Cosmology

with torsion and nonmetricity

Metric-Affine Gravity

GGI 10/07/2026

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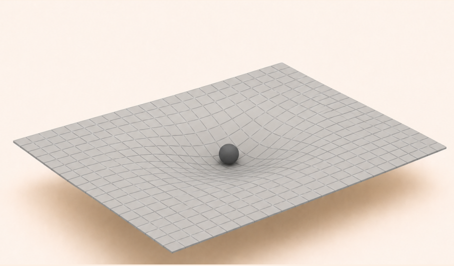
A generic lagrangian

$$\mathcal{L} = \mathcal{L}_g + \mathcal{L}_m$$

General relativity

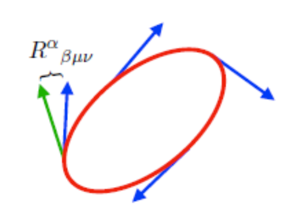
Metric $g_{\mu\nu}$

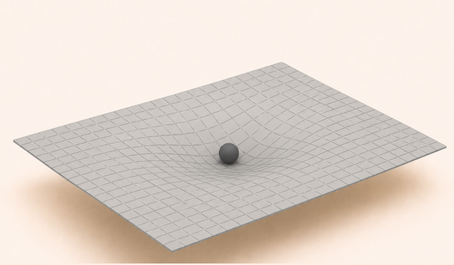


General relativity

Metric $g_{\mu\nu}$

Curvature R





General relativity

$\boxed{T_{\mu\nu}}$

Stress - energy tensor

$$T^{\alpha\beta} \equiv +\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_M)}{\delta g_{\alpha\beta}}$$

General relativity

$\tilde{\Gamma}^{\alpha}{}_{\mu\nu}[g_{\mu\nu}]$ symmetric

$$\nabla_{\alpha} g_{\mu\nu} = 0$$

General relativity

$\cancel{\tilde{\Gamma}^{\alpha}{}_{\mu\nu}[g_{\mu\nu}] \text{ symmetric}}$

$\cancel{\nabla_\alpha g_{\mu\nu} = 0}$

General relativity

$\cancel{\tilde{\Gamma}^{\alpha}{}_{\mu\nu}[g_{\mu\nu}] \text{ symmetric}}$ $\cancel{\nabla_\alpha g_{\mu\nu} = 0}$

$$\downarrow$$

Metric affine gravity

Metric affine gravity

$g_{\mu\nu}$ Γ

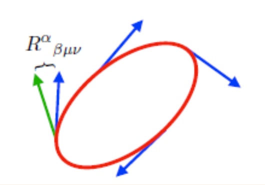
$\boxed{\text{Curvature R}}$

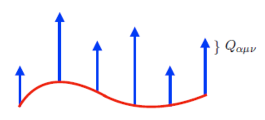
$+$

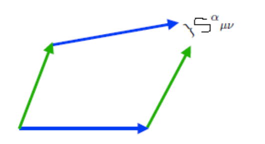
$\boxed{\text{Nonmetricity Q}}$

$+$

$\boxed{\text{Torsion S}}$







Metric affine gravity

$$\boxed{T_{\mu\nu}}$$

$$\boxed{\Delta_{\lambda}^{\mu\nu}}$$

Stress - energy tensor

HYPERMOMENTUM

$$T^{\alpha\beta} \equiv +\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_M)}{\delta g_{\alpha\beta}}$$

$$\Delta_{\lambda}{}^{\mu\nu} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_M)}{\delta \Gamma^{\lambda}{}_{\mu\nu}}$$

ΛCDM model

- Baryons + radiation + Λ + CDM
- GR as theory of gravity
- CP = homogeneity and isotropy
- 6 free parameters
- Spatial flatness $k_u = 0$
- Inflationary initial conditions

General relativity

$$\mathcal{L} = \tilde{R}[\tilde{\Gamma}[g]] + \mathcal{L}_m[g]$$

$$\downarrow$$

$$\delta_{g}\mathcal{L} = 0$$

General relativity

$$3\,H^2 = \kappa\,\rho$$

$$2\dot{H} + 3H^2 = -\kappa\,\mathrm{w}\,\rho$$

$$\dot{\rho} + 3\,H(1+\mathrm{w})\rho = 0$$

$$p = \mathrm{w}\,\rho$$

Perfect fluid

$$\boxed{\dot{\rho} \pm \sqrt{3\,\kappa}\,(1 + \mathrm{w})\,\rho^{3/2} = 0}$$

$$\rho(t) = \frac{\rho_0}{\left(1 \pm \frac{\sqrt{3\,\kappa\,\rho_0}}{2}\,(1 + \mathrm{w})\,(t - t_0)\right)^2}$$

How the energy density dilutes

$$\boxed{2\dot{H} + 3\left(1 + w\right) H^2 = 0}$$

$$H(t) = \frac{H_0}{1 + \frac{3H_0}{2}\left(1 + w\right)\left(t - t_0\right)}$$

How the universe expands

Metric affine gravity

$$\mathcal{L} = R[g, \Gamma] + \mathcal{L}_m[g, \Gamma]$$

$$\delta_{g}\mathcal{L} = 0 \qquad\qquad \delta_{\Gamma}\mathcal{L} = 0$$

Metric affine gravity

perfect fluid

+

hypermomentum

$$3H^2 = \kappa\rho + \kappa\rho_h$$

$$2\dot{H} + 3H^2 = -\kappa p - \kappa p_h$$

$$\dot{\rho} + 3H(\rho + p) = -\dot{\rho}_h - 3H\left(\rho_h + p_h\right)$$

$$p = \mathrm{w}\,\rho$$

$$p_h = \mathrm{w}_h\,\rho_h$$

Metric affine gravity

$$3H^2 = \kappa\rho + \kappa\rho_h$$

$$\boxed{\rho_h[\sigma, \Sigma_1, \Sigma_2]}$$

$$2\dot{H} + 3H^2 = -\kappa p - \kappa p_h$$

$$\boxed{p_h[\sigma, \Sigma_1, \Sigma_2]}$$

$$\dot{\rho} + 3H(\rho + p) = -\dot{\rho}_h - 3H\left(\rho_h + p_h\right)$$

$$p = \mathrm{w}\,\rho$$

$$p_h = \mathrm{w}_h\,\rho_h$$

Metric affine gravity

$$\Delta_\lambda{}^{\mu\nu} \equiv -\frac{2}{\sqrt{-g}} \frac{\delta\left(\sqrt{-g}\mathcal{L}_M\right)}{\delta \Gamma^\lambda{}_{\mu\nu}}$$

σ Σ_1 Σ_2

$$\boxed{\rho_h[\sigma, \Sigma_1, \Sigma_2]}$$

$$\boxed{p_h[\sigma, \Sigma_1, \Sigma_2]}$$

Metric affine gravity

$$3H^2 = \kappa\rho + \kappa\rho_h$$

$$\boxed{\rho_h[\sigma, \Sigma_1, \Sigma_2]}$$

$$2\dot{H} + 3H^2 = -\kappa p - \kappa p_h$$

$$\boxed{p_h[\sigma, \Sigma_1, \Sigma_2]}$$

$$\dot{\rho} + 3H(\rho + p) = -\dot{\rho}_h - 3H\left(\rho_h + p_h\right)$$

$$p = \mathrm{w}\,\rho$$

$$p_h = \mathrm{w}_h\,\rho_h$$

$$w_{eff} \equiv \frac{p+p_h}{\rho+\rho_h}$$

$$\dot{\rho} \pm \sqrt{3\kappa}\,(1 + \mathbf{w})\,\rho^{3/2} = 0 \qquad\qquad 2\dot{H} + 3\,(1 + \mathbf{w})\,H^2 = 0$$

$$\mathbf{w} \longrightarrow \mathbf{w}_\rho \qquad\qquad \mathbf{w} \longrightarrow \mathbf{w}_{\mathrm{eff}}$$

$$\dot{\rho} \pm \sqrt{3\kappa}\,(1 + \mathbf{w}_\rho)\,\rho^{3/2} = 0 \qquad\qquad 2\dot{H} + 3\,(1 + \mathbf{w}_{\mathrm{eff}})\,H^2 = 0$$

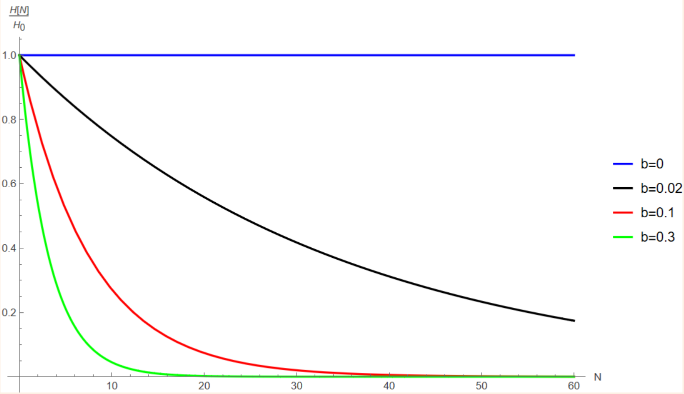
$$\dot{\rho} \pm \sqrt{3\kappa}\,(1 + \mathrm{w}_\rho)\,\rho^{3/2} = 0$$

$$2\dot{H} + 3\,(1 + \mathrm{w}_{\mathrm{eff}})\,H^2 = 0$$

How the energy density dilutes

How the universe expands

$$\sigma \propto b\sqrt{\rho}$$



$$w = -1$$

$$\mathrm{w_{eff}} = \frac{2\,\mathrm{w} - b}{2 + 3b}$$

$$dS \nrightarrow \rho(t) = \rho_0$$

$$\nrightarrow H \text{ constant}$$

Thank you

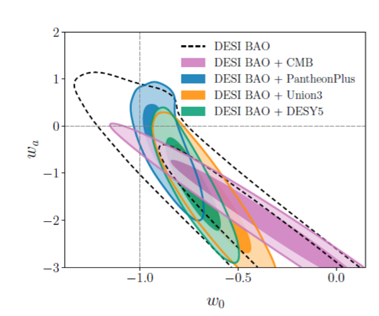
- Phys.Rev.D 111 (2025) 6, 064063
- J.Phys.Conf.Ser. 3177 (2026) 1, 012033

Metric affine gravity

$$
\begin{tikzcd}
& \mathcal{L}_g[g,\Gamma] + \mathcal{L}_M[g,\Gamma,\phi_M] \arrow[dl] \arrow[dr] & \\
\delta_g(\mathcal{L}_g+\mathcal{L}_M)=0 & & \delta_\Gamma(\mathcal{L}_g+\mathcal{L}_M)=0 \arrow[d, "\boxed{1}"] \\
& & \text{Constrains!!!} \arrow[ull, bend left, "\boxed{2}"] \arrow[uul, bend right, "\boxed{3}"']
\end{tikzcd}
$$

Why we need to go beyond ΛCDM?

- H_0 tension
- S_8 tension
- Dark energy
- Dark matter



$$w(z) = w_0 + w_a \frac{z}{1+z}$$

$$\text{DESI} \rightarrow w_0 \neq -1, w_a \neq 0$$

Modified Friedmann equations assumptions:

- Cosmological principle
- Projective invariance
- Parity even

Cosmological principle

- $\boxed{Q \to 3 \text{ dofs}}$
- $\boxed{S \to 2 \text{ dofs}}$
- $\boxed{\Delta \to 5 \text{ dofs}}$

- ~~*Weyssenhoff fluid*~~
- FLRW metric
- Perfect fluid

Projective invariance

$$\Gamma^{\lambda}_{\mu\nu} \mapsto \Gamma^{\lambda}_{\mu\nu} + \delta^{\lambda}_{\mu}\xi_{\nu}$$

$$\mathcal{L}_g = R[\tilde{\Gamma}] \qquad \Longrightarrow \qquad \Delta_{\alpha} = 0$$

- $\boxed{\Delta \rightarrow 4 \text{ dofs}}$
- $\boxed{Q \rightarrow 2 \text{ dofs}}$

Parity even

$$\mathcal{L}_g = R[\tilde{\Gamma}] \qquad \Longrightarrow \qquad \mathcal{L}_m \text{ even}$$

- $\boxed{\Delta \to 3 \text{ dofs}}$
- $\boxed{S \to 1 \text{ dof}}$