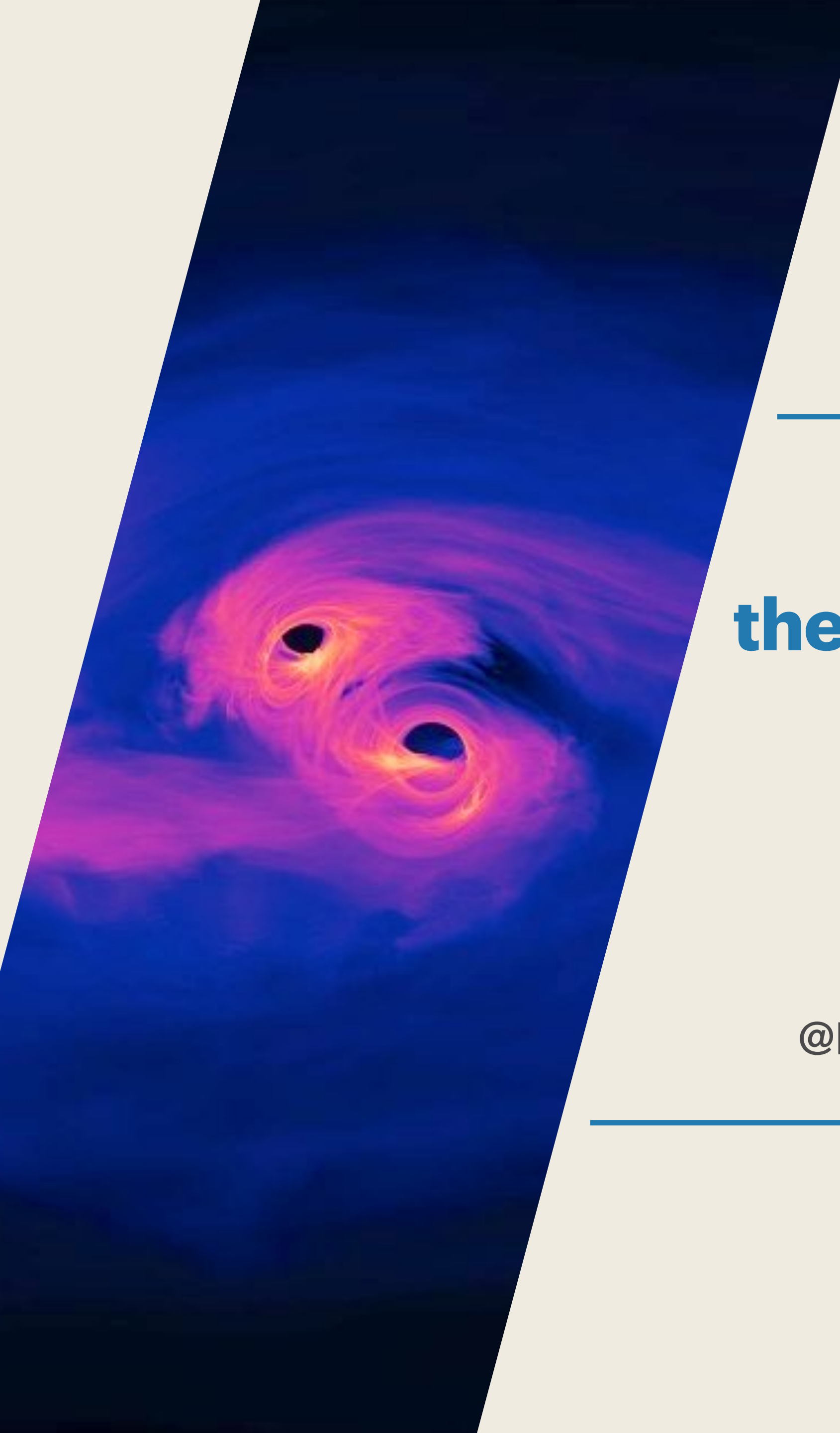




FC & M.Oguri

Phys. Rev. D**112**(2025)083502



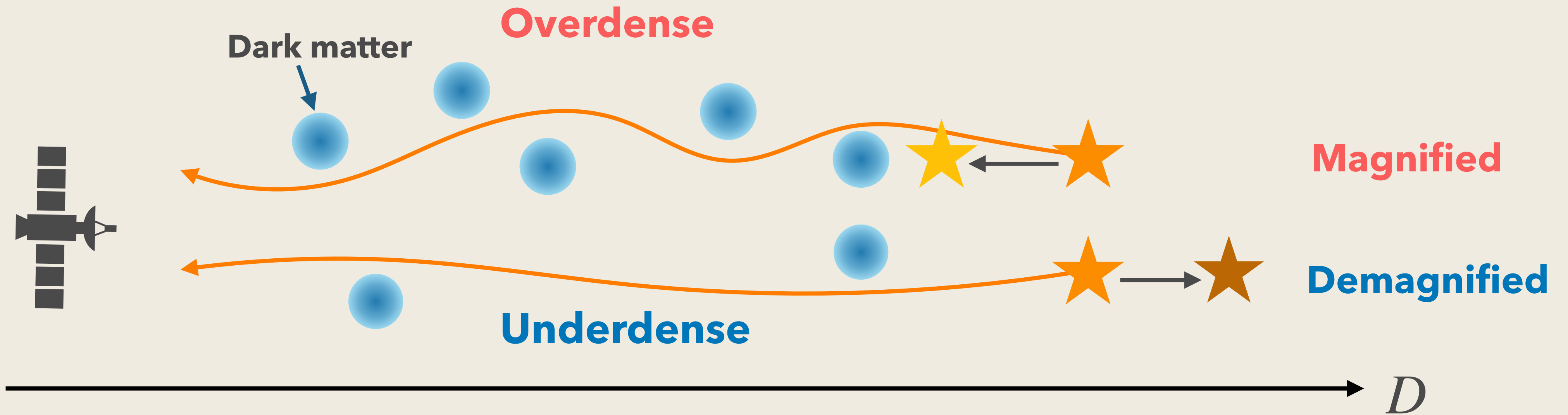
Constraining the lensing dispersion from the angular clustering of binary black hole mergers

Fumihiro Chuman (中馬 史博) Chiba Univ.

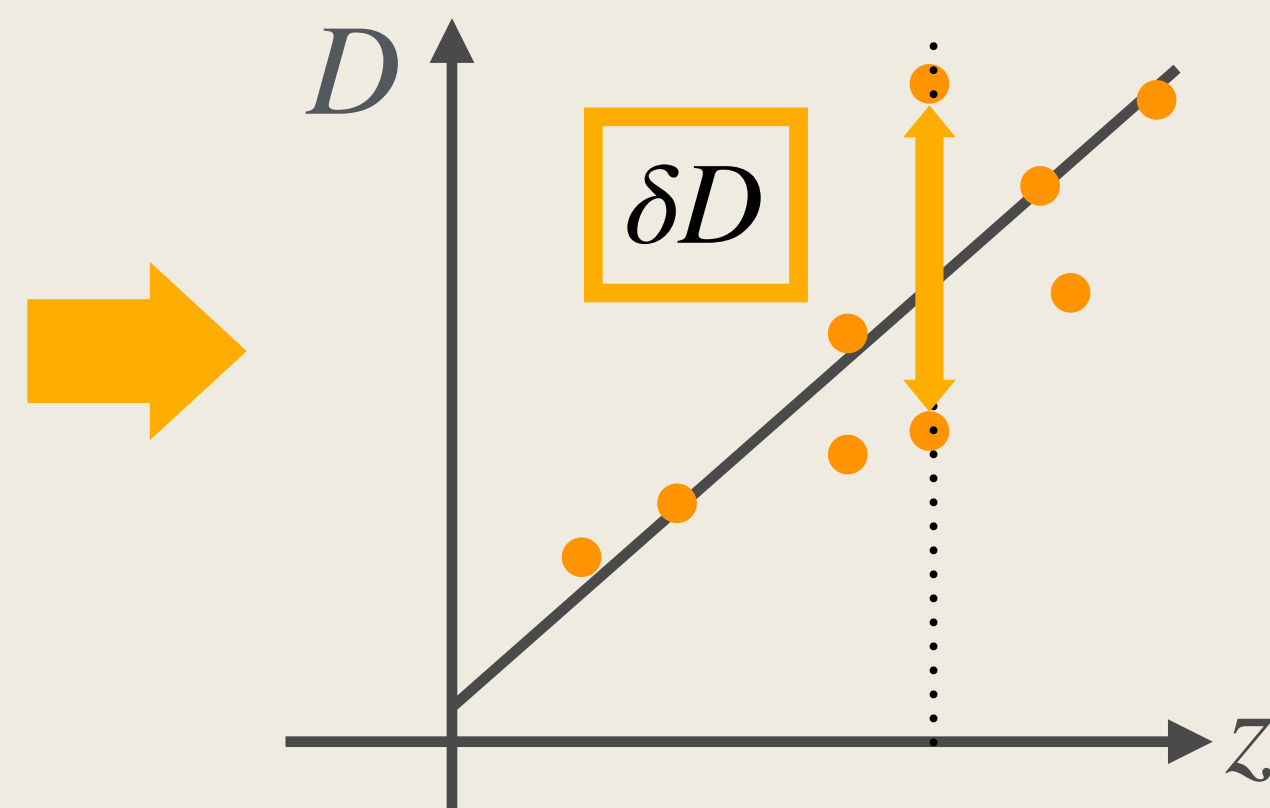
Collaborator: Masamune Oguri (大栗 真宗) Chiba Univ.

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What is the lensing dispersion?



The distance–redshift relation

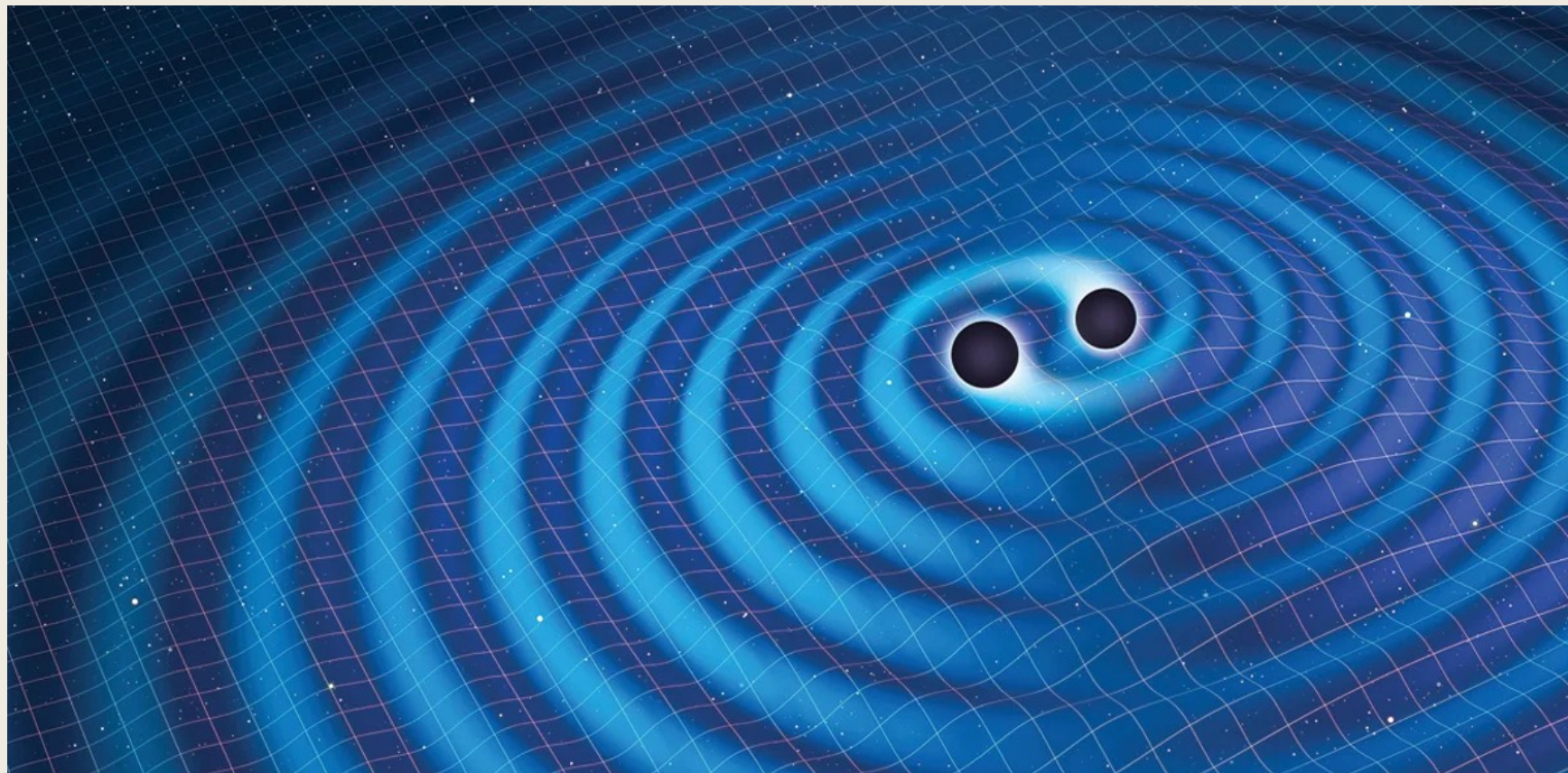


The lensing dispersion

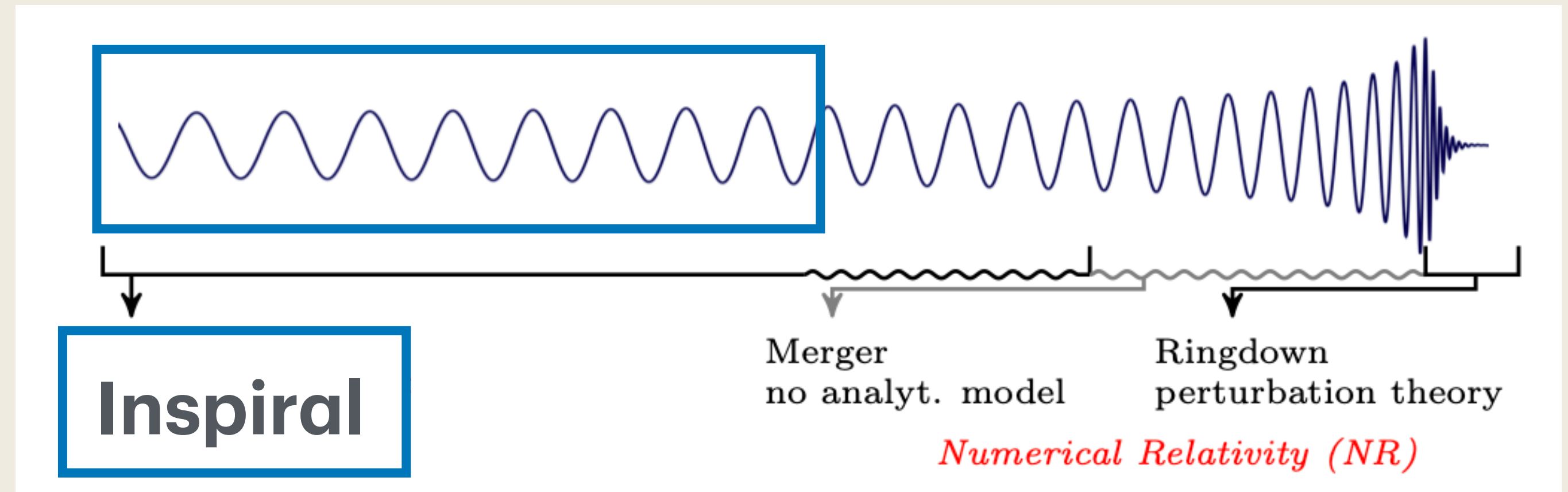
$$\langle \kappa^2 \rangle \sim \langle \kappa(\theta, z) \kappa(\theta', z') \rangle$$
$$(\because \theta \sim \theta', z \sim z')$$

Small-scale
density fluctuations

Measuring luminosity distance D_L from GW



MARK GARLICK/SPL/Getty



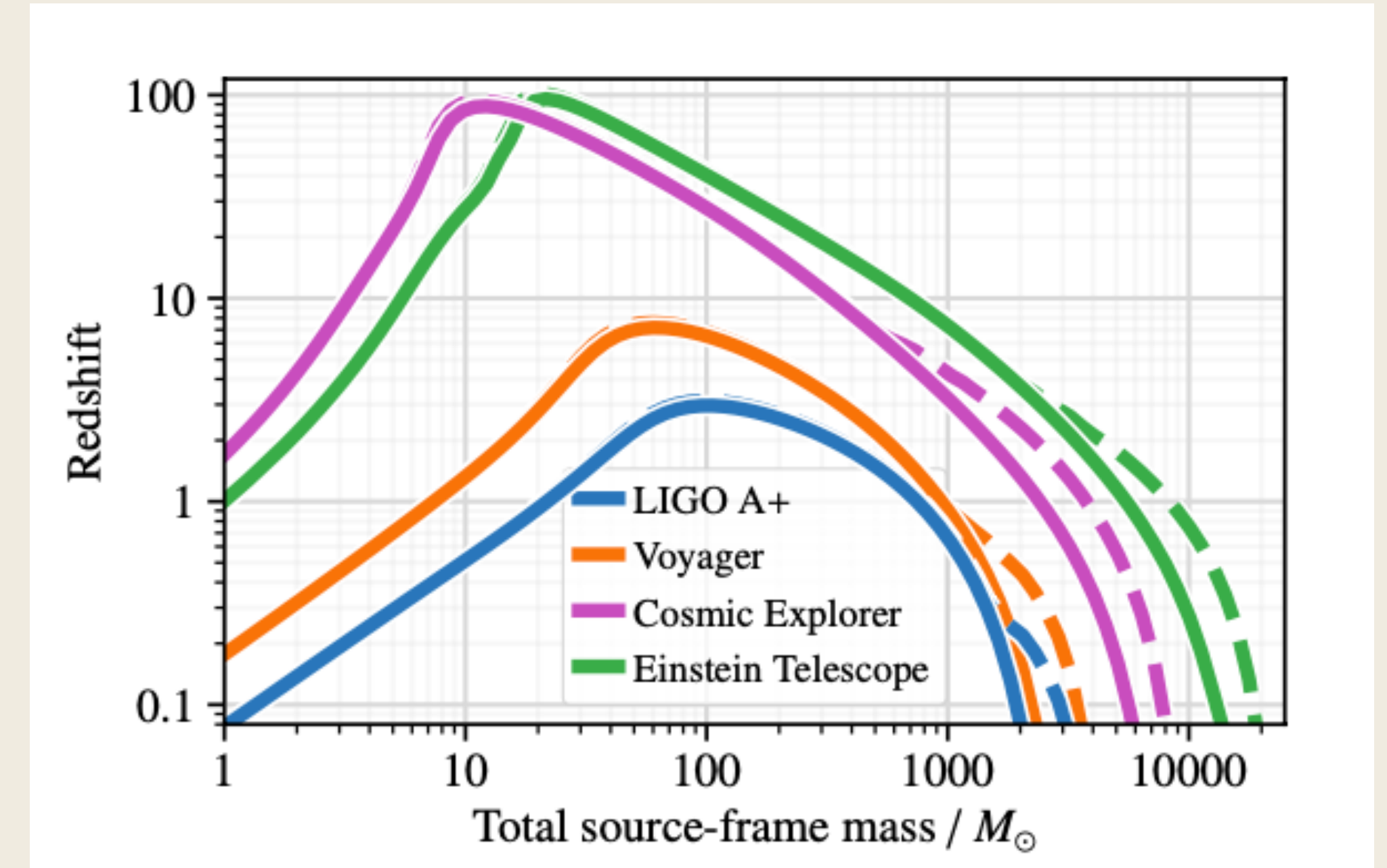
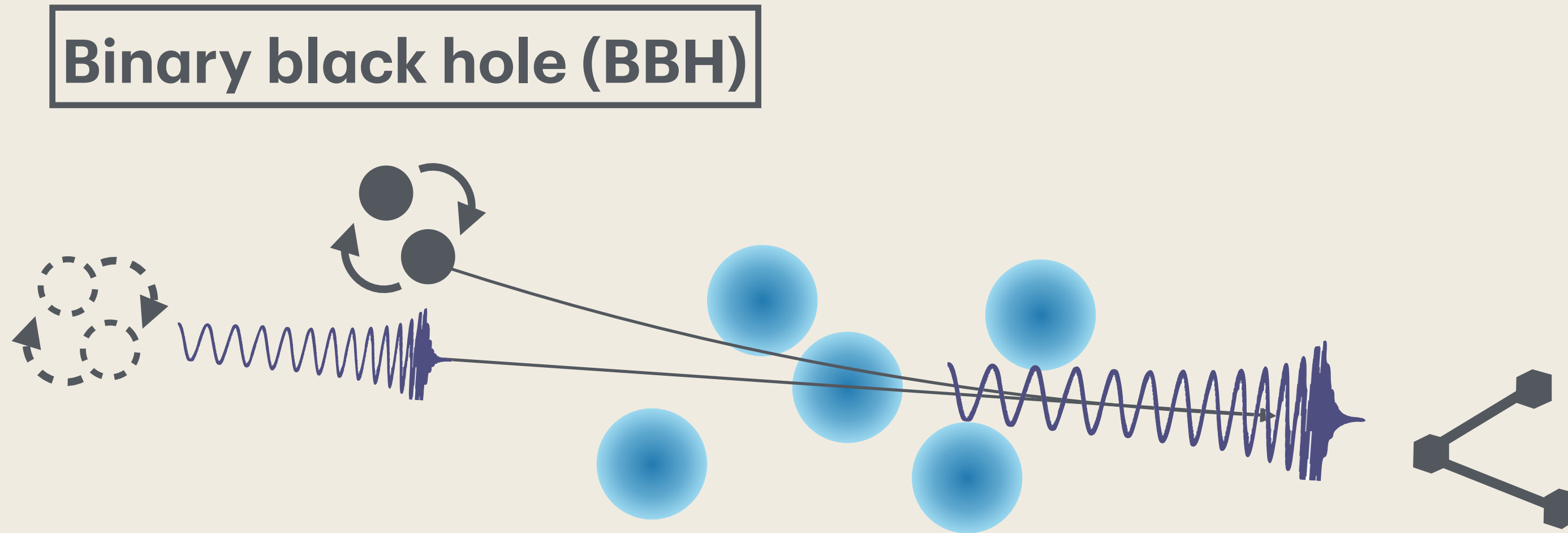
Ohme 2012

$$\Rightarrow \dot{f}(t) \propto M_z^{5/3} f^{11/3} \quad h_{+ \times}(t) \propto \frac{M_z^{5/3} f^{2/3}}{D_L} \quad (M_z \equiv (1+z)M_c)$$

$f, \dot{f}$: frequency, frequency evolution $h_+, h_{\times}$: strain of plus mode, cross mode M_c : chirp mass

Gravitational waves provide direct distance measurements!

Advantages of using gravitational waves from binary black holes



Hall 2022

1. Obtain luminosity distances with small systematics
2. Probe higher redshifts $z > 2$ than supernova surveys

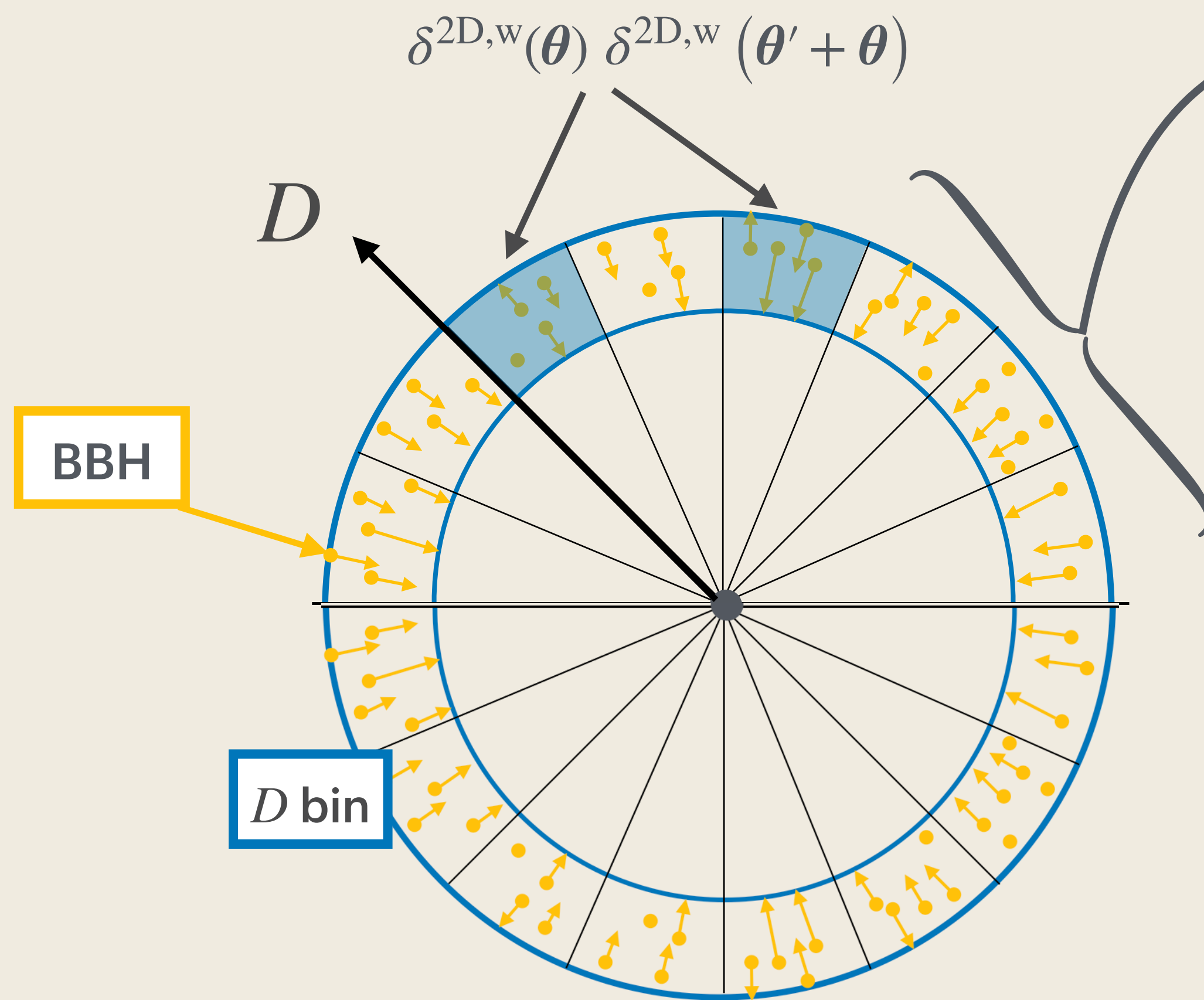
Identifying the BBH's redshift is challenging

1. **Dark Sirens: Mergers without an electromagnetic counterpart.**
2. **Poor Localization of the gravitational wave source on the sky.**

 **We cannot measure BBH's redshift directly**

Aim: Measure lensing dispersion using gravitational waves, even when redshifts are unknown.

Method : Using angular clustering of GWs on the sky



Large scale : $\ell \sim 100 - 300$

➡ **GW's density distribution = linear bias**

$$\delta_{\text{GW}} = b_{\text{GW}} \delta_{\text{m}} \quad b_{\text{GW}} = 1.5$$

δ_{m} : matter density distribution

Angular clustering signal

Auto-correlation angular power spectrum

$C^{\text{ww}}(\ell) + \text{lensing effect}$

➡ **Lensing-induced $C^{\text{ww}}(\ell)$ deviation**

➡ **Lensing dispersion $\langle \kappa^2 \rangle$**

Method: Lensing effect changes D of the GW source

Weak lensing effect on luminosity distance

$$D = \frac{\bar{D}}{\sqrt{\mu}} \simeq \frac{\bar{D}}{\sqrt{1 + 2\kappa + 3\kappa^2 + \gamma^2}} \quad \mu \simeq 1 + 2\kappa + 3\kappa^2 + \gamma^2$$

$\bar{D}$: Standard luminosity distance in a uniform isotropic universe

μ : magnification , κ : lensing convergence, γ : shear field

We compute the angular clustering signal, including lensing effects up to second order, and capture $\langle \kappa^2 \rangle$ contribution.

Method: New power spectrum formula of GW sources with $\langle \kappa^2 \rangle$

NEW

$$C^{\text{ww}}(\ell) = C^{\text{ss}}(\ell) + C^{\text{st}}(\ell) + C^{\text{su}}(\ell) + C^{\text{tt}}(\ell)$$

intrinsic clustering

$$C^{\text{ss}}(\ell) = \int_0^\infty dz W^{\text{s}}(z) W^{\text{s}}(z) \frac{H(z)}{\chi^2} b_{\text{GW}}^2 P_{\text{m}} \left(\frac{\ell + 1/2}{\chi}; z \right)$$

intrinsic clustering **×** **lensing effect**

$$C^{\text{st}}(\ell) = -2 \int_0^\infty dz W^{\text{s}}(z) W^{\text{t}}(z) \frac{H(z)}{\chi^2} b_{\text{GW}}^2 P_{\text{m}} \left(\frac{\ell + 1/2}{\chi}; z \right) \langle \kappa^2 \rangle$$

$$C^{\text{su}}(\ell) = 2 \int_0^\infty dz W^{\text{s}}(z) W^{\text{u}}(z) \frac{H(z)}{\chi^2} b_{\text{GW}}^2 P_{\text{m}} \left(\frac{\ell + 1/2}{\chi}; z \right) \langle \kappa^2 \rangle$$

b_{GW} : bias of GW sources

χ : comoving distance

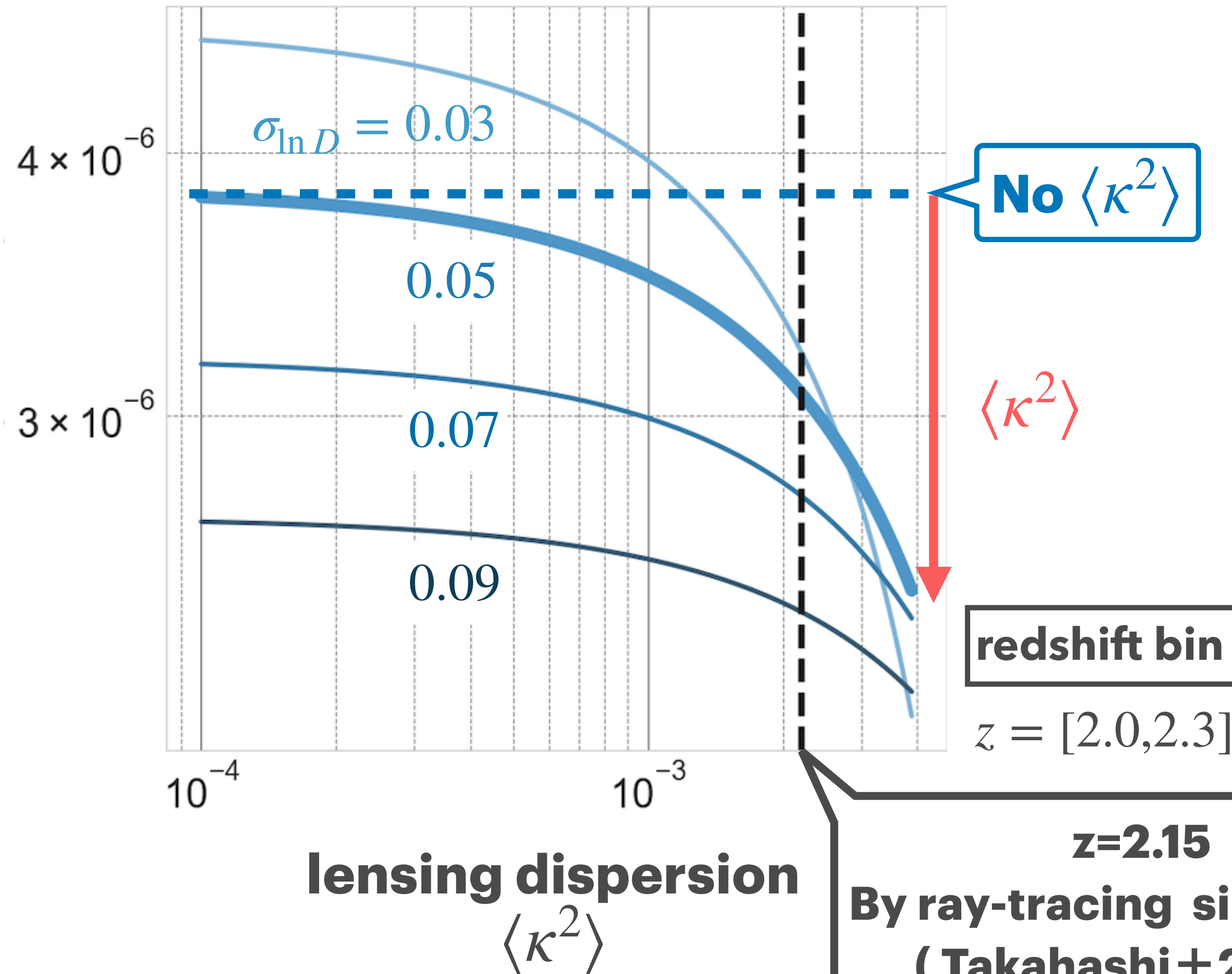
P_{m} : linear matter power spectrum

lensing effect

$$C^{\text{tt}}(\ell) = \int_0^\infty dz \int_0^\infty dz' W^{\text{t}}(z) W^{\text{t}}(z') \int_0^{\min(z, z')} dz'' W^{\kappa}(z''; z) W^{\kappa}(z''; z') \frac{H(z'')}{\chi''^2} P_{\text{m}} \left(k = \frac{\ell + 1/2}{\chi''}, z'' \right)$$

Result: Relation between $\langle \kappa^2 \rangle$ & $C^{\text{ww}}(\ell)$

auto-correlation $C^{\text{ww}}(\ell = 100)$



Relation between $\langle \kappa^2 \rangle$ & $C^{\text{ww}}(\ell)$

$$\langle \kappa^2 \rangle \uparrow \Rightarrow C^{\text{ww}}(\ell) \downarrow$$

$\langle \kappa^2 \rangle$ induced ΔC_ℓ indicates sensitivity to the dispersion.

Result: Ratio of signal to noise S/N

Ratio of signal to noise

$$\sigma^2 \sim \text{Cov} = \delta_{\ell\ell'} \frac{2}{2\ell + 1} \left(C_{\ell}^{\text{ww}} + \frac{1}{\bar{n}_{\text{GW}}} \right)^2$$

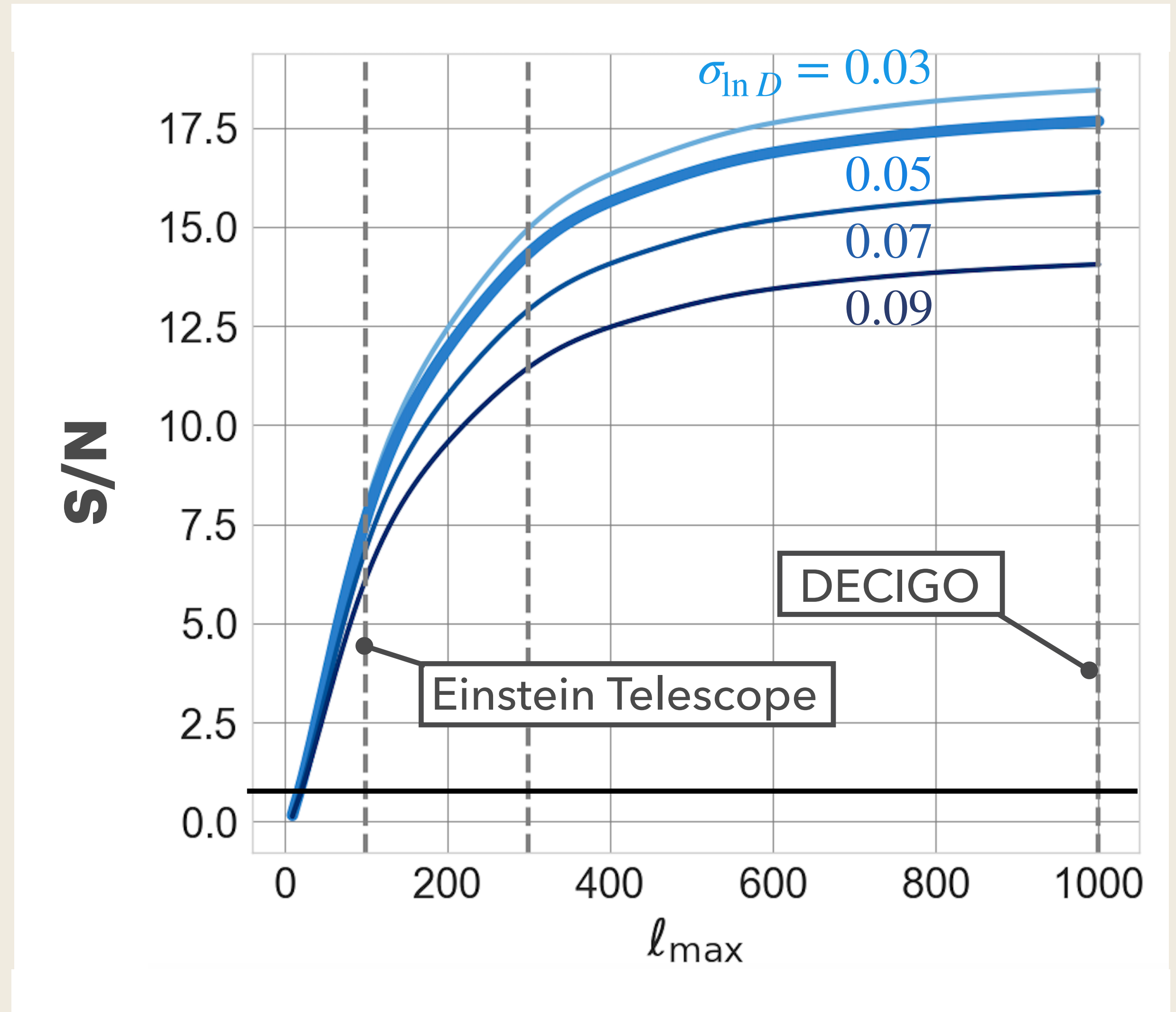
$\bar{n}_{\text{GW}}$: average number density of GWs

$$\text{S/N} \equiv \sqrt{\sum_{\ell=0}^{\ell_{\text{max}}} \frac{C_{\ell}^{\text{ww}2}}{\text{Cov}}}$$

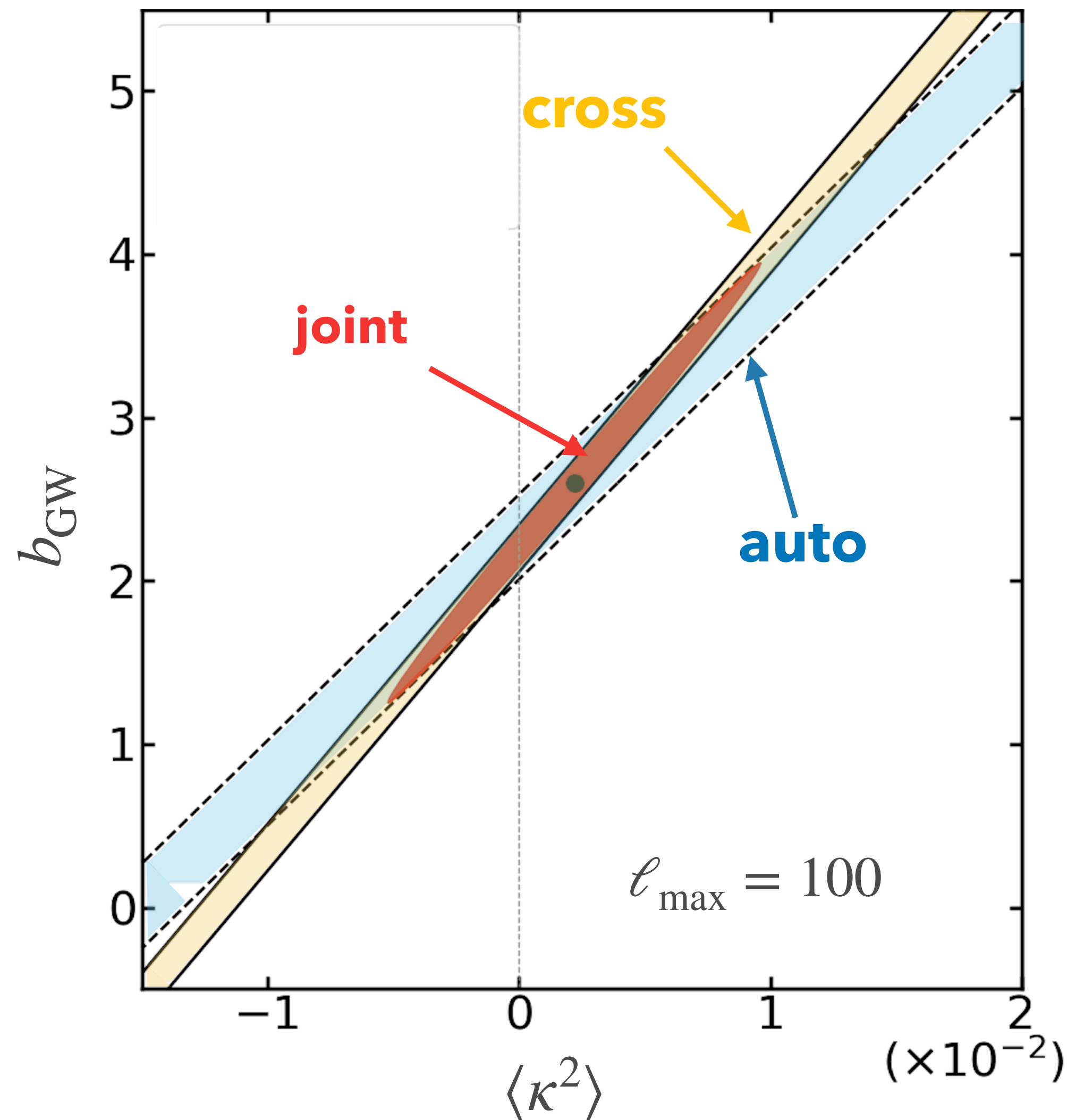
Angular resolution

$$\theta \sim \frac{\pi}{\ell_{\text{max}}}$$

The correlation signal is observable with next-generation observatories!



Result: 1σ confidence region in the $\langle \kappa^2 \rangle$ - b_{GW} plane



Joint analysis

Auto-correlation of BBHs

Cross-correlation between BBHs and spec-z galaxy

➡ The degeneracy is partially broken

Opening a path to constrain $\langle \kappa^2 \rangle$!

Fiducial value

$$\langle \kappa^2 \rangle = 2.2 \times 10^{-3} \quad \sigma_{\ln D} = 0.05 \quad b_{\text{GW}} = 2.6$$

Future Work : Toward Better Constraints on $\langle \kappa^2 \rangle$

Approach 1 : Utilizing the scale dependence of the bias parameter

Effect of $\langle \kappa^2 \rangle \sim$ scale-independent $\neq \delta_{\text{GW}} = b_{\text{GW}}(\boxed{k})\delta_{\text{m}}$

Approach 2 : Constraining formation channels at low-z and predicting b_{GW} at high-z

At low-z {
More precise luminosity distances
Better sky localization
Weaker gravitational lensing impact

➡ BBH formation channel ➡ predicting b_{GW} to high-z

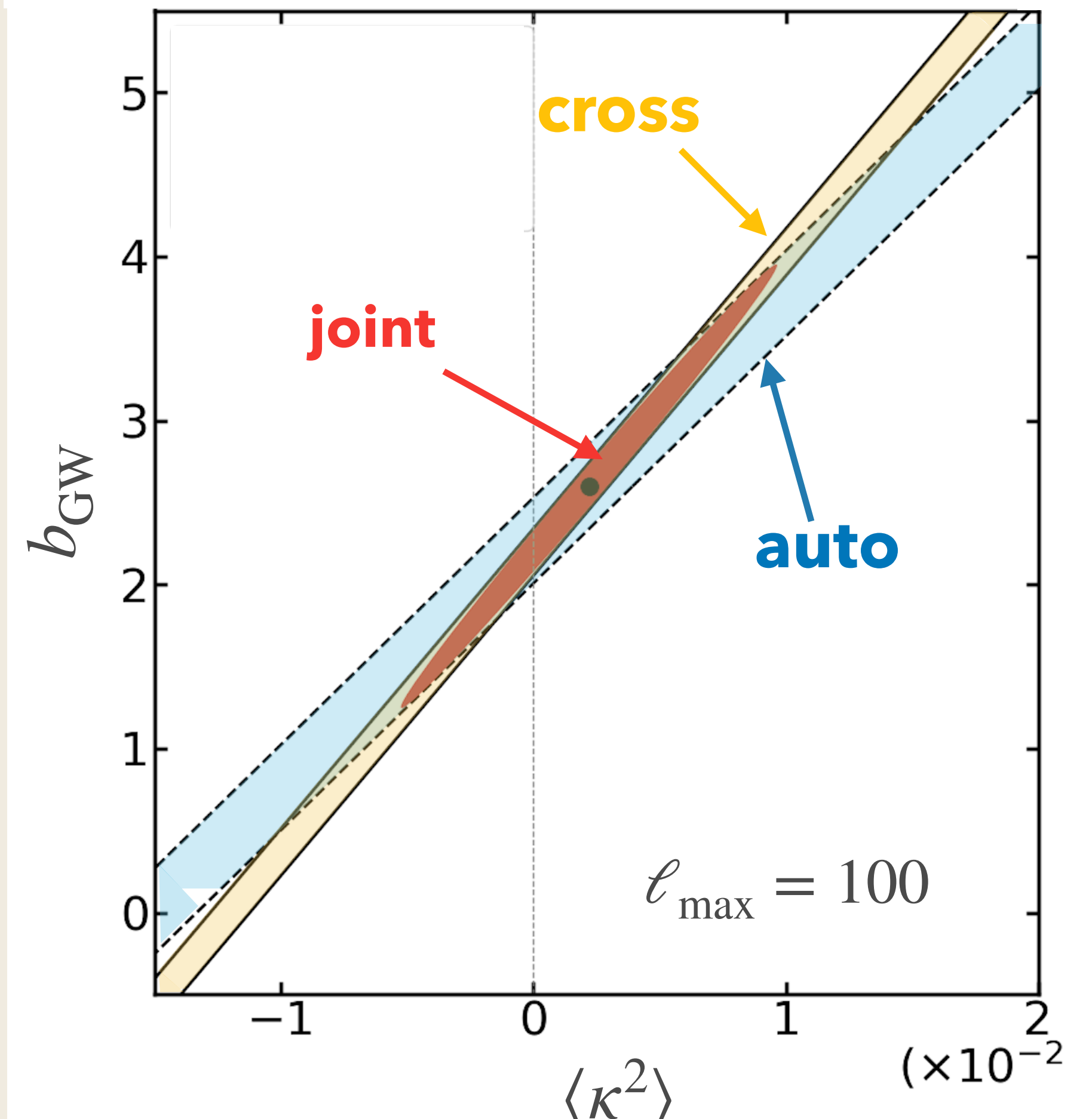
Better knowledge of b_{GW} enables significantly tighter constraints on $\langle \kappa^2 \rangle$ with this method

Summary

- GW can measure luminosity distance directly + probe lensing dispersion to higher redshifts.
- Since many events are dark sirens, we propose a method utilizing angular clustering in luminosity-distance bins, excluding redshifts.
- The amplitudes of the angular power spectra of gravitational wave sources: decreasing function of the lensing dispersion.
- The signal should be observable with next-generation detectors.
- Joint analysis partially breaks degeneracies, opening a path to constrain lensing dispersion.

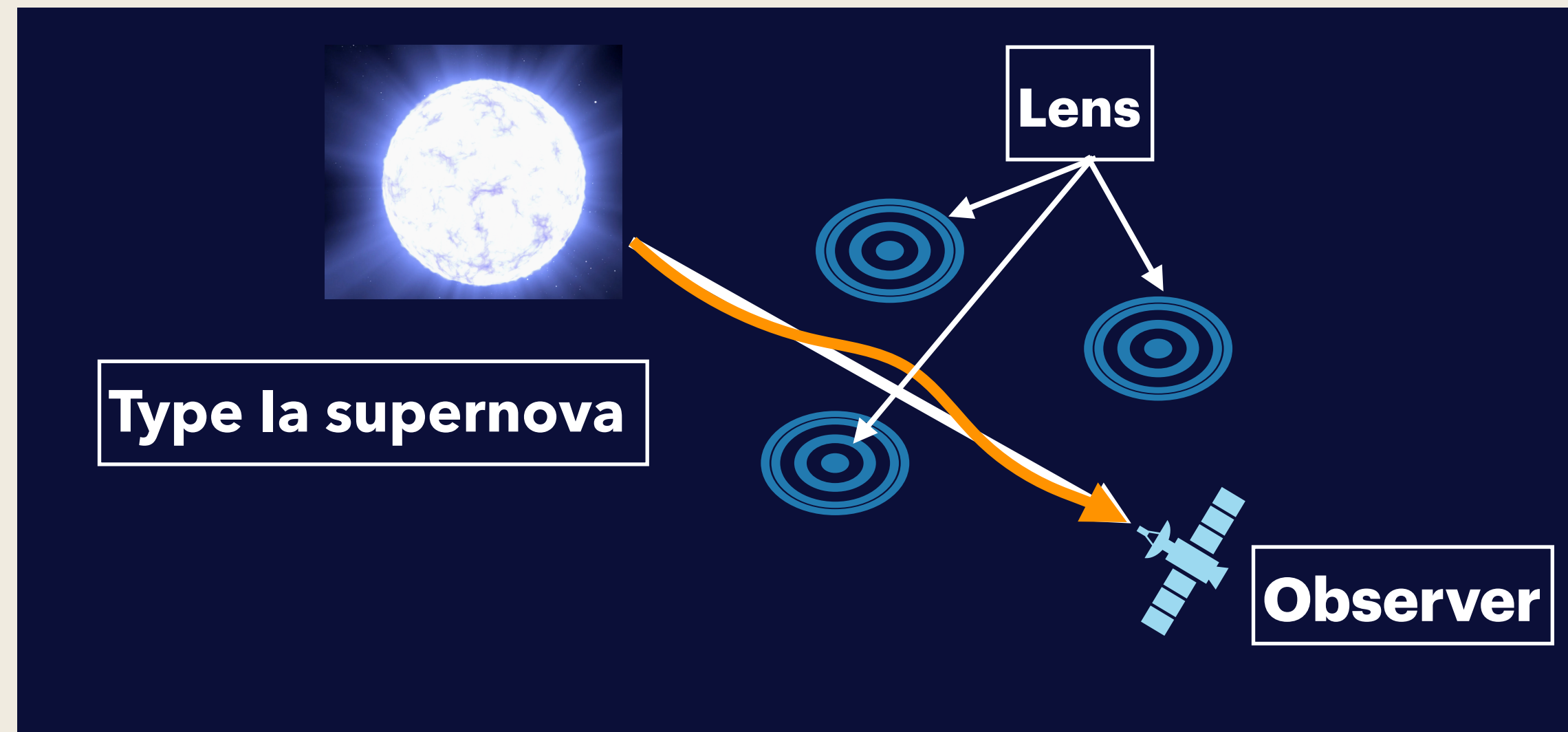
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The lensing dispersion $\langle \kappa^2 \rangle$ > useful cosmological probe

Observation of $\langle \kappa^2 \rangle$ by Type Ia supernova

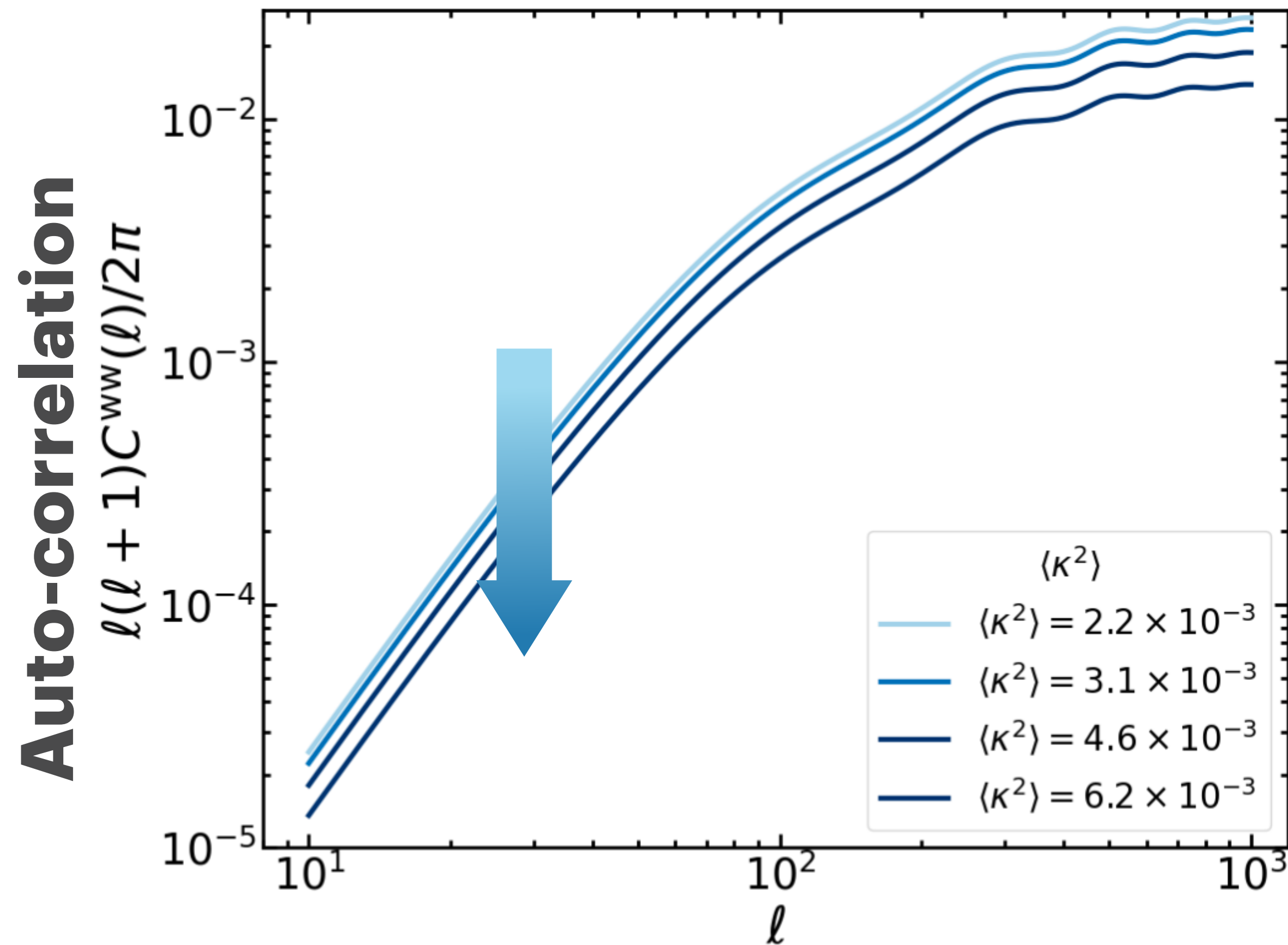


For instance.....

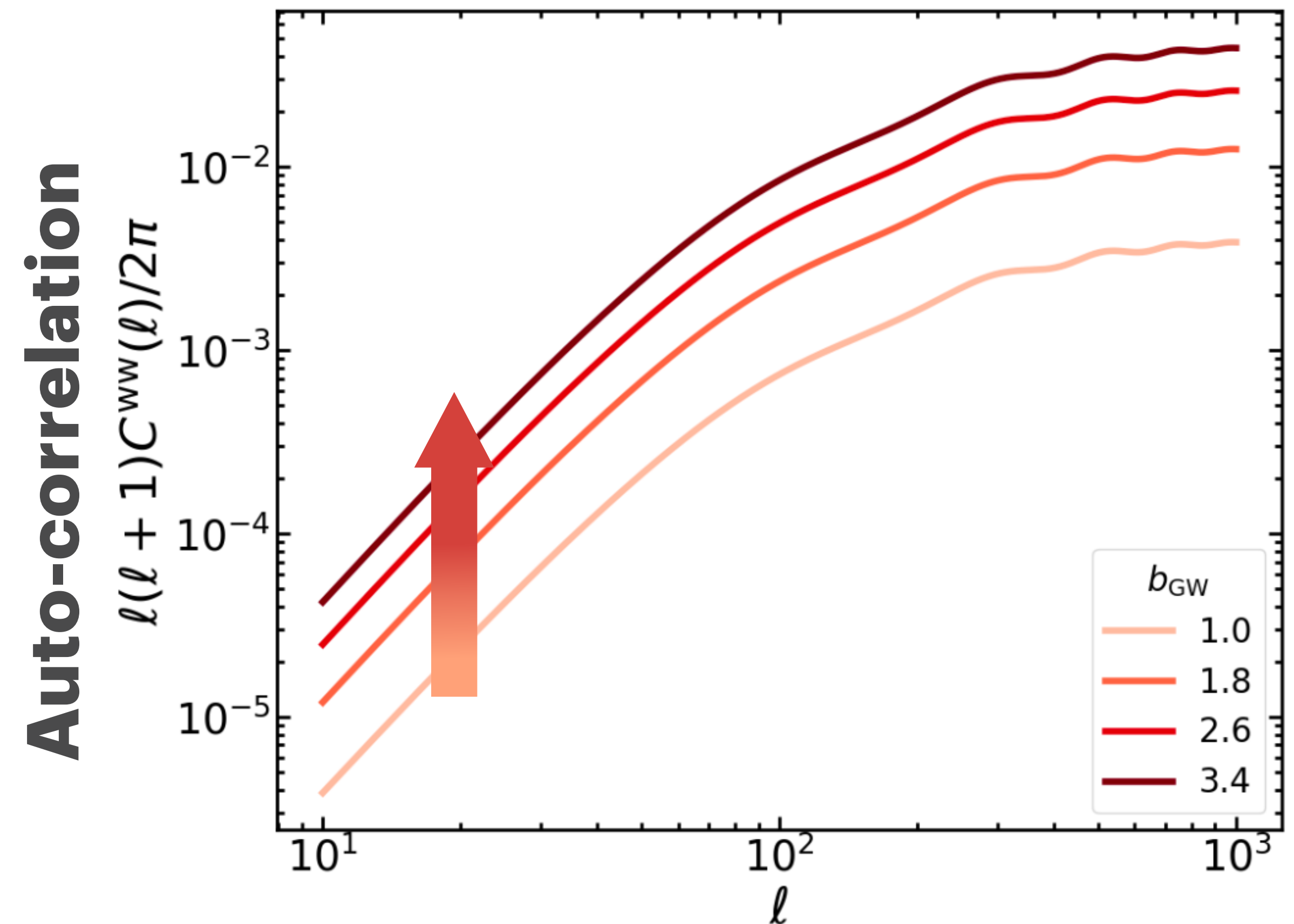
- Abundance of primordial black holes (PBH) (Zumalacárregui & Seljak 2018)
- Upper limit on neutrino masses (Hada and Futamase, 2016; Agrawal et al., 2019)
- Small-scale cosmological density power spectrum (Ben-Dayana & Takahashi 2016)

Result : Strong degeneracy between $\langle \kappa^2 \rangle$ vs b_{GW}

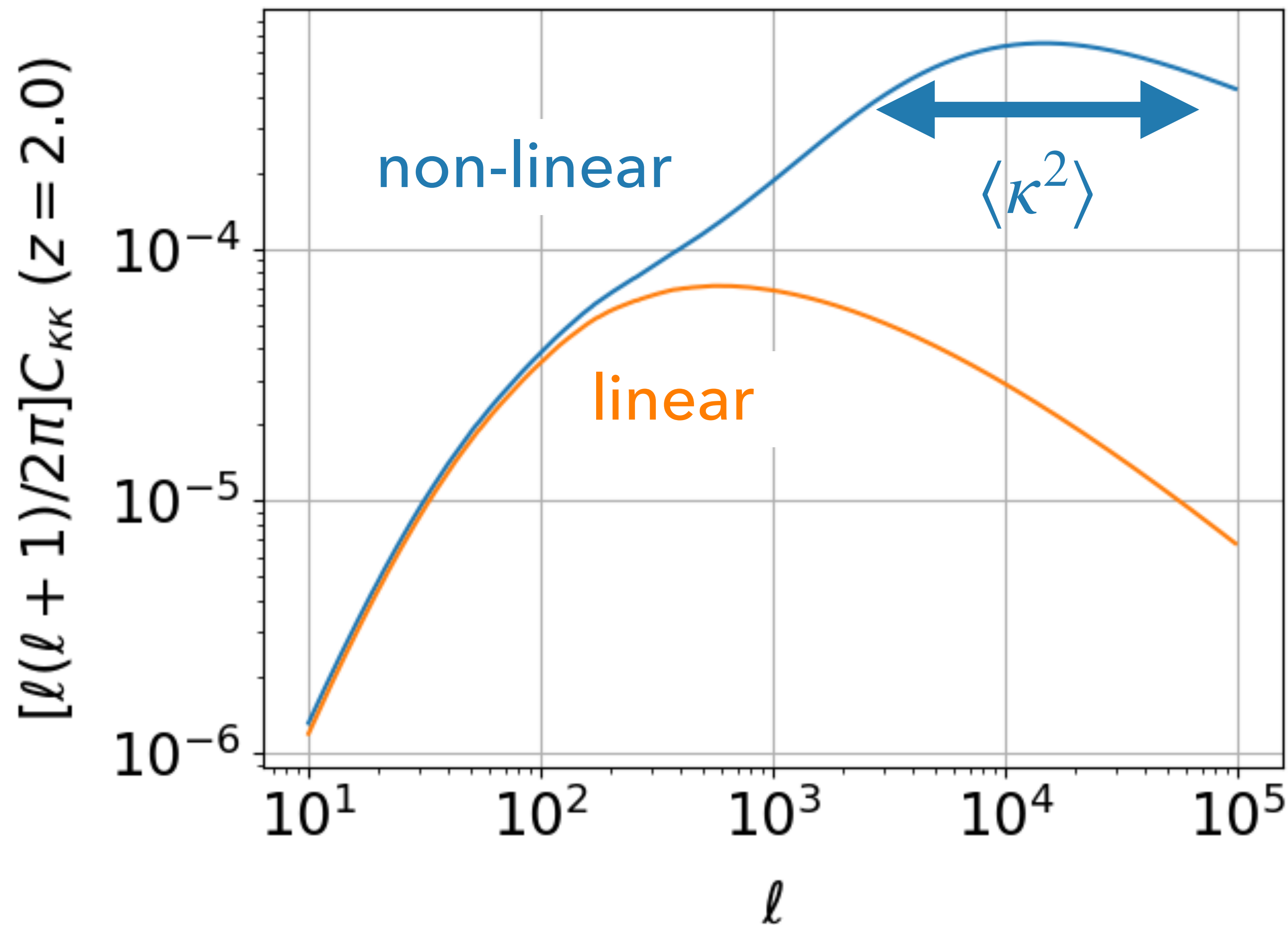
$\langle \kappa^2 \rangle$ variation (b_{GW} fixed)



b_{GW} variation ($\langle \kappa^2 \rangle$ fixed)



The correlation of lensing effect



Takahashi et al., 2012

Preparation for auto-correlation angular power spectrum

Observational error : log-normal distribution

$$p(D_{\text{obs}} | D) = \frac{1}{\sqrt{2\pi}\sigma_{\ln D}} \exp \left[-x^2(D_{\text{obs}}) \right] \frac{1}{D_{\text{obs}}} \quad x(D_{\text{obs}}) \equiv \frac{\ln D_{\text{obs}} - \ln D}{\sqrt{2}\sigma_{\ln D}}$$

The average projected number density of GW sources $\bar{n}_i^{\text{w}}$:

$$\bar{n}_i^{\text{w}} = \int_0^\infty dz \frac{\chi^2}{H(z)} S_i(z) T_{\text{obs}} \frac{\dot{n}_{\text{GW}}(z)}{1+z}$$

The duration of the observation : $T_{\text{obs}} = 5$ (yr)

BH merger rate : $\dot{n}_{\text{GW}} = 10^{-5} \text{ (} h^3 \text{Mpc}^{-3} \text{yr}^{-1} \text{)}$

Liner bias : $\delta_{\text{GW}} = b_{\text{GW}} \delta_m \quad b_{\text{GW}} = 2.6$

Limber approximation:

When $\theta \sim 1/l \ll 1$, only the power spectrum of sources at the same radial distance from the observation point contributes to the calculation.

SELECTION FUNCTION

The zero order $S(z) \equiv \frac{1}{2} \left[\text{erfc} \left\{ x \left(D_{\min} \right) \right\} - \text{erfc} \left\{ x \left(D_{\max} \right) \right\} \right]$

Expand to the second order with a small amount of Δx

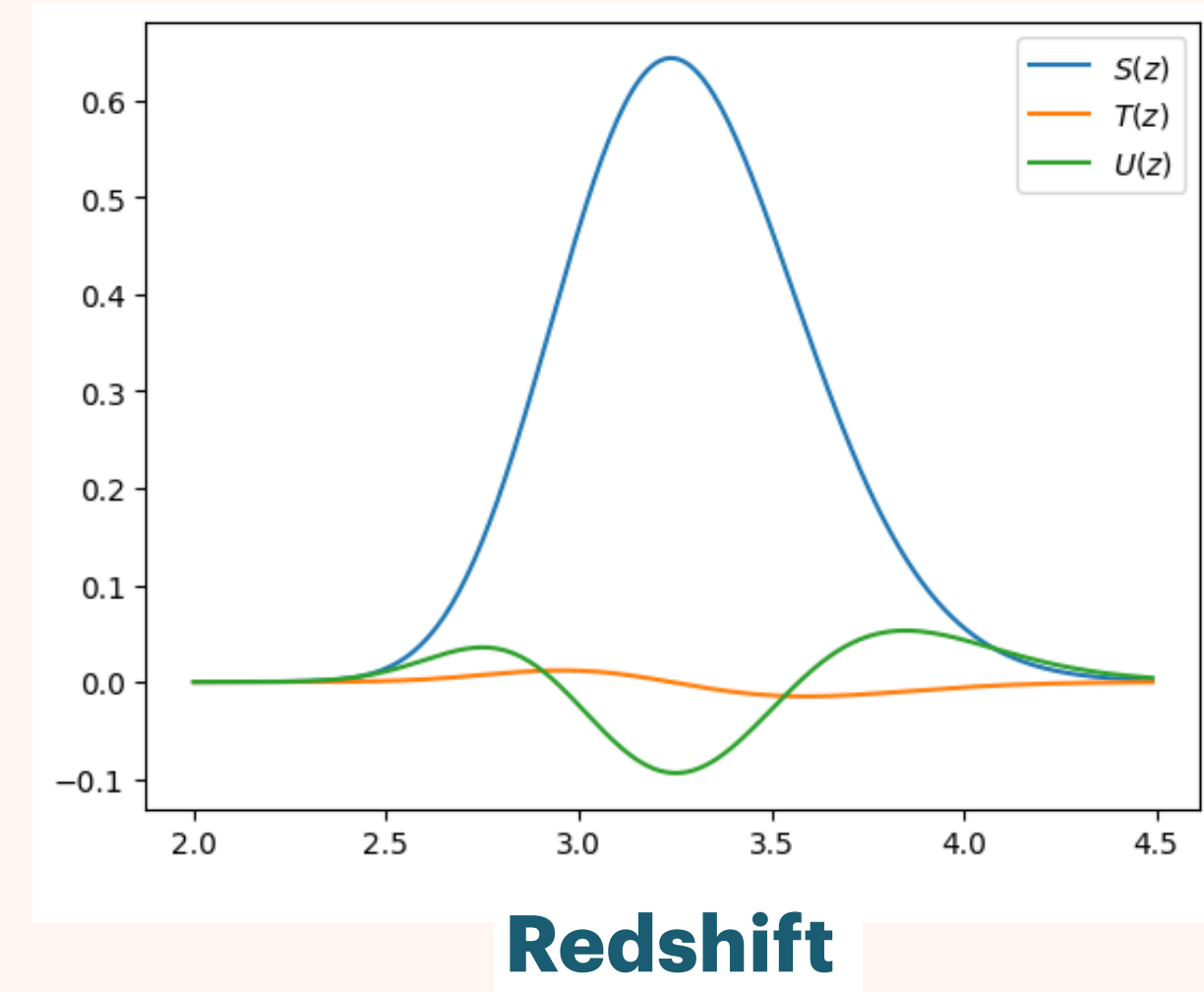
$$x \left(D_{\text{obs}} \right) \simeq \bar{x} + \frac{1}{\sqrt{2} \sigma_{\ln D}} \left(\kappa + \frac{1}{2} \kappa^2 + \frac{1}{2} \gamma^2 \right) = \Delta x$$

$$S(z) = \bar{S}(z) + T(z) \left(\kappa + \frac{\kappa^2}{2} + \frac{\gamma^2}{2} \right) + U(z) \kappa^2$$

The first order $T(z) = \frac{1}{\sqrt{2\pi} \sigma_{\ln D}} \left[-\exp \left\{ -x^2 \left(D_{\min} \right) \right\} + \exp \left\{ -x^2 \left(D_{\max} \right) \right\} \right]$

The second order $U(z) = \frac{1}{2\sqrt{\pi} \sigma_{\ln D}^2} \left[\bar{x} \left(D_{\min} \right) \exp \left\{ -\bar{x}^2 \left(D_{\min} \right) \right\} - \bar{x} \left(D_{\max} \right) \exp \left\{ -\bar{x}^2 \left(D_{\max} \right) \right\} \right]$

$$W^s = \frac{1}{\bar{n}^w} \frac{\chi^2}{H(z)} \bar{n}_{\text{GW}}(z) S(z) \quad W^t(z) = \frac{1}{\bar{n}^w} \frac{\chi^2}{H(z)} \bar{n}_{\text{GW}}(z) T(z) \quad W^u(z) = \frac{1}{\bar{n}^w} \frac{\chi^2}{H(z)} \bar{n}_{\text{GW}}(z) U(z)$$



cross-correlation of GWs and Galaxy

cross-correlation power spectrum

$$C^{\text{wg}}(\ell) = C^{\text{sg}}(\ell) + C^{\text{tg}}(\ell) + C^{\text{ug}}(\ell)$$

$$C^{\text{sg}}(\ell) = \int_0^\infty dz W^{\text{s}}(z) W^{\text{g}}(z) \frac{H(z)}{\chi^2} b_{\text{GW}} b_{\text{g}} P_{\text{m}} \left(\frac{\ell + 1/2}{\chi}; z \right)$$

$$C^{\text{tg}}(\ell) = - \int_0^\infty dz W^{\text{t}}(z) W^{\text{g}}(z) \frac{H(z')}{\chi'^2} b_{\text{GW}} b_{\text{g}} P_{\text{m}} \left(\frac{\ell + 1/2}{\chi'}; z' \right) \langle \kappa^2 \rangle$$

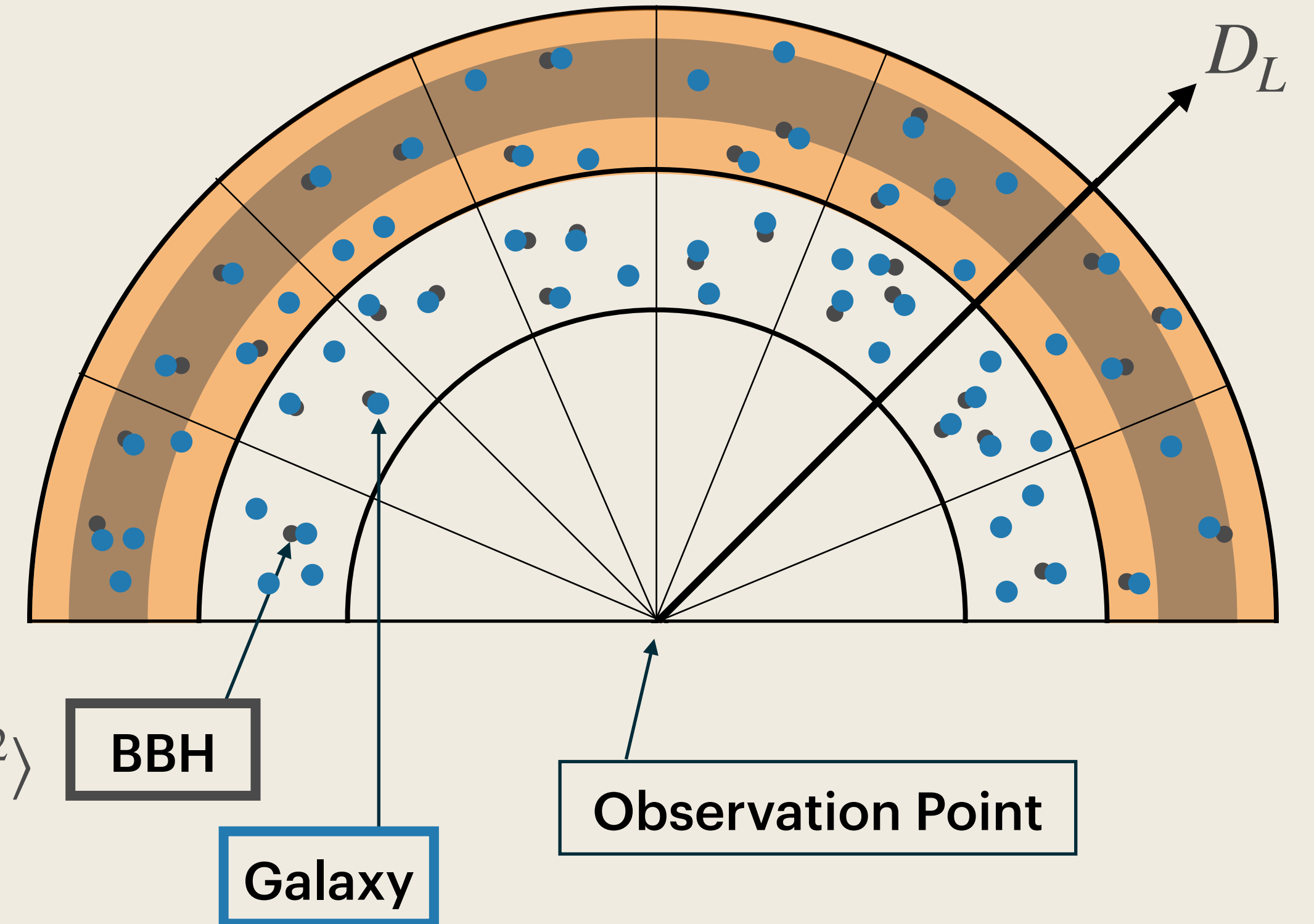
$$C^{\text{ug}}(\ell) = \int_0^\infty dz W^{\text{u}}(z) W^{\text{g}}(z) \frac{H(z')}{\chi'^2} b_{\text{GW}} b_{\text{g}} P_{\text{m}} \left(\frac{\ell + 1/2}{\chi'}; z' \right) \langle \kappa^2 \rangle$$

χ : comoving distance

b_{GW} : bias of GW sources

b_{g} : bias of galaxies

P_{m} : linear matter power spectrum



Why is Log-normal distribution chosen?

GW observation

		Error
GWXXXXXX1 : SN=13	○	→ ±13 %
GWXXXXXX2 : SN=1	×	
GWXXXXXX3 : SN=20	○	→ ±20 %
GWXXXXXX4 : SN=5	×	

error propagation

$$h_{+ \times}(t) \propto \frac{1}{D_L} \longrightarrow \sigma_{h_{\pm}} = \sigma_D$$

→ Luminosity distance error is a **relative error**

relative error → **log-normal distribution**

absolute error → **normal distribution**

$$x(D_{\text{obs}}) \equiv \frac{\ln D_{\text{obs}} - \ln D}{\sqrt{2}\sigma_{\ln D}}$$

Lensing convergence and its dispersion

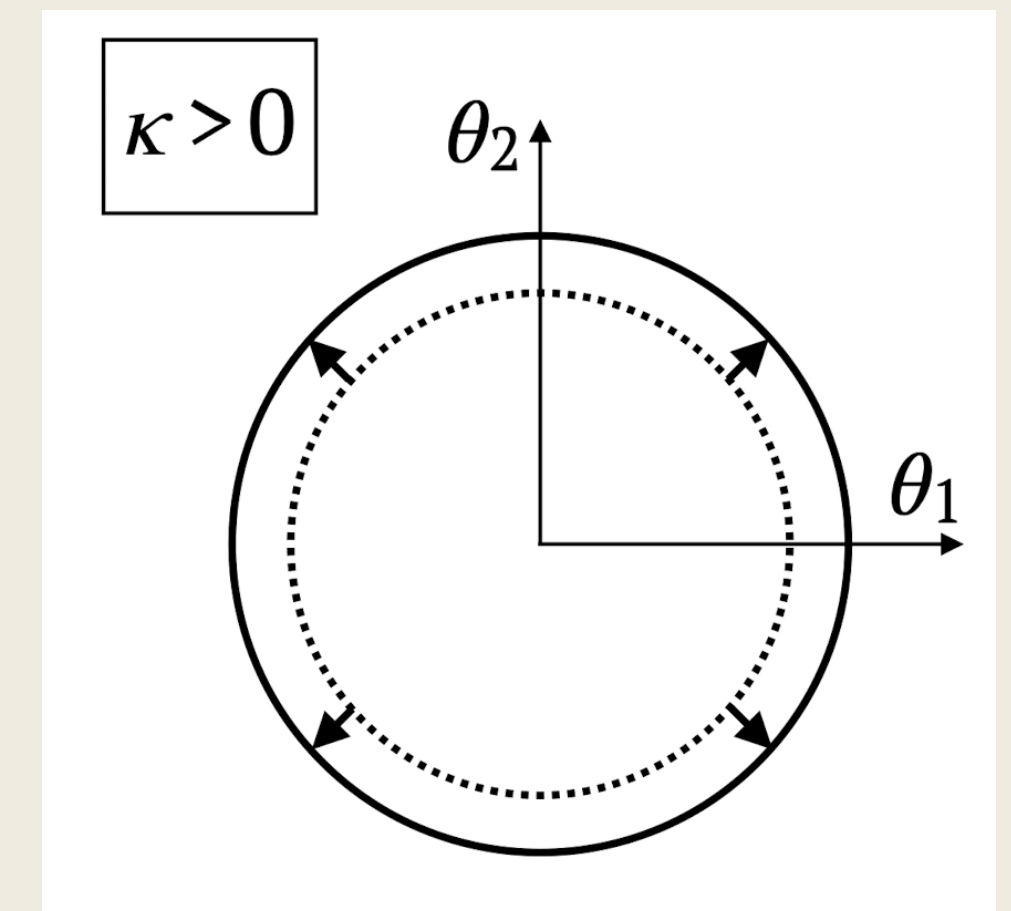
The lensing convergence field $\kappa(\boldsymbol{\theta}, z)$ is the projection of the matter density field $\bar{\rho}_m(z)$ and magnify the images.

$$\kappa(\boldsymbol{\theta}, z) = \int_0^z dz' \frac{\bar{\rho}_m(z')}{H(z')(1+z')\Sigma_{\text{crit}}(z'; z)} \delta_m(\boldsymbol{\theta}, z') \equiv \int_0^z dz' W^\kappa(z'; z) \delta_m(\boldsymbol{\theta}, z')$$

$$\langle \kappa(\boldsymbol{\theta}, z) \kappa(\boldsymbol{\theta}', z') \rangle = \int_0^z dz'' W^\kappa(z''; z) \delta_m(\boldsymbol{\theta}, z'') \int_0^{z'} dz''' W^\kappa(z'''; z') \delta_m(\boldsymbol{\theta}', z''')$$

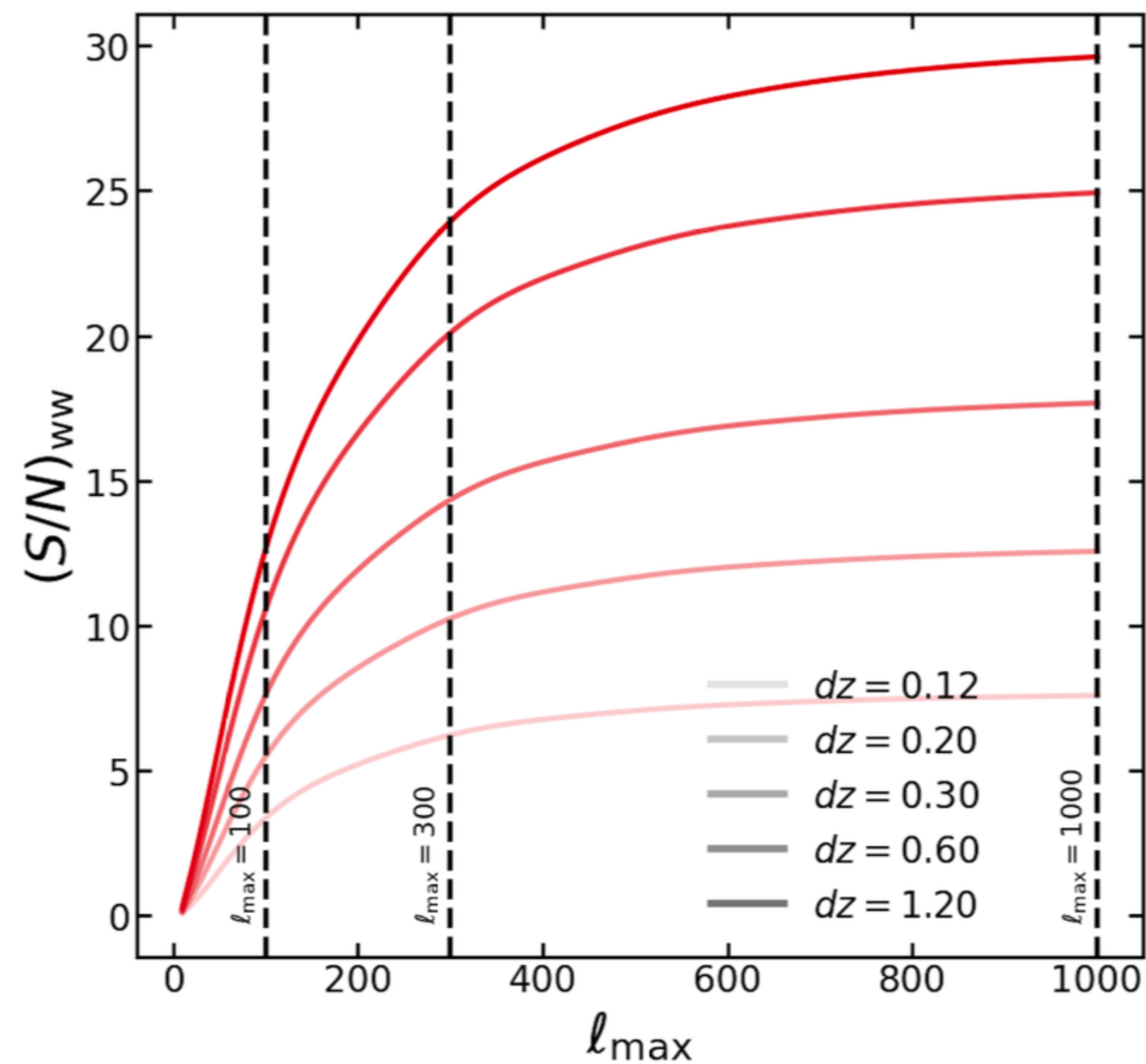
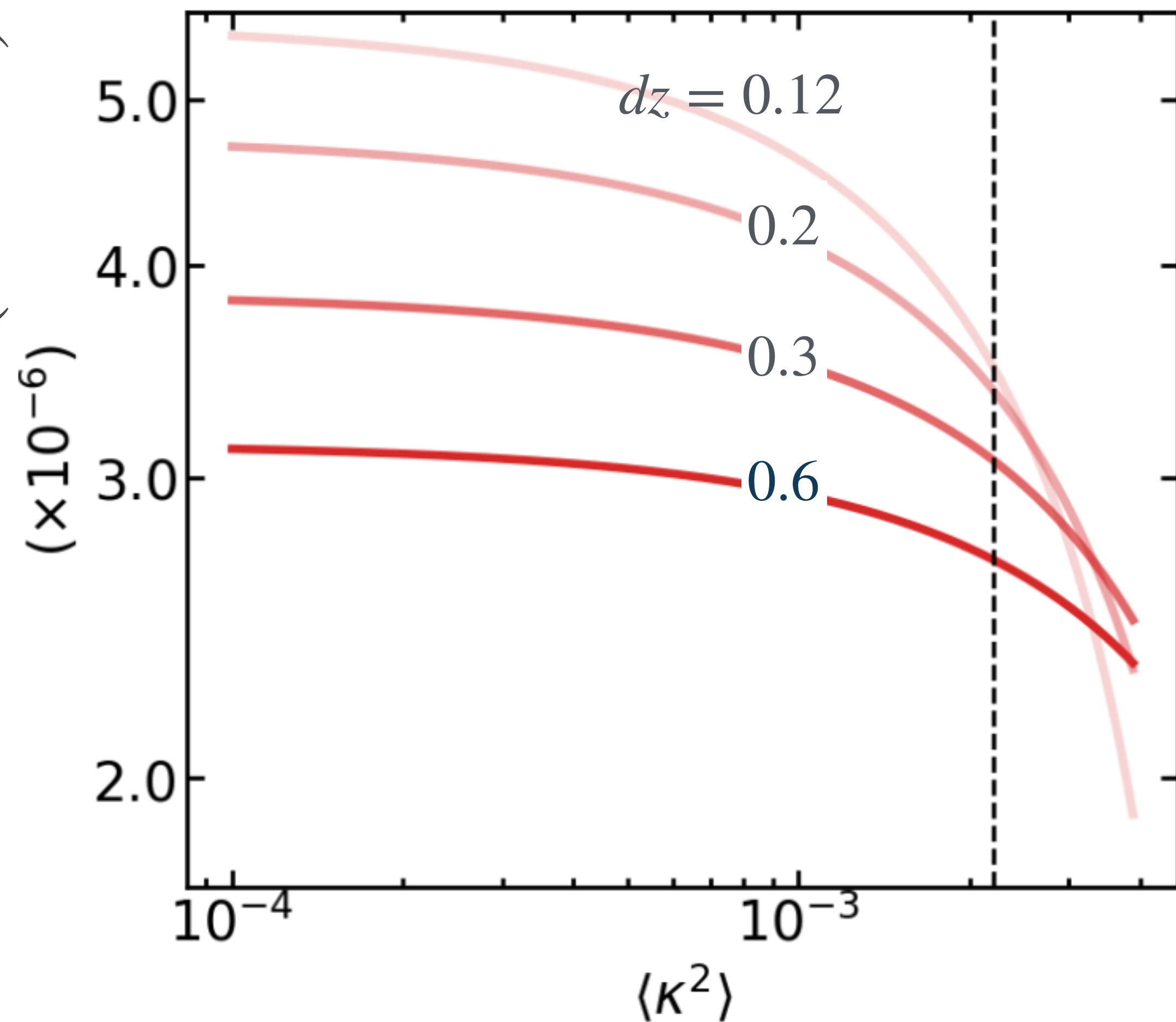
From the ray tracing simulation

$$\langle \kappa \rangle = -2\langle \kappa^2 \rangle, \langle \gamma^2 \rangle = \langle \kappa^2 \rangle \text{ (Takahashi+2011)}$$



Bin size dependent of the results

auto-correlation $C^{ww}(\ell = 100)$



LENSING CONVERGENCE DISPERSION $\langle \kappa^2 \rangle$

Magnification is approximate as follows

$$\mu = \frac{1}{(1 - \kappa)^2 - \gamma^2} \simeq 1 + 2\kappa + 3\kappa^2 + \gamma^2$$

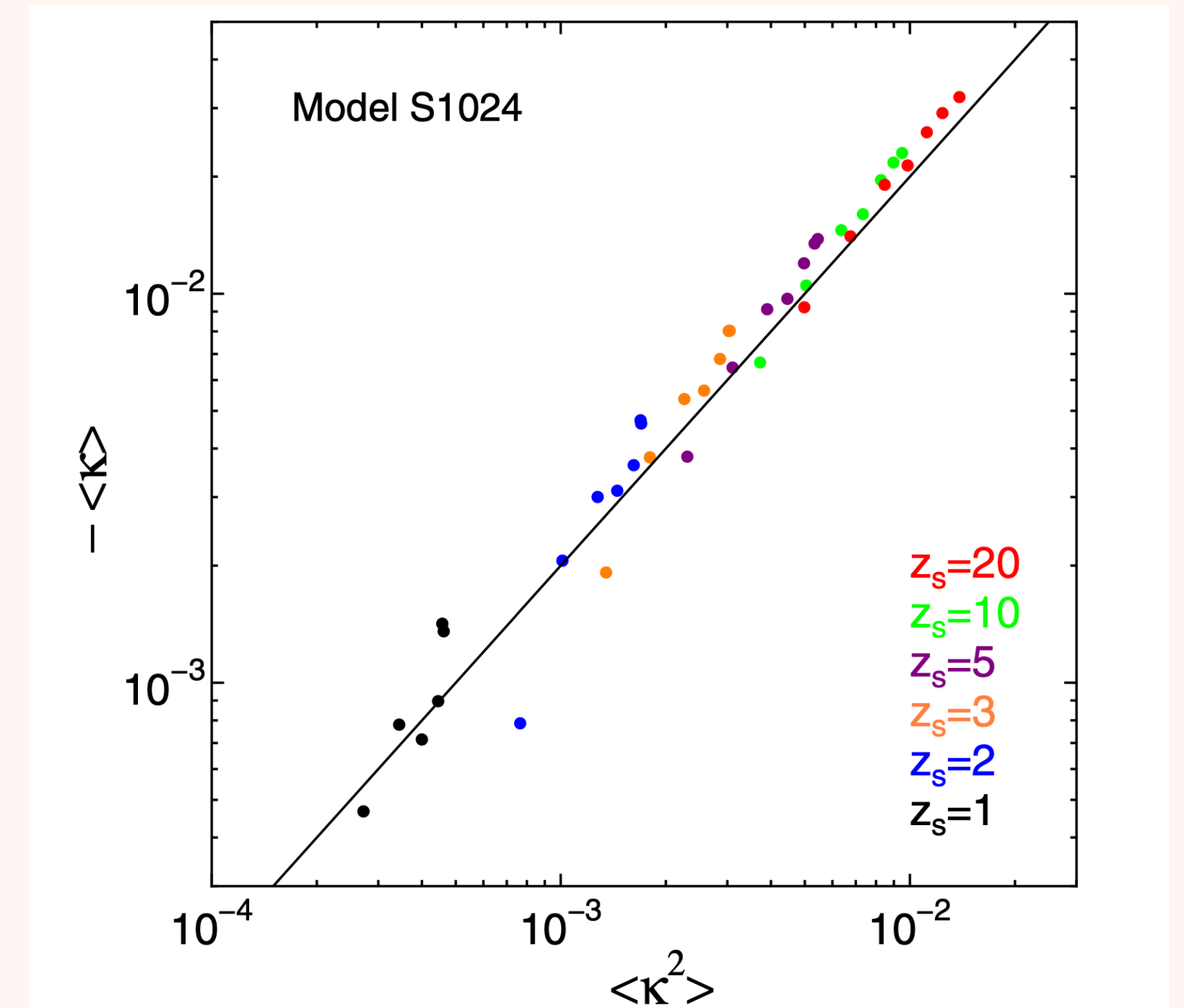
Average expansion rate

in terms of source $\langle \mu \rangle = 1$

$$\langle \mu \rangle = 1 \simeq 1 + 2\langle \kappa \rangle + 3\langle \kappa^2 \rangle + \langle \gamma^2 \rangle$$

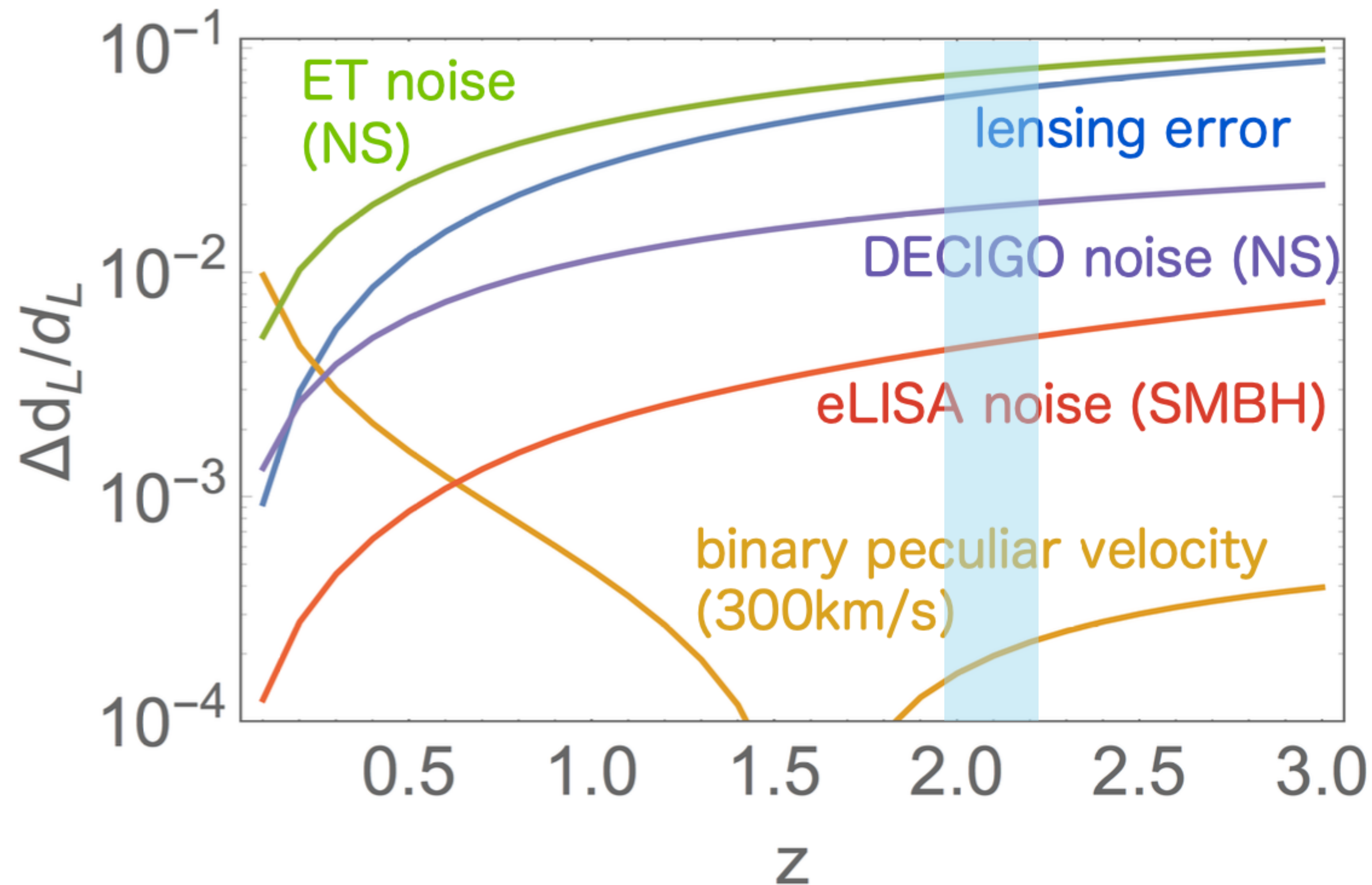
**From the approximation of
the weak gravity lens effect $\langle \kappa^2 \rangle = \langle \gamma^2 \rangle$**

$$\langle \kappa \rangle \simeq -2\langle \kappa^2 \rangle$$



**Ray-tracing Simulationsによる結果
(Takahashi+2011)**

The error of luminosity distance



$$\frac{\Delta d_L(z)}{d_L(z)} = \sqrt{\sigma_{inst}^2(z) + \sigma_{lens}^2(z) + \sigma_{pv}^2(z)}$$

The relative magnitude of errors

$$\sigma_{inst}^2(z) > \sigma_{lens}^2(z) \gg \sigma_{pv}^2(z)$$

Comparison with another way of incorporating lensing effects

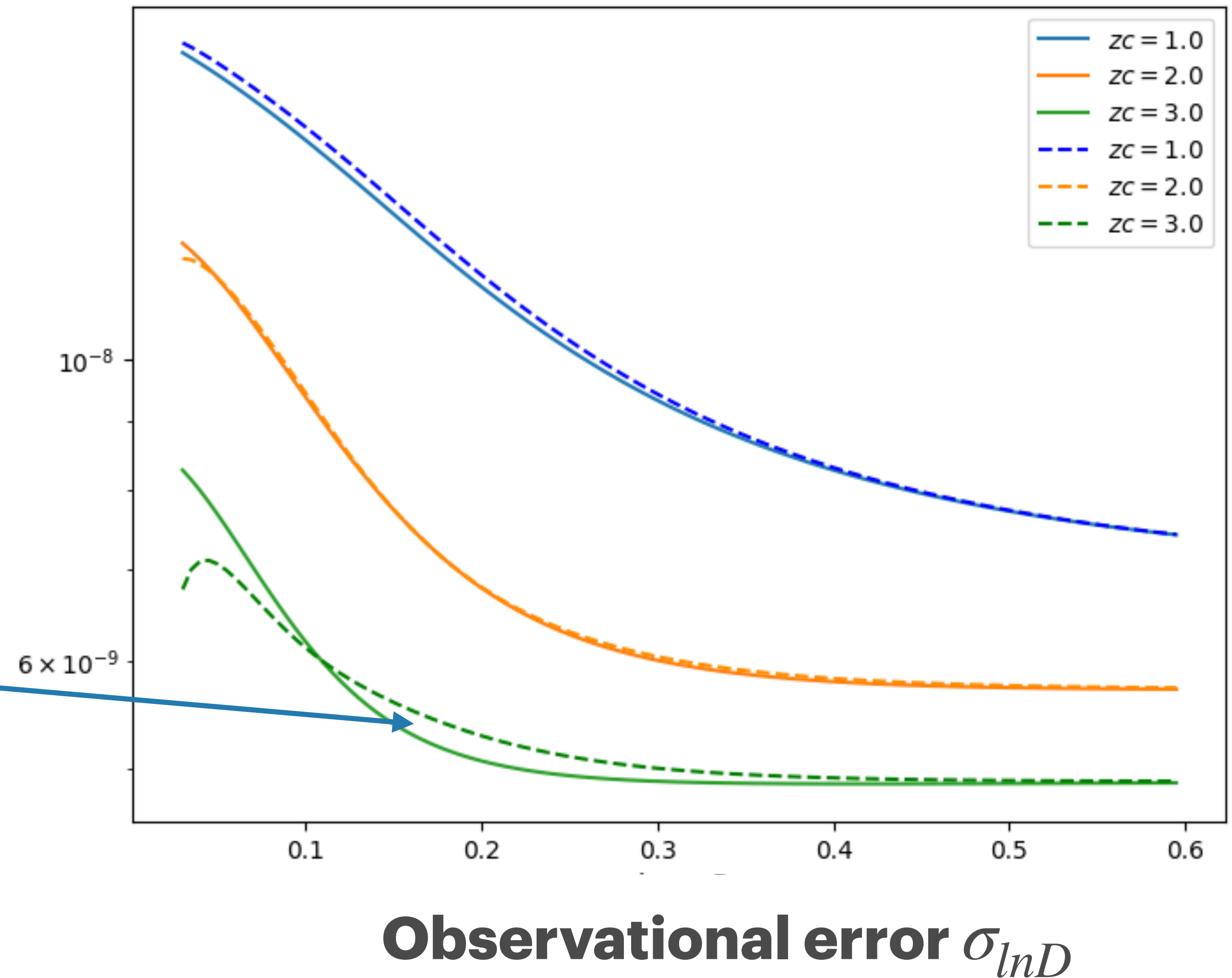
the gravitational lensing effect is expressed by increasing the probability distribution dispersion.

$$\sigma_{lnD} \longrightarrow \sigma = \sqrt{\sigma_{lnD}^2 + \sigma_{\kappa}^2}$$

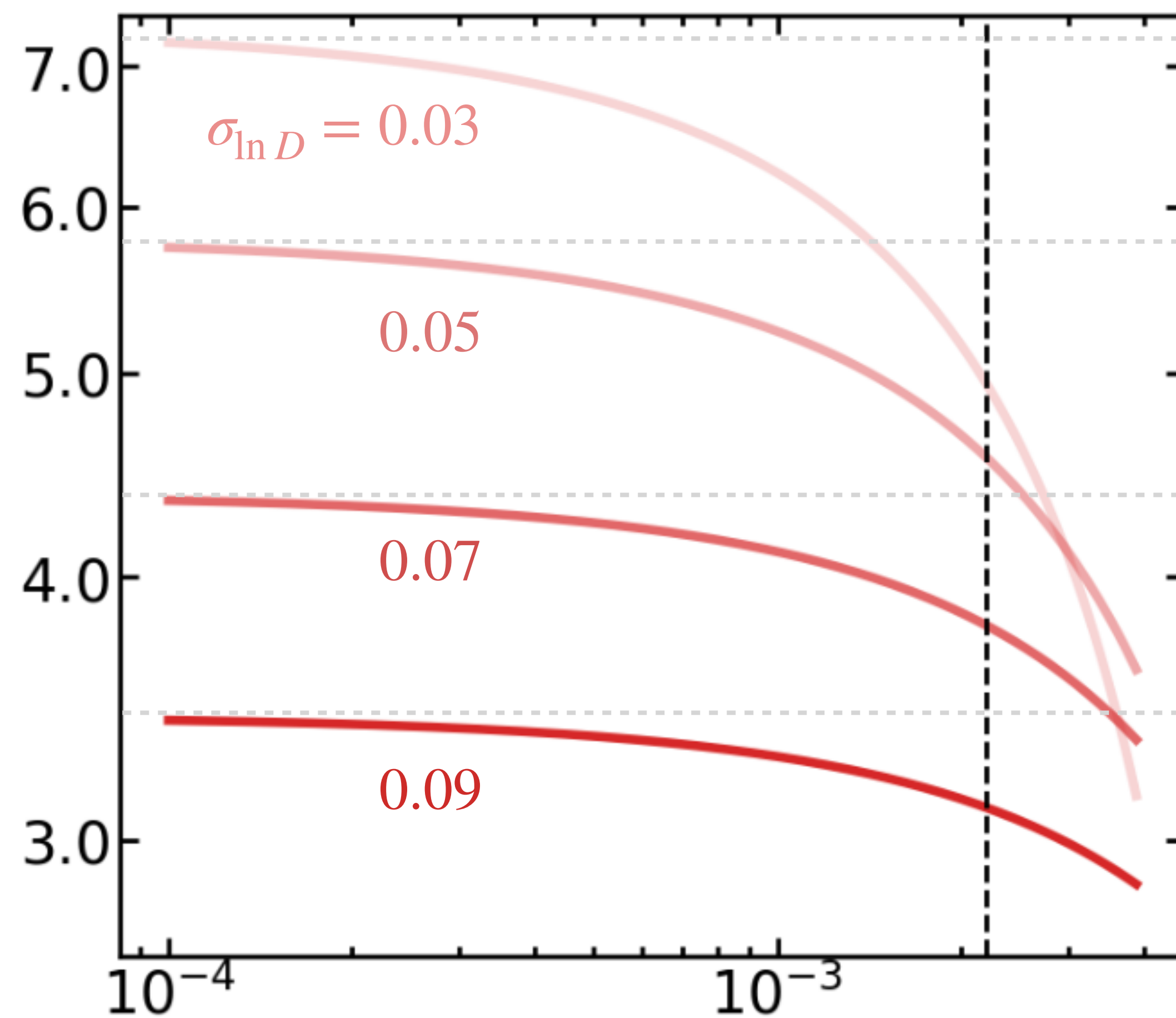
Differences can be seen where the observation error is small.

The distribution is uniformly expanded, so the distribution's distortion cannot be fully expressed in this way?

auto-correlation $C^{ww}(\ell = 100)$



相互相関角度パワースペクトル



2 次の重力レンズ効果