LARGE DEVIATIONS FOR HALOS & VOIDS : BEYOND PERTURBATIVE NON-GAUSSIANITIES

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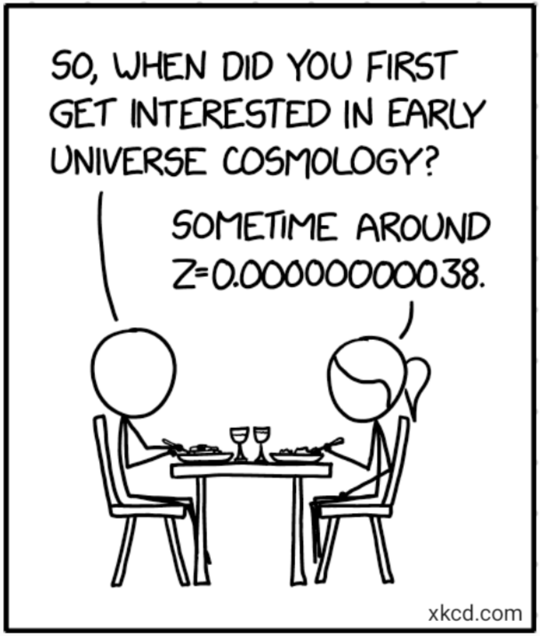
with Ruth DURRER, Julien GRAIN, Killian MARTINEAU, Aurélien BARRAU

based on ARXIV: 2607.16152



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COMMON THREAD

HOW CAN WE CONSTRAIN EARLY NON-GAUSSIANITIES FROM LATE-TIME OBSERVABLES?

HALO AND VOID COUNTING

Q: HOW MANY HALOS OF GIVEN MASS M ARE THERE?

EXCURSION SET THEORY: [Bond et al. 1991]

- SOLVES CLOUD-IN-CLOUD PROBLEM (NO HALO OVERCOUNTING)
- IS SUITABLE FOR NUMERICAL SIMULATIONS
- CAN MODEL UNDERDENSITIES (VOIDS)

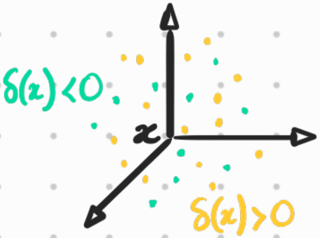
EXCURSION SET THEORY

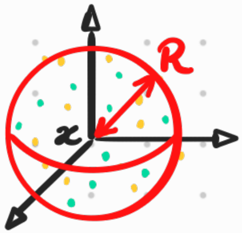
Q° HOW MANY HALOS OF GIVEN MASS M ARE THERE ?

① LOCAL DENSITY CONTRAST $\delta(x)$

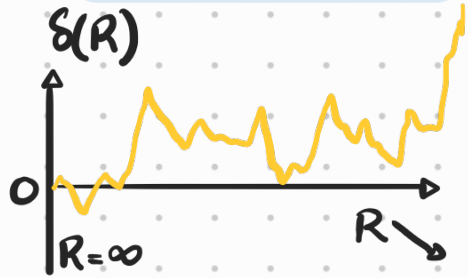
② SMOOTH CONTRAST

③ SHRINK WINDOW: RANDOM WALK





$$\delta(R) = \int d^3y\, \delta(y) \times \mathrm{Window}(y, R)$$



EXCURSION SET THEORY

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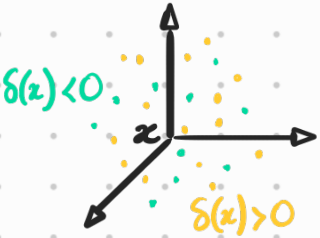
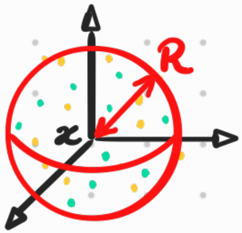


② SMOOTH CONTRAST



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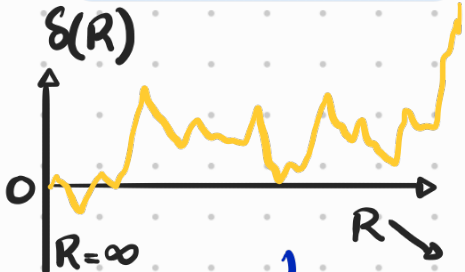
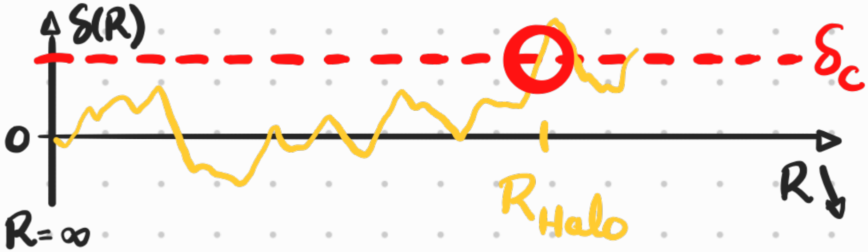
③ SHRINK WINDOW: RANDOM WALK



④ HALO FORMATION : FIRST-PASSAGE AT BARRIER



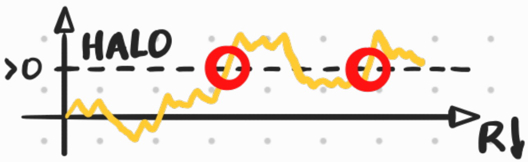
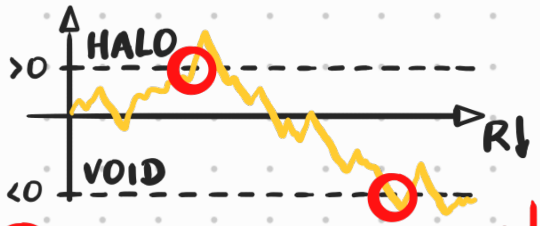
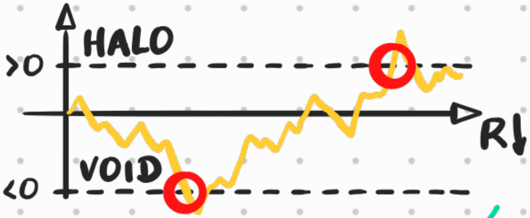
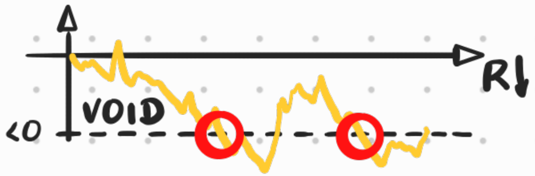
⑤ $M_{Halo} \propto \rho_m R^3_{Halo}$

EXCURSION SET : VOIDS

VOIDS = GROWING FIELD FOR CONSTRAINING COSMOLOGY

[Pisani et al. 1903.05161,
Verza et al. 2401.14451,...]

BUT HARDER FOR EXCURSION SET THEORY:

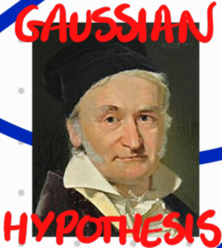
"X IN Y PROBLEM"	CLOUD IN ...	VOID IN ...
... CLOUD	 SOLVED BY EXCURSION SET ✓	 NO VOID WILL FORM!
... VOID	 REGULAR HALO ✓	 SOLVED BY EXCURSION SET ✓

BUT...

THE PERKS OF BEING A GAUSSIAN FIELD

① LOCAL DENSITY CONTRAST

$\delta(x)$ OR $\hat{\delta}_k$



② SMOOTH CONTRAST

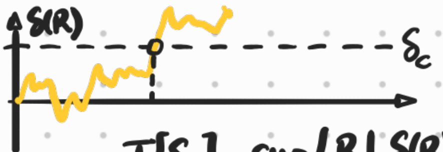
$\delta(R)$ = WEIGHTED SUM OF $\delta(x)$ (OR $\hat{\delta}_k$)

③ RANDOM WALK

INDEPENDENT STEPS

$\delta(R) \longrightarrow \delta(R-dR)$

④ FIRST-PASSAGE TIME



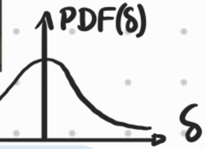
$T[\delta_c] = \sup\{R \mid \delta(R) \geq \delta_c\}$

⑤ HALO/VOID MASS FUNCTION

$$\frac{dn_{HALO}(M)}{d\ln M}$$

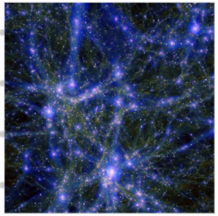
PRIMORDIAL NON-GAUSSIANITIES

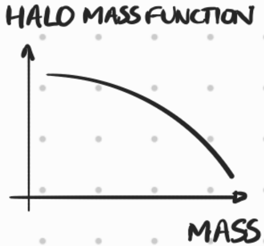
STANDARD LORE :



GAUSSIAN I.C.

EXCURSION SET





PRIMORDIAL NON-GAUSSIANITIES

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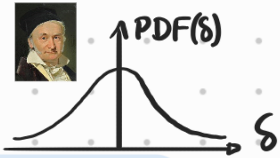

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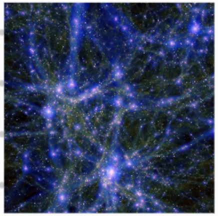
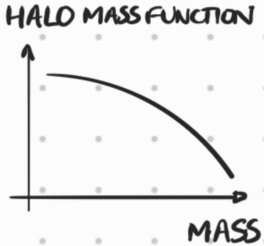




MULTIFIELD INFLATION

[Pinol HAL03610789 ...]

STOCHASTIC INFLATION

[Vennin et al. 1506.04732
Cruces 2203.13852]

COSMOLOGICAL COLLIDER

[Arkani-Hamed & Maldacena 1503.0843]

ULTRA SLOW-ROLL

[Renaux-Petel 1508.06740,...]

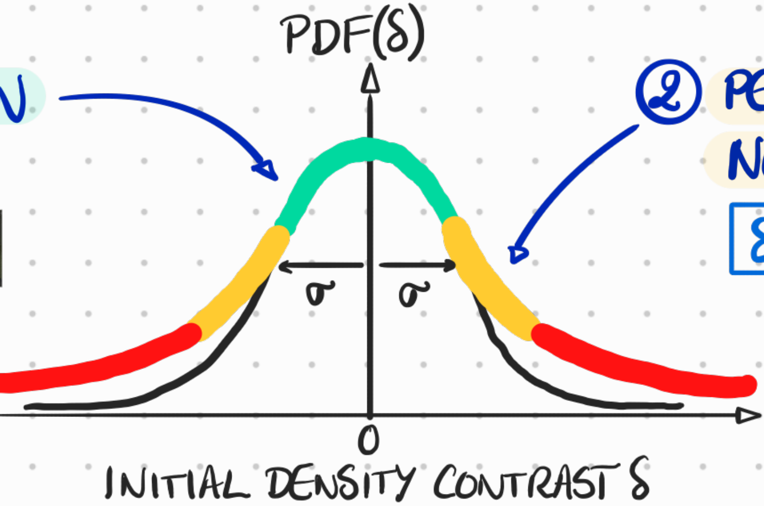
δN-FORMALISM

[Sasaki et al. 9507001]

INITIAL CONDITIONS LIKELY PRESENT NON-GAUSSIAN FEATURES

Q° HOW TO BE PREDICTIVE WITH STRONG NON-GAUSSIANITIES ?

PERTURBATIVE OR NON-PERTURBATIVE



1. GAUSSIAN REGIME

$\boxed{\delta \ll \sigma}$ 

2. PERTURBATIVE NON-GAUSSIANITIES

$\boxed{\delta \sim \sigma}$

[Maggiore & Riotto 0903.1251, D'Amico et al. 1005.1203,...]

↳ BISPECTRUM $\langle\delta\delta\delta\rangle$, TRISPECTRUM $\langle\delta\delta\delta\delta\rangle$

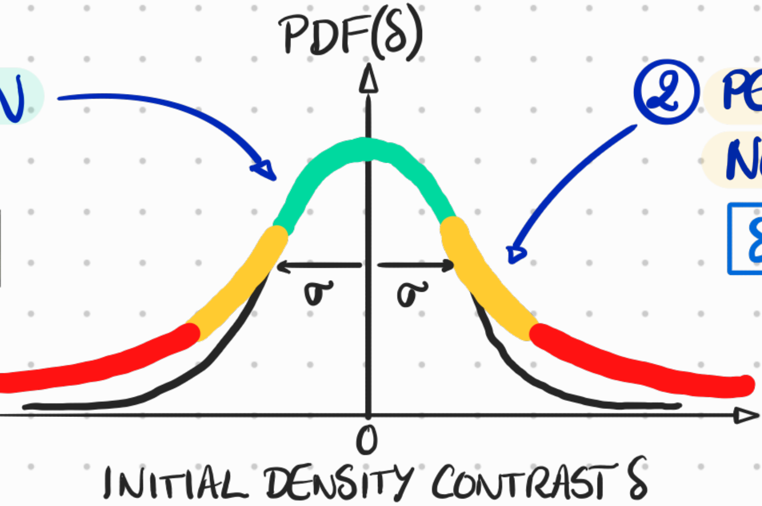
↳  + f_{NL}  ^{2} + g_{NL}  ^{3} + ...

↳ CMB, LSS, ...

PERTURBATIVE OR NON-PERTURBATIVE

① GAUSSIAN REGIME

$\boxed{\delta \ll \sigma}$ 



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↳  $+ f_{NL}$  $^2 + g_{NL}$  $^3 + \dots$

↳ CMB, LSS, ...

③ NON-PERTURBATIVE NON-GAUSSIANITIES

$\boxed{\delta \gg \sigma}$ (RARE EVENTS)

↳ LESS CONSTRAINED, HARD TO SAMPLE & MODEL

↳ BUT VERY OFTENLY PREDICTED

[Pi & Sasaki 2211.13932, Gow et al. 2211.08348, Ezquiaga et al. 2207.06317,...]

OUTLINE

1. MOTIVATIONS FROM INFLATION
2. THE METHOD OF LARGE DEVIATIONS
3. RESULTS: HALO AND VOID SIZE FUNCTIONS
4. CONCLUSION: LET'S DISCUSS!

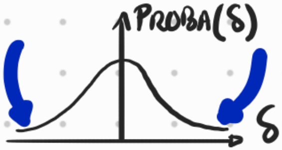
THE

METHODOLOGY

LARGE DEVIATIONS – QUALITATIVE

- CORE OBJECT : **TAILS** OF DISTRIBUTIONS

[Dembo & Zeitouni 2009, Uhlemann et al. 1512.05793,...]



- CORE METHODOLOGY :

PROBABILITY THEORY PROBLEMS

OPTIMIZATION PROBLEMS

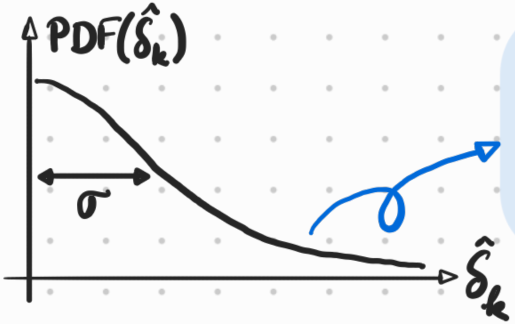
HARD

EASIER

LARGE DEVIATIONS – QUANTITATIVE

Q°1 HOW TO MODEL RARE EVENTS?

[Touchette 2009]



$$\mathrm{PDF}(\hat{\delta}_k) \underset{\varepsilon \to 0}{\simeq} e^{-\frac{1}{\varepsilon} I(\hat{\delta}_k)}$$

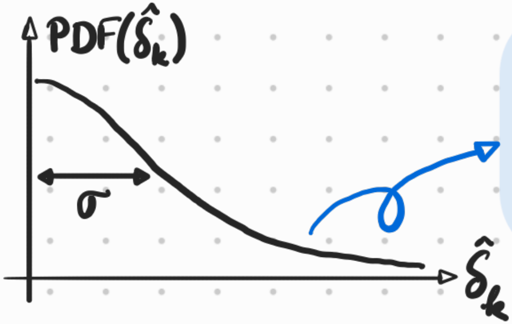
SHAPE OF THE TAIL

ENCODES BEING IN THE TAIL : $\varepsilon \to 0 \iff \delta \gg \sigma = \sqrt{\mathrm{Var}(\delta)}$

LARGE DEVIATIONS – QUANTITATIVE

Q°1 HOW TO MODEL RARE EVENTS?

[Touchette 2009]



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SHAPE OF THE TAIL

ENCODES BEING IN THE TAIL : $\varepsilon \to 0 \iff \delta \gg \sigma = \sqrt{\mathrm{Var}(\delta)}$

- GAUSSIAN CASE

$$I(\hat{\delta}_k) \propto |\hat{\delta}_k|^2$$

- INFLATION (δN-FORMALISM)

$$I(\hat{\delta}_k) \propto |\hat{\delta}_k|^1$$ [Pi & Sasaki 2211.13932]

$$I(\hat{\delta}_k) \propto |\hat{\delta}_k|^{3/2}$$ [Coulton et al. 2406.15546,...]

- OUR ANALYSIS

$$I(\hat{\delta}_k) \propto |\hat{\delta}_k|^q$$

$$q > 0$$

CONTRACTION PRINCIPLE

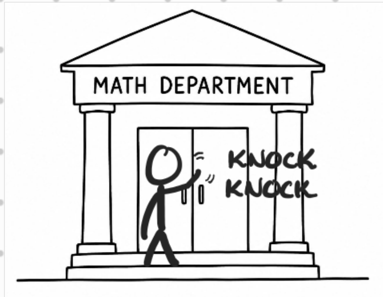
Q°2 HOW TO CONNECT $\hat{\delta}_k$ TO $\delta(R) = \int d^3k\, e^{ikx}\, \hat{\delta}_k \times \hat{W}\text{INDOW}_k(R)$?

Q°: PDF OF $\delta(R)$?

HARD PROBLEM

$\Longrightarrow$

Q°: TAIL SHAPE OF $\delta(R)$?



CONTRACTION PRINCIPLE

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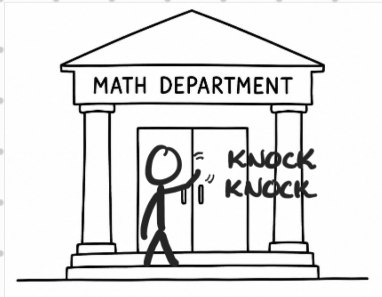
Q°: PDF OF $\delta(R)$? $\Longrightarrow$ Q°: TAIL SHAPE OF $\delta(R)$?

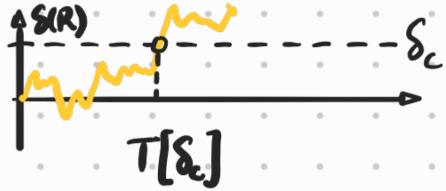
HARD PROBLEM



OPTIMIZATION!

$$\log \mathrm{PDF}(\delta(R)) \propto I(\delta(R)) = \inf \left\{ \sum_k I(\hat{\delta}_k),\ (\hat{\delta}_k)_k \,\middle|\, F[(\hat{\delta}_k)_k] = \delta(R) \right\}$$

FIRST-PASSAGE TIME $T[\delta_c]$



Q°3 HOW TO CONNECT $\delta(R)$ TO THE 1st PASSAGE AT δ_c ?

STRATEGY 1

"LARGE DEVIATION FOR REVERSE PROBLEMS"

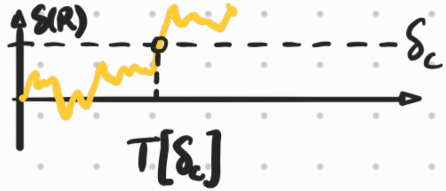
[Puhalskii & Whitt 1997]

STRATEGY 2

"REFLECTION PRINCIPLE"

Symmetry $\mathrm{PDF}(\delta) = \mathrm{PDF}(-\delta)$

FIRST-PASSAGE TIME $T[\delta_c]$



Q°3 HOW TO CONNECT $\delta(R)$ TO THE 1st PASSAGE AT δ_c ?

STRATEGY 1

"LARGE DEVIATION FOR REVERSE PROBLEMS"

[Puhalskii & Whitt 1997]

EXACT
+
GAUSSIANITY NOT REQUIRED

STRATEGY 2

"REFLECTION PRINCIPLE"

Symmetry $\mathrm{PDF}(\delta) = \mathrm{PDF}(-\delta)$

$\Downarrow$

$$\mathrm{PDF}(T[\delta_c]) = \mathrm{PDF}\left(\frac{\delta_c^2}{\delta^2(R=1)}\right)$$

SUMMARY

⓪ EXPONENTIAL TAILS

① LOCAL DENSITY CONTRAST

$\delta(x)$ OR $\hat{\delta}_k$

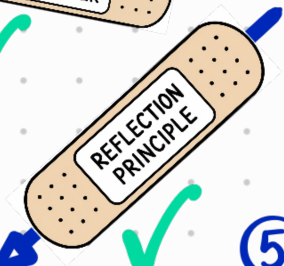
② SMOOTH CONTRAST

$\delta(R)$ = WEIGHTED SUM OF $\delta(x)$ (OR $\hat{\delta}_k$)

③ RANDOM WALK

INDEPENDENT STEPS

$\delta(R) \longrightarrow \delta(R-dR)$



④ FIRST-PASSAGE TIME

$T[\delta_c] = \sup\{R \mid \delta(R) \geq \delta_c\}$

⑤ HALO/VOID MASS FUNCTION

$$\frac{dn_{HALO}(M)}{d\ln M}$$

THE RESULTS

EXPONENTIAL TAILS

RECALL: SHAPE OF THE TAIL

WE SOLVE $I(\delta(R)) = \inf\left\{\sum_k |\hat{\delta}_k|^q,\ (\hat{\delta}_k)_k \text{ reproducing } \delta(R)\right\}$

$+$

USING HÖLDER INEQUALITY $\quad \|fg\|_1 \leq \|f\|_q \times \|g\|_{\frac{q}{q-1}}$

$+$

CONVEXITY ARGUMENTS $\quad F(\sum x_i) \leq \sum_i F(x_i)$

$+$

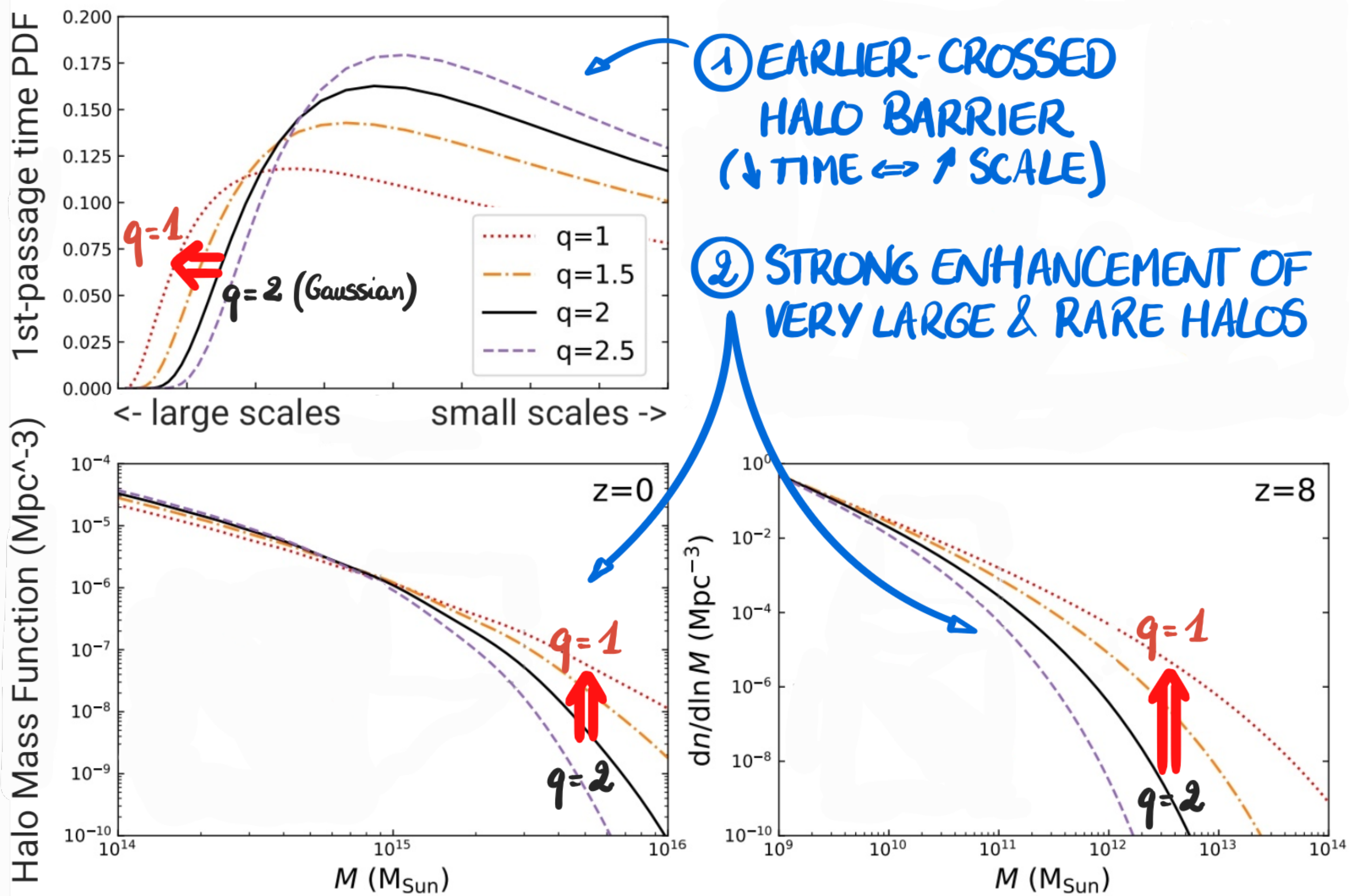
MEASURE THEORY $\quad \int \mathbb{1}_{\{k \in A\}}\, d\mu = \mu(A)$

$=$

$$\log \mathrm{PDF}(\delta(R)) \propto I(\delta(R)) = C_q |\delta(R)|^q R^{3q-3}$$

HALO MASS FUNCTION

RECALL : $PDF(\delta) \propto \exp(-|\delta|^{q})$



① EARLIER-CROSSED HALO BARRIER (↓ TIME ⇔ ↑ SCALE)

② STRONG ENHANCEMENT OF VERY LARGE & RARE HALOS

DOUBLE-BARRIER SOLUTION FOR VOIDS

RESULTS KNOWN $\begin{cases} \text{FOR GAUSSIAN } (q=2) \\ \text{PERTURBATIVELY} \end{cases}$

[Sheth & Van de Weygaert 0311260]

[D'Amico et al. 1011.1229]

DOUBLE-BARRIER SOLUTION FOR VOIDS

RESULTS KNOWN
- FOR GAUSSIAN ($q=2$) [Sheth & Van de Weygaert 0311260]
- PERTURBATIVELY [D'Amico et al. 1011.1229]

OUR RESULT: NON-PERTURBATIVE AND $q \neq 2$

USING: LARGE DEVIATION PRINCIPLE $p(\delta) \sim e^{-I(\delta)/\varepsilon}$

+

LAPLACE TRANSFORMS, LAPLACE'S METHOD $\int e^{\lambda f(x)} dx \sim e^{\lambda f(x_*)} \sqrt{\frac{2\pi}{\lambda f''(x_*)}}$

=

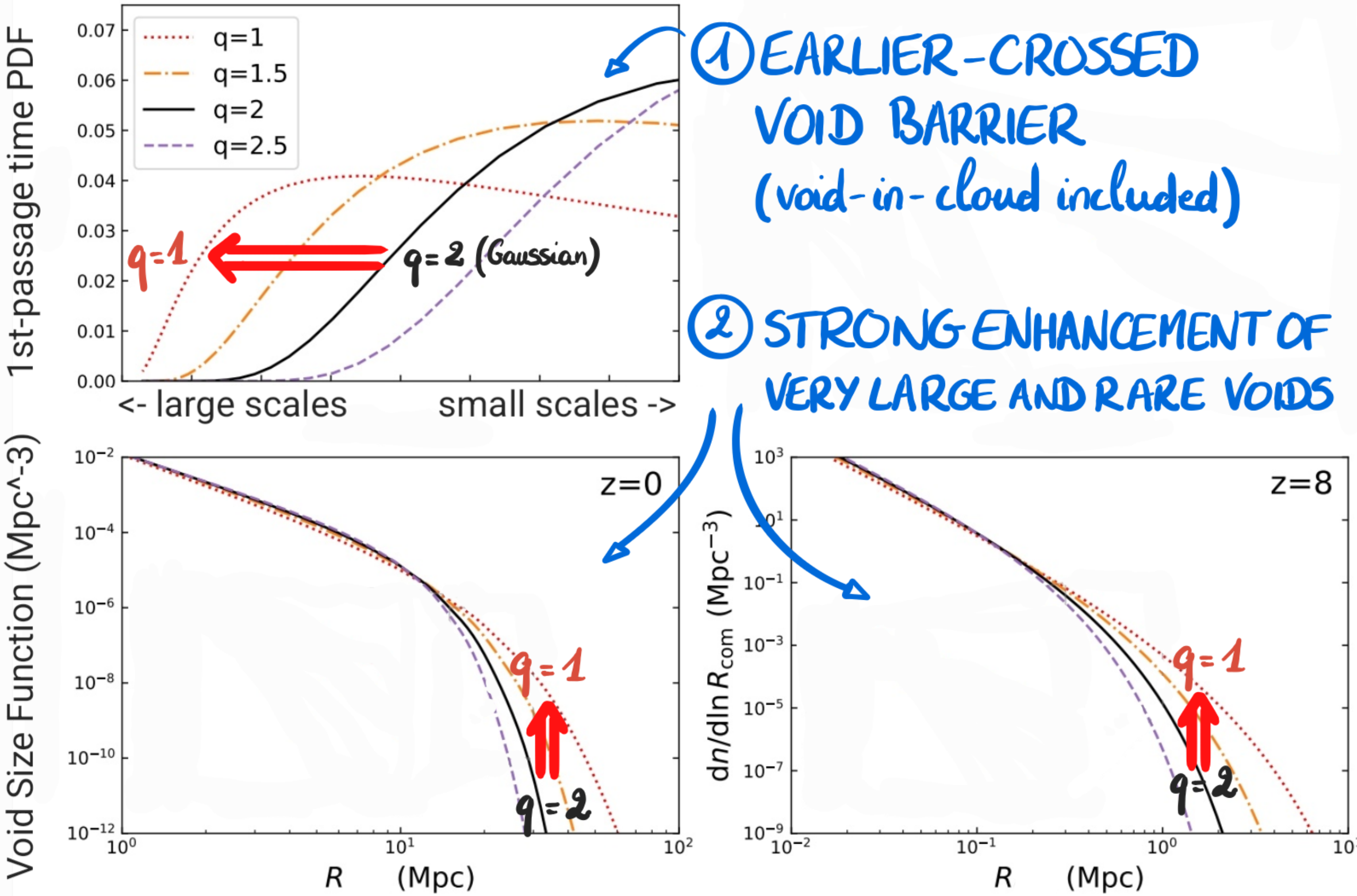
1ST-PASSAGE TIME PDF FOR LARGE VOIDS

$$f^{\rm FPT}_{\rm void}(t) = \frac{q}{2\sqrt{\pi}t} \sum_{n\geq 0} \left[Q^n \left(\frac{c_n \gamma}{\sqrt{t}}\right)^{q/2} \exp\left(-\left(\frac{c_n\gamma}{\sqrt{t}}\right)^q\right) - Q^{n+1/2} \left(\frac{d_n \gamma}{\sqrt{t}}\right)^{q/2} \exp\left(-\left(\frac{d_n\gamma}{\sqrt{t}}\right)^q\right)\right]$$

$$c_n \equiv (|a|^{2Q} + 2n|b-a|^{2Q})^{1/(2Q)} \quad , \quad d_n \equiv (|b|^{2Q} + (2n+1)|b-a|^{2Q})^{1/(2Q)}$$

VOID SIZE FUNCTION

RECALL : $PDF(\delta) \propto \exp(-|\delta|^q)$



① EARLIER-CROSSED VOID BARRIER (void-in-cloud included)

② STRONG ENHANCEMENT OF VERY LARGE AND RARE VOIDS

THE CONCLUSION

TAKE-AWAYS

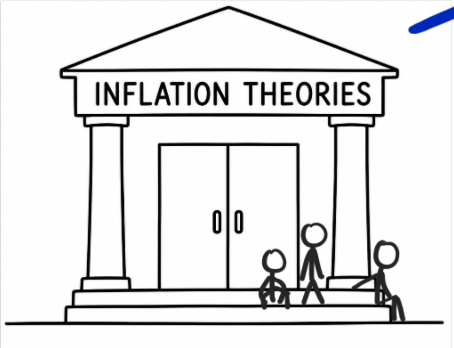
LARGE DEVIATION FORMALISM CAN BRING THEORETICAL PREDICTIONS IN REGIMES UNACCESSIBLE TO PERTURBATIVE NON-GAUSSIANITIES

TAKE-AWAYS

LARGE DEVIATION FORMALISM CAN BRING THEORETICAL PREDICTIONS IN REGIMES UNACCESSIBLE TO PERTURBATIVE NON-GAUSSIANITIES

EXPONENTIAL TAILS CAN BOOST ABUNDANCE OF LARGE CLUSTERS AND LARGE VOIDS BY MANY ORDERS OF MAGNITUDE

PERSPECTIVES



- STOCHASTIC INFLATION
- NUMERICAL TREATMENTS
- PRIMORDIAL BLACK HOLES
- ...

LET'S DISCUSS!

- VOID CATALOGS
- JUST LITTLE RED DOTS
- DEGENERACIES
- ...

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PERTURBATIVE OR NON-PERTURBATIVE

Q° HOW TO BE PREDICTIVE WITHOUT GAUSSIANITY?

 $+ f_{NL}$  $^2 + g_{NL}$  $^3 + \dots$

PERTURBATIVE

NUMERICAL APPROACH

[Coulton et al. 2406.15546,...]

FAR FROM GAUSSIAN
→ NO EXPANSION

NON-PERT.
RARE EVENTS

HARD TO SAMPLE → NUMERICS
IS NOT AN EASY SOLUTION

MARKOVIANITY

$\{\hat{\delta}_k,\ k \in \mathbb{R}^3\}$ JOINTLY GAUSSIAN $\Longrightarrow$ $\{\hat{\delta}_k,\ k \in \frac{1}{2}\mathbb{R}^3\}$ INDEPENDENT

$\not\Longleftarrow$

remove half of modes $(\delta_{-k} = \delta_k^*)$

REAL-SPACE LOCAL NON-GAUSSIANITIES

$$\delta_{NG}(x) = F[\delta_G(x)] \underset{e.g.}{=} \delta_G(x) + \frac{3}{5} f_{NL} \left(\delta_G^2(x) - \langle \delta_G^2 \rangle\right)$$

INDEPENDENCE IS LOST

FOURIER-SPACE LOCAL NON-GAUSSIANITIES

$$\hat{\delta}_{NG}(k) = F[\hat{\delta}_G(k)]$$

INDEPENDENCE REMAINS

MARKOVIANITY

CORRELATIONS CAN BE INCORPORATED PERTURBATIVELY IN THE LARGE DEVIATION PRINCIPLE

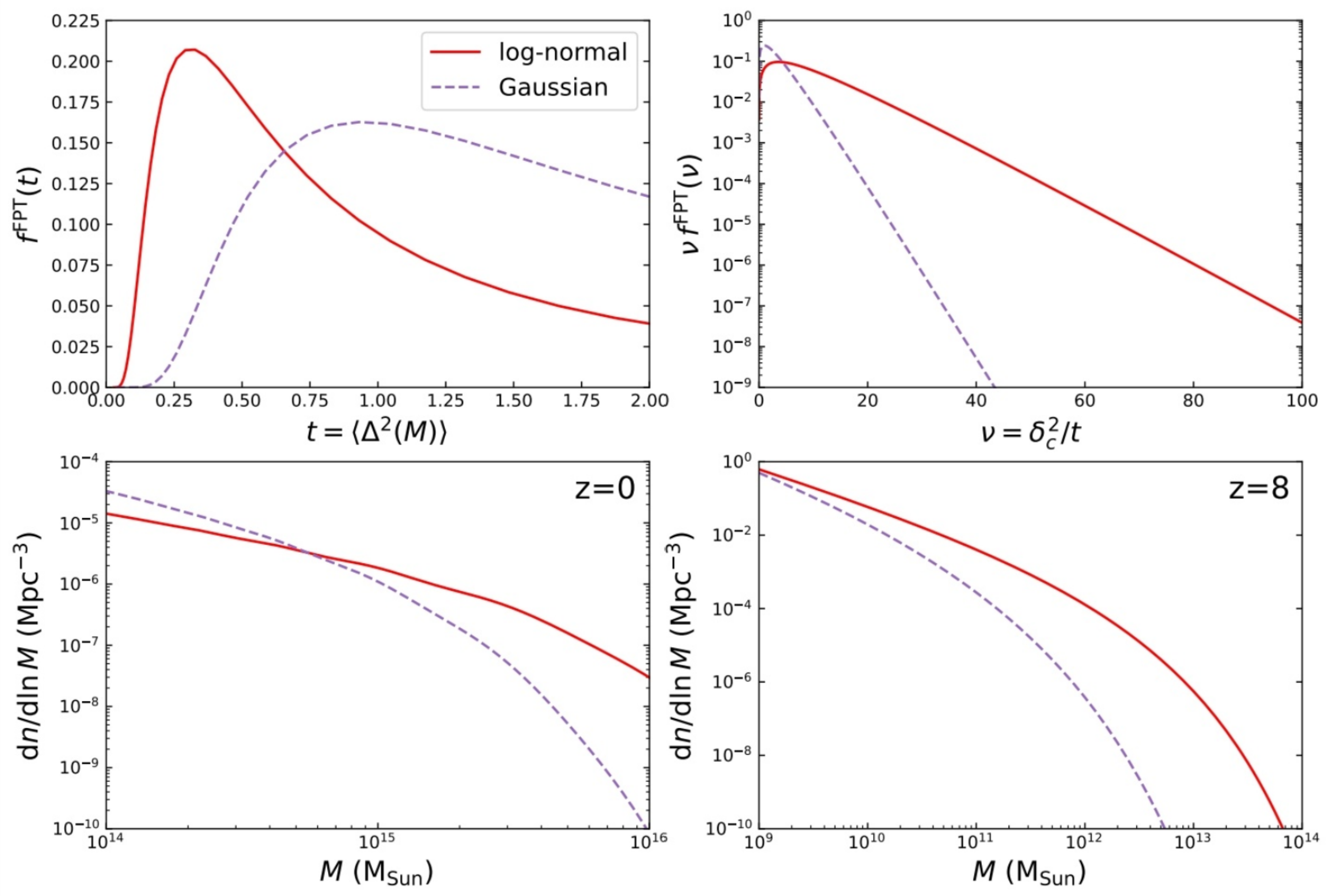
$$\langle \exp(\mathrm{Re}(\boldsymbol{v}\cdot\Delta)/\varepsilon)\rangle = \sum_{n=0}^{\infty} \frac{1}{2^n\varepsilon^n n!} \int_{K_{1/2}(R)} \mathrm{d}^3k_1\cdots\mathrm{d}^3k_n \sum_{b\in\{0,1\}^n} \prod_{i=1}^{n} v_{k_i}^{(1-b_i)} \langle \prod_{i=1}^{n} \Delta_{k_i}^{(b_i)}\rangle\ , \quad (3.14)$$

where $z_i^{(b_i)}$ means z_i whenever $b_i = 0$ and z_i^* whenever $b_i = 1$. Another strategy is to perform an Edgeworth-like expansion [18] of the joint probability, without relying on the Gärtner-Ellis lemma. One writes (for simplicity we directly consider the distribution of the joint moduli $|\boldsymbol{\Delta}| = \{|\Delta_k|\}_k$) $p_{|\boldsymbol{\Delta}|}$ using the probability distributions $p_{|\Delta_k|}$'s of each individual mode,

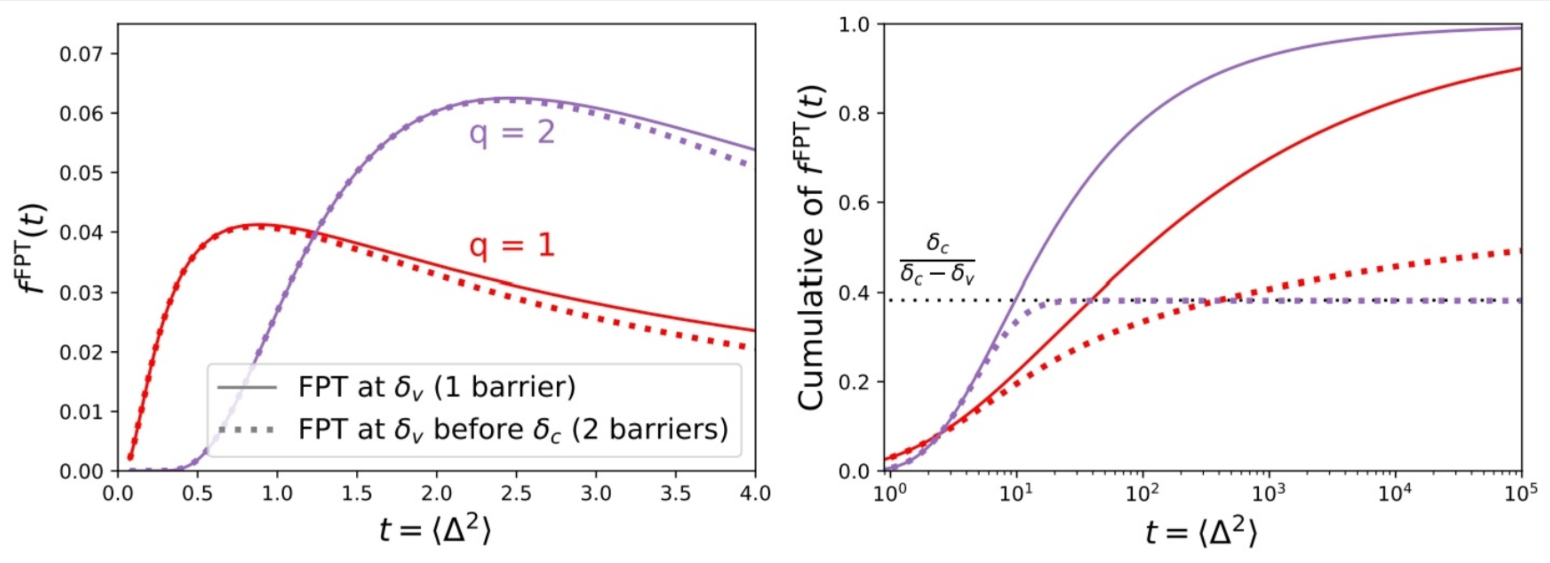
$$p_{|\boldsymbol{\Delta}|}(|\boldsymbol{z}|) = \left(\prod_{k\in K_{1/2}(R)} p_{|\Delta_k|}(|z_k|)\right) \times \left[1 + \sum_{\boldsymbol{m}=\{m_k\}_k} C(\boldsymbol{m}) \prod_{k\in K_{1/2}(R)} Q_k^{(m_k)}(|z_k|)\right] . \quad (3.15)$$

$\Rightarrow$ LDP $\begin{cases} \text{NON-PERTURBATIVE IN THE PDF} \\ \text{PERTURBATIVE (AS OF NOW) IN CORRELATIONS} \end{cases}$

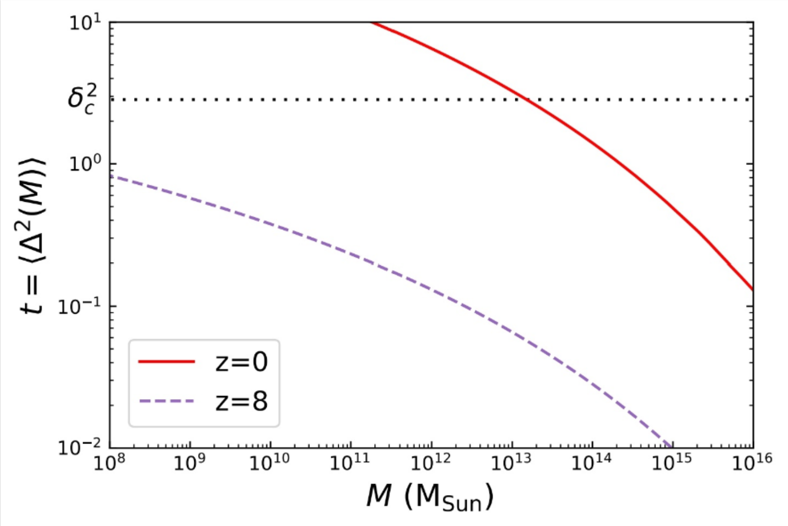
LOG-NORMAL & MOVING BARRIERS

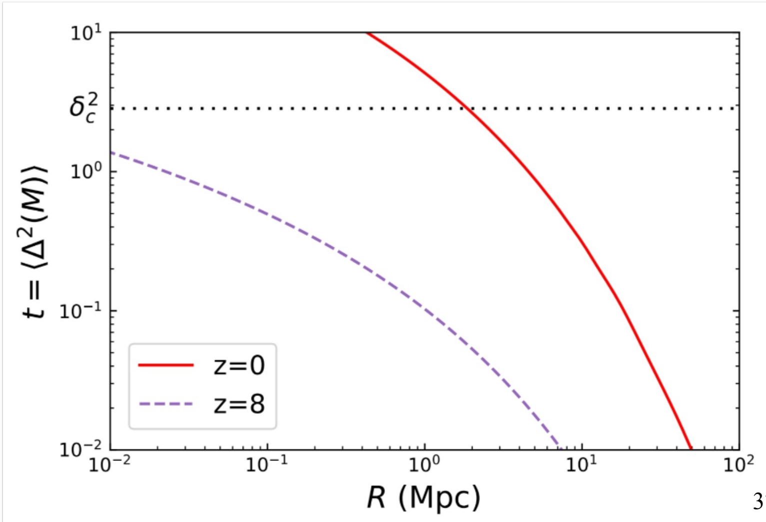


1 V.S. 2 BARRIERS



VARIANCE V.S. SCALE





PDF V.S. HMF

HALOS :

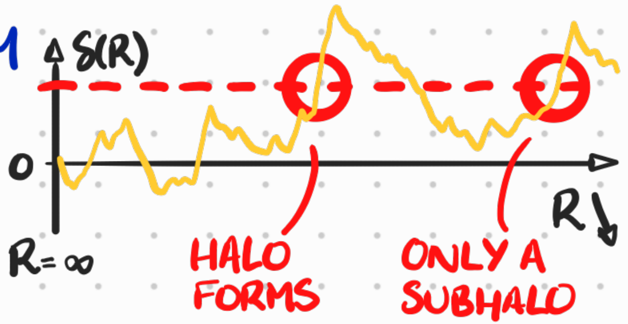
$$\frac{\mathrm{d}n(z)}{\mathrm{d}\ln M} = \frac{\bar{\rho}_{0,m}}{M} f^{\mathrm{FPT}}(t(z)) \frac{1}{3} \left| \frac{\mathrm{d}t(z)}{\mathrm{d}\ln R} \right|$$

VOIDS :

$$\frac{\mathrm{d}n(z)}{\mathrm{d}\ln R_{\mathrm{com}}} = \frac{V(R)}{V(R_{\mathrm{com}})} \frac{3}{4\pi(cR)^3} f_v^{\mathrm{FPT}}(t) \left| \frac{\mathrm{d}t(z)}{\mathrm{d}\ln R} \right|_{R=R_{\mathrm{com}}/1.7}$$

EXCURSION SET : WHAT FOR?

- SOLVE CLOUD-IN-CLOUD PROBLEM (NO HALO OVERCOUNTING)



- ONLY REQUIRE LINEAR THEORY

- SUITABLE FOR NUMERICAL SIMULATIONS

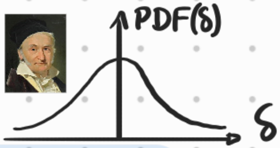
- CAN MODEL UNDERDENSITIES (VOIDS)

THE MOTIVATIONS

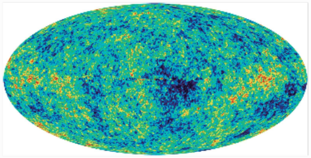
PRIMORDIAL NON-GAUSSIANITIES

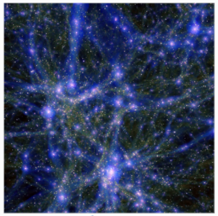
STANDARD LORE:

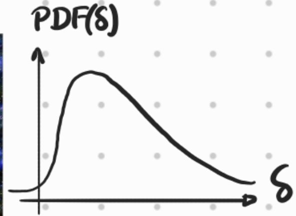
INFLATION



GAUSSIAN I.C.
FOR DENSITY
CONTRAST δ







late-time non-linearities
$\Rightarrow$ late-time non-Gaussianities)

MULTIFIELD INFLATION

[Pinol HAL03610789 ...]

ULTRA SLOW-ROLL

[Renaux-Petel 1508.06740,...]

δN-FORMALISM

[Sasaki et al. 9507001]

STOCHASTIC INFLATION

[Vennin et al. 1506.04732
Cruces 2203.13852]

COSMOLOGICAL COLLIDER

[Arkani-Hamed &
Maldacena 1503.0843]

INITIAL CONDITIONS
LIKELY PRESENT
NON-GAUSSIAN
FEATURES