Lagrangian models of interacting dark matter and dark energy

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GGI Workshop: Exploring New Frontiers in Cosmology

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Image credit: ESA and the Planck Collaboration

Outline

Part 1 – Theory

- Introduction & motivation
- Perfect fluids & Lagrangians
- Interacting Lagrangian models
- Entropy coupled dark sector

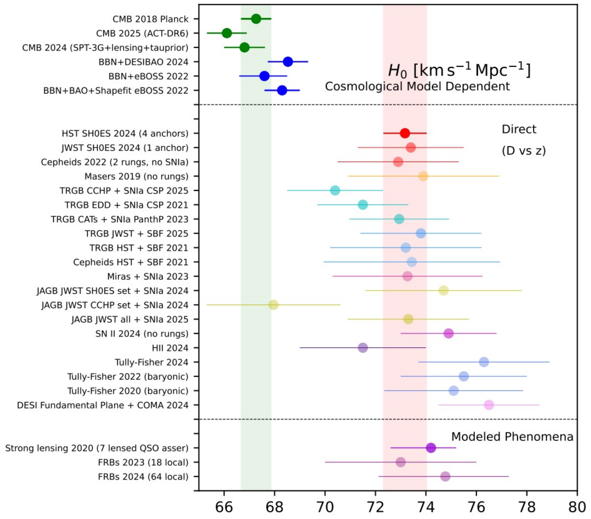
Part 2 – Cosmology

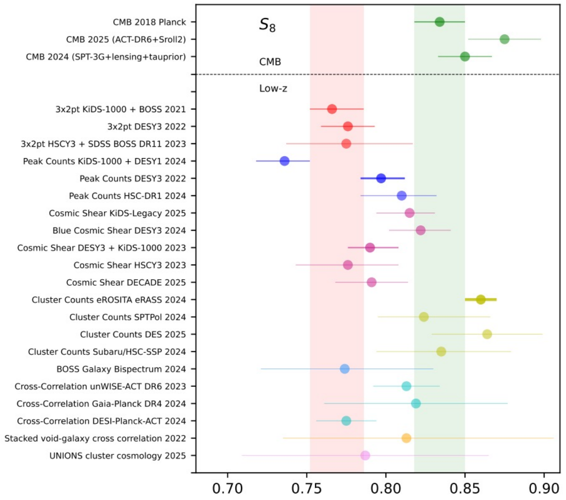
- Background & perturbations
- Observational signatures

Introduction

Challenges to ΛCDM

- Unknown nature of dark matter and dark energy
- Tensions & discrepancies in cosmology





CosmoVerse White Paper [arXiv:2504.01669]

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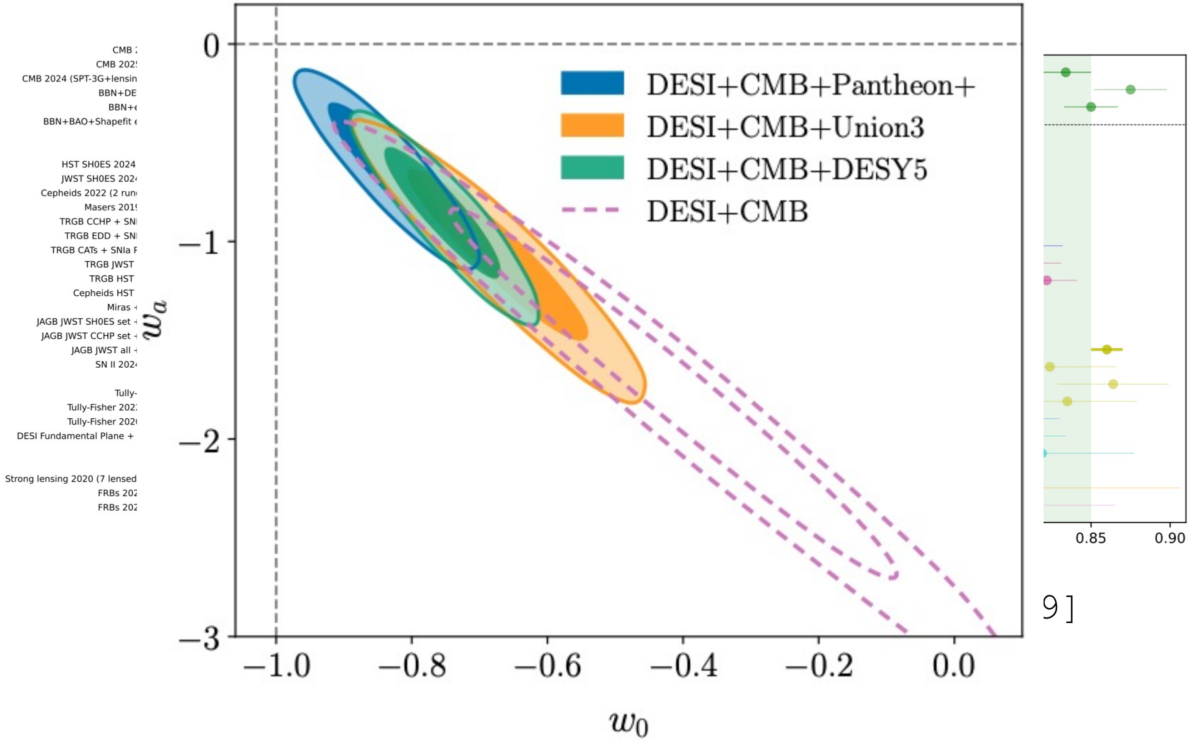

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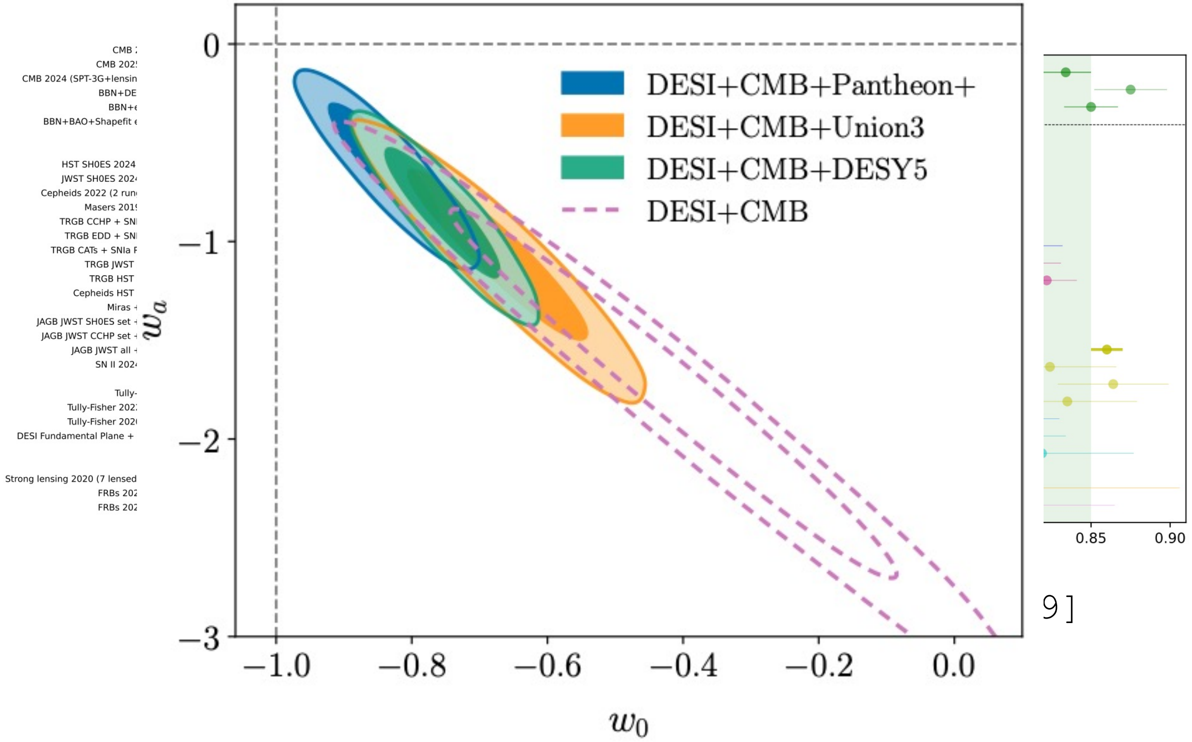

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Dark sector interactions

- **Interactions** in the dark sector explored as possible modifications to ΛCDM

$$\boxed{\begin{aligned} \dot{\rho}_c + 3H(\rho_c + p_c) &= Q \\ \dot{\rho}_{DE} + 3H(\rho_{DE} + p_{DE}) &= -Q \end{aligned}}$$

- Can give effective phantom-crossing behaviour

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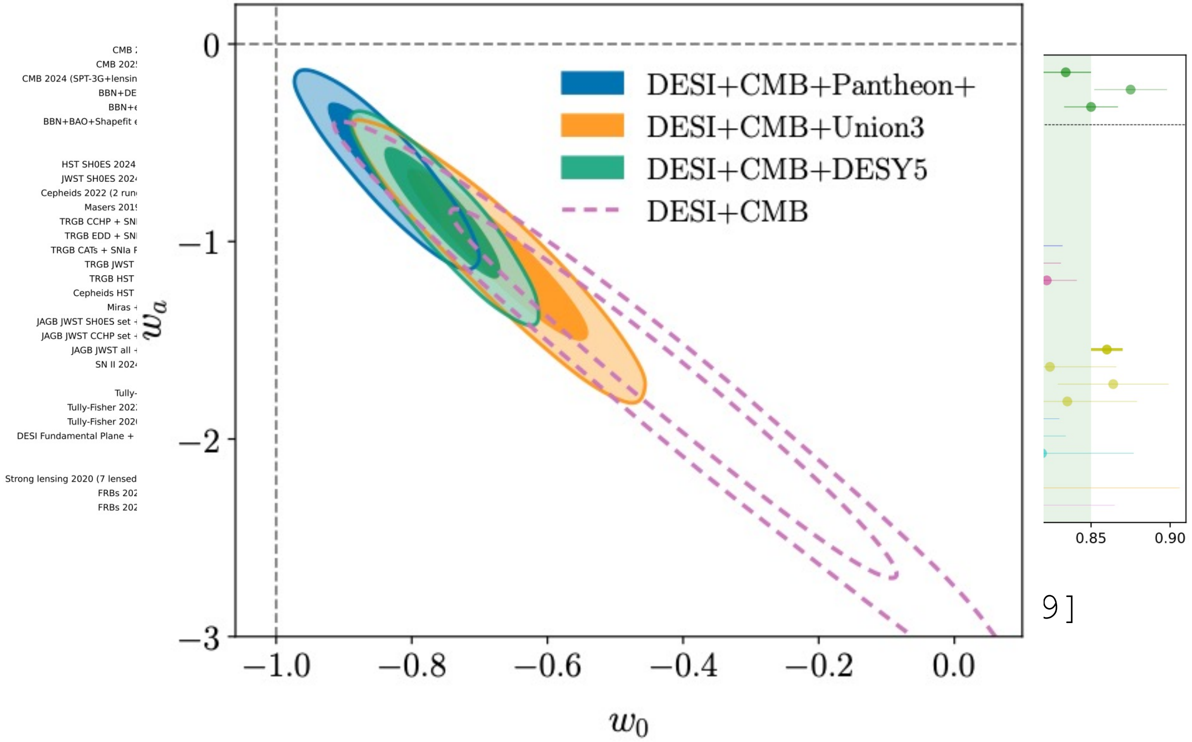

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Problems with phenomenological parameterizations:

- Difficult to investigate degrees of freedom
- Often prone to instabilities `Valiviita et al. [arXiv:0804.0232]`

Motivates **Lagrangian** models

Phenomenological models & instabilities

Well-known cosmological stability conditions (no-ghost & no gradient instability) for Lagrangian models:

- Barotropic cosmological perfect fluid/canonical scalar field (single degree of freedom)

$$\boxed{\rho + p > 0} \qquad \boxed{\text{NEC}}$$

- Same result for general scalar field Lagrangian $L(\phi, \nabla\phi)$ + non-kinetic interactions Vikman [arXiv:0407107]

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Trivial example: **phantom-crossing models**, which require exotic scenarios to be healthy:

- Interactions
- Higher-order terms
- Modified gravity theories
- Additional fields (e.g., quintom)

Generically changes the perturbative structure!

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Recent non-trivial example: **sign-switching DE** Bouhmadi-López [arXiv:2607.05044]

- Violates null energy condition (generic property of non-interacting sign-switching) $\dot{\rho}_{DE} > 0$
- DE Perturbations assumed to be ***perfect-fluid-like*** with fixed propagation $c_s^2 = 1$
- Generically expect ghosts – but hidden in numerical evolution

Perfect cosmological fluids

Perfect fluid energy-momentum tensor $\boxed{T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu}}$ with EoM $\boxed{\nabla_\mu T^{\mu\nu} = 0}$

Fluid dictionary

Fundamental d.o.f

- *number density* n
- *intrinsic entropy per particle* s

Thermodynamic variables

- *energy density* $\rho(n, s)$
- *pressure* $p(n, s) = n\frac{\partial \rho}{\partial n}|_s - \rho$

Perturbations

$$\delta p = c_a^2 \delta\rho + \frac{\partial p}{\partial s}\bigg|_\rho \delta s$$

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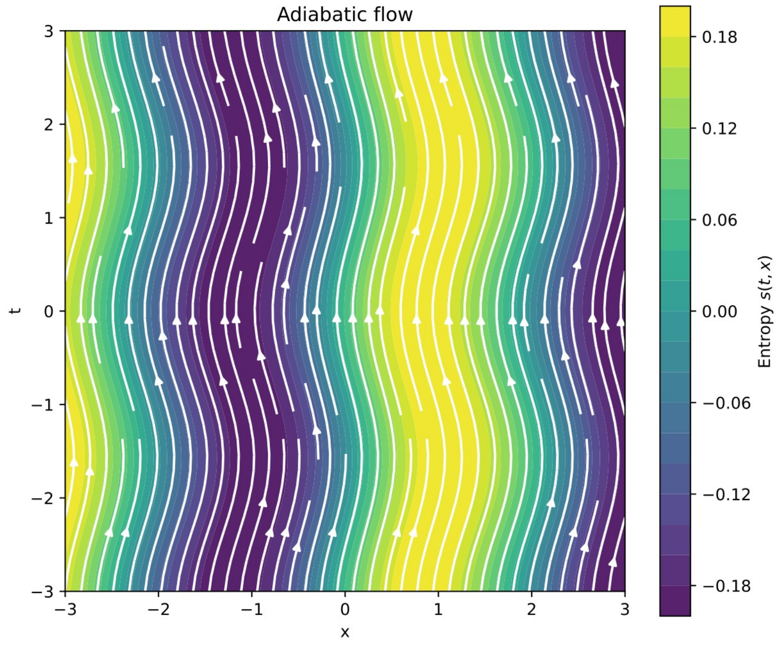
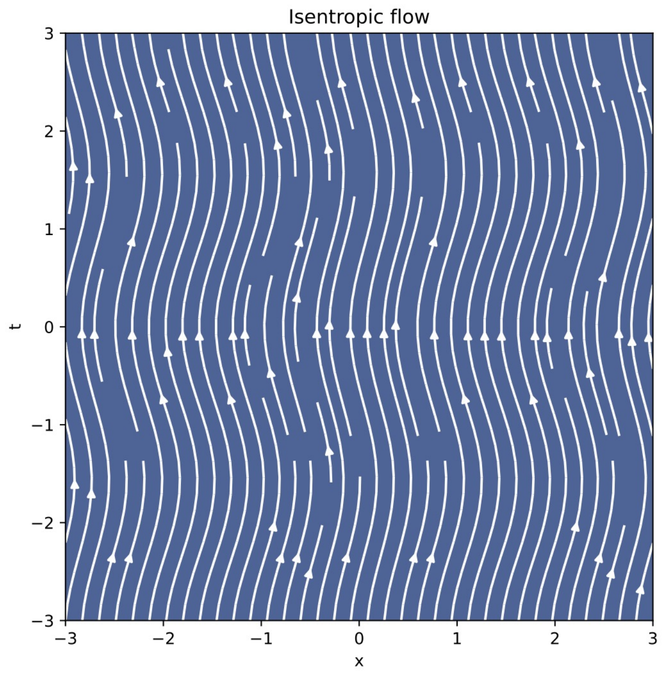
- Conservation of number density $\nabla_\mu(nu^\mu) = 0$ and entropy flux $\nabla_\mu(nsu^\mu) = 0$

Implies *adiabatic flow* $u^\mu \nabla_\mu s = 0$ ***not*** *isentropic flow* $\nabla_\mu s = 0$





Perfect fluid action

Brown perfect fluid Lagrangian `Brown [arXiv:9304026]`

$$S_{\text{Brown}} = \int \sqrt{-g}\Big[-\rho(n,s) + \overbrace{nu^{\mu}(\partial_{\mu}\varphi + s\partial_{\mu}\Theta)}^{\text{constraints}}\Big] d^4x$$

Lagrange multipliers

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Lagrange multipliers

Variations produce perfect fluid $T_{\mu\nu}$ and fluid equation of motion

- Metric $g_{\mu\nu}$

$$\boxed{T_{\mu\nu} = (\rho + p)u_{\mu}u_{\nu} + pg_{\mu\nu}}$$

- Lagrange multipliers φ,Θ

$$\boxed{\nabla_{\mu}(nu^{\mu}) = 0 \quad u^{\mu}\nabla_{\mu}s = 0}$$

- Fluid variables n, s

$$\boxed{\nabla_{\mu}T^{\mu\nu} = 0}$$

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Perfect fluid action stability

Expand Brown action to quadratic order in scalar perturbations and integrate out δn and δs

$$S_{\text{Brown}} = \int dt\, d^3k \left(\dot{\vec{\chi}}^{\mathrm{T}} \mathbf{K} \dot{\vec{\chi}} - \vec{\chi}^{\mathrm{T}} \mathbf{V} \vec{\chi} - \vec{\chi}^{\mathrm{T}} \mathbf{B} \dot{\vec{\chi}} \right)$$

with $\vec{\chi} = (\delta\varphi_k, \delta\Theta_k)$

In the large k-limit we obtain the sound speeds:

- Standard propagating fluid mode $\boxed{c_1^2 = \dfrac{n^2 \rho_{,nn}}{\rho + p}}$
- Non-propagating entropy mode $\boxed{c_2^2 = 0}$

e.g., barotropic fluid $p = w\rho$ with constant w has $c_1^2 = w$

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- Entropy condition $\rho_{,ss} > 0$
- Thermodynamic condition $\rho_{,nn} > \frac{(\rho_{,s} - n\rho_{,ns})^2}{n^2 \rho_{,ss}}$

With absence of gradient instabilities $c_1^2 \geq 0$ gives

- Null energy condition $\rho + p > 0$
- For barotropic fluids this rules out phantom behaviour $w > -1$

Non-minimal couplings

Model interactions with non-minimal couplings to scalar field dark energy

$$\underbrace{S_{\mathrm{EH}} + S_{\mathrm{Brown}}}_{\text{DM}} + \underbrace{S_{\phi}}_{\text{DE}} + \underbrace{\int f(n, s, \phi, \ldots)\sqrt{-g}\,d^4x}_{\text{Interactions}}$$

Modified EFE equations and matter energy-momentum conservation

$$\boxed{G_{\mu\nu} = \kappa\left(T_{\mu\nu} + T_{\mu\nu}^{(\phi)} + T_{\mu\nu}^{\mathrm{int}}\right)}$$

$$\boxed{\nabla^{\mu}\left(T_{\mu\nu} + T_{\mu\nu}^{(\phi)} + T_{\mu\nu}^{\mathrm{int}}\right) = 0}$$

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Number density & adiabatic constraints preserved (from multipliers)

$$\boxed{\nabla_\mu(nu^\mu) = 0 \quad u^\mu\nabla_\mu s = 0}$$

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Automatically implies the energy-momentum tensor is conserved along the fluid flow

$$\boxed{\nabla_\mu(nu^\mu) = 0 \quad u^\mu\nabla_\mu s = 0} \Rightarrow \boxed{u^\mu\nabla^\nu T_{\mu\nu} = 0}$$

Non-minimal couplings – current literature

Model interactions with non-minimal couplings to scalar field dark energy

$$S_{\text{EH}} + \underbrace{S_{\text{Brown}}}^{\textit{DM}} + \overbrace{S_\phi}^{\textit{DE}} + \overbrace{\int f(n, s, \phi, \ldots)\sqrt{-g}\,d^4x}^{\textit{Interactions}}$$

Various models considered in the literature:

- Algebraic $f(n, \phi)$ and derivative $f(n)u^\mu \nabla_\mu \phi$ Boehmer et al. [arXiv:1501.06540, arXiv:1502.04030]
- Velocity-entrainment models $f(\mathcal{Z})$ with $\mathcal{Z} = g_{\mu\nu} u_c^\mu u_{DE}^\nu$ Jiménez et al. [arXiv:2012.12204]
- `**Type-1,2,3 models**' $f(n, \phi, Z)$ with four-velocity coupling $Z = u^\mu \nabla_\mu \phi$ Pourtsidou et al. [arXiv:1307.0458]

Type-1: $ne^{\alpha(\phi)}$ **Type-2**: $nh(Z)$ **Type-3**: $\gamma(Z) + f(n)$

- **Entropy-couplings** $f\left(s, \phi, \nabla_\mu s \nabla^\mu \phi\right)$ Jensko et al. [arXiv:2603.10622]

Non-minimal couplings – pure-momentum transfer

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- **Type-3** *chooses* $T_{\mu\nu}$ and therefore describes a pure-momentum transfer

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- **Type-3** *chooses* $T_{\mu\nu}$ and therefore describes a pure-momentum transfer
- This relies on the Brown formulation! In Schutz approach this isn't true Boehmer et al. [arXiv: 2605.16220]

$\boxed{u^\mu \nabla^\nu T_{\mu\nu} = 0 \quad \text{Brown approach}}$ $\boxed{h^\mu_\sigma \nabla^\nu T_{\mu\nu} = 0 \quad \text{Schutz approach}}$

Pure-entropy couplings

Consider **pure-entropy interactions** in the dark sector with *algebraic* and *derivative* couplings

$$S_{\text{int}} = \int f(s, \phi, \mathcal{S})\sqrt{-g}\, d^4x \qquad \text{where } \mathcal{S} := \nabla^\mu \phi \nabla_\mu s$$

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Interaction tensor

$$\boxed{T_{\mu\nu}^{\text{int}} = -g_{\mu\nu} f + 2 f_{,\mathcal{S}} \nabla_{(\mu} \phi \nabla_{\mu)} s}$$

- Define total effective dark energy tensor $\tilde{T}_{\mu\nu}^{\text{DE}} := T_{\mu\nu}^{(\phi)} + T_{\mu\nu}^{\text{int}}$

- Pure-momentum transfer

$$\boxed{u^\mu \nabla^\nu T_{\mu\nu} = 0 \quad , \quad h_\nu^\mu \nabla^\lambda T_{\mu\lambda} = -J_\nu \neq 0}$$

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- Define total effective dark energy tensor $\tilde{T}^{\text{DE}}_{\mu\nu} \coloneqq T^{(\phi)}_{\mu\nu} + T^{\text{int}}_{\mu\nu}$
- Coupling acts only in the perpendicular direction

$$\boxed{J_\nu \coloneqq \nabla^\mu \tilde{T}^{\text{DE}}_{\mu\nu} = \nabla_\mu\left(f_{,\mathcal{S}} \nabla^\mu \phi\right)\nabla_\nu s - f_{,s}\nabla_\nu s}$$

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- Compare to Type-1,2,3 models:

Model	Coupling current	Continuity coupling δ'
Type-1	$J_\mu \propto \nabla_\mu \phi$	$\propto \phi'$
Type-2	$J_\mu \propto \nabla_\mu \phi$	$\propto \phi'' - \phi'$
Type-3	$J_\mu \propto h^\lambda_\mu \nabla_\lambda \phi + h^\lambda_\mu \nabla_\lambda (u^\nu \nabla_\nu \phi) + u^\nu \nabla_\mu u_\nu$	0

Stability – algebraic couplings (preliminary!)

Quadratic action for the scalar cosmological perturbations for ***algebraic couplings*** $f(s, \phi)$

$$S_{\text{quad}} = \int dt\, d^3k \left(\dot{\vec{\chi}}^{\mathrm{T}} \mathbf{K} \dot{\vec{\chi}} - \vec{\chi}^{\mathrm{T}} \mathbf{V} \vec{\chi} - \vec{\chi}^{\mathrm{T}} \mathbf{B} \dot{\vec{\chi}} \right)$$

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- Thermodynamic condition $\rho_{,nn} > \frac{(f_{,s} - n\rho_{,ns} + \rho_{,s})^2}{n^2(\rho_{,ss} + f_{,ss})}$

With absence of gradient instabilities $c_1^2 > 0$ gives

- Null energy condition $\rho + p > 0$

Generalises results in EFT approach for non-adiabatic perfect fluids `Ballesteros et al. [arXiv:1210.1561]`

Theoretically viable

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UCL

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Cosmology – background

Conformal time FLRW metric

$$ds^2 = -a(\tau)^2 d\tau^2 + a(\tau)^2(dx^2 + dy^2 + dz^2) \quad \text{with} \quad \mathcal{H} = a'/a$$

- Entropy conservation $u^\mu \nabla_\mu s = 0$ $\quad \boxed{s = \text{const}}$
- Number conservation $\nabla_\mu (n u^\mu) = 0$ $\quad \boxed{n \propto a^{-3}}$
- Standard continuity equation $\quad \boxed{\rho_c' + 3\mathcal{H}(p_c + \rho_c) = 0}$

Unchanged background dynamics from uncoupled quintessence

$$\boxed{\rho_{\mathrm{DE}} = \frac{1}{2a^2}\phi'^2 + \hat{V} \quad , \quad p_{\mathrm{DE}} = \frac{1}{2a^2}\phi'^2 - \hat{V}}$$

with effective potential $\hat{V}(\phi) \coloneqq V(\phi) + f$

Viable background evolution

Conformal Newtonian gauge

$$ds^2 = -a(\tau)^2(1+2\Phi)d\tau^2 + a(\tau)^2(1-2\Psi)(dx^2+dy^2+dz^2)$$

- Entropy conservation $u^\mu \nabla_\mu s = 0$

$$\boxed{(s' - s'\Phi + \delta s') = 0}$$

$$\boxed{s = \text{const}} \qquad \boxed{\delta s' = 0}$$

Frozen entropy perturbations

Cosmology – linear perturbations

Conformal Newtonian gauge

$$ds^2 = -a(\tau)^2(1+2\Phi)d\tau^2 + a(\tau)^2(1-2\Psi)(dx^2+dy^2+dz^2)$$

- Entropy conservation $u^\mu \nabla_\mu s = 0$

$$\boxed{(s' - s'\Phi + \delta s') = 0}$$

$$\boxed{s = \text{const}} \qquad \boxed{\delta s' = 0}$$

Frozen entropy perturbations

Choose *entropic-CDM* fluid $\rho = n\gamma(s)$ (with $\delta p \; = p = 0$)

Continuity Eq.

$$\delta_c' + \theta_c - 3\Psi' = 0$$

Euler Eq.

$$\theta_c' + \theta_c \mathcal{H} - \; k^2\Phi = -k^2 Q_s(\tau)\delta s(k)$$

Fifth-force term

Modified Euler equation with **scale-dependent source term**

Quasi-static limit

Quasi-static limit

Effective density source *Fifth-force term*

$$\delta_c'' + \mathcal{H}\delta_c' - 4\pi G a^2[\rho_c\delta_c + \Delta\rho_s(k,\tau)\delta s(k)] \simeq k^2 Q_s(\tau)\delta s(k)$$

with effective density source $\Delta\rho_s(k,\tau)$ and fifth-force $Q_s(\tau)$

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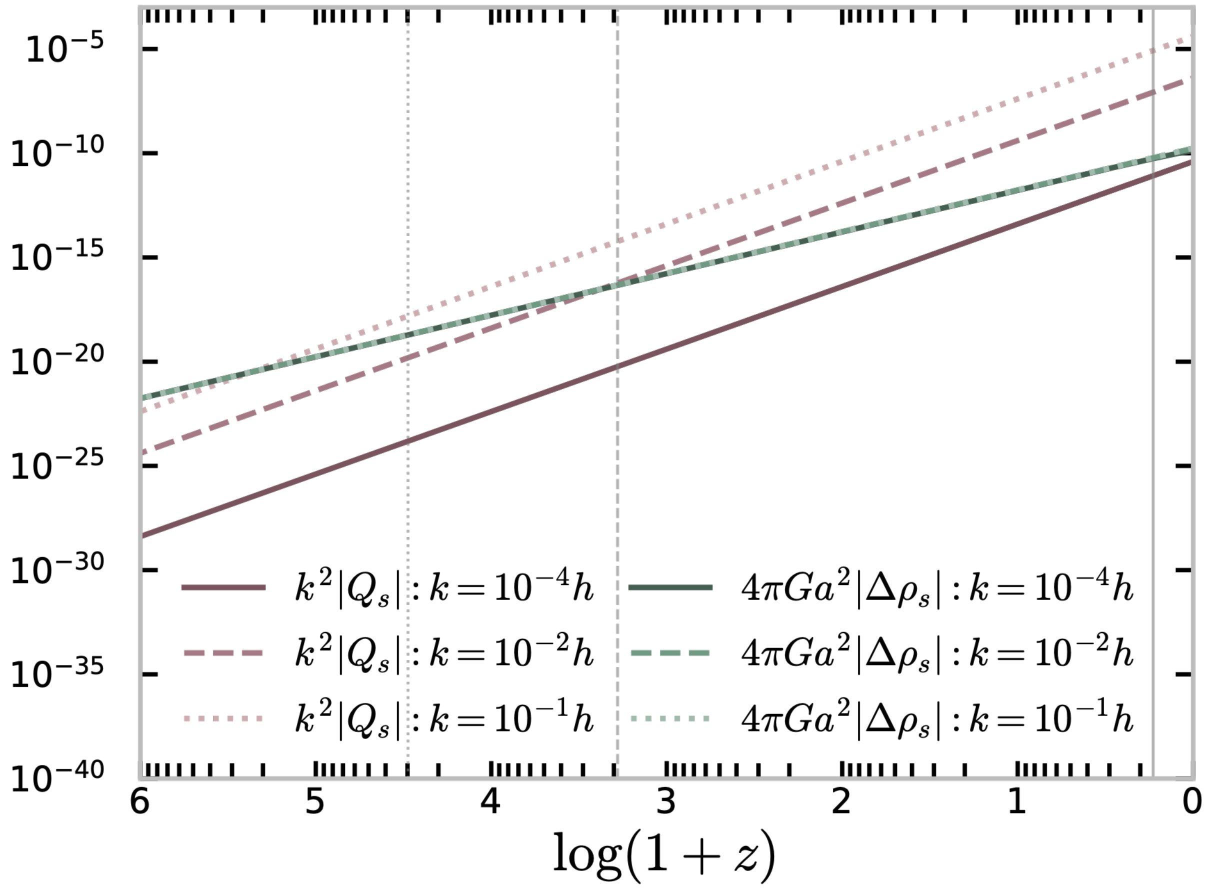
- Fifth-force term dominates on sub-Hubble scales
- Expect late-time changes to $P(k)$ on small scales



CLASS implementation

- **Phenomenological parametrization** of intrinsic entropy perturbations with power-law spectrum & UV cutoff

$$\boxed{\langle \delta s_{\mathbf{k}}\, \delta s^{*}_{\mathbf{k}'} \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k} - \mathbf{k}')\, P_s(k)}$$

$$\boxed{\mathcal{P}_s(k) = \mathcal{A}_{\mathrm{e}} \left(\frac{k}{k_p}\right)^{n} \exp\left[-\left(\frac{k}{k_c}\right)^{p_c}\right]}$$

- Inspired by isocurvature-like parameterisations
- Regularises $\mathcal{P}_s(k)$ on small scales

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- Inspired by isocurvature-like parameterisations
- Regularises $\mathcal{P}_s(k)$ on small scales

We then fix:

- Scale-invariant entropy perturbations $n = 0$ with $\mathcal{A}_e = 1$
- UV-suppression with cutoff $k_c = 1$ Mpc^{-1} and $p_c = 2$

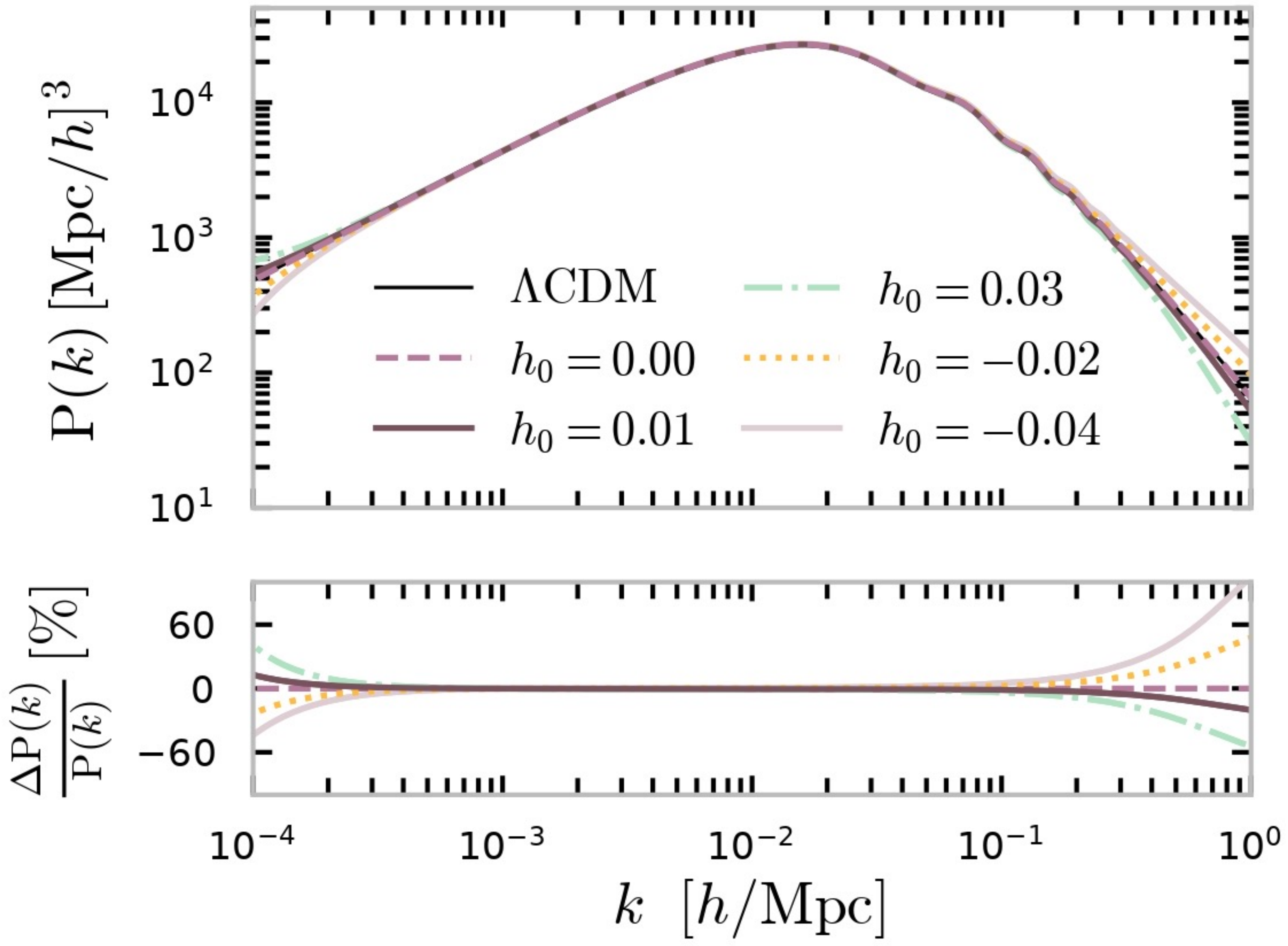
Consider linear couplings:

- Algebraic models $f = g_0 \phi s$
- Derivative models $f = h_0 \nabla^\mu \phi \nabla_\mu s$

And exponential potential $\hat{V} \propto \exp(-\lambda\phi)$

Observational signatures

Matter power spectrum $P(k)$



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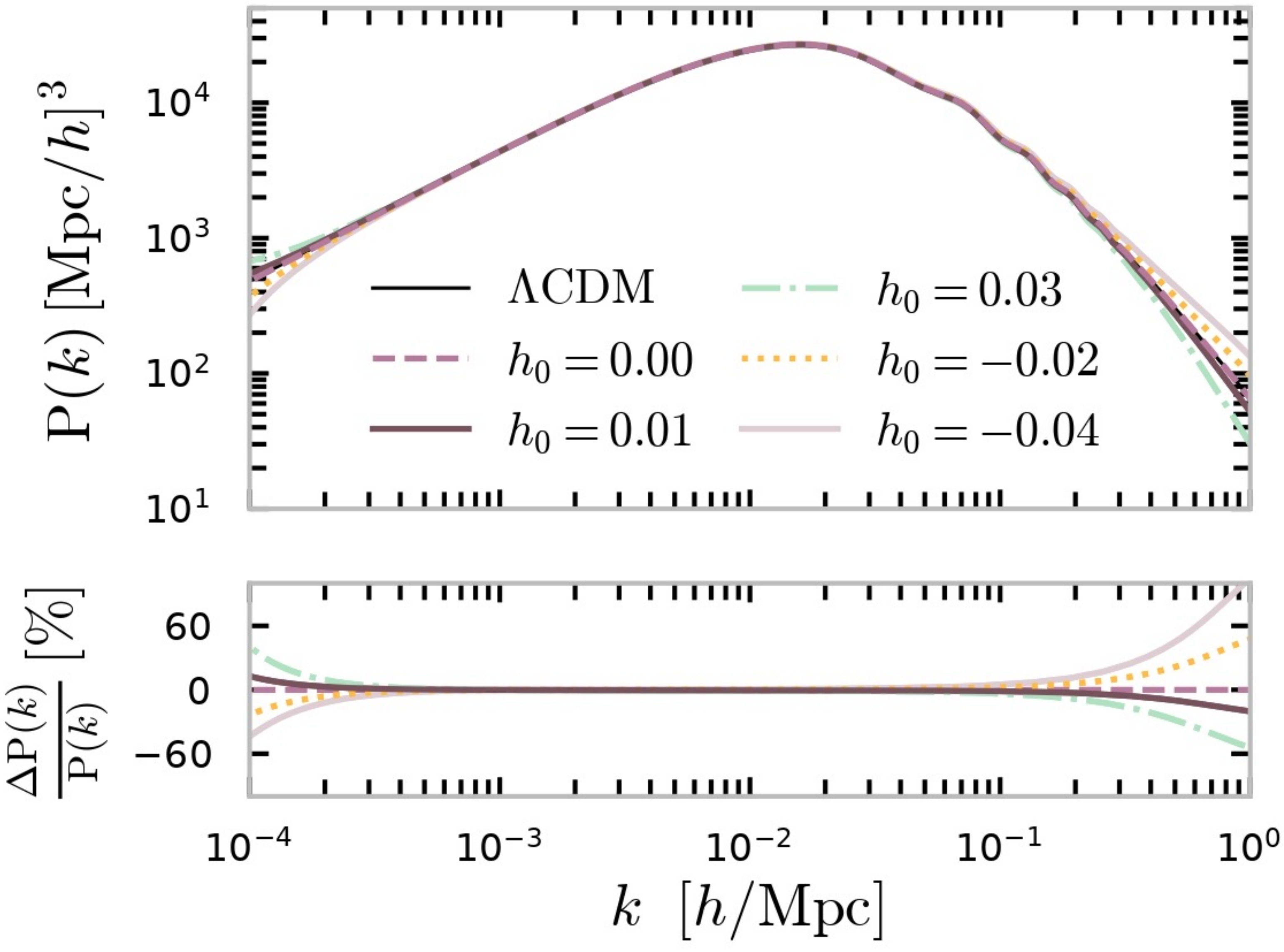


Small scale suppression/enhancement

$$\theta_c' + \theta_c \mathcal{H} - k^2 \Phi = -k^2 Q_s(\tau) \delta s$$

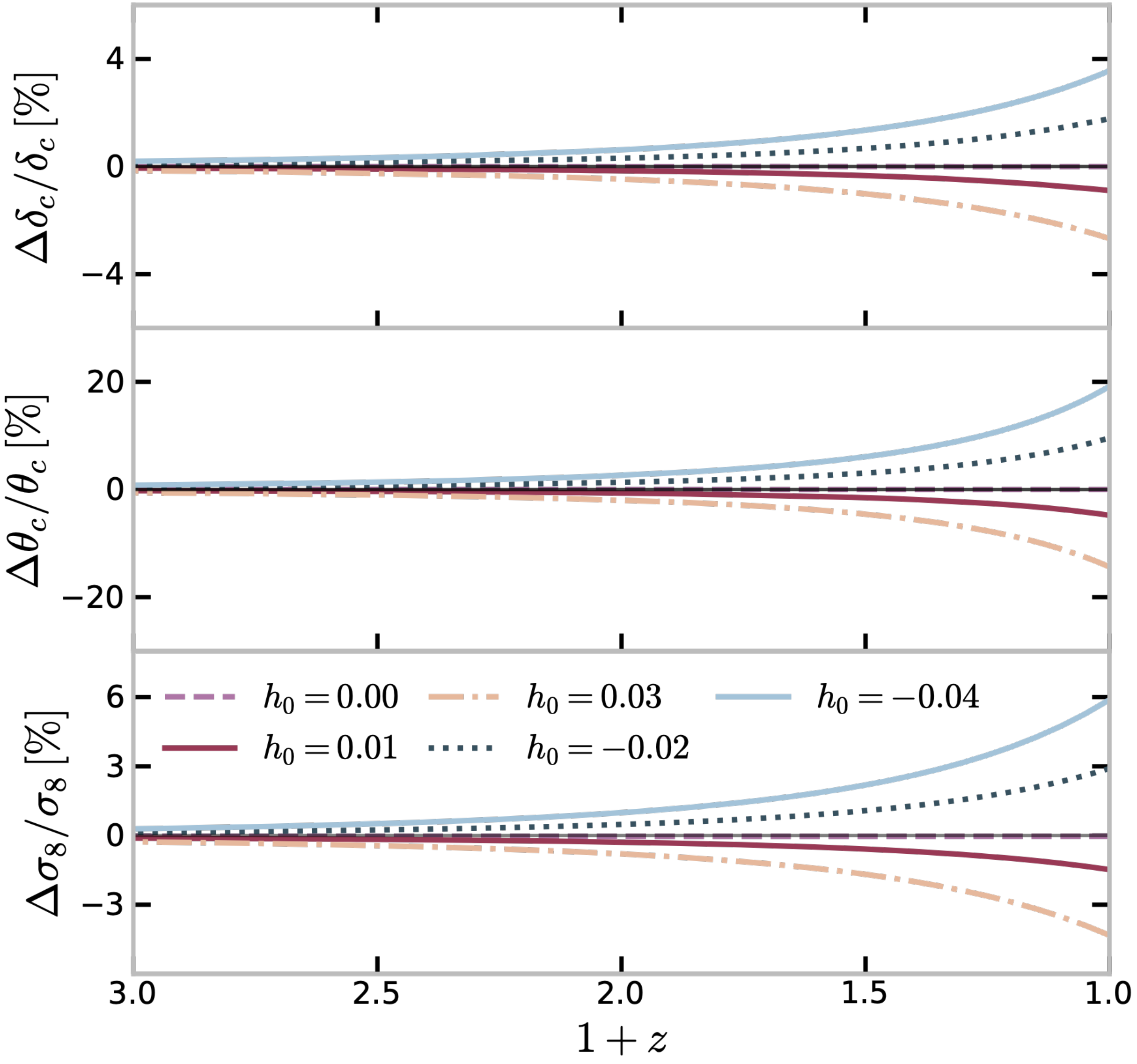
$$Q_s = -\frac{h_0}{\rho_c} \hat{V}_{,\phi}$$

Modified time-evolution of metric potentials → large-scale features

$$\Psi' + \mathcal{H}\Phi \approx \frac{4\pi G a^2}{k^2} \theta_c \rho_c$$

Observational signatures

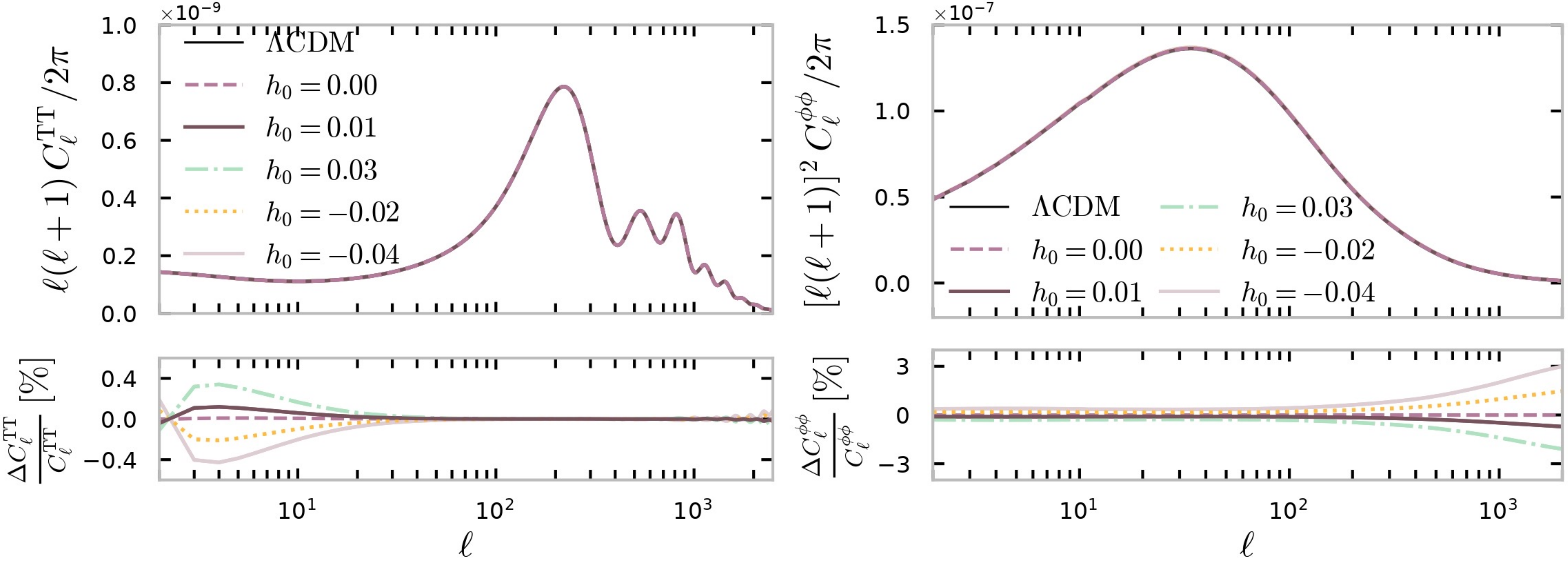
CDM density and velocity perturbations, and amplitude of matter fluctuations σ_8



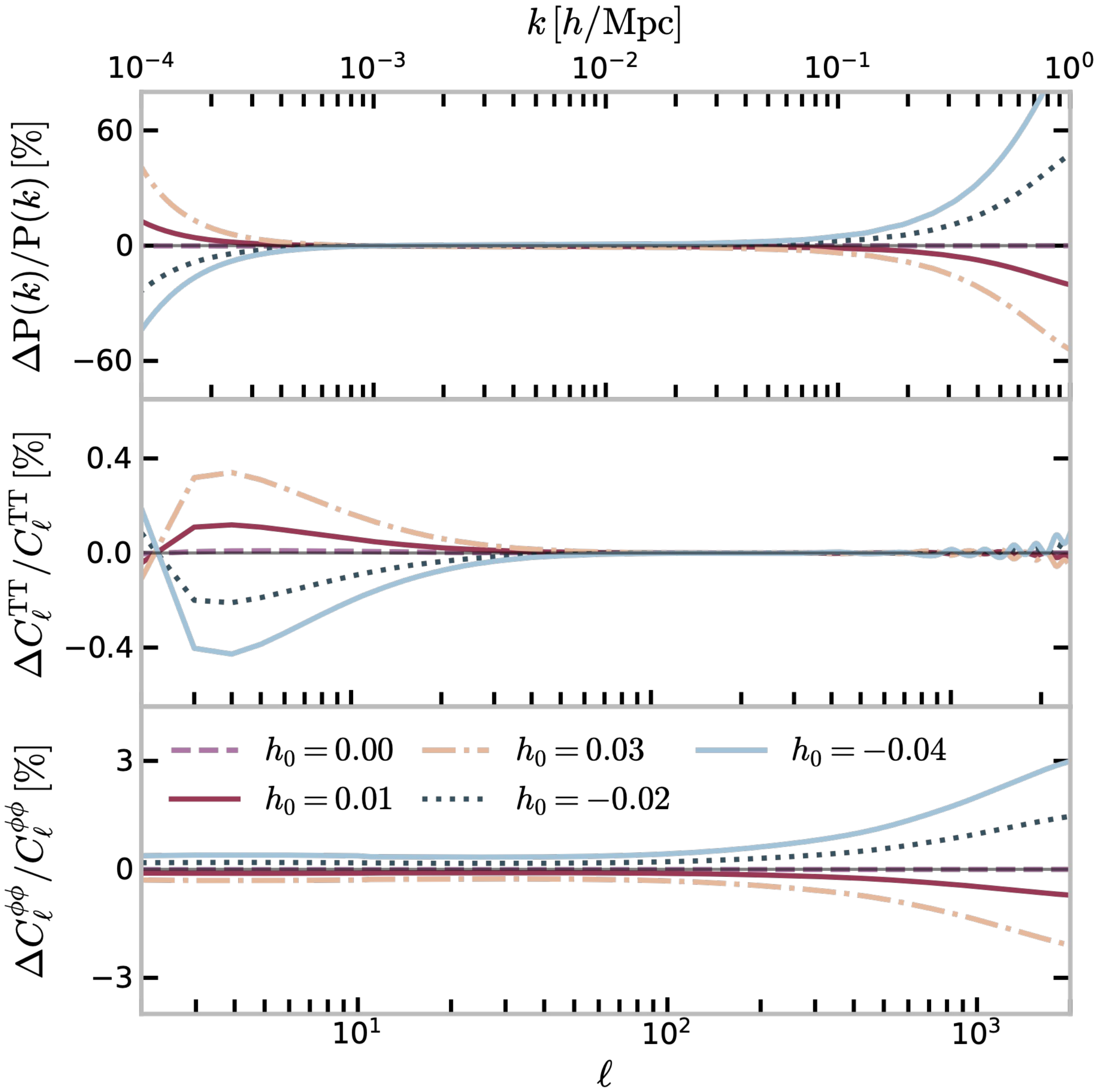
Observational signatures

UCL

CMB lensing spectrum $C_\ell^{\phi\phi}$ and temperature spectrum C_ℓ^{TT}



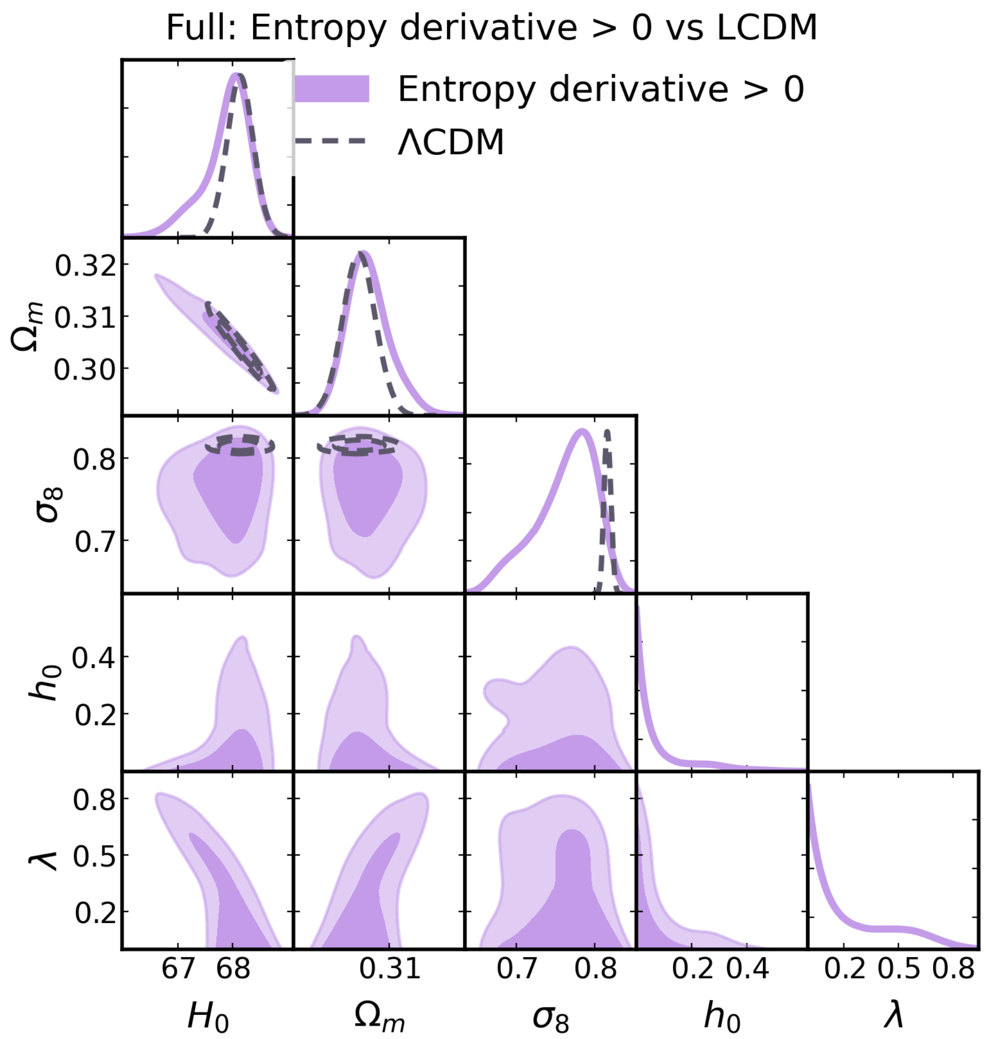
Observational signatures

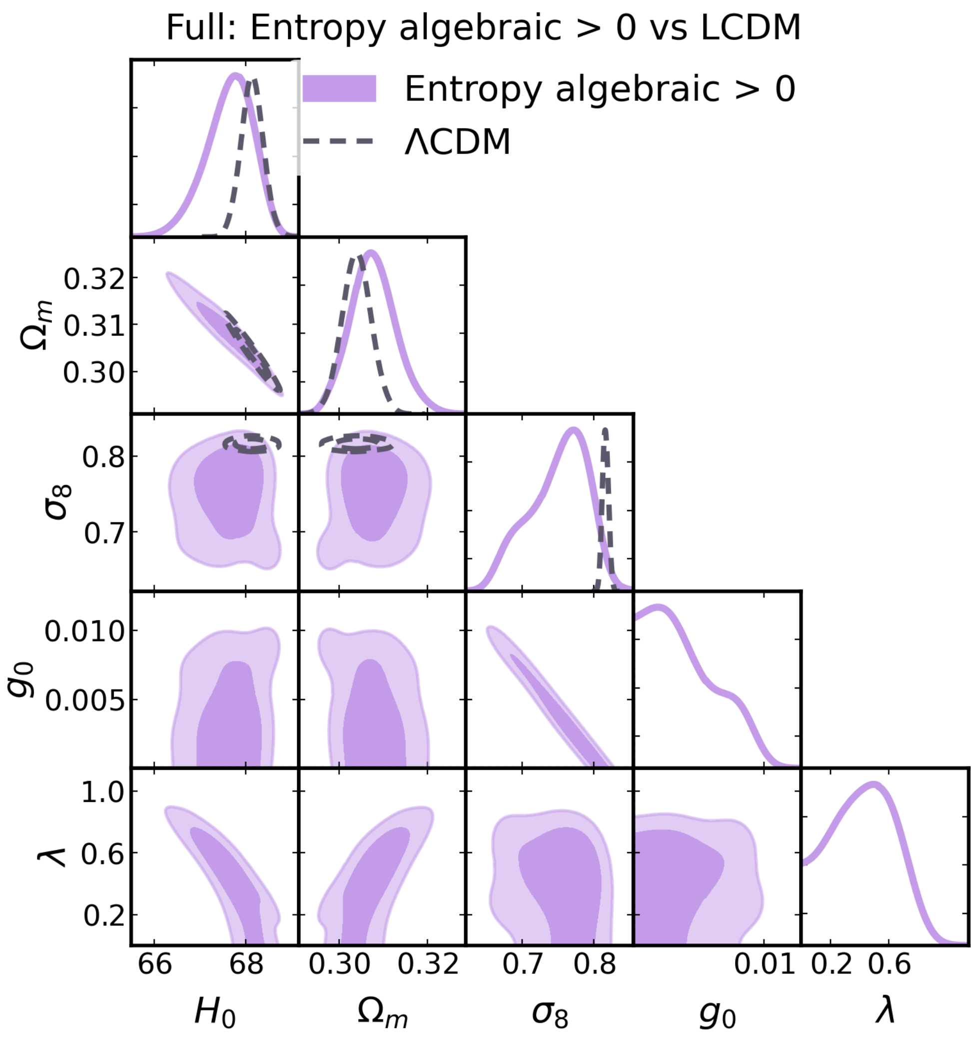


- Late-time suppression/enhancement of matter power spectrum on small/large scales
- Minimal changes to TT spectrum
- Potential observable signatures in lensing potentials

Observationally viable phenomenology with late-time suppression of matter clustering

Observational signatures (preliminary!)





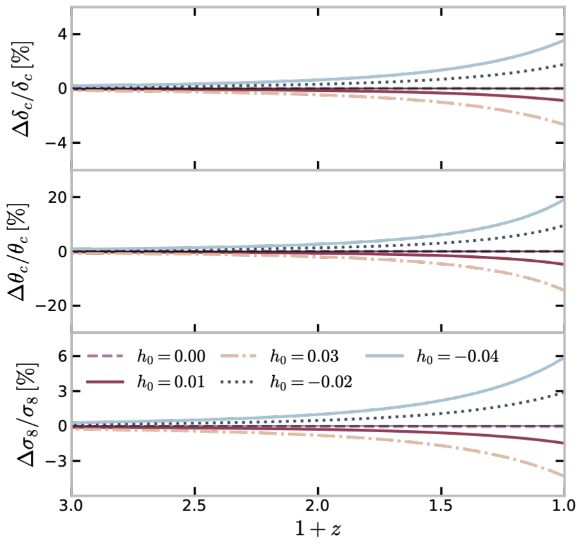
Summary

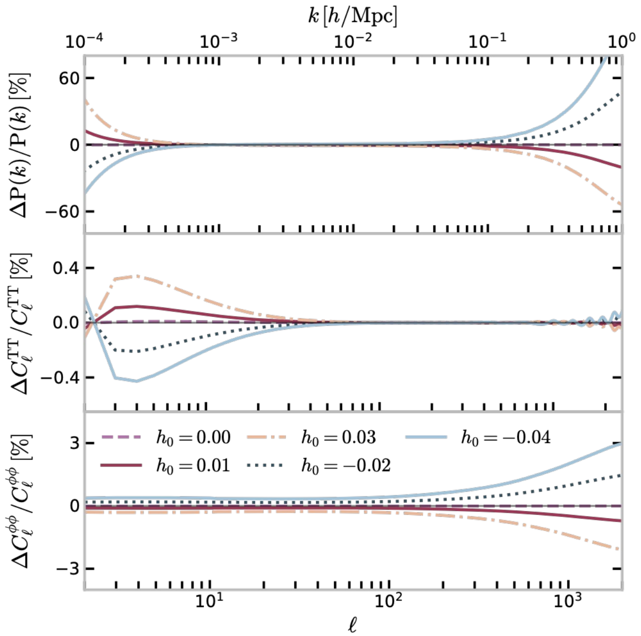
Results:

- New Lagrangian formulation of interacting dark sector
- Interactions from **coupling DM intrinsic entropy** s to DE ϕ
- Unchanged background expansion & modified cosmological perturbations
- Unique realisation of **pure-momentum transfer**

Ongoing work:

- Detailed study of stability & ghosts 
- Observational constraints on allowed interactions `(with E. M. Teixeira, V. Poulin, T. Smith & Anna Chiara Ferri)`
- Complimentary EFT approach `(with J. Beltrán Jiménez, L. J. Garay & M. Pérez Garrote)`





Thanks for listening

`E. Jensko, E. M. Teixeira, V. Poulin, Interacting dark sector from intrinsic entropy couplings [arXiv:2603.10622]`

Extra slides

Quasi-static limit

Effective density source *Fifth-force term*

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with effective density source $\Delta\rho_s(k,\tau)$ and fifth-force $Q_s(\tau)$

Choose algebraic models $f = g_0\phi s$

$$\boxed{Q_s^{\mathrm{alg}} = \frac{g_0\phi}{\rho_c}} \qquad \boxed{\Delta\rho_s^{\mathrm{alg}} = g_0\phi - \frac{a^2 g_0}{k^2 + a^2\widehat{V_{,\phi\phi}}}\widehat{V_{,\phi}}}$$

$$\boxed{\hat{V} \propto \exp(-\lambda\phi)}$$

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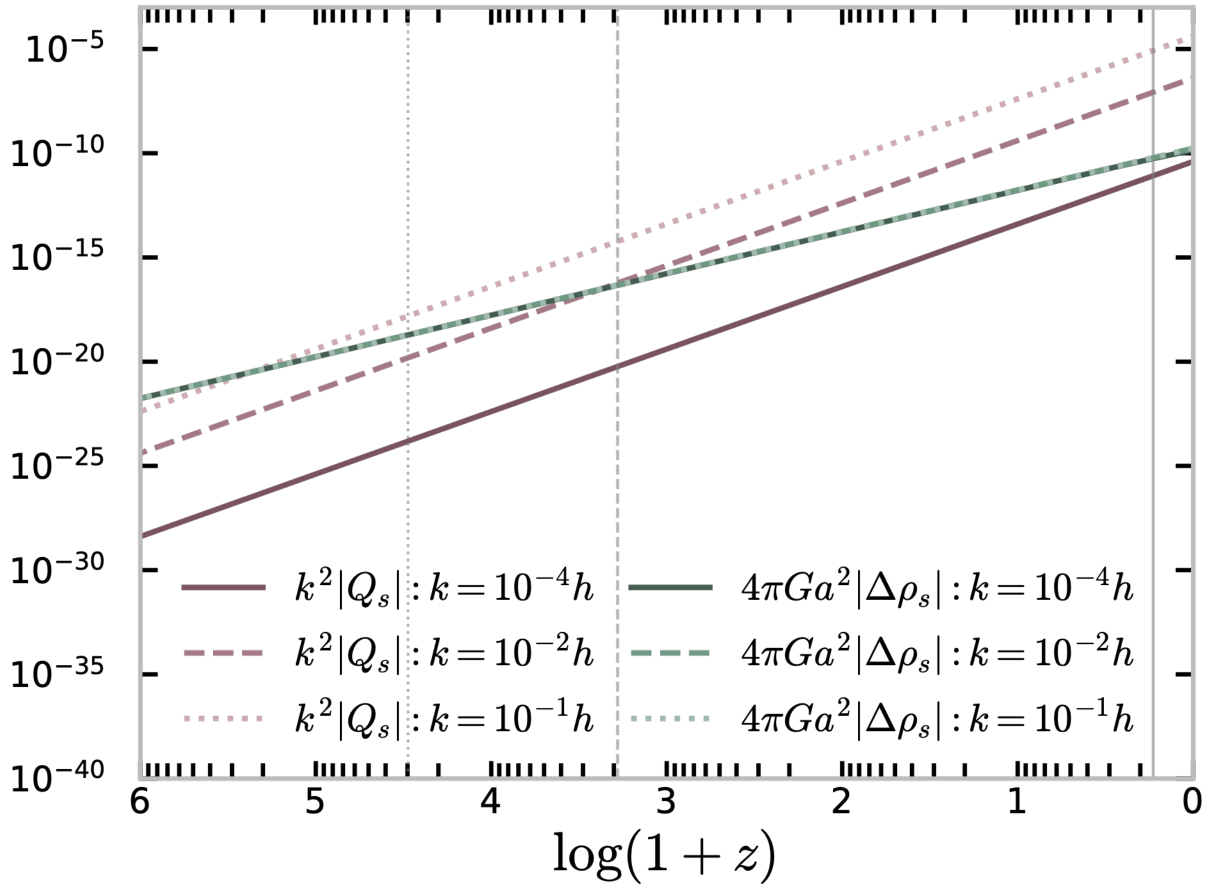
- Expect late-time changes to $P(k)$ on small scales



Large-scale limit

- Momentum constraint modified by θ_c – coupling via Euler equation
- Time-evolution of metric potentials modified at small-k

$$\Psi' + \mathcal{H}\Phi = \frac{4\pi G a^2}{k^2} \sum_{c,b,r,\mathrm{DE}} \theta_I(\rho_I + p_I)$$

- Coupling un-suppressed on large scales

$$\theta_c' + \theta_c\mathcal{H} - k^2\Phi = k^2 Q_s(\tau)\delta s(k)$$

- Expect late-time changes to $P(k)$ on large scales