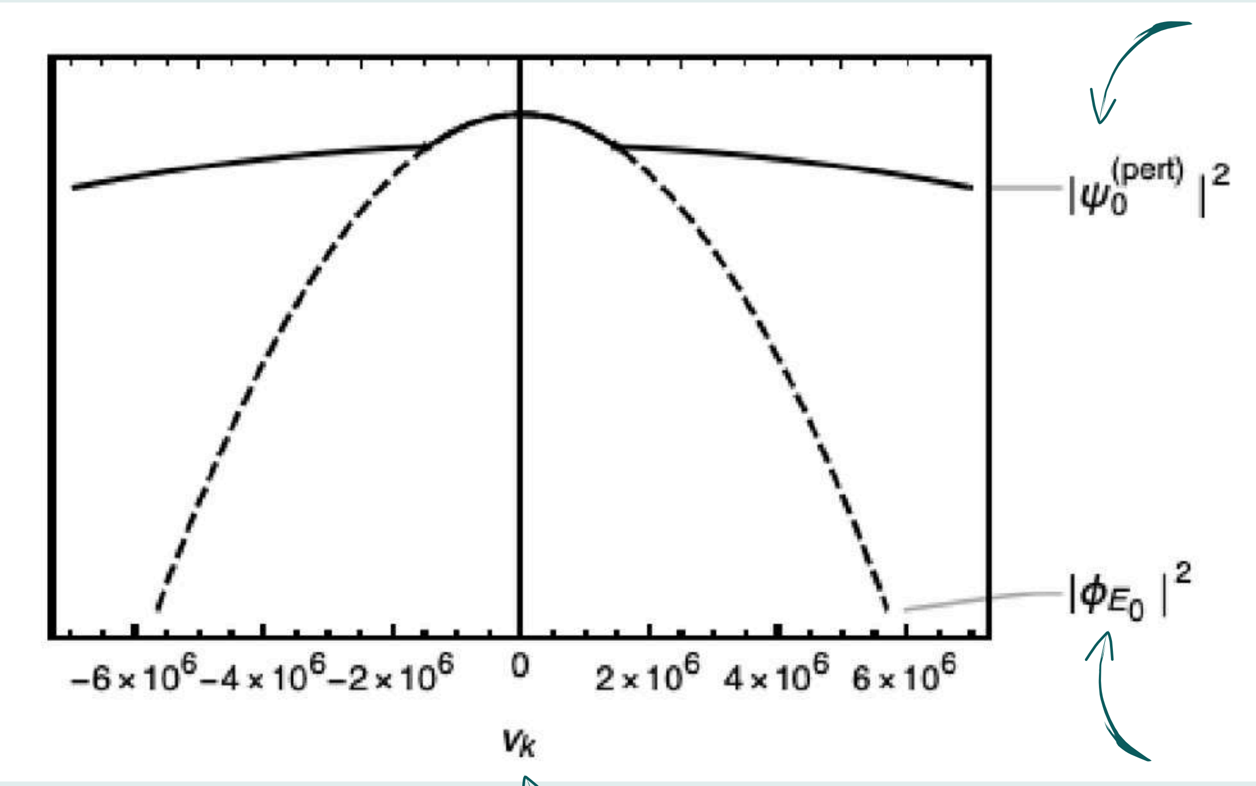
$\propto H$



$\propto a$

- Entanglement of perturbations between states

total perturbations



single state $\Psi = \psi_0$

perturbation

EXAMPLE: DOUBLE SLIT EXPERIMENT

- Weak measurements – reconstruct trajectories

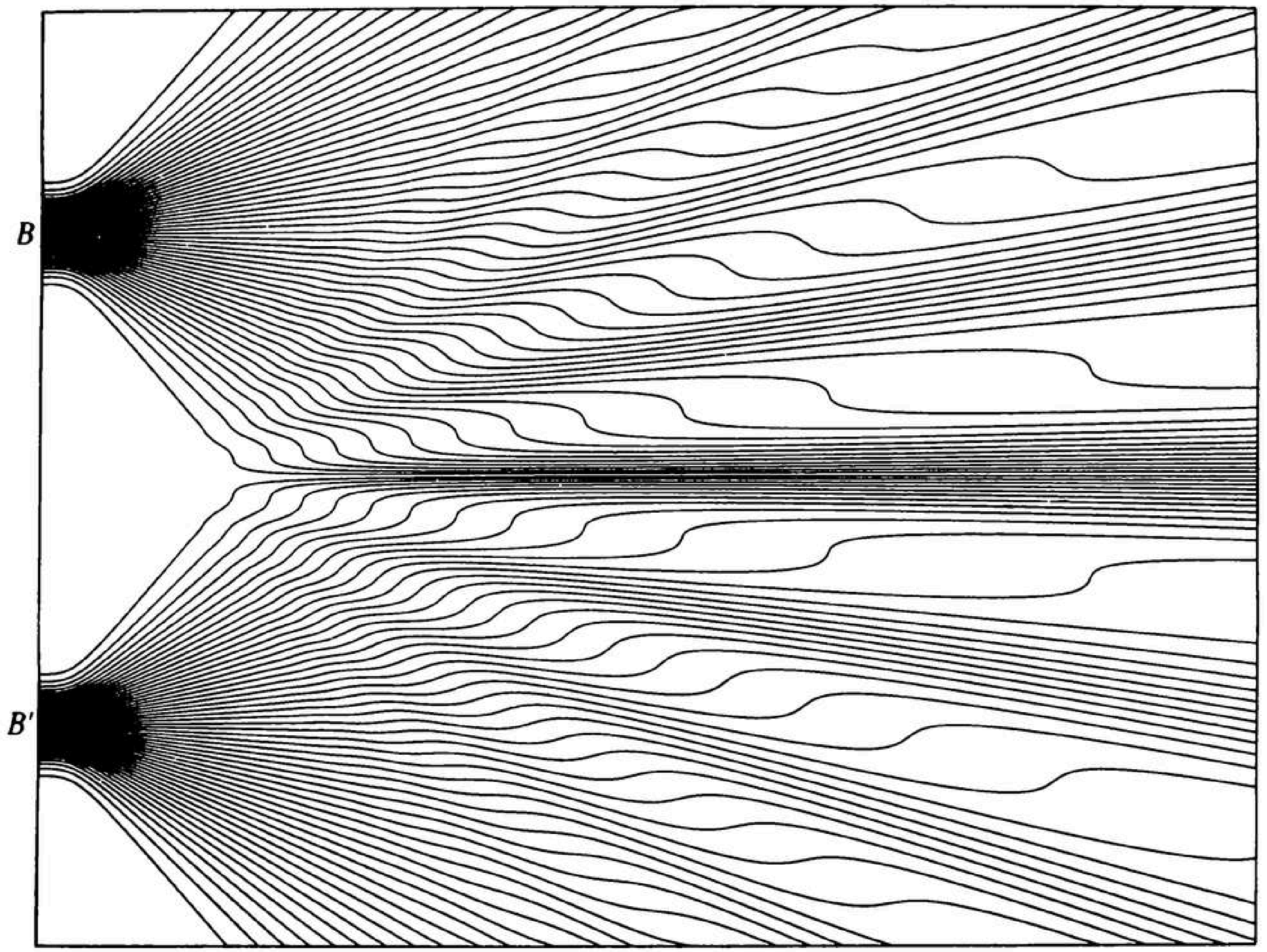


Fig. 5.7 Trajectories for two Gaussian slits with a Gaussian distribution of initial positions at each slit. The probability density is proportional to the number of lines per unit length in the *y*-direction (from Philippidis *et al.* (1982)).