# Gravitational Waves from Inflation


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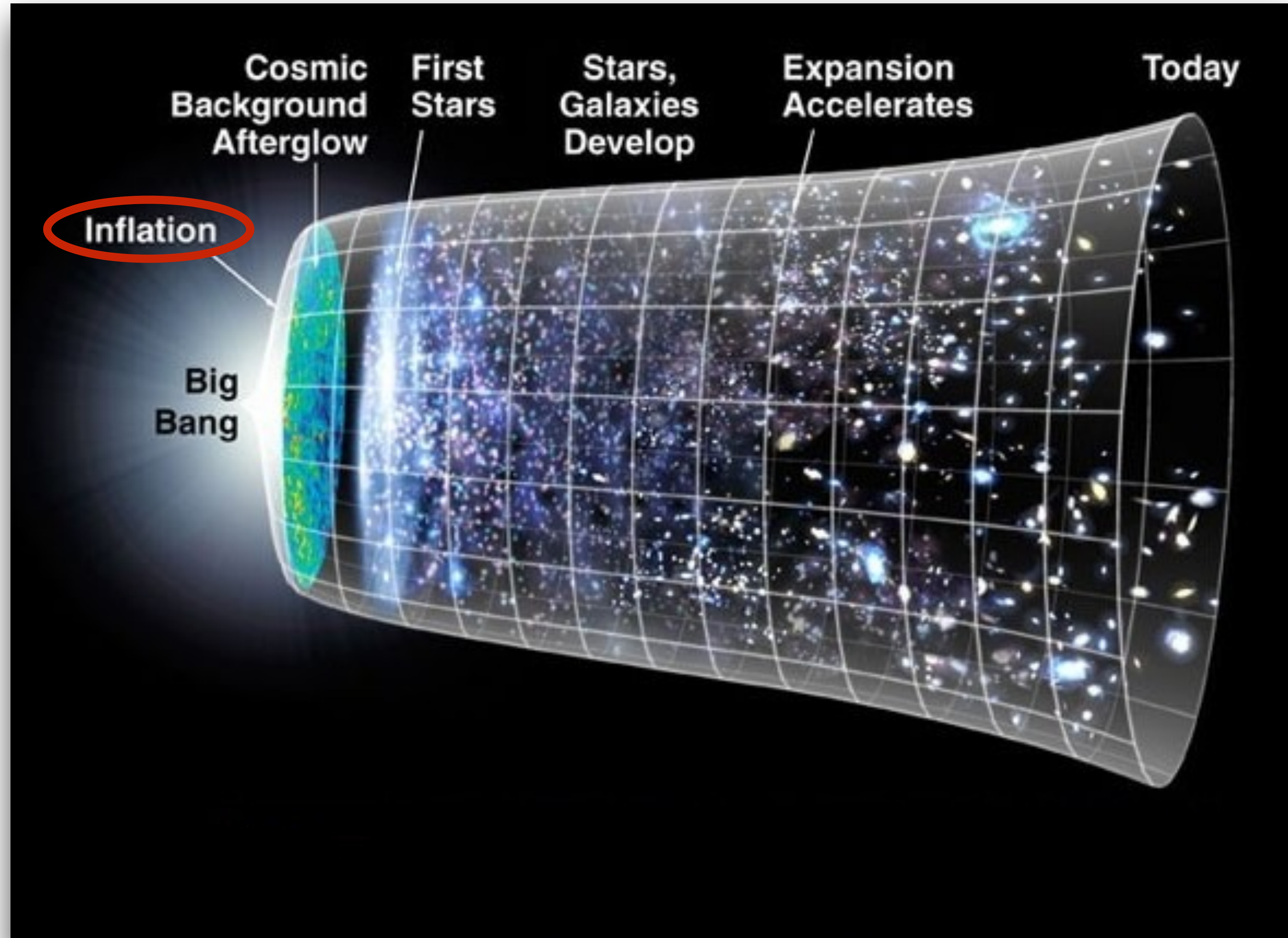
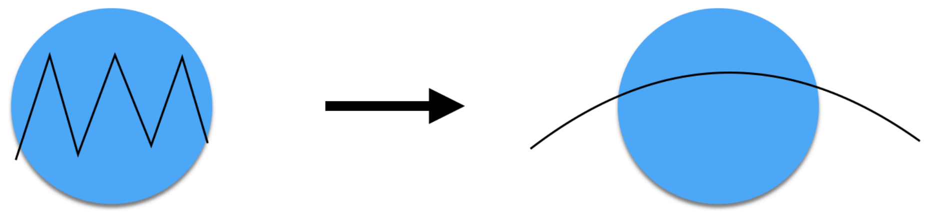
*The University of Groningen*


**GGI workshop: Exploring New Frontiers in Cosmology - July 24th 2026**

# Inflation


Cosmic Background Afterglow
First Stars
Stars, Galaxies Develop
Expansion Accelerates
Today
Inflation
Big Bang

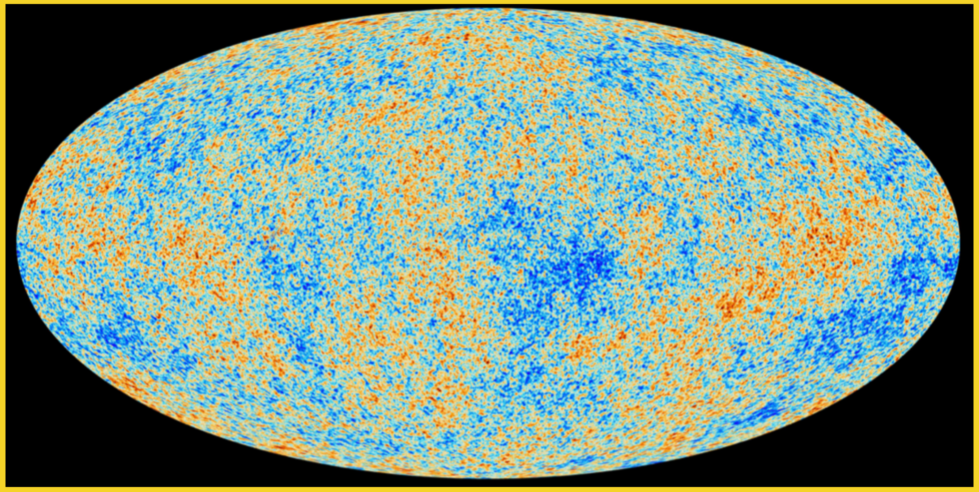
# CMB temperature anisotropies from inflaton fluctuations



$$\lambda(t) = a(t)\,\lambda_0$$

quantum fluctuations in the inflaton field gets stretched to cosmic scales
becoming classical density perturbations

from $\delta\phi$ to $\Delta T$

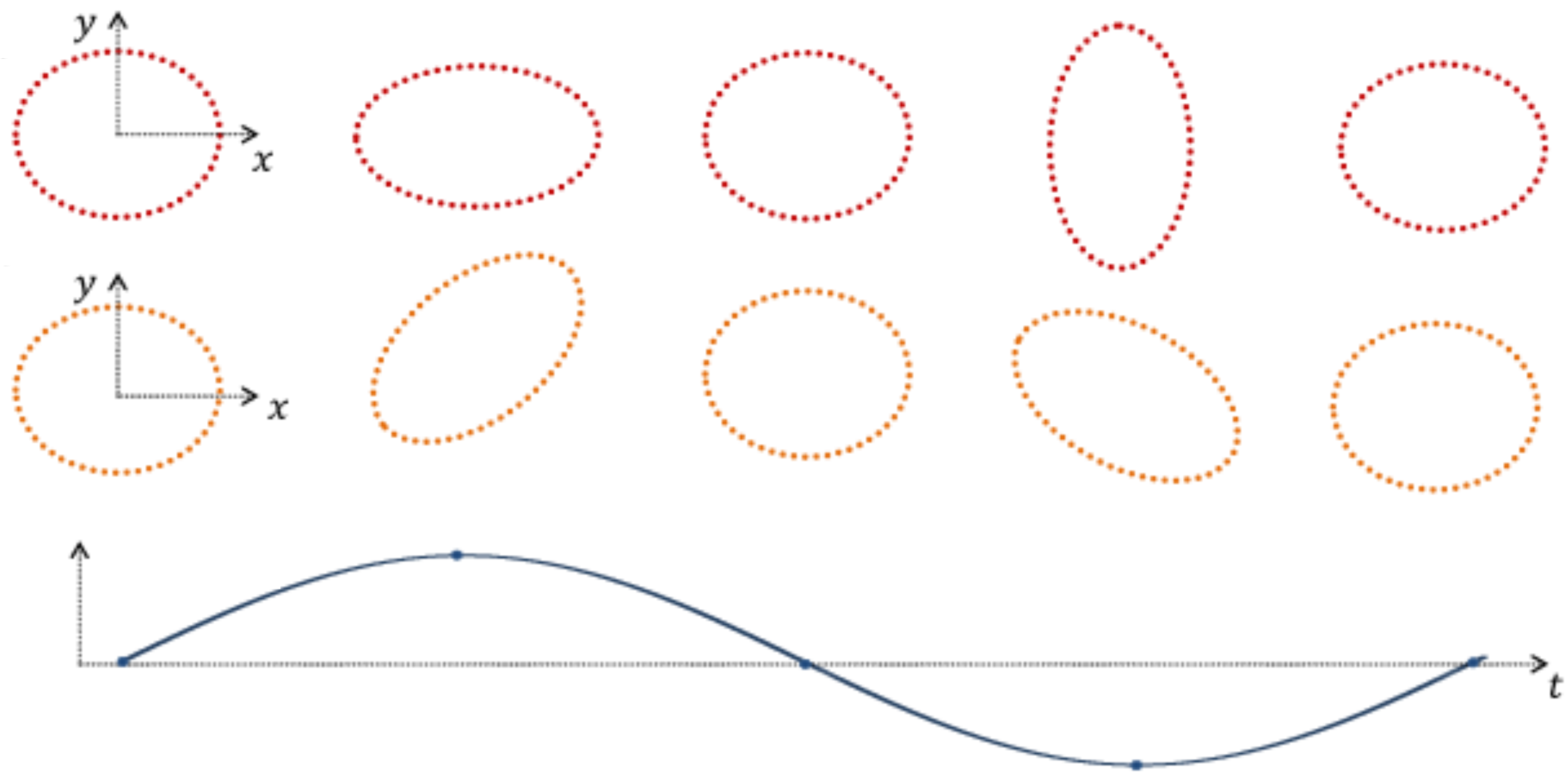


So far the universe looks remarkably similar
to what inflation predicts

A fundamental piece of the inflationary picture remains unconfirmed

If inflation happened, it should have also produced a gravitational-wave background

# Gravitational waves




Tensor perturbation on top of a homogenous & isotropic background:

$$ds^2 = -dt^2 + a^2(t)\,(\delta_{ij} + \gamma_{ij})\,dx^i dx^j$$

$$\gamma^i_i = \partial_i \gamma_{ij} = 0 \longrightarrow$$ **two polarization states of the graviton: $+$, $\times$**

traceless    transverse

# Gravitational waves from inflation

Einstein equations: $\underbrace{G_{\mu\nu}}_{\text{geometry}} = 8\pi G \underbrace{T_{\mu\nu}}_{\text{matter/source}}$

$$\ddot{\gamma}_{ij} + 3H\dot{\gamma}_{ij} + k^2\gamma_{ij} = 16\pi G\,\Pi_{ij}^{TT}$$

- **homogeneous** solution: GWs from **vacuum fluctuations**

Production of gravitons out of the vacuum
in an expanding universe!

- **inhomogeneous** solution: GWs from **sources**

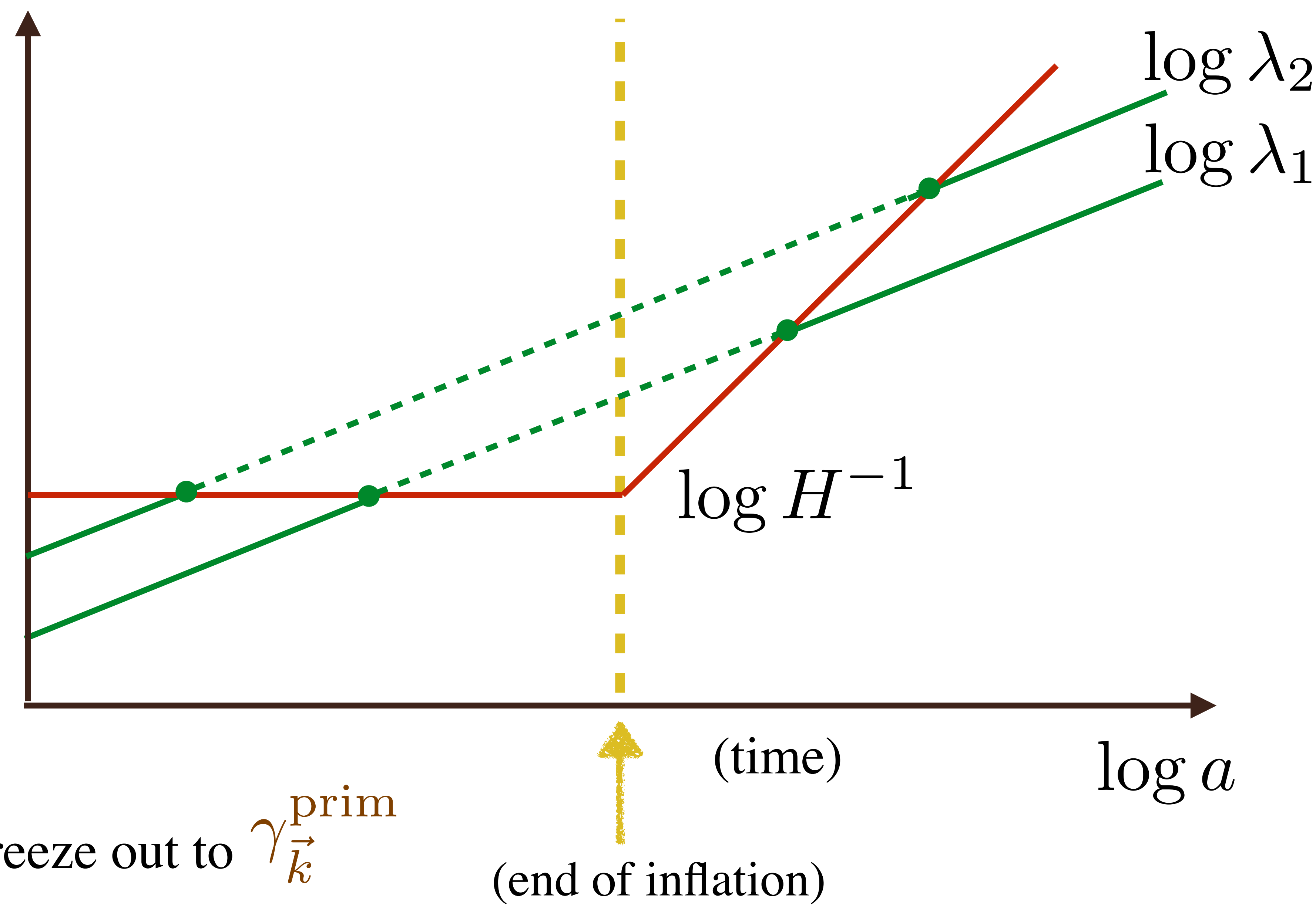
$$\Pi_{ij}^{TT} \propto \{\text{scalar fields, vector fields, fermions, tensors} \ldots\}$$

# Modes and Scales

… they re-enter after inflation
and become dynamical again

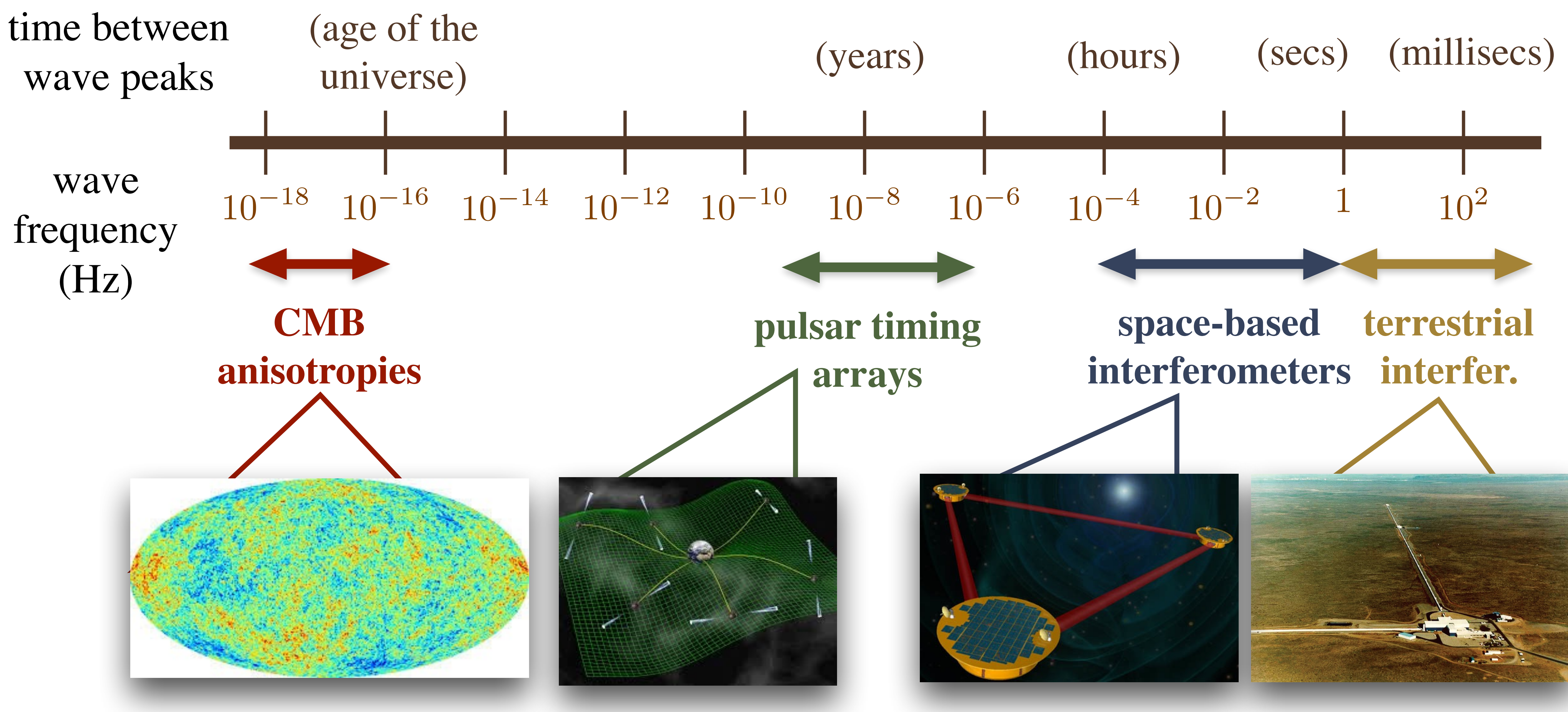
$$\gamma_{\vec{k}}(\tau) = T(\tau, k)\gamma_{\vec{k}}^{\text{prim}}$$

transfer function
(subhorizon evolution)

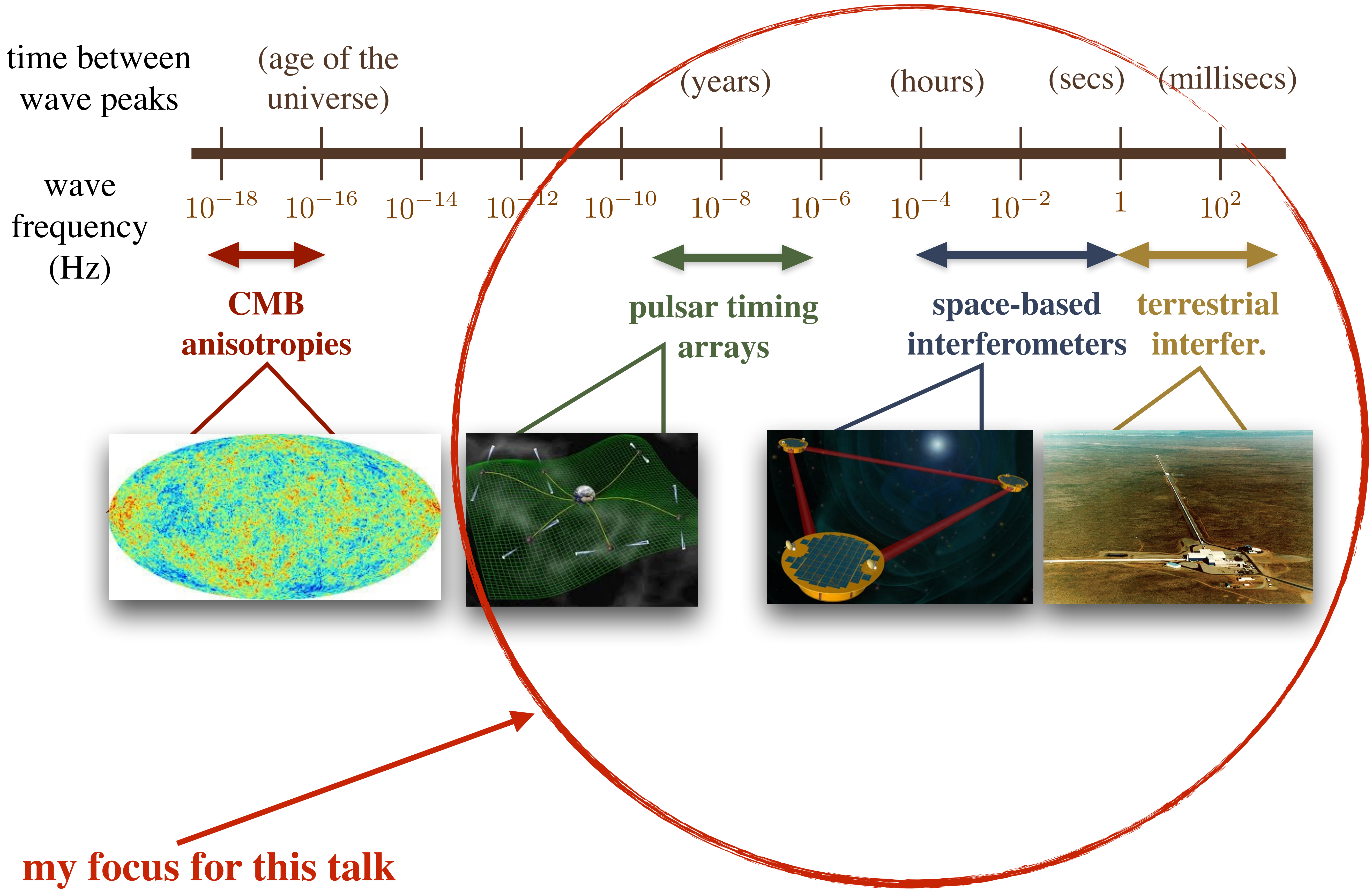



Perturbation
modes leave
horizon during
inflation and freeze out to $\gamma_{\vec{k}}^{\text{prim}}$

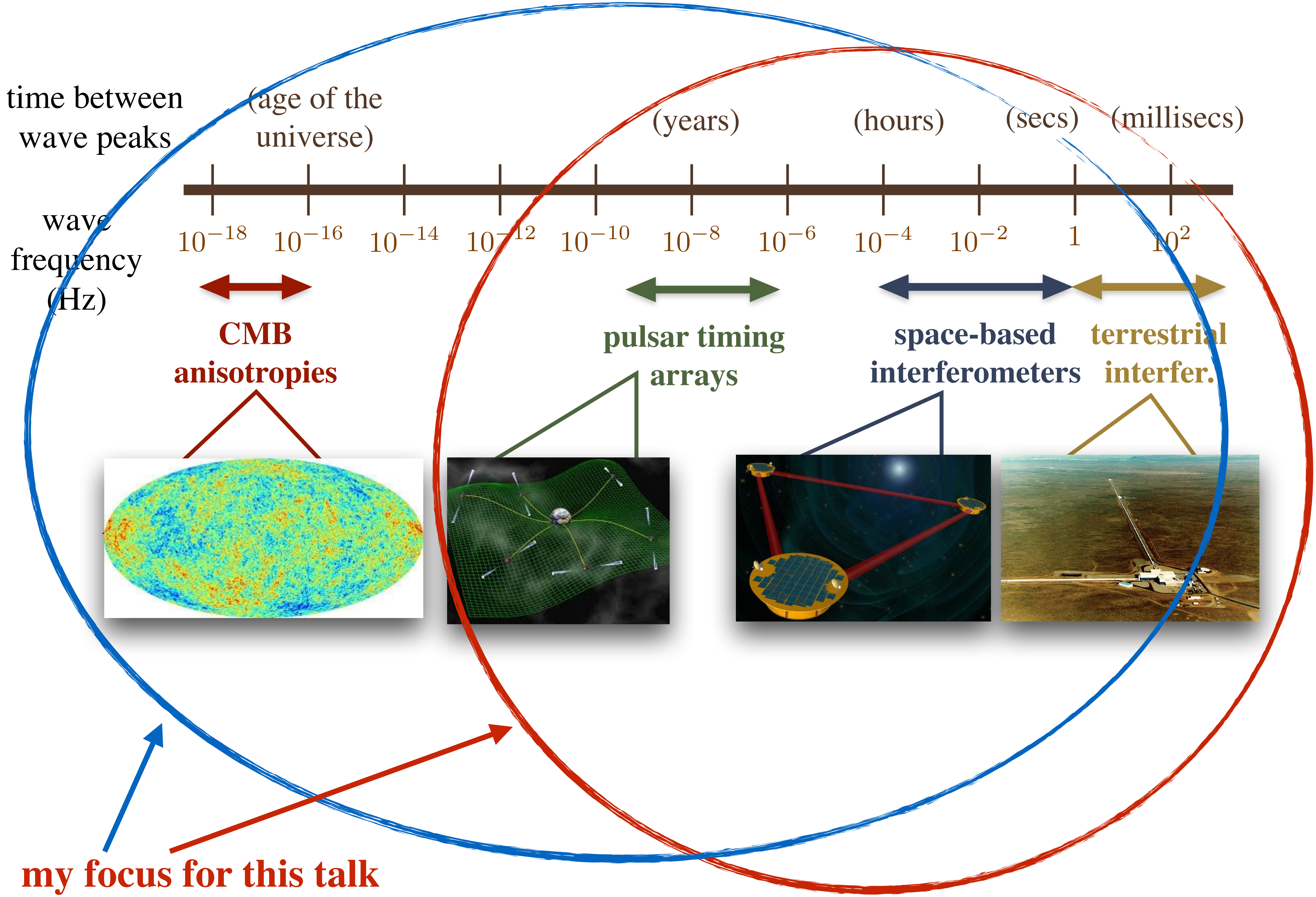
# Scales — Experiments



# Scales — Experiments



# Scales — Experiments



# Gravitational waves from inflation

- **homogeneous** solution: GWs from **vacuum fluctuations**

Production of gravitons out of the vacuum in an expanding universe!

**energy scale of inflation**

$$\mathcal{P}_\gamma^{\text{vacuum}}(k) = \frac{2}{\pi^2}\frac{H^2}{M_{\text{pl}}^2}\left(\frac{k}{k_*}\right)^{n_T}$$

$n_T = -r/8$

**red** tilt: amplitude decreases as we go towards smaller scales

$$\boxed{r \equiv \frac{\mathcal{P}_\gamma}{\mathcal{P}_\zeta} \quad \textbf{tensor-to-scalar ratio}}$$

# Tensor Power Spectrum

fractional GW energy density today per logarithmic interval of frequency

$$\Omega_{\mathrm{GW}}(k,\eta_0) \equiv \frac{1}{\rho_c}\frac{d\rho_{\mathrm{GW}}}{d\ln k}$$

$$\Omega_{\mathrm{GW}}(k,\eta_0) = \frac{k^2}{12a_0^2H_0^2}T^2(k,\eta_0)\,\mathcal{P}_\gamma(k)$$

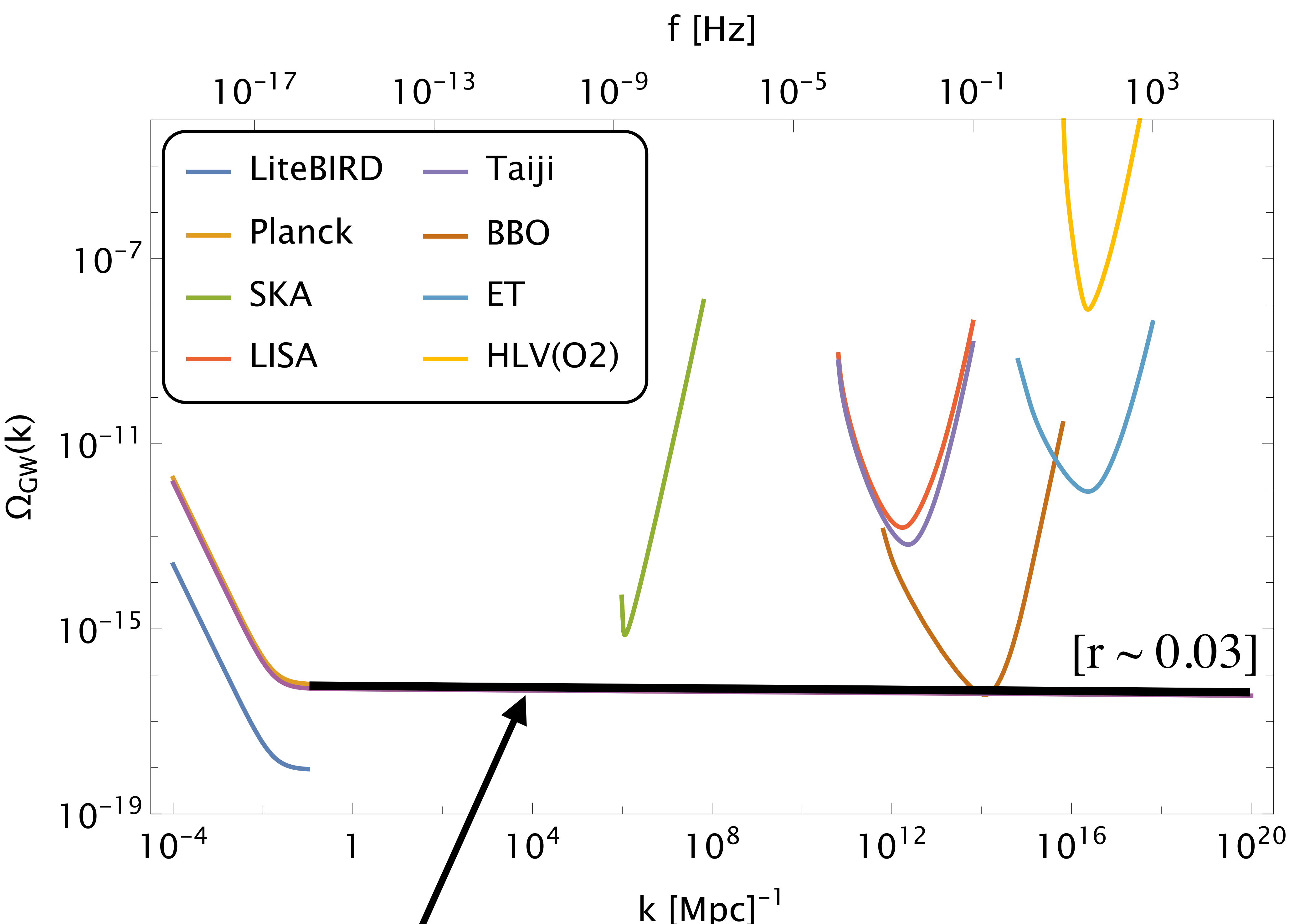
tensor transfer function (encodes evolution from horizon exit until today)

primordial power spectrum

Observed gravitational waves spectrum sensitive to

**primordial power spectrum** + **post-inflationary expansion**

# Prediction and sensitivity limits




Standard SFSR would go undetected at small scales (**red tilt**)

# Gravitational waves from inflation

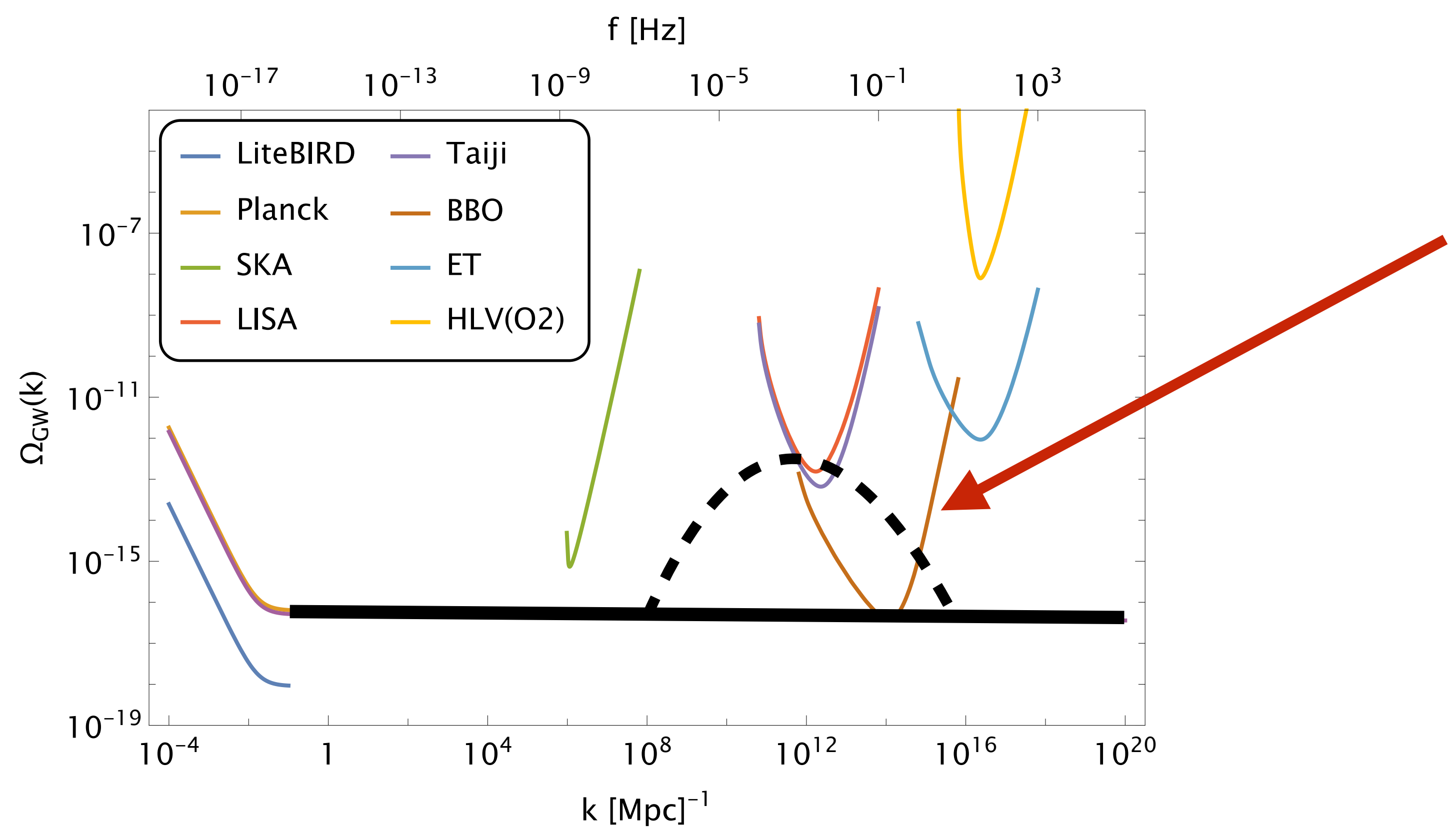
- **inhomogeneous** solution: GWs from **sources**

$$\ddot{\gamma}_{ij} + 3H\dot{\gamma}_{ij} + k^2\gamma_{ij} = 16\pi G\,\Pi_{ij}^{TT}$$

anisotropic stress-energy tensor

Axion-gauge field models $\frac{\lambda\chi}{4f}F\tilde{F}$

[Anber - Sorbo 2009, Cook - Sorbo 2011, Barnaby - Peloso 2011, Adshead - Wyman 2011, Maleknejad - Sheikh-Jabbari, 2011, ED - Fasiello - Tolley 2012, ED - Peloso 2012, Namba - ED - Peloso 2013, Adshead - Martinec -Wyman 2013, ED - Fasiello - Fujita 2016 Agrawal - Fujita - Komatsu 2017, Caldwell - Devulder 2017, Domcke et al. 2018, … ]



# Can we detect a primordial background?

**Challenge:**

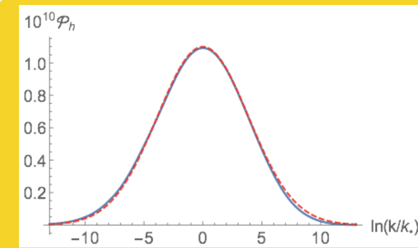
We may have a variety of **overlapping backgrounds from cosmological sources**
(inflation … first order phase transitions, topological defects…)

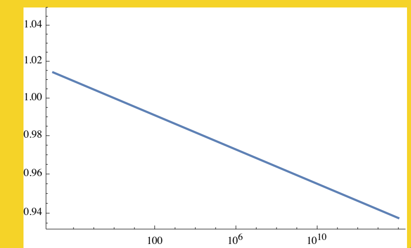
as well as a (possibly larger) **astrophysical background**
from the superposition of signals from a large number of astrophysical sources
(e.g. mergers of black holes, neutron stars)

**What can we do about it?**

# Can we detect a primordial background?

— Frequency profile

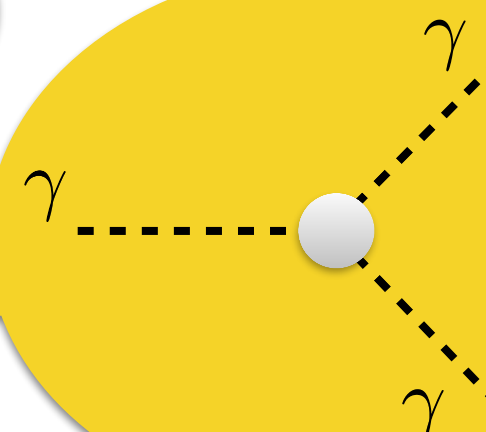


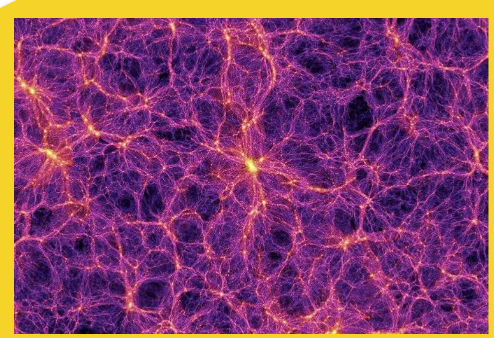
— Chirality

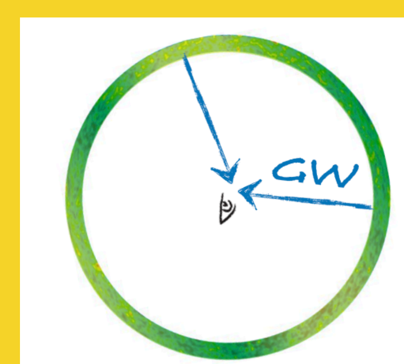
$$P_L \stackrel{?}{=} P_R$$

— Non-Gaussianity




— Anisotropies





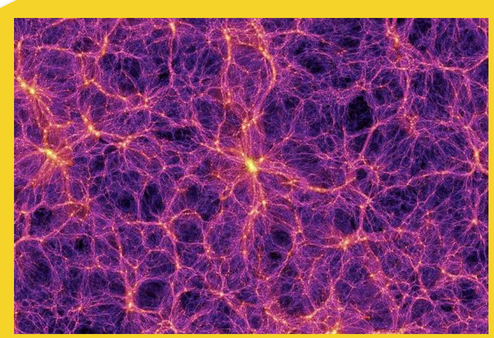
# Can we detect a primordial background?

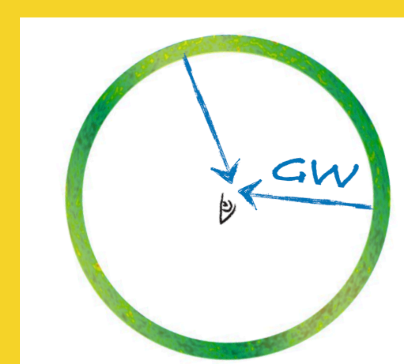
— Frequency profile

— Chirality

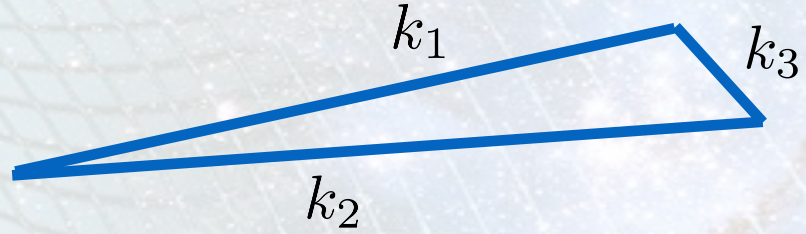
— Non-Gaussianity

— Anisotropies

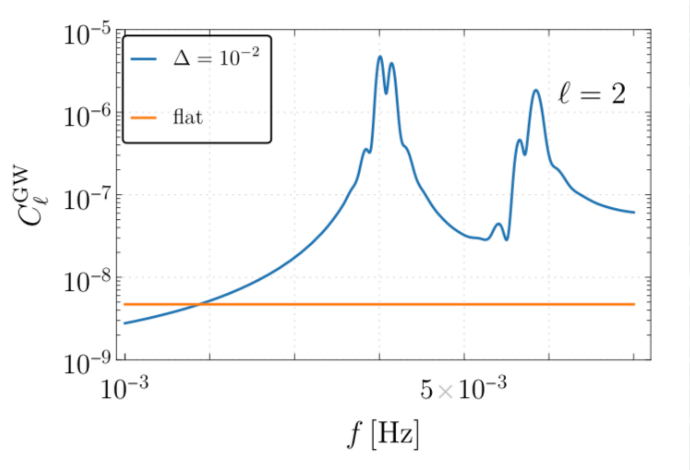




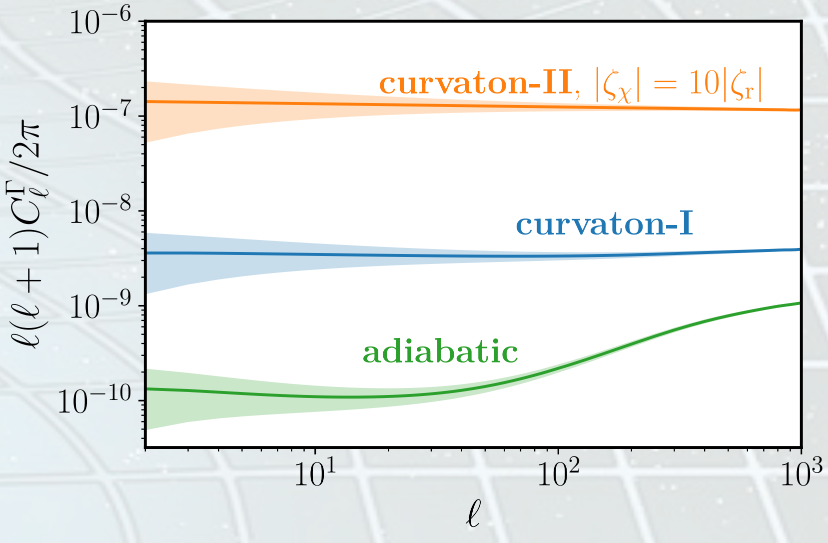
- Anisotropies as a probe of **squeezed non-Gaussianity**



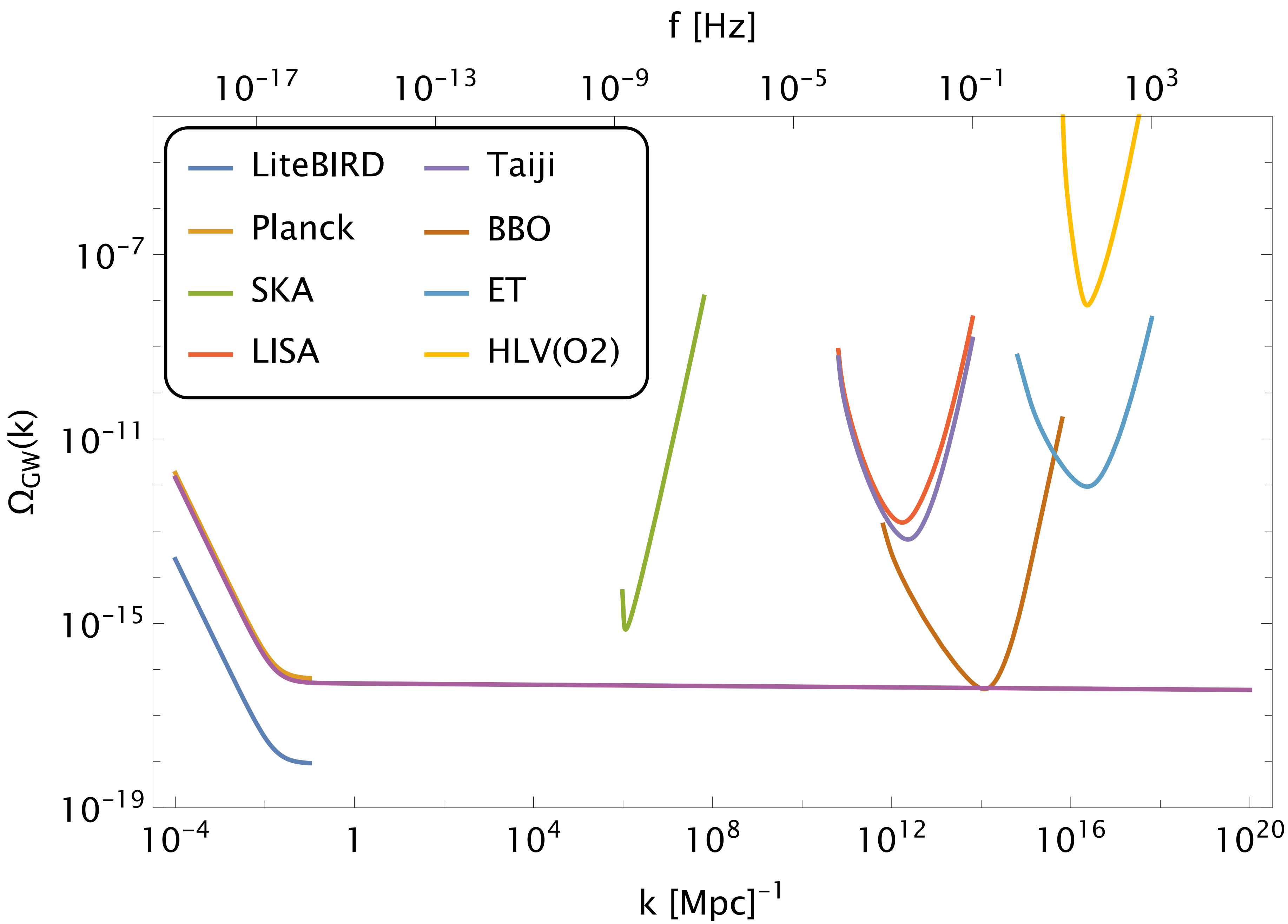

- Anisotropies as a probe of **SIGW** and **primordial black holes** physics




- Anisotropies as a probe of **isocurvature perturbations**



# Anisotropies in the GW background




Next step: <u>angular information</u>:
looking for spatial variations in the contributions to the energy density spectrum

# Anisotropies in the GW background

$$\Omega_{\mathrm{GW}}(k, \hat{\boldsymbol{n}}) \equiv \frac{1}{\rho_c} \frac{d^3 \rho_{\mathrm{GW}}}{d \ln k \, d^2 \hat{\boldsymbol{n}}} = \frac{\bar{\Omega}_{\mathrm{GW}}(k)}{4\pi} \left[1 + \delta_{\mathrm{GW}}(k, \hat{\boldsymbol{n}})\right]$$

directional GW
energy density spectrum

isotropic GW
energy density spectrum

anisotropy
(GW analogue of
CMB $\frac{\Delta T}{T}$)

k = comoving wavenumber
(~ observed frequency)

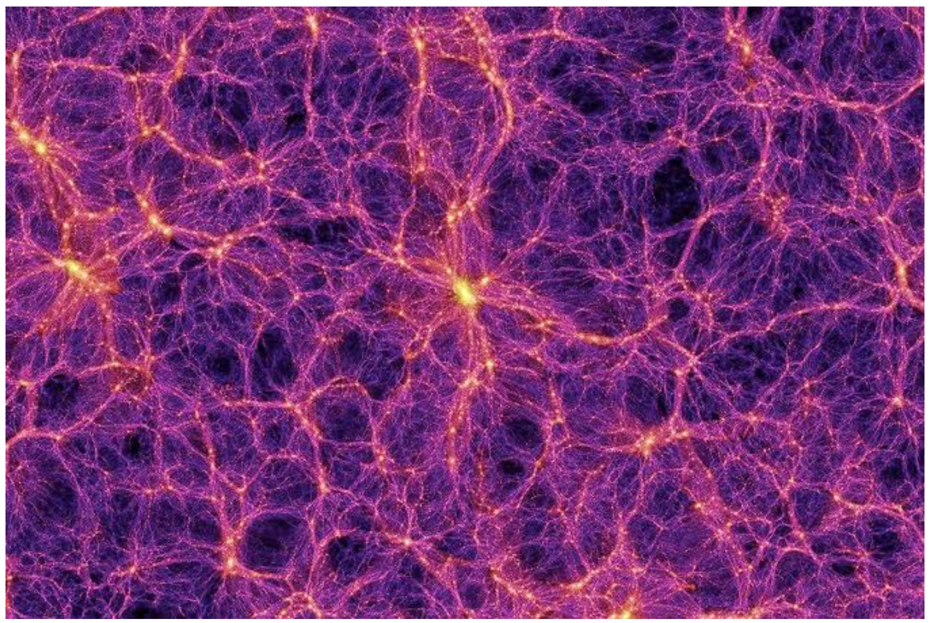
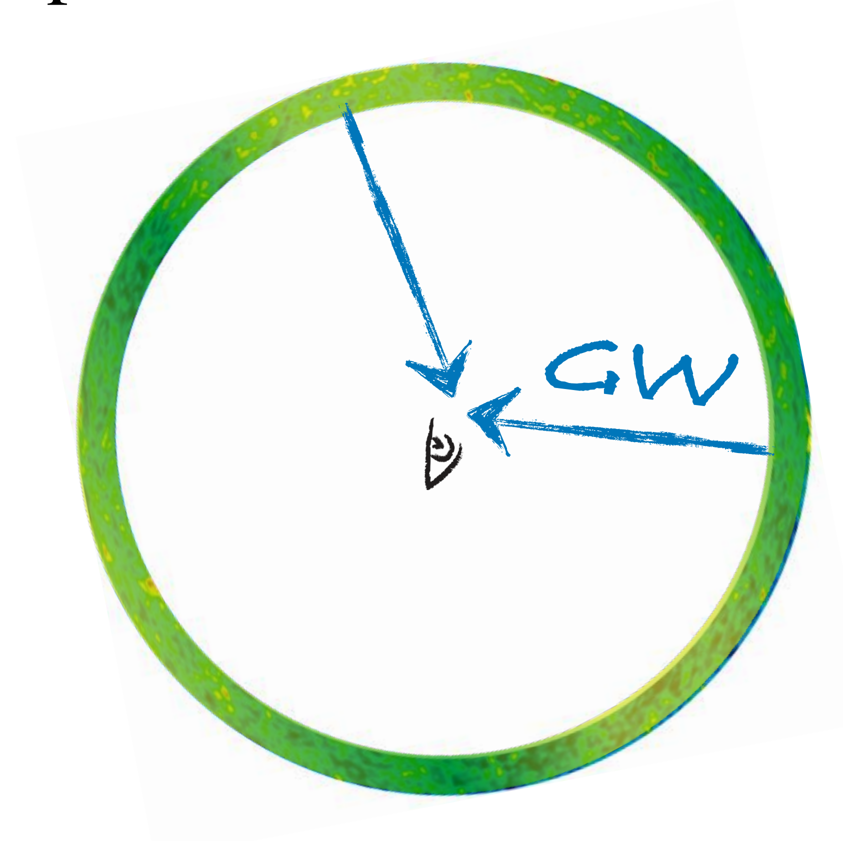
$$\bar{\Omega}_{\mathrm{GW}}(k) \equiv \frac{1}{\rho_c} \frac{d\bar{\rho}_{\mathrm{GW}}}{d \ln k}$$

**Angular power spectrum** for the anisotropies:

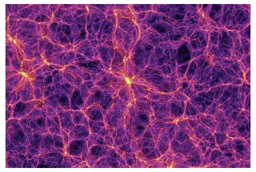
$$\delta_{\mathrm{GW}}(k, \hat{n}) = \sum_{\ell m} \delta^{\mathrm{GW}}_{\ell m} Y_{\ell m}(\hat{n})$$

$$\langle \delta^{\mathrm{GW}}_{\ell m} \delta^{\mathrm{GW}}_{\ell' m'} \rangle = \delta_{\ell\ell'} \delta_{mm'} C^{\mathrm{GW}}_{\ell}$$

# Origin of the anisotropies

| From propagation through the perturbed universe | Intrinsic to the production mechanism |
| --- | --- |
|  |  |

# Origin of the anisotropies: propagation through inhomogeneities



GW propagate through the perturbed universe

↓

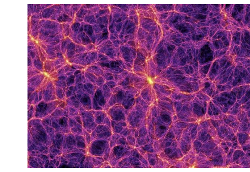
subject to Sachs-Wolfe / integrated Sachs-Wolfe …,
similarly to CMB photons

Sachs-Wolfe:

Different directions on the GW sky sample regions with
different gravitational potentials on the effective initial surface.

GWs emerging from deeper potential wells undergo a larger gravitational redshift than those emerging from shallower wells, producing anisotropies

# Origin of the anisotropies: propagation through inhomogeneities



- Gravitational redshift:

| |
|---|
| Hierarchy of scales between GW frequency and scale of the perturbations |
| GW background = collection of massless particles emitted at early times |

$$ds^2 = a^2(\eta)\left[-d\eta^2 + (1+2\zeta)\delta_{ij}dx^i dx^j - \frac{4}{5aH}\partial_i\zeta d\eta dx^i\right]$$

(uniform density gauge in matter domination)

Geodesic equation for the graviton

graviton's 4-momentum: $P^\mu = \frac{dx^\mu}{d\lambda}$

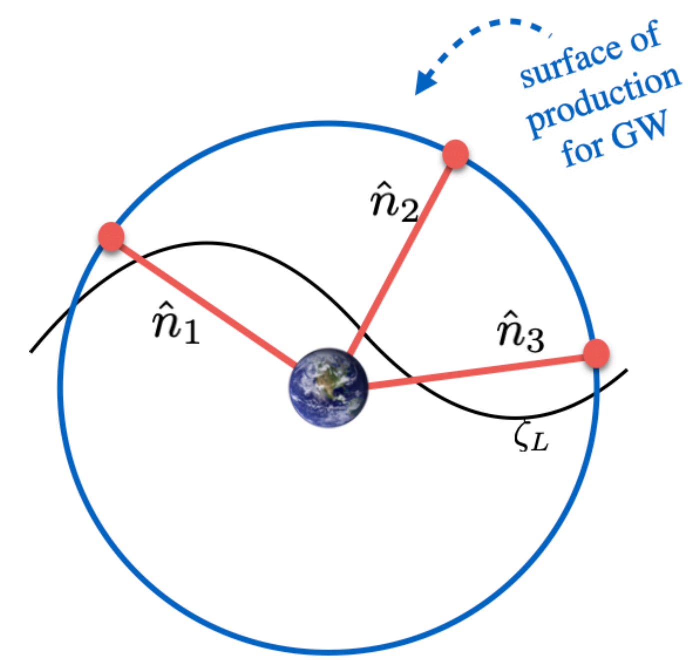
$$\frac{d^2x^\mu}{d\lambda^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\lambda}\frac{dx^\beta}{d\lambda} = 0$$

affine parameter along the graviton's geodesic

$$P_0(\eta) = P_0(\eta_{\rm i})\left[1 + \frac{1}{5}\left(\zeta(\eta,\mathbf{x}) - \zeta(\eta_{\rm i},\mathbf{x}_i)\right)\right]$$

value at observer's location (common to emissions from all directions) -> it drops out

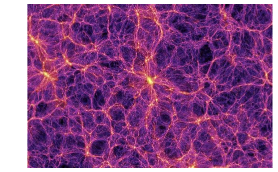
value at "emission" (direction dependent)

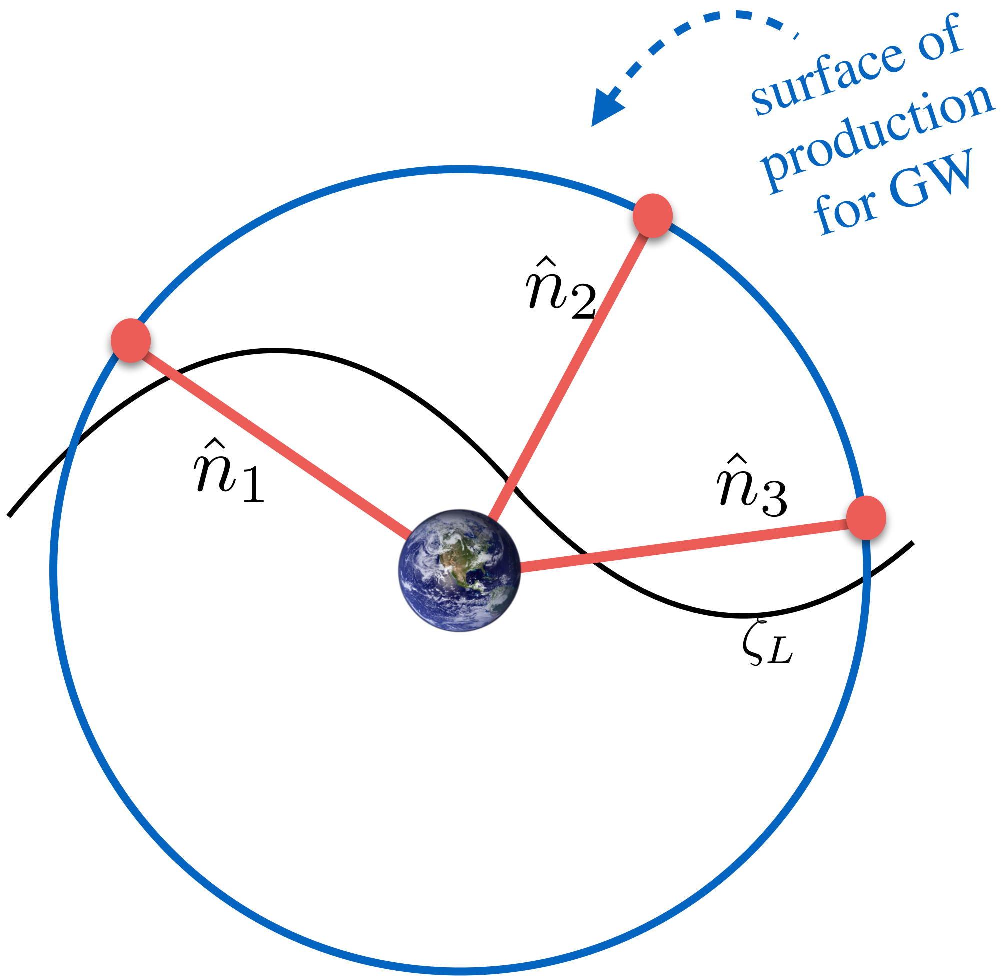





[Alba - Maldacena, 2015]

# Origin of the anisotropies: propagation through inhomogeneities






Large-scales: SW dominates

$$\frac{\delta f(\hat{n})}{f} \propto \left[\zeta_{L(\text{today})} - \zeta_L(\hat{n}\cdot\eta_0)\right]$$

$$\zeta_L(\hat{n}_1\cdot\eta_0) \neq \zeta_L(\hat{n}_2\cdot\eta_0)$$

Direction-dependent frequency shift

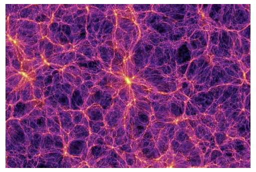
Anisotropy in the GW energy density

$$\delta_{\rm GW} \sim \left(\frac{\partial \ln \Omega_{\rm GW}}{\partial \ln k}\right)\zeta_{\rm L}$$

$$\delta_{\rm GW} \simeq \mathcal{O}(1)\,\zeta_{\rm L} \simeq 10^{-5}$$ (for SFSR Inflation)

**[Alba - Maldacena, 2015]**

# Origin of the anisotropies: original calculation (adiabatic perturbations)



<u>Hierarchy of scales</u> between GW frequency and scale of the perturbations

GW background = collection of <u>massless particles</u> emitted at early times

$$ds^2 = a^2(\eta)\left[-d\eta^2 + (1+2\zeta)\delta_{ij}dx^i dx^j - \frac{4}{5aH}\partial_i\zeta d\eta dx^i\right]$$

(<u>uniform density gauge</u> in <u>matter domination</u>)

- Field theory derivation:

Massless field h in a perturbed universe:

$$S = \frac{1}{2}\int d^3x\, d\eta\, a^2\,(1+3\zeta)\left[(-\partial_\eta h)^2 + (1-2\zeta)\left(\vec{\partial} h\right)^2 - \frac{4(\partial_\eta h)}{5aH}\vec{\partial}\zeta\cdot\vec{\partial}h\right]$$

$$\downarrow$$

$$\mathcal{L}_{\rm int} \propto a^2\zeta\left[3\,(\partial_\eta h)^2 - \left(\vec{\partial}h\right)^2\right] + \frac{4a(\partial_\eta h)}{5H}\vec{\partial}\zeta\cdot\vec{\partial}h$$

$$\downarrow$$

correction to the two point function of h proportional to $\zeta$

**[Alba - Maldacena, 2015]**

**[see also Bethke, Figueroa, Rajantie, 2014]**

# Origin of the anisotropies: Boltzmann formalism

<u>Hierarchy of scales</u> between GW frequency and scale of the perturbations

GW background = collection of <u>massless particles</u> emitted at early times and described by a <u>distribution function</u> $f(x^\mu,\, p^\mu)$

$$ds^2 = a^2(\eta)\left[-(1+2\Phi)d\eta^2 + (1-2\Psi)\delta_{ij}dx^i dx^j\right]$$

$$\frac{df}{d\lambda} = \underbrace{C[f(\lambda)]}_{\text{collision term}} + \underbrace{I[f(\lambda)]}_{\text{injection term}} \simeq 0 \longrightarrow \frac{df}{d\eta} = \frac{\partial f}{\partial \eta} + \frac{\partial f}{\partial x^i}\underbrace{\frac{dx^i}{d\eta}}_{n^i} + \frac{\partial f}{\partial q}\underbrace{\frac{dq}{d\eta}}_{q\left[\frac{\partial \Psi}{\partial \eta} - \frac{\partial \Phi}{\partial x^i}n^i\right]} = 0$$

collision term = 0 [gravitons effectively decoupled below Mp]

injection term = 0 [accounted for as an initial condition for f]

(focusing on free-streaming, SW and iSW terms)

graviton's <u>4-momentum:</u>

$$p^\mu = \frac{dx^\mu}{d\lambda} \qquad q \equiv |\vec{p}|a$$

[Contaldi, 2017- Bartolo et al 2019
see also: Pitrou et al, 2020 - Schulze et al, 2023]

# Origin of the anisotropies: Boltzmann formalism

$$\boxed{\frac{\partial f}{\partial \eta} + \frac{\partial f}{\partial x^i} n^i + q \frac{\partial f}{\partial q}\left[\frac{\partial \Psi}{\partial \eta} - \frac{\partial \Phi}{\partial x^i} n^i\right] = 0}$$

$$f(\eta_0, \vec{x}_0, \hat{n}, q) = \bar{f}(q) - q \frac{\partial \bar{f}}{\partial q} \Gamma(\eta_0, \vec{x}_0, \hat{n}, q)$$

(linear expansion for the distribution function)

$$\rho_{\rm GW}(\underline{\eta_0, \vec{x}_0}) = \int d^3p\, p\, f(\eta_0, \vec{x}_0, q, \hat{n}) = \rho_{cr} \int d\ln q\, \Omega_{\rm GW}(\eta_0, \vec{x}_0, q)$$

observer's time/location

$$\boxed{\delta_{\rm GW}(\eta_0, \vec{x}_0, \hat{n}, q) = \left[4 - \frac{\partial \ln \overline{\Omega}_{\rm GW}(\eta_0, q)}{\partial \ln q}\right] \Gamma(\eta_0, \vec{x}_0, \hat{n}, q)}$$

$$\boxed{\Gamma(\eta_0, \vec{x}_0, \hat{n}, q) = \Gamma(\eta_{\rm i}, \vec{x}_i, q) + \Phi(\eta_{\rm i}, \vec{x}_i) + \int_{\eta_{\rm i}}^{\eta_0} d\eta (\Phi' + \Psi')}$$

initial perturbation

gravitational redshift at the initial surface

iSW

**[Contaldi, 2017- Bartolo et al 2019 see also: Pitrou et al, 2020]**

# Origin of the anisotropies: Boltzmann formalism

$$\Gamma(\eta_0, \vec{x}_0, \hat{n}, q) = \underbrace{\Gamma(\eta_\mathrm{i}, \vec{x}_i, q)}_{\Gamma_I} + \Phi(\eta_\mathrm{i}, \vec{x}_i) + \int_{\eta_\mathrm{i}}^{\eta_0} d\eta (\Phi' + \Psi')$$

For <u>adiabatic</u> primordial perturbations: $$\frac{\delta\rho_i}{(1+w_i)\bar{\rho}_i} = \frac{\delta\rho_j}{(1+w_j)\bar{\rho}_j}$$

(for any two species “i” and “j”)

applying this to photons and gravitons fluids:

$$w_i = w_j = 1/3$$

$$\frac{\delta\rho_\gamma}{\bar{\rho}_\gamma} = -2\Phi$$ (from Einstein equ. in RD, superhorizon)

$$\Gamma_I = \frac{1}{4}\frac{\delta\rho_{\mathrm{GW}}}{\bar{\rho}_{\mathrm{GW}}}$$ (by definition)

$$\Gamma_I = -\frac{1}{2}\Phi$$

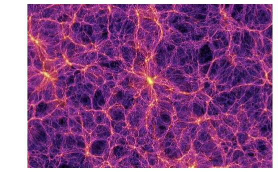
$$\Gamma = -\frac{1}{2}\Phi + \Phi + \int_{\eta_i}^{\eta_0} d\eta \left(\Phi^{'} + \Psi^{'}\right)$$

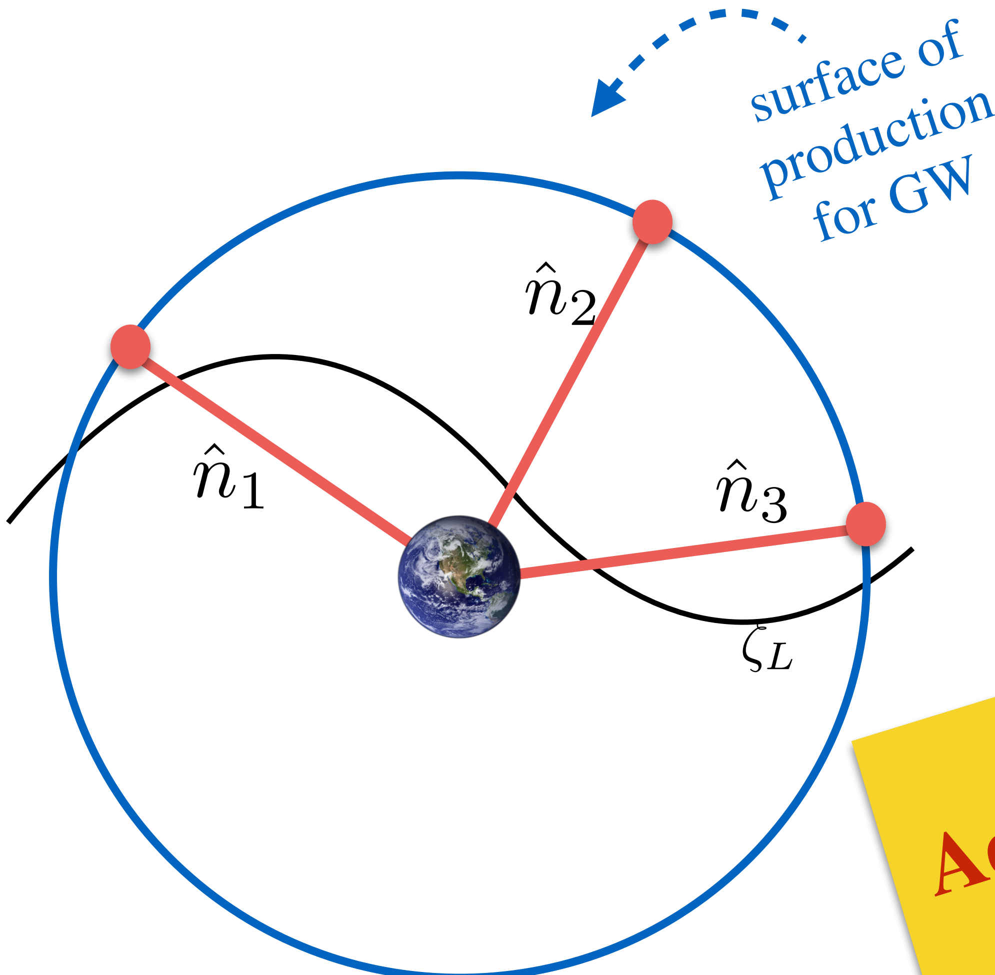
$$= \frac{1}{2}\Phi + \int_{\eta_i}^{\eta_0} d\eta \left(\Phi^{'} + \Psi^{'}\right)$$

$$= -\frac{1}{3}\zeta + \ldots$$

$$\Phi = -\frac{2}{3}\zeta$$

# Origin of the anisotropies: propagation through inhomogeneities






Large-scales: SW dominates

$$\frac{\delta f(\hat{n})}{f} \propto [\zeta$$

**Adiabatic primordial perturbations**

**Small spectral tilt for tensors**

**No intrinsic anisotropies**

Anisotropy in the G… gy density

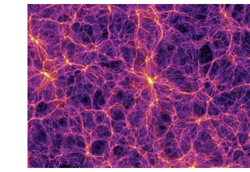
$$\delta_{\rm GW} \sim \left( \frac{\partial \ln \Omega_{\rm GW}}{\partial \ln k} \right) \zeta_{\rm L}$$

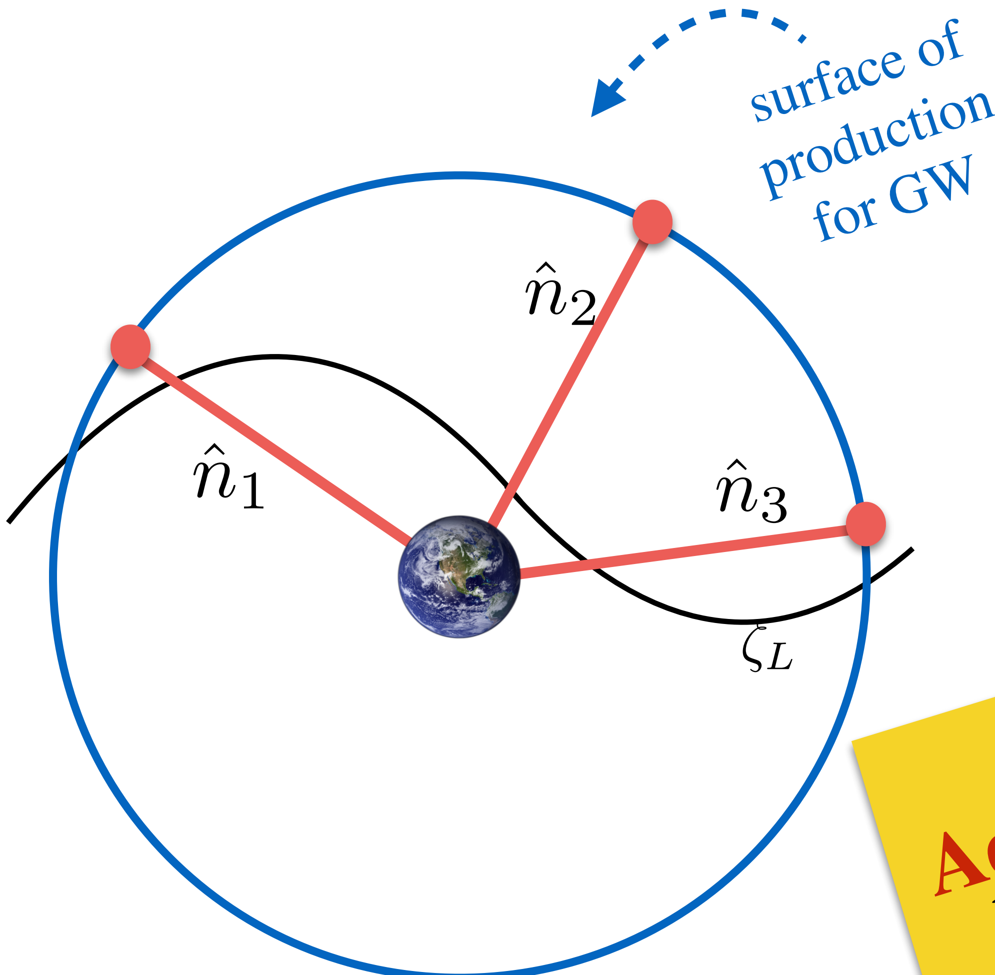
$$\delta_{\rm GW} \simeq \mathcal{O}(1)\, \zeta_{\rm L} \simeq 10^{-5}$$

(for SFSR Inflation)

**[Alba - Maldacena, 2015]**

# Origin of the anisotropies: propagation through inhomogeneities






Large-scales: SW dominates

~~Adiabatic primordial perturbations~~

~~Small spectral tilt for tensors~~

~~No intrinsic anisotropies~~

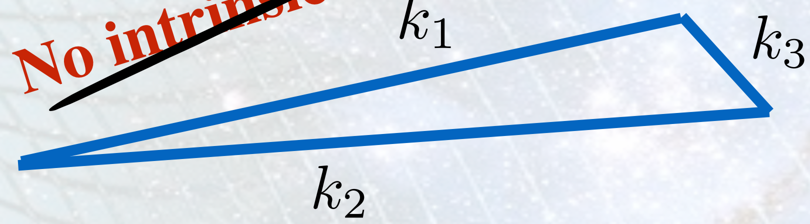
Anisotropy in the G… …gy density

$$\delta_{\rm GW} \sim \left(\frac{\partial \ln \Omega_{\rm GW}}{\partial \ln k}\right) \zeta_{\rm L}$$

$$\delta_{\rm GW} \simeq \text{?}$$

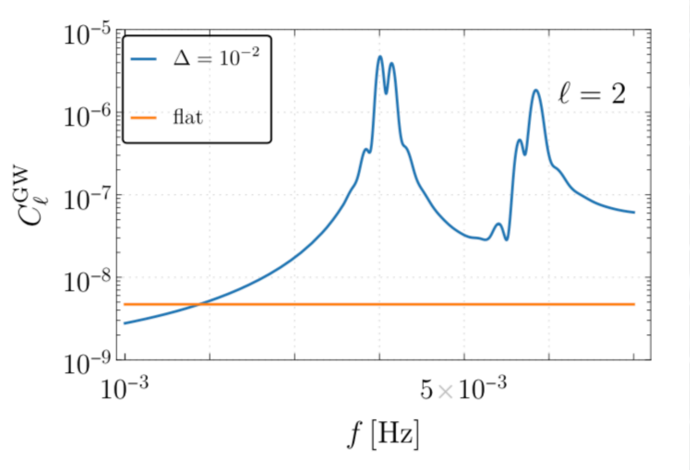
- Anisotropies as a probe of **squeezed non-Gaussianity**

**~~No intrinsic anisotropies~~**



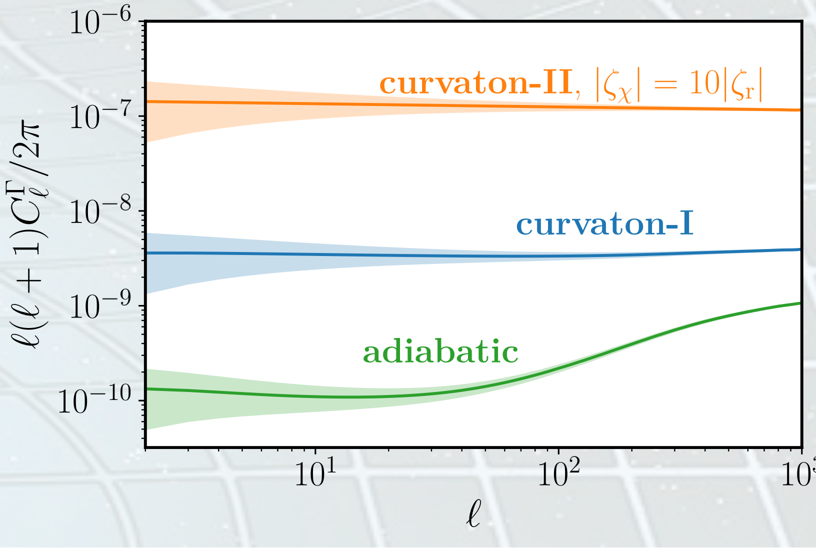

- Anisotropies as a probe of **SIGW** and **primordial black holes** physics

**~~Small spectral tilt for tensors~~**

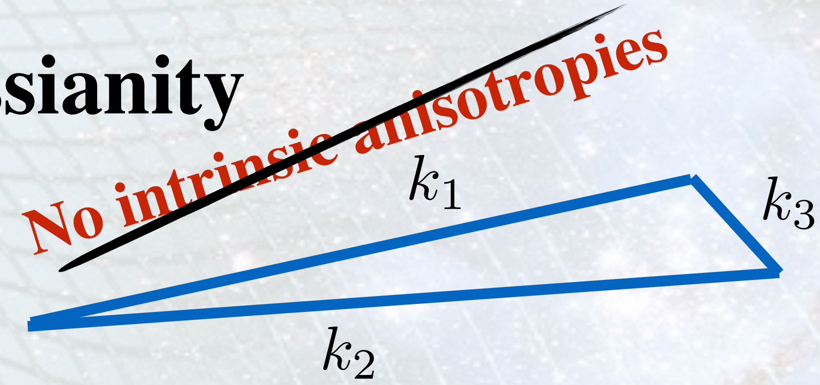



- Anisotropies as a probe of **isocurvature perturbations**

**~~Adiabatic primordial perturbations~~**



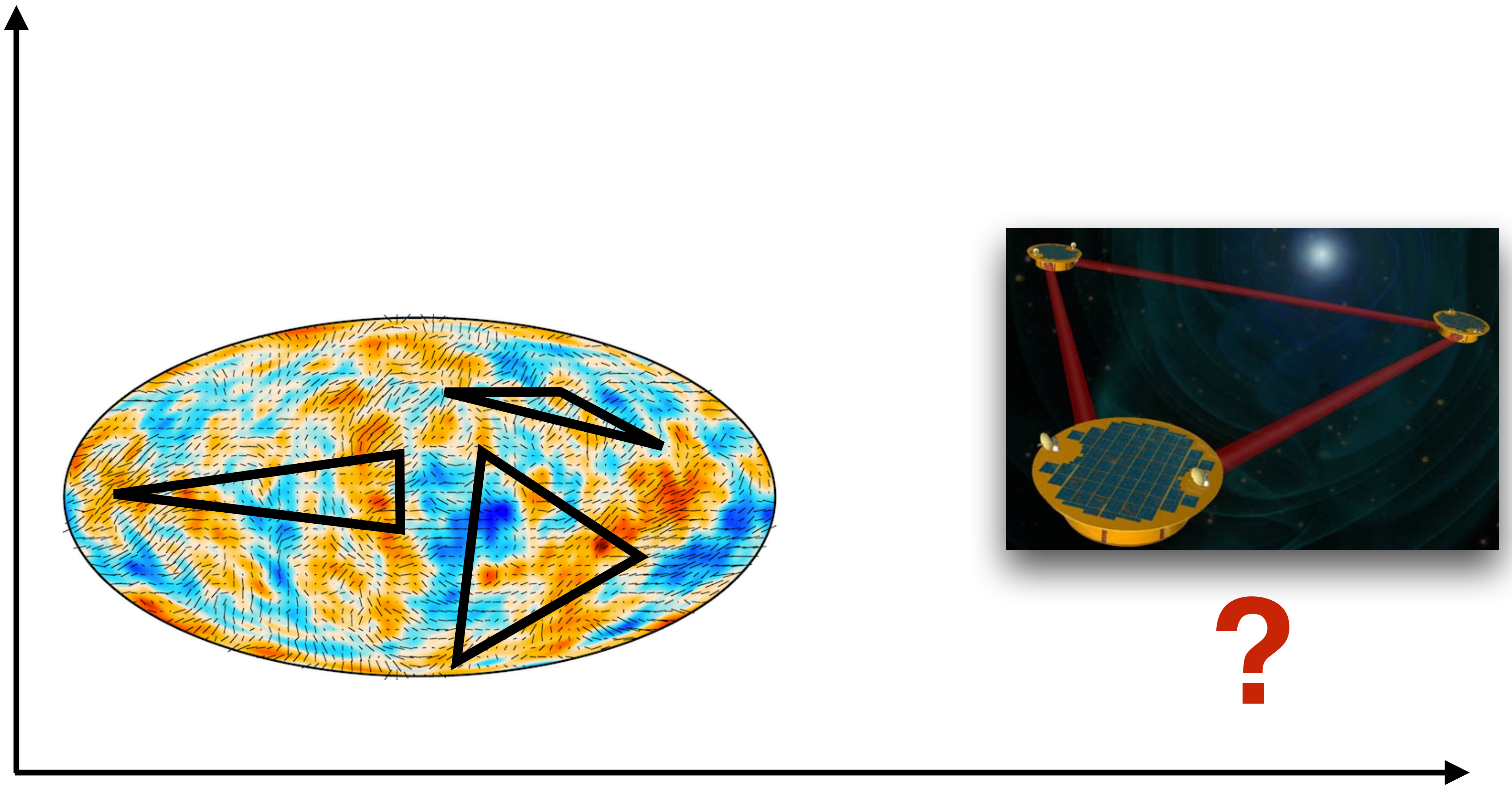
- Anisotropies as a probe of **squeezed non-Gaussianity**




- Anisotropies as a probe of **SIGW** and **primordial black holes** physics

- Anisotropies as a probe of **isocurvature perturbations**

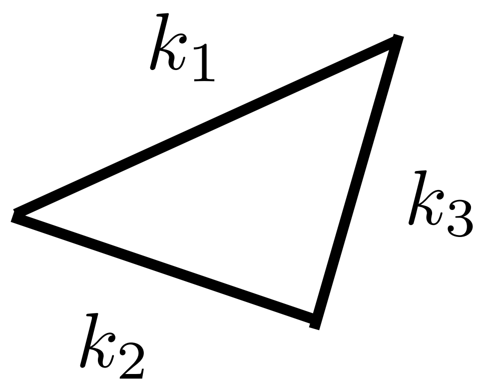
# Primordial non-Gaussianity

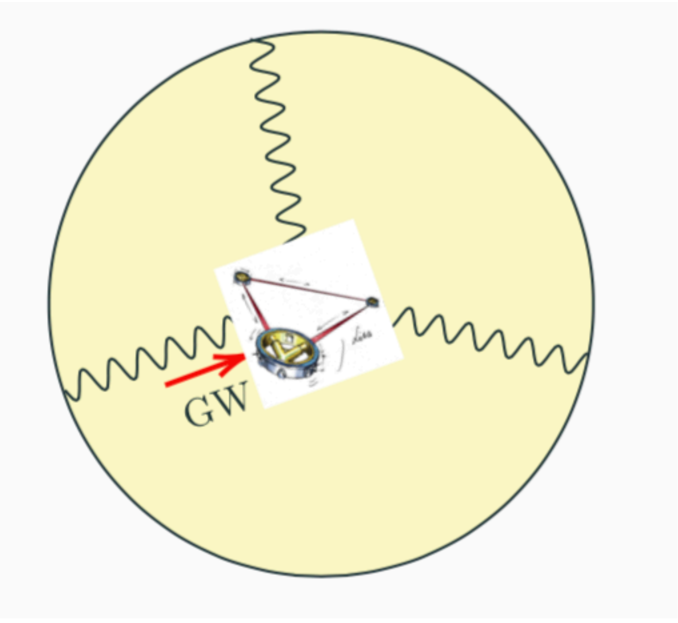


$k_{\rm CMB} \sim 10^{-3}{\rm Mpc}^{-1}$ $k_{\rm GW} \sim 10^{12}{\rm Mpc}^{-1}$

# Non-Gaussianity at interferometers

Measuring $\langle \gamma^3 \rangle$ directly is not possible: phase decorrelation from propagation in an inhomogeneous universe




[Bartolo et al. 2018]

# Non-Gaussianity at interferometers

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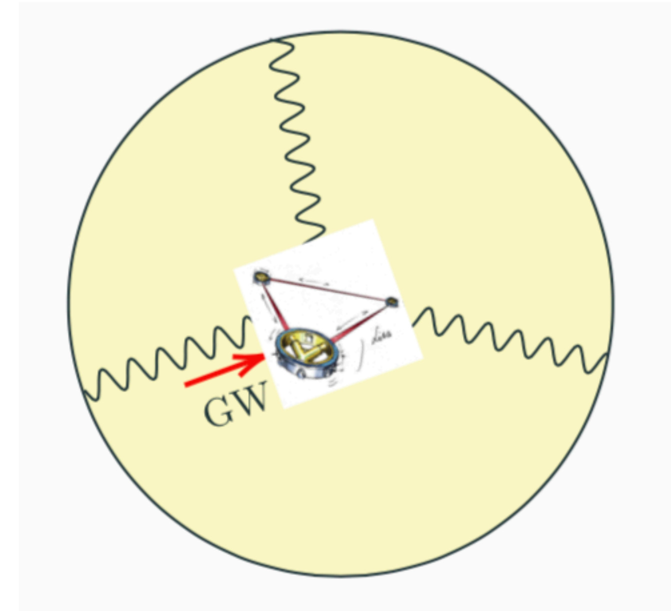



Shapiro time delay:

$$\gamma'' + 2\mathcal{H}\gamma' - [1 + (12/5)\zeta]\,\gamma_{,kk} = 0$$

GW propagating in FRW background
+ long-wavelength perturbations

$$\gamma_{ij} = A_{ij}\, e^{ik\tau + ik\cdot 2\int^{\tau} d\tau'\, \zeta[\tau', (\tau' - \tau_0)\hat{k}]}$$

GW from different directions
undergo different phase shifts
due to intervening structure

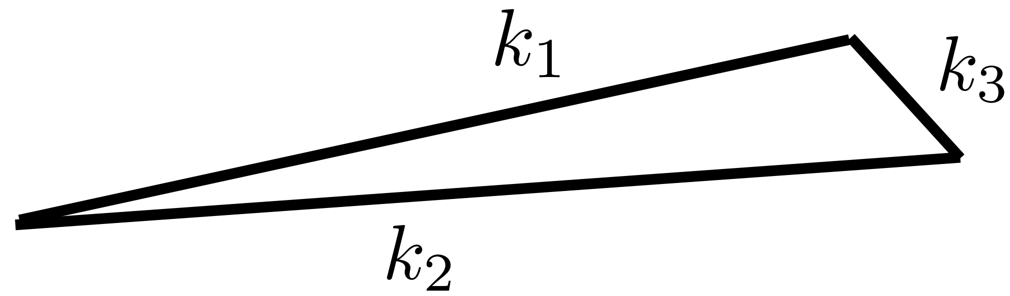
$\longrightarrow$ initial non-Gaussianity wiped out by propagation

[Bartolo et al. 2018]

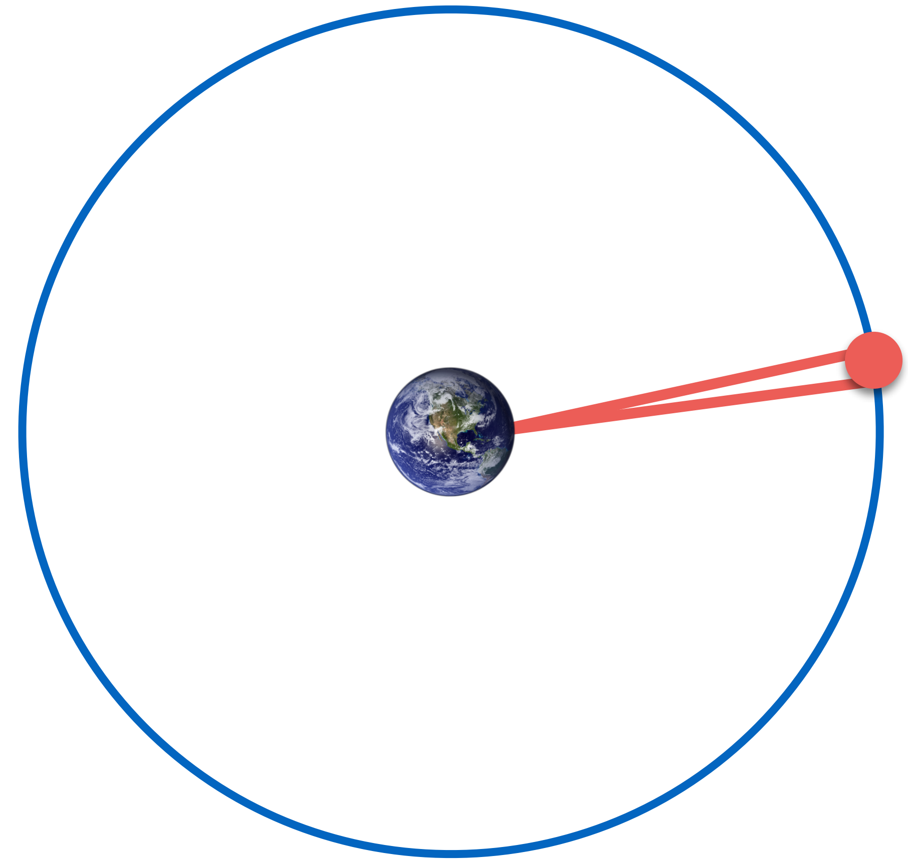
CLT: signal measured by an interferometer arises from the superposition of signals from a large number of Hubble patches at production

[Adshead, Lim 2009 — Caprini, Figueroa 2018 — Bartolo et al 2018]

# Non-Gaussianity at interferometers




Correlation among two short-wavelength modes (e.g. interferometer scale) and 1 very long-wavelength mode: signals originate from the same patch!





[ED, Fasiello, Tasinato, PRL 124(2020)6 061302]

# Non-Gaussianity at interferometers

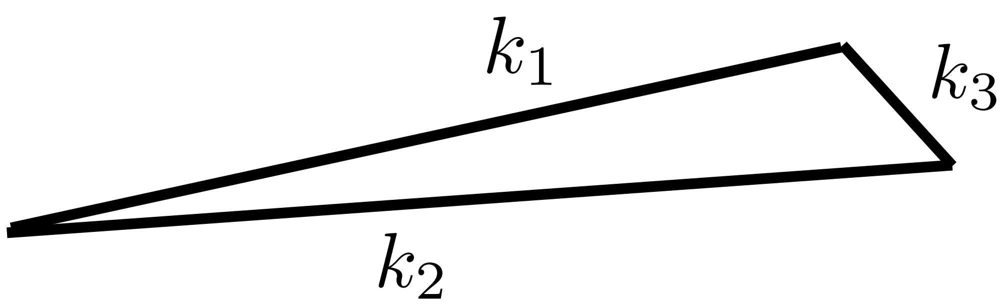
<u>Based on</u>: PRL 124(2020)6 061302 with

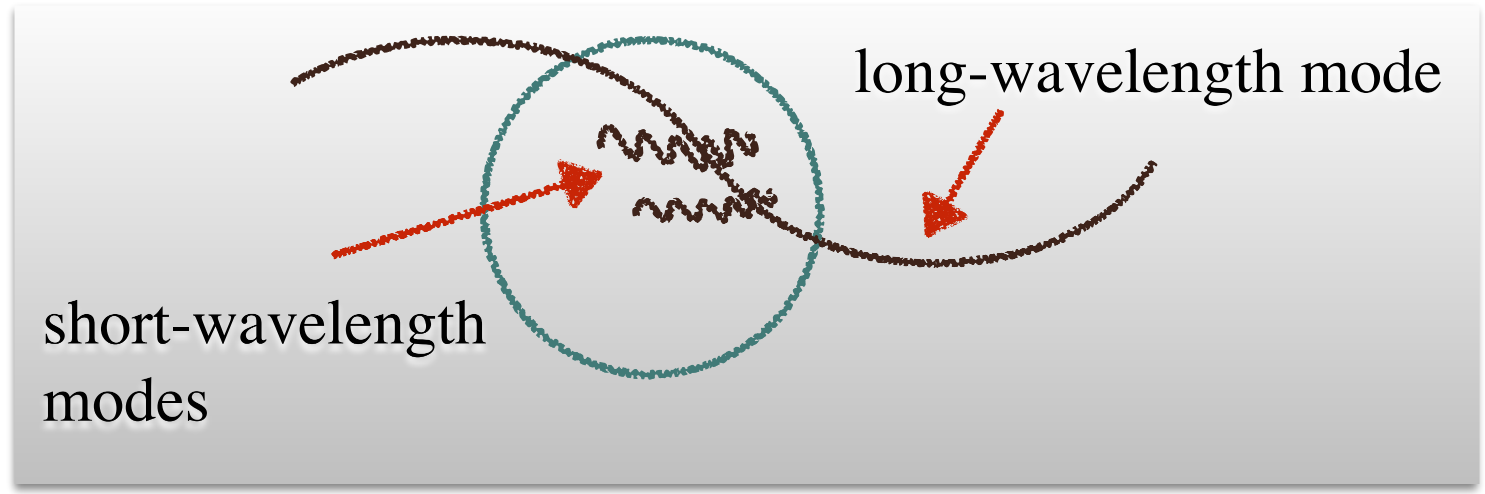
Matteo Fasiello
(IFT Madrid)

Gianmassimo Tasinato
(Swansea)

# Soft limits and ‘fossils’

$$k_1 \simeq k_2 \gg k_3$$




long wavelength modes introduces a modulation
in the primordial power spectrum of the short wavelength modes

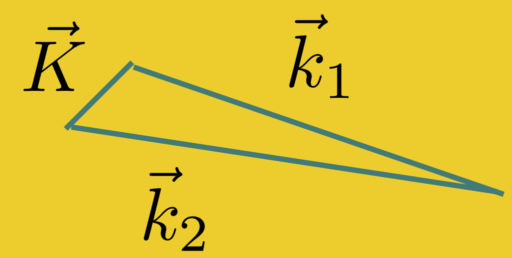
$$B^{F\gamma\gamma} \equiv \langle F_L \gamma_S \gamma_S \rangle' \sim F_L \cdot \langle \gamma_S \gamma_S \rangle'_{F_L}$$

$$\delta \langle \gamma_S \gamma_S \rangle \equiv \langle \gamma_S \gamma_S \rangle_{F_L} \sim \frac{B^{F\gamma\gamma}}{P_F(k_3)} \cdot F_L^* = P_\gamma(k_1) \cdot \overbrace{\boxed{\frac{B^{F\gamma\gamma}}{P_F(k_3) P_\gamma(k_1)}}}^{f_{\mathrm{NL}}^{F\gamma\gamma}} \cdot F_L^*$$

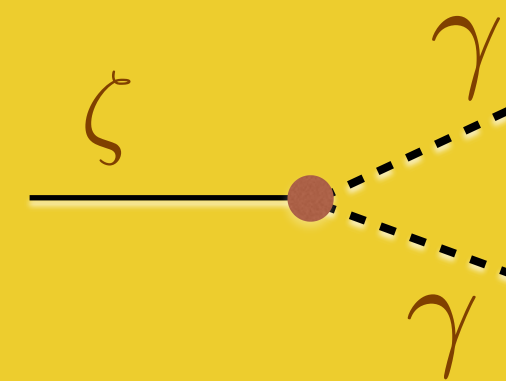
$$\boxed{\langle \gamma_S \gamma_S \rangle'_{\mathrm{total}} = P_\gamma(k_1) \left( 1 + f_{\mathrm{NL}}^{F\gamma\gamma} \cdot F_L^* \right)}$$

[ED, Fasiello, Jeong, Kamionkowski - 2014, ED, Fasiello, Kamionkowski - 2015, …]

# Soft limits and fossils




from




$$\delta_{\rm GW}(k,\hat{n}) = \int \frac{d^3q}{(2\pi)^3} e^{-id\hat{n}\cdot\mathbf{q}} \zeta(\mathbf{q}) F_{\rm NL}^{\rm stt}(\mathbf{k},\mathbf{q})$$

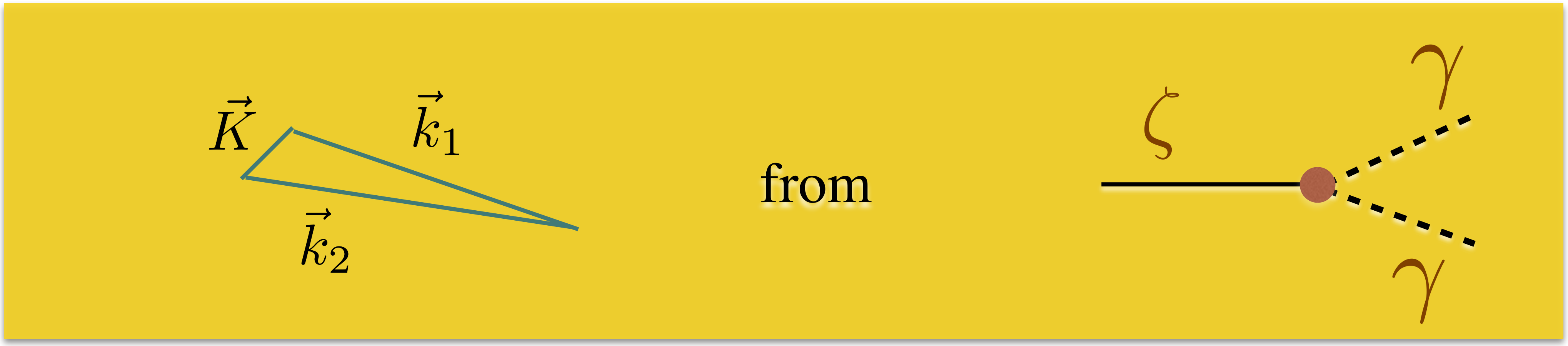
large scale variation in the energy density of GW

$$\mathbf{d} = -(\eta_0 - \eta_{\rm in})\hat{n}$$



[ED, Fasiello, Tasinato, PRL 124(2020)6 061302]

# Soft limits and fossils





**[ED, Fasiello, Tasinato, PRL 124(2020)6 061302]**

for derivation with in-in formalism and applications:

see: **[ED, Fasiello, Pinol, 2022]**

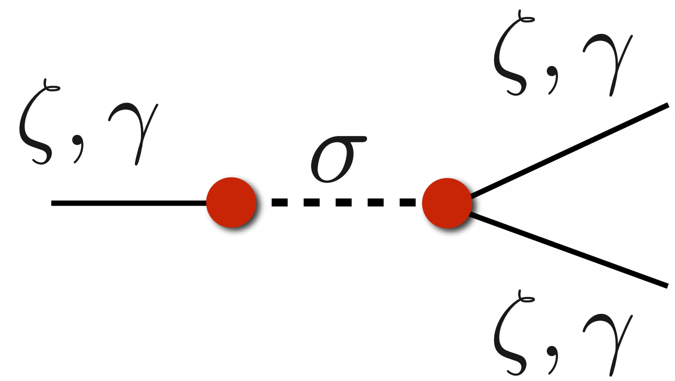
# Soft limits in inflation

- ***Extra*** *fields* / superhorizon ***evolution***

  [Maldacena 2003, Creminelli - Zaldarriaga 2004, Chen - Wang 2009, Baumann - Green 2011, Chen et al 2013, Pajer et al. 2013, ED - Fasiello - Kamionkowski 2015, …]

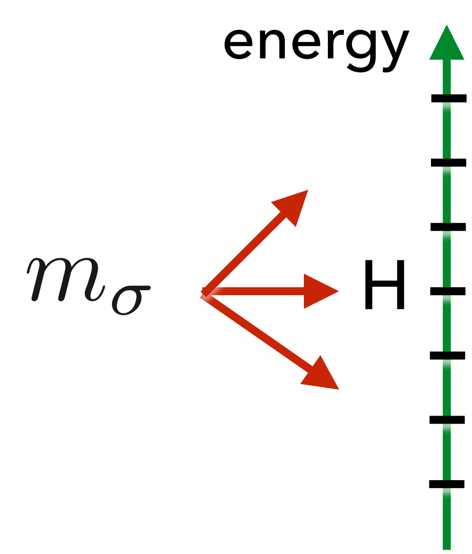
# Soft limits in inflation

- ***Extra*** *fields*




Soft limits reveal (extra) fields mediating inflaton or graviton interactions

squeezed bispectrum delivers
info on mass spectrum



# Soft limits in inflation

- ***Extra*** *fields* / superhorizon ***evolution***

  [Maldacena 2003, Creminelli - Zaldarriaga 2004, Chen - Wang 2009, Baumann - Green 2011, Chen et al 2013, Pajer et al. 2013, ED - Fasiello - Kamionkowski 2015, …]

- ***Non-Bunch Davies*** initial states

  [Holman - Tolley 2007, Ganc - Komatsu 2012, Brahma - Nelson - Shandera 2013, …]

- ***Broken space diffs***
  (e.g. space-dependent background)

  [Endlich et al. 2013, ED - Fasiello - Jeong - Kamionkowski 2014, Celoria - Comelli - Pilo - Rollo 2021…]

Ideal probe for (extra) fields, pre-inflationary dynamics, (non-standard) symmetry patterns

# GW anisotropies from squeezed non-Gaussianity

Based on work with:

Matteo Fasiello
(IFT Madrid)

Ameek Malhotra
(Swansea)

JCAP 03 (2021) 088

JCAP 02 (2022) 02, 040

Maresuke Shiraishi
(Suwa University)

Daan Meerburg
(Groningen)

Giorgio Orlando
(Jagiellonian University)

# GW anisotropies from squeezed non-Gaussianity

- Typical amplitude of these anisotropies:

$$\delta^{\rm tss}_{\rm GW} \sim F^{\rm tss}_{\rm NL}\sqrt{A_{\rm S}}$$

$$\delta^{\rm ttt}_{\rm GW} \sim F^{\rm ttt}_{\rm NL}\sqrt{r\, A_{\rm S}}$$

scalar power spectrum amplitude at CMB scales

tensor-to-scalar ratio

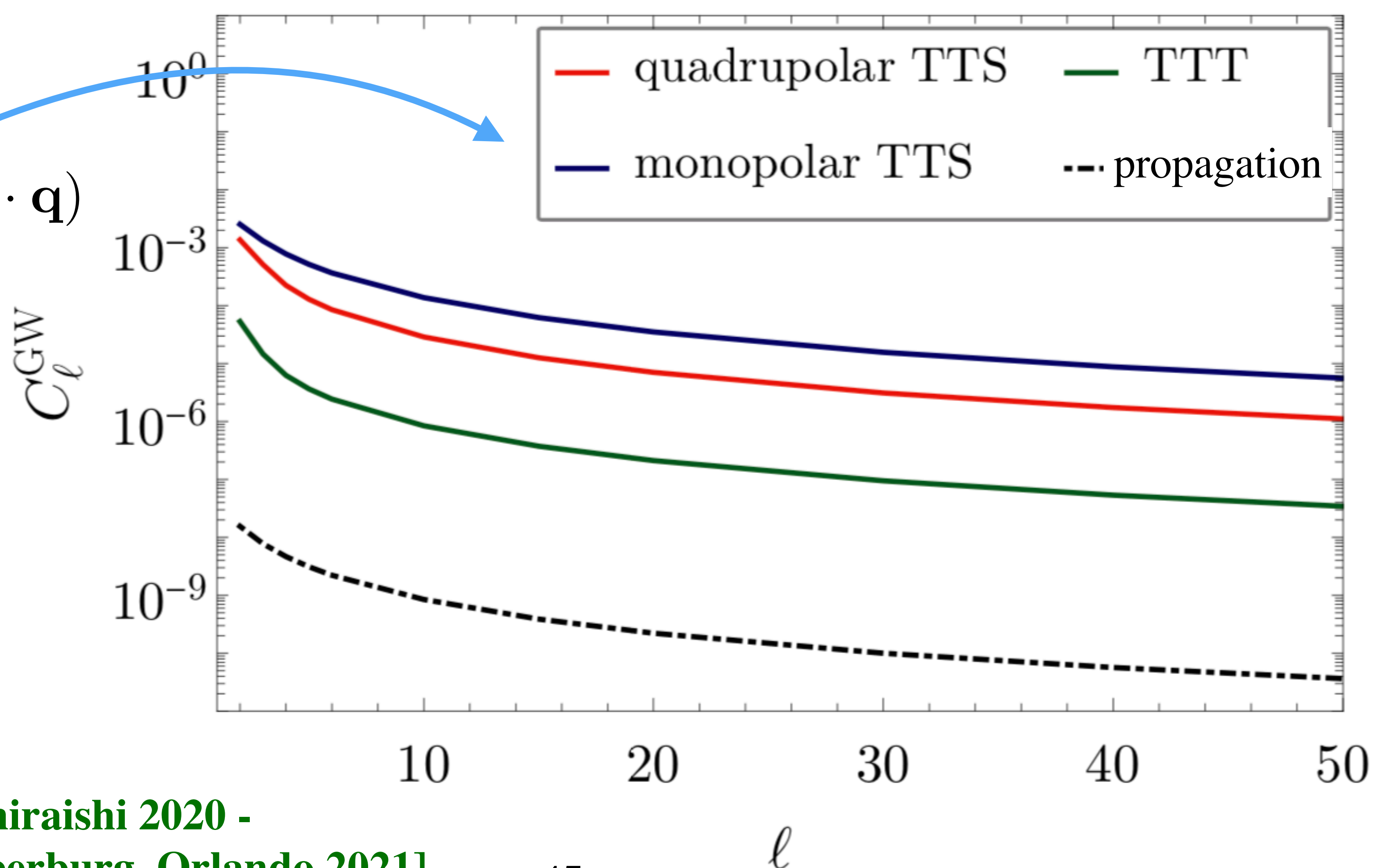
angular dependence:

$$F_{\rm NL}(\mathbf{k},\mathbf{q}) = \tilde{F}_{\rm NL}\mathcal{P}_\ell(\mathbf{k}\cdot\mathbf{q})$$

$$\begin{cases} \ell = 0 & \text{monopole} \\ \ell = 2 & \text{quadrupole} \end{cases}$$

(model dependent)

* scale-invariant case



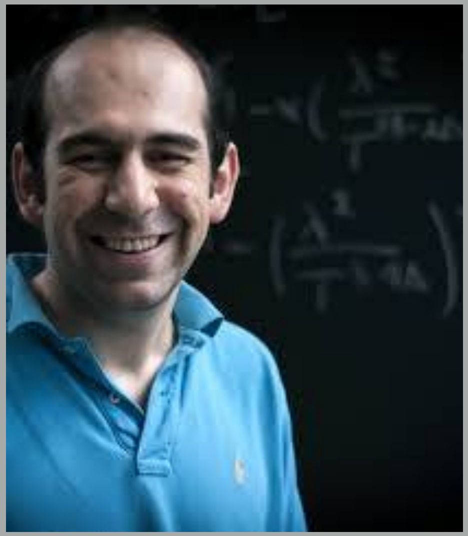

[Malhotra, ED, Fasiello, Shiraishi 2020 -
ED, Fasiello, Malhotra, Meerburg, Orlando 2021]

# Cross-correlations of GW and CMB anisotropies

Based on: Phys.Rev.D 103 (2021) 2, 023532 with

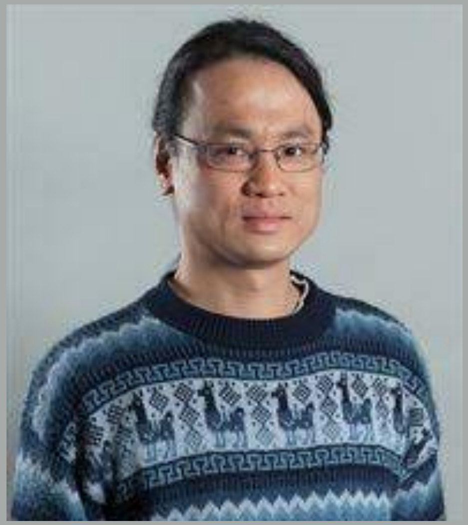
Peter Adshead
(UIUC)

Niayesh Afshordi
(Waterloo/PI)



Matteo Fasiello
(IFT Madrid)

Eugene Lim
(KCL)



Gianmassimo Tasinato
(Swansea)

# Cross-correlations of GW and CMB anisotropies

$$\left.\begin{aligned}\delta_{\rm GW}^{\rm stt} &\sim F_{\rm NL}^{\rm stt}\cdot\zeta_L \\ \frac{\Delta T}{T} &\sim \zeta_L\end{aligned}\right\} \quad C_\ell^{\rm GW\text{-}T} \sim F_{\rm NL}^{\rm stt}\cdot C_\ell^{TT}$$

**[Adshead, Afshordi, ED, Fasiello, Lim, Tasinato 2020]**

For forecasts (combining auto- and cross-correlations) and applications to specific models see:

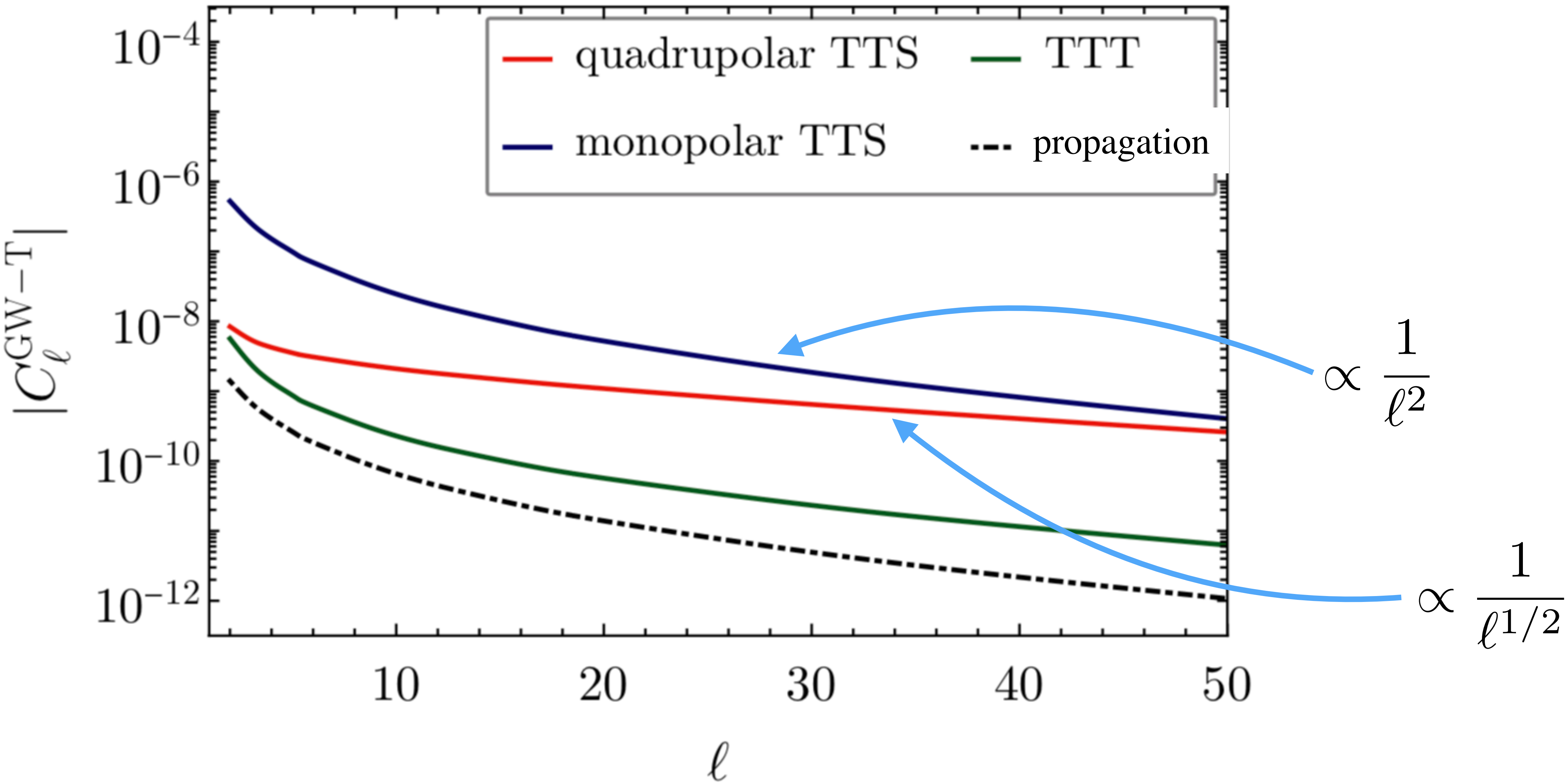
**[Malhotra, ED, Fasiello, Shiraishi 2020]**
**[ED, Fasiello, Malhotra, Meerburg, Orlando 2021]**

On large scales, anisotropies in the astrophysical GW background do not correlate strongly with CMB (cross-correlations with LSS observables much more effective) **[Ricciardone et al, 2021]**

$\downarrow$

There is great potential in GW-CMB correlations as a probe of cosmological GW background

# Cross-correlations of GW and CMB anisotropies

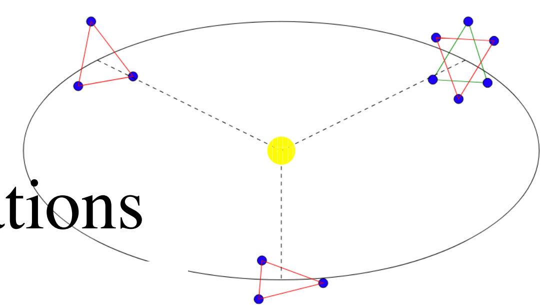



[ED, Fasiello, Malhotra, Meerburg, Orlando 2021]

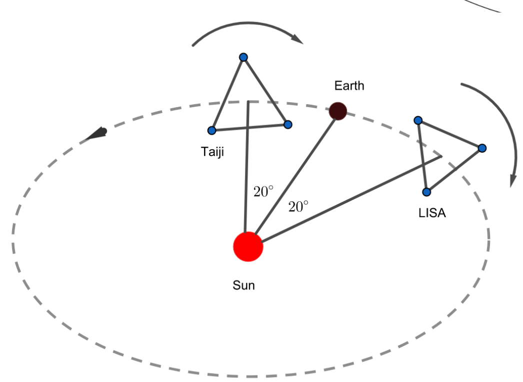
# Projected constraints on $F_{\rm NL}^{\rm tss}$

$$F_{ij} = \sum_{XY} \sum_{\ell=\ell_{\rm min}}^{\ell_{\rm max}} \frac{\partial C_\ell^X}{\partial \theta_i} \left(\mathcal{C}_\ell^{XY}\right)^{-1} \frac{\partial C_\ell^Y}{\partial \theta_j} \qquad X, Y = \{\text{TT,GW,GW-T}\}$$

$$\mathscr{C}_\ell = \frac{2}{2\ell+1} \begin{bmatrix} (C_\ell^{\rm TT})^2 & (C_\ell^{\rm GW-T})^2 & C_\ell^{\rm TT} C_\ell^{\rm GW-T} \\ (C_\ell^{\rm GW-T})^2 & (C_\ell^{\rm GW})^2 & C_\ell^{\rm GW} C_\ell^{\rm GW-T} \\ C_\ell^{\rm TT} C_\ell^{\rm GW-T} & C_\ell^{\rm GW} C_\ell^{\rm GW-T} & \frac{1}{2}(C_\ell^{\rm GW-T})^2 + \frac{1}{2} C_\ell^{\rm TT} C_\ell^{\rm GW} \end{bmatrix}$$



- BBO: 4 LISA-like constellations




- LISA+Taiji
- ET + CE
- SKA (assumed 50 identical pulsars)

$$C_\ell^{\rm TT} \simeq \frac{2\pi A_S}{25\ell(\ell+1)},$$

$$C_\ell^{\rm GW} = C_\ell^{\rm GW,tts} + C_\ell^{\rm GW,ind} + N_\ell^{\rm GW}$$

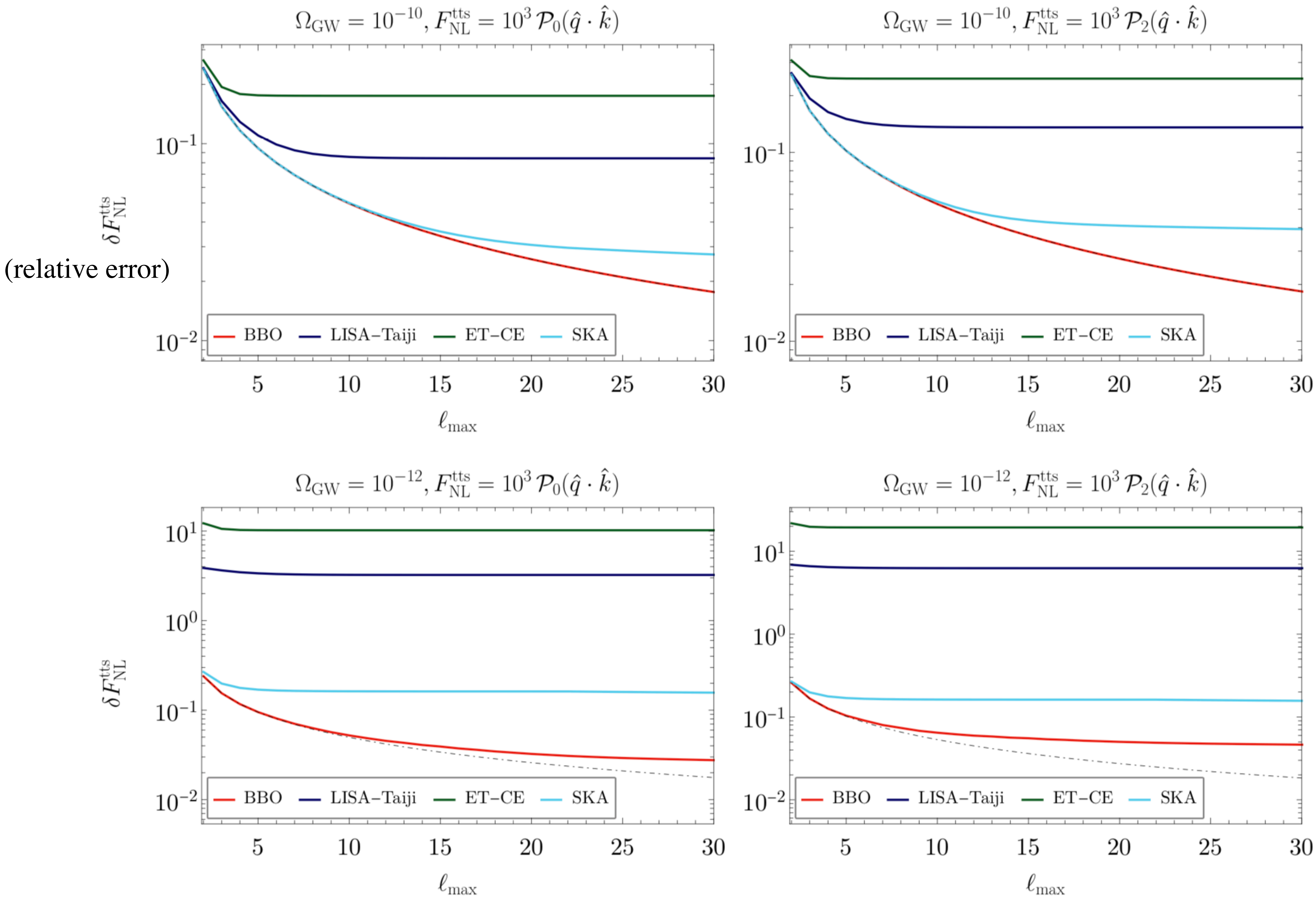
$$C_\ell^{\rm GW-T} = C_\ell^{\rm GW-T,tts} + C_\ell^{\rm GW-T,ind}$$

Noise angular power spectra computations based on

**[Alonso et al. 2020]**

**[Malhotra, ED, Fasiello, Shiraishi 2020]**
**[ED, Fasiello, Malhotra, Meerburg, Orlando 2021]**

# Projected constraints on $F_{\mathrm{NL}}^{\mathrm{tss}}$




 **[ED, Fasiello, Malhotra, Meerburg, Orlando 2021]**

# What have we learned so far?

- Tensor non-Gaussianity not directly observable with interferometers due to propagation of GW through the inhomogeneous universe

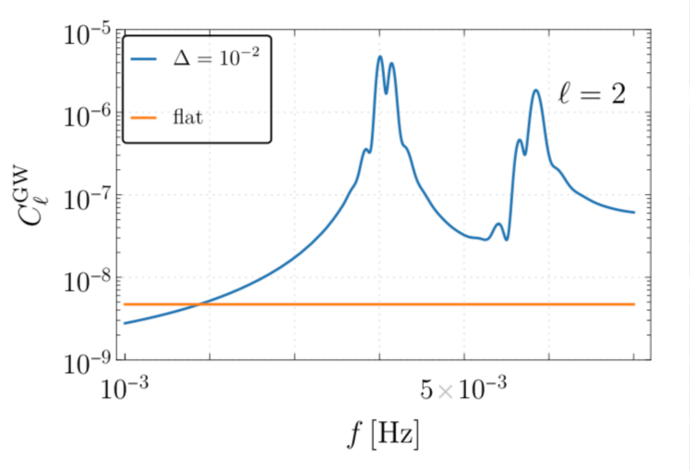
- (Tensor and mixed) primordial non-Gaussianity of the squeezed type induces anisotropies in the GW background

- Inflationary models with detectable GW background and significant squeezed non-Gaussianity can be tested at interferometer scales thanks to these anisotropies
  $\longrightarrow$ GW anisotropies as a probe of inflationary fields and interactions

- Anisotropies as a probe of **squeezed non-Gaussianity**

- Anisotropies as a probe of **SIGW** and **primordial black holes** physics

~~**Small spectral tilt for tensors**~~




- Anisotropies as a probe of **isocurvature perturbations**

# Anisotropies for peaked spectra

Based on: JCAP 01 (2023) 018 with

Matteo Fasiello
(IFT Madrid)

Ameek Malhotra
(Swansea)

Gianmassimo Tasinato
(Swansea)

# Anisotropies for peaked spectra

Anisotropies from propagation:
amplitude proportional to the slope of the spectrum

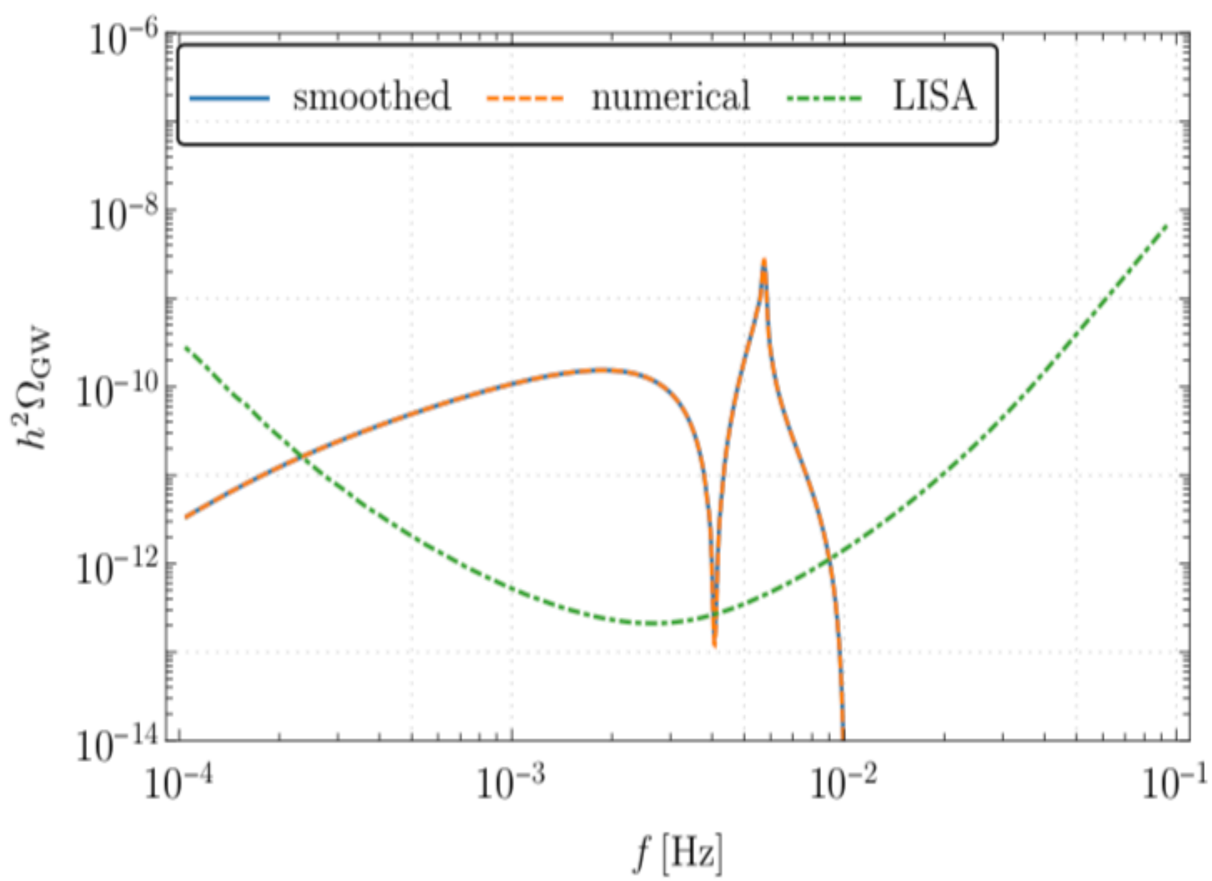
$$\boxed{\delta_{\rm GW} \sim \left( \frac{\partial \ln \Omega_{\rm GW}}{\partial \ln k} \right) \zeta_{\rm L}}$$

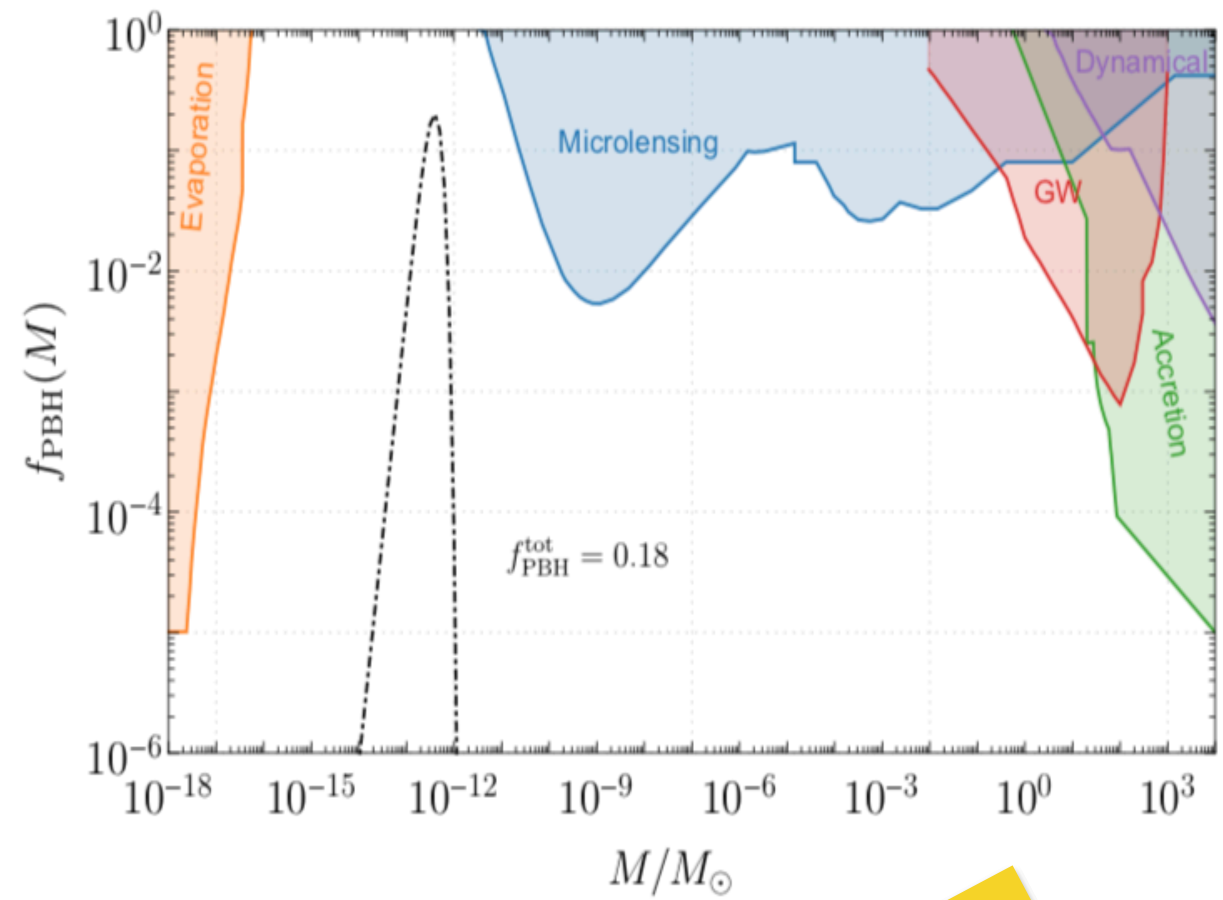
pronounced slope $\longrightarrow$ enhanced anisotropies

# Anisotropies for peaked spectra

Models with sharp peaks in the scalar power spectrum (e.g. PBH production)

- a large GW background with sharp peaks induced at second order from scalar perturbations **[Ananda - Clarkson - Wands 2006, Baumann et al. 2007]**



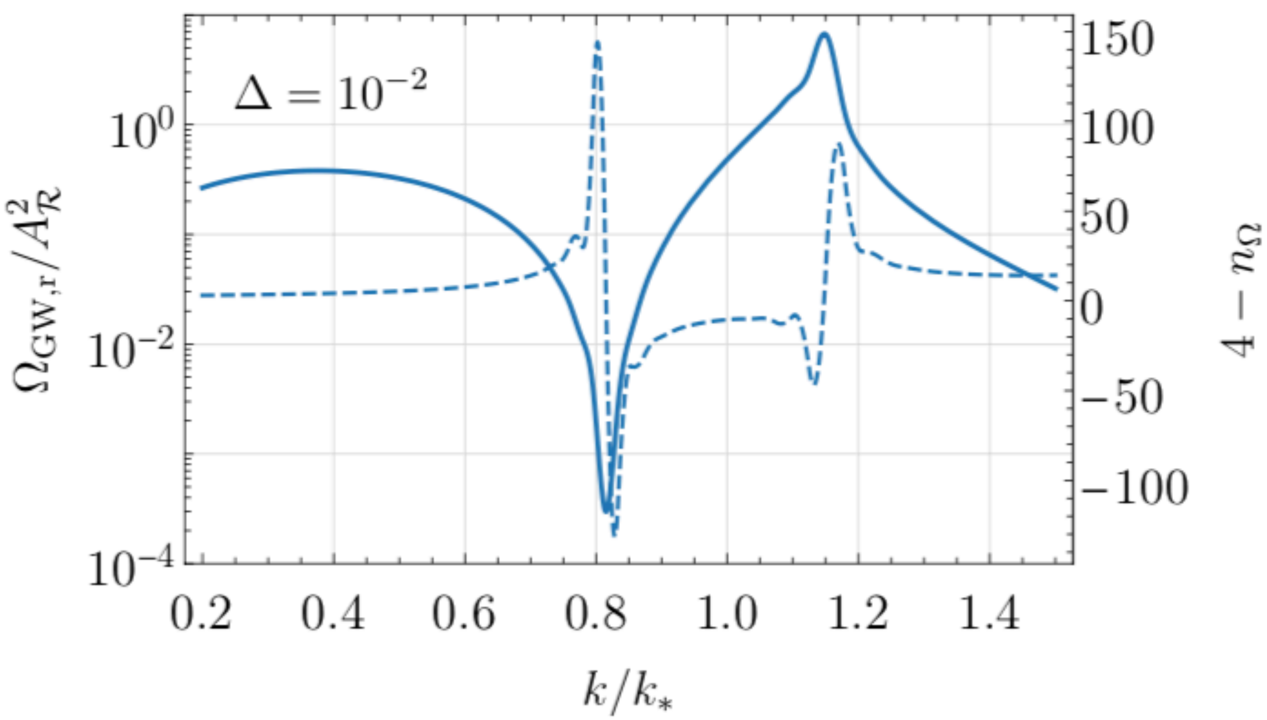

$$\Omega_{\rm GW,r}(k) = 3\int_0^\infty dv \int_{|1-v|}^{1+v} du \frac{\mathcal{T}(u,v)}{u^2v^2}\mathcal{P}_\mathcal{R}(vk)\mathcal{P}_\mathcal{R}(uk)\,,$$

$$\mathcal{P}_\mathcal{R}(k)|_{k\gg k_{\rm CMB}} = \frac{A_\mathcal{R}}{\sqrt{2\pi}\Delta}\exp\left[-\frac{\ln^2(k/k_*)}{2\Delta^2}\right]$$

| | |
|---|---|
| $\Delta$ | $10^{-2}$ |
| $f_*$ | $5\times10^{-3}$ Hz |
| $k_*$ | $3\times10^{12}$ Mpc$^{-1}$ |
| $A_\mathcal{R}$ | $7.5\times10^{-3}$ |

# Anisotropies for peaked spectra

Models with sharp peaks in the scalar power spectrum (e.g. PBH production)

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| | |
|---|---|
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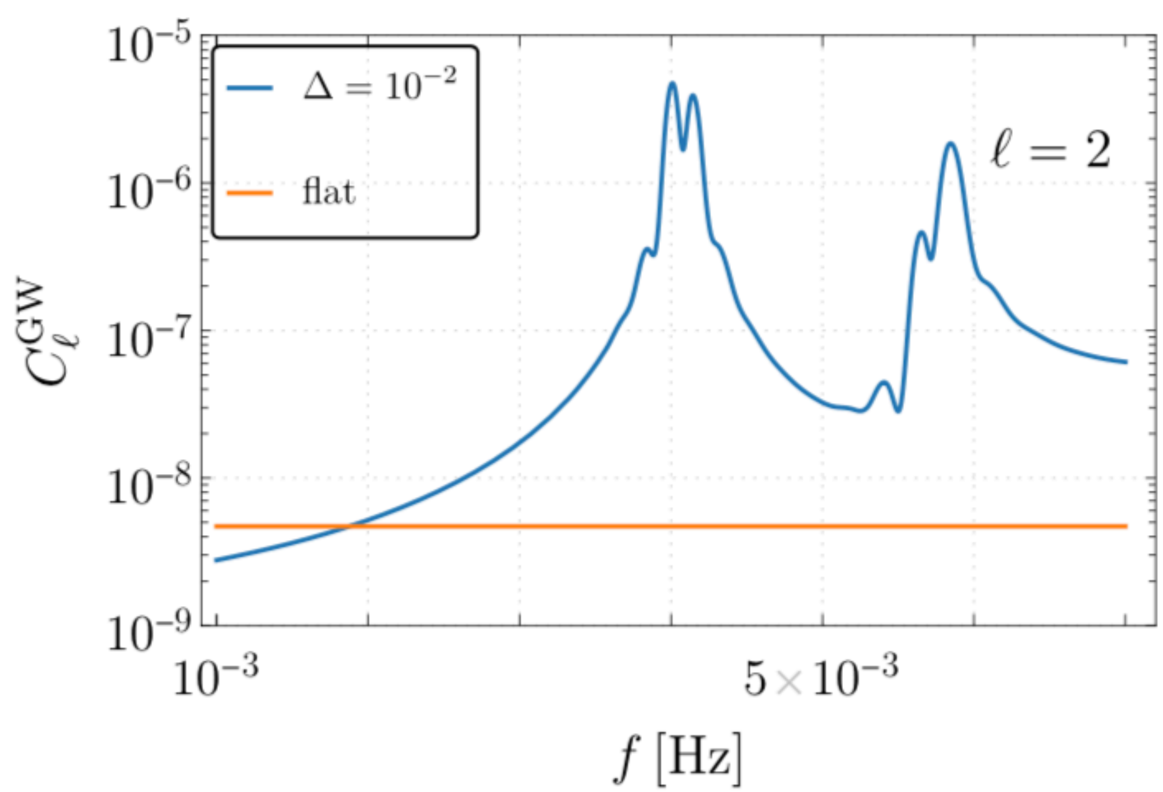
| $k_*$ | $3\times10^{12}\,$Mpc$^{-1}$ |
| $A_{\mathcal{R}}$ | $7.5\times10^{-3}$ |

- the anisotropies can be typically enhanced by O(10-100)

$$\delta_{\rm GW} \sim \left(\frac{\partial \ln \Omega_{\rm GW}}{\partial \ln k}\right)\zeta_{\rm L}$$

- the angular power spectrum of the GW anisotropies inherits the frequency dependence

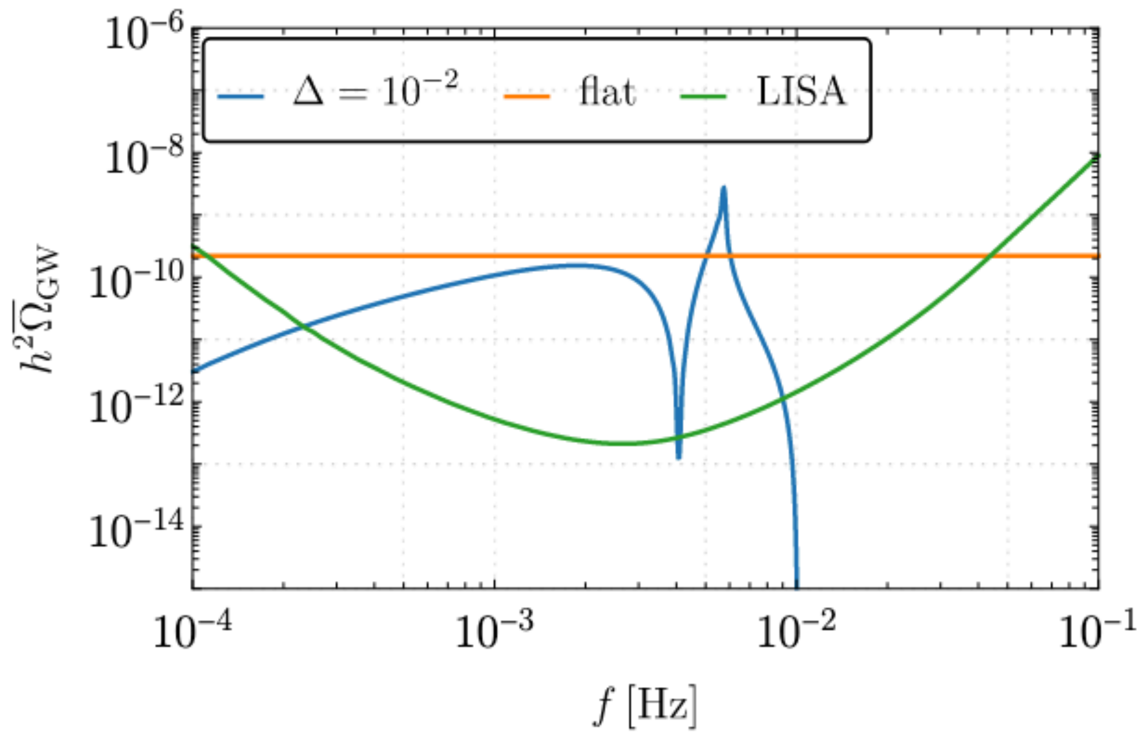




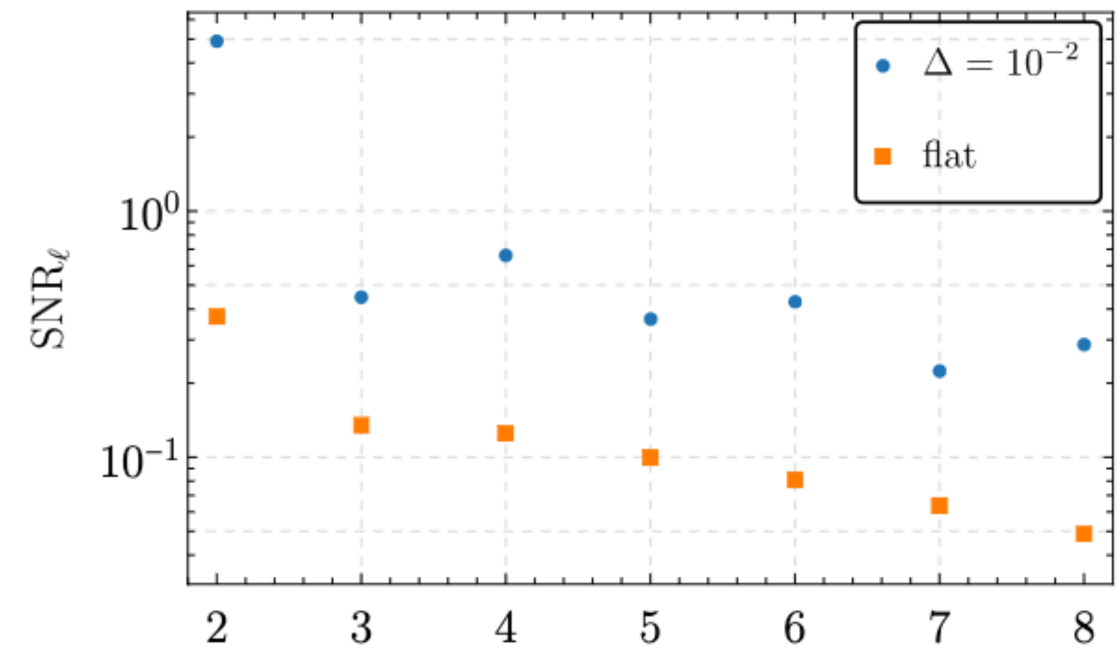

**[ED, Fasiello, Malhotra, Tasinato 2022]**

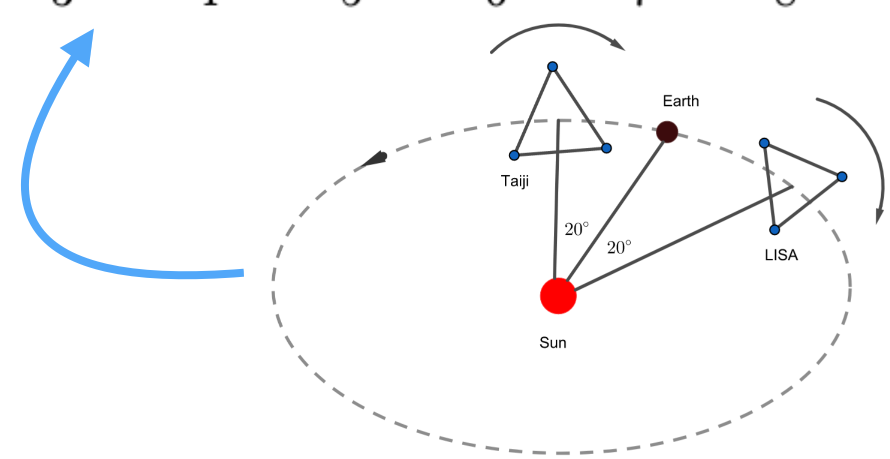
# Anisotropies for peaked spectra

Considering a flat spectrum with the same SNR for monopole as peaked spectrum:




Signal to noise ratio for the individual multipoles:



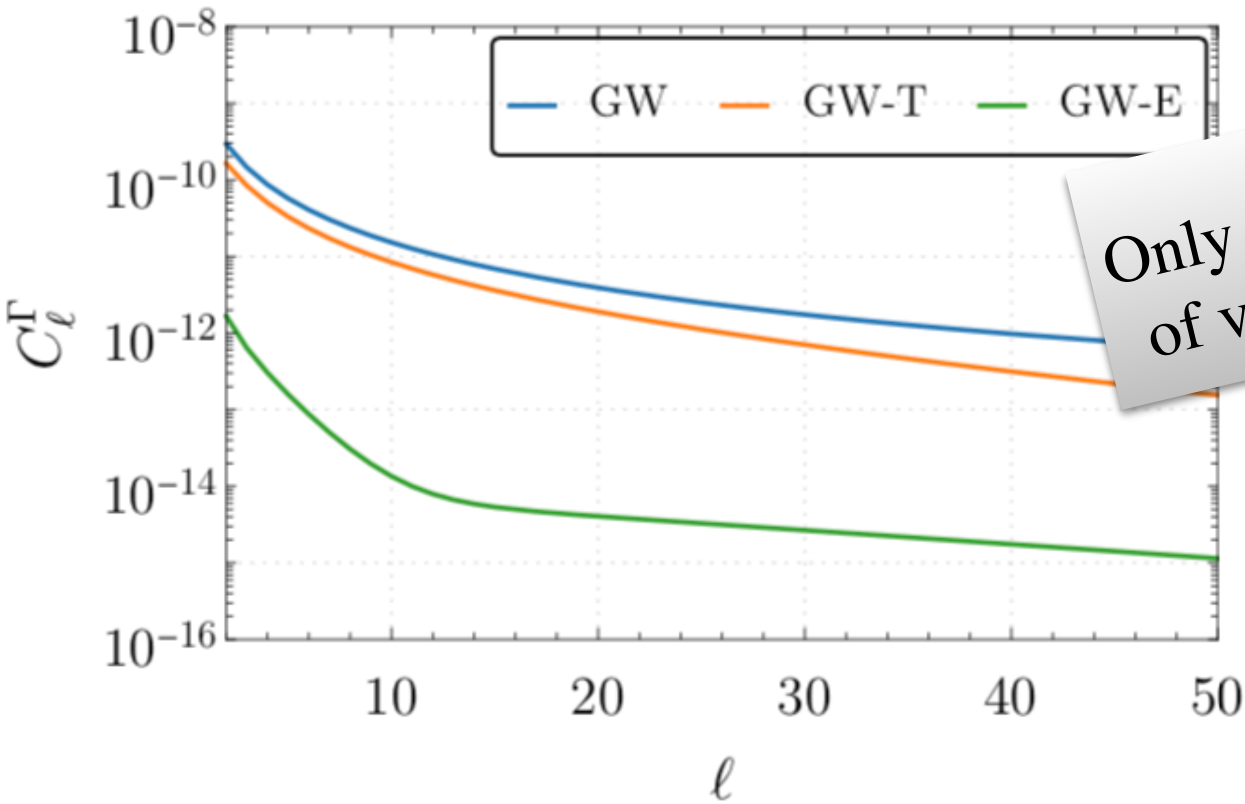

anisotropies are easier to detect compared to those of a flat spectrum!

**[ED, Fasiello, Malhotra, Tasinato 2022]**

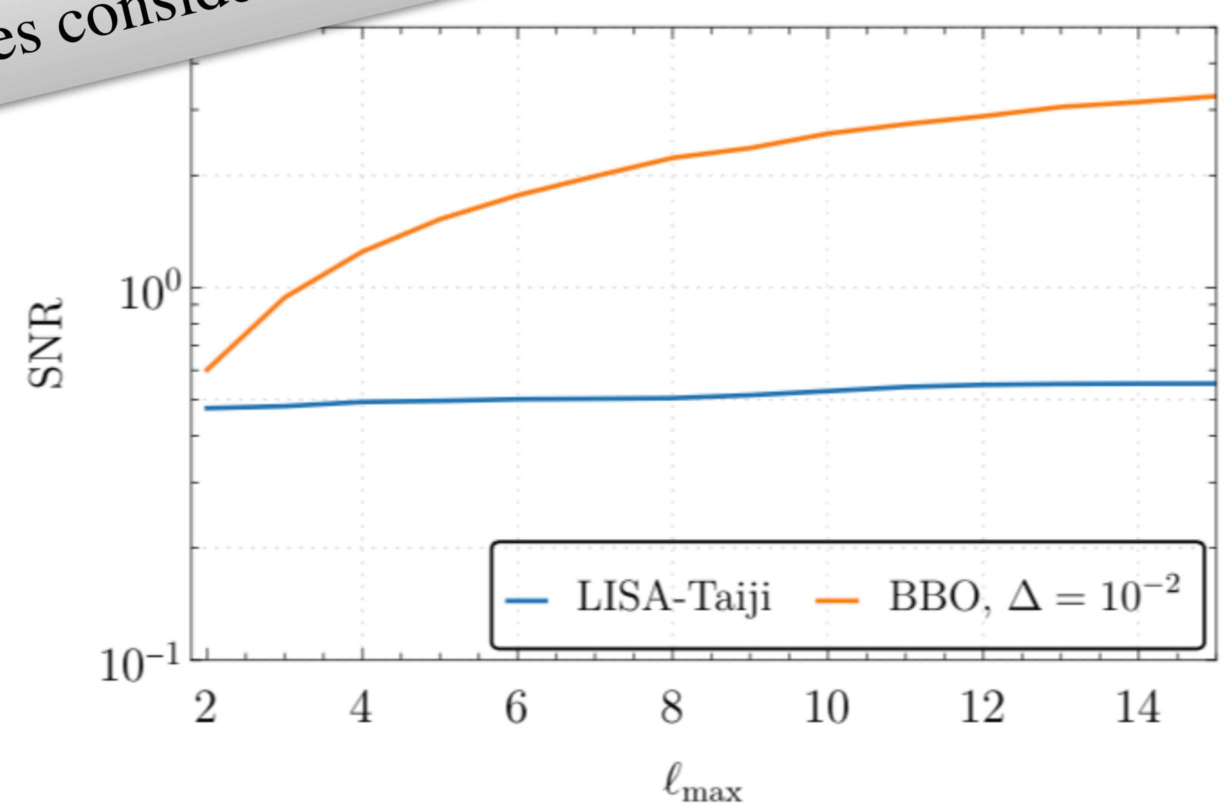
# Cross-correlations

$$\mathrm{SNR}^2 = \sum_{\ell=2}^{\ell_{\max}} \sum_{\mathrm{X=T,E}} (2\ell+1) \frac{\left(C^{\mathrm{X}\Gamma}\right)^2}{\left(C^{\mathrm{X}\Gamma}\right)^2 + \left(C_\ell^{\Gamma} + N_\ell^{\Gamma}\right) C_\ell^{\mathrm{X}}}$$

(assuming full sky maps for both CMB and GW anisotropies)




Only valid for the specific set of values considered earlier




**[ED, Fasiello, Malhotra, Tasinato 2022]**

# What have we learned so far?

- For sharply peaked GW spectra (e.g. common to PBH formation scenarios), GW background anisotropies are enhanced w.r.t. power-law GW spectra

- In spite of enhancement being limited to a small range of scales, anisotropies easier to detect relative to those of power-law spectra.

- For a representative case, a LISA-Taiji network would be able to detect quadrupole, BBO would detect GW-CMB cross-correlations

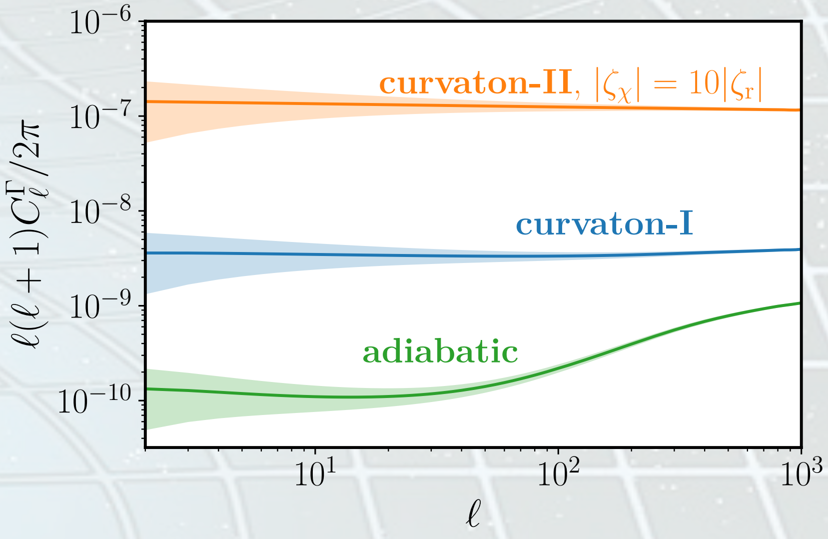
- The distinct frequency dependence of these GW anisotropies potentially useful to separate these anisotropies from those associated with other (cosmological or astrophysical) GW backgrounds

- Anisotropies as a probe of **squeezed non-Gaussianity**

- Anisotropies as a probe of **SIGW** and **primordial black holes** physics

- Anisotropies as a probe of **isocurvature perturbations**

~~**Adiabatic primordial perturbations**~~



# Anisotropies in the presence of isocurvature perturbations

Based on: Phys.Rev.D 107 (2023) 10, 103502 with

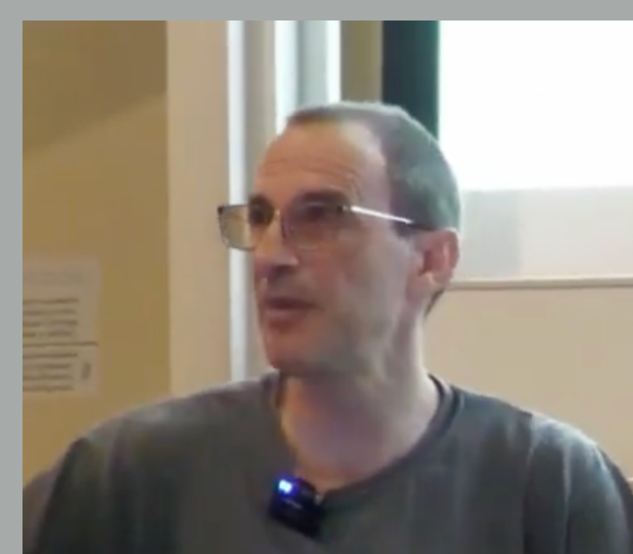

Guillem Domenech
(Hannover)

Ameek Malhotra
(Swansea)



Matteo Fasiello
(IFT Madrid)

Gianmassimo Tasinato
(Swansea)

# Our initial question:

The observed GW background is in general determined both by:
(1) the specific source
(2) the evolution of the universe after production

In particular, an early non-standard cosmic history (e.g. early matter domination) could affect the frequency profile of $\Omega_{\mathrm{GW}}$ in ways that are fully degenerate w.r.t. the imprints of the GW source.

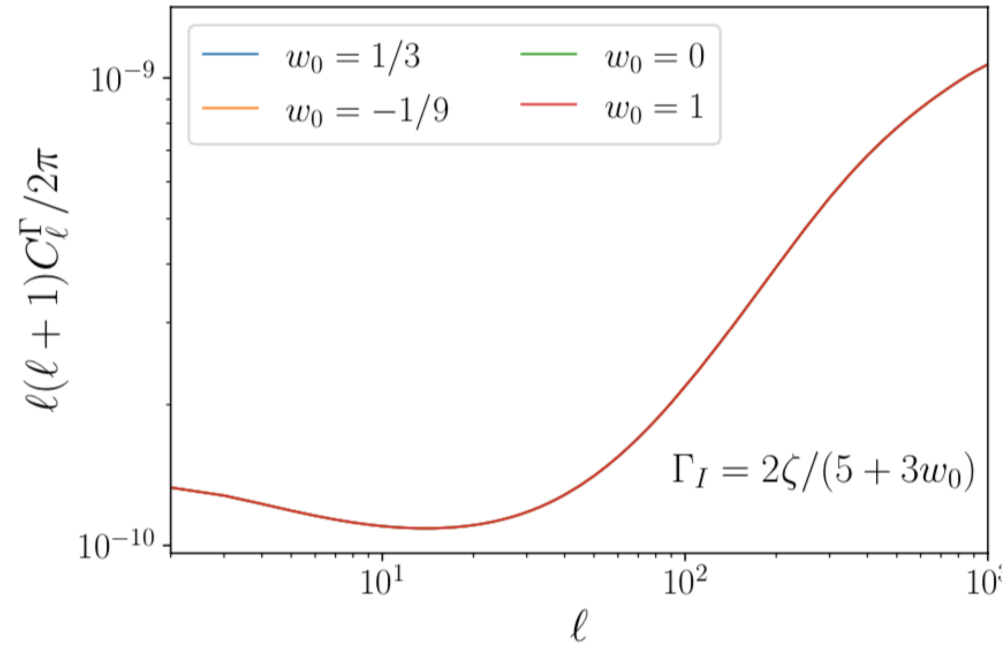
Can GW background anisotropies break this degeneracy?

<u>How do propagation anisotropies respond to a modified cosmic evolution?</u>

# Our findings:

<u>universality of behaviour of anisotropies for adiabatic modes</u>

Anisotropies unaffected by an early non-standard phase of cosmic evolution so long as the primordial fluctuations are adiabatic




unsurprinsing: conservation of superhorizon curvature perturbation

$\downarrow$ in turn:

A <u>departure from adiabaticity</u> would break universality condition

<u>Anisotropies as a probe of isocurvature modes</u>

# Anisotropies in the presence of isocurvature perturbations

Discussed earlier: for <u>adiabatic</u> primordial perturbations: $\boxed{\Gamma_I = -\frac{1}{2}\Phi = \frac{\zeta}{3}}$

If there is an isocurvature perturbation of GW w.r.t. another component (e.g. radiation):

$$S_{\mathrm{GW},r} = 3\left(\zeta_{\mathrm{GW}} - \zeta_r\right) = \frac{3}{4}\left(\delta_{\mathrm{GW}} - \delta_{\mathrm{r}}\right)$$

$$\zeta = \sum_i f_i \zeta_i$$

$$f_i \equiv \frac{\rho_i}{\rho_{\mathrm{tot}}}$$ (fractional density for the "i" component)

$$\zeta_{\mathrm{GW}} = -\Psi - \mathcal{H}\frac{\delta\rho_{\mathrm{GW}}}{\rho'_{\mathrm{GW}}}$$

(curvature perturbation of GW)

$$\Gamma_I = \frac{1}{4}\frac{\delta\rho_{\mathrm{GW}}}{\bar{\rho}_{\mathrm{GW}}}$$ (Initial condition for GW anisotropies)

$$\longrightarrow \boxed{\Gamma_I = \frac{1}{3}\left(1 - f_{\mathrm{GW}}\right) S_{\mathrm{GW},r} + \frac{\zeta}{3}}$$



[Kumar, Sundrum, Tsai 2021]

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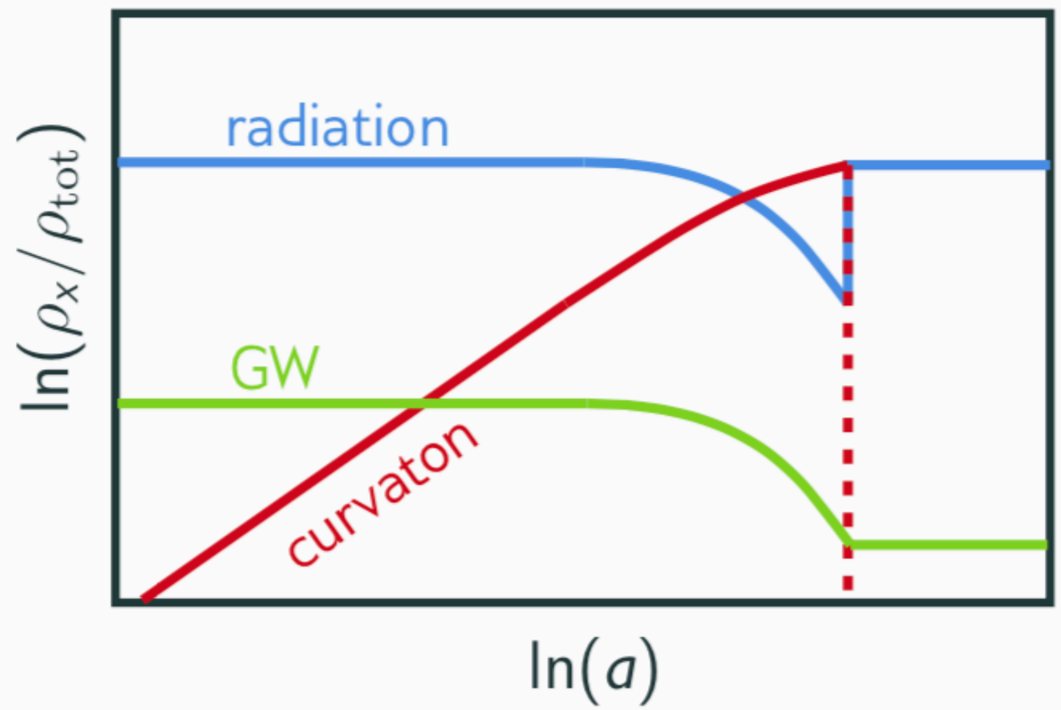
## Case I:

- Curvaton dominates $\rho_{\rm tot}$ then decays *entirely* into radiation
- Fluctuation amplitude *fixed* by CMB normalisation




$$C_\ell \propto \Big[\underbrace{-\frac{4}{3}\zeta_r j_\ell(k\eta_0)}_{4\times\ \text{adiabatic term}}\Big]^2 + \text{standard ISW}$$

* Expect the GW background anisotropy map to be fully correlated with CMB

[Ameek Malhotra, ED, Guillem Domènech, Matteo Fasiello, Gianmassimo Tasinato 2023]

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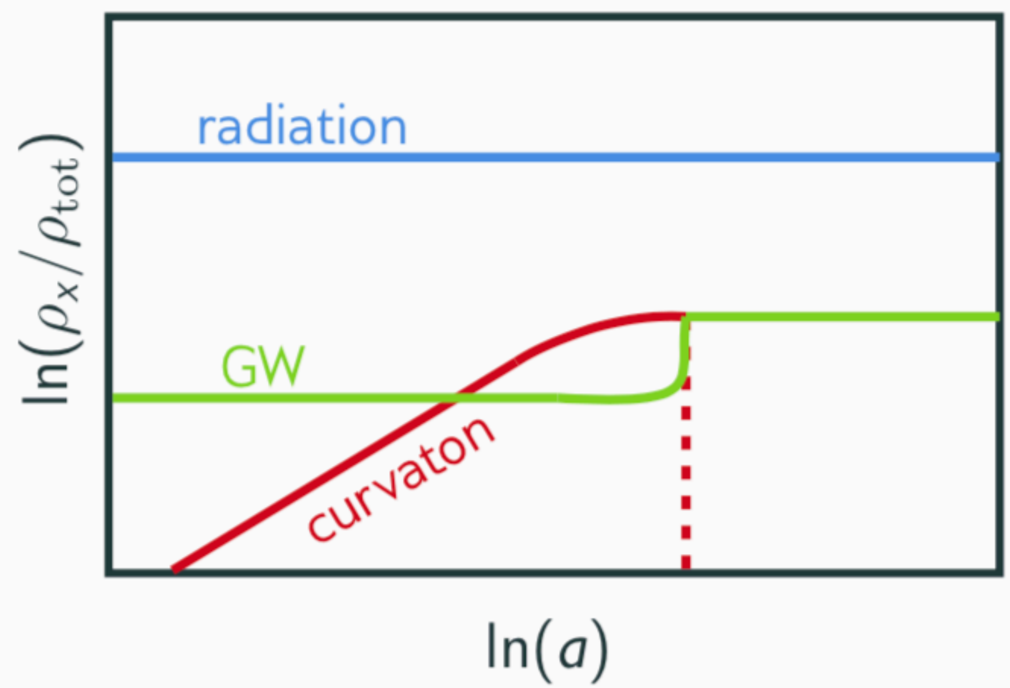
## Case II:

- Curvaton remains subdominant and decays *entirely* into GW
- Fluctuation amplitude *not fixed*




- $f_\chi^b \zeta_{\chi,\rm ini} \ll \zeta_r$ , $\zeta_{\chi,\rm ini} \gg \zeta_r$

$$C_\ell \propto \left\{ \left[ \frac{(1+w_\chi)}{(1+w_r)} \zeta_{\chi,\rm ini} - \frac{1}{3}\zeta_r \right] j_\ell(k\eta_0) \right\}^2 + \text{standard ISW}$$

* Expect little correlations between GW background and CMB anisotropies

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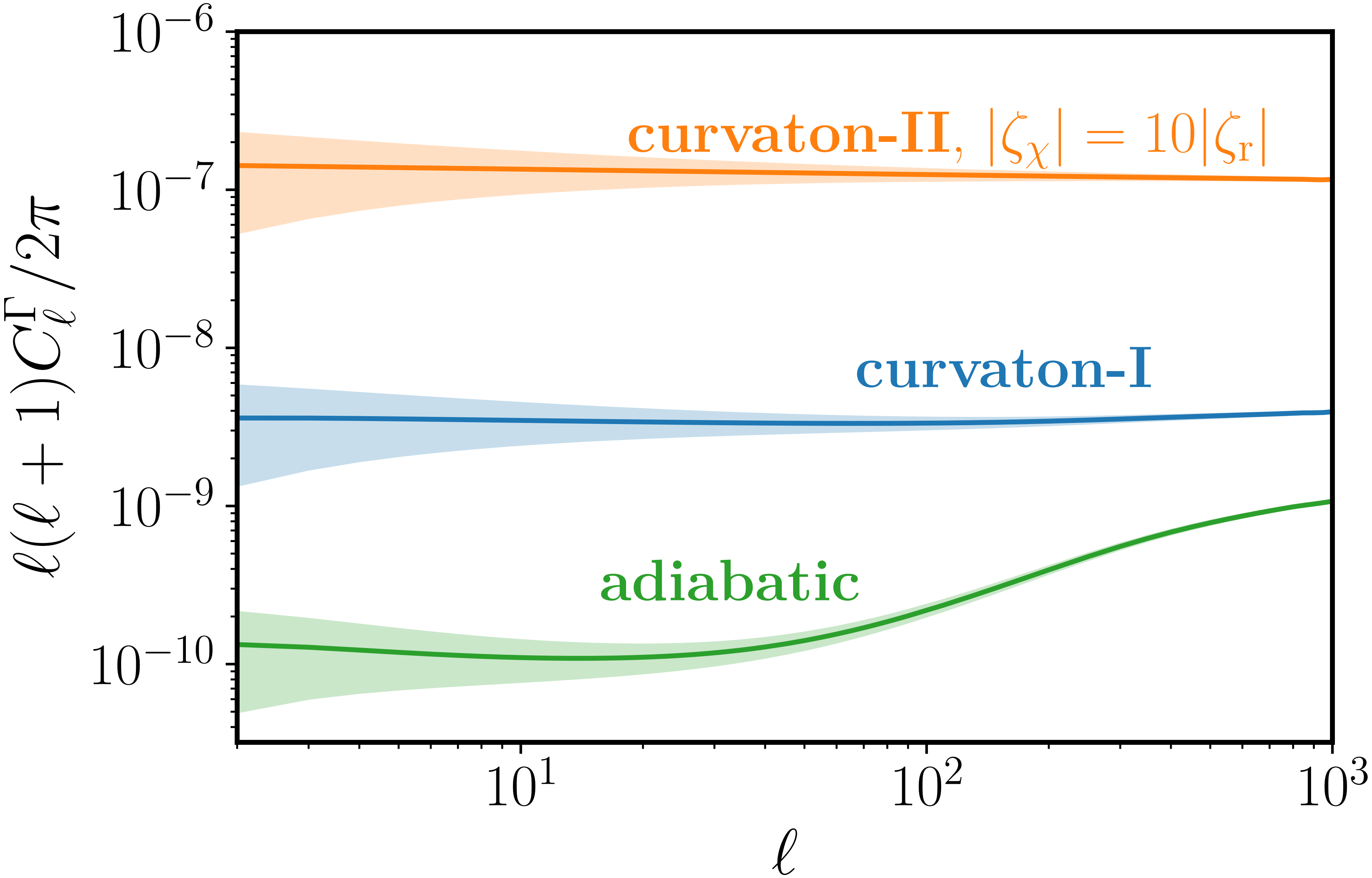


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# What have we learned so far?

- For adiabatic primordial fluctuations, GW anisotropies are insensitive to the EoS of the early universe $\longrightarrow$ universal behaviour
- Deviations from this universal behaviour is an indication of the presence of isocurvature fluctuations
- Isocurvature fluctuation can lead to a sizeable enhancement of the GW anisotropies w.r.t. the adiabatic case
- GW-CMB cross-correlations can also be an effective probe for curvaton models

# Final Thoughts

- Primordial gravitational waves offer a unique probe of the physics of inflation.
- Different production mechanisms during inflation generate distinct GW signatures.
- Key observables (*spectral shape*, *chirality*, *non-Gaussianity*, *anisotropies*) can be used as diagnostic tools to identify these sources.
- These features can help separate inflationary GWs from backgrounds produced after inflation.
- A detection would open a new observational window onto the earliest moments of the universe.