

Waving in the Dark: Warm, Noisy and Spinning Fields

Mustafa A. Amin



what is wave dark matter ?

how is it produced?

what does it do?



what *really* is wave dark matter?

what is wave dark matter ?

DM mass

$$m \ll \text{eV}$$

$$\hbar = c = 1$$

* see for example Hui (2101.11735)

what is wave dark matter ?

$$\hbar = c = 1$$

DM mass

$$m \ll \text{eV}$$

scales

$$\lambda_{\text{dB}} \sim \frac{1}{mv}$$

$$\tau_{\text{dB}} \sim \frac{1}{mv^2}$$

$$\mathcal{N}_{\text{dB}} \sim \bar{n} \lambda_{\text{dB}}^3$$

* see for example Hui (2101.11735)

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$$\mathcal{N}_{\text{dB}} \sim \bar{n} \lambda_{\text{dB}}^3$$

$$m = 10^{-19} \text{ eV}$$

$$v = 10^{-3}$$

$$\rho = 0.3 \text{ GeV cm}^{-3}$$

$$0.1 \text{ pc}$$

$$100 \text{ yrs}$$

$$10^{79}$$

what is wave dark matter

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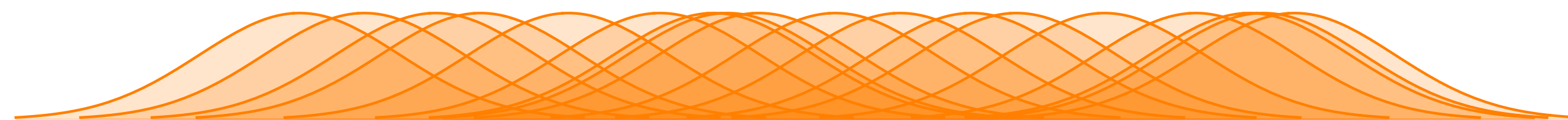
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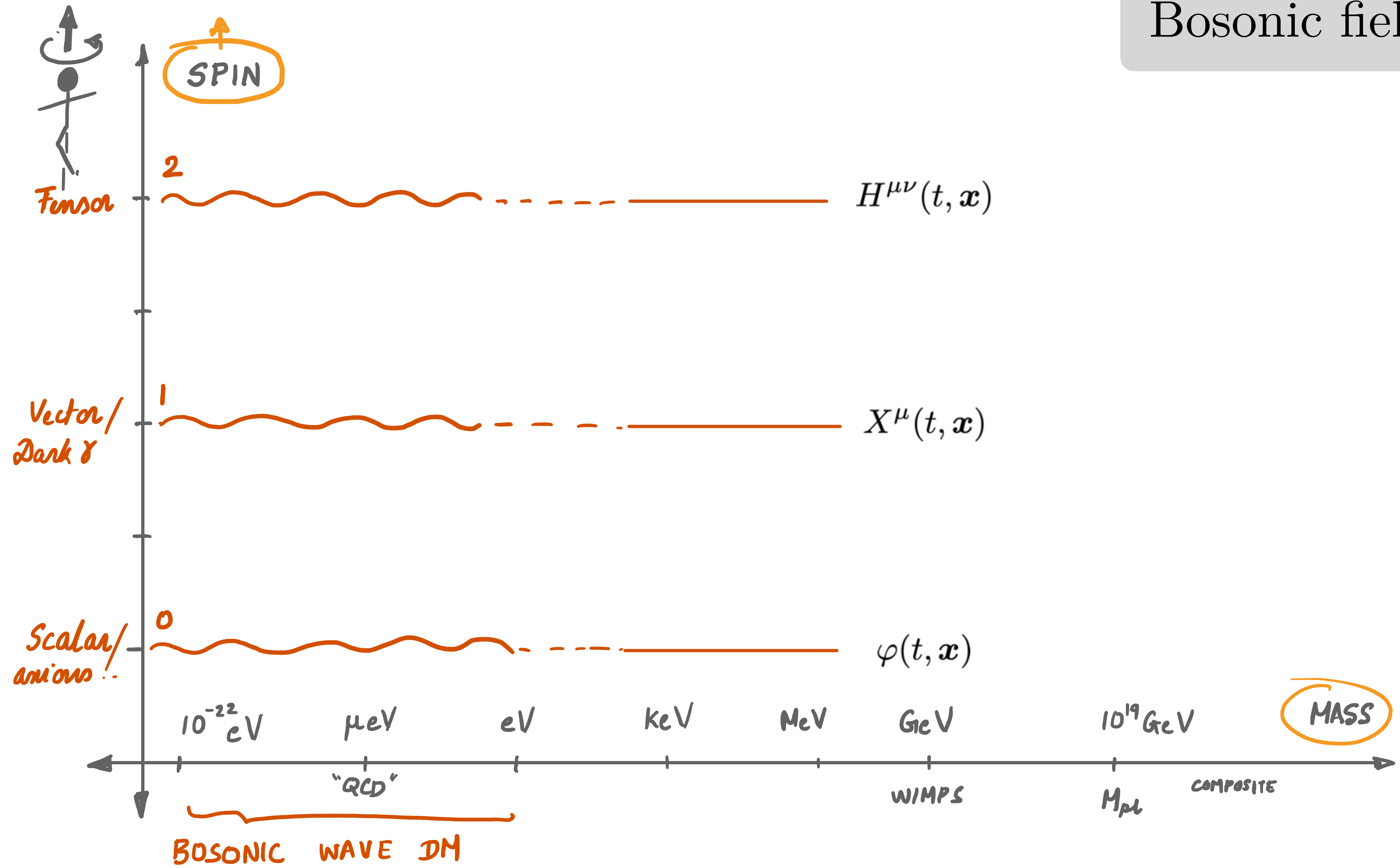
Bosonic “field”



$$\bar{n}^{-\frac{1}{3}} \ll \lambda_{\text{dB}}$$

* caution large occupation number not always sufficient to get a classical single-field description

Bosonic field



scalar = spin-0 wave dark matter

* assuming classical fields

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2} \nabla^\mu \varphi \nabla_\mu \varphi - \frac{1}{2} m^2 \varphi^2 + \frac{1}{8\pi G} R + \dots \right] + \text{non-grav, interactions}$$

$$\nabla^\mu \nabla_\mu \varphi + m^2 \varphi = 0 \quad G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

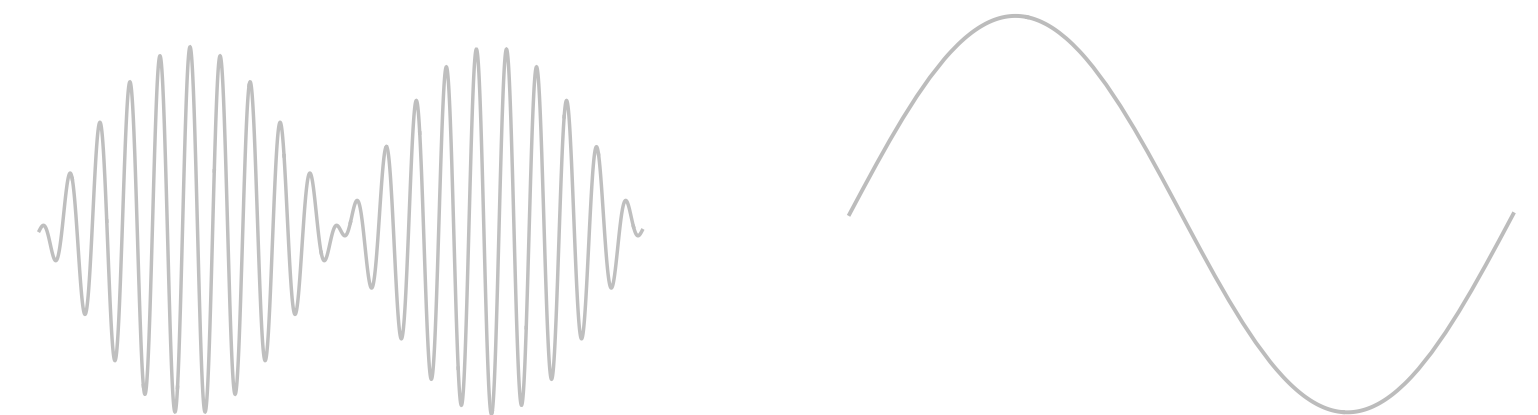
* can be a QCD axion, can be string-theory axions or some generic axion-like particle

scalar = spin-0 wave dark matter

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2} \nabla^\mu \varphi \nabla_\mu \varphi - \frac{1}{2} m^2 \varphi^2 + \frac{1}{8\pi G} R + \dots \right] + \text{non-grav, interactions}$$

non-relativistic limit

$$\varphi(t, \boldsymbol{x}) \equiv \frac{1}{\sqrt{2m}} \Re \left[\Psi(t, \boldsymbol{x}) e^{-imt} \right]$$



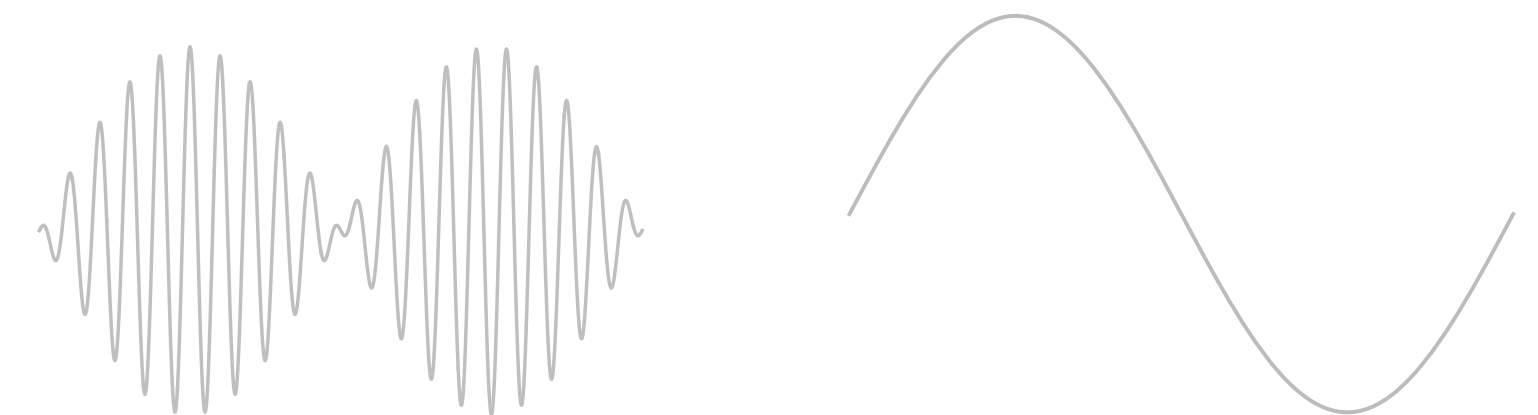
split in “fast” and “slow” parts

scalar = spin-0 wave dark matter

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non-relativistic limit

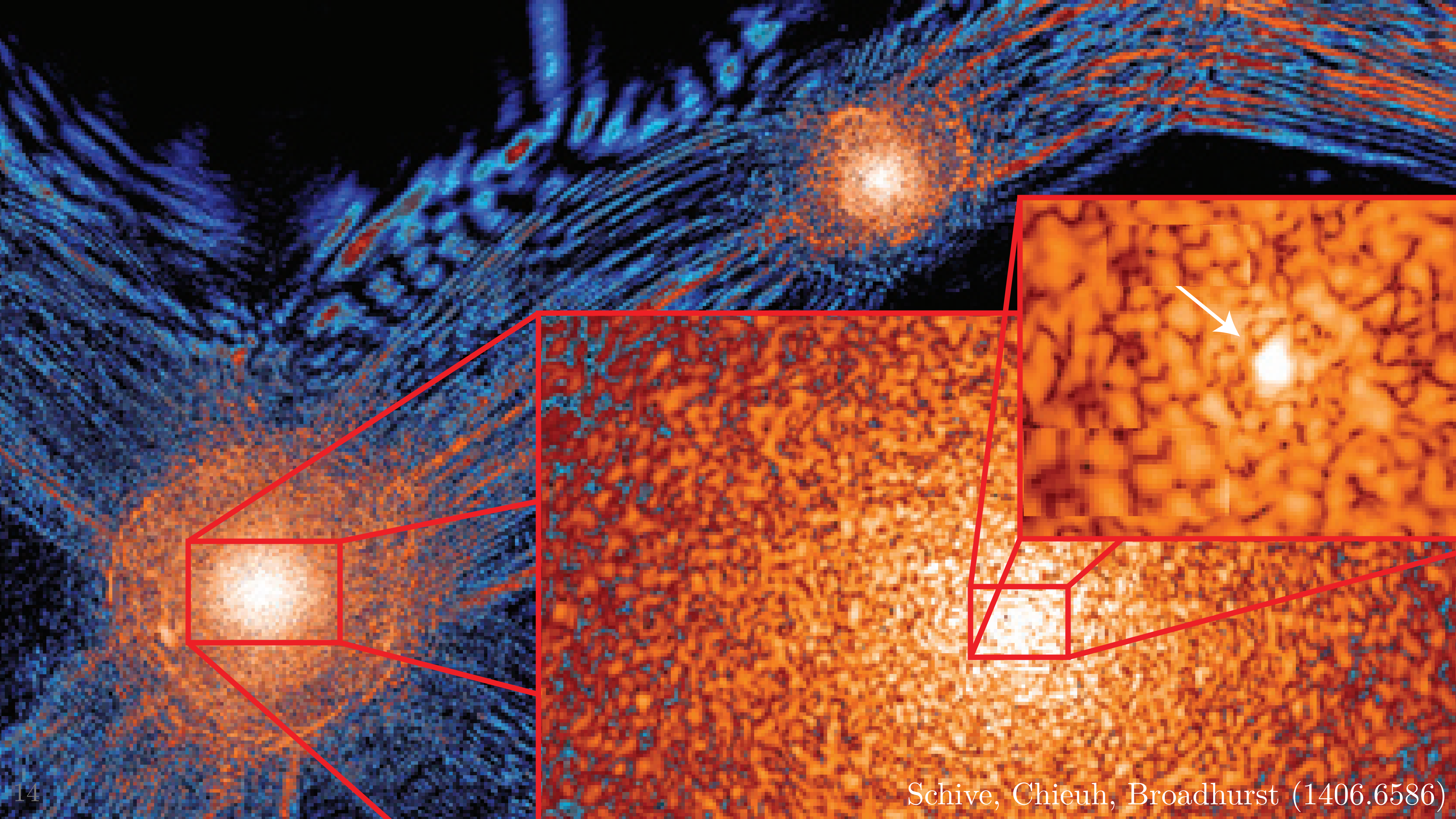
$$\varphi(t, \mathbf{x}) \equiv \frac{1}{\sqrt{2m}} \Re [\Psi(t, \mathbf{x}) e^{-imt}]$$



split in “fast” and “slow” parts

Schrodinger-Poisson system

$$i\hbar \frac{\partial}{\partial t} \Psi = -\frac{\hbar^2}{2m} \nabla^2 \Psi + m\Phi \Psi, \quad \nabla^2 \Phi = 4\pi G m \Psi^* \Psi$$



vector = spin-1 wave dark matter

* assuming classical fields

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{4} X^{\mu\nu} X_{\mu\nu} + \frac{1}{2} m^2 X^\mu X_\mu + \frac{1}{8\pi G} R + \dots \right] + \text{non-grav, interactions}$$

$$X_{\mu\nu} = \nabla_\mu X_\nu - \nabla_\nu X_\mu$$

* inflationary production (Graham et. al), or post-inflationary (Agrawal et. al)

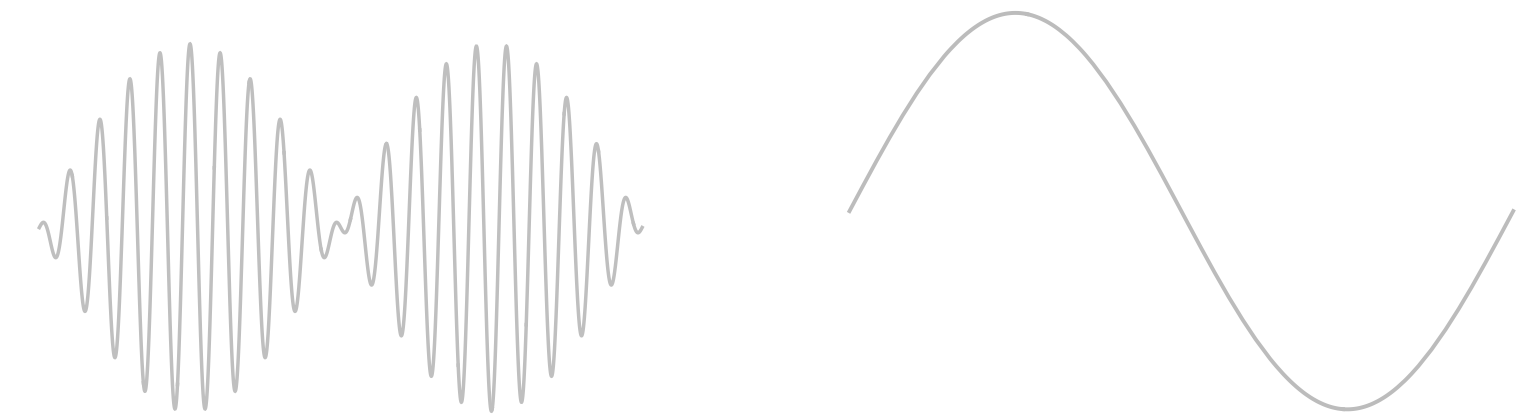
vector = spin-1 wave dark matter

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{4} X_{\mu\nu} X^{\mu\nu} + \frac{1}{2} \frac{m^2 c^2}{\hbar^2} X_\mu X^\mu + \frac{c^2}{8\pi G} R + \dots \right] + \text{non-grav, interactions}$$

$$X_{\mu\nu} = \nabla_\mu X_\nu - \nabla_\nu X_\mu$$

non-relativistic limit

$$\mathbf{X}(t, \mathbf{x}) = \frac{1}{\sqrt{2m}} \Re [\boldsymbol{\Psi}(t, \mathbf{x}) e^{-imt}]$$



split in “fast” and “slow” parts

Schrodinger-Poisson system

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$$\nabla^2 \Phi = 4\pi G m \boldsymbol{\Psi}^\dagger \boldsymbol{\Psi}$$

Spin - 1 Adshead & Lozanov (2101.07265)

Spin - 2s+1 Jain & MA (2109.04892)

non-relativistic limit = multicomponent Schrödinger-Poisson

$$i\partial_t \boldsymbol{\Psi} = -\frac{\nabla^2 \boldsymbol{\Psi}}{2m} + m\Phi \boldsymbol{\Psi}, \quad \nabla^2 \Phi = 4\pi G m \boldsymbol{\Psi}^\dagger \boldsymbol{\Psi}$$

$[\boldsymbol{\Psi}]_i = \psi_i$ with $i = 1$ scalar case

$[\boldsymbol{\Psi}]_i = \psi_i$ with $i = 1, 2, 3$ vector case

at this level this is just $2s+1$ equal-mass scalar fields
but not when non-gravitational interactions are included

non-relativistic limit = multicomponent Schrödinger-Poisson

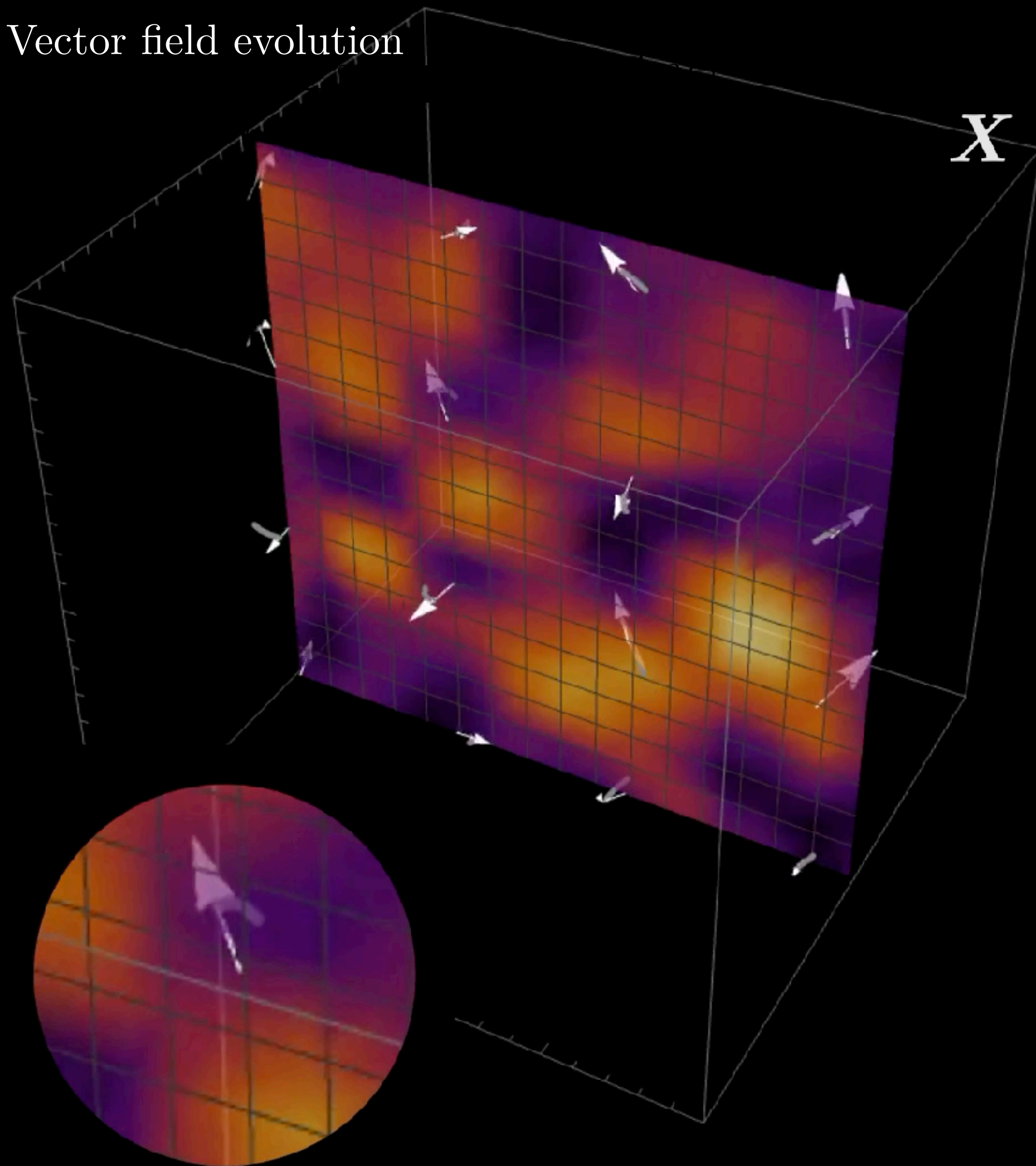
$$i\partial_t \boldsymbol{\Psi} = -\frac{\nabla^2 \boldsymbol{\Psi}}{2m} + m\Phi \boldsymbol{\Psi}, \quad \nabla^2 \Phi = 4\pi G m \boldsymbol{\Psi}^\dagger \boldsymbol{\Psi}$$

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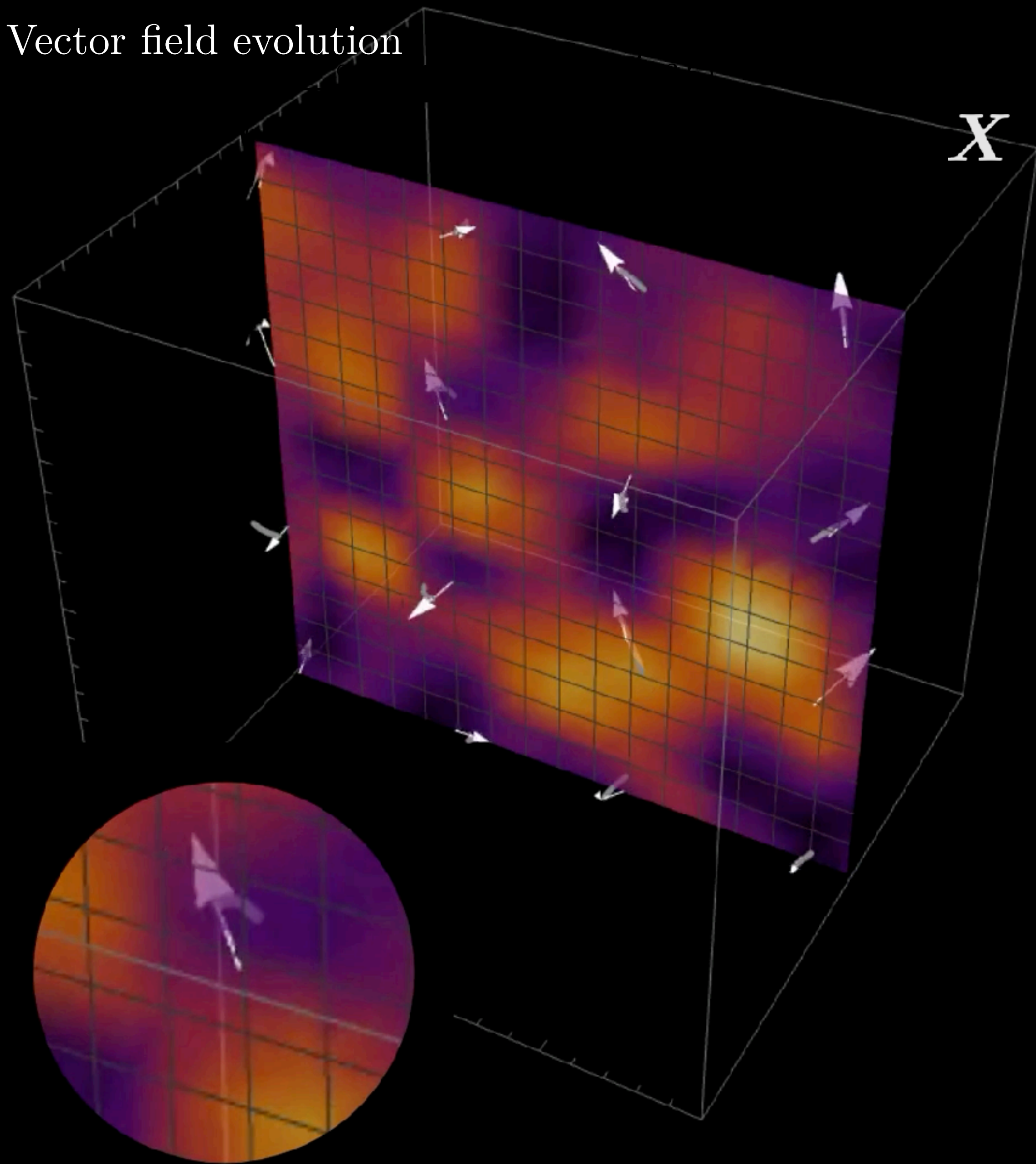
$[\boldsymbol{\Psi}]_i = \psi_i$ with $i = 1, 2, 3$ vector case

Spin angular momentum $S = \hbar \int d^3x i \boldsymbol{\Psi} \times \boldsymbol{\Psi}^\dagger$

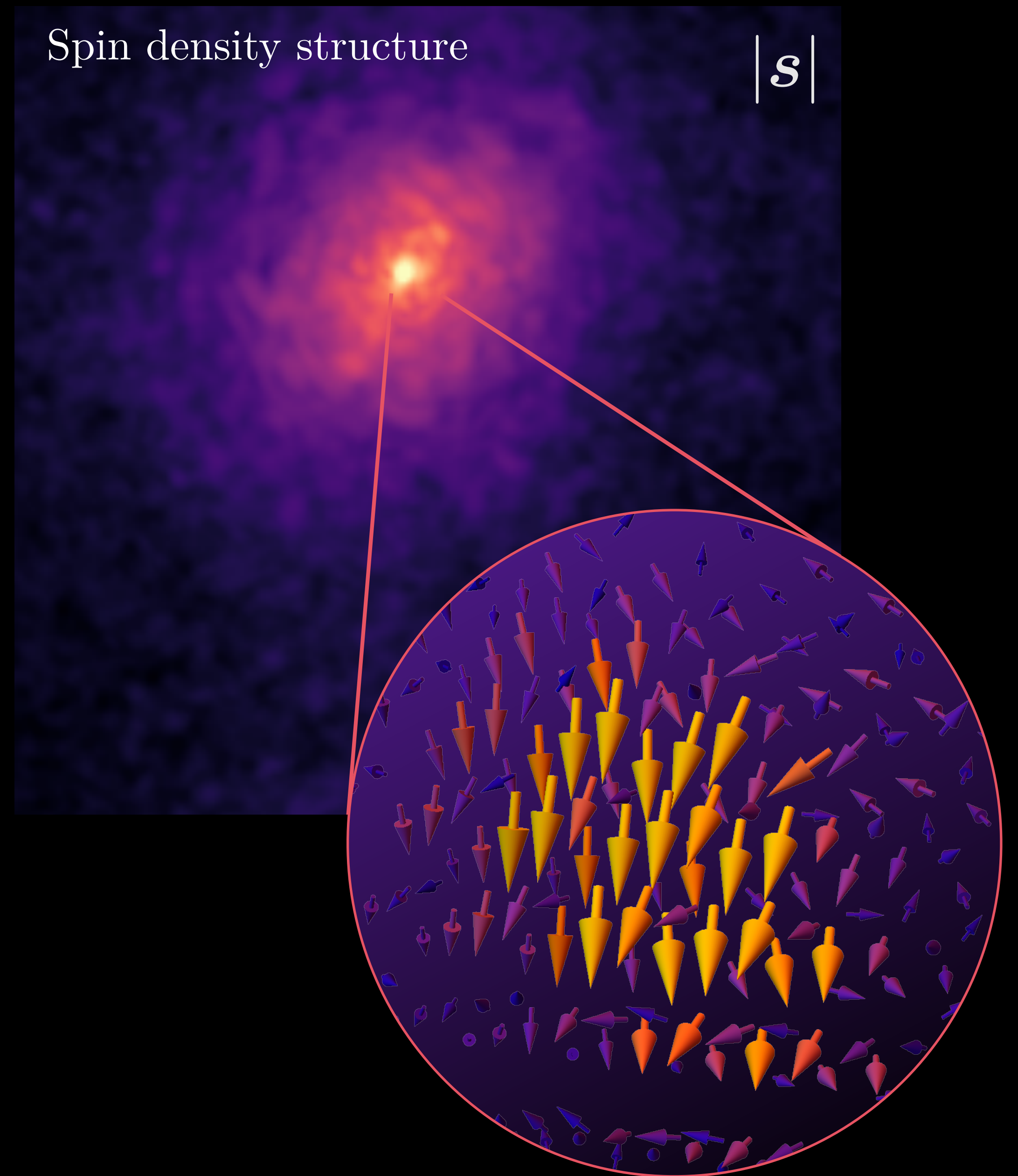
Vector field evolution



Vector field evolution



Spin density structure



what is wave dark matter ?



how is it produced?

what does it do?

what *really* is wave dark matter?

what does wave dark matter do

scale-dependent growth of structure

wave interference

soliton+halo profiles

Bose effects

interactions



wave interference

wave interference

- dynamical heating of stars

$$m \gtrsim \frac{1}{(2s+1)^{1/3}} [3 \times 10^{-19} \text{eV}]$$

Dalal & Kratsov (2203.05750)

MA, Jain, Karur & Mocz (2203.11935)

what does wave dark matter do

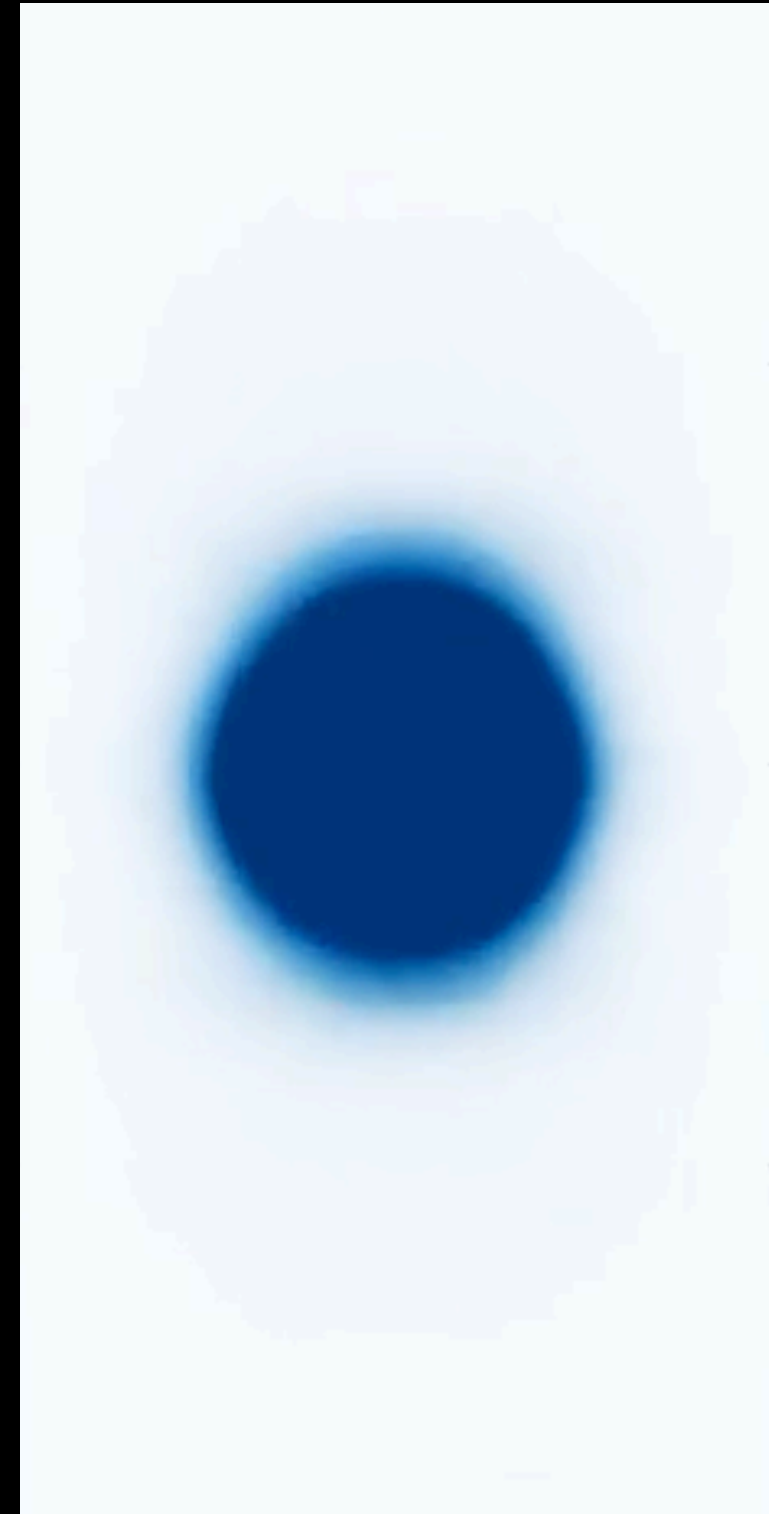
scale-dependent growth of structure

wave interference

soliton+halo profiles

Bose effects

interactions



with Mou, Schiappacasse & Long (2009.11337, 2103.12082)

Amaral, Schiappacasse, Zhang (2509.08932)

what does wave dark matter do

scale-dependent growth of structure

wave interference

dynamical heating, lensing etc.

soliton+halo profiles

cored halos, small scale structure

Bose effects

condensation, superradiance, Bosenovas

interactions

CMB birefringence, resonant photon prod.

$$\lambda\varphi^4, \quad \varphi F\tilde{F}, \quad FF' \quad (X \cdot X)F\tilde{F},$$

what does wave dark matter do

scale dependent growth of structure

wave interference

dynamical heating, lensing etc.

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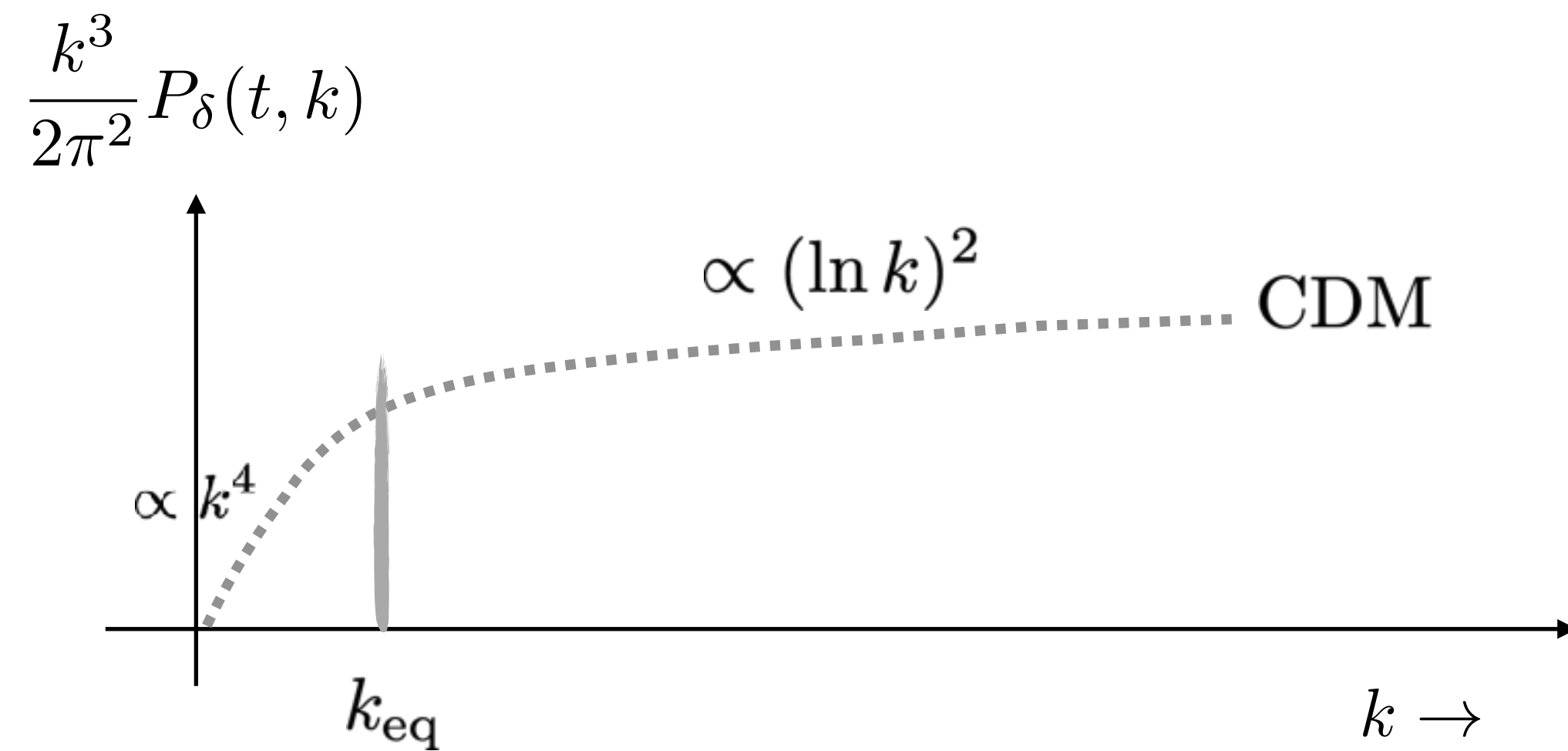
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what does wave dark matter do

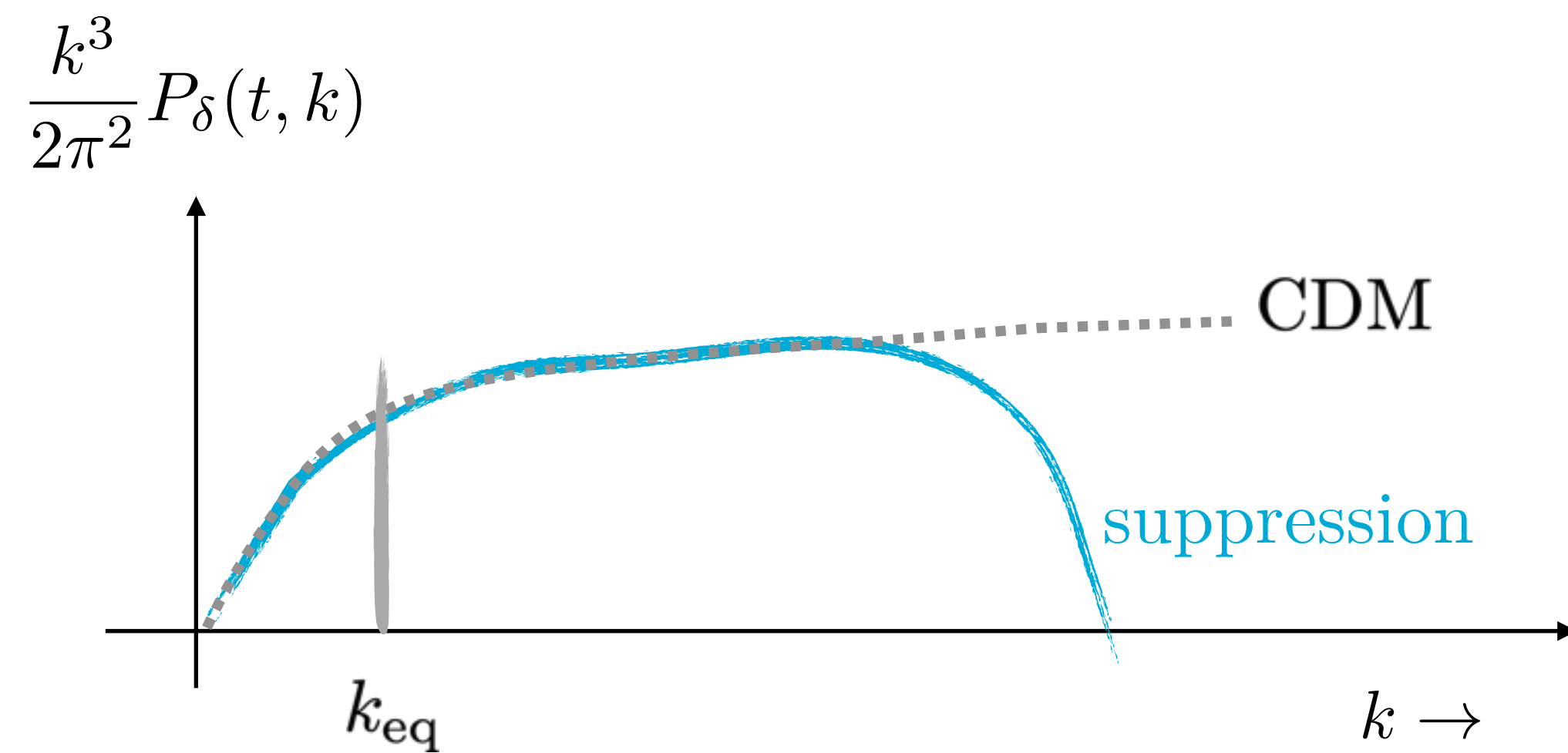
scale dependent growth of structure



CDM

what does wave dark matter do

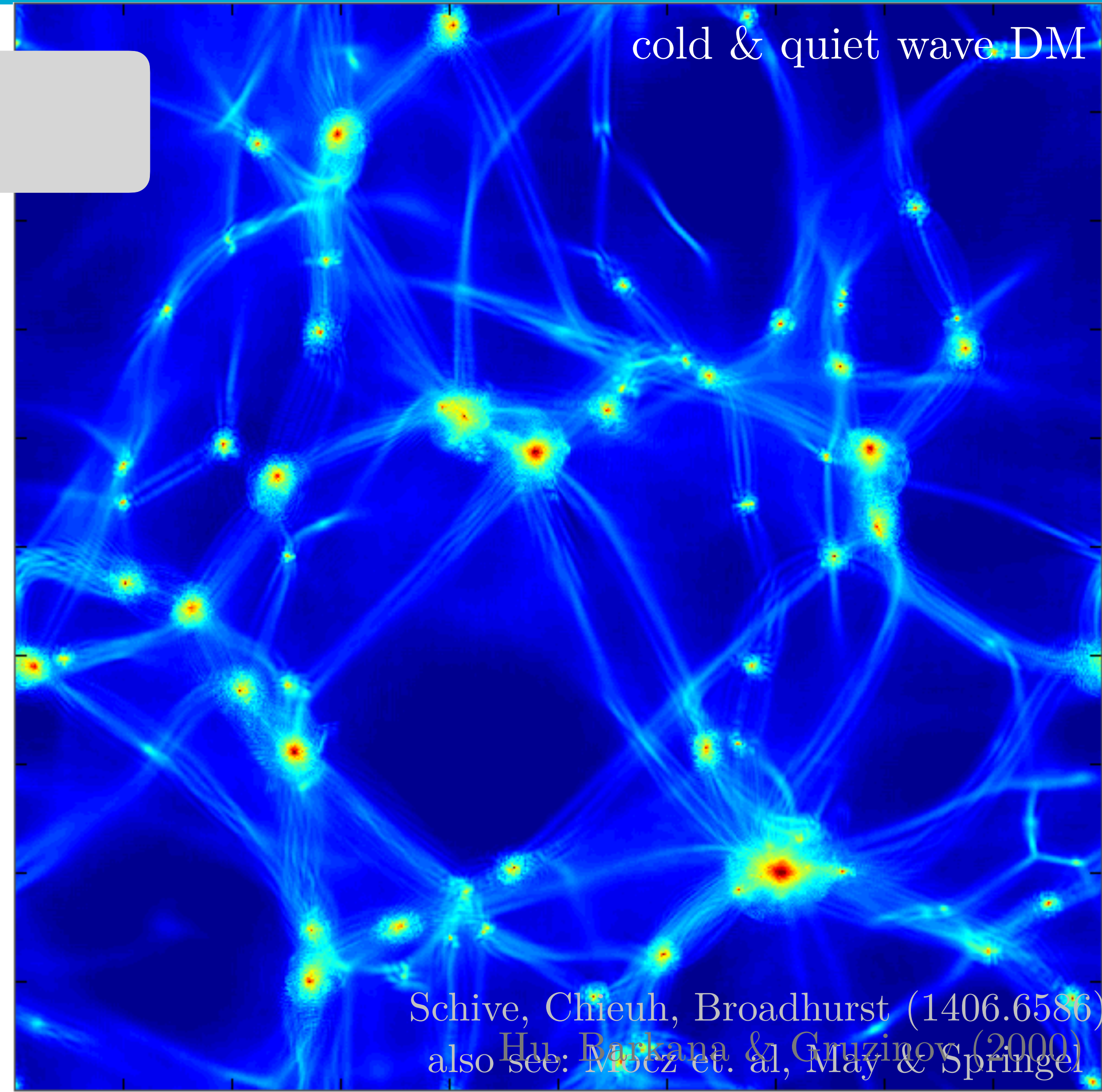
scale dependent growth of structure



Wave-dynamical “quantum” Jeans

$$k_j \sim 200 \text{ Mpc}^{-1} \sqrt{\frac{m}{10^{-19} \text{ eV}}}$$

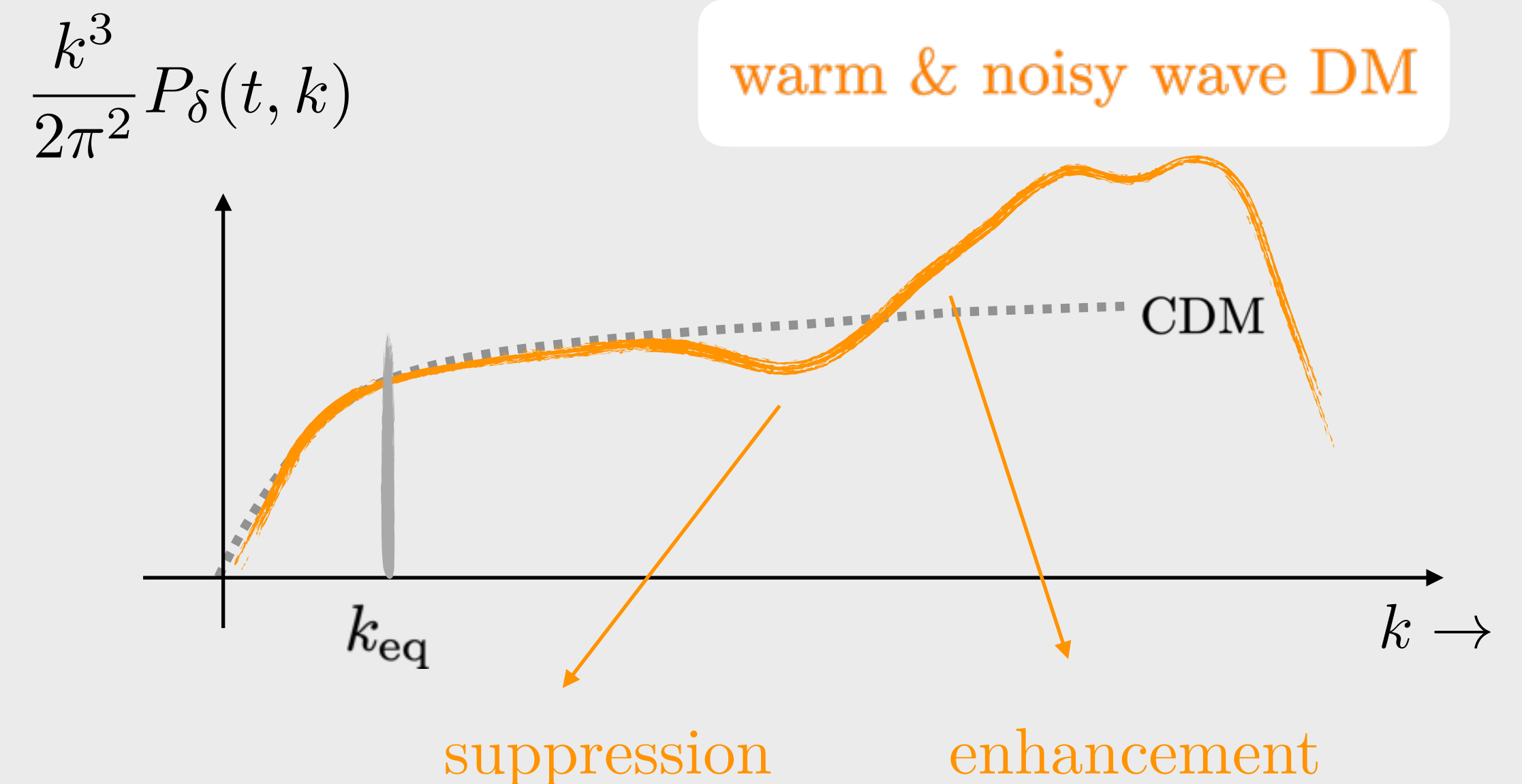
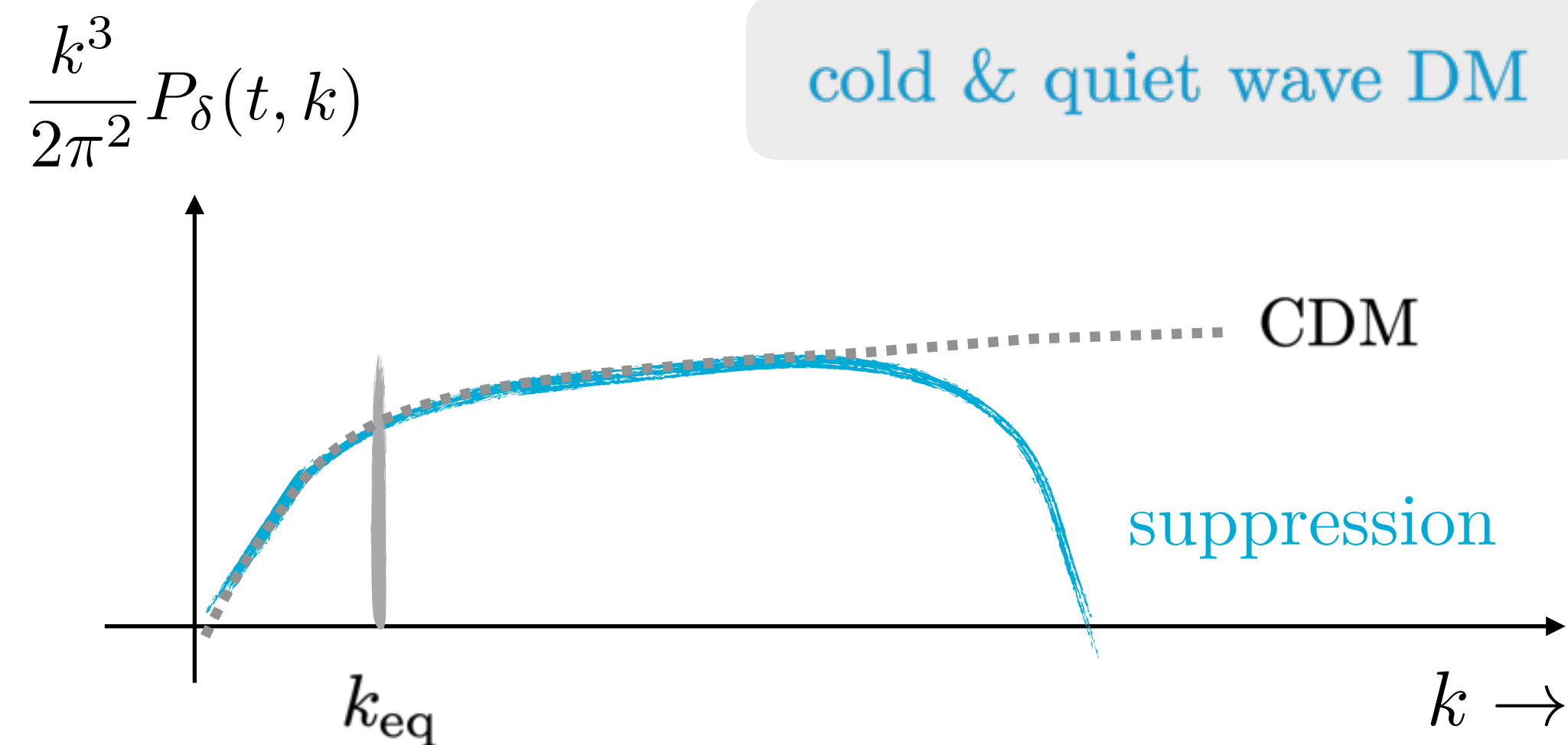
Hu, Barkana & Gruzinov (2000)



what does wave dark matter do

scale-dependent growth of structure

Lya, galaxy satellite populations etc.



what is wave dark matter ?



how is it produced?

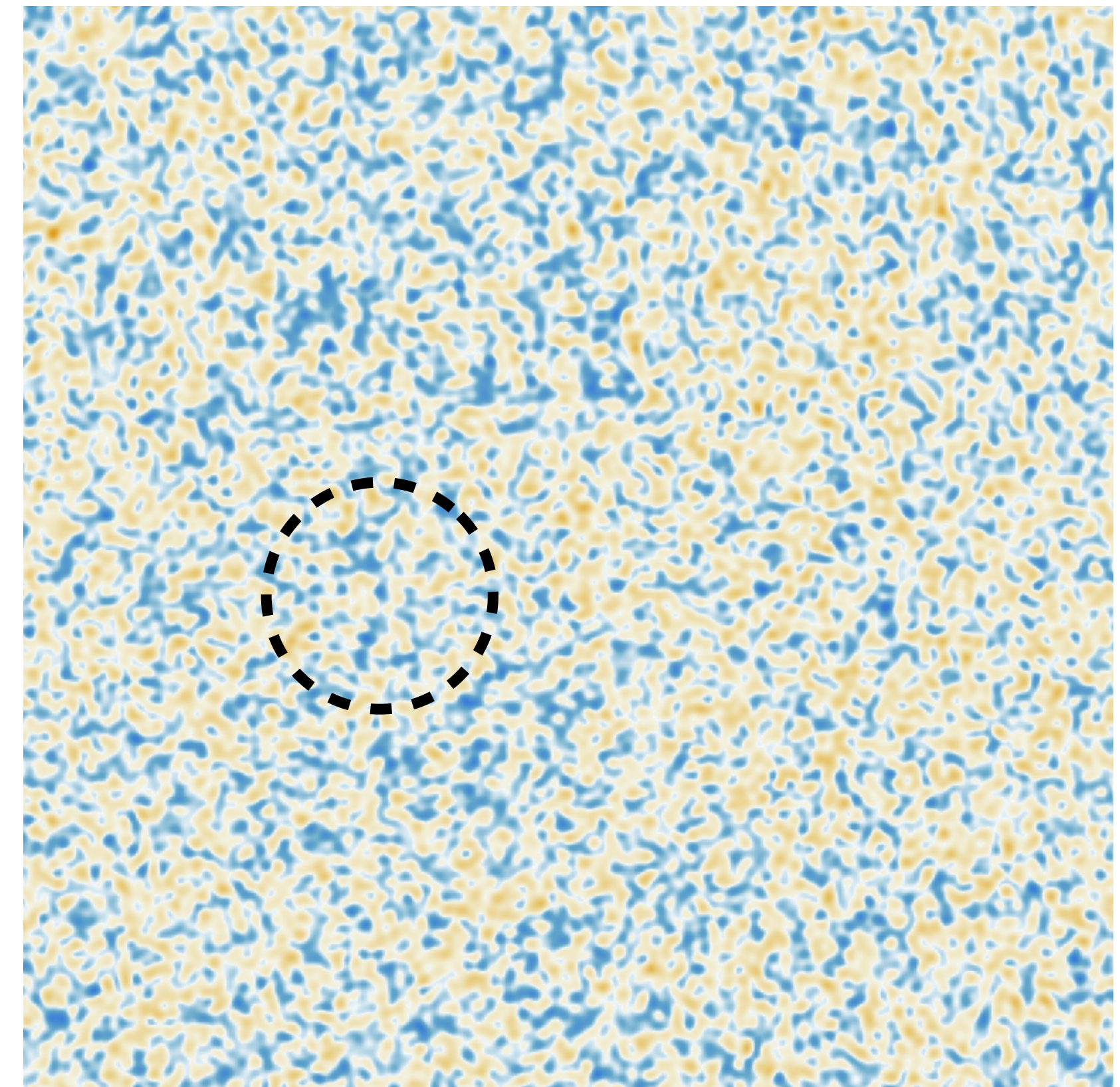
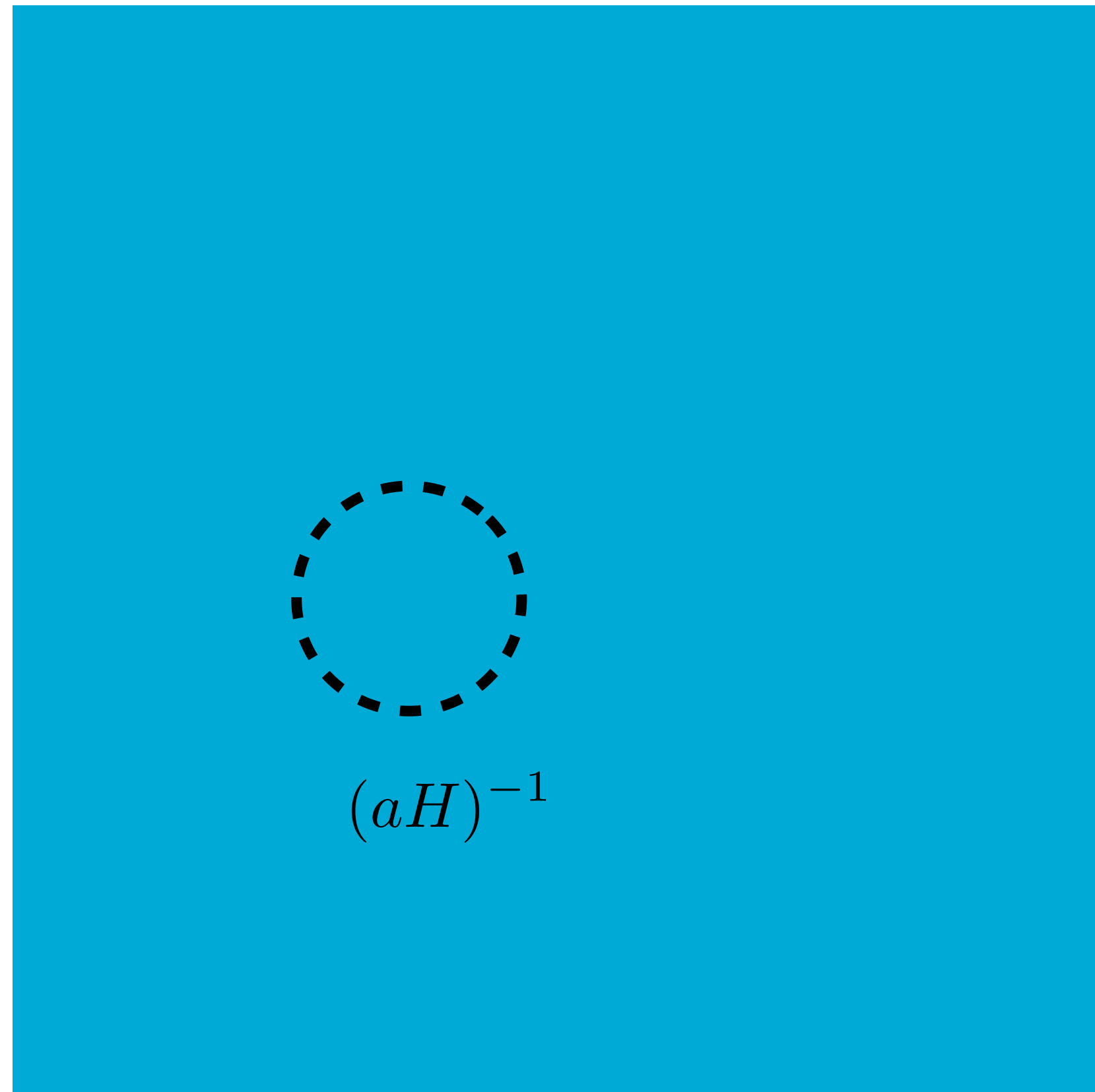
and its consequences

what does it do?



what *really* is wave dark matter?

production mechanisms

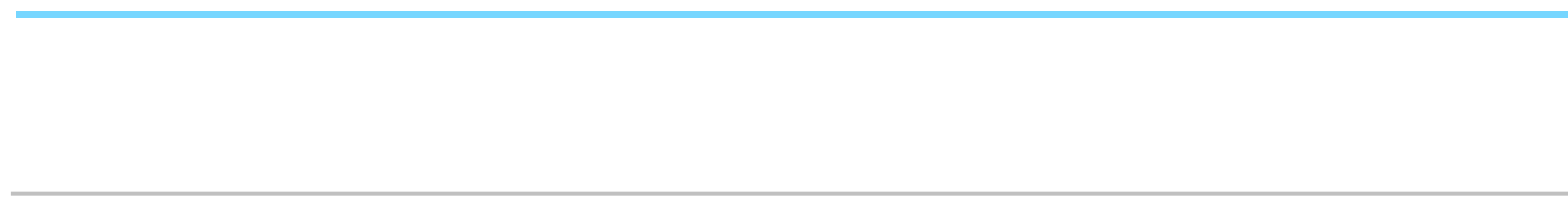


inflationary vs. **post-inflationary**

Reviews for axions: by O'hare (2403.17697), Luzzio, Giannotti, Nardi & Visinelli (2003.01100)

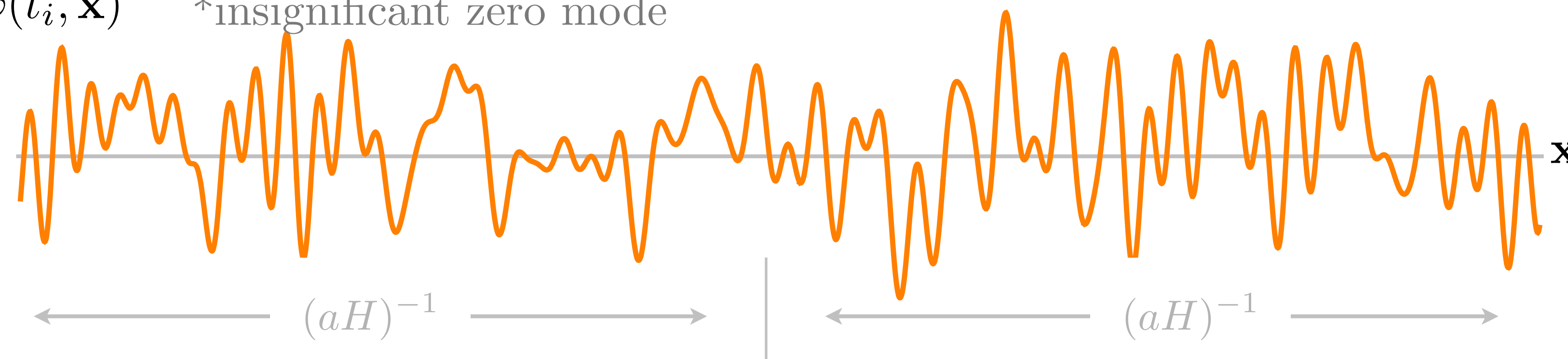
Quiet vs. Noisy Fields

$\varphi(t_i, \mathbf{x})$ * significant zero mode



eg. axions with $\mathbb{PQ}$ before inflation
inflaton just at end of inf.

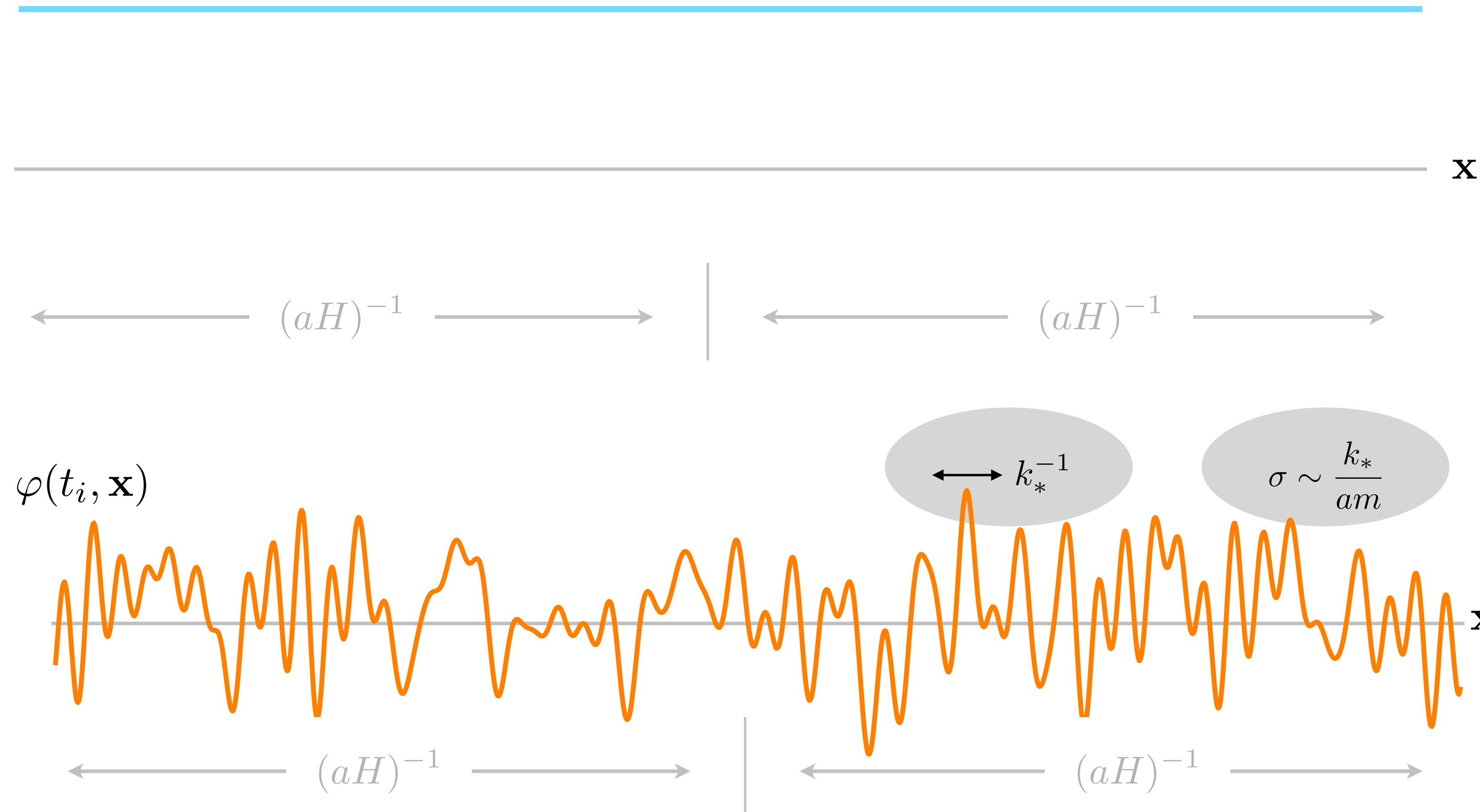
$\varphi(t_i, \mathbf{x})$ * insignificant zero mode



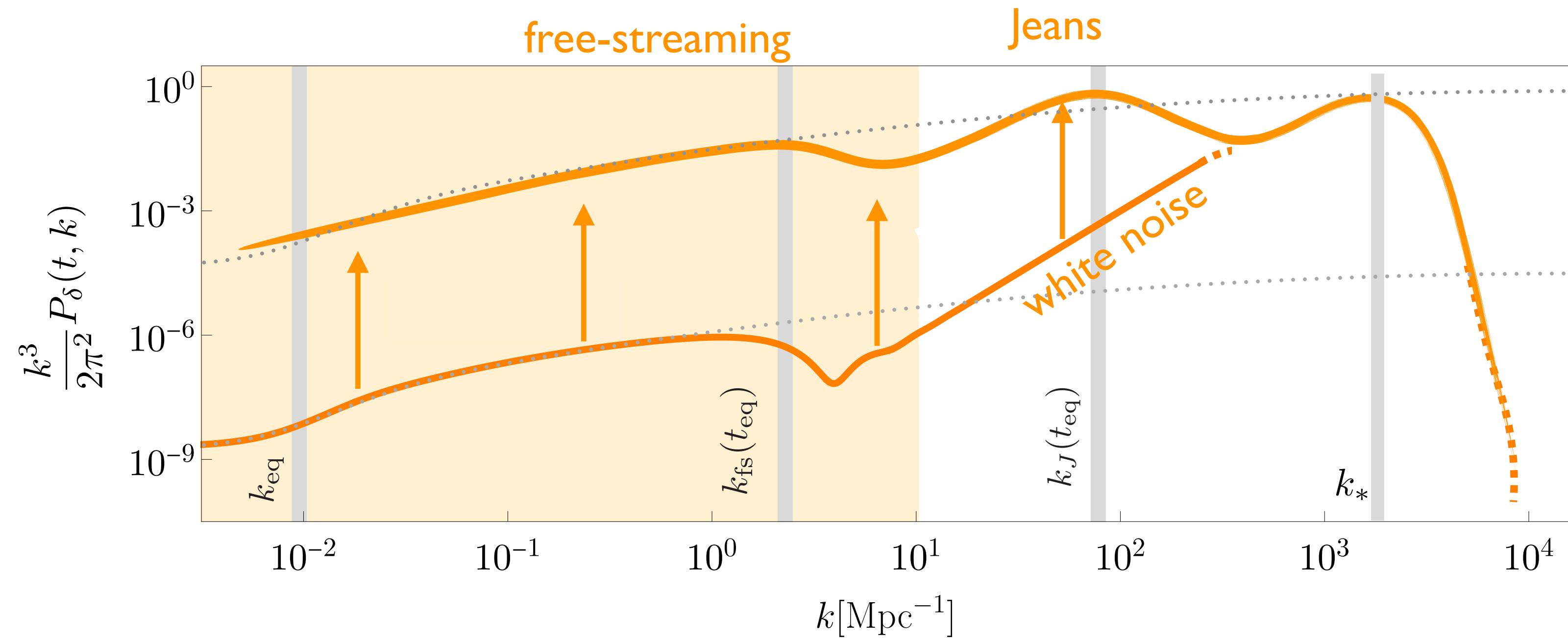
eg. axions with $\mathbb{PQ}$ after inflation
field after inflation fragmentation

Cold & Quiet vs. Warm & Noisy Fields

$\varphi(t_i, \mathbf{x})$

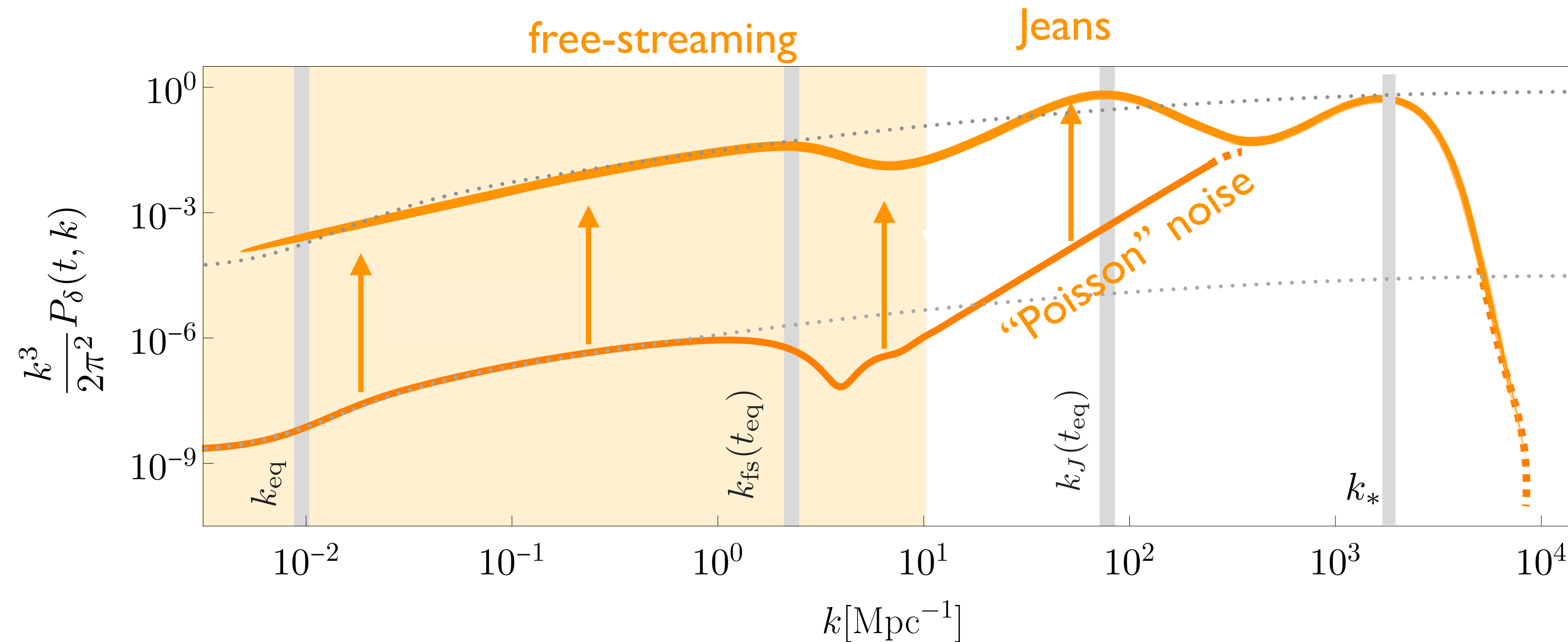


warm & noisy wave dark matter



$\longleftrightarrow k_*^{-1}$
 $\sigma \sim \frac{k_*}{am}$

warm & noisy wave dark matter



$$\longleftrightarrow k_*^{-1} \quad \sigma \sim \frac{k_*}{am}$$

free-streaming

$$k_{\text{fs}}(a_{\text{eq}}) \sim 10 \text{ Mpc}^{-1} \left(\frac{m}{10^{-19} \text{ eV}} \right) \left(\frac{10^3 \text{ Mpc}^{-1}}{k_*} \right)$$

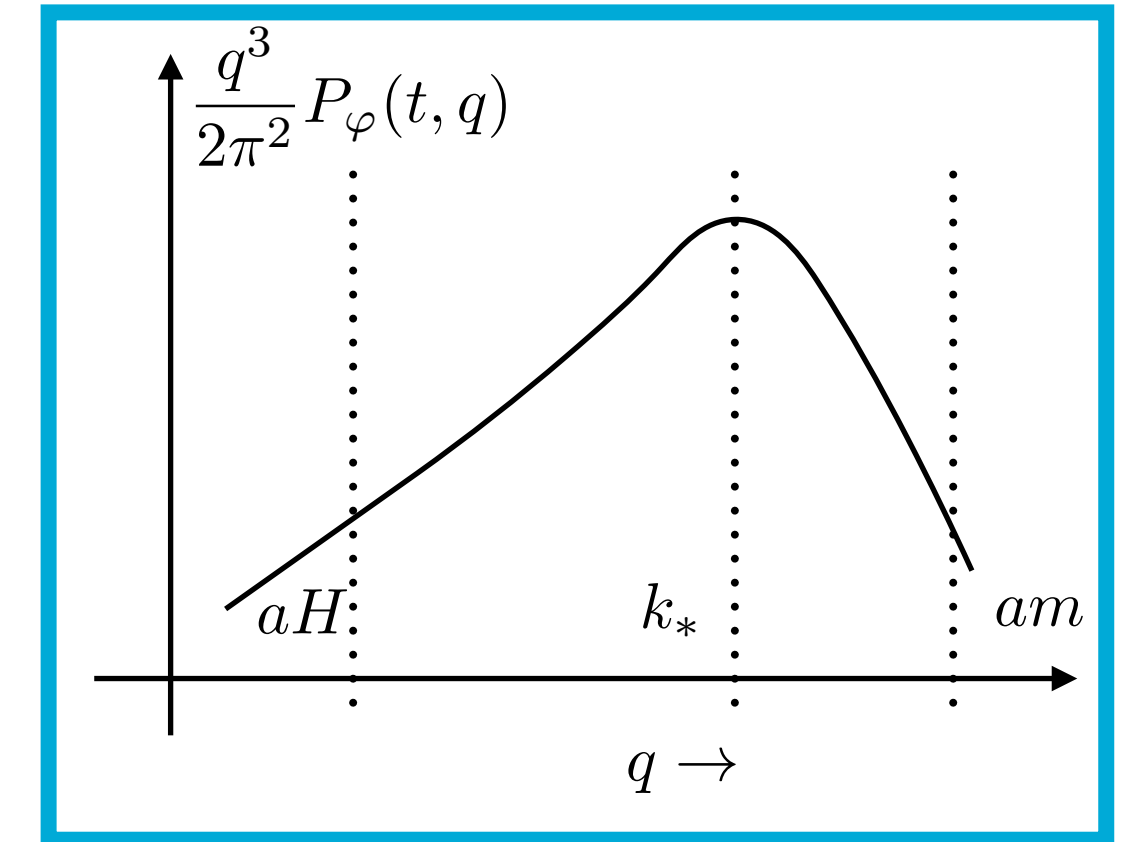
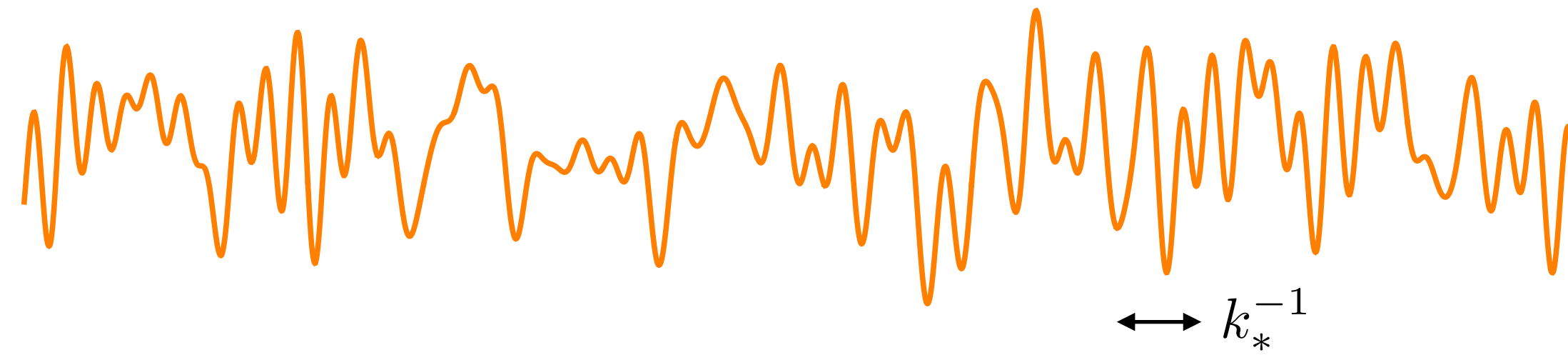
Jeans

$$k_J(a_{\text{eq}}) \sim 50 \text{ Mpc}^{-1} \left(\frac{m}{10^{-19} \text{ eV}} \right) \left(\frac{10^3 \text{ Mpc}^{-1}}{k_*} \right)$$

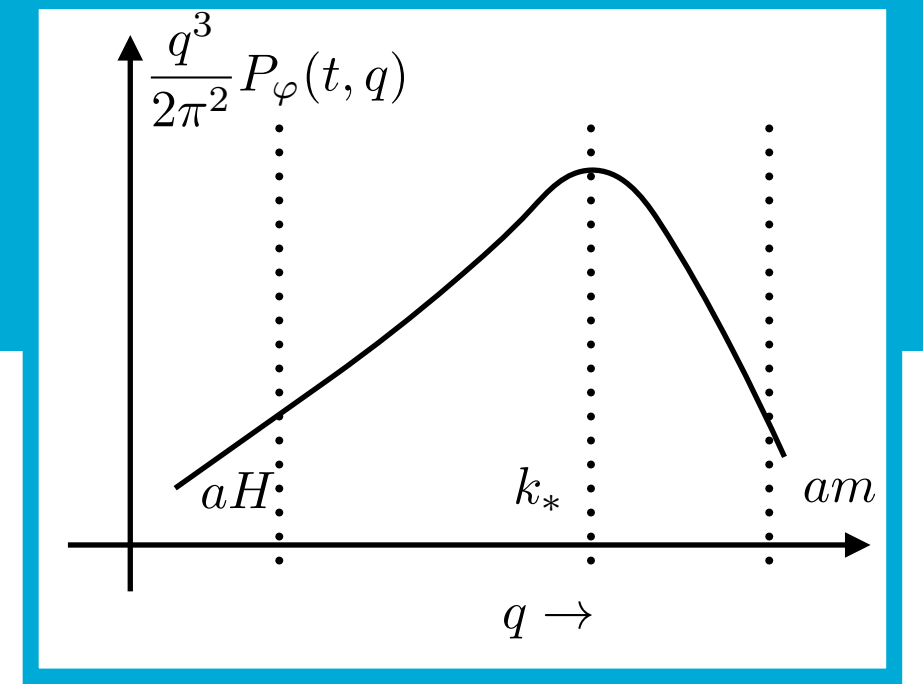
“Poisson” noise

$$(k/k_*)^3, \quad k_* \gtrsim 10^3 \text{ Mpc}^{-1}$$

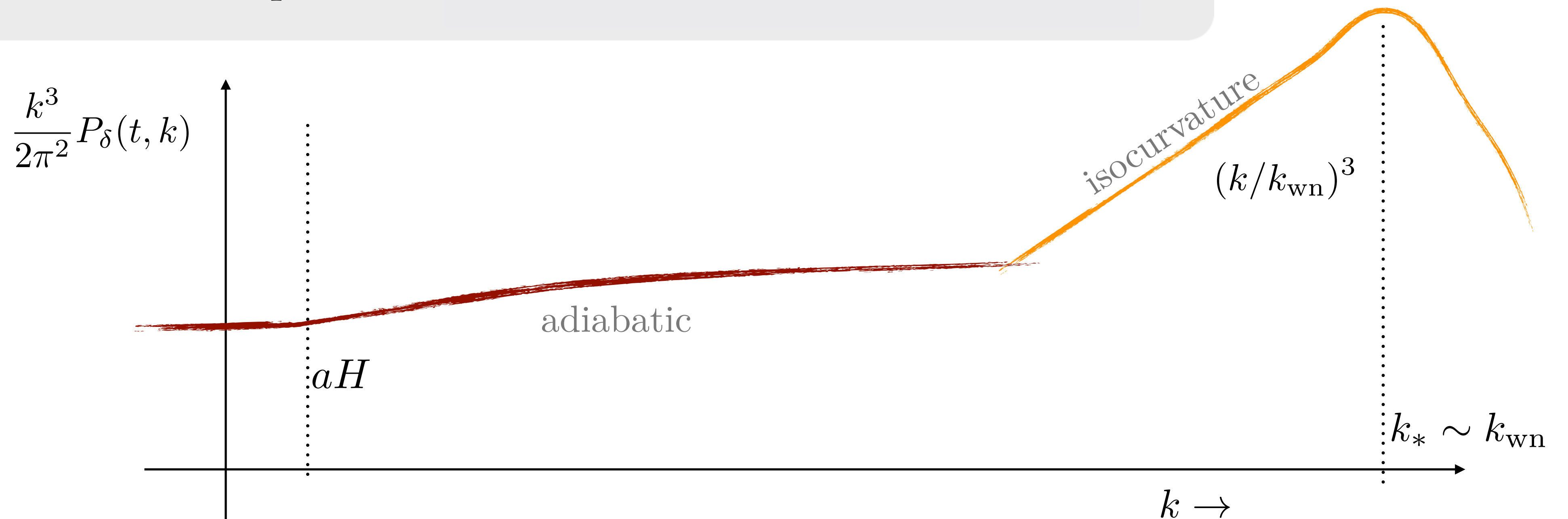
field power spectrum of noisy cosmological fields



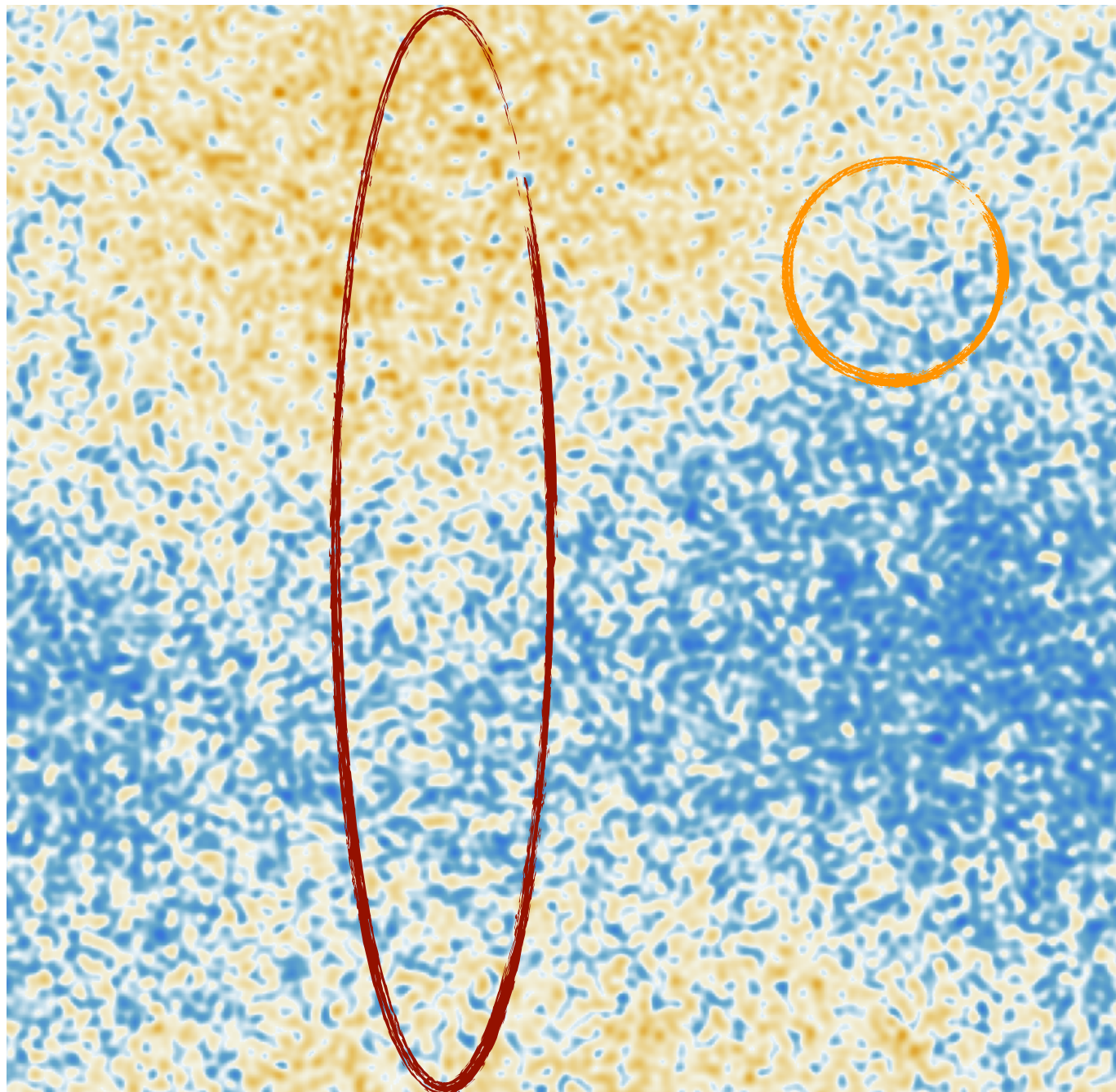
initial conditions



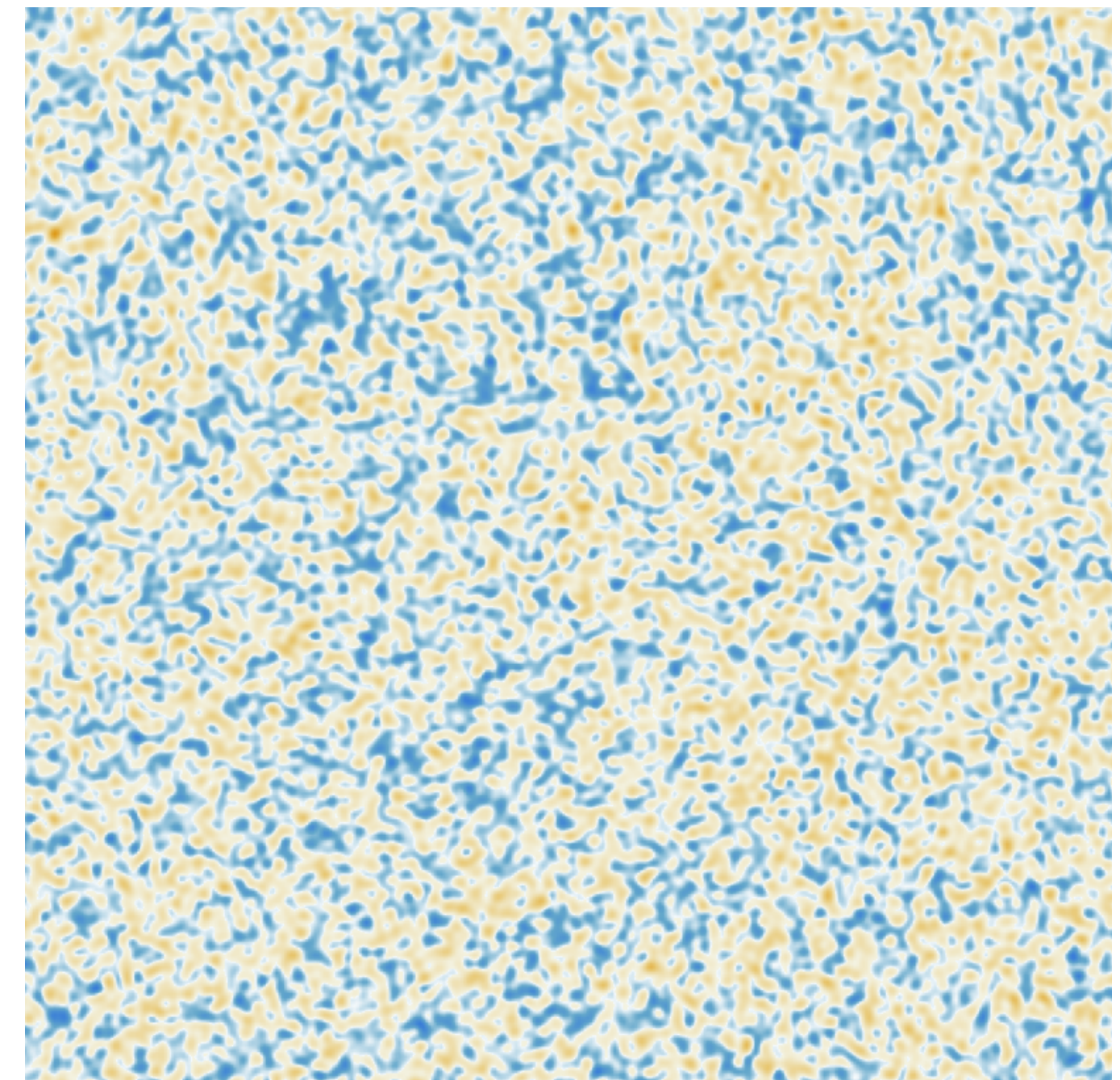
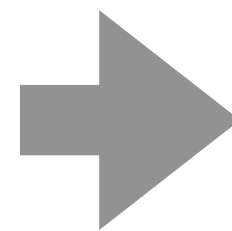
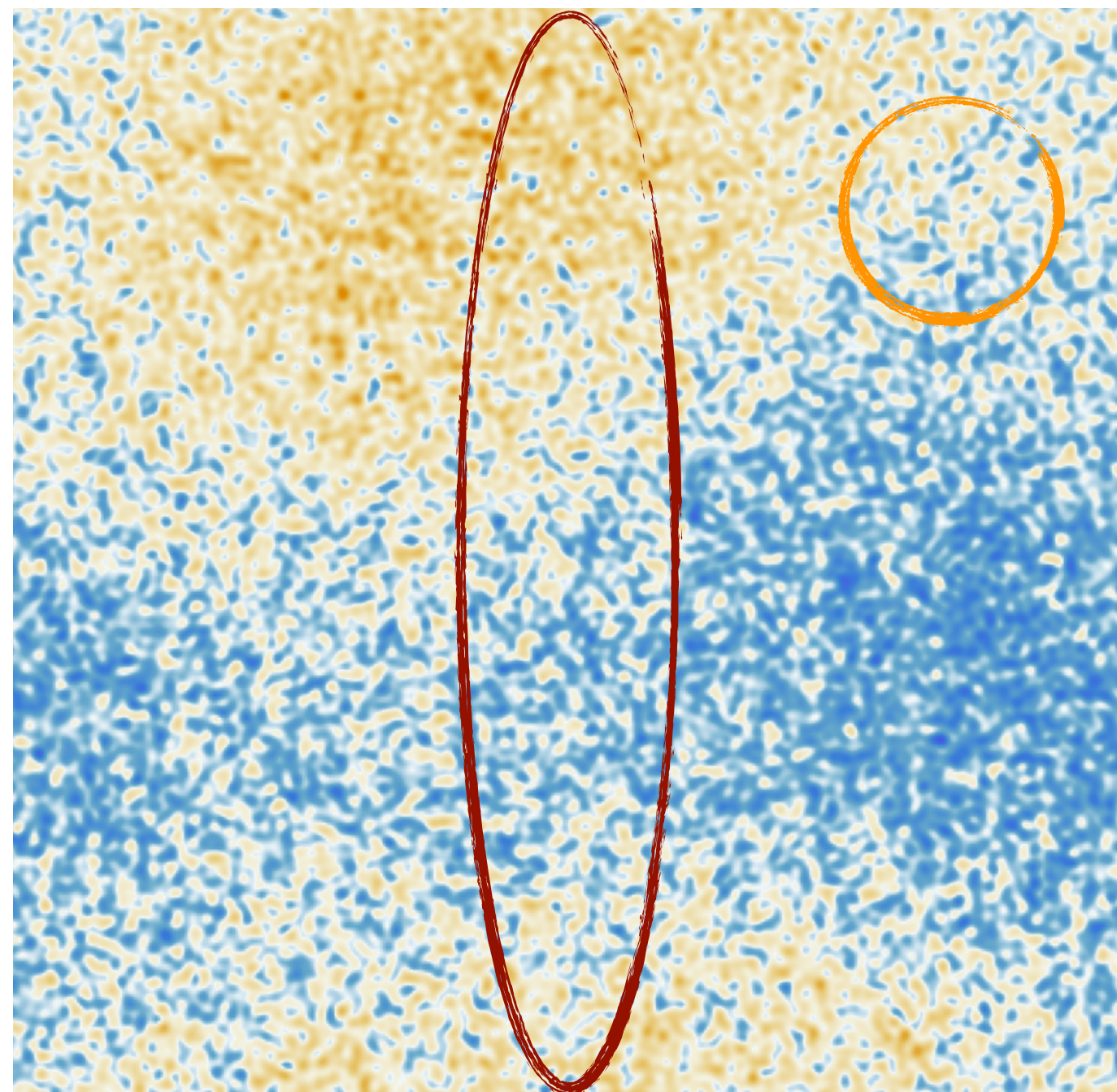
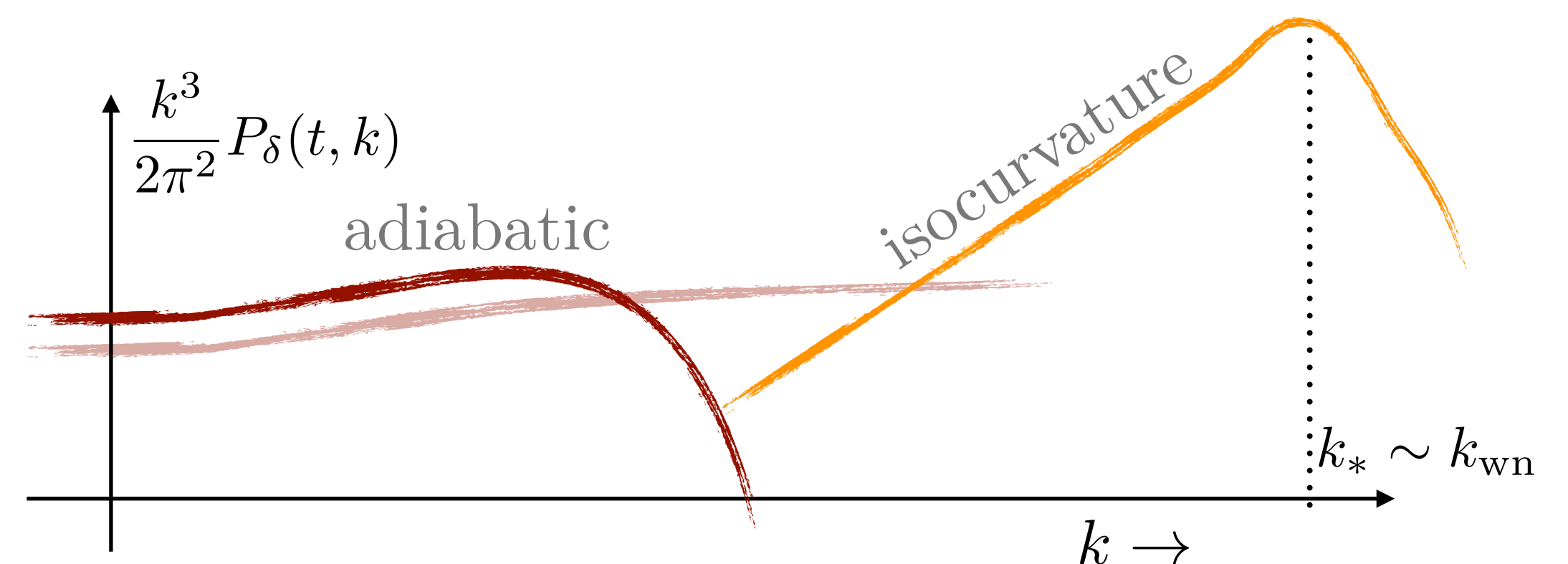
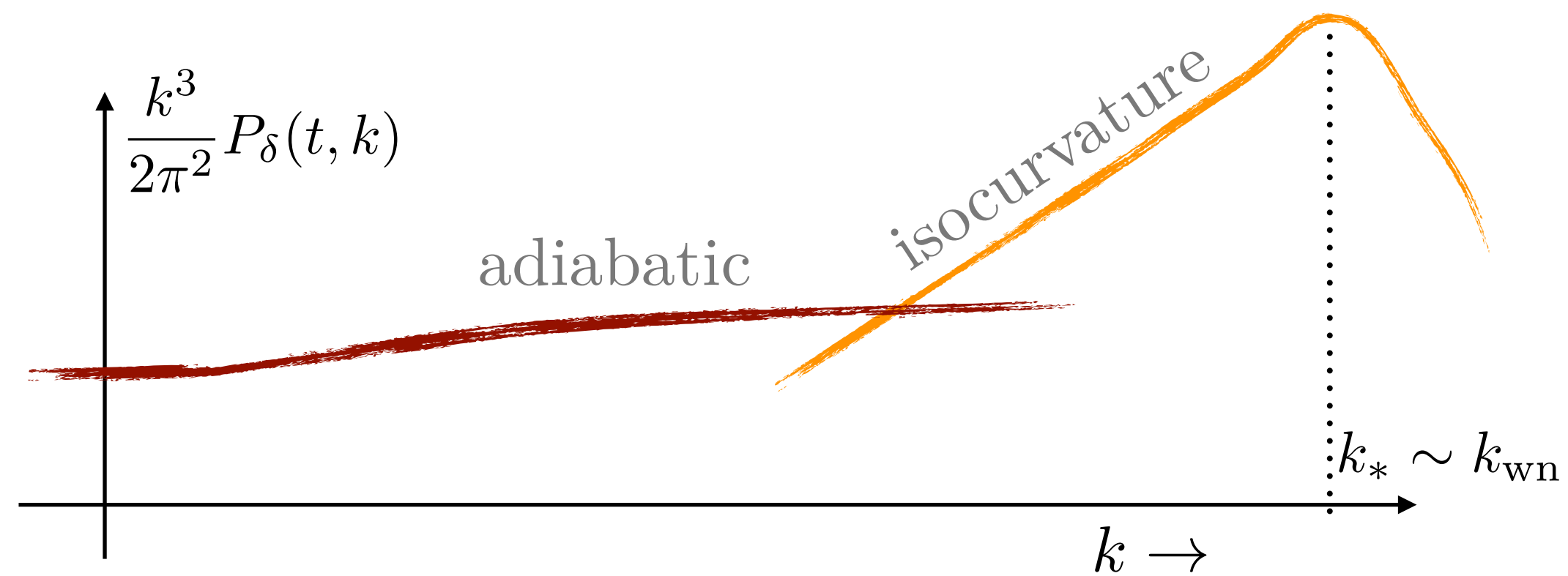
1. small length scale “isocurvature” component
2. larger scale **adiabatic** component



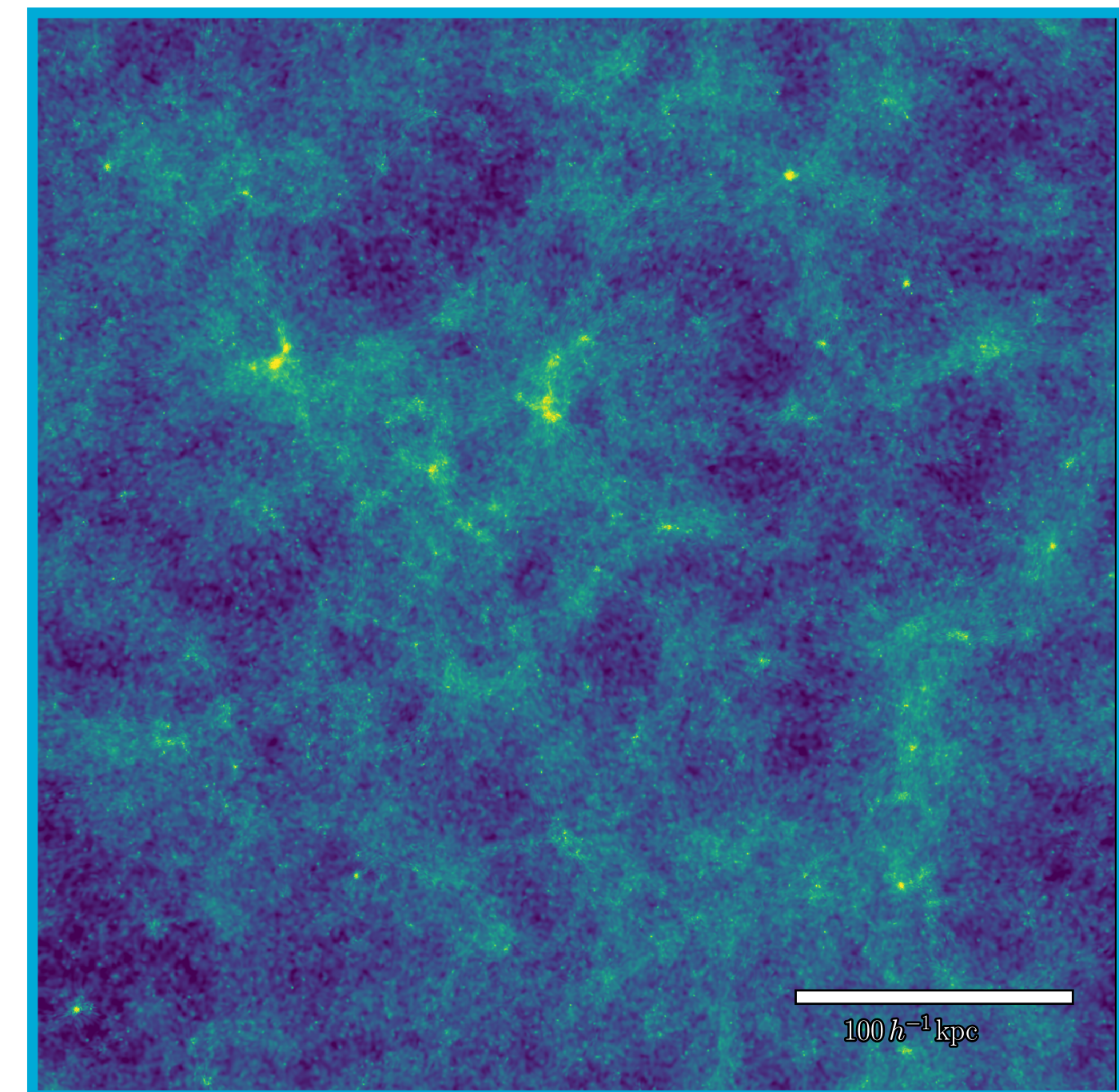
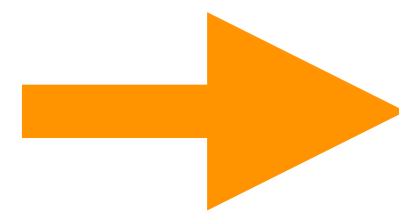
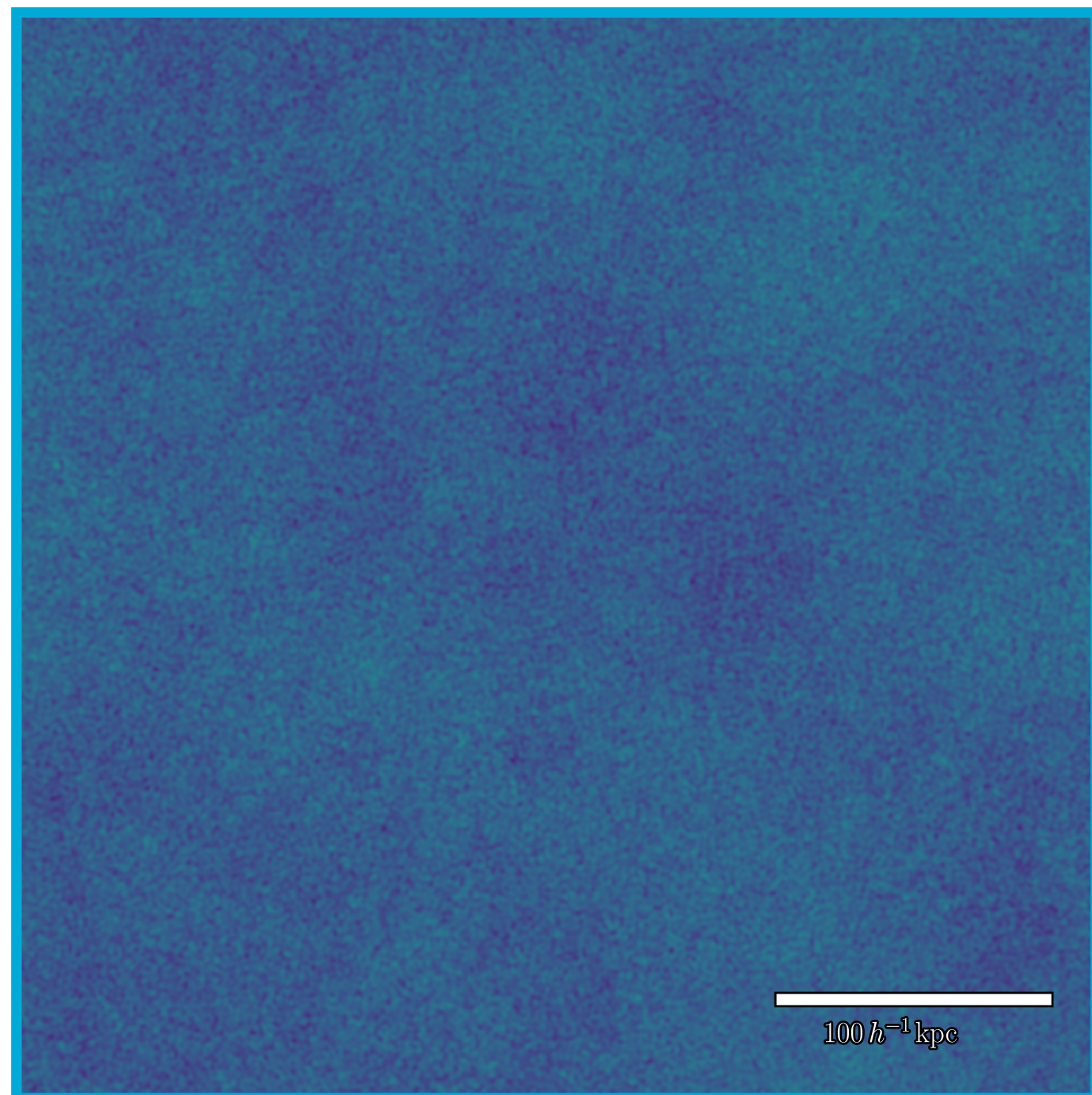
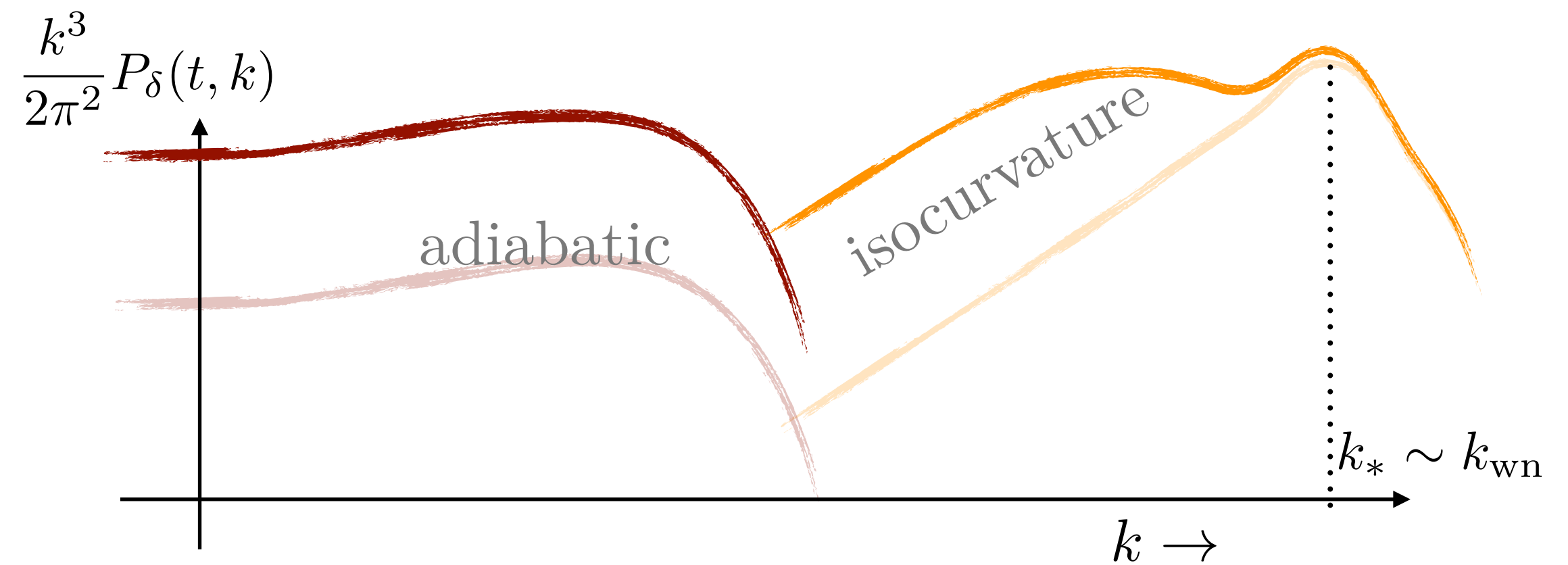
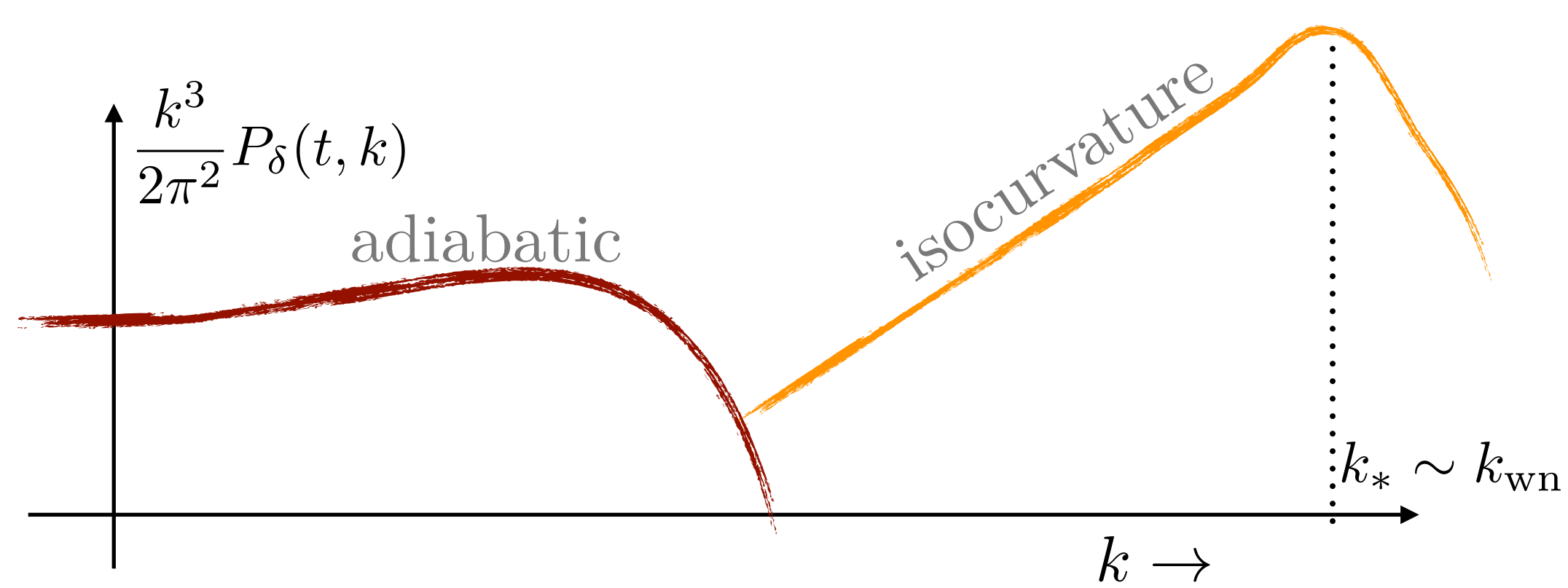
density evolution during radiation domination



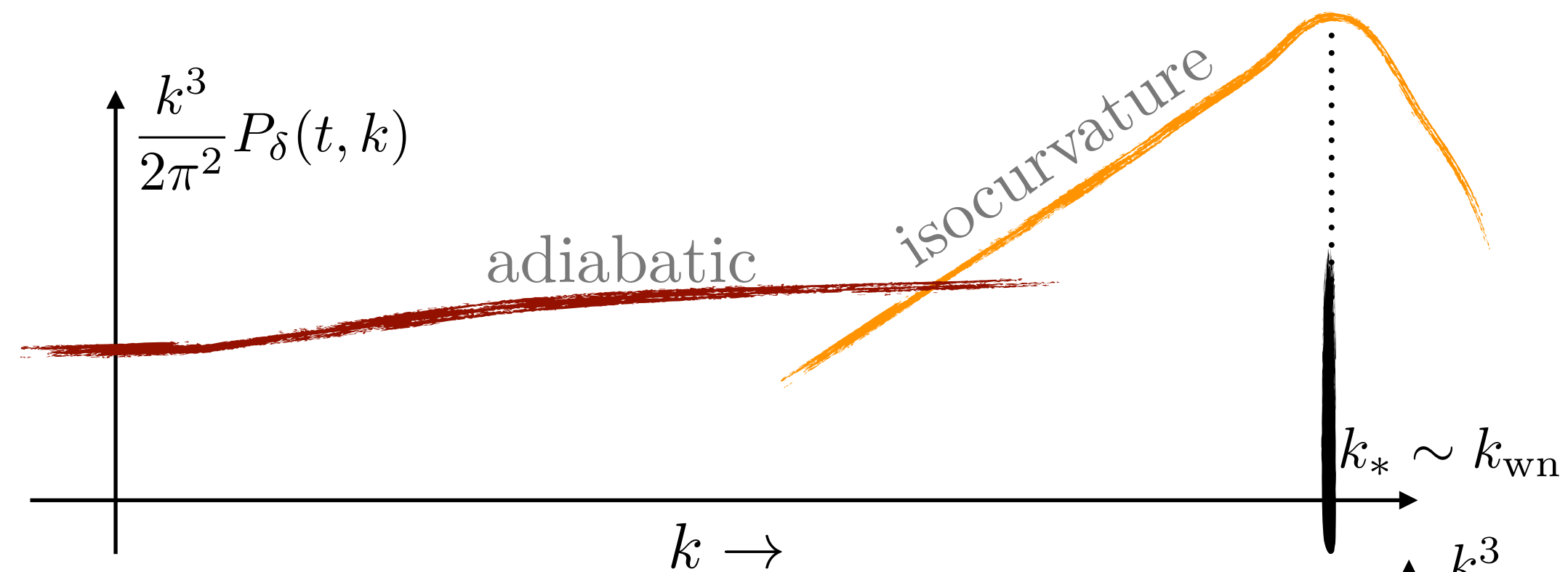
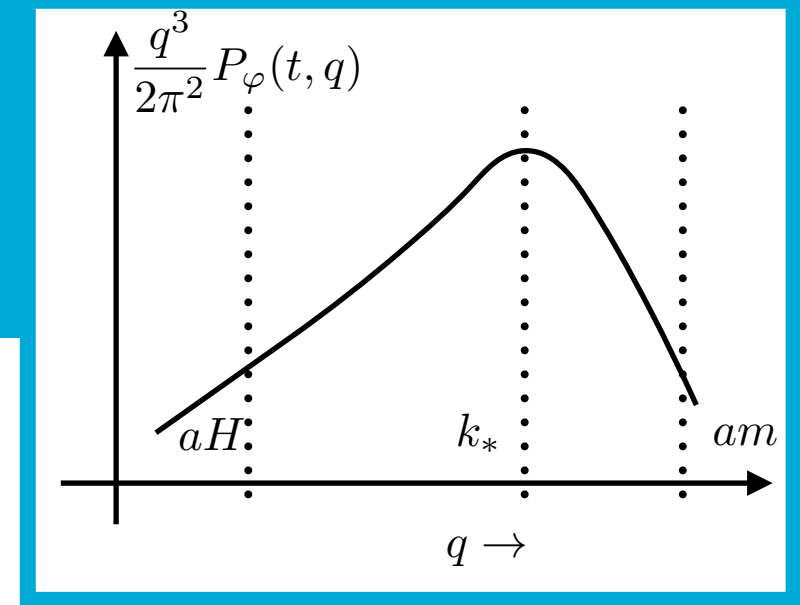
density evolution during radiation domination



density evolution during matter domination

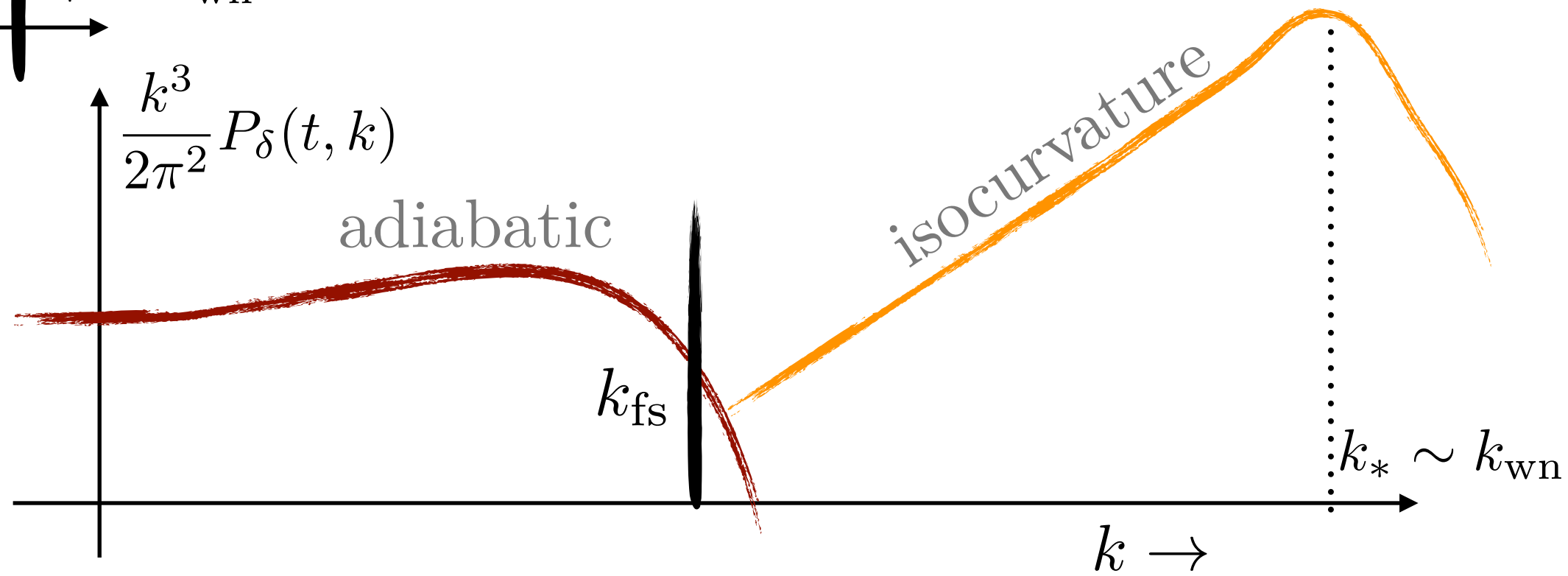


evolution of the matter power spectrum

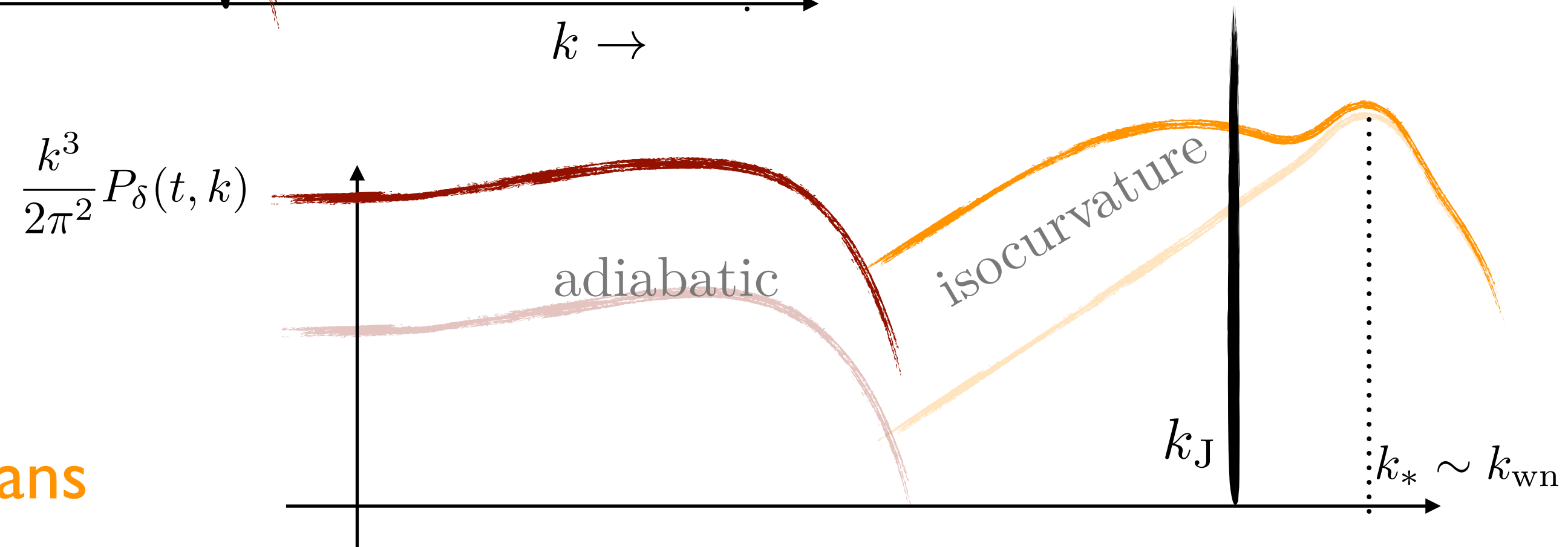


white noise

free-streaming



Jeans



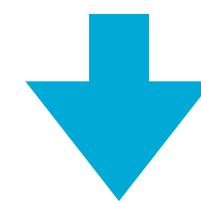
the lower bound on wave DM mass



Then in density power spectrum:

1. white-noise excess
2. free-streaming suppression

1. and 2. not seen for $k < k_{\text{obs}} \sim 10 \text{ Mpc}^{-1}$



$$m \gtrsim 10^{-19} \text{ eV}$$

MA & Mirbabayi (2022/24)
2211.09775

comparison with literature

$$m \gtrsim 1 \times 10^{-21} \text{ eV}$$

Banik et. al (2019) — Streams

$$m \gtrsim 3 \times 10^{-21} \text{ eV}$$

Nadler et. al (2021) — MW satellites

$$m \gtrsim 2 \times 10^{-20}$$

Rogers and Peiris (2021) — $\text{Ly}\alpha$

$$m \gtrsim 4 \times 10^{-21} \text{ eV}$$

Powell et. al (2023) — lensing

$$m \gtrsim 3 \times 10^{-19} \text{ eV}$$

Dalal & Kravtsov (2022) — dynamical heating
+May, Dalal & Kravtsov (2025)

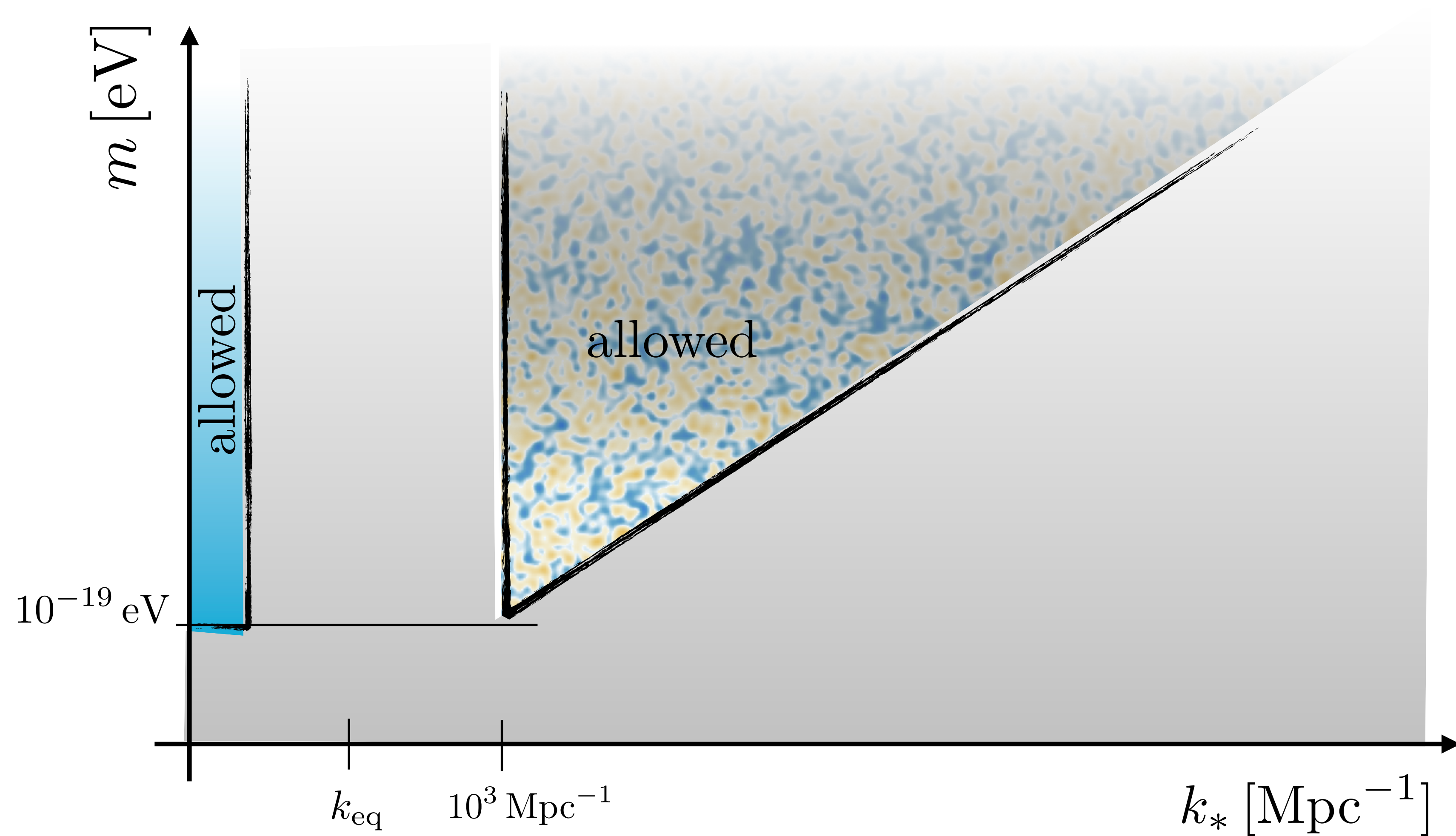
$$m \gtrsim 10^{-19} \text{ eV}$$

MA & Mirbabayi (2022/24)

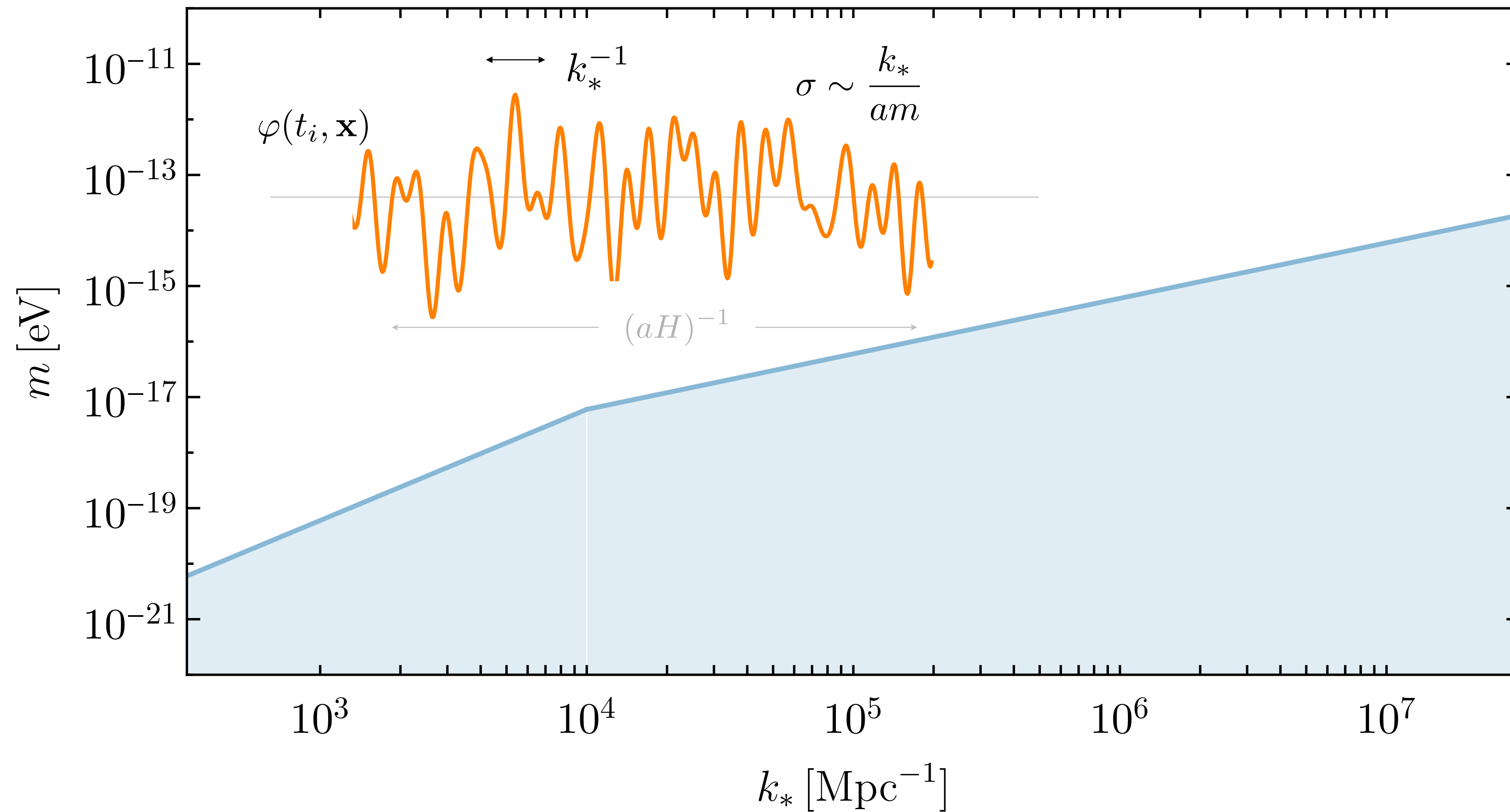
* constraints come from different observations and assumptions about production mechanisms, compare with care.

* For thermal production, bound is $\sim$ few keV — why the difference ? — Ask me.

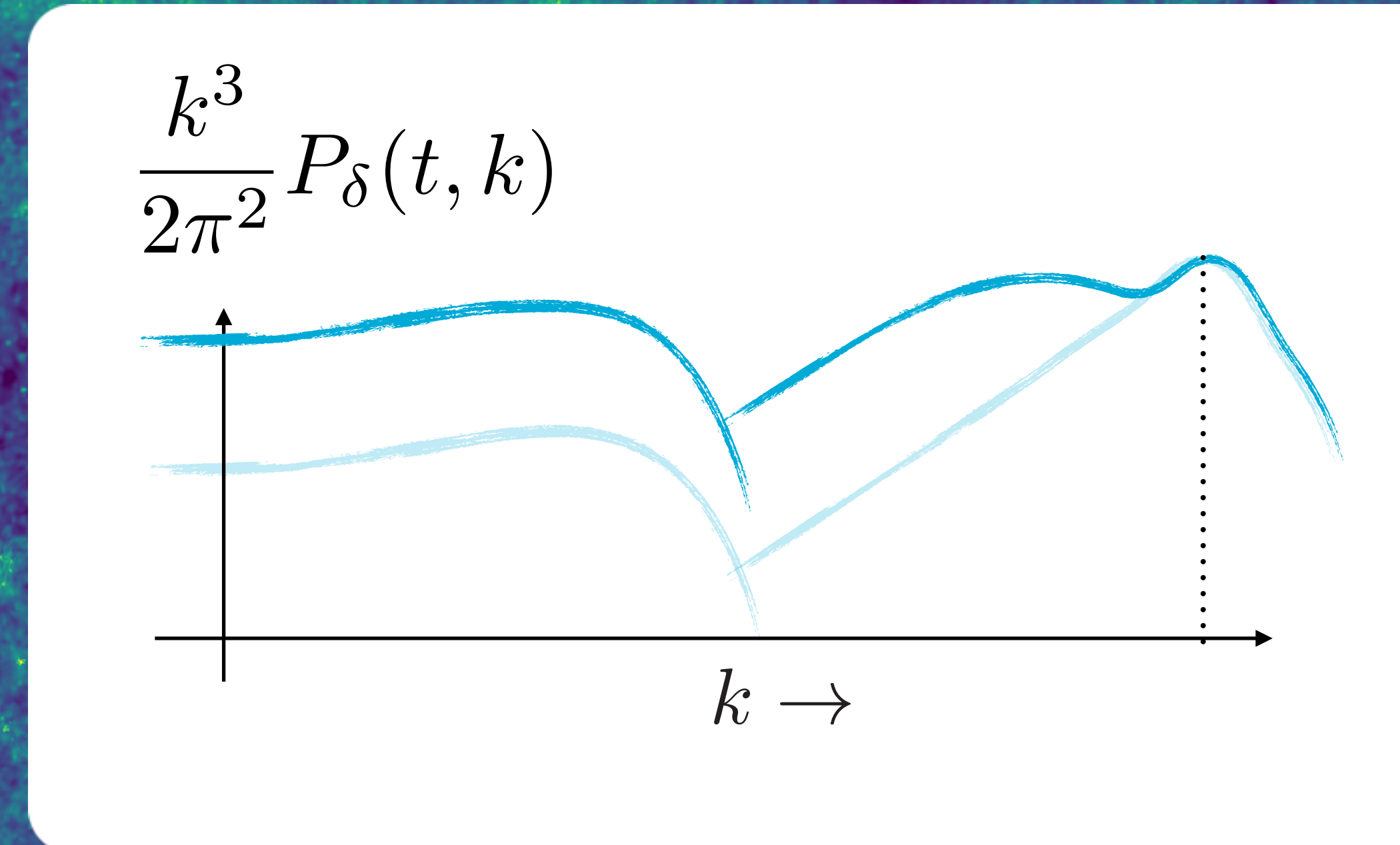
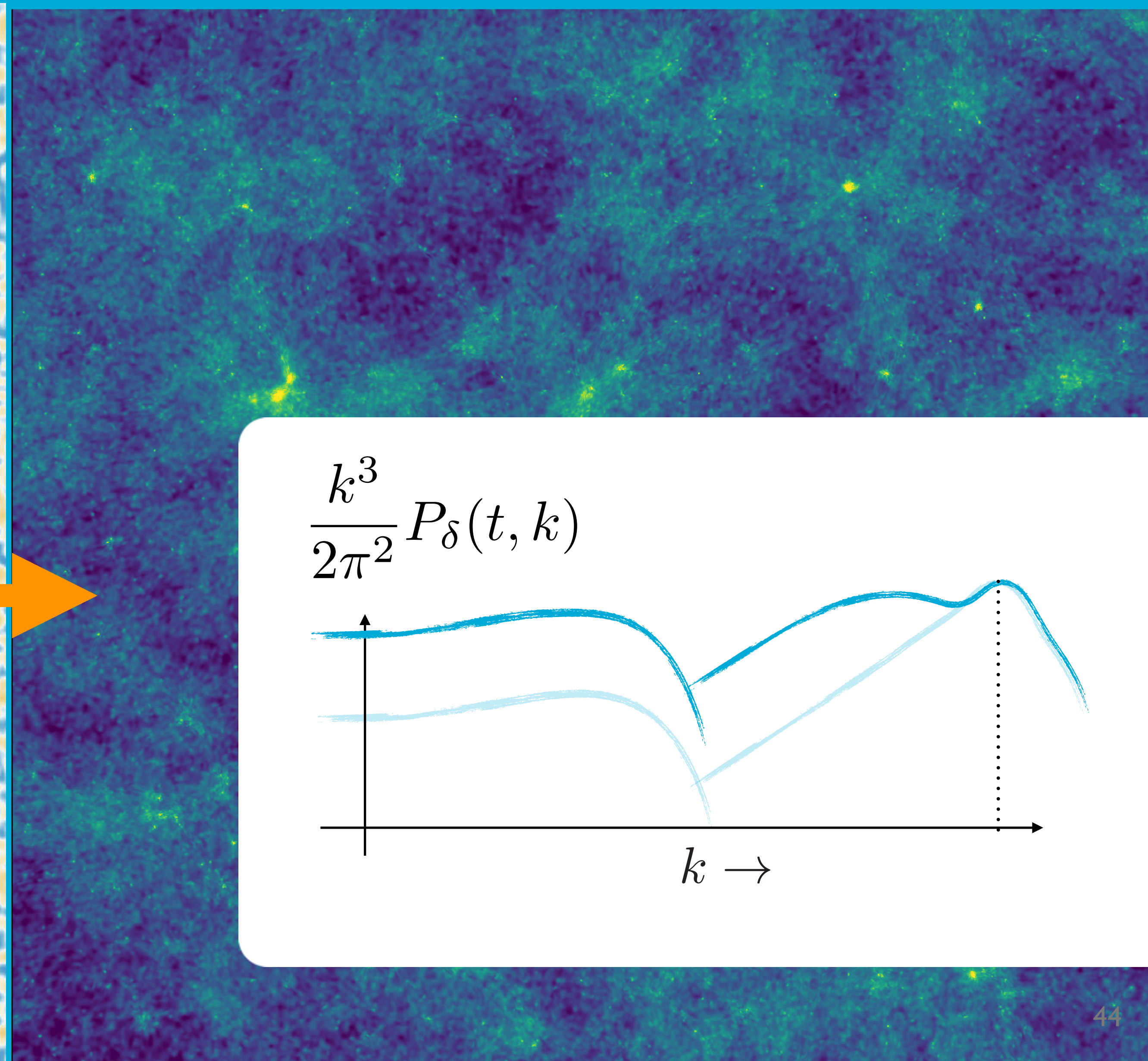
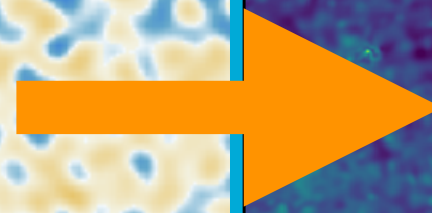
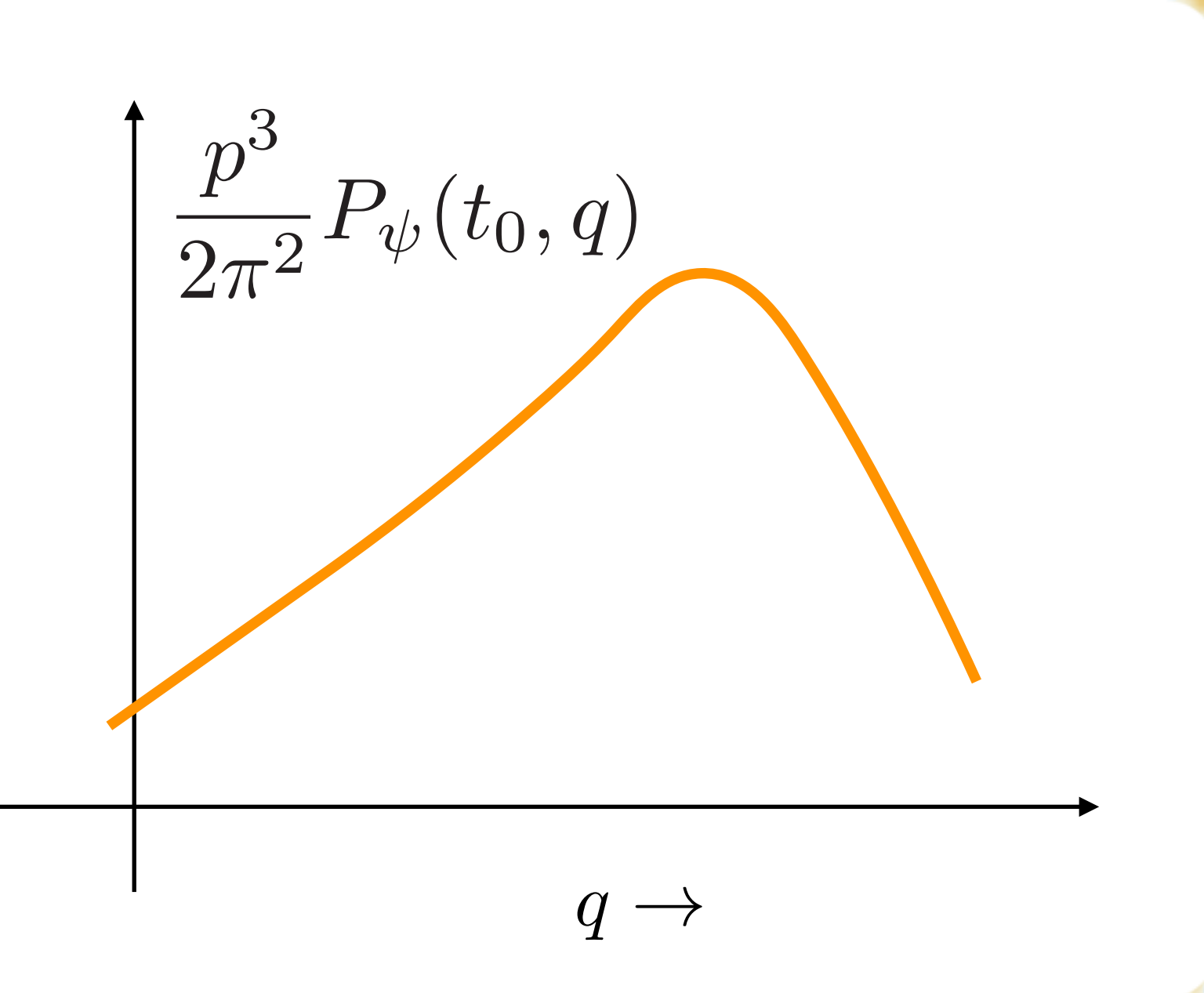
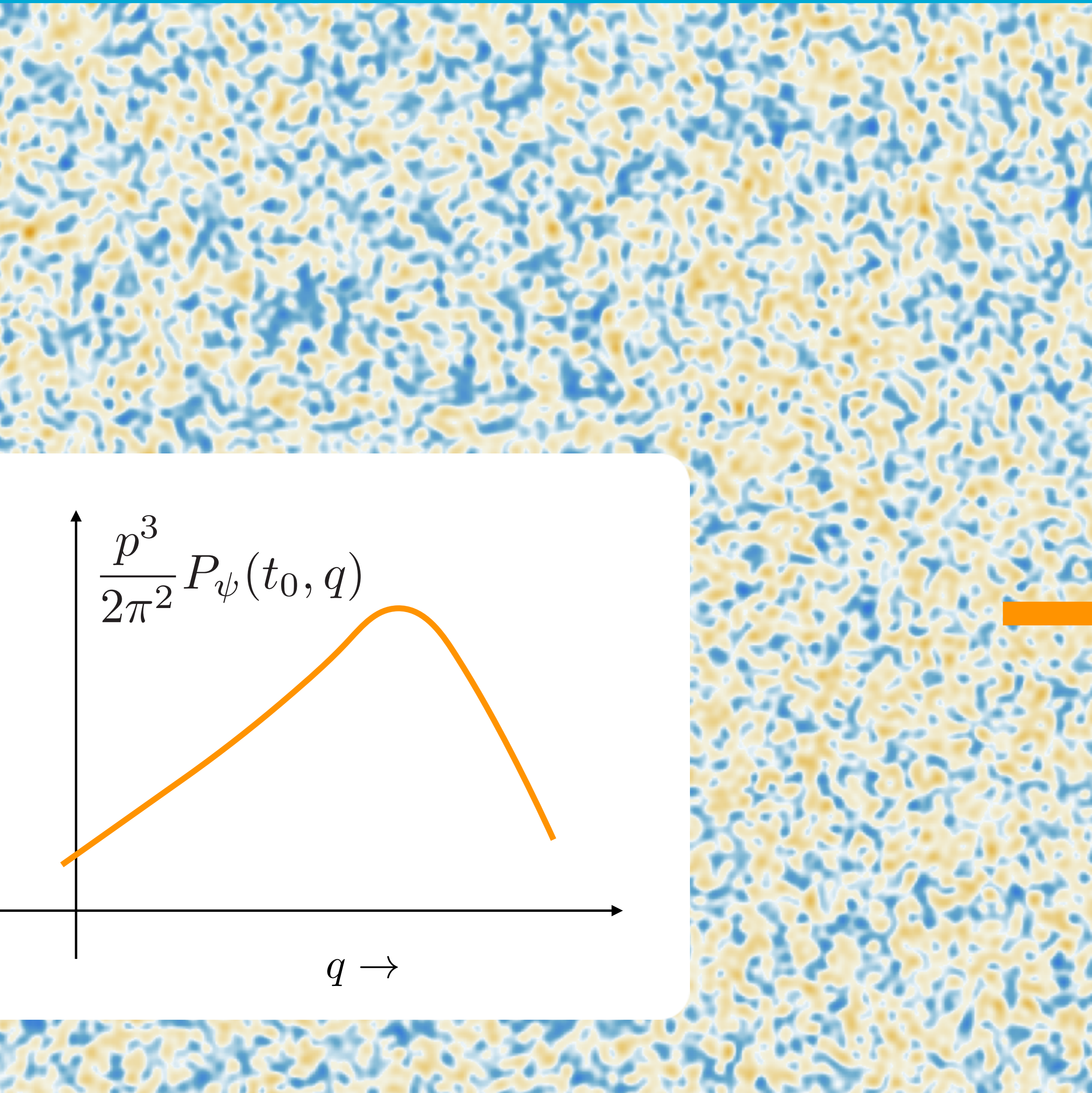
the lower bound on wave DM mass



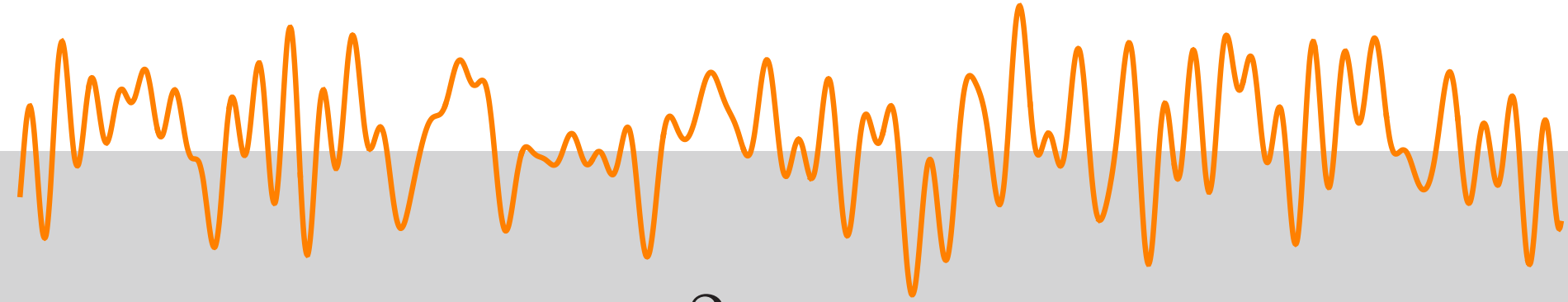
from MW satellites



key input and output



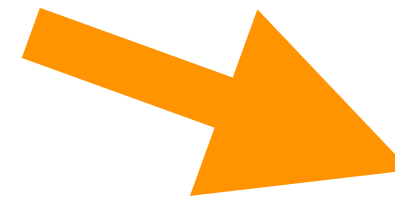
how is the calculation done?



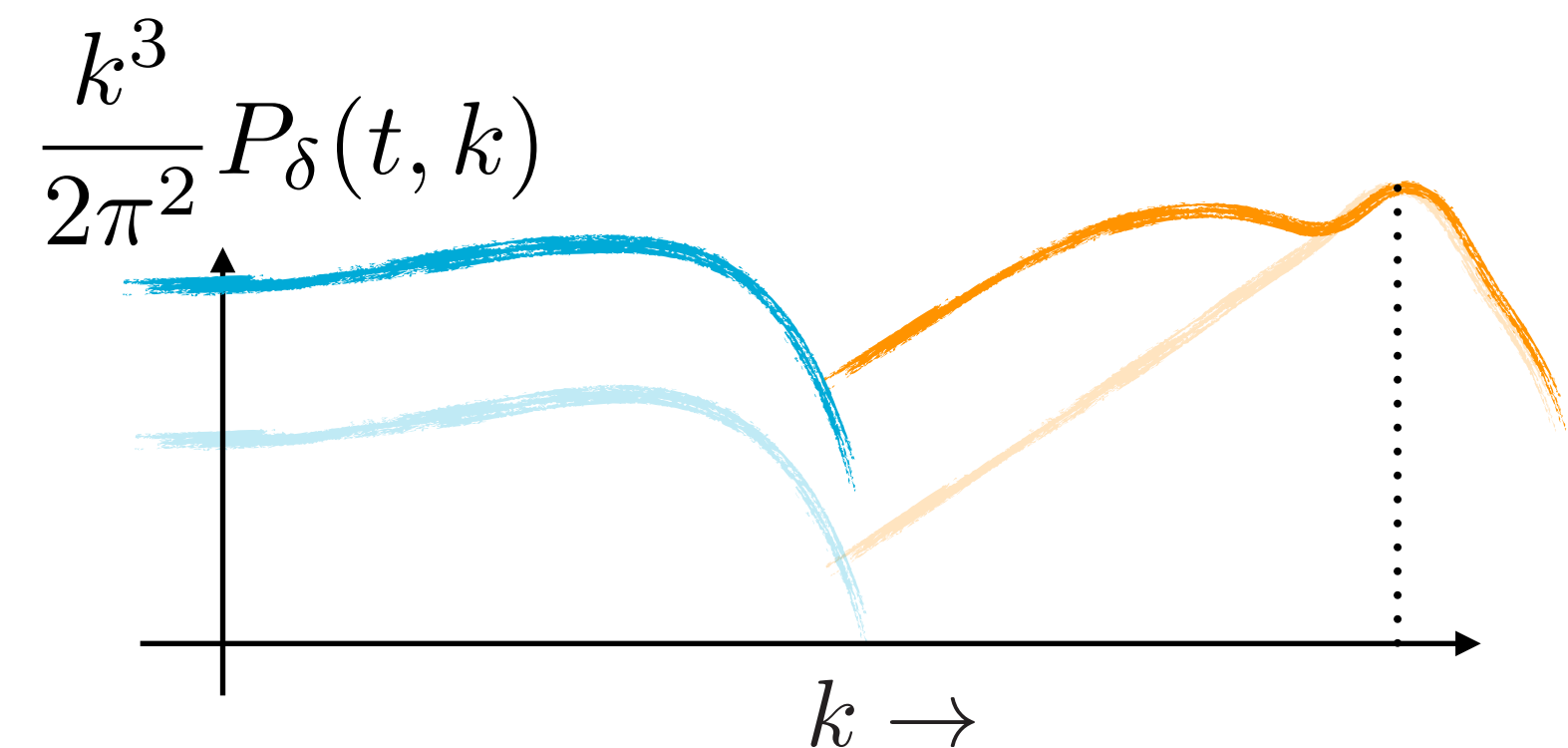
$$i\partial_\eta\psi = -\frac{\nabla^2}{2m}\psi + ma^2\Phi\psi$$

$$\nabla^2\Phi = \frac{4\pi G}{a}(\rho - \bar{\rho}), \quad \rho = m|\psi|^2$$

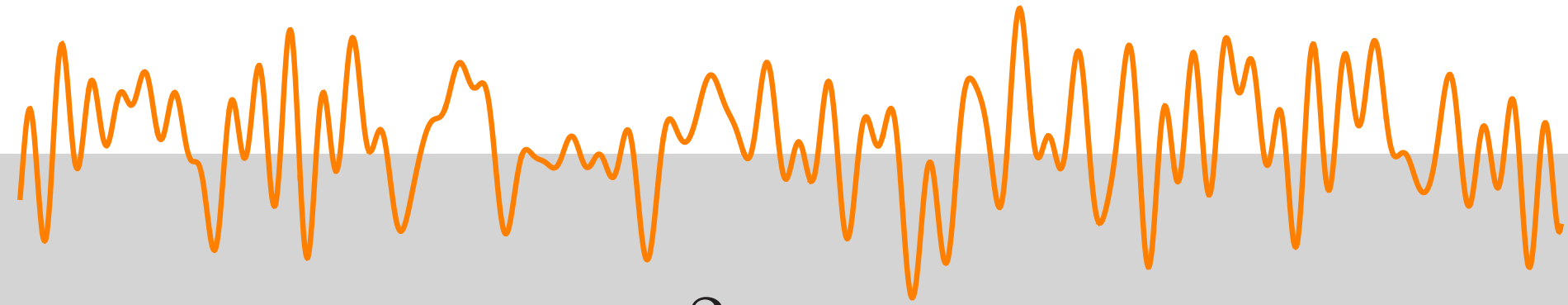
$$d\eta = a^{-2}dt$$



$$\langle \delta_{\mathbf{k}} \delta_{\mathbf{k}'} \rangle \propto P_\delta(k)$$



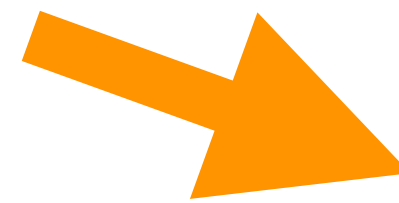
2-pt density correlation function = 4 pt field corr. func.



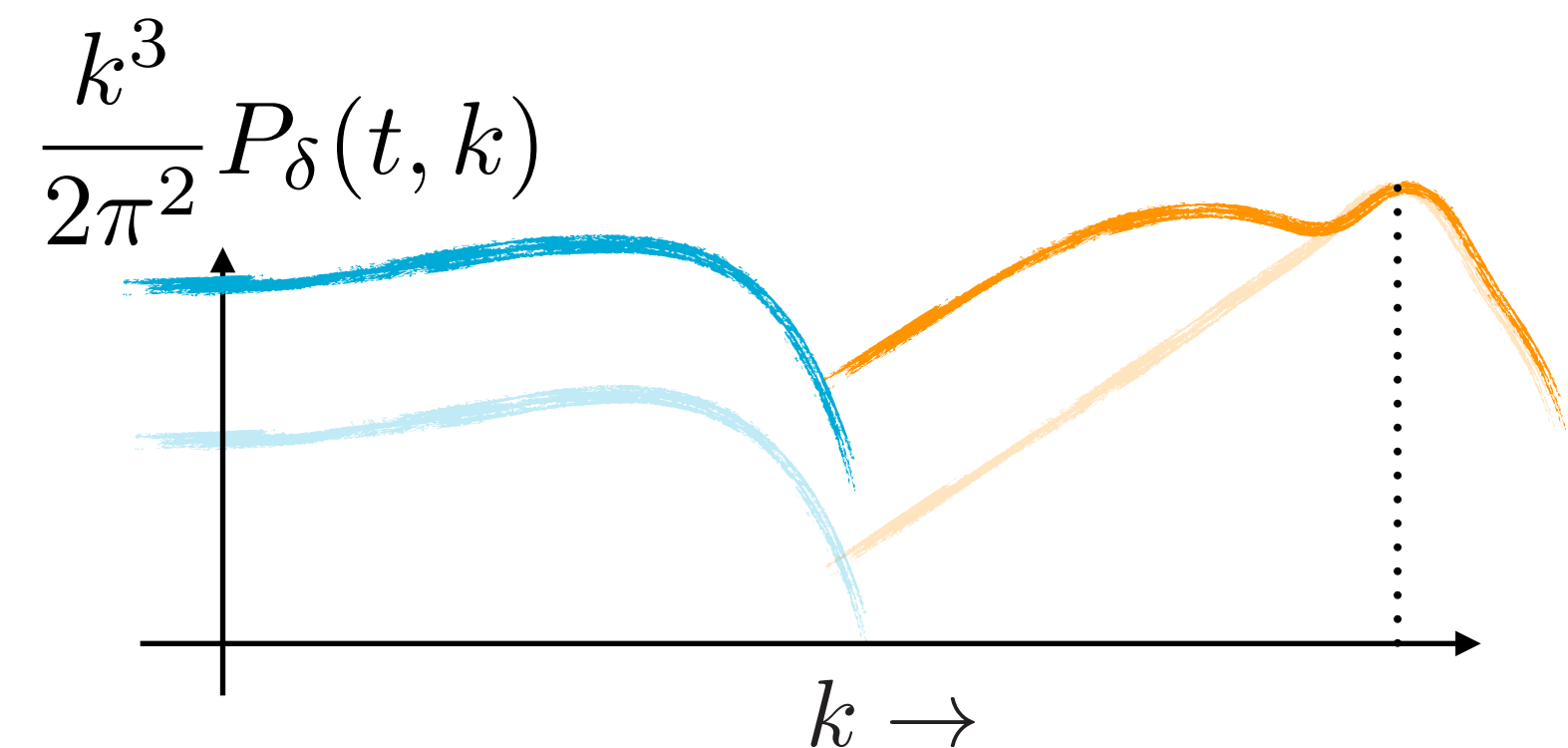
$$i\partial_\eta\psi = -\frac{\nabla^2}{2m}\psi + ma^2\Phi\psi$$

$$\nabla^2\Phi = \frac{4\pi G}{a}(\rho - \bar{\rho}), \quad \rho = m|\psi|^2$$

$$d\eta = a^{-2}dt$$



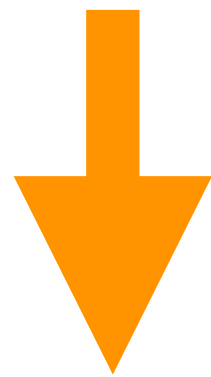
$$\langle\psi\psi\psi\psi\rangle \sim \langle\delta_{\mathbf{k}}\delta_{\mathbf{k}'}\rangle \propto P_\delta(k)$$



field eq. to eqns. for distribution (correlation) functions

$$i\partial_\eta \psi_{\mathbf{q}} = \frac{q^2}{2m} \psi_{\mathbf{q}} + ma^2 \int_{\mathbf{p}} \Phi_{\mathbf{p}} \psi_{\mathbf{q}-\mathbf{p}}$$

$$\Phi_{\mathbf{q}} = -\frac{4\pi G}{aq^2} \rho_{\mathbf{q}}$$



$$(2\pi)^3 \delta_D(\mathbf{k} + \mathbf{k}') \partial_\eta f(p) = \frac{3\bar{H}_0^2}{2} a \int_{\mathbf{k}, \mathbf{q}} \Delta_{\mathbf{k}} g_{\mathbf{k}\mathbf{k}'}(\mathbf{q}, \mathbf{p}) \approx 0$$

$$\begin{aligned} \partial_\eta g_{\mathbf{k}\mathbf{k}'}(\mathbf{p}, \mathbf{p}') + i \left(\frac{\mathbf{p} \cdot \mathbf{k}}{m} + \frac{\mathbf{p}' \cdot \mathbf{k}'}{m} \right) g_{\mathbf{k}\mathbf{k}'}(\mathbf{p}, \mathbf{p}') - \frac{3}{2} \bar{H}_0^2 a \left[\Delta_{\mathbf{k}} f(p) \int_{\mathbf{l}} g_{\mathbf{k}\mathbf{k}'}(\mathbf{l}, \mathbf{p}') + \Delta_{\mathbf{k}'} f(p') \int_{\mathbf{l}} g_{\mathbf{k}'\mathbf{k}}(\mathbf{l}, \mathbf{p}) \right] \\ \approx \frac{3}{2} \bar{H}_0^2 a [\Delta_{\mathbf{k}} f(p) f(|\mathbf{p}' + \mathbf{k}/2|) f(|\mathbf{p}' - \mathbf{k}'/2|) + (1 \leftrightarrow 2)] (2\pi)^3 \delta_D(\mathbf{k} + \mathbf{k}'). \end{aligned}$$

$$\mathcal{L}\psi = 0 \Rightarrow \begin{aligned} \mathcal{L}_1 f &= S[g] \\ \mathcal{L}_2 g &\approx S[f] \end{aligned}$$

similar to truncated BBGKY

ignore connected 3~pt

power spectrum evolution

$$P_\delta(\eta, k) = P_\delta(\eta_0, k) + 3\bar{H}_0^2 \int_{\eta_0}^{\eta} d\eta' a(\eta') T_k^{(b)}(\eta, \eta') T_k^{(c)}(\eta, \eta')$$

$$T_k^{(i)}(\eta, \eta_0) = T_k^{\text{fs}(i)}(\eta, \eta_0) + \frac{3\bar{H}_0^2}{2} \int_{\eta_0}^{\eta} d\eta' a(\eta') T_k^{\text{fs}(b)}(\eta, \eta') T_k^{(i)}(\eta', \eta_0) \quad i = \text{b, c}$$

pure free-streaming gravitational clustering (with free-streaming)

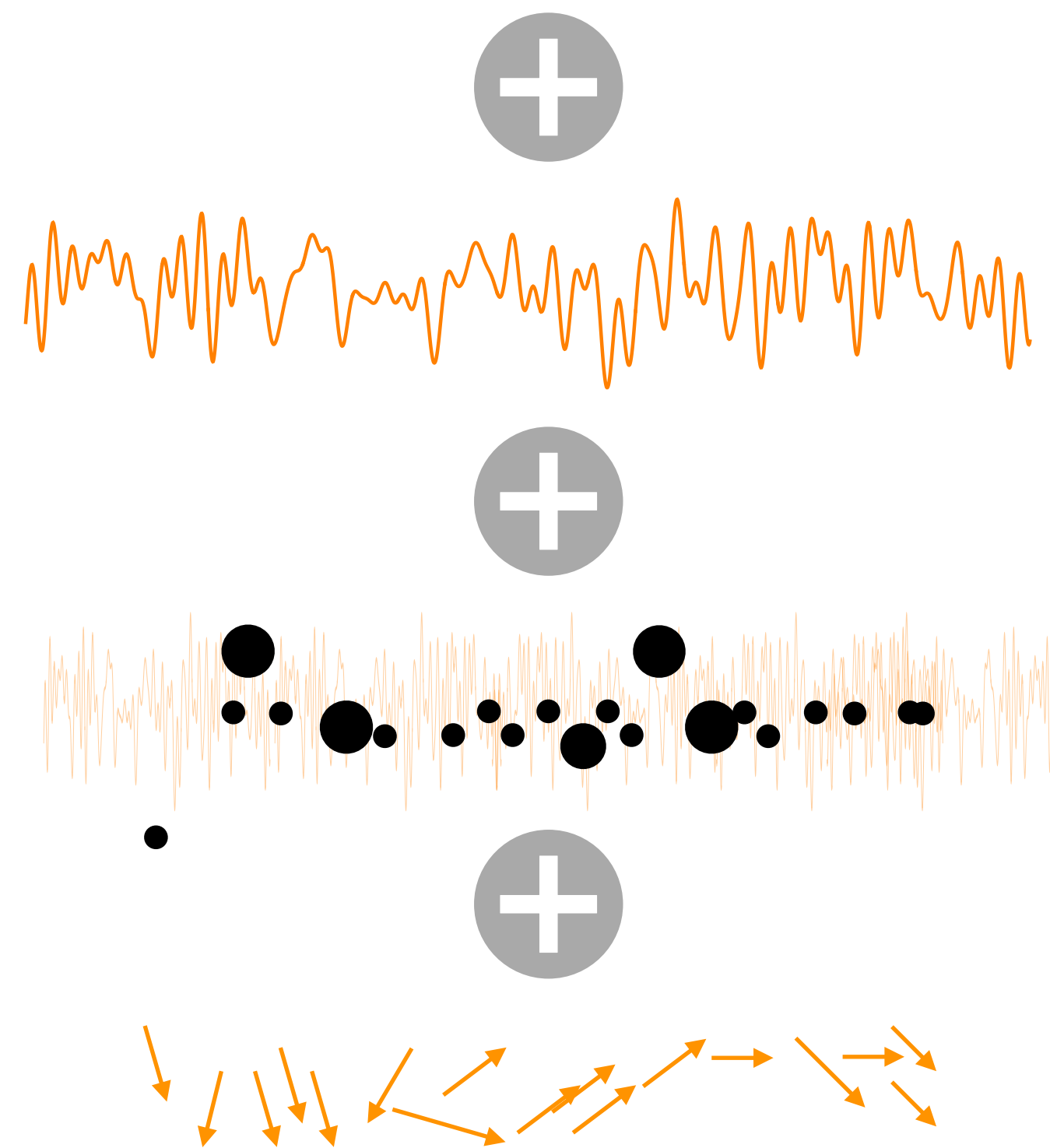
$$T_k^{\text{fs}(b)}(\eta, \eta') = \frac{2m}{k^2} \sin \left[\frac{k^2}{2m} (\eta - \eta') \right] \int_{\mathbf{p}} f(p) e^{-i \frac{\mathbf{p} \cdot \mathbf{k}}{m} (\eta - \eta')},$$

$$T_k^{\text{fs}(c)}(\eta, \eta') = \int_{\mathbf{p}} f(|\mathbf{p} + \mathbf{k}/2|) f(|\mathbf{p} - \mathbf{k}/2|) e^{-i \frac{\mathbf{p} \cdot \mathbf{k}}{m} (\eta - \eta')}$$

$$\bar{H}_0^2 = (8\pi G/3)\bar{\rho} \quad \Delta_{\mathbf{k}} f(p) \equiv (-im/k^2) [f(|\mathbf{p} + \mathbf{k}/2|) - f(|\mathbf{p} - \mathbf{k}/2|)] \quad 48$$

generality of the framework

<https://github.com/mustafaaamin/warm-and-random>



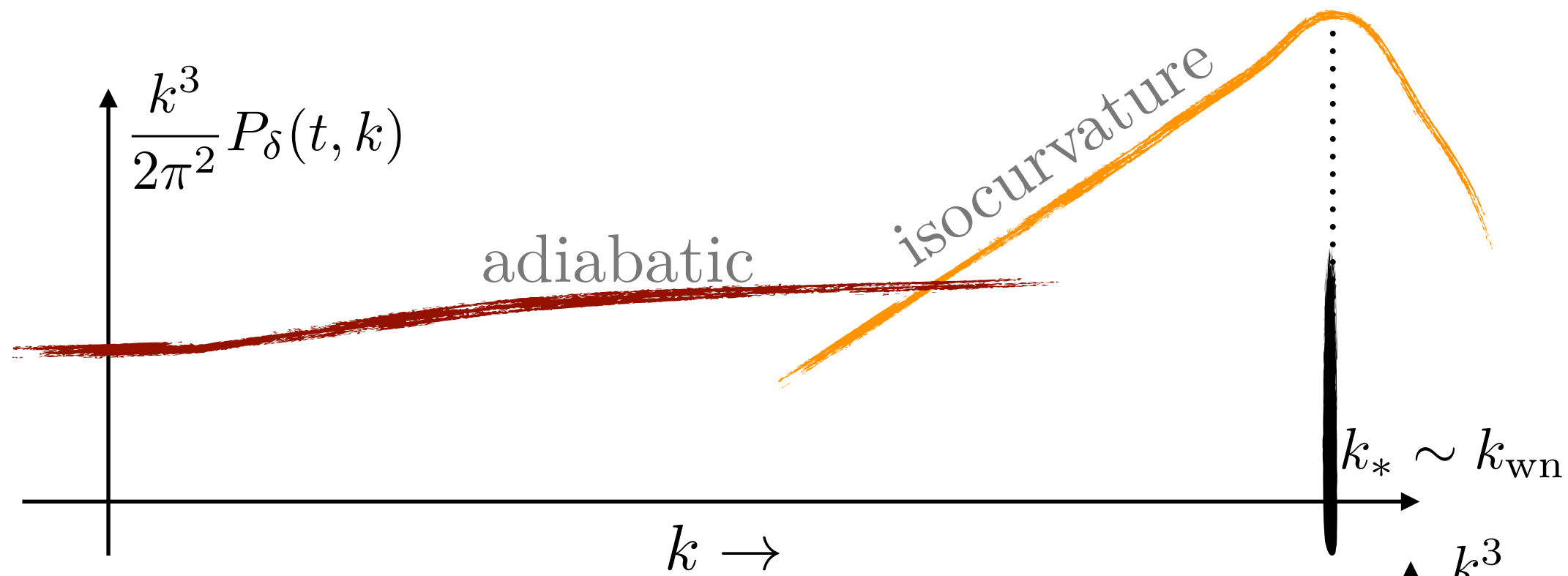
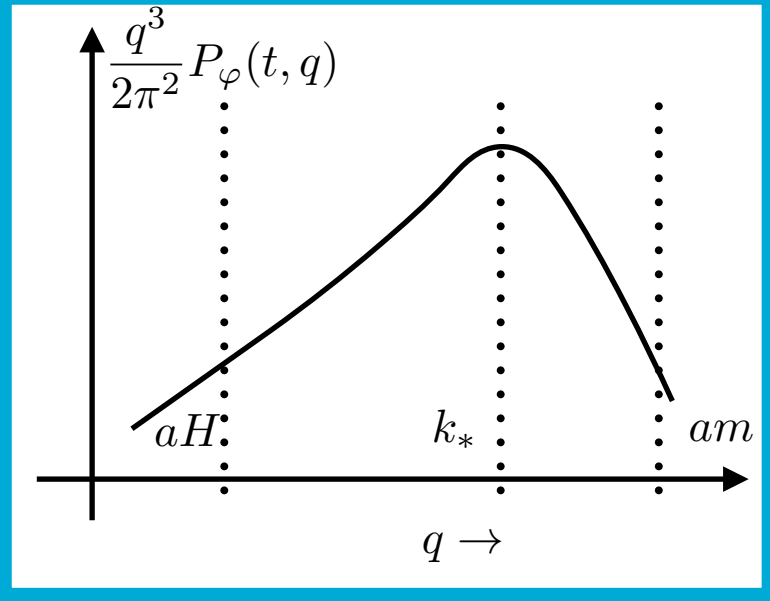
- works for point particles, solitons and fields with **arbitrary initial spectra**
- includes effects of **noise and warmth** (not captured by CLASS, AxiCAMB etc.)*
- works for **arbitrary # of components**
- can include non-gravitational interactions*

2211.09775, 2408.05591, 2503.20881, 2506.12131, 2510.15046, 2510.17977

with Delos, Ling, May, Mirbabayi & Yang

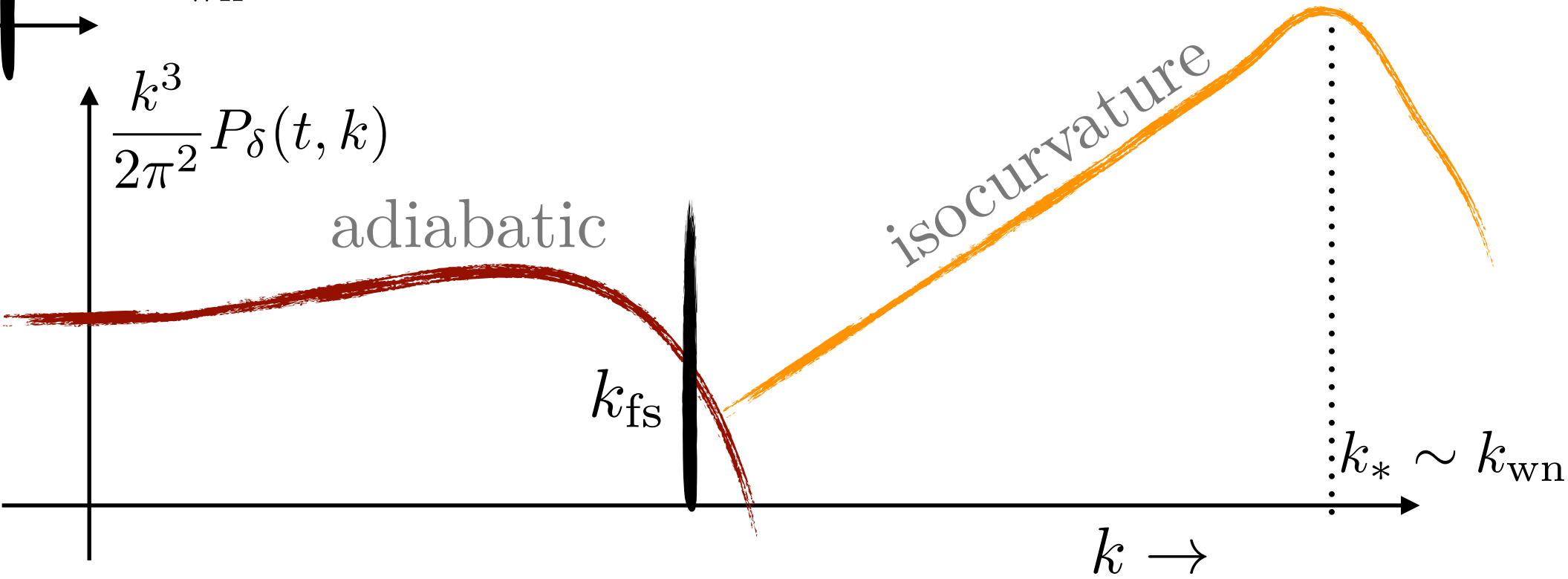
recently applied to high-redshift obs. by Gorghetto et. al (2025), Niemeyer & Kulkarni (in progress)
Jens et. al (I expect it will be in progress)

nonlinear structures

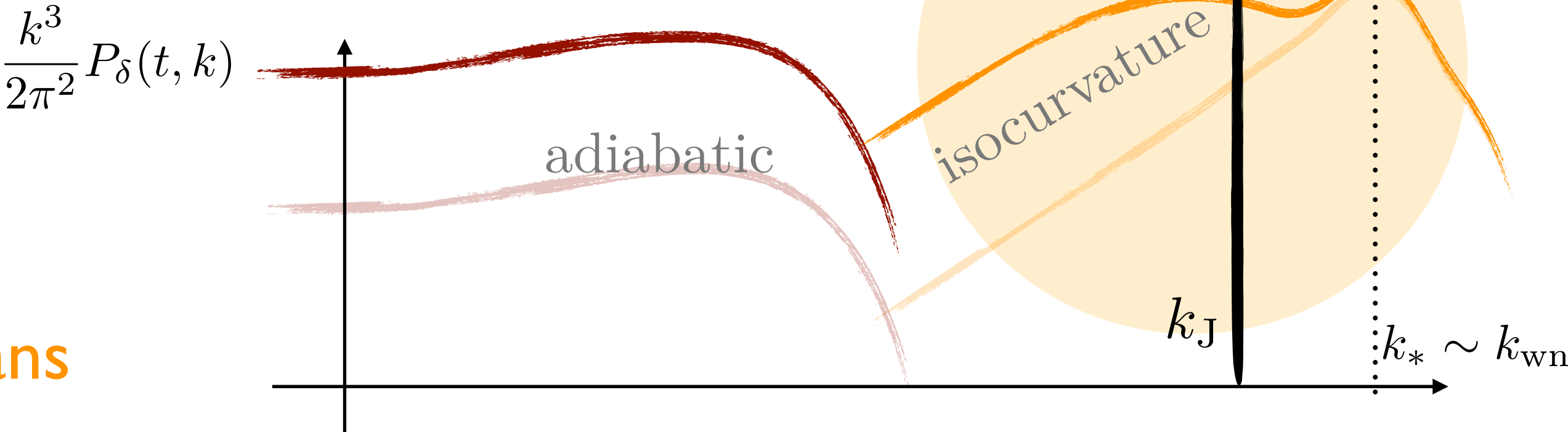


white noise

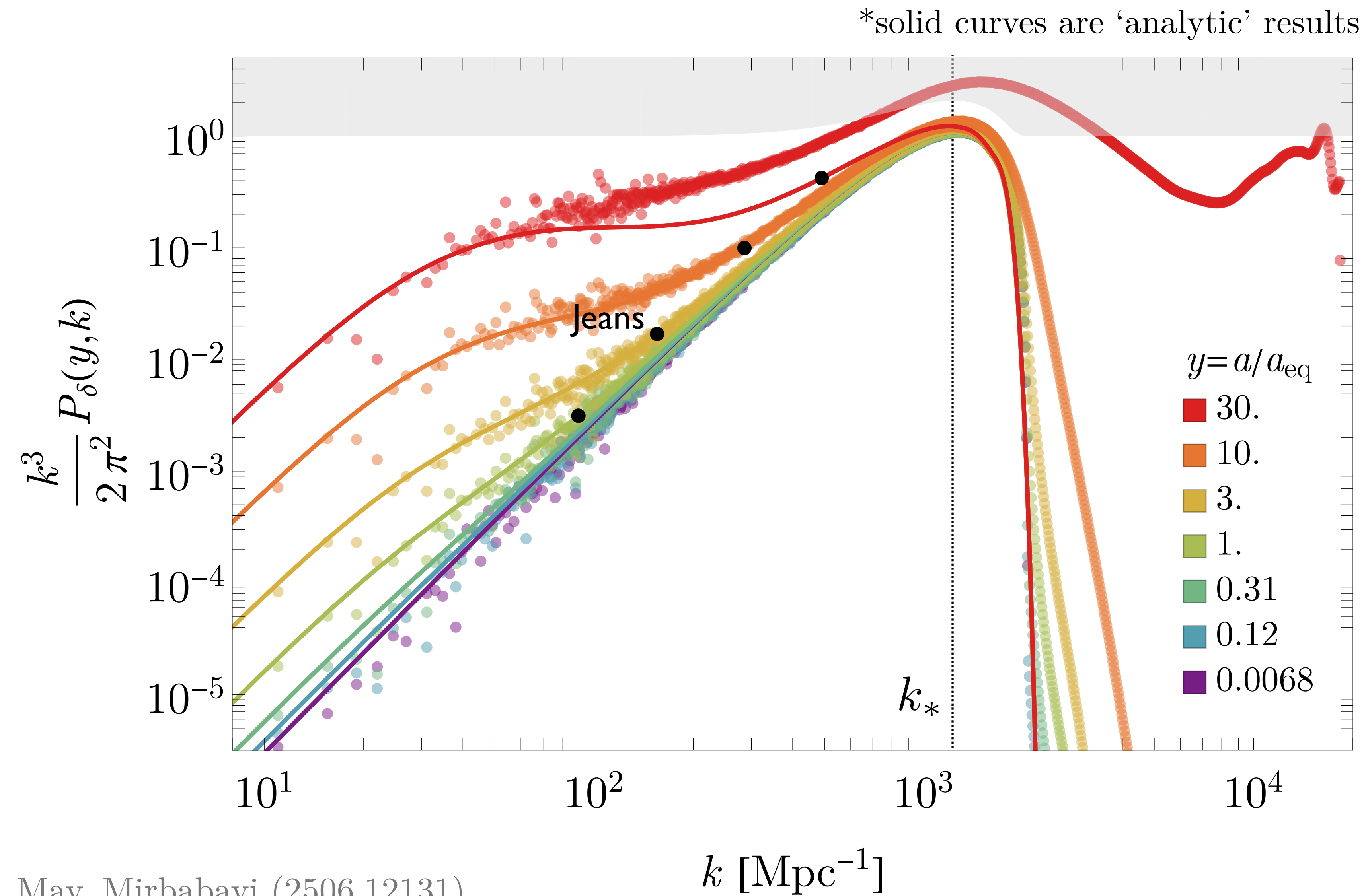
free-streaming



Jeans



comparison with simulations



$a/a_{\text{eq}} = 0.85$

$100 h^{-1} \text{ kpc}$

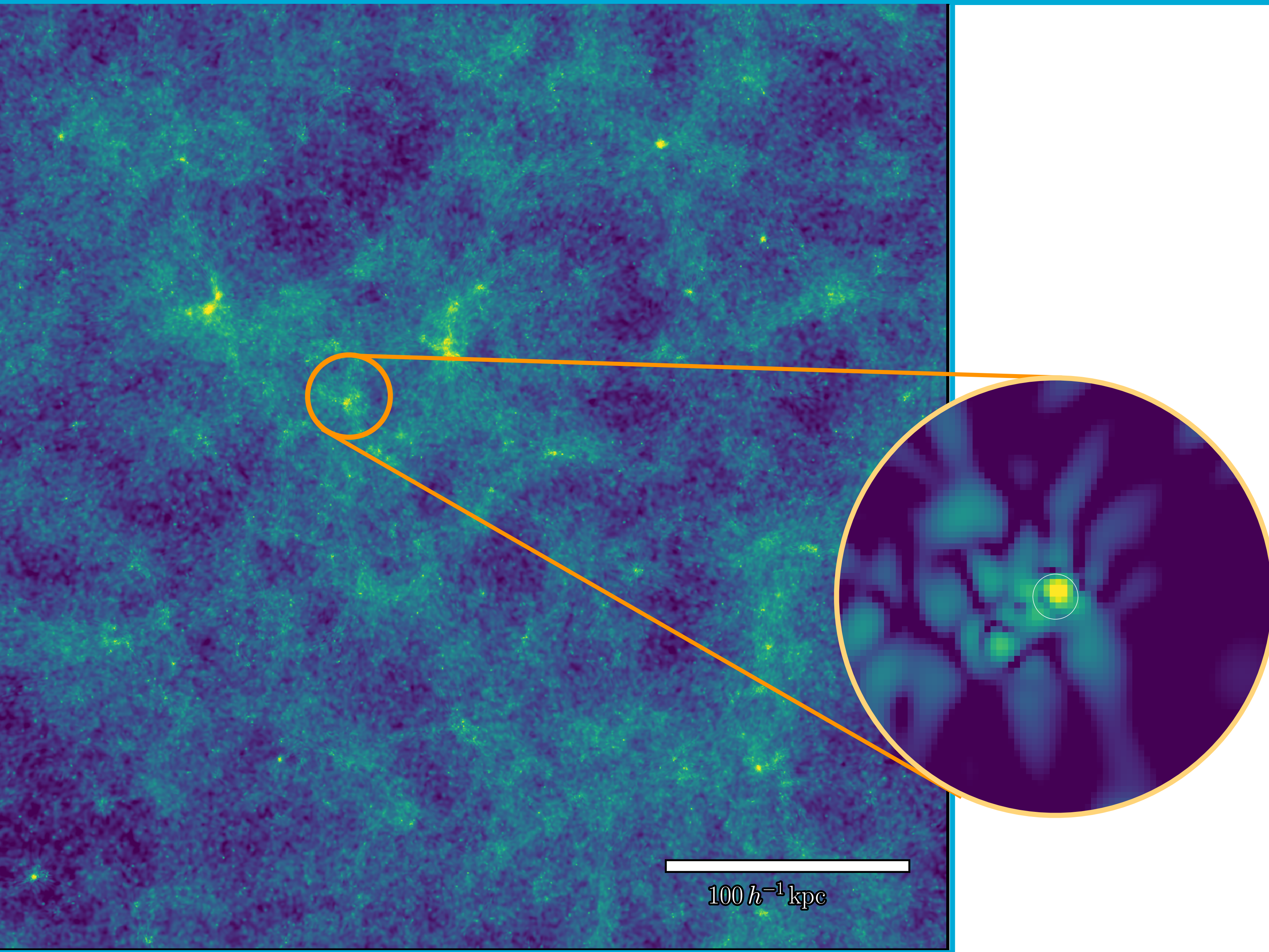
$a/a_{\text{eq}} = 5.6$

$100 h^{-1} \text{ kpc}$

$a/a_{\text{eq}} = 33.8$

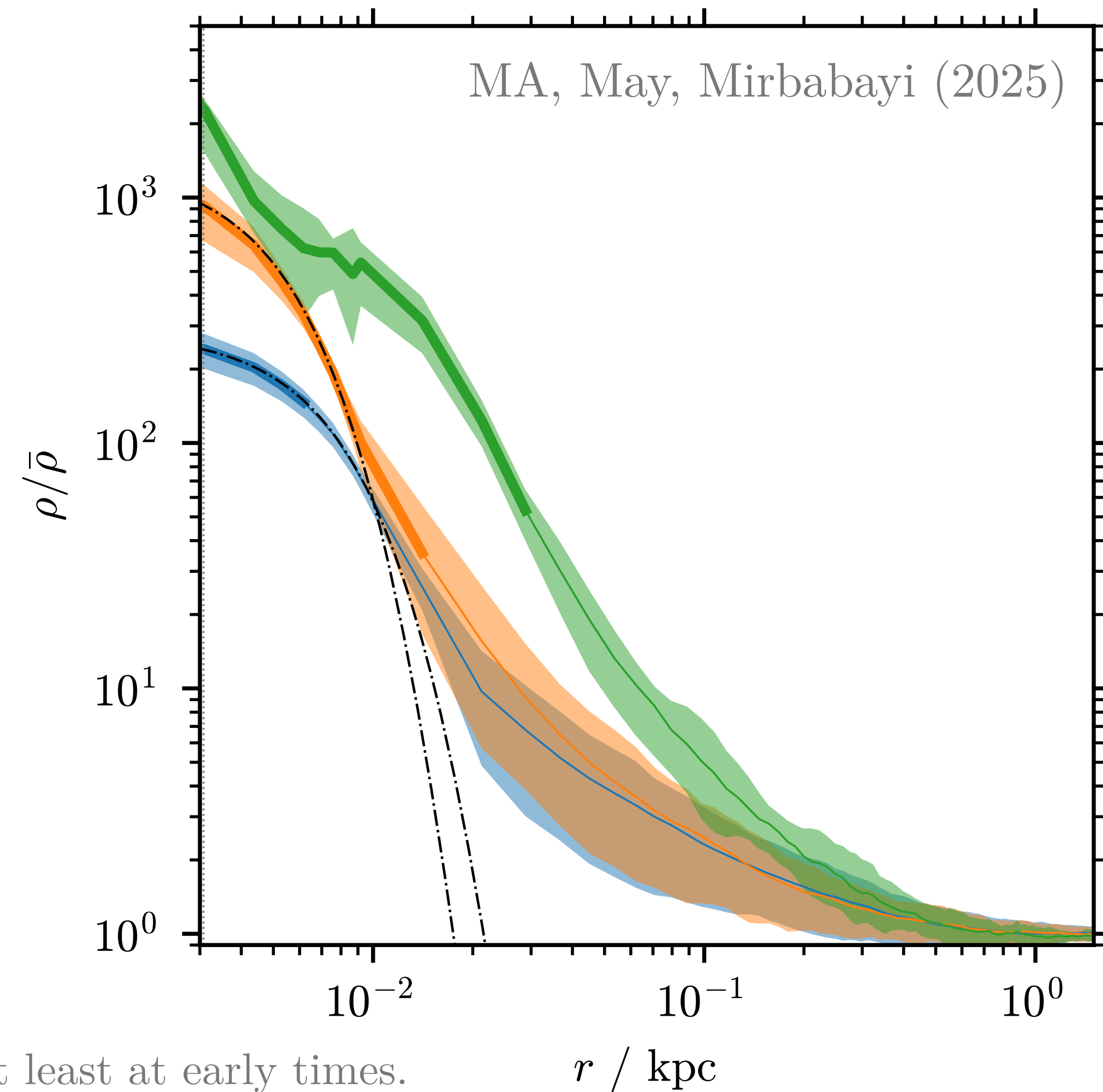
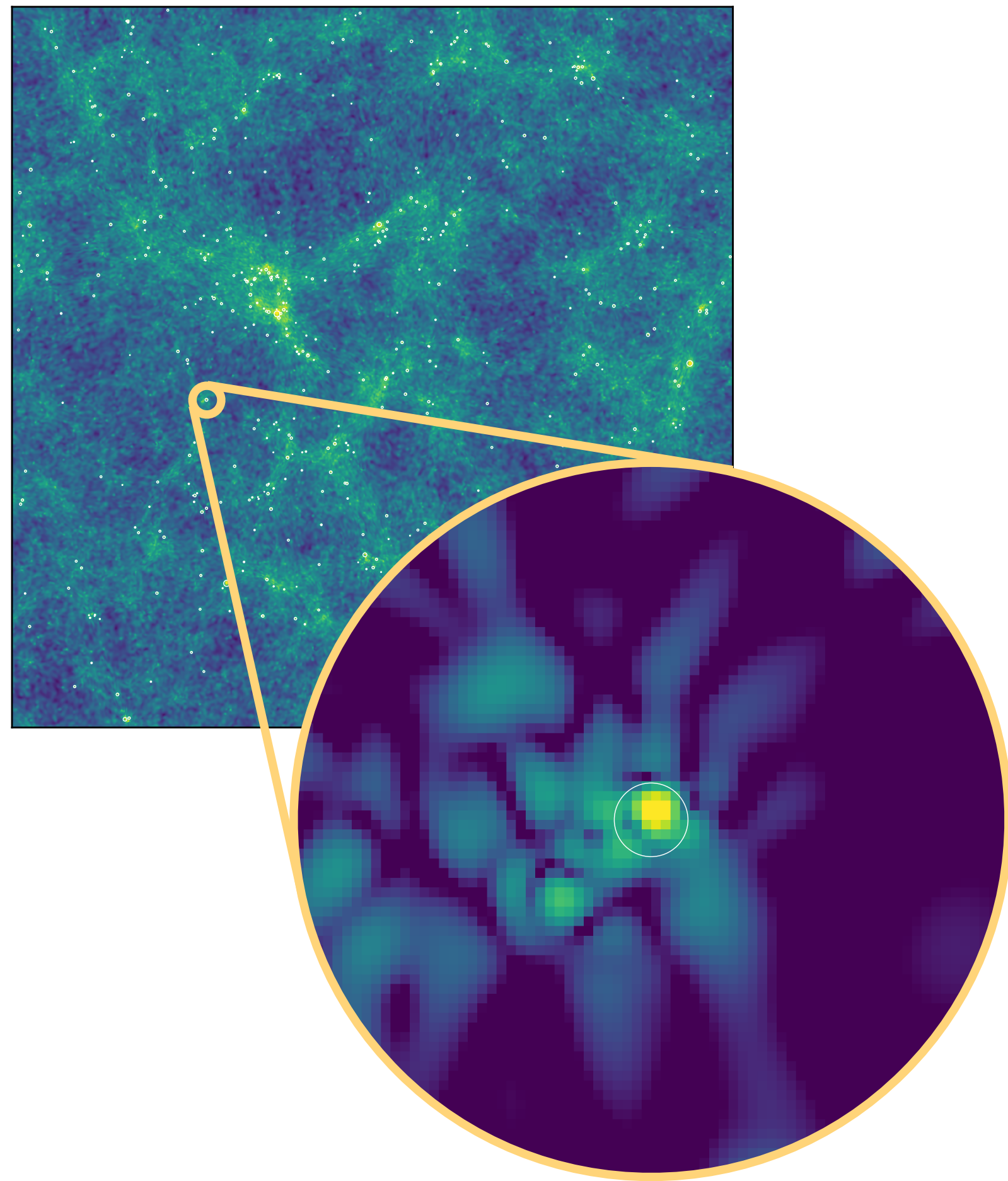
$100 h^{-1} \text{ kpc}$

early nonlinear structure formation



halos+solitons at $z > 100$

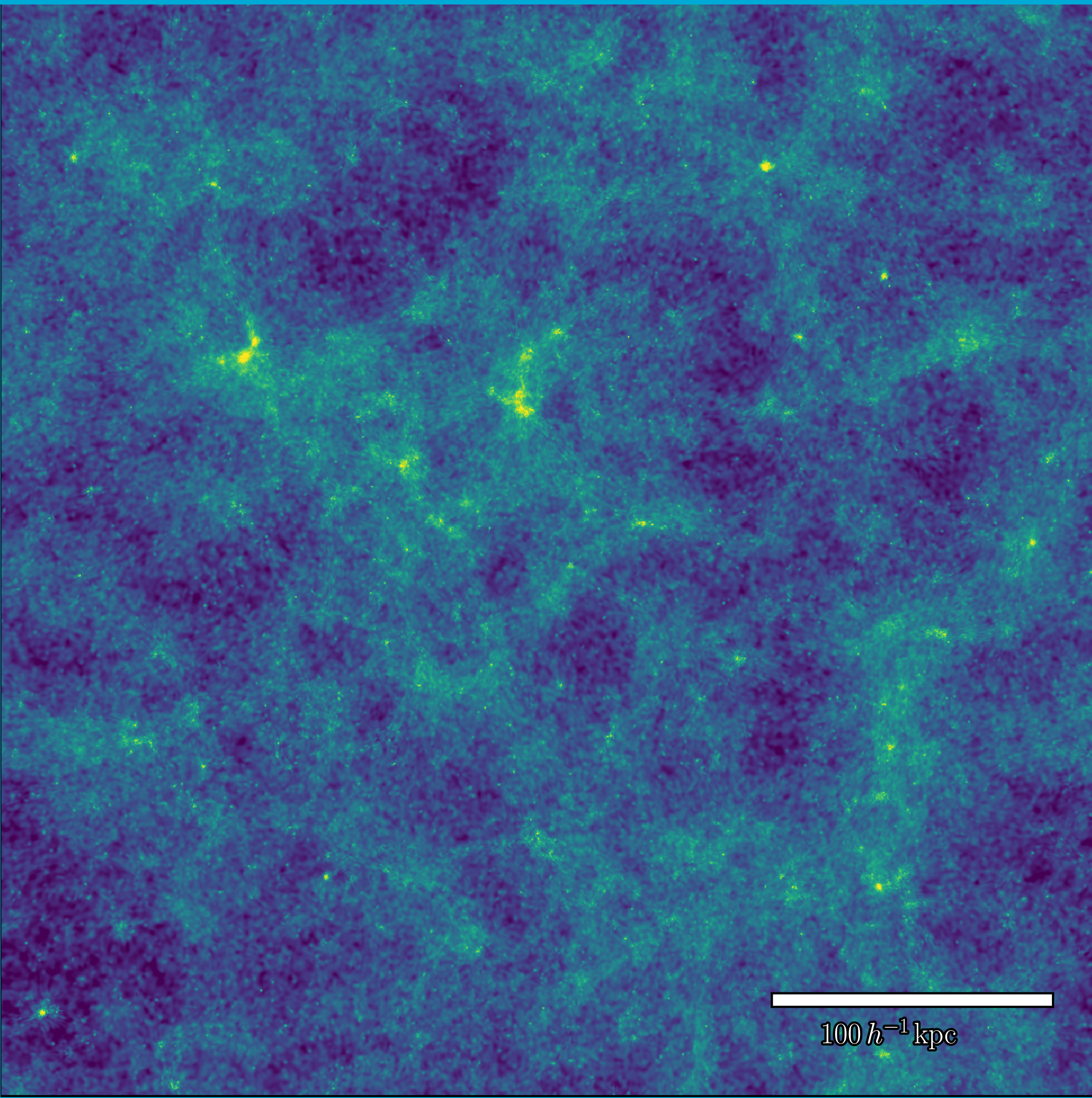
solitons — when do they form, in what fraction ?



* can be order unity fraction of mass in collapsed objects, at least at early times.

also see Gorghetto et. al (2024)

baryons?

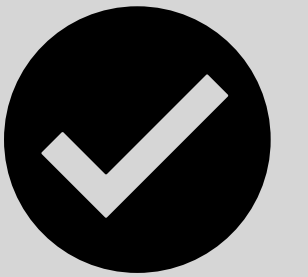


MA & May (in progress)

what is wave dark matter ?



how is it produced?



and its consequences

what does it do?



what *really* is wave dark matter?

what *really* is wave dark matter

$$i\hbar\frac{\partial}{\partial t}\Psi = -\frac{\hbar^2}{2m}\nabla^2\Psi + m\Phi\Psi, \quad \nabla^2\Phi = 4\pi Gm\Psi^*\Psi$$



what *really* is wave dark matter



$$i\hbar \frac{\partial}{\partial t} \Psi = -\frac{\hbar^2}{2m} \nabla^2 \Psi + m\Phi\Psi, \quad \nabla^2 \Phi = 4\pi G m \Psi^* \Psi$$

is single-field Schrödinger-Poisson the correct evolution ?

what *really* is wave dark matter



$$i\hbar \frac{\partial}{\partial t} \Psi = -\frac{\hbar^2}{2m} \nabla^2 \Psi + m\Phi\Psi, \quad \nabla^2 \Phi = 4\pi G m \Psi^* \Psi$$

single-field Schrödinger-Poisson equation ?

finite quantum coherence (may not be a single condensate)

what have I learnt

waving in the dark is fun

observationally accessible*

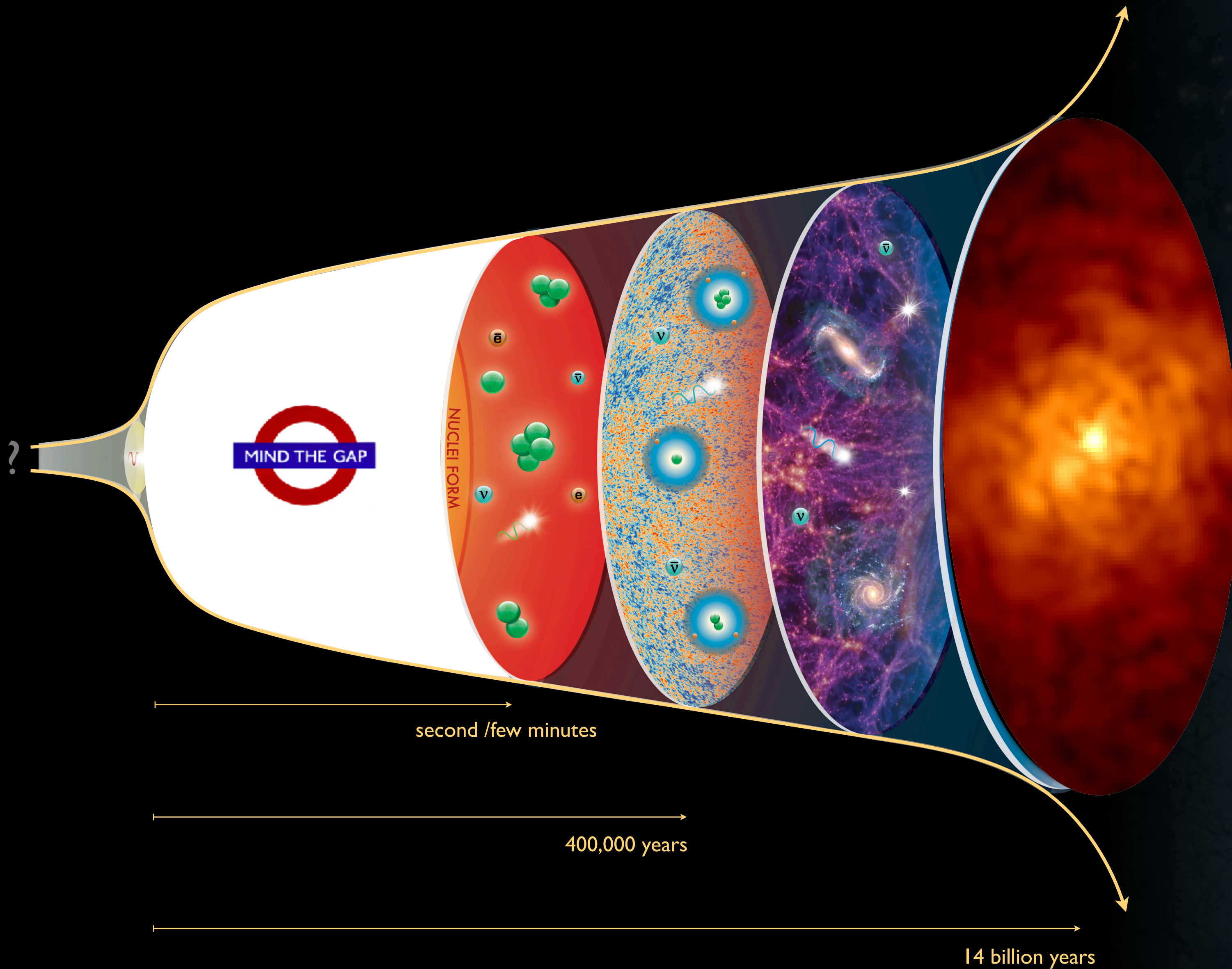
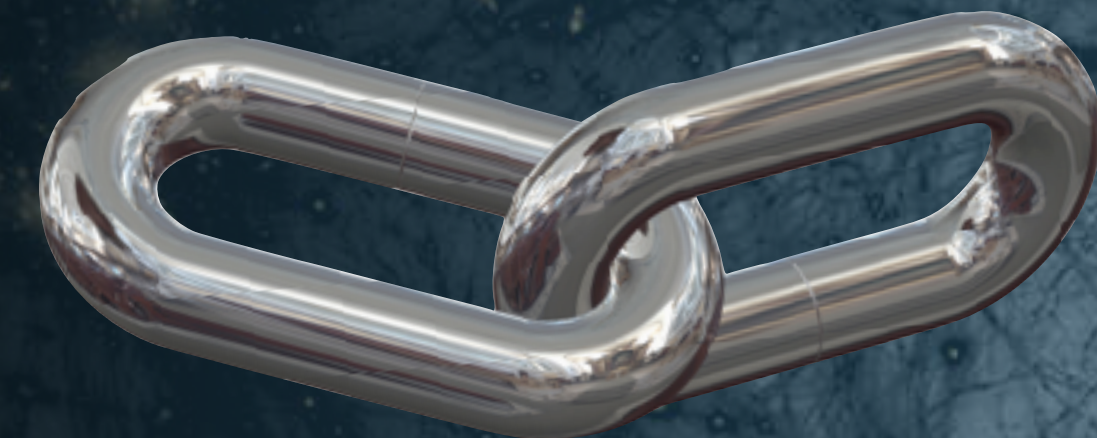
forces me to think globally

window into fundamental questions

**dark matter
distribution**

production

microscopic properties



what is wave dark matter ?



how is it produced?

what does it do?



what *really* is wave dark matter?

what is wave dark matter ?



how is it produced?

what does it do?

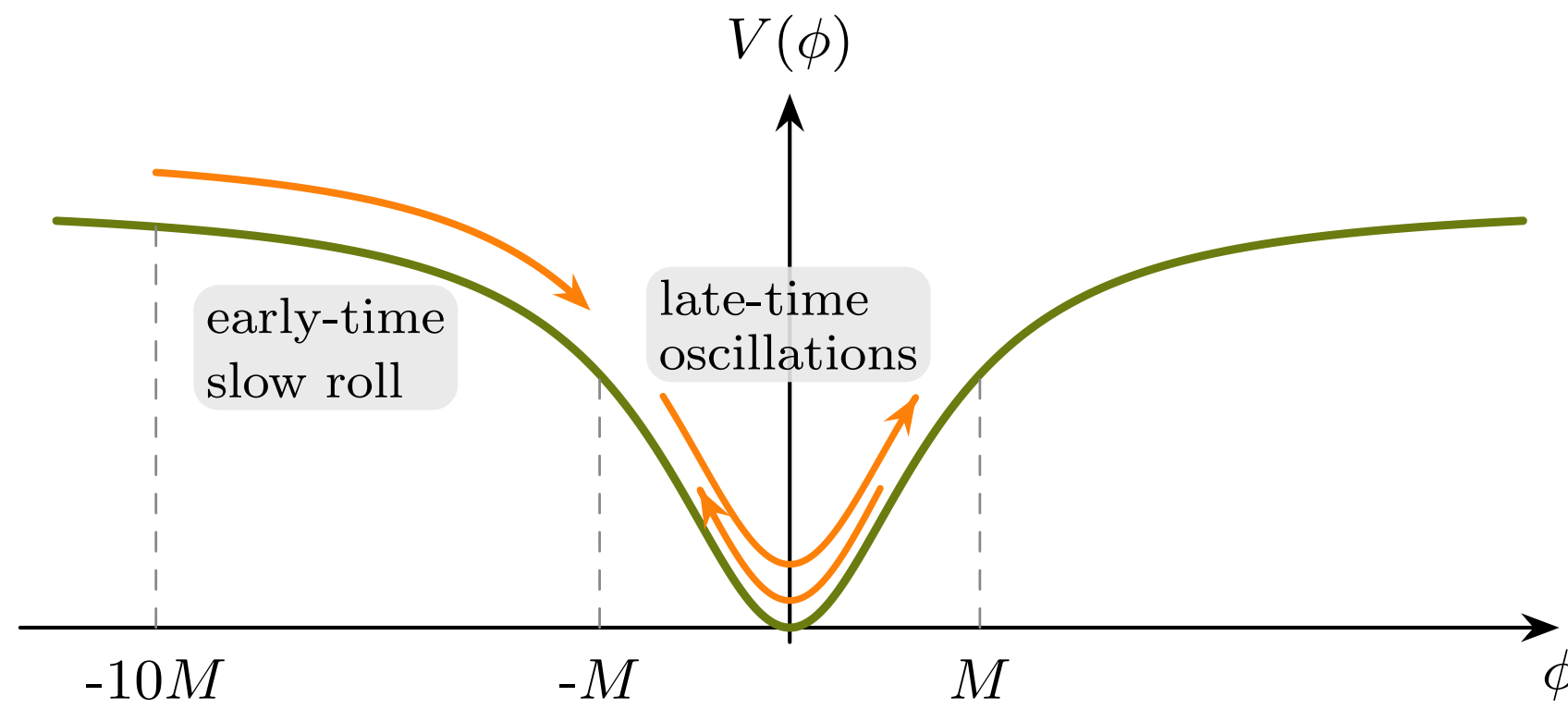


Extra slides about evolving DE

Late-time oscillating DE in light of DESI

MA Bertschinger & Zukin
Jiang, MA & Shafieloo

arXiv: 1108.1793
arXiv: 2606.24221



Best fit: $M \sim 10^{-2} m_{\text{pl}}$, $V''(\phi = 0) \sim 10^4 H_0^2$

