GGI, Florence, 21.07.2026

Theories of Large-Scale Structure and Newtonian Motion gauges

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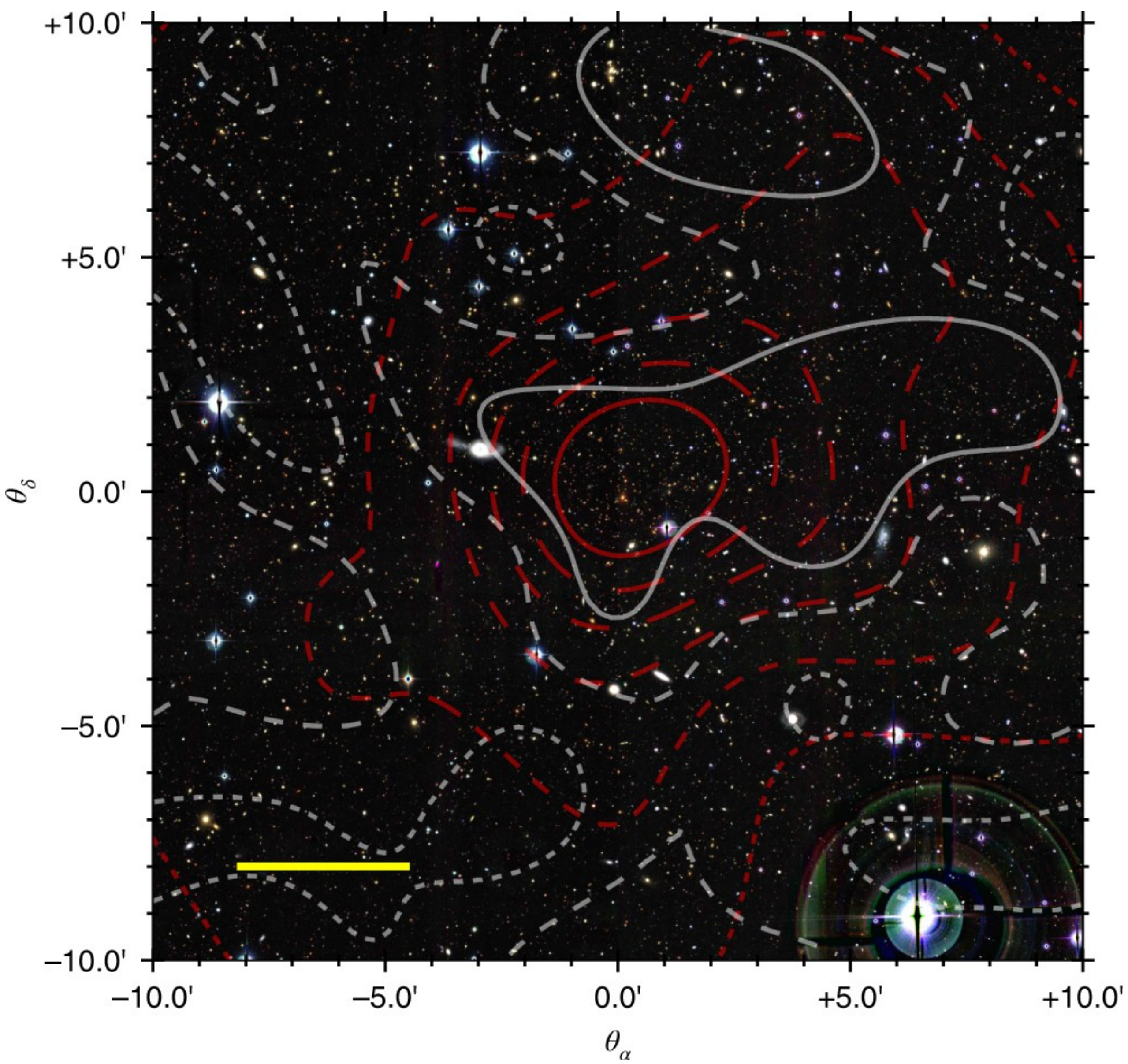
based on 2605.05114, together with

Christian Fidler, Antonia Mates, Azadeh Moradinezhad, Simon Neuland

Structure formation can be described at O(1) in metric perturbations

- need $O(1)$ for $\delta g_{\mu\nu}$, high order for $\delta T_{\mu\nu}$
- higher order for $\delta g_{\mu\nu}$ only necessary around black holes, no impact on LSS
- consistent with observed velocities ($\psi \sim v_{\rm esc}^2/c^2 \sim 10^{-5}$)
- confirmed by relativistic N-body codes like gevolution, Adamek et al. astro-ph/1604.0665, github.com/gevolution-code/gevolution-1.2
- In $\delta g_{\mu\nu}$, some d.o.f. evolve strictly like in linear theory, some get enhanced perturbatively on small scales

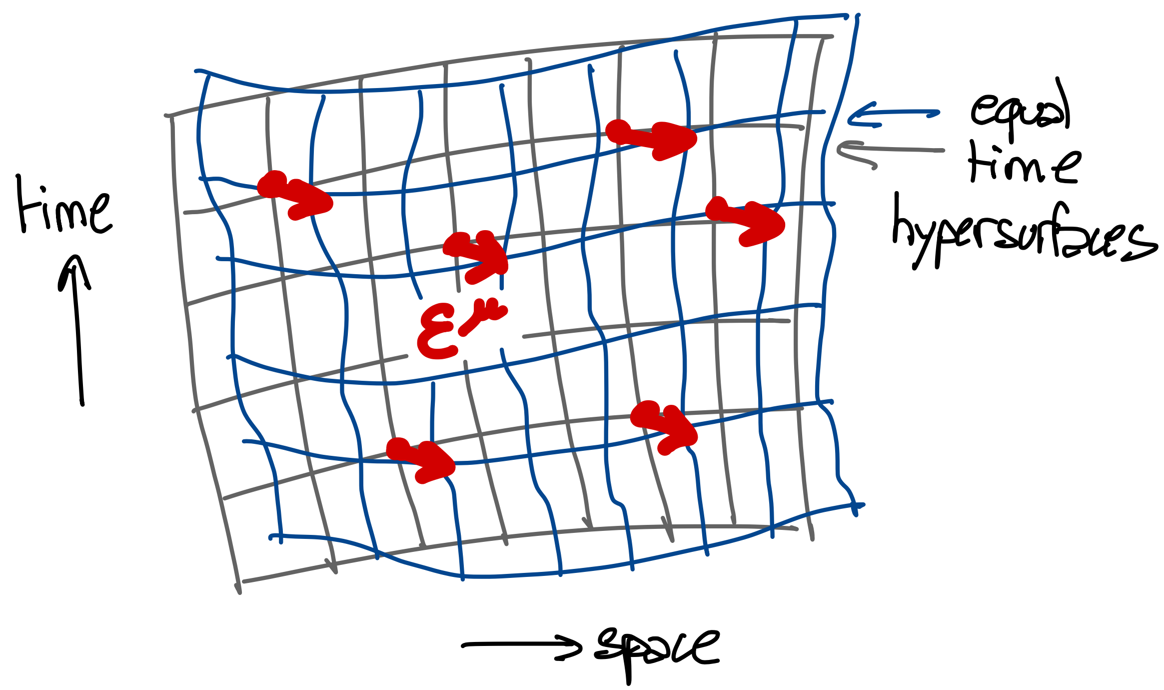


Linear gauge transformations applicable to structure formation

- gauge transformation induced by coordinate transformation / displacement field $\epsilon^\mu(x^\alpha)$

- transformation of $\delta g_{\mu\nu}$ and $\delta T_{\mu\nu}$ given by Lie derivative along $\epsilon^\mu(x^\alpha)$ (linear transformation)

$$\delta T'_{\mu\nu} \quad = \quad \delta T_{\mu\nu} - \frac{\partial \epsilon^\sigma}{\partial x^\mu} T_{\sigma\nu} - \frac{\partial \epsilon^\tau}{\partial x^\nu} T_{\mu\tau} - \epsilon^\gamma \, \frac{\partial T_{\mu\nu}}{\partial x^\gamma}$$



- employed to cancel some degrees of freedom /reduce number of equations / match perturbations in one gauge to gauge-independent combinations…
  - ~~Newtonian~~ / ~~longitudinal~~ / Poisson
  - synchronous
  - comoving ($\theta_{\text{tot}} = kB$, $H_T = 0$) to make contact with observations in real/redshift space

Durrer et al., Yoo et al., Jeong et al., …

- formalism for gauge transformations at $O(2)$ exists, relevant only for specific problems (PNG, SD…)

Most N-body codes implement Newtonian gravity in expanding background

- pure non-relativistic matter, Newtonian gravity, Poisson equation

- neglect: density and shear of photons and neutrinos, $\phi \neq \psi$, time-derivative of metric: "GR and radiation correction"

- justified in small-scale limit, but GR and radiation correction potentially interesting for large-volume survey

- small impact, can be studied with gevolution or COSIRA (GADGET $\leftrightarrow$ CLASS)

  Fidler et al. 1505.04756, 1606.05558, 1702.03221, 1708.0769

  (with Tram, Rampf, Crittenden, Koyama, …)

  Brandbyge et al. 1610.04236

  Adamek et al. 1703.08585

- Another approach: N-body / N-boisson gauge

  [Same authors]

Getting GR / radiation corrections from N-body codes "for free"

- define gauge in which e.o.m for $\delta_{\rm cb}$ including GR / radiation corrections become identical to Newtonian e.o.m for pure non-relativistic matter: N-boisson gauge

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- use linear Boltzmann code to find:

  - gauge transformation from e.g. arbitrary gauge, e.g. Poisson gauge, to N-boisson gauge at each time; get $\epsilon^{\mu}(x^{\alpha})$

  - adiabatic ICs in Poisson gauge → special ICs in N-boisson gauge; linear spectrum $P_{\mathrm{L}}^{(\mathrm{NM})}(k, z_{\mathrm{ini}})$

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- use $P_{\mathrm{L}}^{(\mathrm{NM})}(k, z_{\mathrm{ini}})$, use it to initialise particle positions in N-body code

- run Newtonian N-body code without radiation

- Take snapshot, displace particles according to $-\epsilon^\mu(x^\alpha)$, measure power spectrum in, e.g., Poisson or comoving gauge

- Method validated by comparing to gevolution and COSIRA

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Adding massive neutrinos corrections "for free"

- define gauge in which e.o.m including radiation and massive neutrino perturbations become identical to Newtonian e.o.m without radiation and massive neutrinos: Newtonian Motion gauges (pick up one: time at which NM gauge = N-boisson gauge, e.g. today)

- In NM gauge, single self-gravitating fluid cb with Poisson equation

$$-k^2\Psi = 4\pi G a^2 \bar{\rho}_{\rm cb}\delta_{\rm cb}$$

$\Rightarrow$ growth rate becomes scale-independent (e.g. during MD: $\delta_{\rm cb} \propto a^{1-\frac{3}{5}f_\nu}$)

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- use linear Boltzmann code to find gauge transformation, $\epsilon^\mu(x^\alpha)$, ICs, $P_{\rm L}^{\rm (NM)}(k, z_{\rm ini})$

- initialise particle positions, run Newtonian N-body code without radiation nor massive neutrinos

- take snapshot, displace particles according to $-\epsilon^\mu(x^\alpha)$, measure power spectrum in, e.g., Poisson or comoving gauge; get spoon-shape suppression "for free" !

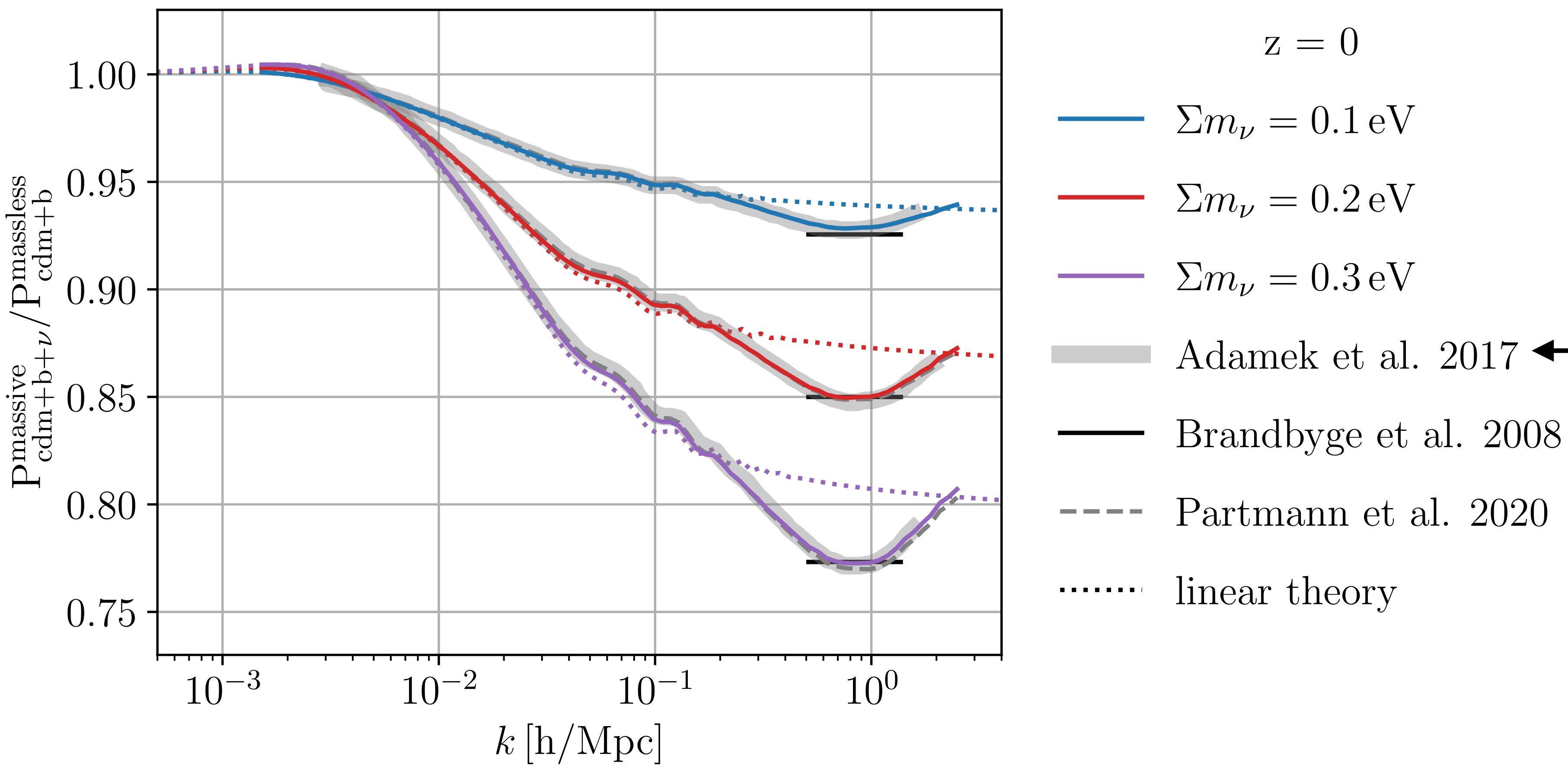
Fidler et al. 1807.03701, 1810.12019

Partmann et al. 2003.07387

Heuschling et al. 2201.13186

Adding massive neutrinos corrections "for free"

- NM gauge method fully validated using gevolution (or gadget on small scales)



brute-force N-body with cdm+ν particles

only cdm particles, NM gauge method

1. modified ICs
2. modified Poisson
3. particle displacement

Heuschling, Partmann, Fidler, 2201.13186

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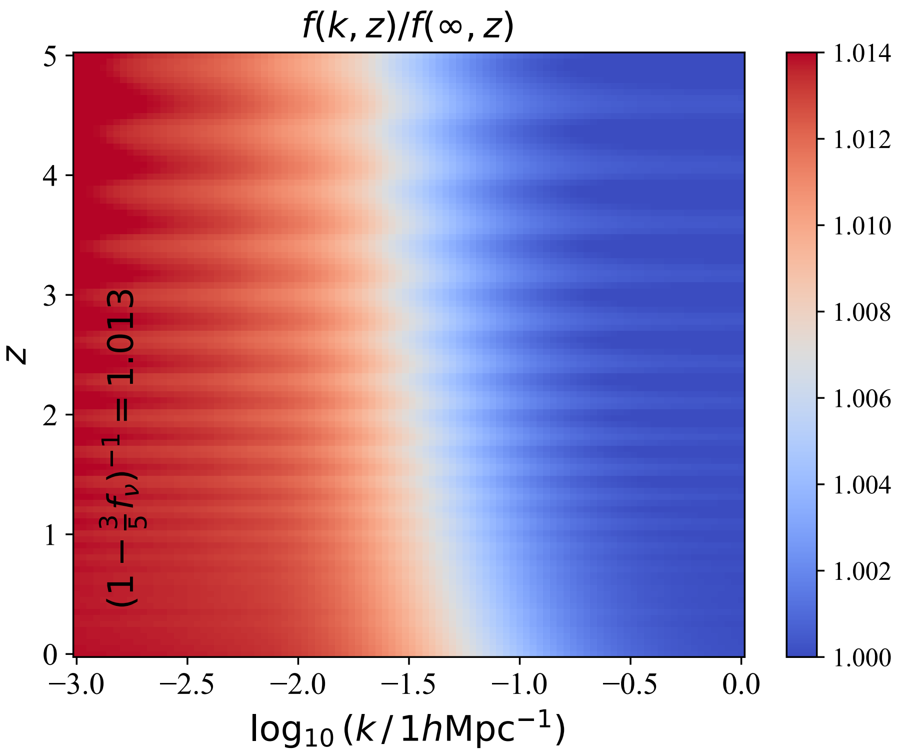
- for massive neutrinos: growth factor has two asymptotes: e.g. during MD,
  - small scales: $\delta_{\rm cb} \propto a^{1-\frac{3}{5}f_\nu} \Rightarrow f = 1 - \dfrac{3}{5}f_\nu$
  - large scales: $\delta_{\rm cb} \propto a \Rightarrow f = 1$
  - gauge transformation absorbs scale-dependence by imposing the small-scale f on all scales



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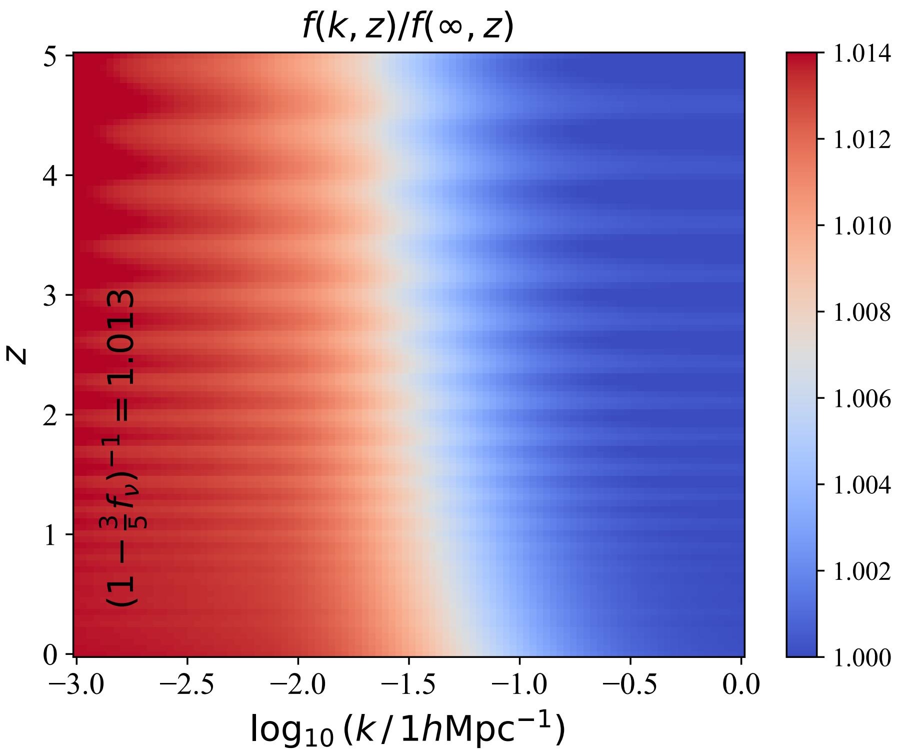

- applies to many other interesting models: f(R), WDM, CWDM, but not all (e.g. if all CDM decays or interacts at late time: requires infinite gauge transformation fields)

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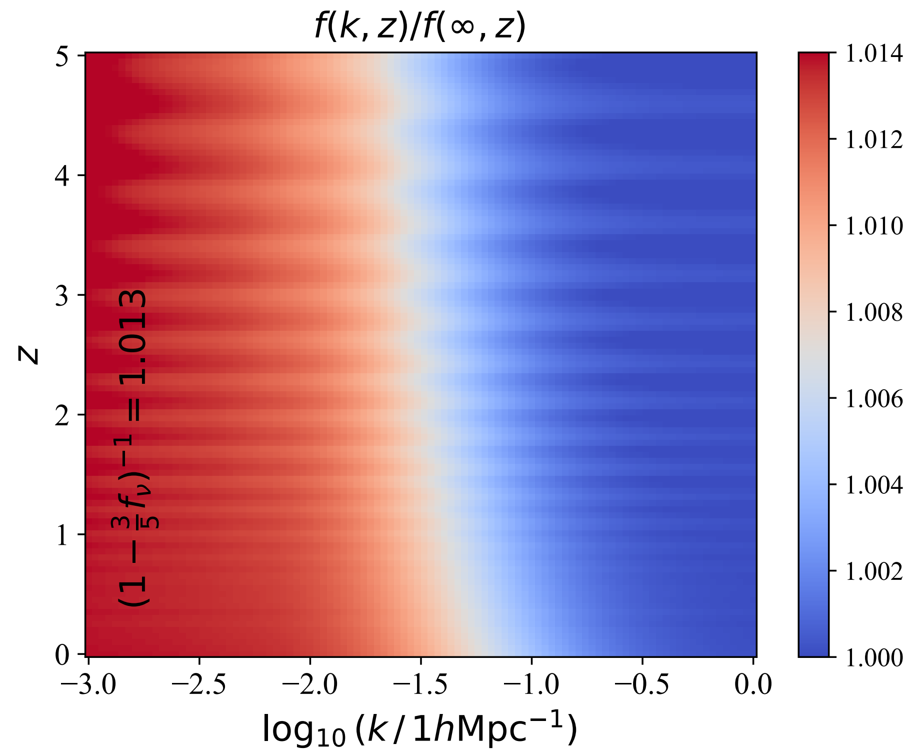

- applies to many other interesting models: f(R), WDM, CWDM, but not all (e.g. if all CDM decays or interacts at late time: requires infinite gauge transformation fields)
- can we transpose this to theories of large scale structure formation?

Theories of Large Scale Structure

- Standard Perturbation Theory (SPT), EFTof LSS, etc., are all based on Newtonian e.o.m

Continuity:
$$\dot{\delta}_{\rm cb}(\mathbf{k}) = -\,\theta_{\rm cb}(\mathbf{k}) + \cancel{3\dot{\phi}(\mathbf{k})} + \int d^3\mathbf{q}\,\alpha(\mathbf{q},\mathbf{k}-\mathbf{q})\,\theta_{\rm cb}(\mathbf{q})\,\delta_{\rm cb}(\mathbf{k}-\mathbf{q}) \quad ,$$

Euler:
$$\dot{\theta}_{\rm cb}(\mathbf{k}) = -\frac{\dot{a}}{a}\theta_{\rm cb}(\mathbf{k}) + k^2\psi(\mathbf{k}) - \int d^3\mathbf{q}\,\beta(\mathbf{q},\mathbf{k}-\mathbf{q})\,\theta_{\rm cb}(\mathbf{q})\,\theta_{\rm cb}(\mathbf{k}-\mathbf{q}) \; ,$$

Einstein $\longrightarrow$ Poisson:
$$-k^2\psi + \cancel{3\frac{\dot{a}}{a}\left(\dot{\phi}+\frac{\dot{a}}{a}\psi\right)} = 4\pi G a^2 \rho_{\rm cb}\bar{\delta}_{\rm cb} + 4\pi G a^2 \cancel{\left[\bar{\rho}\delta - 3(\bar{\rho}+\bar{p})\sigma\right]_{\rm other}}$$

- gives matter power spectrum / bispectrum at N-loop order
- RSD from additional expansion in velocity field
- galaxy power spectrum from additional bias expansion
- potentially useful to fit full-shape galaxy power spectrum of spectroscopic surveys

Theories of Large Scale Structure: EdS limit

- simplest case: pure CDM, no Λ: Einstein-De Sitter universe, $\delta_{\rm cb} \propto a \Rightarrow f = 1$

$$\Psi_a = (\delta_{\rm cb}\,, \quad -\theta_{\rm cb}/\mathscr{H}f)$$

$$\partial_\eta \Psi_a + \Omega_{ab}\Psi_b = \int K_{abc}\,\Psi_a\Psi_c \qquad \text{with} \qquad \Omega_{ab} = \begin{pmatrix} 0 & -1 \\ -\frac{3}{2} & \frac{1}{2} \end{pmatrix}$$

- leads to separability of $\Psi_a(k,\tau)$ in τ and k

- famous time-independent EdS kernels $F_2(\mathbf{q},\mathbf{k})$, $G_2(\mathbf{q},\mathbf{k})$, $F_3(\mathbf{q},\mathbf{k})$, $G_3(\mathbf{q},\mathbf{k})$,…

- gives matter power/bispectrum

- additional time-independent kernels for galaxy, RSD…

Theories of Large Scale Structure: ΛCDM

- next level of complexity: pure ΛCDM (but no GR / radiation / massive neutrino corrections): growth rate $f(\tau)$ modified, but still scale-independent

$$\Omega_{ab} = \begin{pmatrix} 0 & -1 \\ -\frac{3}{2}\frac{\Omega_{\rm bc}(\tau)}{f^2(\tau)} & \frac{3}{2}\frac{\Omega_{\rm bc}(\tau)}{f^2(\tau)} - 1 \end{pmatrix}$$ with $\frac{\Omega_{\rm bc}(\tau)}{f^2(\tau)}$ growing by $\sim 14\,\%$ during ΛD

- $\Psi_a(k,\tau)$ and kernels no longer separable in τ and k

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- $\Psi_a(k,\tau)$ and kernels no longer separable in τ and k

- exact and approximate methods giving time-dependent kernels (Bernardeau astro-ph/9312026, Garny & Taule 2008.00013, Green function method of Donath & Senatore 2005.04805 implemented in PyBird…)

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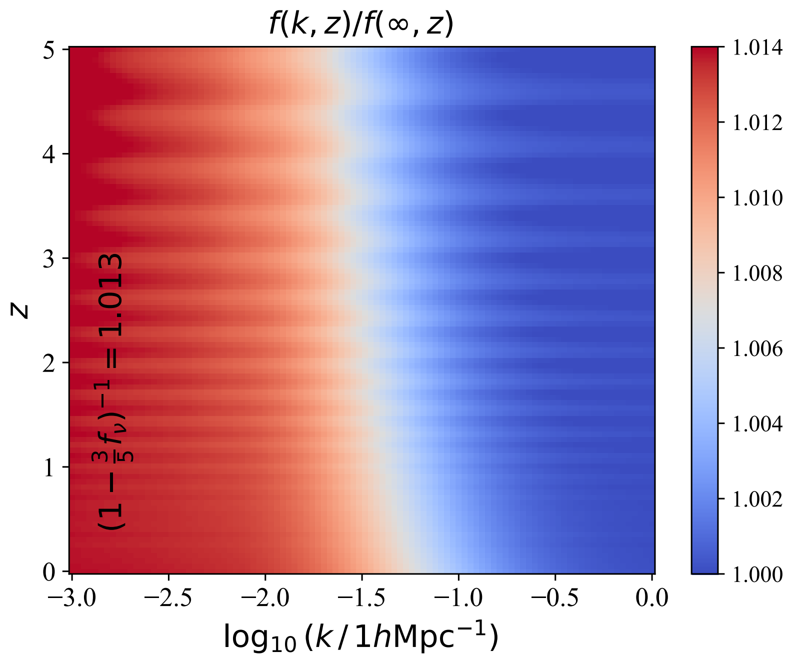
- Common EdS approximation: $\frac{\Omega_{\rm bc}(\tau)}{f^2(\tau)} \simeq 1$
  - Λ-effect partially taken into account by using the right input $P_{\rm L}(k,z)$
  - but EdS kernel $\Rightarrow$ incorrect "memory of the past", NL corrections slightly underestimated
  - error shown to be small (but not *that* small: see later)

Theories of Large Scale Structure: ΛCDM+scale-dependent growth

- additional level of complexity: massive neutrino corrections: scale-dependent growth rate



- 2-fluid problem (cb + ν), Ω_{ab} is a 4×4 matrix containing $f(k, \tau)$, which has two asymptotes on small/large scales

- full 2-fluid problem with kernels integrated over time solved by Garny & Taule 2008.00013, 2205.11533. Slow and expensive method.

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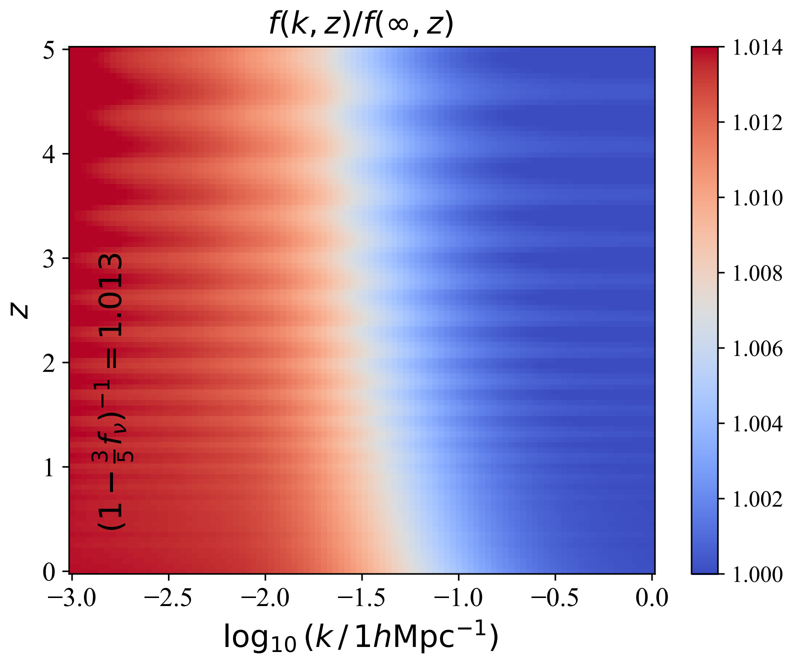

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- several intermediate levels of approximation (e.g., effective 1-fluid problem as in Aviles et al. 2012.05077, …, implemented in FOLPs, used in DESI full-shape analysis)

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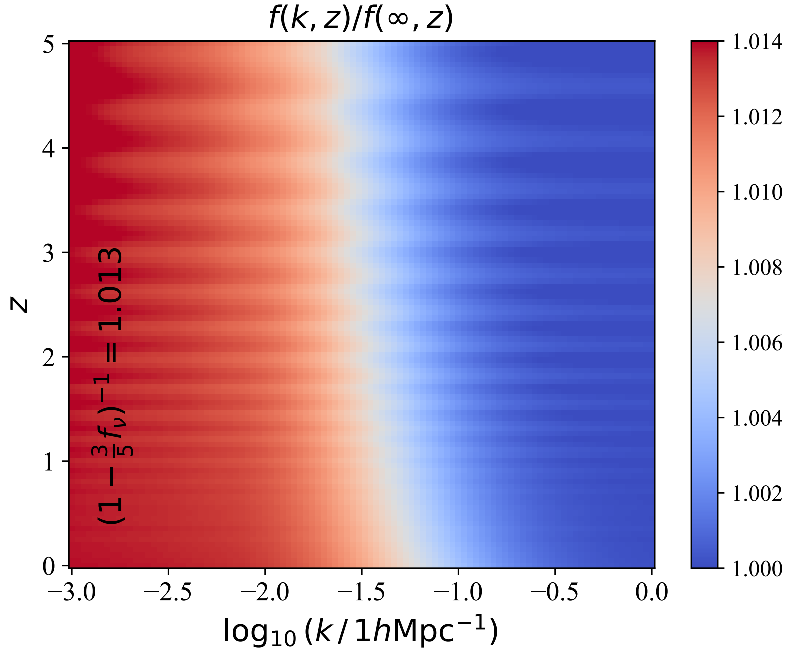

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- new idea: get these corrections + GR / radiation corrections "for free" using NM gauge !

Finding the gauge transformation fields

- Metric scalar fluctuations:

$$
\begin{aligned}
g_{00} &= -a^2(1+2A) \ , \\
g_{0i} &= -a^2\hat{\nabla}_i B \ , \\
g_{ij} &= a^2\left[\delta_{ij}(1+2H_{\mathrm{L}}) - 2\left(\hat{\nabla}_i\hat{\nabla}_j + \frac{\delta_{ij}}{3}\right)H_{\mathrm{T}}\right]
\end{aligned}
$$

- What we really need is $H_{\mathrm{T}}^{(\mathrm{NM})}$:
  - Sufficient to compute all relevant gauge transformations, in particular:

$$\delta_{\mathrm{cb}}^{(\mathrm{NM})} \longrightarrow \delta_{\mathrm{cb}}^{(\mathrm{C})}, \qquad \theta_{\mathrm{cb}}^{(\mathrm{NM})} \longrightarrow \theta_{\mathrm{cb}}^{(\mathrm{C})}$$

  - $H_{\mathrm{T}}^{(\mathrm{NM})}$ is blind to non-linear evolution

Finding the gauge transformation fields

Two methods allow to compute $H_{\rm T}^{\rm (NM)}(k,\tau)$ in a Boltzmann code (for us: modified CLASS):

1. Solve full system including Einstein equations in NM gauge: gives directly $H_{\rm T}^{(NM)}(k,\tau)$

$$(\partial_\tau^2 + \mathcal{H}\partial_\tau)H_{\rm T}^{\rm (NM)} + 4\pi G a^2 \bar{\rho}_{\rm cb}(3\zeta^{\rm (NM)} - H_{\rm T}^{\rm (NM)}) = S \ ,$$

$$S = 4\pi G a^2 \delta\rho_{\rm other}^{\rm (NM)} + 3\frac{\mathcal{H}}{k^2}(\bar{\rho}+\bar{p})_{\rm other}(\theta_{\rm tot}^{\rm (NM)} - \partial_\tau H_{\rm T}^{\rm (NM)}) + 8\pi G a^2 \Sigma_{\rm other}^{\rm (NM)},$$

2. Solve in parallel:
   - full system in arbitrary gauge (e.g. Poisson, …)
   - equations in Newtonian limit [Nl] for self-gravitating cb fluid with scale-independent growth factor (= small-scale limit of usual growth factor) and appropriate (non-trivial) ICs:

$$\dot{\delta}_{\rm cb}^{\rm [Nl]} = -\theta_{\rm cb}^{\rm [Nl]} \ ,$$

$$\dot{\theta}_{\rm cb}^{\rm [Nl]} = -\frac{\dot{a}}{a}\theta_{\rm cb}^{\rm [Nl]} + k^2\Psi^{\rm [Nl]} \ ,$$

$$\Psi^{\rm [Nl]} \equiv -\frac{3}{2}\frac{\mathcal{H}^2}{k^2}(\Omega_{\rm c}+\Omega_{\rm b})\,\delta_{\rm cb}^{\rm [Nl]} \ .$$

   Then, compare the two:

$$H_{\rm T}^{\rm (NM)} = \delta_{\rm cb}^{\rm (P)} - \delta_{\rm cb}^{\rm [Nl]} - 3\phi$$

- In both case, self-consistency condition: $|H_{\rm T}^{(NM)}(k,\tau)| \ll 1$ at all (k,τ)

Finding the one-loop power spectrum

- feed $P^{[\mathrm{Nl}]}_{\mathrm{cb,L}}(k,z)$ to e.g. EFTofLSS algorithm (for us: CLASS-Oneloop) to get $P^{[\mathrm{Nl}]}_{\mathrm{cb,NL}}(k,z)$

  (already differs from usual through non-trivial ICs and scale-independent/time-dependent growth factor reflected in time/dependent kernels)

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- we can infer the non-linear cb power spectrum in real space and comoving gauge:

  - $P^{(\mathrm{C})}_{\mathrm{cb,NL}}(k,z) = \langle\, |\, \delta_{\mathrm{cb,NL}} + \Delta^{(\mathrm{C})}\,|^2 \,\rangle$ where $\delta_{\mathrm{cb,NL}}$ can be expanded in powers of $\delta_{\mathrm{cb,L}}$

  - knowing that $\Delta^{(\mathrm{C})}(k,z)$ remains linear and negligible on small scales, we prove:

$$P^{(\mathrm{C})}_{\mathrm{cb,NL}} = P^{[\mathrm{Nl}]}_{\mathrm{cb,NL}} \; + \; 2\sqrt{P_{\Delta^{(\mathrm{C})}}}\sqrt{P^{[\mathrm{Nl}]}_{\mathrm{cb,L}}} \; + \; P_{\Delta^{(\mathrm{C})}}$$

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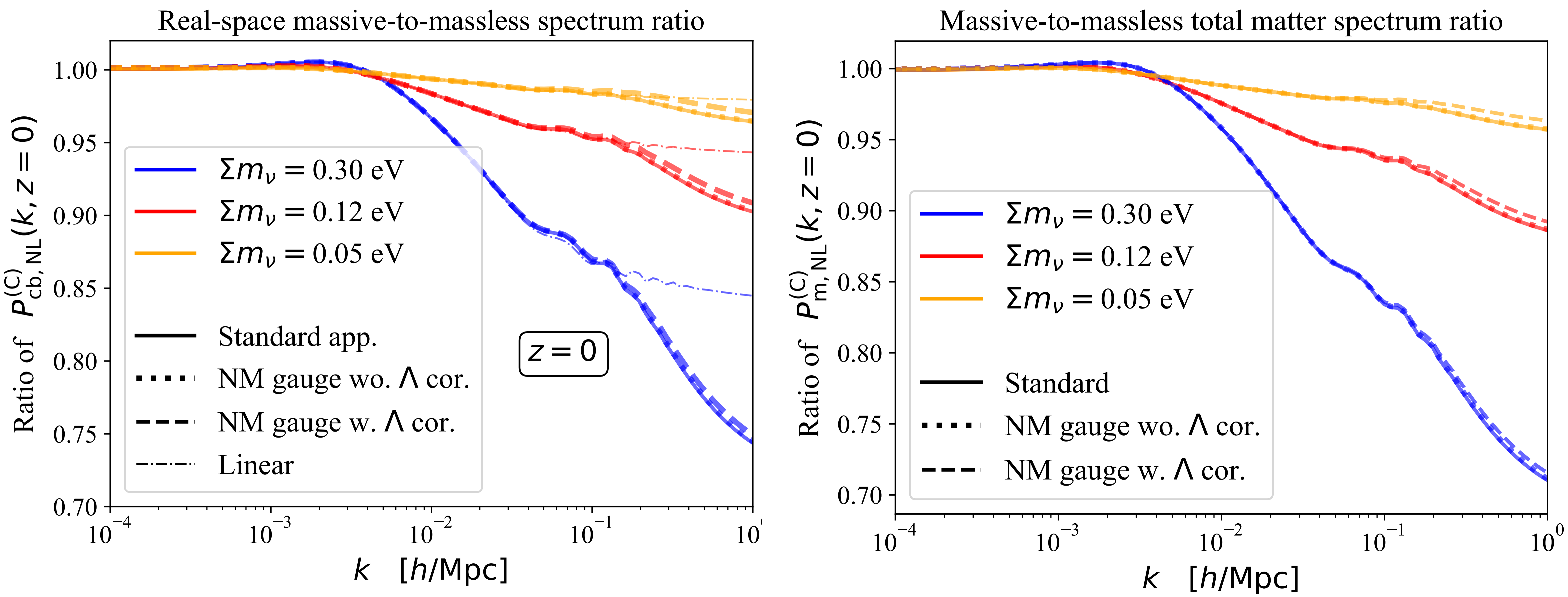
- we can infer similarly the total matter power spectrum in real space and comoving gauge:

  - knowing that $\delta^{(\mathrm{C})}_{\nu}(k,z)$ remains linear and negligible on small scales, we prove:

$$P^{(\mathrm{C})}_{\mathrm{m,NL}} = \left(1-f_\nu\right)^2 P^{(\mathrm{C})}_{\mathrm{cb,NL}} + 2f_\nu \left(1-f_\nu\right)\sqrt{P^{(\mathrm{C})}_{\nu,L}}\sqrt{P^{(\mathrm{C})}_{\mathrm{cb,L}}} + f_\nu^2 \, P^{(\mathrm{C})}_{\nu,L}$$

Finding the one-loop power spectrum



Fidler et al. 2605.05114

Going to redshift space

- in redshift space, linear term with Kaiser formula plus loop correction:

$$P^{(\mathrm{C})}_{\mathrm{cb,NL}}(k,\mu,z) = \left\langle \left| \delta^{[\mathrm{Nl}]}_{\mathrm{cb}} - \frac{\mu^2}{\mathcal{H}} \theta^{[\mathrm{Nl}]}_{\mathrm{cb}} + \Delta^{(\mathrm{RSD})} \right|^2 \right\rangle + \Delta P_{\mathrm{loops}} \left[\delta^{[\mathrm{Nl}]}_{\mathrm{cb}} + \Delta^{(\mathrm{C})}, \theta^{[\mathrm{Nl}]}_{\mathrm{cb}} - \dot{H}^{(\mathrm{NM})}_{\mathrm{T}} \right]$$

- using linearity of the gauge transformation field, we prove:

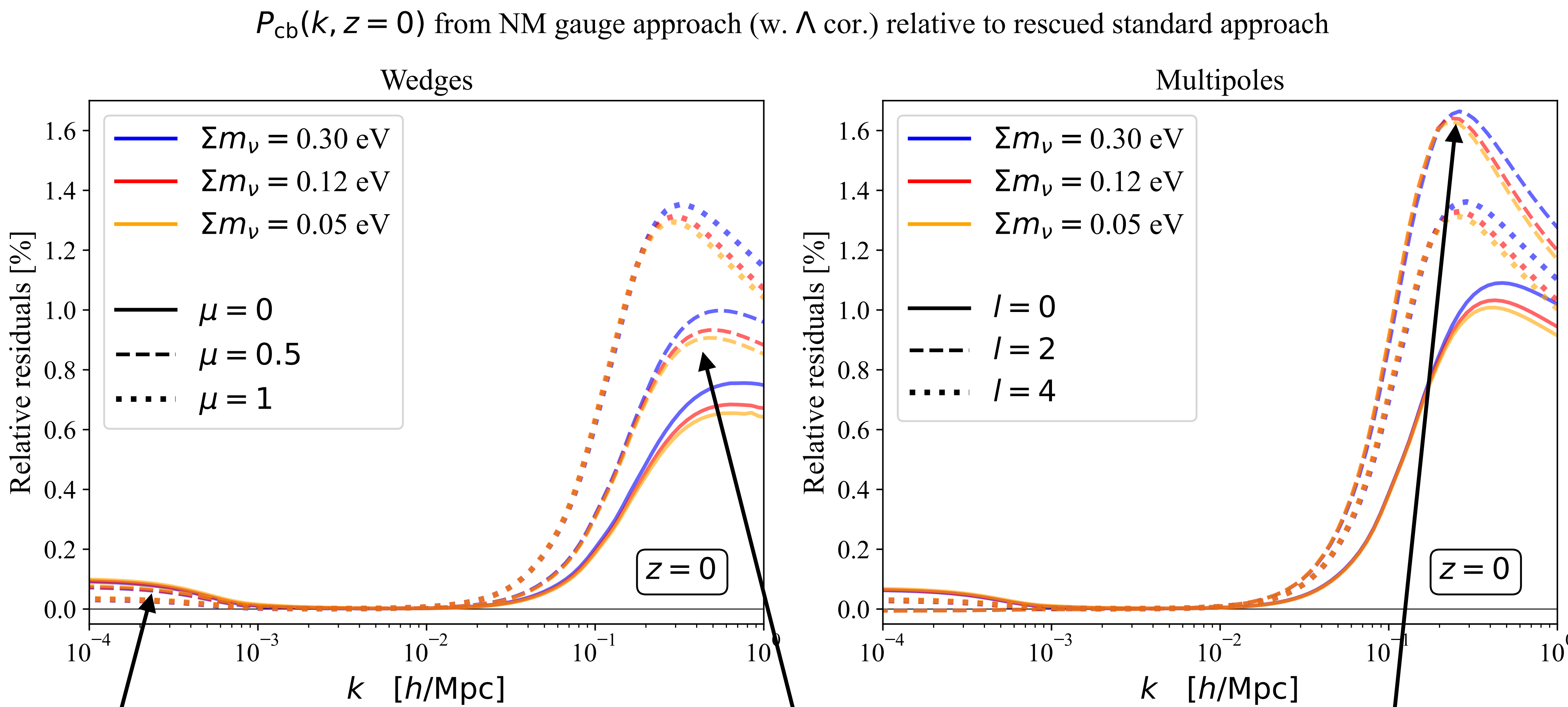
$$P^{(\mathrm{C})}_{\mathrm{cb,NL}}(k,\mu,z) \simeq P^{[\mathrm{Nl}]}_{\mathrm{cb,NL}}(k,\mu,z) + 2\sqrt{P_{\Delta^{(\mathrm{RSD})}}(k,\mu,z)}\sqrt{P^{[\mathrm{Nl}]}_{\mathrm{cb,NL}}(k,\mu,z)} + P_{\Delta^{(\mathrm{RSD})}}(k,\mu,z)\ .$$

- finally, we can infer the spectrum multipoles

Summary of modifications to CLASS-Oneloop

- linear part (perturbations module):
  - solve Newtonian equations in parallel to usual perturbation equations
  - find correct (non-trivial) ICs for $\delta_{\rm cb,L}^{\rm [Nl]}(k,\tau)$
  - find gauge transformation field $H_{\rm T}^{\rm (NM)}(k,\tau)$
- non-linear part (oneloop module):
  - input is $P_{\rm cb,L}^{\rm [Nl]}(k,z)$
  - kernels (F_2, G_2, F_3, G_3) incorporate a time-dependent but scale-independent $\frac{\Omega_{\rm bc}(\tau)}{f^2(\tau)}$ accounting for both Λ and neutrino mass effect
- finally (fourier module):
  - final cb non-linear spectrum combined with linear spectrum of gauge transformation fields
  - result: one-loop cb / matter power spectra in real or redshift space

Results: z=0



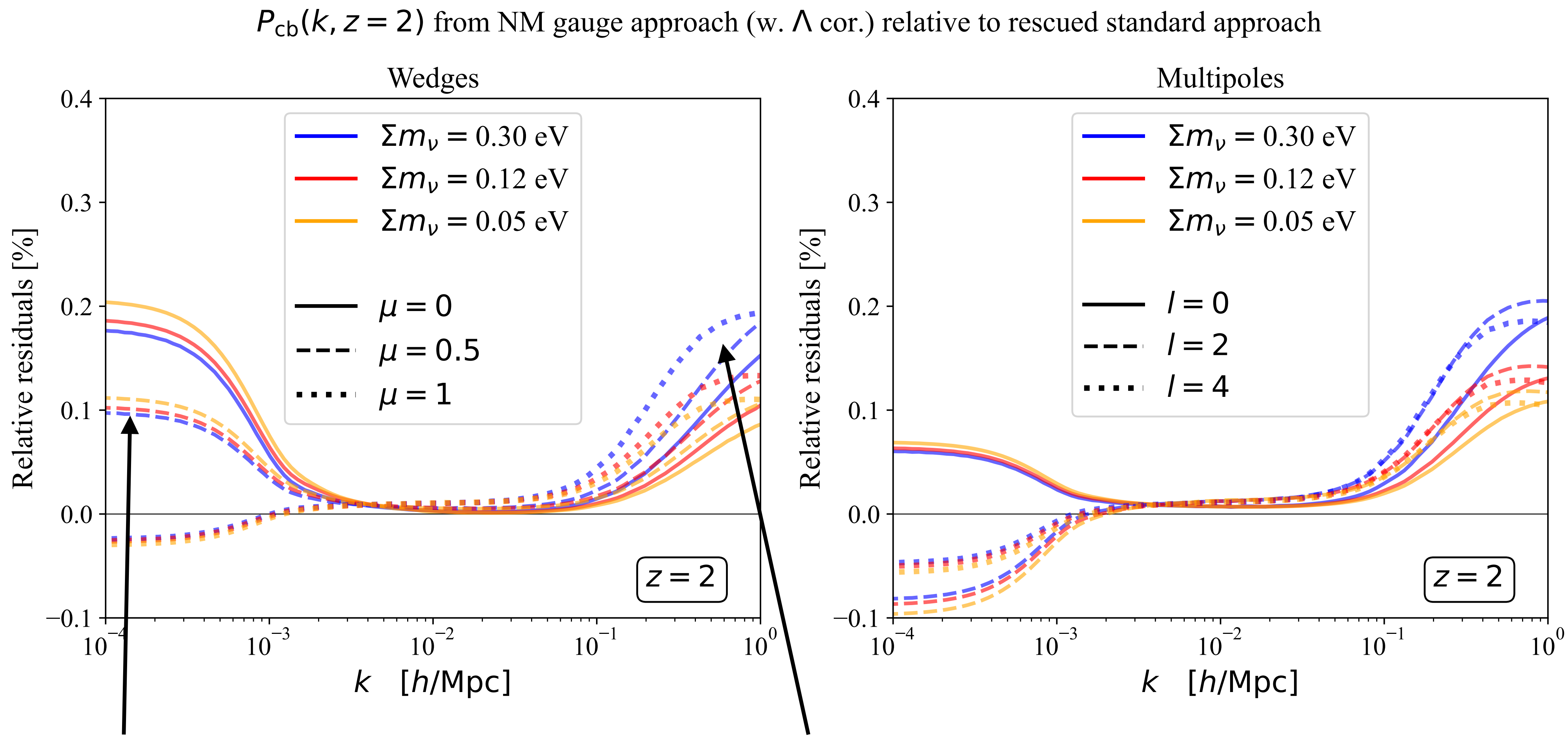
GR/radiation corrections: O(0.1%)
Come through gauge
transformation field

$\Lambda + \Sigma m_\nu$ corrections: O(1%)
Come through modified
ICs and kernels
(85% Λ, 15% Σm_ν)

dipole most sensitive
(growth of $\theta_{\rm cb}$)

- all scales: highly consistent with gevolution
- small scales: highly consistent with full calculation of Garny & Taule

Results: z=2



GR/radiation corrections: O(0.1%)
Come through gauge
transformation field

Σm_ν corrections: O(0.15%)
Come through modified
ICs and kernels

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- small scales: highly consistent with full calculation of Garny & Taule

Confirming some prescriptions in standard method

- In standard approach, accurate results require some choices/prescriptions:
  - Non-linear corrections computed for $P_{\rm cb}$, not $P_{\rm m}$
  - Time-scaling of the loops should be based on $D(\infty, z),\ f(\infty, z)$ excepted for Kaiser term rescaled by full $f(k, z)$
- In NM gauge, everything uniquely defined, no choices/prescriptions

Conclusion

- Effect of fully taking massive neutrino effects into account (compared to usual EdS approximation) is very small: just a confirmation
- Effect of fully taking Λ effects into account at kernel level is not that small
- Generalisation to galaxy spectrum with bias expansion under progress
- Method could be applied to several extended models (CWDM with light WDM, fraction of decaying DM, two-body decaying DM, modified gravity…)
- Method is generic and straightforward (extended models only needs to be implemented at linear level in CLASS)
- Potentially very useful for enhanced analysis of DESI / Euclid full-shape