Galileo Galilei Institute for Theoretical Physics

Arcetri, Florence

Exploring New Frontiers in Cosmology

Conference week – 20th-26th July 2026

Istituto Nazionale di Fisica Nucleare

THE UNIVERSAL ROTATION CURVE OF DISK GALAXIES: PROBING DARK MATTER AND THE LOCAL EXPANSION RATE

Speaker: Dr. Tiziano Schiavone (SISSA, Trieste)

20/07/2026

Project in collaboration with Prof. Paolo Salucci and Dr. Sandeep Haridasu (SISSA, Trieste)

SISSA Scuola Internazionale Superiore di Studi Avanzati

MUR PRIN 2022 PNRR

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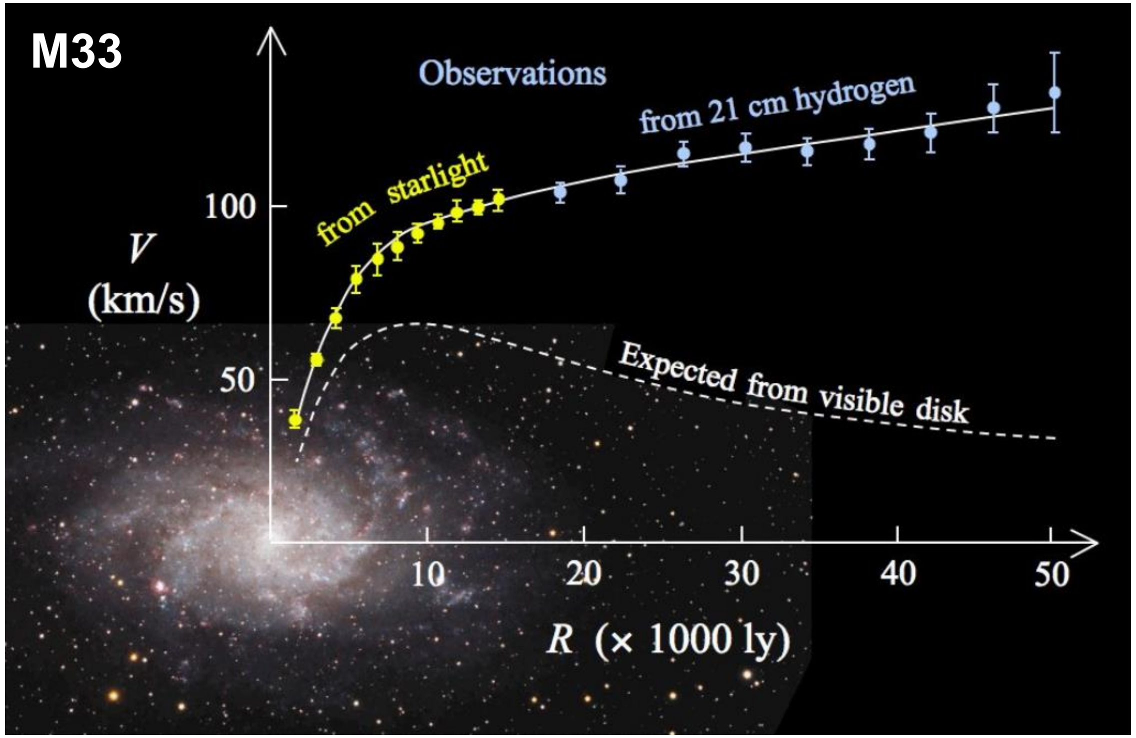
CONTENTS

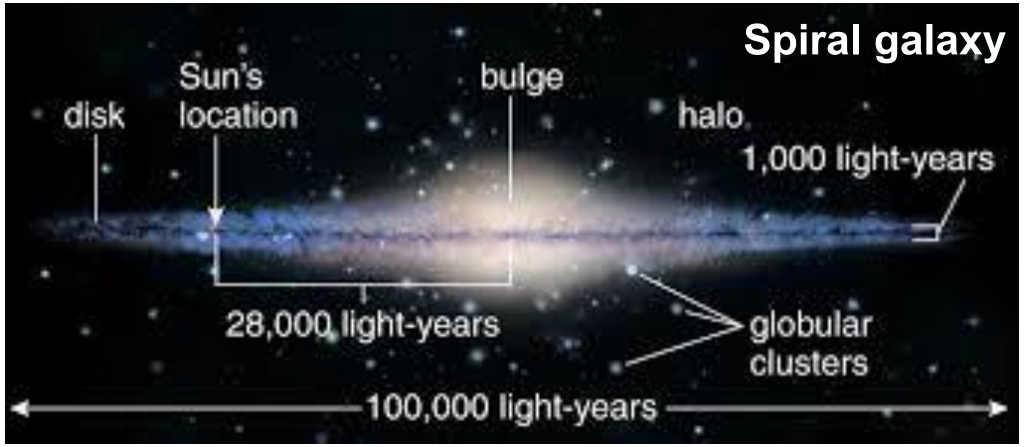
RCs AS PROBES OF GALAXY MASS DISTRIBUTION

Rotation curves (RCs):
plot of rotational velocities of an object around a central mass against its radial distance.



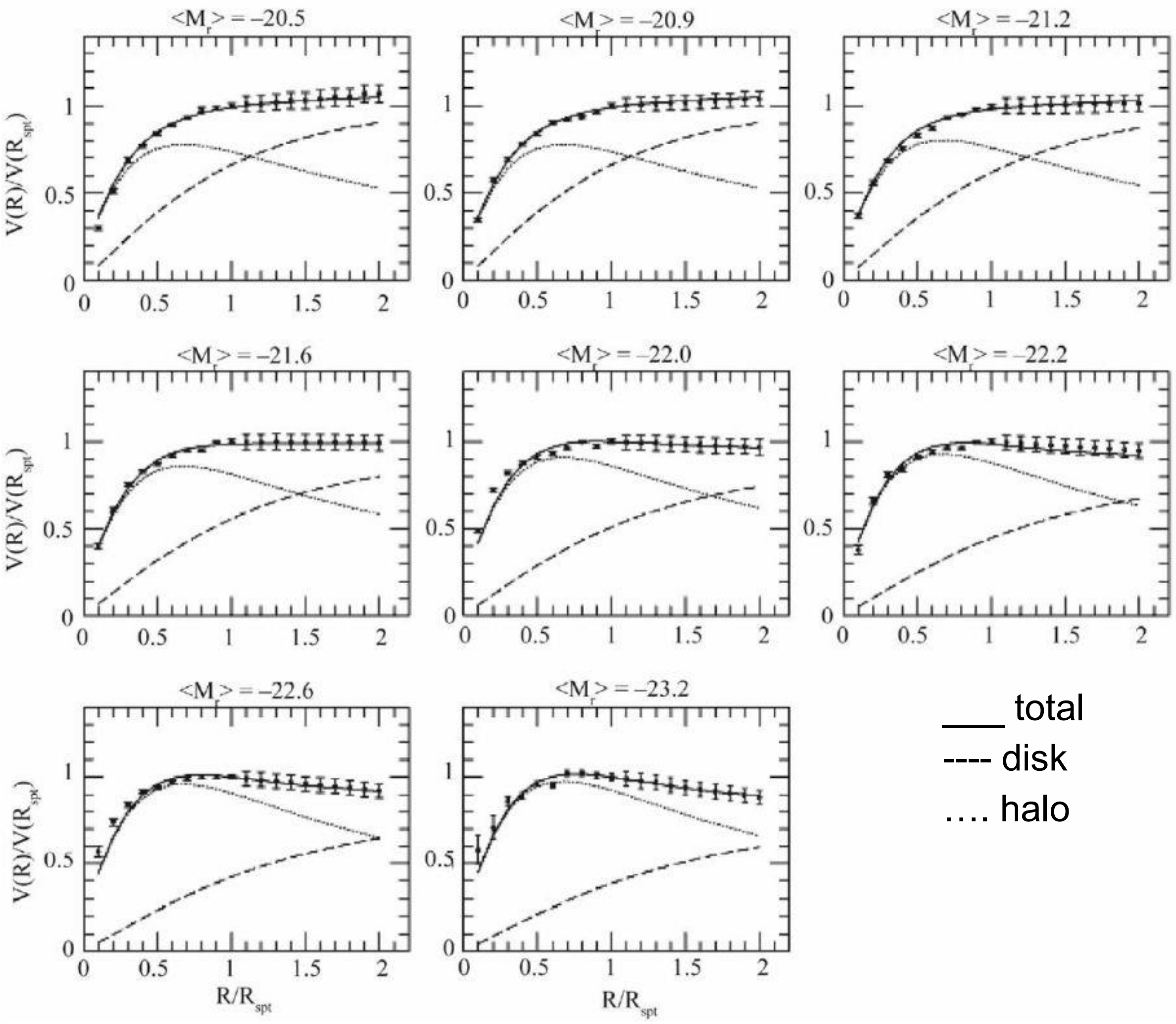
SALUCCI, Astron.Astrophys.Rev. 27 (2019) 1, 2

- Observed velocities in spiral galaxies remain approximately flat beyond the optical disk.
- **Luminous matter** (stellar disk [D], gas [G], stellar bulge [B]) alone cannot explain galaxy kinematics.
- **Dark matter** halos [H] around galaxies become the natural explanation.



RCs provide one of the strongest observational probes of dark matter.

RCs AS PROBES OF GALAXY MASS DISTRIBUTION



____ total
---- disk
…. halo

PERSIC et al. (1996), MNRAS 281, 27-47

- Observed velocities in spiral galaxies remain approximately flat beyond the optical disk.
- **Luminous matter** (stellar disk [D], gas [G], stellar bulge [B]) alone cannot explain galaxy kinematics.
- **Dark matter** halos [H] around galaxies become the natural explanation.
- Rotation curves directly trace the gravitational potential ϕ_{tot}:

$$V_{tot}^2(R) = R\frac{d\phi_{tot}}{dR} = V_D^2 + V_H^2 + V_G^2 + V_B^2$$

$$\phi_{tot} = \phi_D + \phi_H + \phi_G + \phi_B$$

Dynamical equilibrium gravitational VS centrifugal forces

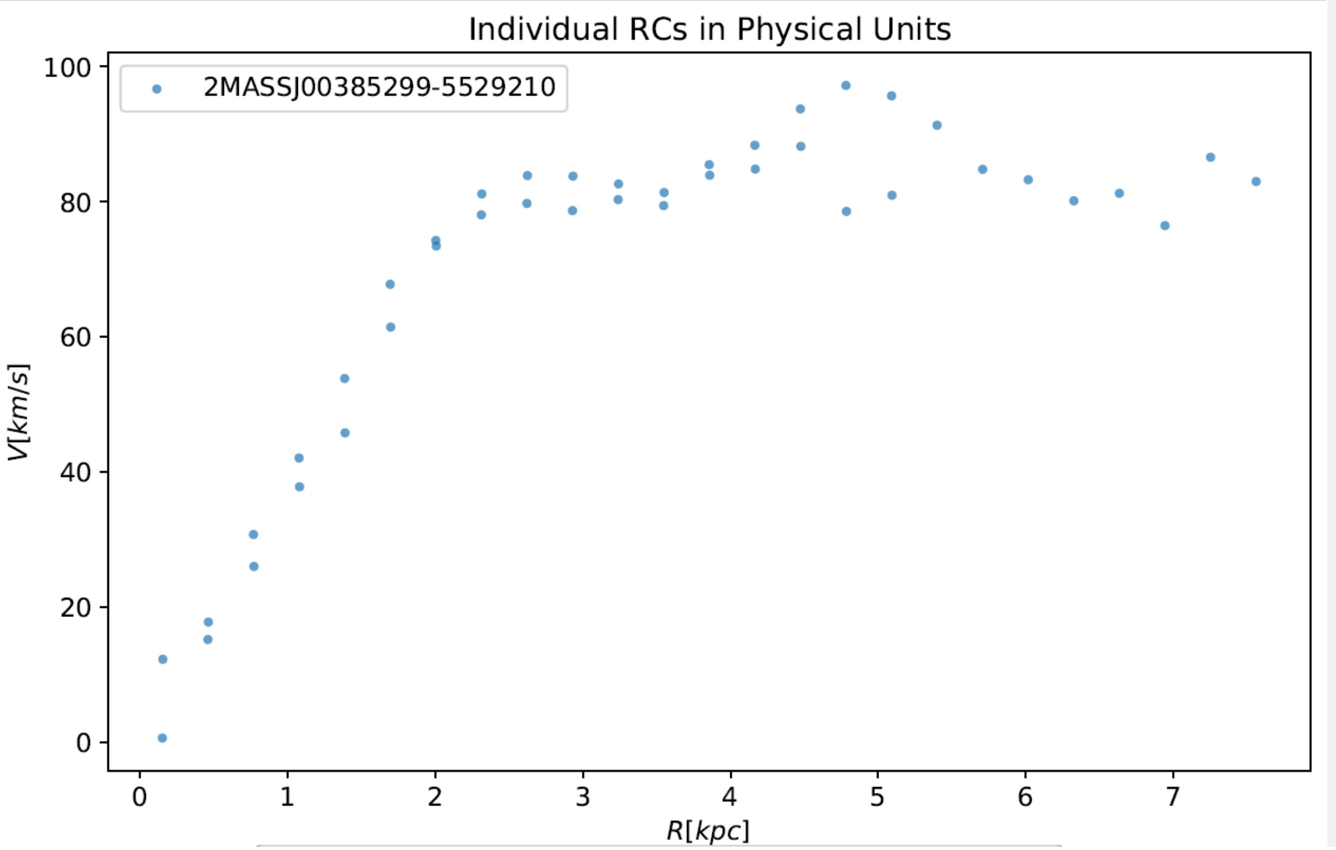
BEYOND INDIVIDUAL ROTATION CURVES

➢ **RCs of N_{gal} galaxies**

Data from **PROBES** Compendium
[https://zenodo.org/records/10456320**]**
Stone *et al.* (2022), ApJS 262 33

Data selection after estimating the kinematic center and shifting the observed radius and velocity.
Good agreement between trailing and receeding arms.
Preliminary independent estimate of the optical radius.

973 Selected galaxies
52966 RC data points

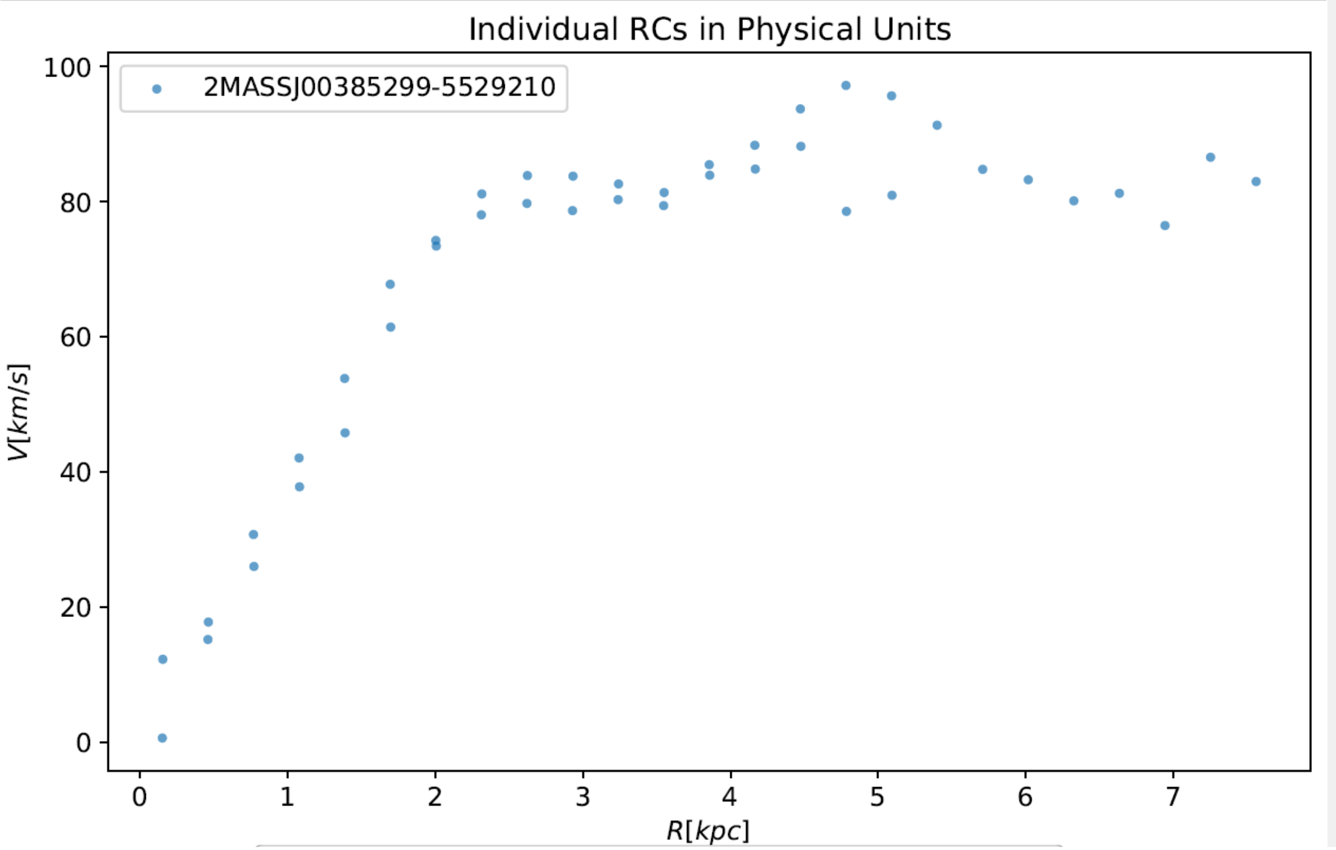


BEYOND INDIVIDUAL ROTATION CURVES

- **RCs of N_{gal} galaxies**

  RCs from individual galaxies can appear very different due to various galactic properties:
  luminosity, size, morphology, stellar mass, surface brightness, metallicity, observational sampling, etc.

  Is there an underlying universal behaviour?



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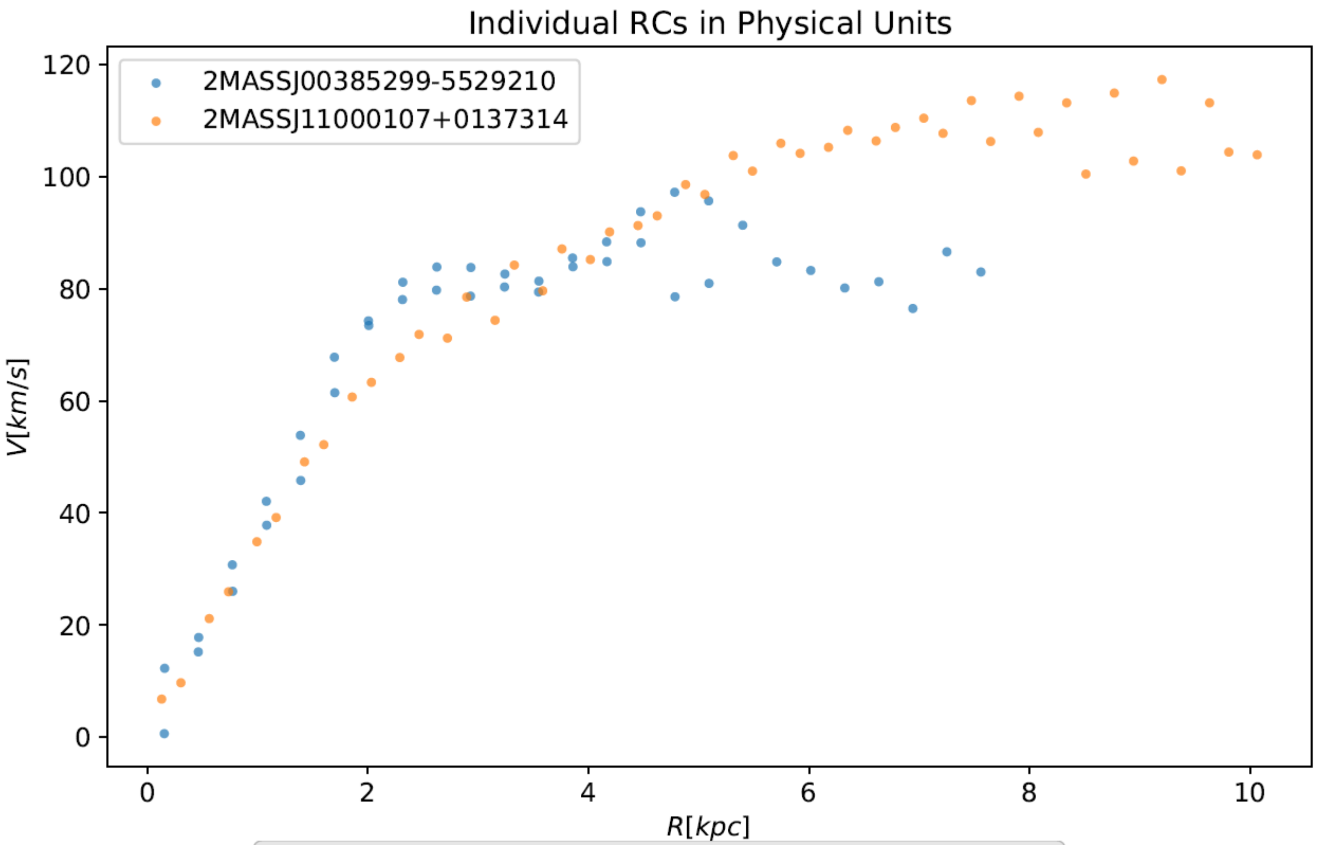

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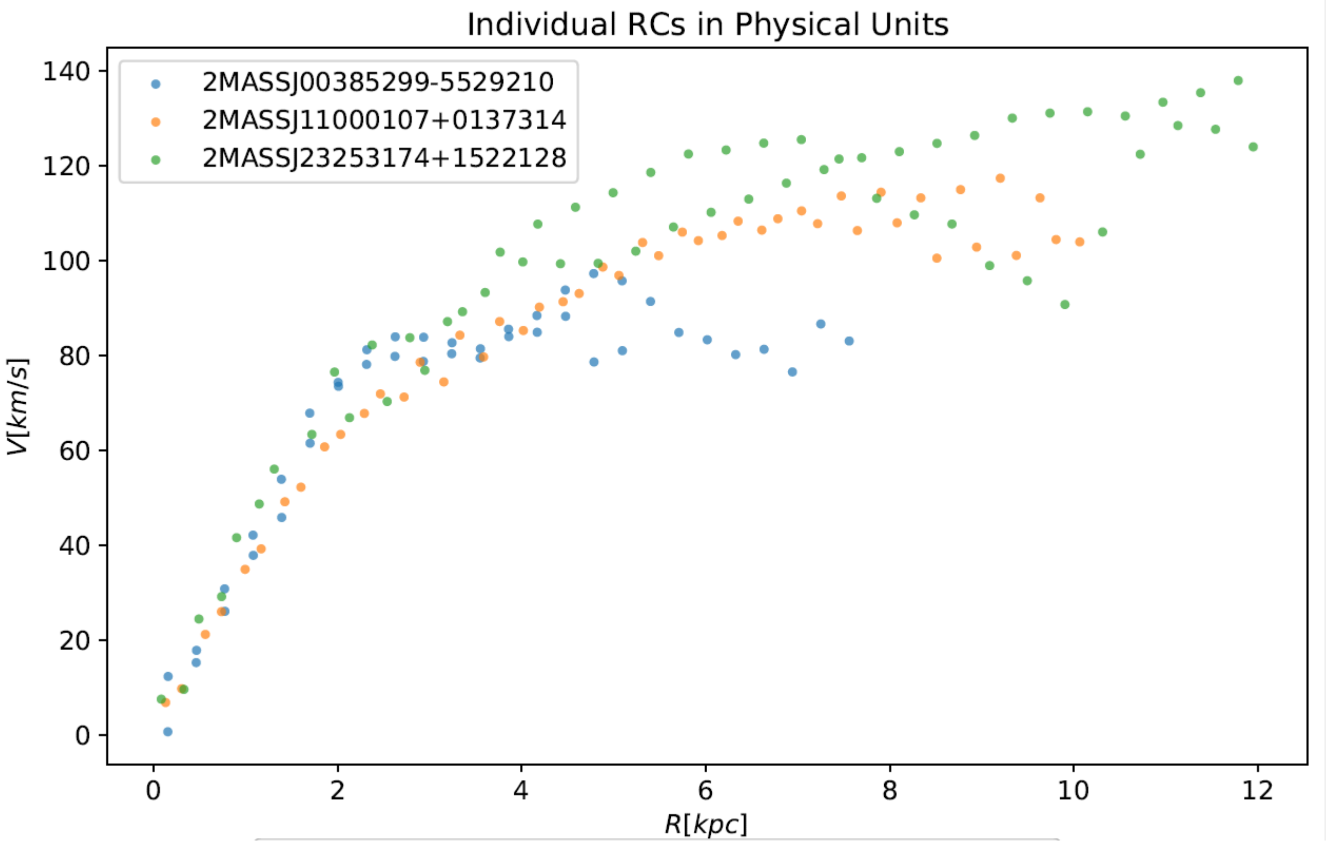

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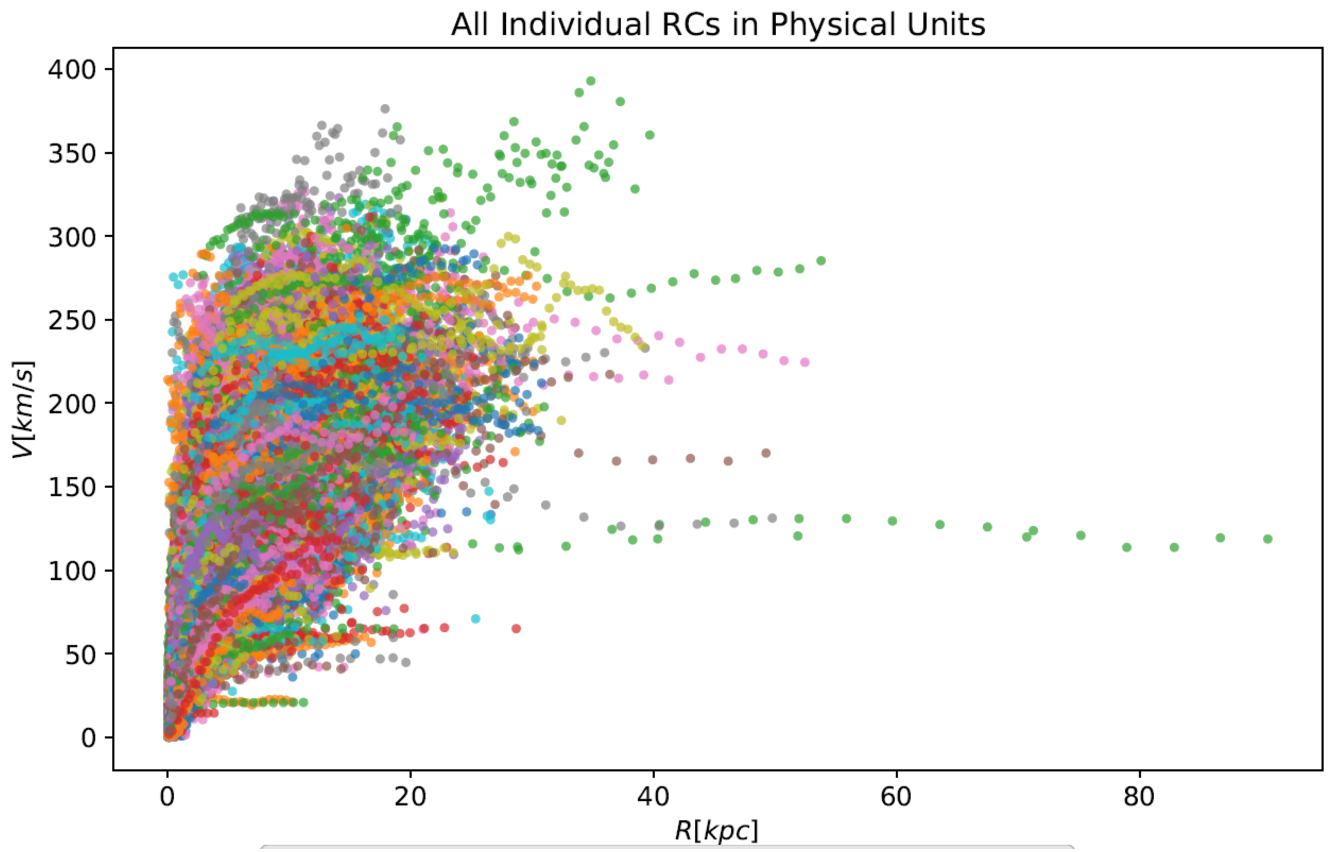


BEYOND INDIVIDUAL ROTATION CURVES

- **RCs of N_{gal} galaxies**
- **Double normalization**

Normalized radius

$$x = \frac{R}{R_{opt}}$$

Normalized velocity

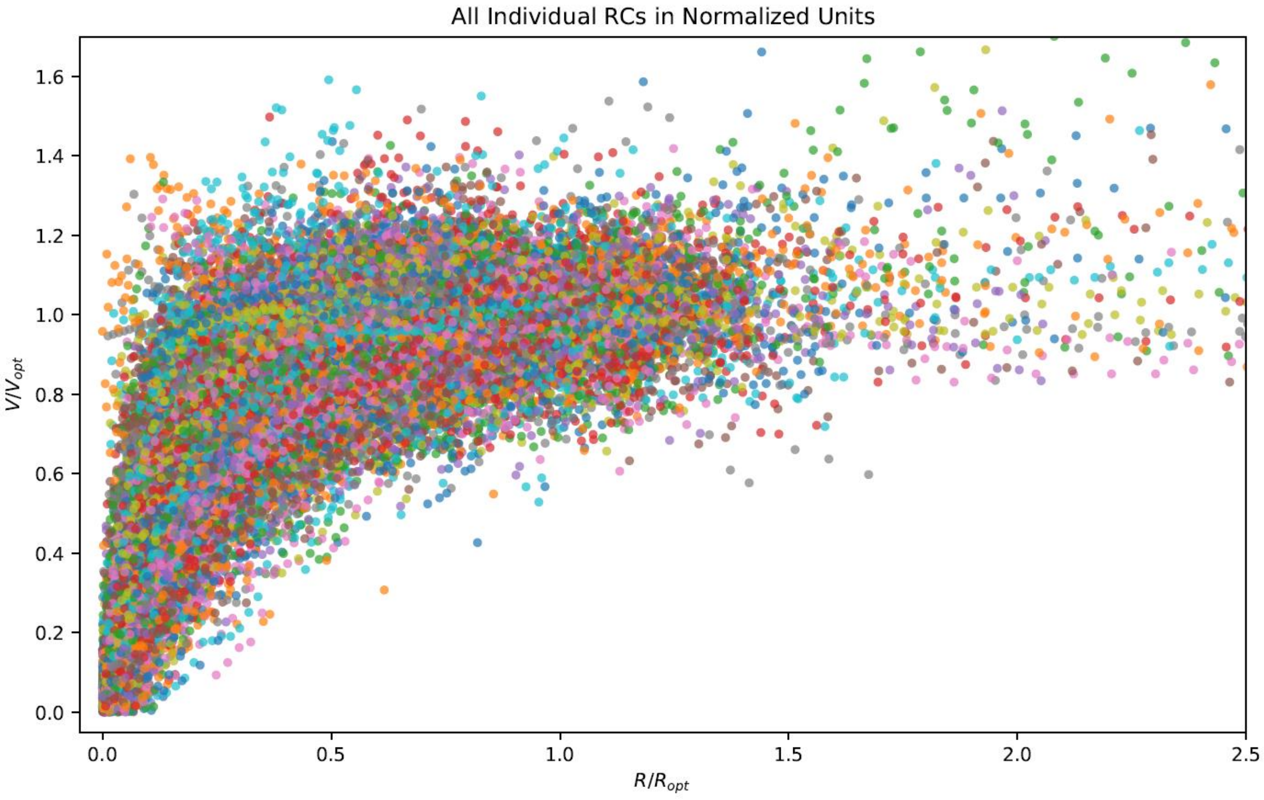
$$v = \frac{V}{V_{opt}}$$

with $R_{opt} = 3.2\, R_D$ and $V_{opt} = V(R_{opt})$

R_{opt}: optical radius encompassing 83% of total luminosity*

R_D: disk radius

* PERSIC & SALUCCI (1991), ApJ 368:60-65



PRESENT WORK (in preparation)

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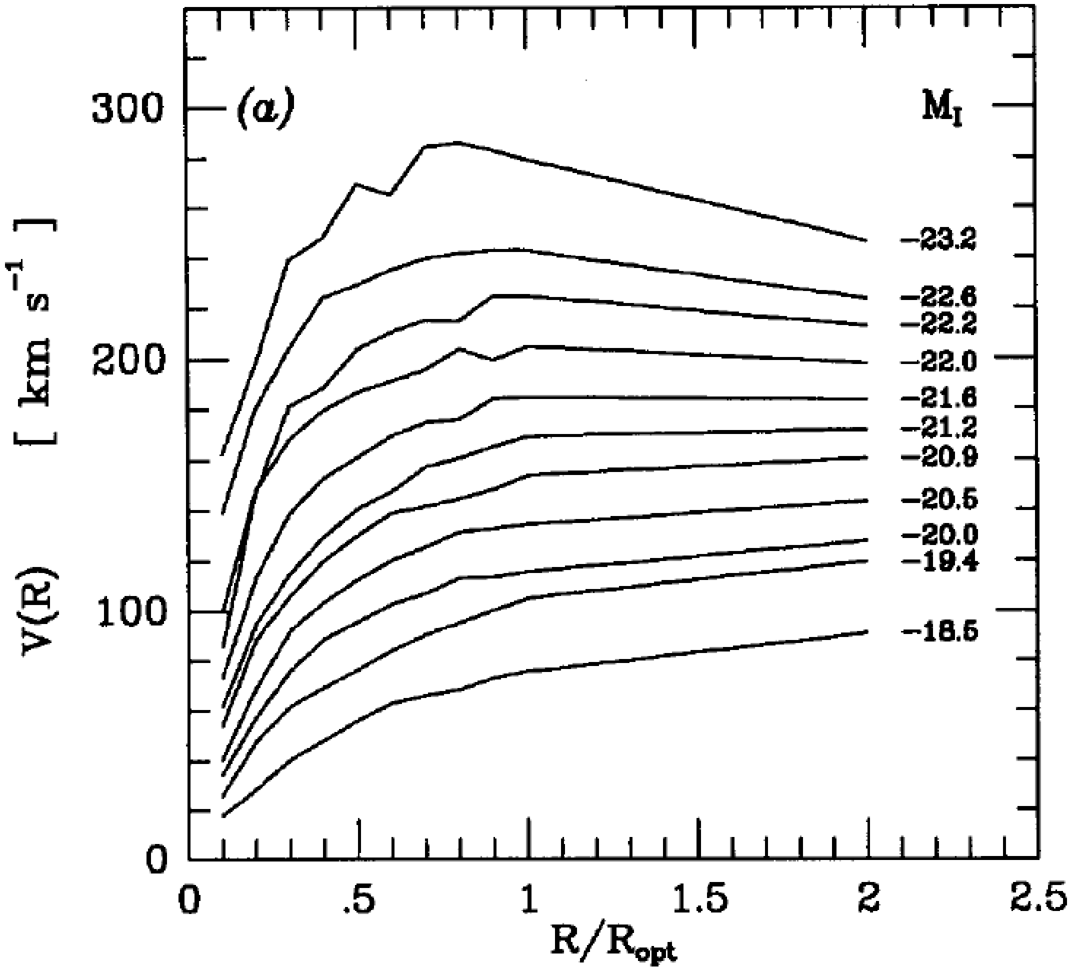

PERSIC et al. (1996), MNRAS 281, 27-47

BEYOND INDIVIDUAL ROTATION CURVES

- **RCs of N_{gal} galaxies**
- **Double normalization**
- **N_{vb} optical velocity bins**

  RCs are collected in N_{vb} optical velocity bins, according to V_{opt} values of individual galaxies
- **N_{rb} radial bins**

  For each optical velocity bin, data points are divided into N_{rb} radial bins
- **Coadded RCs**

  For each optical velocity bin, data points are averaged inside a specific radial bin

$V_{\rm opt}$ Bin	$N_{\rm gal}$	$V_{\rm opt}$ range (km s^{-1})	$\langle V_{\rm opt}\rangle$ (km s^{-1})	$\langle R_{\rm opt}\rangle$ (kpc)
1	57	[17.64, 60.00]	42.25 ± 1.65	4.24 ± 0.39
2	56	[60.00, 85.00]	74.37 ± 1.01	6.62 ± 0.47
3	68	[85.00, 100.00]	93.56 ± 0.54	8.28 ± 0.36
4	63	[100.00, 115.00]	106.51 ± 0.53	9.48 ± 0.44
5	108	[115.00, 130.00]	122.37 ± 0.43	9.82 ± 0.32
6	97	[130.00, 145.00]	136.62 ± 0.40	11.07 ± 0.36
7	84	[145.00, 160.00]	152.33 ± 0.52	11.65 ± 0.37
8	95	[160.00, 175.00]	167.29 ± 0.46	13.20 ± 0.38
9	92	[175.00, 190.00]	182.26 ± 0.45	14.28 ± 0.42
10	103	[190.00, 210.00]	198.65 ± 0.54	14.48 ± 0.46
11	72	[210.00, 235.00]	222.01 ± 0.92	16.80 ± 0.59
12	29	[235.00, 260.00]	244.94 ± 1.24	18.20 ± 0.97
13	26	[260.00, 300.00]	273.95 ± 2.25	18.77 ± 1.02
14	5	[300.00, 422.62]	321.79 ± 8.26	21.58 ± 4.52

10 radial bins in the range [0, 0.75] R_{opt}

6 radial bins in the range [0.75, 2.00] R_{opt}

PRESENT WORK (in preparation)

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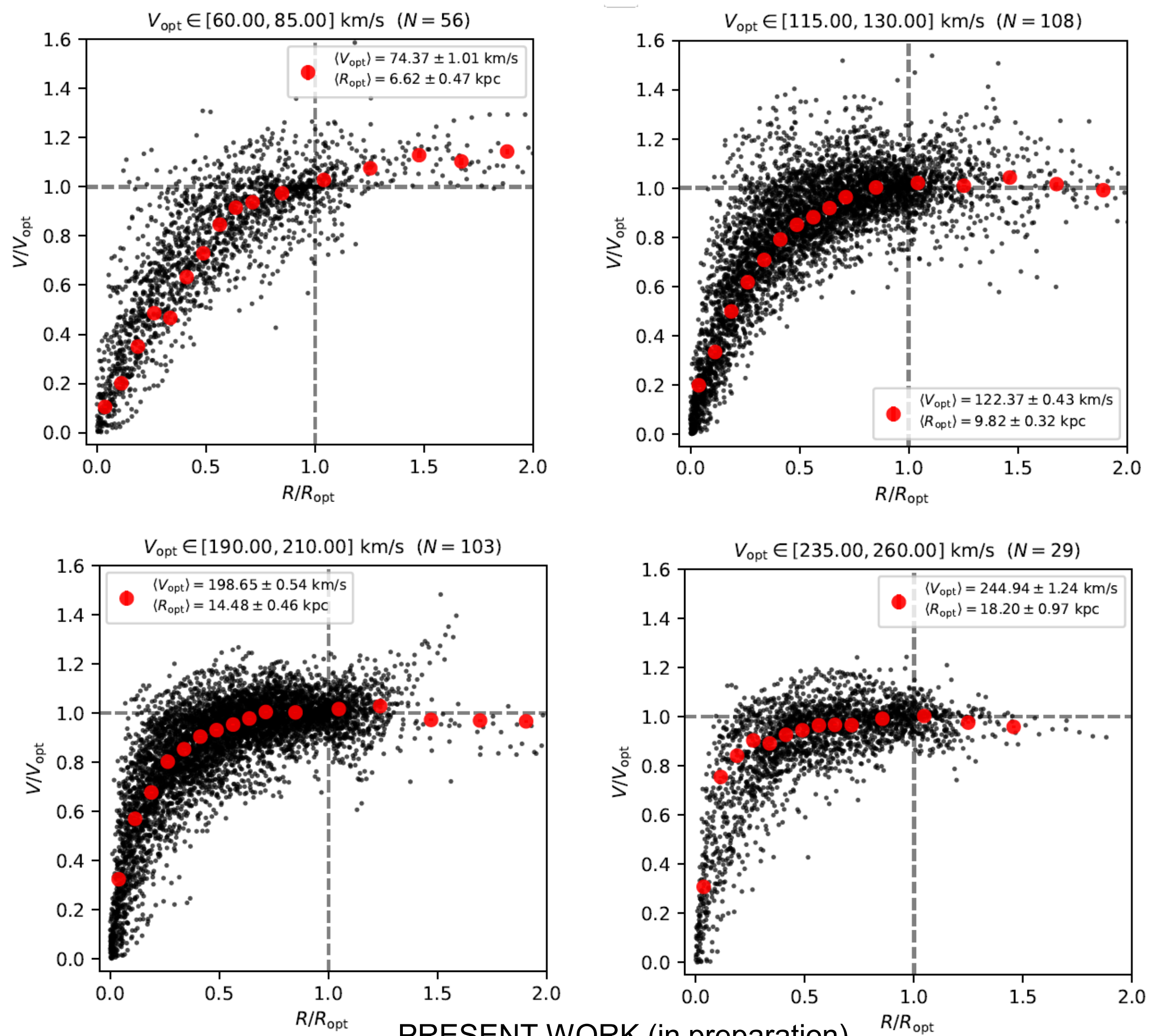

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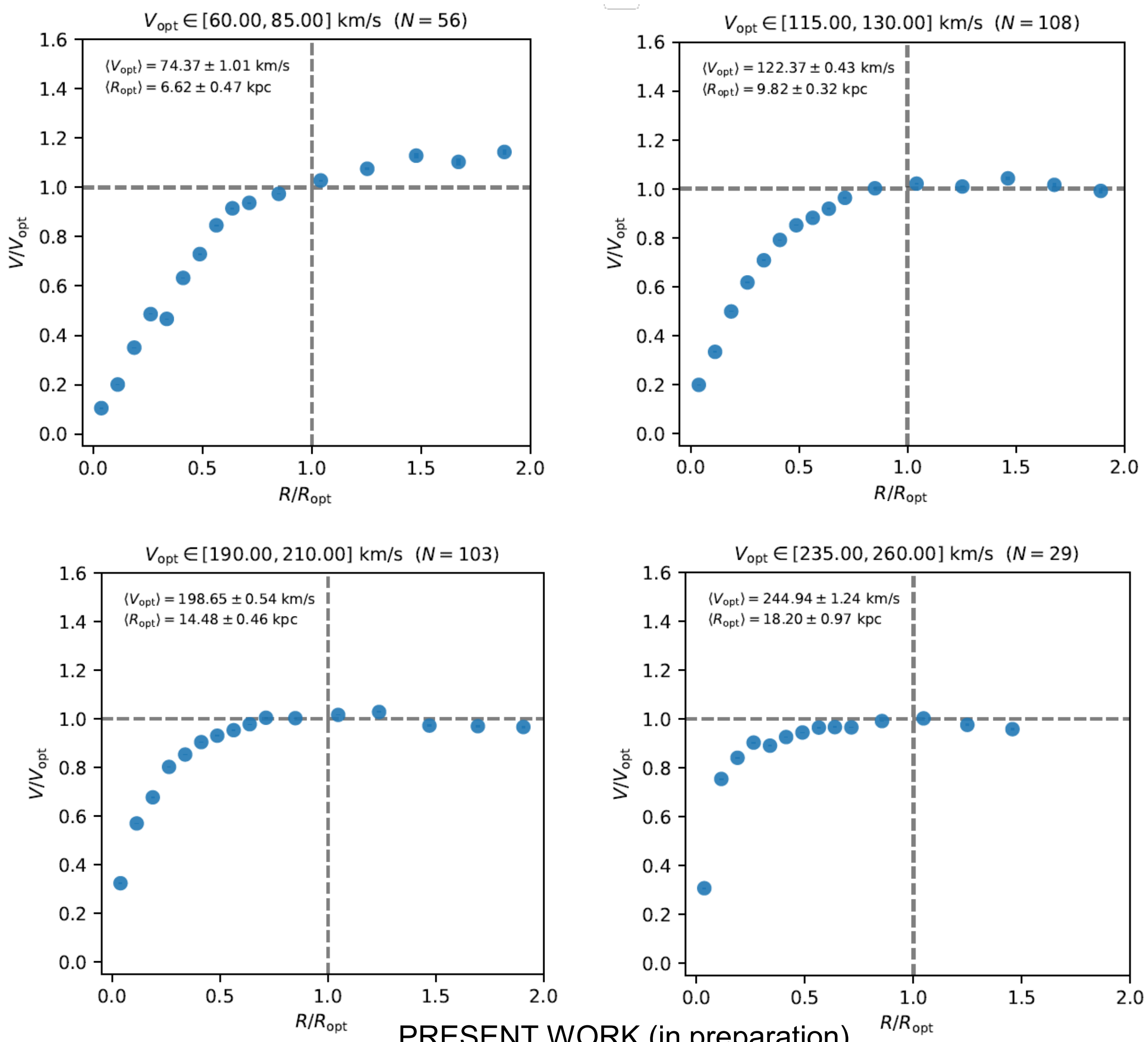

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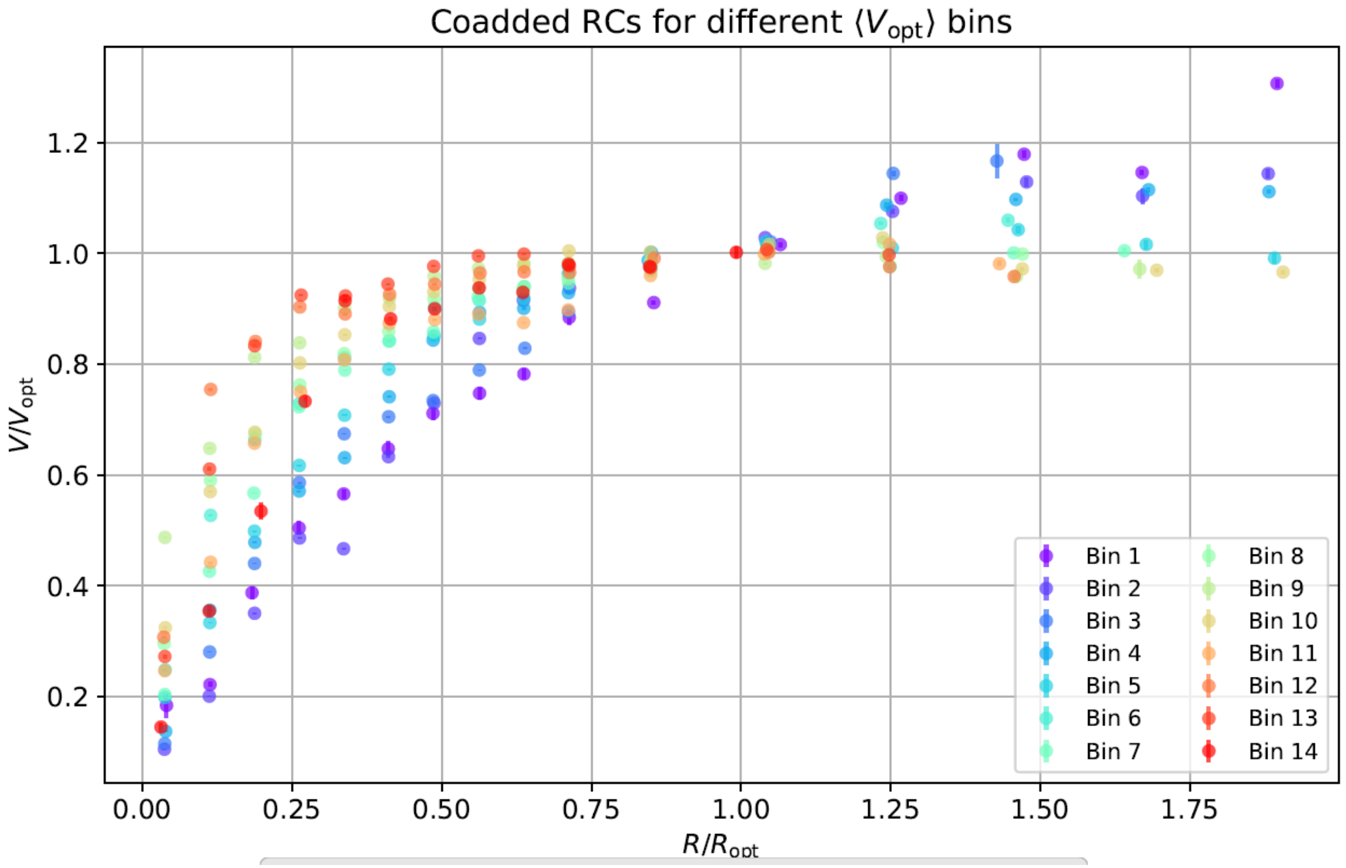


PRESENT WORK (in preparation)

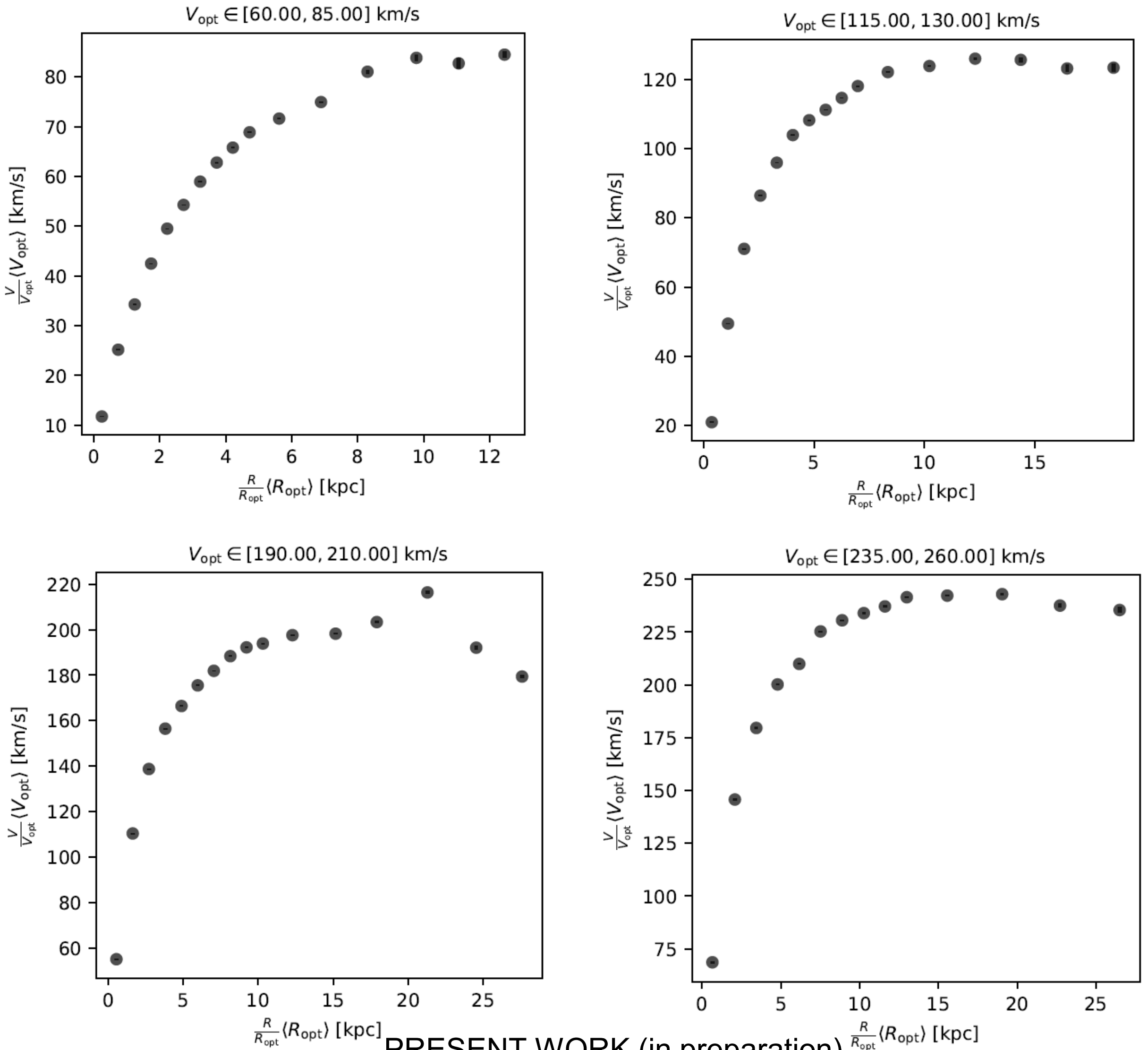
BEYOND INDIVIDUAL ROTATION CURVES

- RCs of N_{gal} galaxies
- Double normalization
- N_{vb} optical velocity bins
- N_{rb} radial bins
- Coadded RCs
- Denormalization

$$\langle V \rangle = v \left\langle V_{opt} \right\rangle$$
$$\langle R \rangle = x \left\langle R_{opt} \right\rangle$$

coadded RCs in physical units: $\langle V \rangle \, [\frac{km}{s}]$ VS $\langle R \rangle \, [kpc]$

$\left\langle V_{opt} \right\rangle$ and $\left\langle R_{opt} \right\rangle$ averaged from galaxies in an optical velocity bin

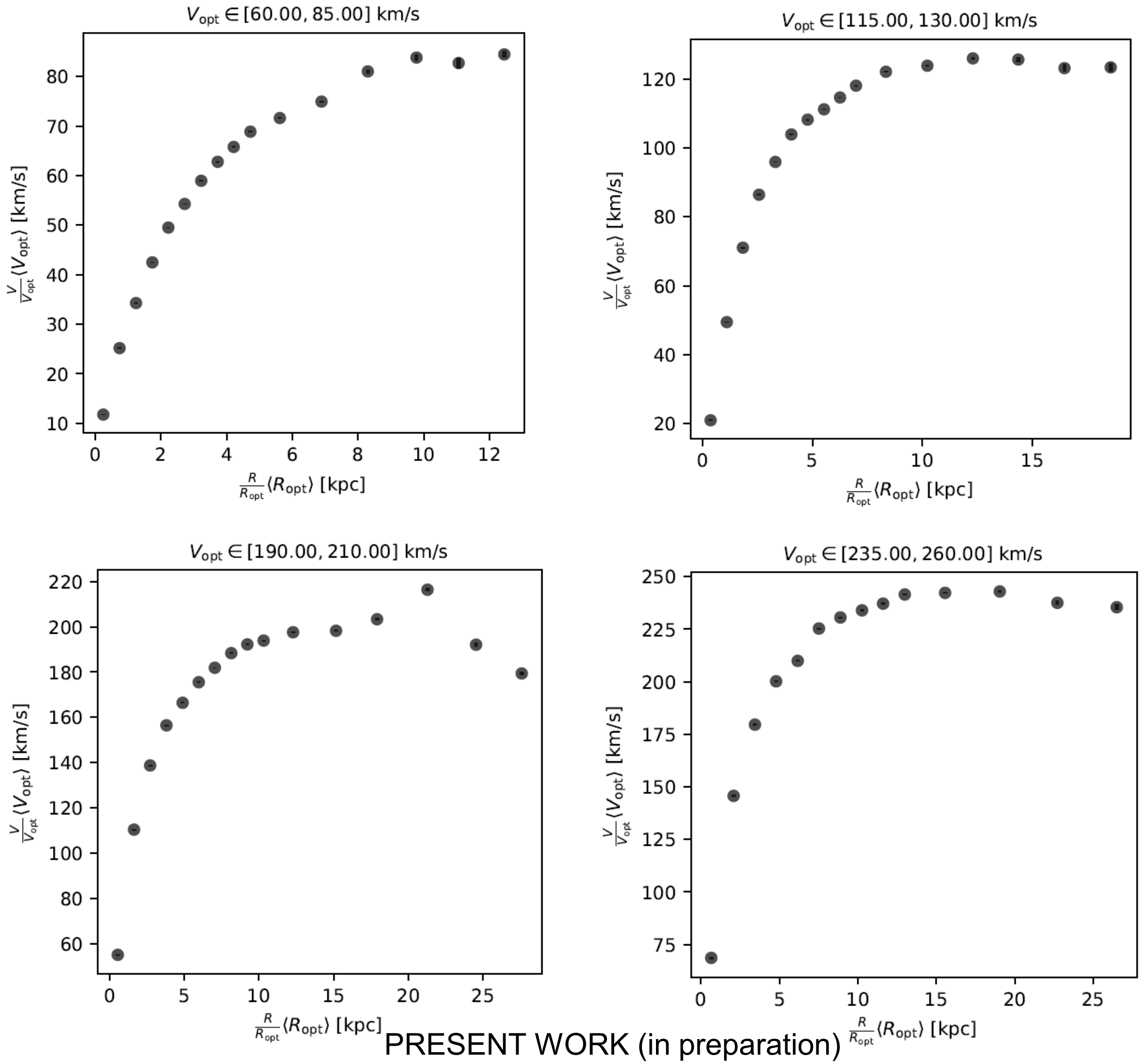


PRESENT WORK (in preparation)

BEYOND INDIVIDUAL ROTATION CURVES

- **RCs of N_{gal} galaxies**
- **Double normalization**
- **N_{vb} optical velocity bins**
- **N_{rb} radial bins**
- **Coadded RCs**
- **Denormalization**

This method partially removes galaxy-to-galaxy diversity and allow us to investigate the underlying dynamical information.

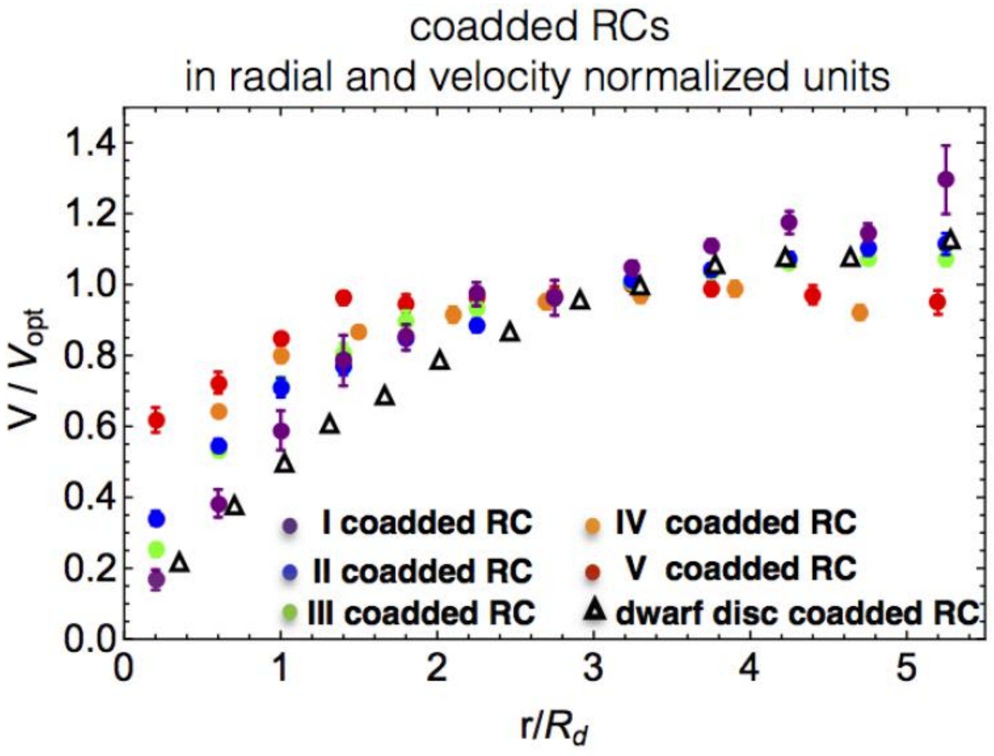


PRESENT WORK (in preparation)

BEYOND INDIVIDUAL ROTATION CURVES

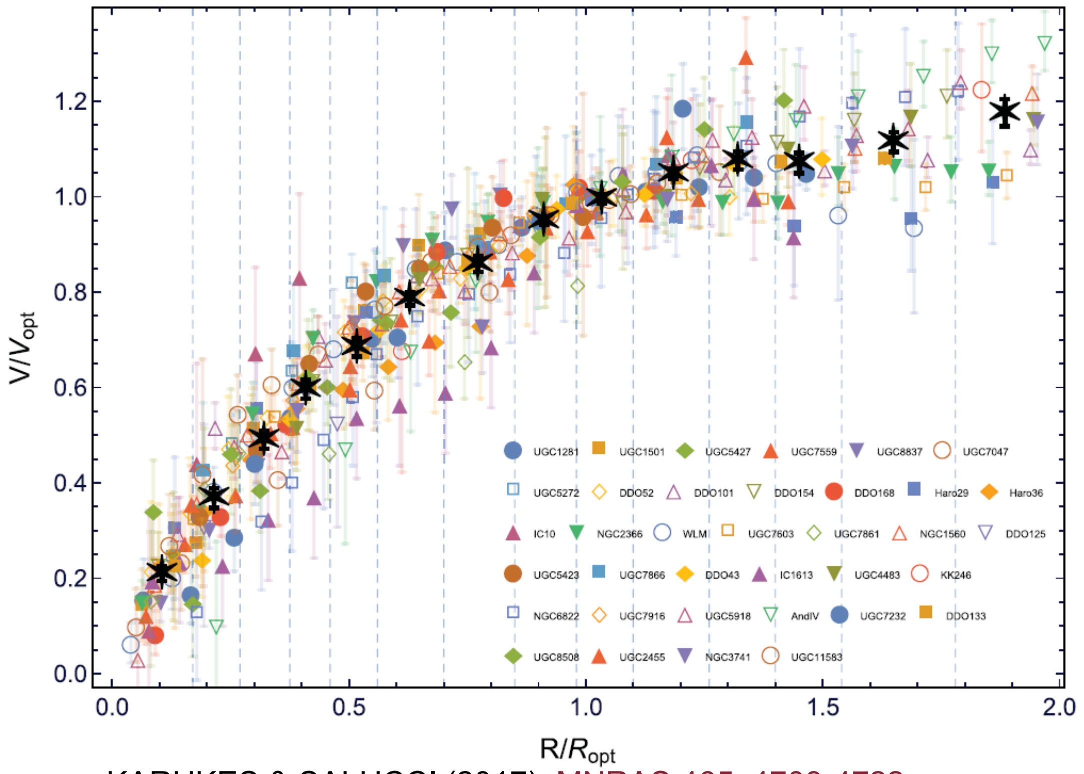
Universality extends across different galaxy populations
(not only spiral galaxies)

Low-surface brightness galaxies



DI PAOLO et al. (2019), MNRAS 490 4, 5451-5477

Dwarf disc galaxies



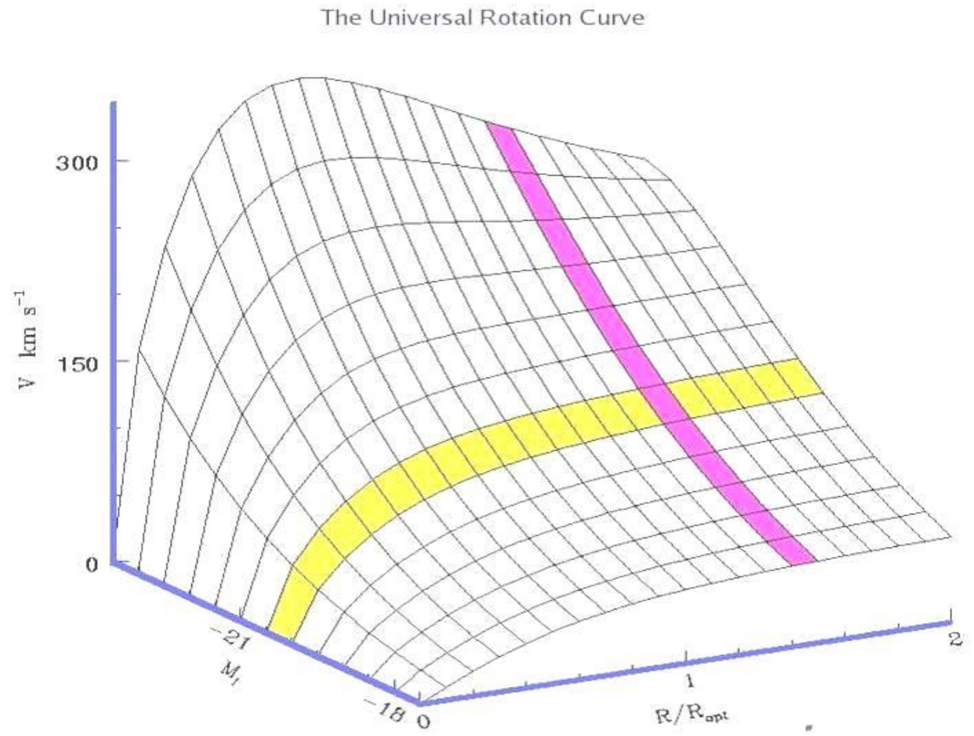
KARUKES & SALUCCI (2017), MNRAS 465, 4703-4722

FROM INDIVIDUAL GALAXIES TO UNIVERSALITY

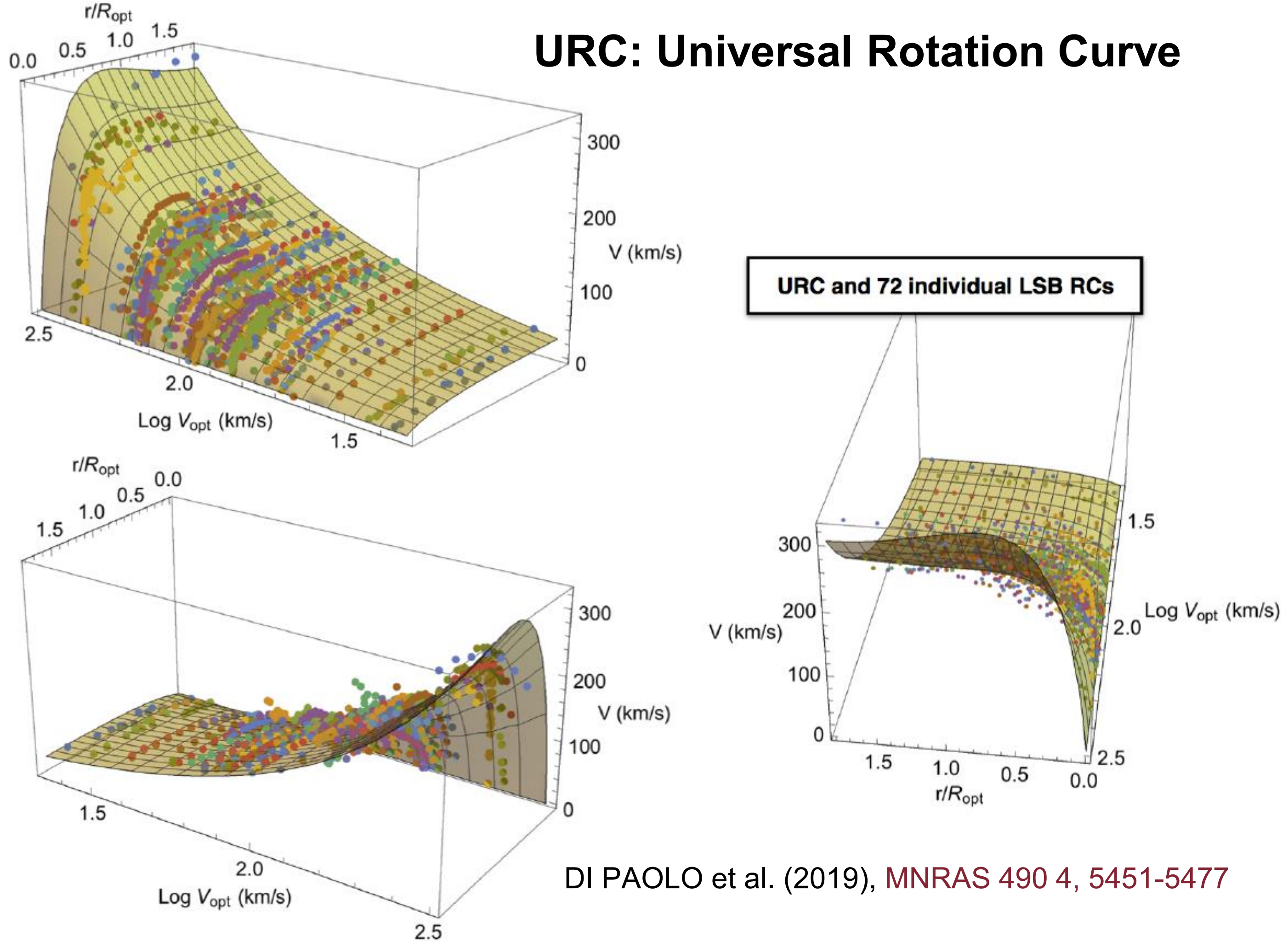
$$V = V\left(\frac{R}{R_{opt}}, V_{opt}\right)$$

$$v = v(x, V_{opt})$$

Universal function to predict the rotation velocity of any galaxy



URC: Universal Rotation Curve



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Every observed rotation curve samples the same surface

MASS MODELLING FROM THE COADDED RCs

Two main components: galaxy disk [D] and dark matter halo [H]

Freeman exponential disk FREEMAN (1970), ApJ, 160, 811

Surface mass density $\Sigma_D(R) = \frac{M_D}{2\pi R_D^2} e^{-\frac{R}{R_D}}$

M_D: disk mass
R_D: disk radius (fixed)

Enclosed mass $\widetilde{M}_D(R) = \int_0^R 2\pi R' \, \Sigma_D(R') \, dR'$

Contribution to rotation velocity

$$V_D^2(R) = G \frac{\widetilde{M}_D(R)}{R} = G \frac{M_D}{R} \left[1 - e^{-\frac{R}{R_D}} \left(1 + \frac{R}{R_D}\right)\right]$$

Dark Matter halo profile

mass density profile of DM $\rho_H(R) = \rho_H(R; \rho_0, R_C)$

Enclosed mass $M_H(R) = \int_0^R 4\pi R'^2 \, \rho_H(R') \, dR'$

Contribution to rotation velocity

$$V_H^2(R) = G \frac{M_H(R)}{R}$$

ρ_0: central density
R_C: core radius

The resulting analytical function $V^2(R)$ is compared with data to constrain the three parameters (M_D, ρ_0, R_C)

$$V^2(R) \approx V_D^2(R) + V_H^2(R)$$

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NFW profile NAVARRO, FRENK, & WHITE (1996), ApJ, 462, 563

mass density profile of DM $$\rho_H^{NFW}(R) = \frac{\rho_0}{\frac{R}{R_C}\left(1 + \frac{R}{R_C}\right)^2}$$

$$M_H^{NFW}(R) = 4\pi \rho_0 R_c^3 \left[\ln\left(1 + \frac{R}{R_C}\right) - \frac{\frac{R}{R_C}}{1+\frac{R}{R_C}} \right]$$

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Burkert (B) profile BURKERT (1995), ApJ 447 L25

mass density profile of DM $$\rho_H^B(R) = \frac{\rho_0}{\left(1 + \frac{R}{R_C}\right)\left(1 + \frac{R^2}{R_C^2}\right)}$$

$$M_H^B(R) = 2\pi \rho_0 R_c^3 \left[\ln\left(1 + \frac{R}{R_C}\right) - \tan^{-1}\left(\frac{R}{R_C}\right) + \frac{1}{2} \ln\left(1 + \frac{R^2}{R_C^2}\right) \right]$$

Contribution to rotation velocity

$$V_H^2(R) = G \frac{M_H(R)}{R}$$

ρ_0: central density
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MASS MODELLING FROM THE COADDED RCs

72 LSB Galaxies

Freeman disk

Burkert DM profile

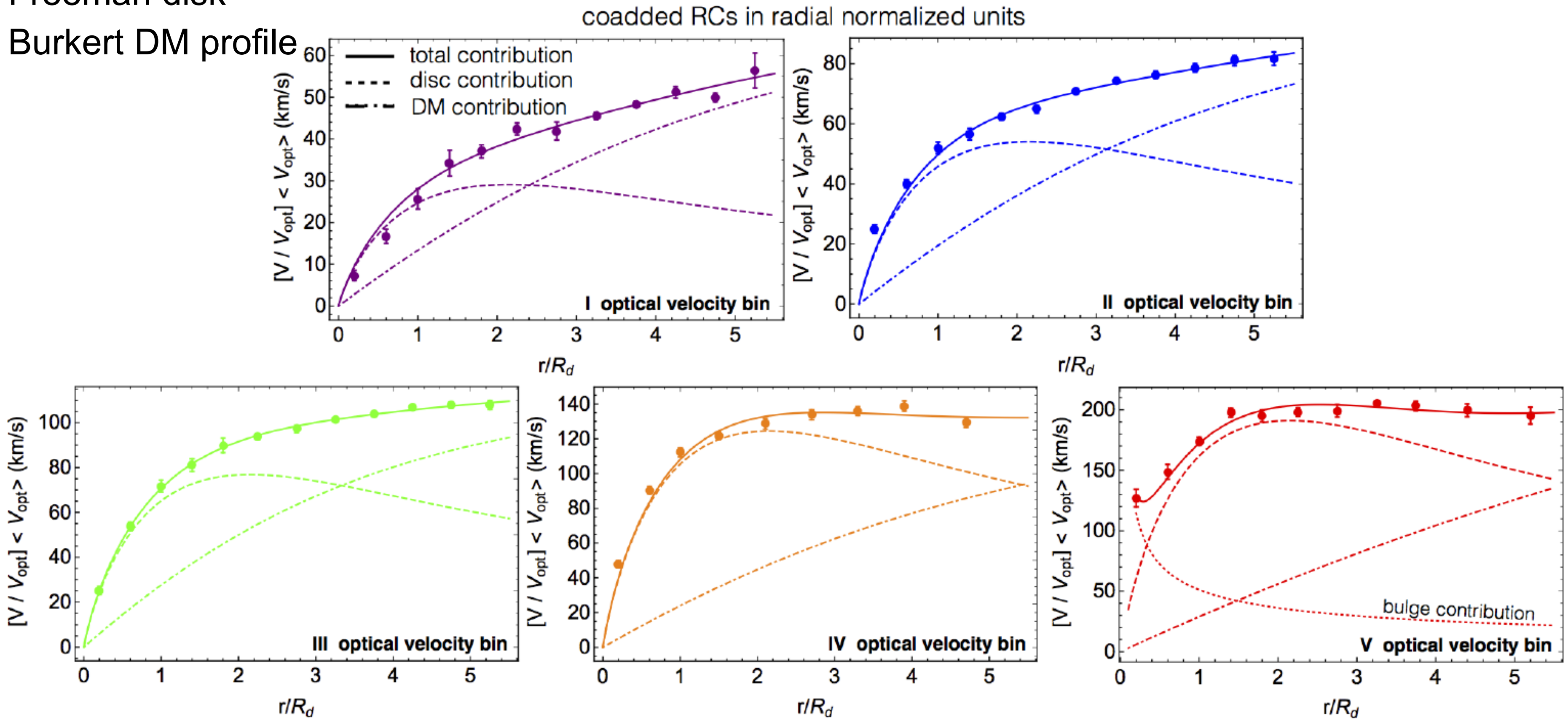

Figure 7. In each of the five panels the velocity best-fit models to the corresponding coadded RCs are shown. The *dashed*, *dot-dashed*, *dotted* and *solid* lines indicate the stellar disc, the DM halo, the stellar bulge and the model contribution to the circular velocities.

DI PAOLO et al. (2019), MNRAS 490 4, 5451-5477

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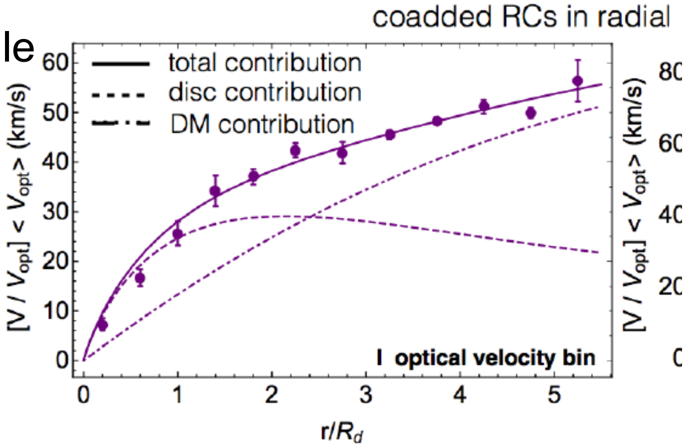
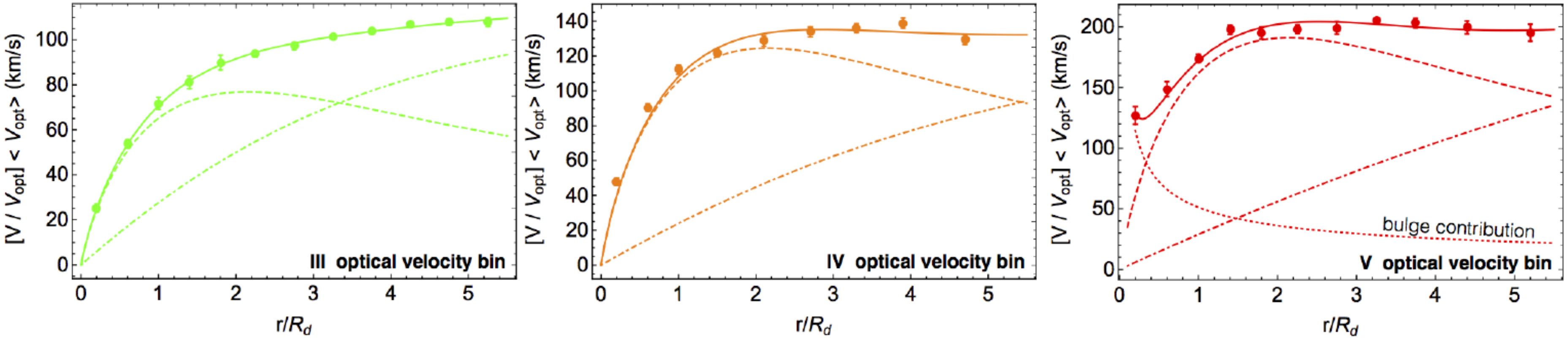


Vel. Bin (1)	$\langle V_{opt} \rangle$ km/s (2)	ρ_0 $10^{-3}\ M_\odot/pc^3$ (3)	R_c kpc (4)	M_d $10^{11}\ M_\odot$ (5)
1	43.5	3.7 ± 1.4	10.7 ± 4.3	$(8.8 \pm 1.8) \times 10^{-3}$
2	73.3	5.1 ± 1.1	12.8 ± 3.0	$(3.8 \pm 0.3) \times 10^{-2}$
3	100.6	3.7 ± 0.5	17.1 ± 1.9	$(13.0 \pm 0.5) \times 10^{-2}$
4	140.6	$1.7^{+1.8}_{-1.1}$	30^{+40}_{-22}	$(4.2 \pm 0.4) \times 10^{-1}$
5	205.6	$0.8^{+0.7}_{-0.4}$	99^{+213}_{-87}	1.7 ± 0.1

Parameter constraints



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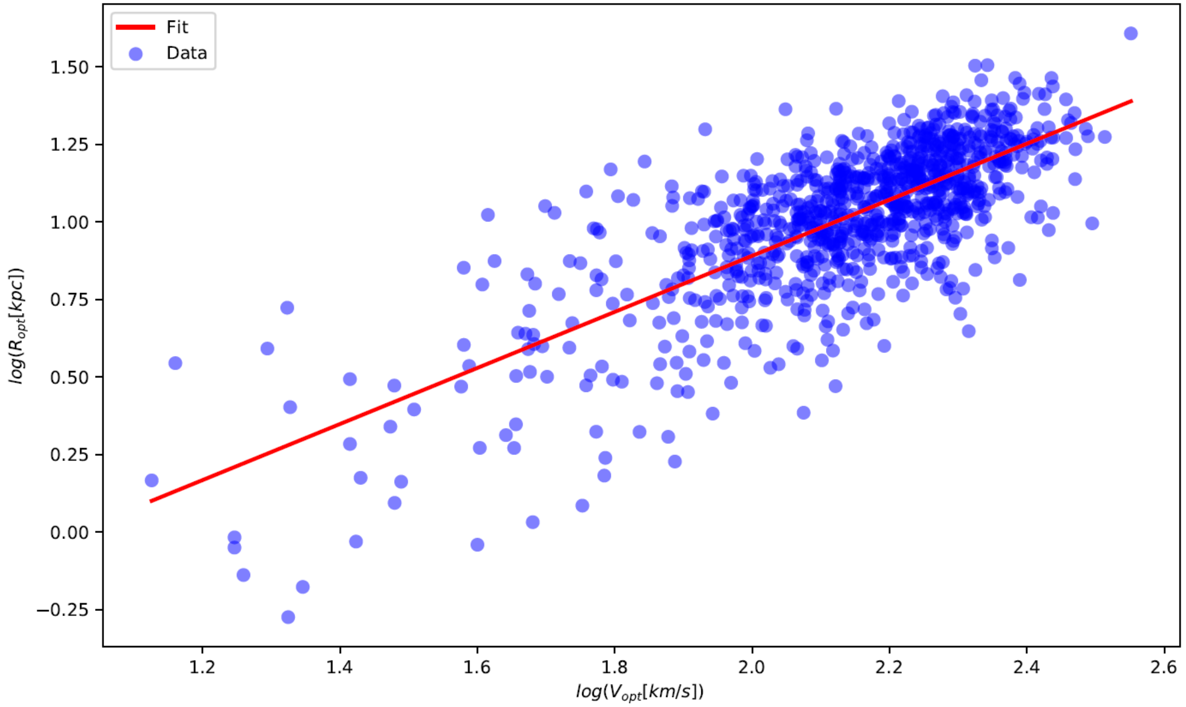
THE COMPACTNESS PARAMETER



$$\log R_{opt}^{(fit)} = a \log V_{opt} + b \qquad i = 1, \ldots, N_{gal}$$

Compactness parameter

$$C_{*,i} = \frac{R_{opt}^{(fit)}(V_{opt,i})}{R_{opt,i}} = \frac{10^{a \log V_{opt,i} + b}}{R_{opt,i}}$$

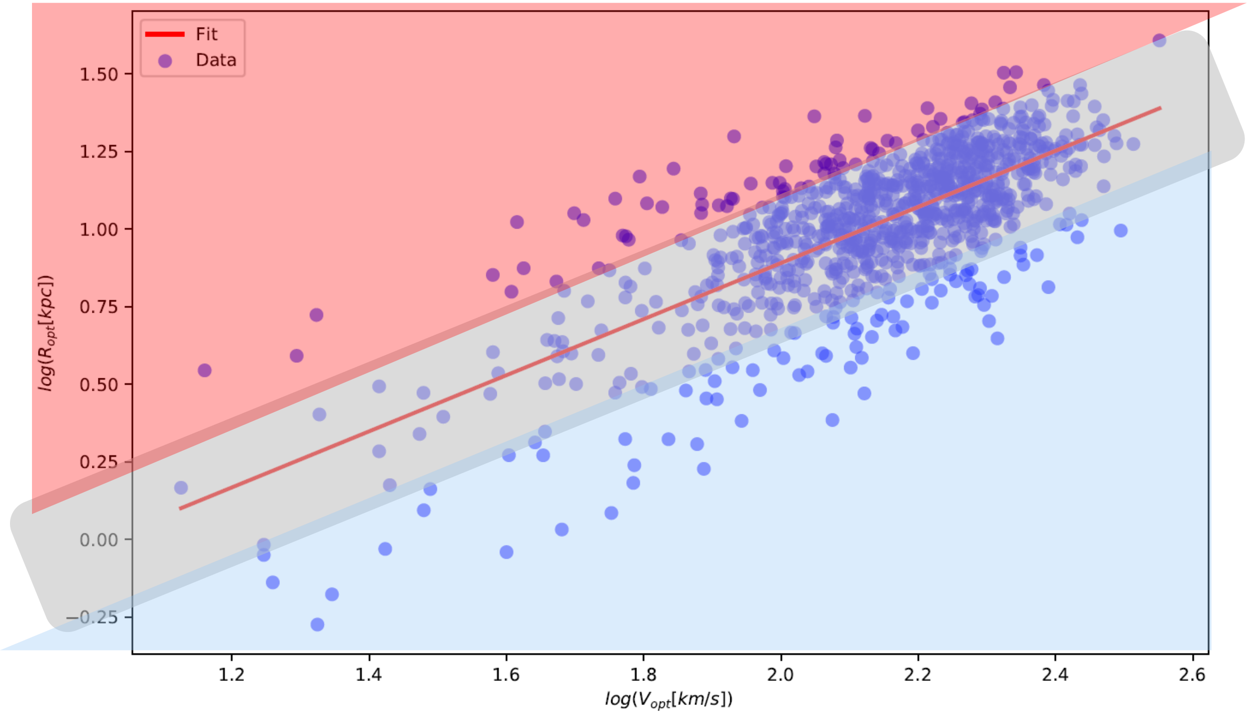
Galaxies with similar V_{opt} value (similar luminosity) can have very different R_{opt}: they can be more or less compact.

It may be necessary to introduce an extra phenomenological parameter that quantifies this compactness and can be relevant for the URC.

$$V = V\left(\frac{R}{R_{opt}}, V_{opt}, C_*\right) \qquad v = v(x, V_{opt}, C_*)$$

Then, we can consider other subsamples of galaxies according to their compactness values C_*

THE COMPACTNESS PARAMETER



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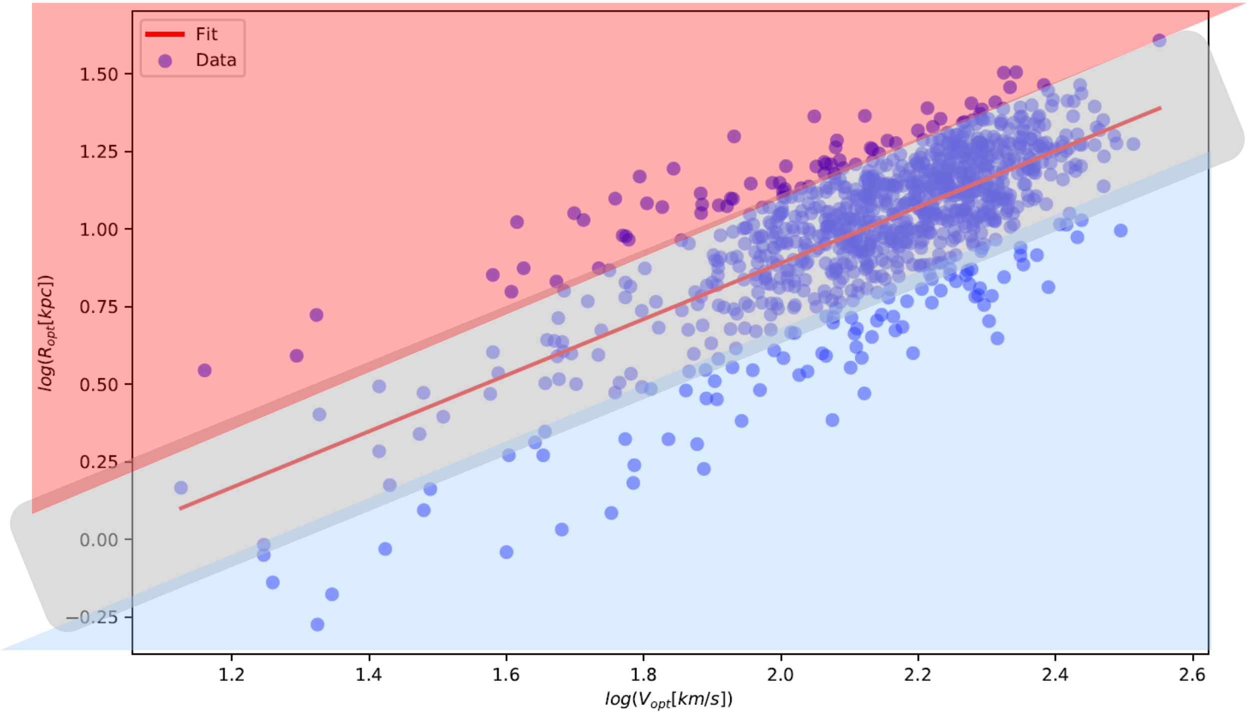

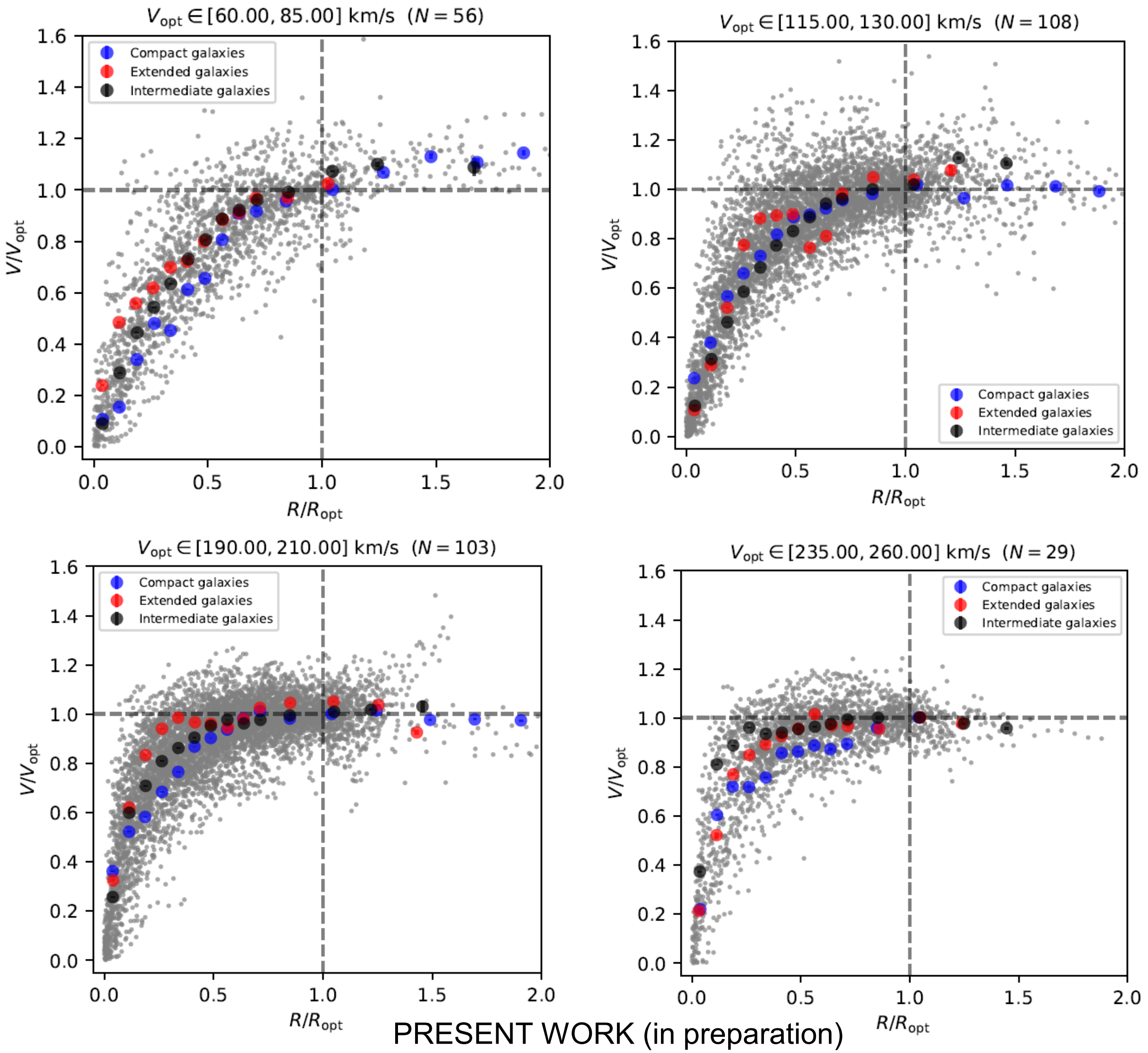

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$$C_{*,i} = \frac{R_{opt}^{(fit)}(V_{opt,i})}{R_{opt,i}} = \frac{10^{a \log V_{opt,i} + b}}{R_{opt,i}}$$

THE COMPACTNESS PARAMETER

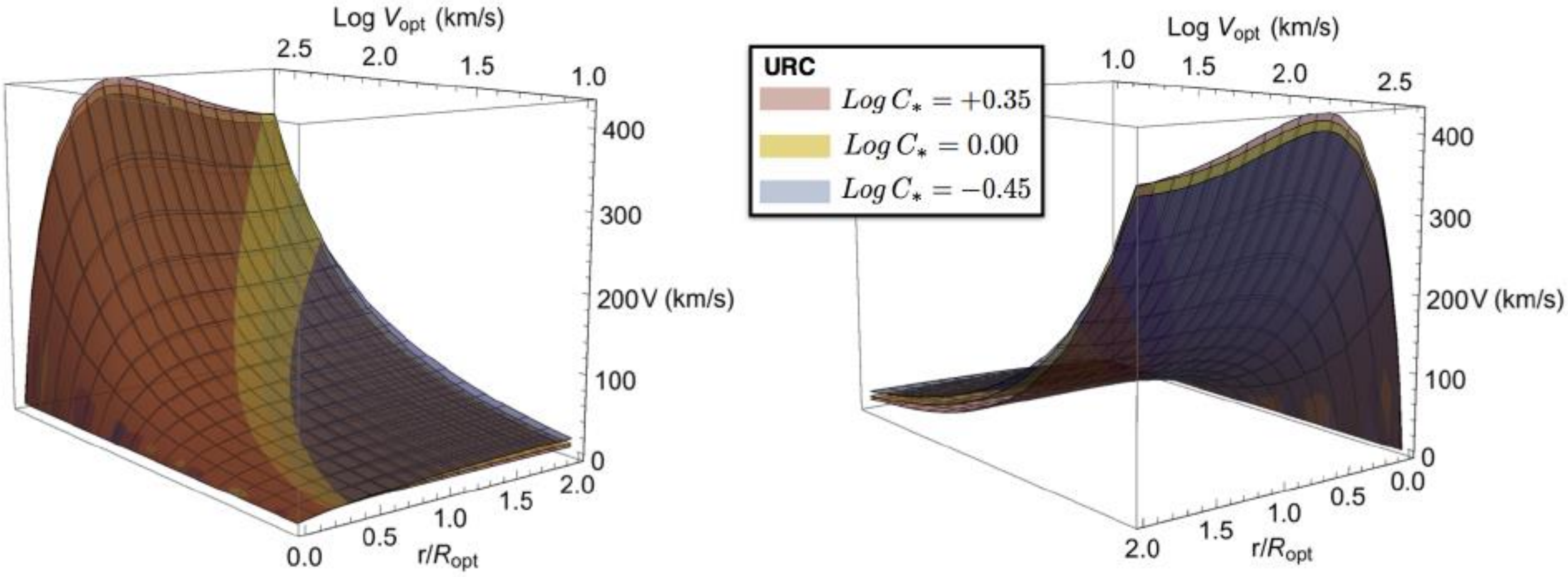


Figure 16. Universal rotation curves (URC) in physical velocity units for three different values of stellar compactness; low ($Log\, C_* = -0.45$), standard ($Log\, C_* = 0.00$) and high ($Log\, C_* = +0.35$) stellar compactness, respectively in *blue*, *yellow* and *red* colors. The figure in the second panel corresponds to that of the first panel when rotated by 180° around the velocity axis.

DI PAOLO et al. (2019),
MNRAS 490 4, 5451-5477

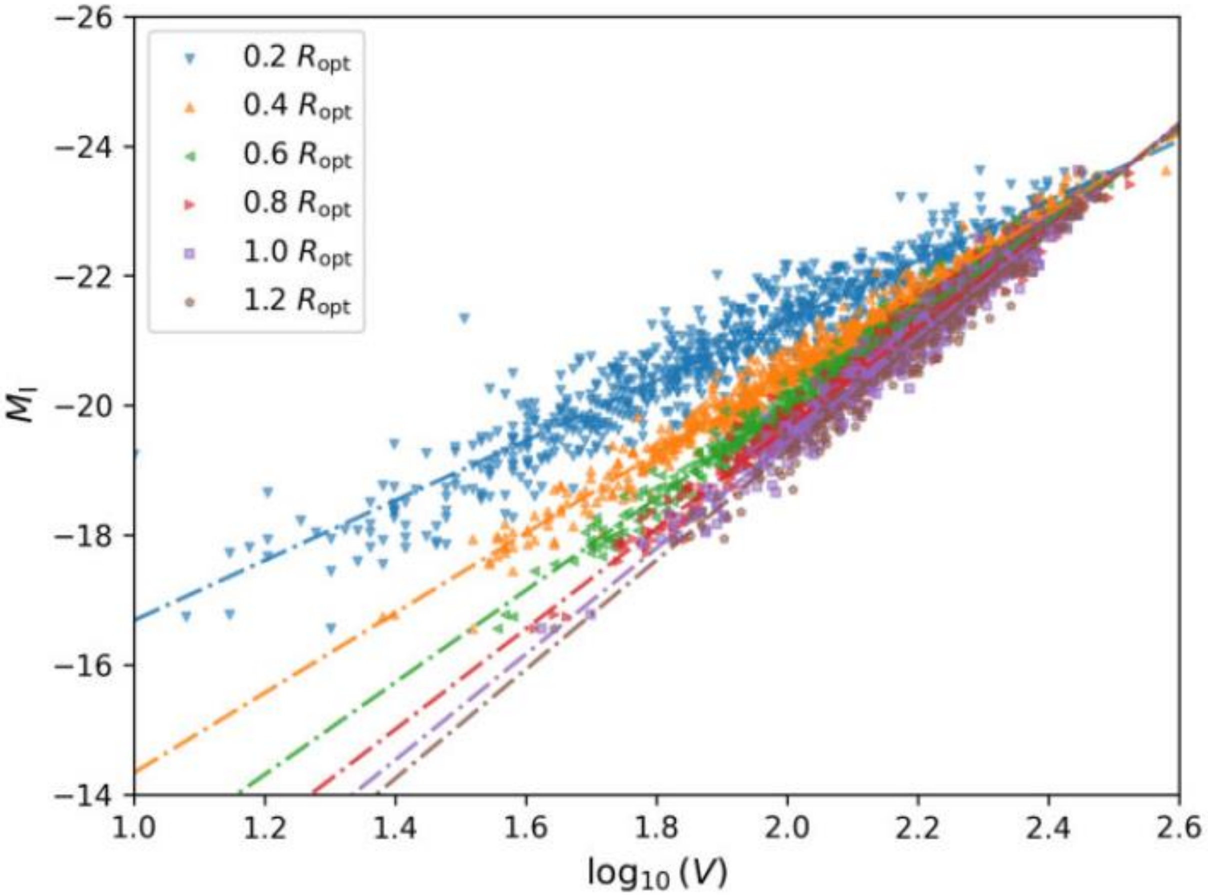
THE RADIAL-TULLY FISHER RELATION

The empirical relation between the magnitude of a spiral galaxy and its maximum rotation velocity is known as Tully-Fisher relation (1977).

$$Mag = A \log V_{max} + B$$

This can be generalized to provide a relation between the magnitude Mag and the velocity measured at a given radius in units of R_{opt}, the radial Tully-Fisher (RTF):

$$Mag = A_i \log V(x_i) + B_i \qquad x_i = \frac{R_i}{R_{opt}}$$



After calibrating the RTF in the local Universe, the RTF can be applied for cosmological purposes.

Distance modulus $\mu = Mag - m = 5 \log d_L(z) + 25$

Luminosity distance $d_L(z) = \frac{1+z}{H_0} \int \frac{dz'}{E(z')}$ $\qquad d_L \approx \frac{1}{H_0}\left[z + \frac{1}{2}(1-q_0)z^2\right]$ for $z \ll 1$

HARIDASU et al. (2024), MNRAS 532, 2234-2247

SUMMARY AND CONCLUSIONS

- RCs are an useful tool to investigate the matter distribution in galaxies.
- Adopting normalized radii and velocities helps to identify an universal trend, and the scatter is significantly reduced in the coadded RCs.
- If we know only two quantities (the optical radius R_{opt} and the optical velocity V_{opt} - or the magnitude) for a given disk galaxy, we are able to predict the rotation velocity $V = V(\frac{R}{R_{opt}}, V_{opt})$ from the Universal Rotation Curve (URC), looking at the surface described by data points in the 3D space $\left(V, \frac{R}{R_{opt}}, V_{opt}\right)$.
- Various dark matter profiles (NFW, Burkert, etc.) in combination with an exponential stellar disk can be tested with the mass modelling and fitting the coadded RCs.
- The URC is related to the radial Tully-Fisher relation (2D projection magnitude VS velocity computed at different radii). After calibrating the relation with local anchors, the RTF can be used to provide an independent local measurement of H_0.
- In preparation: the refined PROBES compendium ($\sim 10^3$ galaxies, $\sim 10^5$ RCs data points) allows us to further investigate the URC, dark matter profiles, the RTF, and the Hubble tension.