Scalar field models for the dark sector

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Work in collaboration with Elsa Teixeira, Gaspard Poulot, Adam Smith, Philippe Brax, Cliff Burgess, Anne Davis, Eleonora Di Valentino, Maria Mylova, Nelson Nunes and Vivian Poulin

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Plan:

1. Motivation
2. A hybrid model for the dark sector
3. A minimal axio-dilaton model
4. Time-varying DM equation of state from DM/DE interactions
5. Outlook

1. Motivation

ΛCDM is an excellent model, with a minimal set of parameters to describe observations. Non-trivial predictions for e.g.

- CMB anisotropies
- Shape of the matter power spectrum
- Equation of state for DE and DM
- The model makes predictions for CMB spectral distortions (at least for the simplest version of ΛCDM)

However, the ΛCDM model relies on

- CDM: pressure-less fluid - origin unknown (new particle(s)…)
- Primordial power spectrum (simplest version: spectral index and amplitude): inflation is the best candidate we have (but needs to be put on firm theoretical basis).
- Origin/interpretation of Λ is unknown (vacuum energy density, properties of empty space…)

From the theoretical perspective we need to bring the model on solid footing; we should not be satisfied with describing DM as a fluid and DE as a constant.

Cosmological tensions may signal anyway that ΛCDM is not the final answer/ incomplete (although a very solid step in the right direction).

Our work is not specifically driven by tensions but rather about modelling of the dark sector (DE and DM). Do DM and DE have a joint origin (i.e. originate from the same sector)? What are the cosmological consequences and what are the observational signatures in such models? Can we address the cosmological constant problem (the so-called Yoga-dark energy models aim to address this...)?

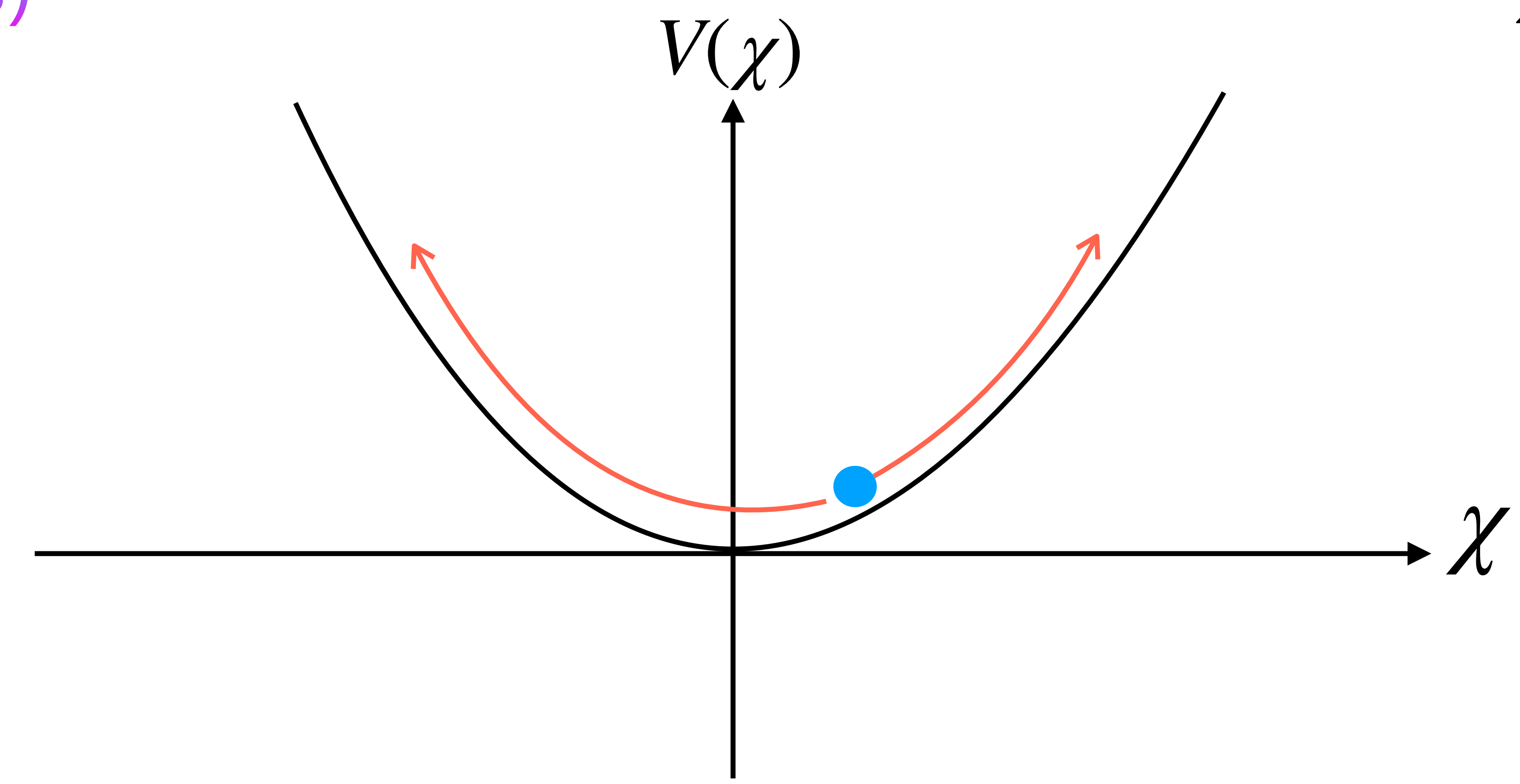
In this talk: Three models of dark sector with DM/DE interactions

1. Interactions mediated by potential (hybrid model for the dark sector)
2. Axio-dilaton model - interactions mediated by kinetic, conformal and couplings via potential energy
3. Interactions between DM and DE are mediated by DM

Scalar field DM - a reminder:

Turner, PRD 28, 1243 (1983)

$$V(\chi) = \frac{m^2}{2}\chi^2$$



Energy density: $\rho_\chi = \frac{1}{2}\dot{\chi}^2 + V(\chi)$

Pressure: $p_\chi = \frac{1}{2}\dot{\chi}^2 - V(\chi)$

If $m \gg H$ then oscillations happen on a time scale much smaller than expansion time scale.

We can average over one oscillation cycle to find

$$\chi(t) = \chi_i \left(\frac{a_i}{a}\right)^{3/2} \sin\left(m\left(t - t_i\right)\right) \longrightarrow$$

$$\langle \rho_\chi \rangle \propto a^{-3}$$

$$\langle p_\chi \rangle = 0$$

This fast oscillating scalar field behaves like DM (at background level).

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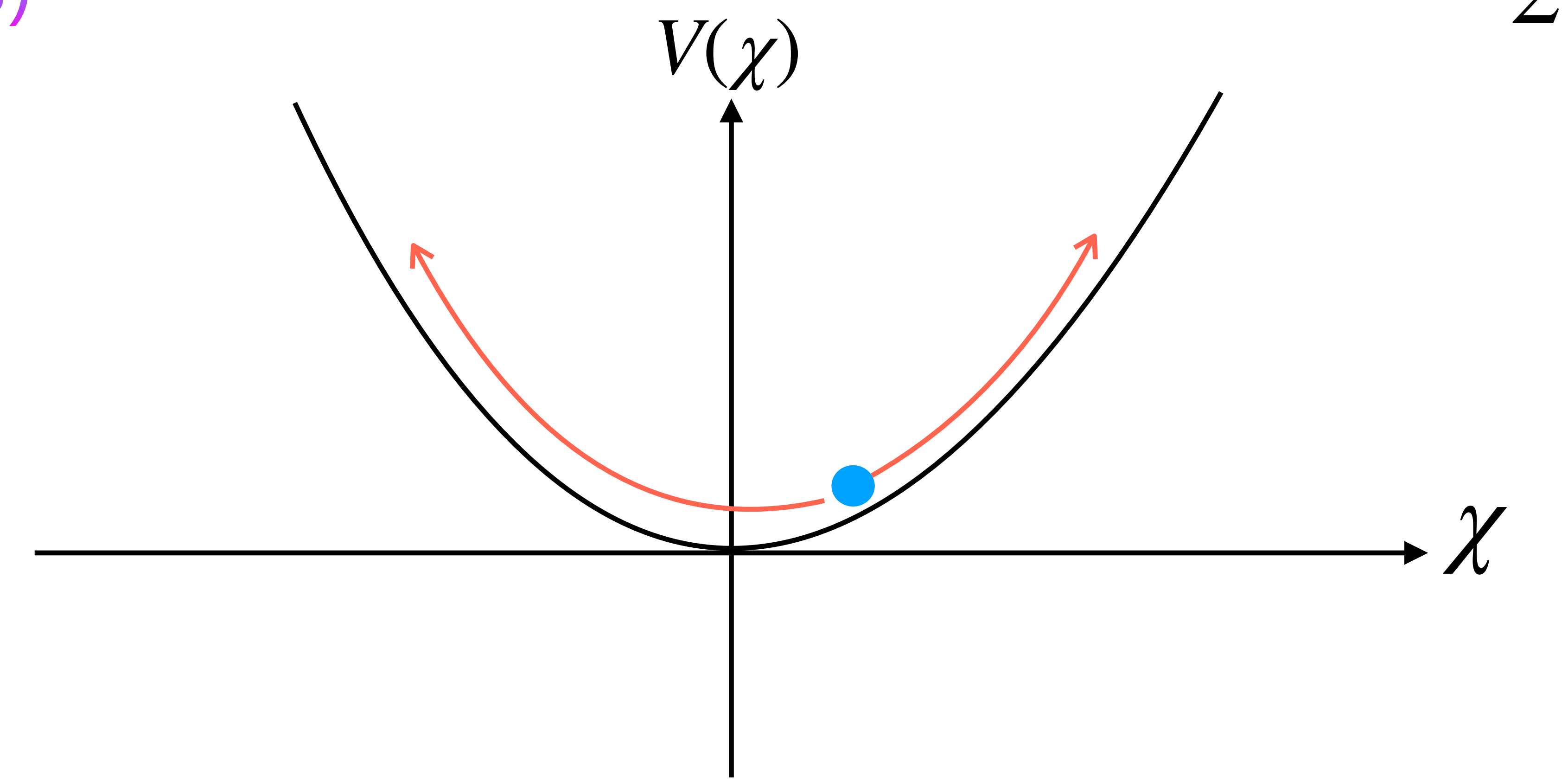


Energy density: $\rho_\chi = \frac{1}{2}\dot{\chi}^2 + V(\chi)$

Pressure: $p_\chi = \frac{1}{2}\dot{\chi}^2 - V(\chi)$

If $m \gg H$ then oscillations happen on a time scale much faster than expansion time scale.

At the level of cosmological perturbations, the sound speed is given by

$$c_a^2 = \frac{\frac{1}{2}\frac{k^2}{a^2m^2}}{\frac{1}{2}\frac{k^2}{a^2m^2} + 2}$$

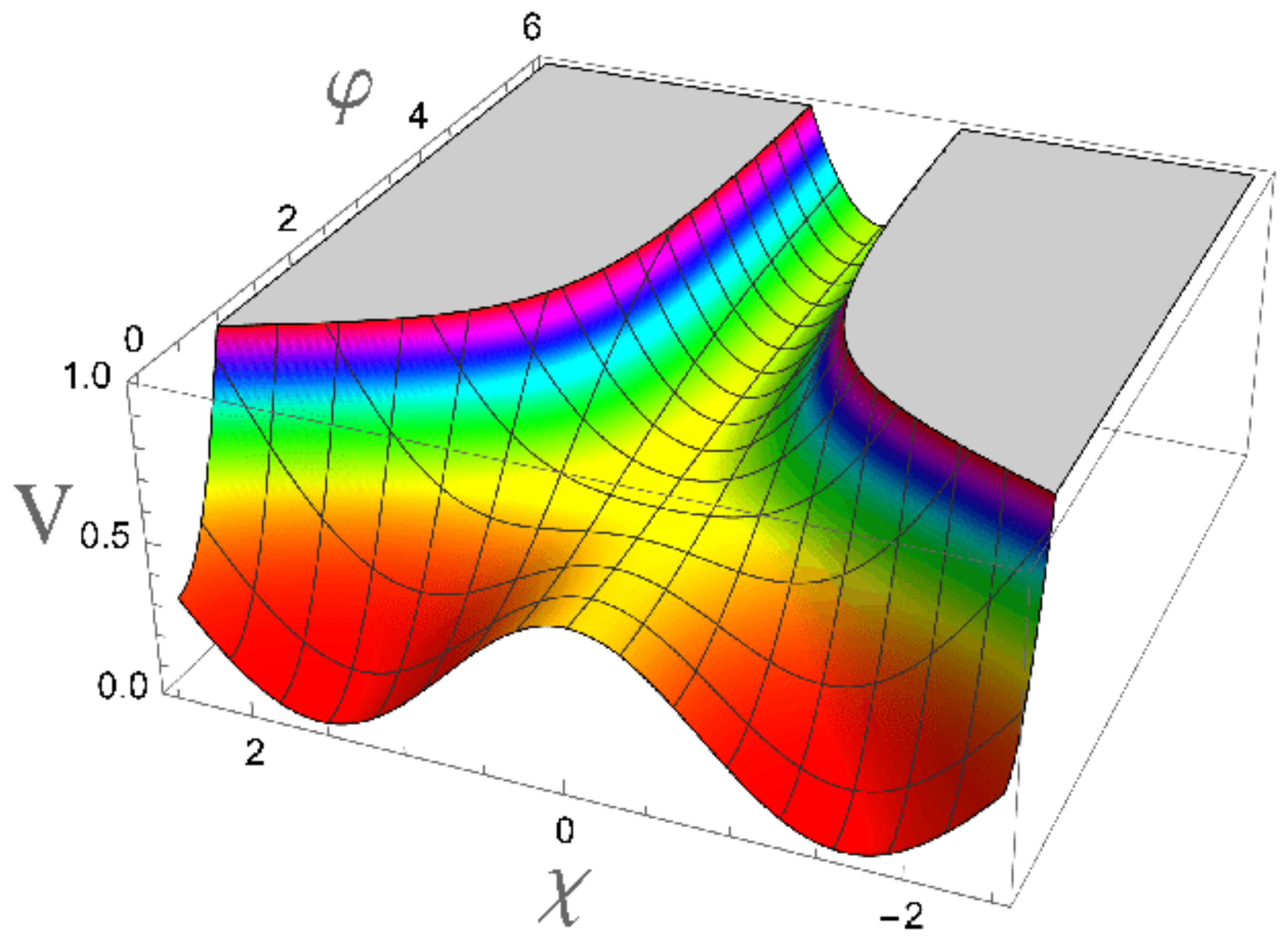
On large scales ($k \ll m$), sound speed vanishes, but is non-zero at small scales ($k \gg m$).

2. A hybrid model for the dark sector

Basic idea is to take a hybrid inflation potential and make this a model for the dark sector. χ is the dark matter field and φ the dark energy field. Of course, the fields are interacting (canonically normalized):

$$V(\phi,\chi) = \frac{\lambda}{4}(M^2-\chi^2)^2 + \frac{1}{2}g^2\phi^2\chi^2 + \frac{1}{2}\mu^2\phi^2$$

$$\equiv V_0 - \frac{1}{2}\lambda M^2\chi^2 + \frac{1}{4}\lambda\chi^4 + \frac{1}{2}g^2\phi^2\chi^2 + \frac{1}{2}\mu^2\phi^2$$



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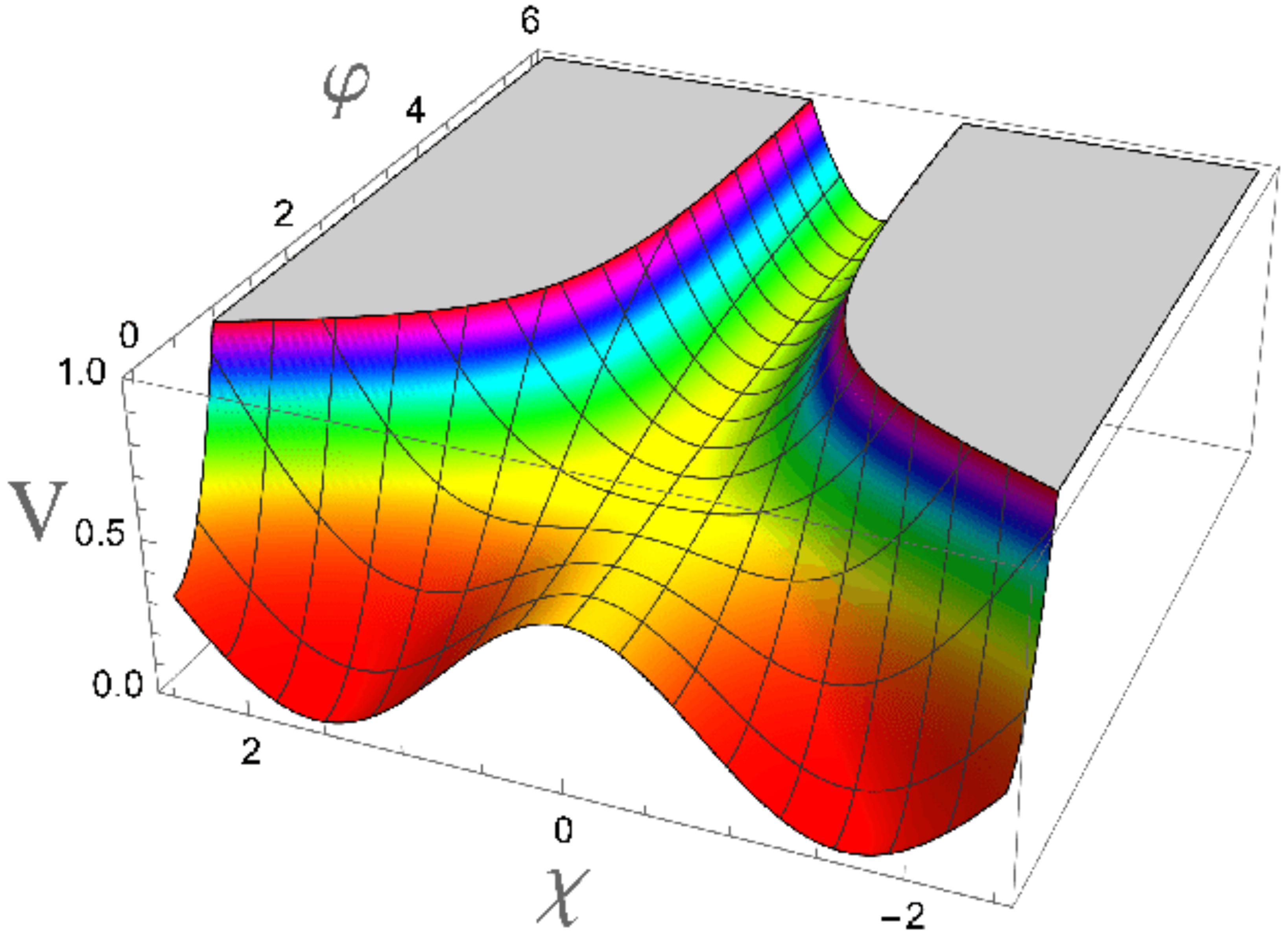

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What is the effective fluid/field description for this dark sector?

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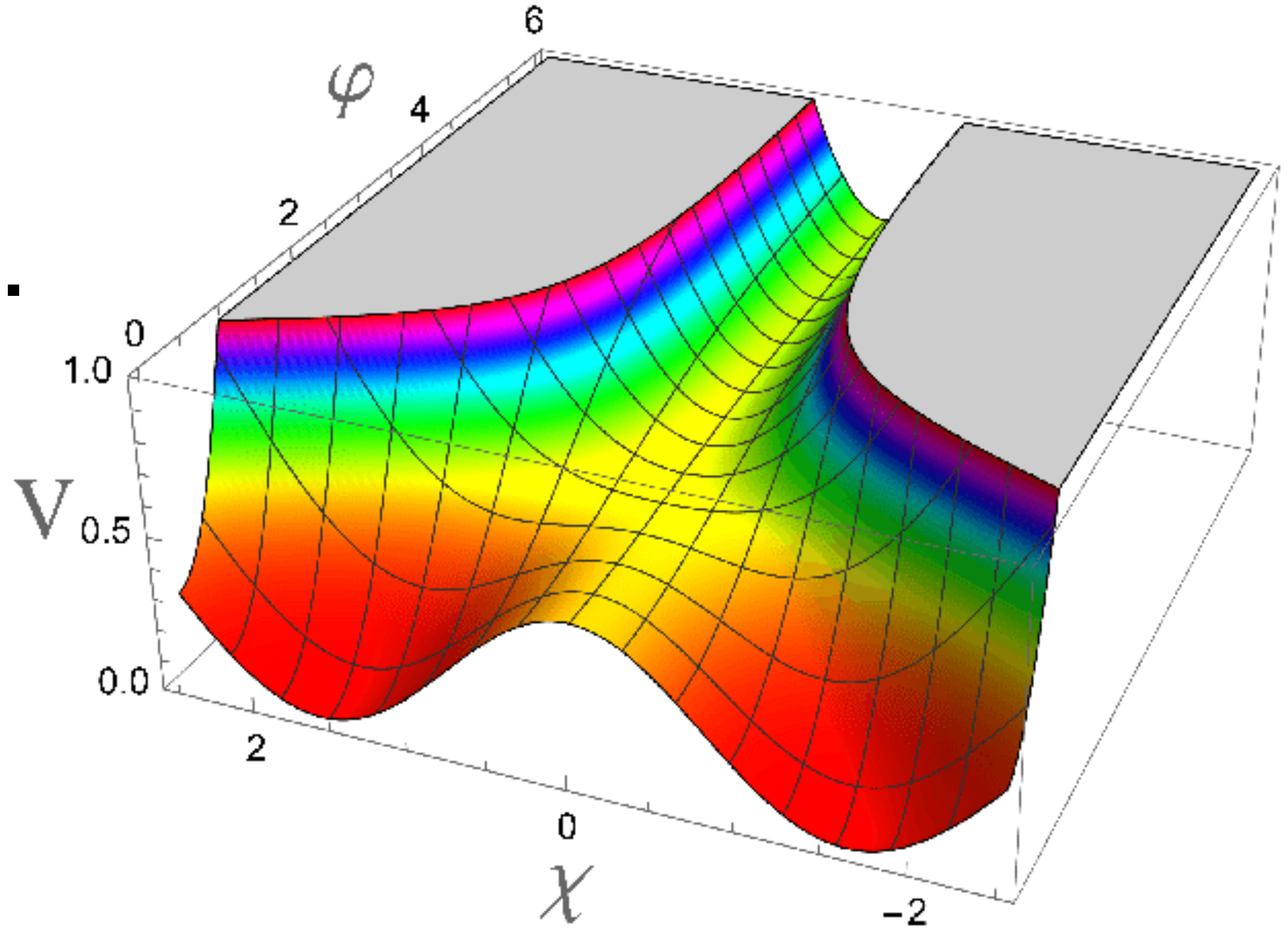
$$V(\phi,\chi) = V_0 - \frac{1}{2}\lambda M^2\chi^2 + \frac{1}{4}\lambda\chi^4 + \frac{1}{2}g^2\phi^2\chi^2 + \frac{1}{2}\mu^2\phi^2$$

The mass of the DM field is effectively

$$m_\chi^2 = g^2\phi^2 - \lambda M^2$$ (need self-interactions to be small).



Energy density of DM defined as

$$\rho_\chi = \frac{1}{2}\dot{\chi}^2 + \frac{1}{4}\lambda\chi^4 + \frac{1}{2}g^2\phi^2\chi^2 - \frac{1}{2}\lambda M^2\chi^2$$

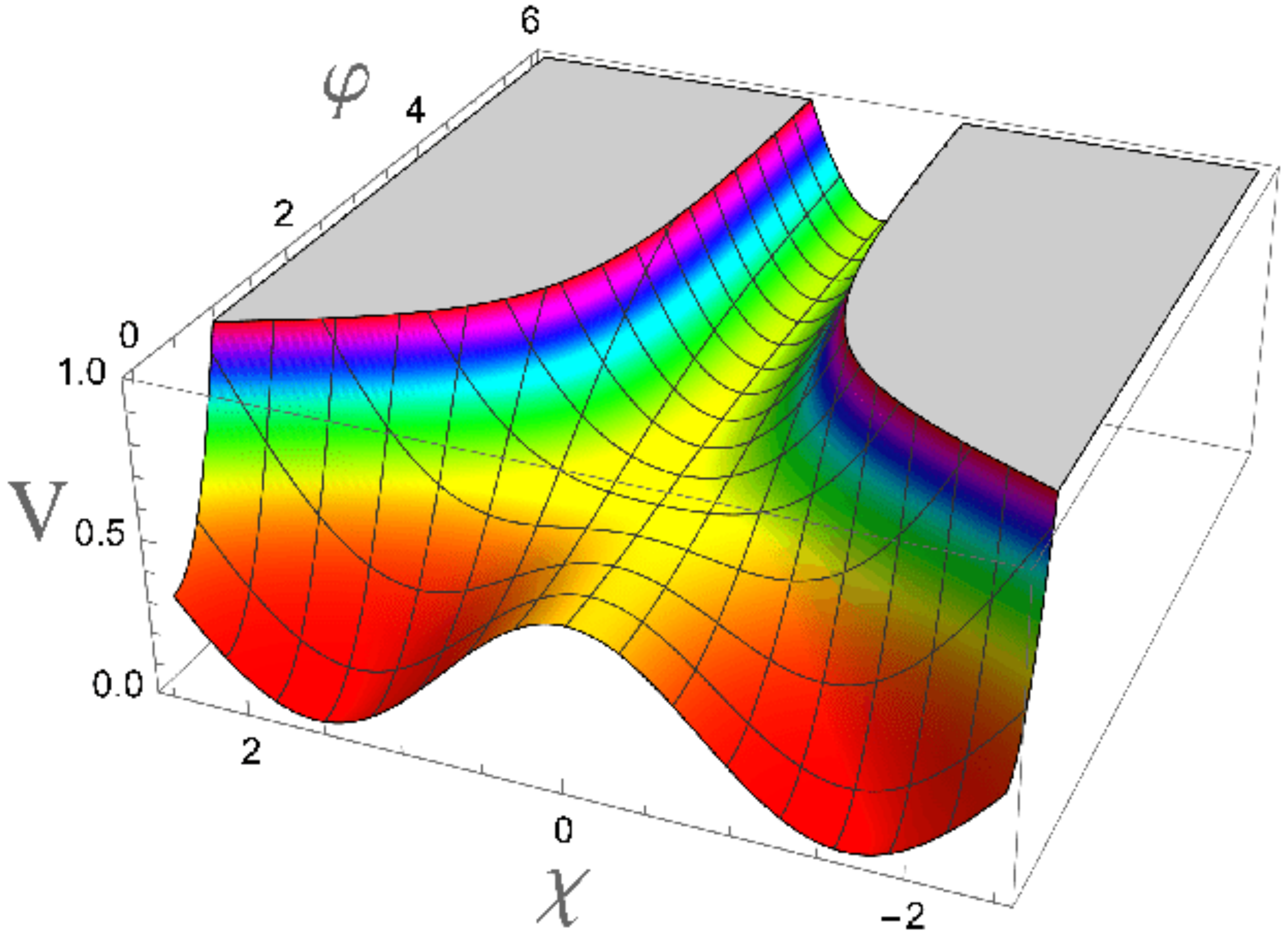
At global minima potential vanishes, therefore period of DE is transient. Ends at

$$\left|\phi_c\right| \approx \frac{\sqrt{\lambda}M}{g}$$

For ϕ to act as dark energy we need

$$V_{\mathrm{DE}}(\phi) = V_0 + \frac{1}{2}\mu^2\phi^2$$

to be of order of the DE energy scale.



This implies that $V_0 = \dfrac{1}{4}\lambda M^4$ is of order of DE scale and μ sufficiently small.

For χ to act as dark matter, we need its mass to be sufficiently large. That is

$$m_\chi \approx g\phi \gg H$$

Using WKB method, one finds

$$\chi(t) = \chi_i \left(\frac{\phi_i}{\phi}\right)^{1/2} \left(\frac{a_i}{a}\right)^{3/2} \sin\left(g\phi\left(t - t_i\right)\right)$$

Averaging over an oscillation period leads to

$$\langle\rho_\chi\rangle \approx \rho_{\chi,i} \left(\frac{\phi}{\phi_i}\right) \left(\frac{a_i}{a}\right)^3$$

So, the energy conservation equation is $$\dot{\rho}_\chi + 3H\rho_\chi = \frac{\dot{\phi}}{\phi}\rho_\chi$$

This is not surprising, since

$$\rho_\chi \propto m_\chi a^{-3}$$

with

$$m_\chi \approx g\phi$$

As a result, DE is described by the following equations:

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{1}{\phi}\rho_\chi \qquad \text{or} \qquad \dot{\rho_\phi} + 3H(\rho_\phi + P_\phi) = -\frac{\dot{\phi}}{\phi}\rho_\chi$$

Theory is *equivalent* to a model with conformal coupling $C(\phi) = \dfrac{\phi^2}{M_{\rm Pl}^2}$.

In our work we have shown that this holds also at the level of cosmological perturbations.

Effective Newton's constant between DM particles can be written as

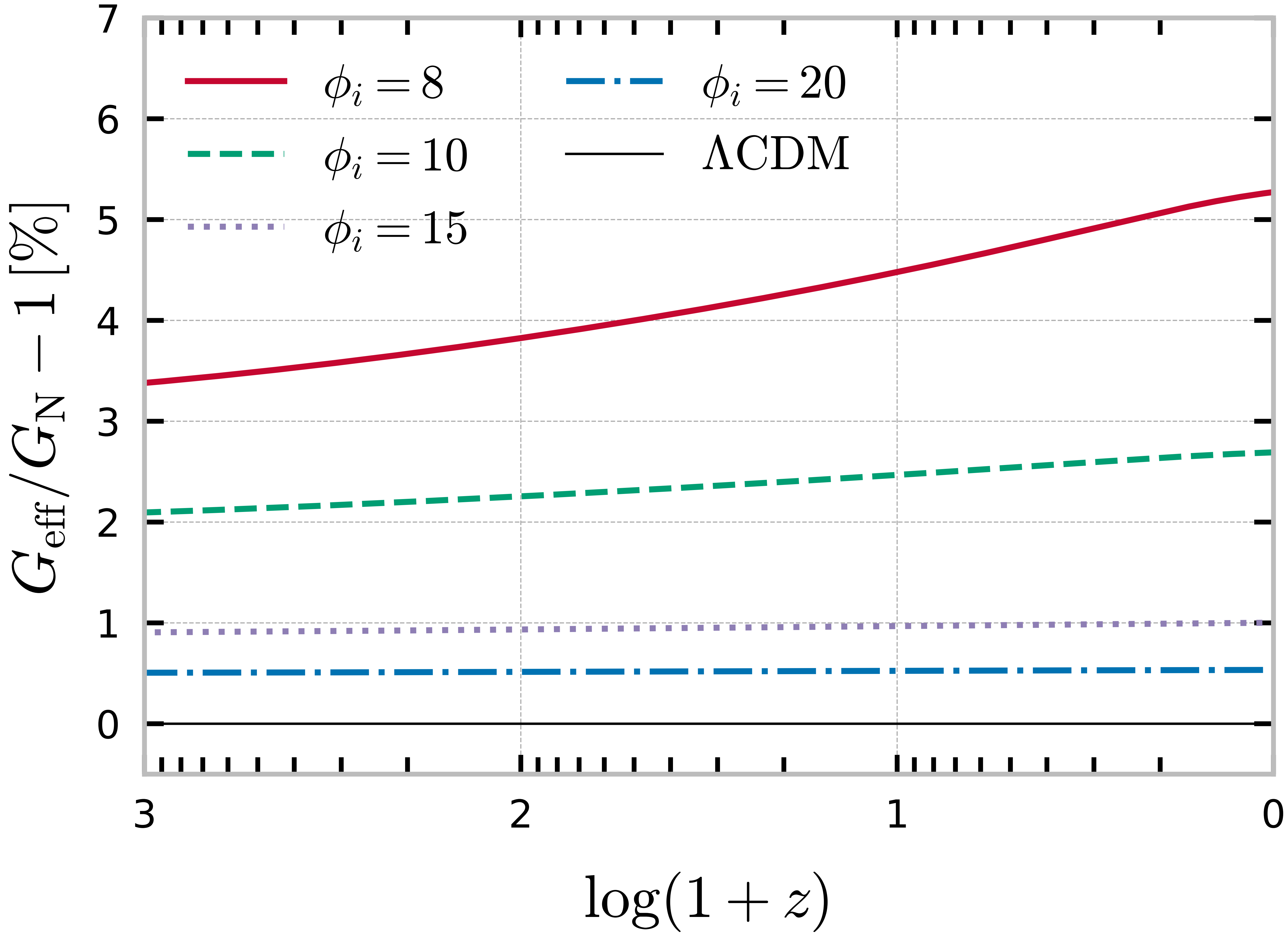
$$G_{\rm eff} \simeq G_N \left(1 + 2\mathrm{M}_{\rm Pl}^2 \frac{Q^2}{\rho_\chi^2} \right)$$

The coupling function is

$$Q = -\frac{C_\phi}{2C}\rho_\chi = -\frac{\rho_\chi}{\phi}$$

One result of this is that ϕ has to be trans-Planckian… and hence the DM field is heavy!

The evolution of $G_{\rm eff}$ (with ϕ in Planck units):



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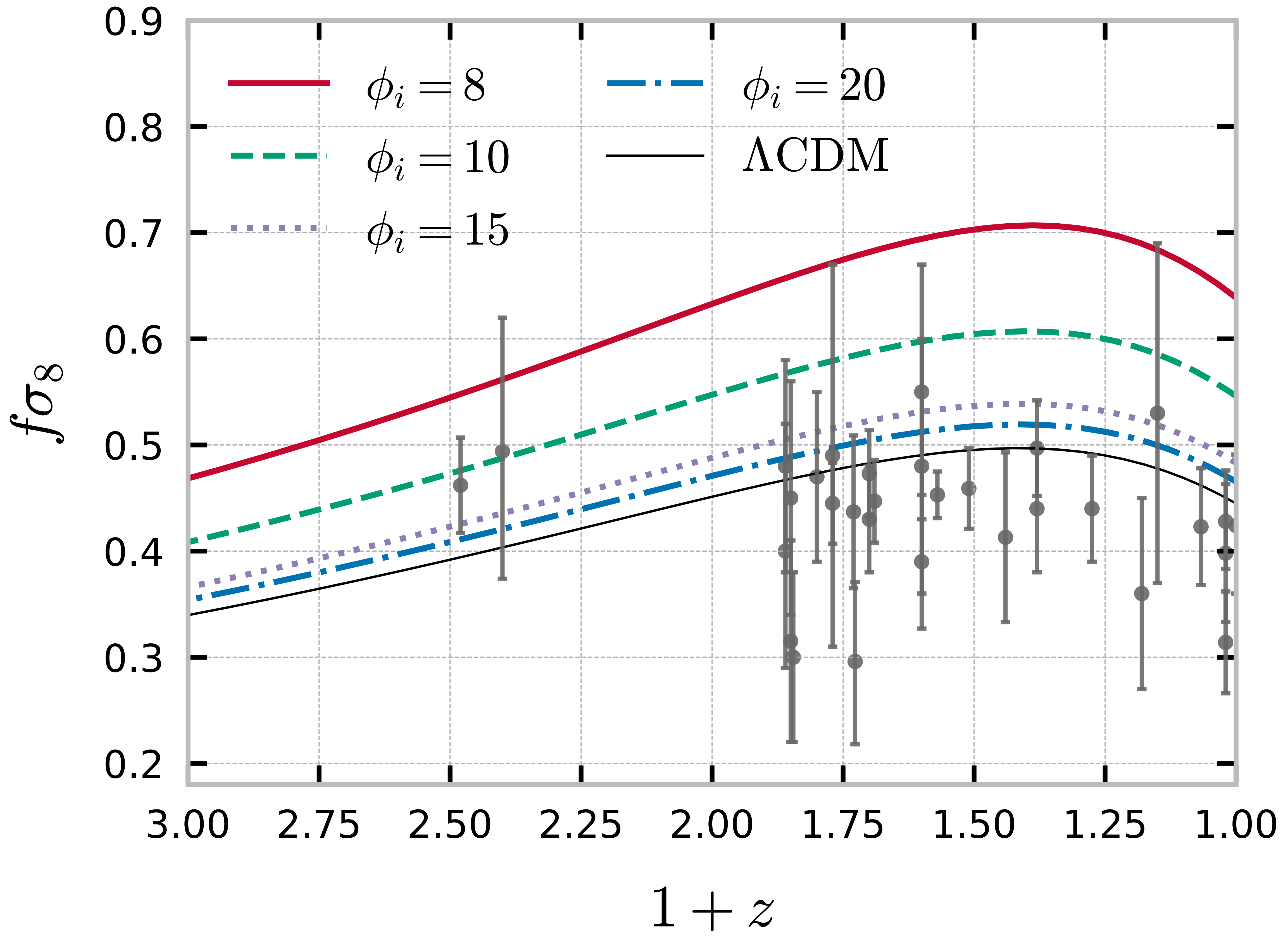
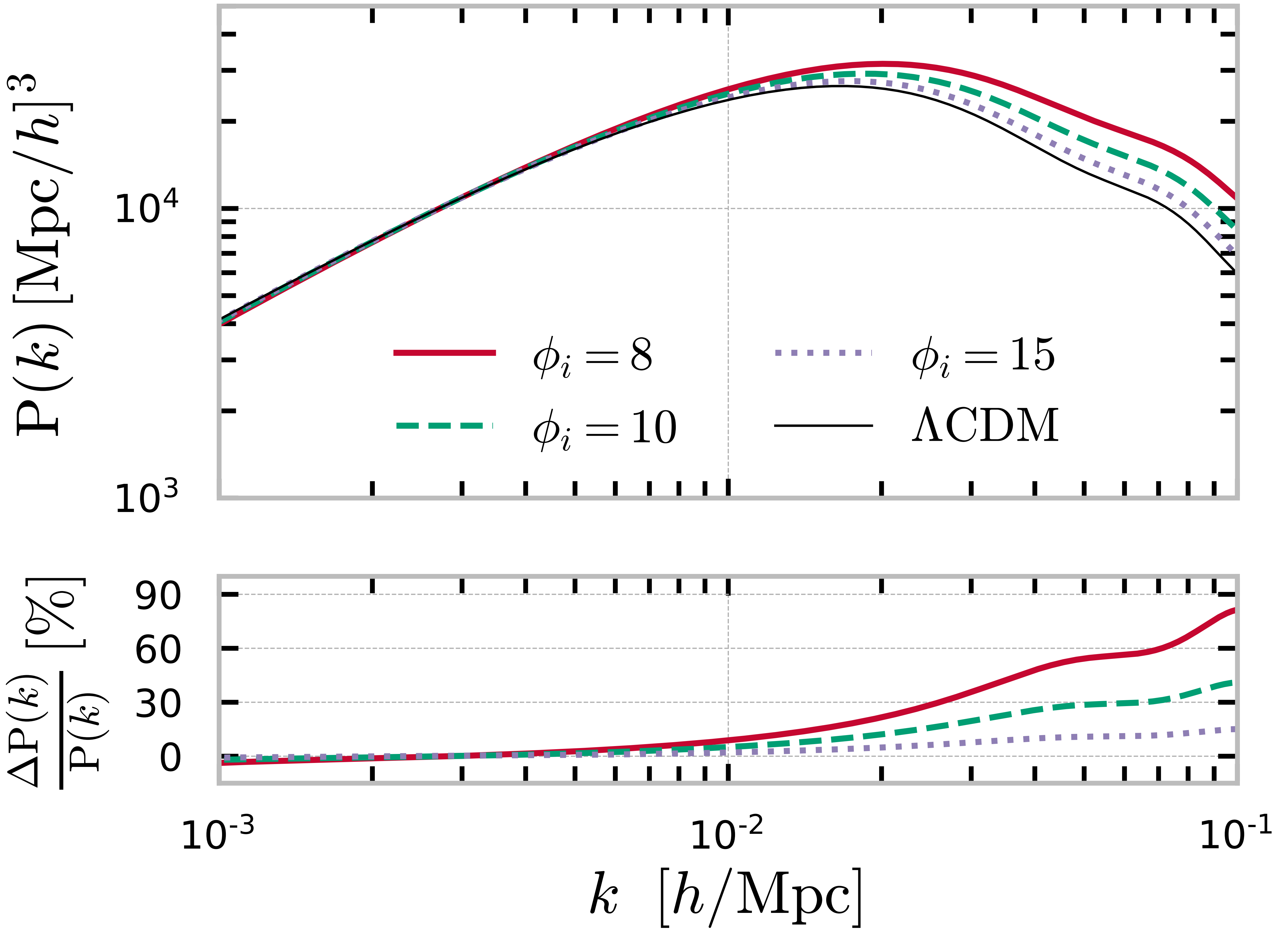
Fixing the cosmological parameter to Planck LCDM best fit:





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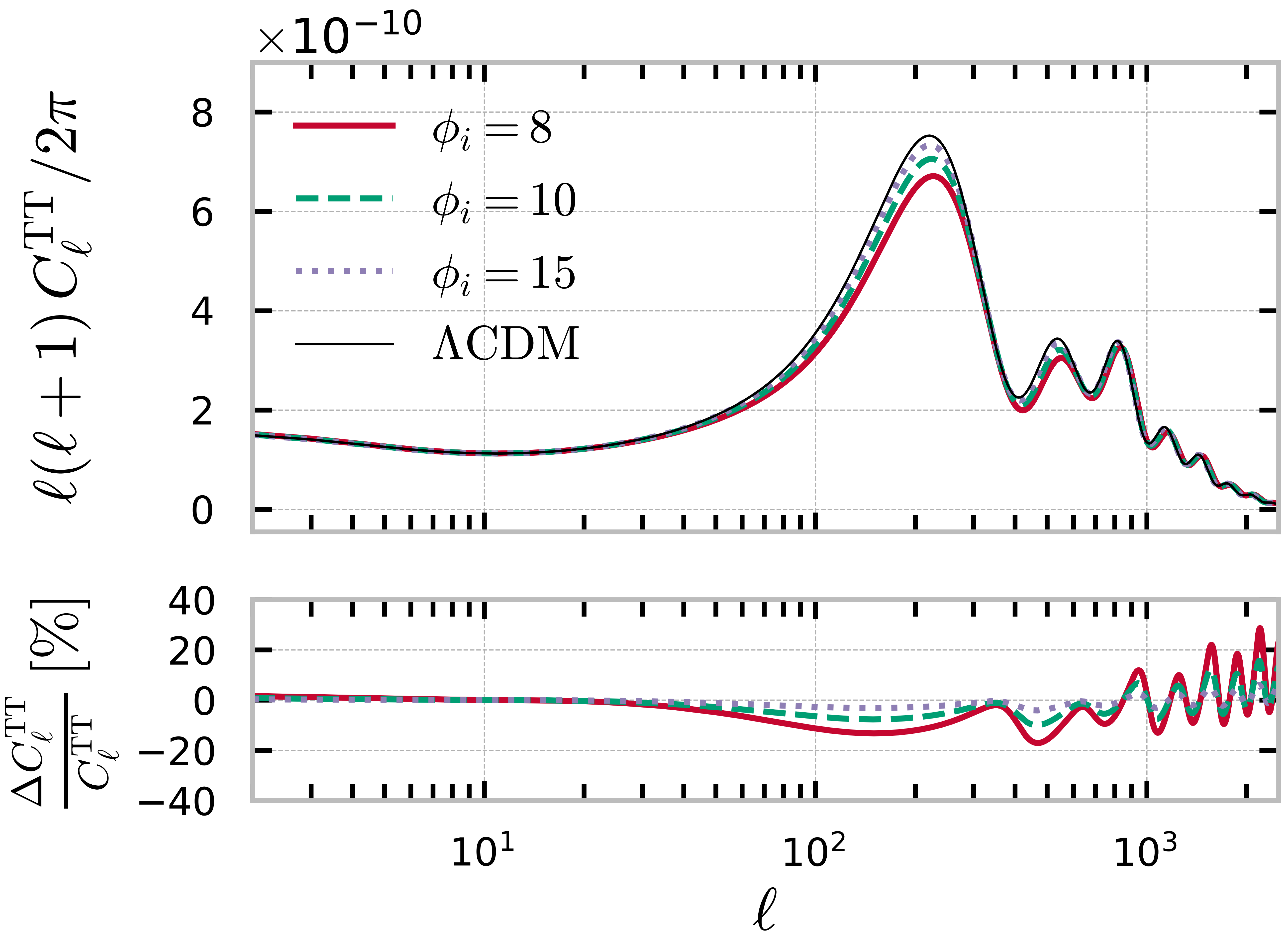

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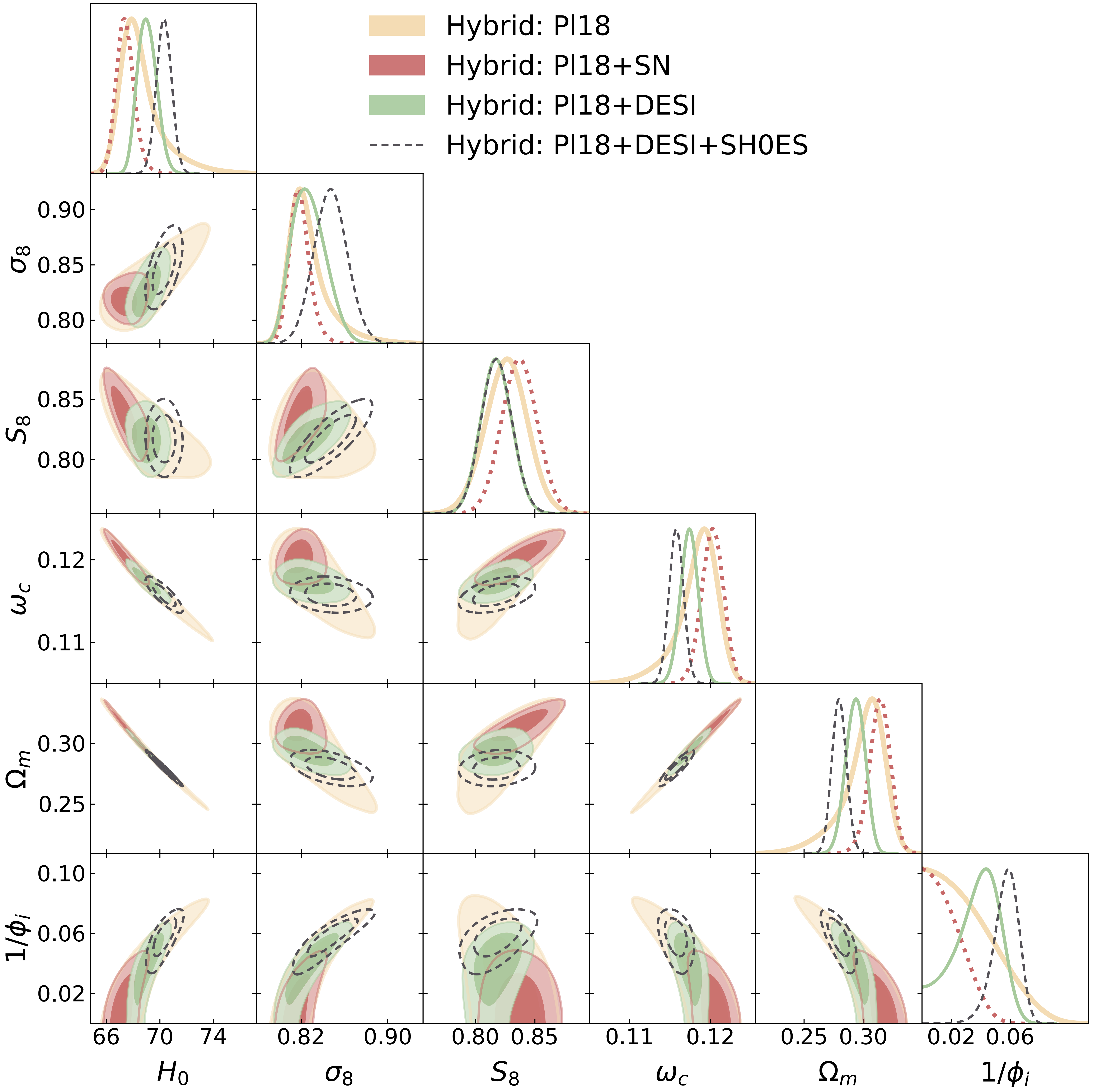
Teixeira, Poulot, van Bruck, Di Valentino, Poulin, arXiv 2412.14139

Constraints:



Teixeira, Poulot, van Bruck, Di Valentino, Poulin,
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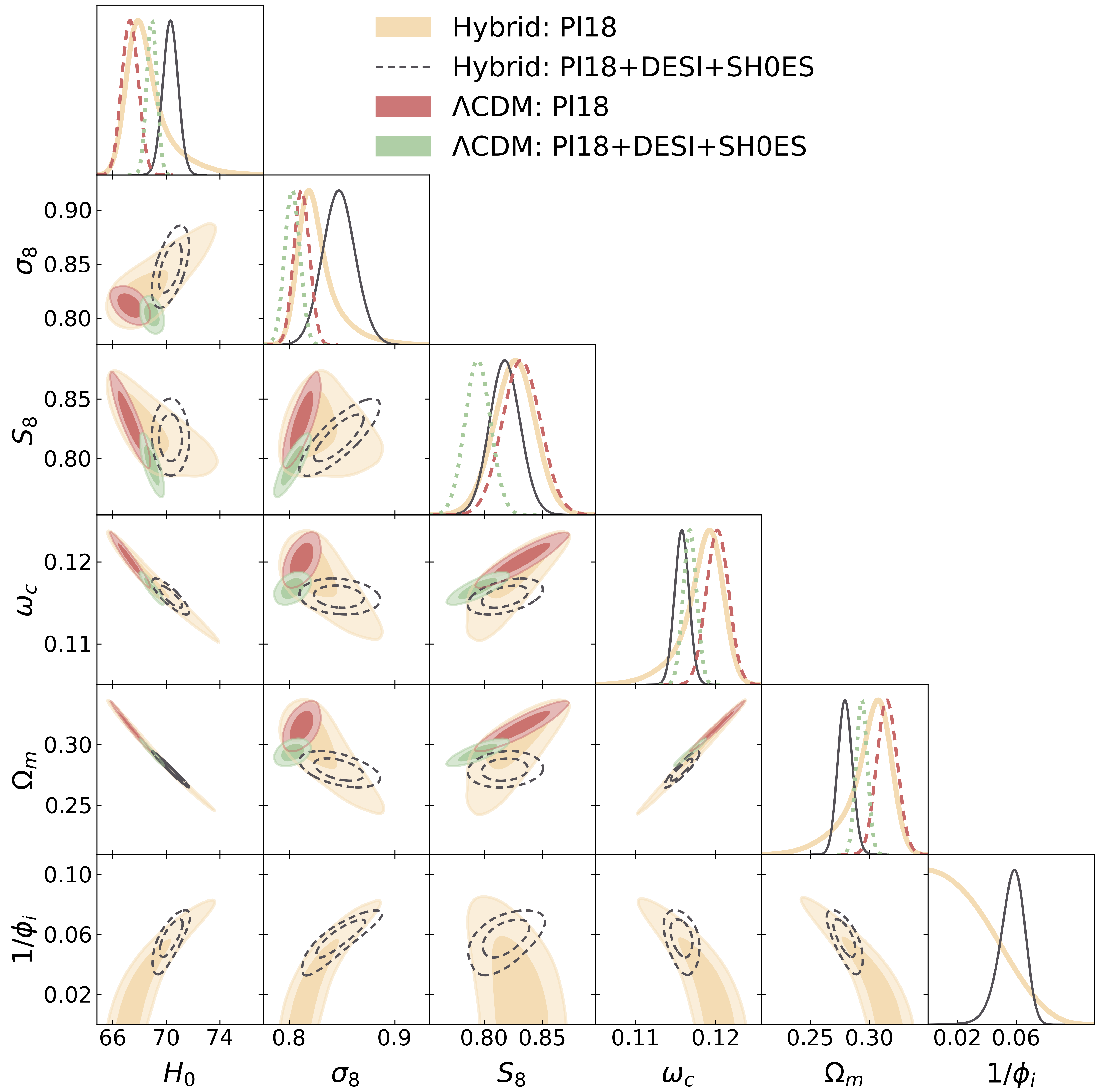

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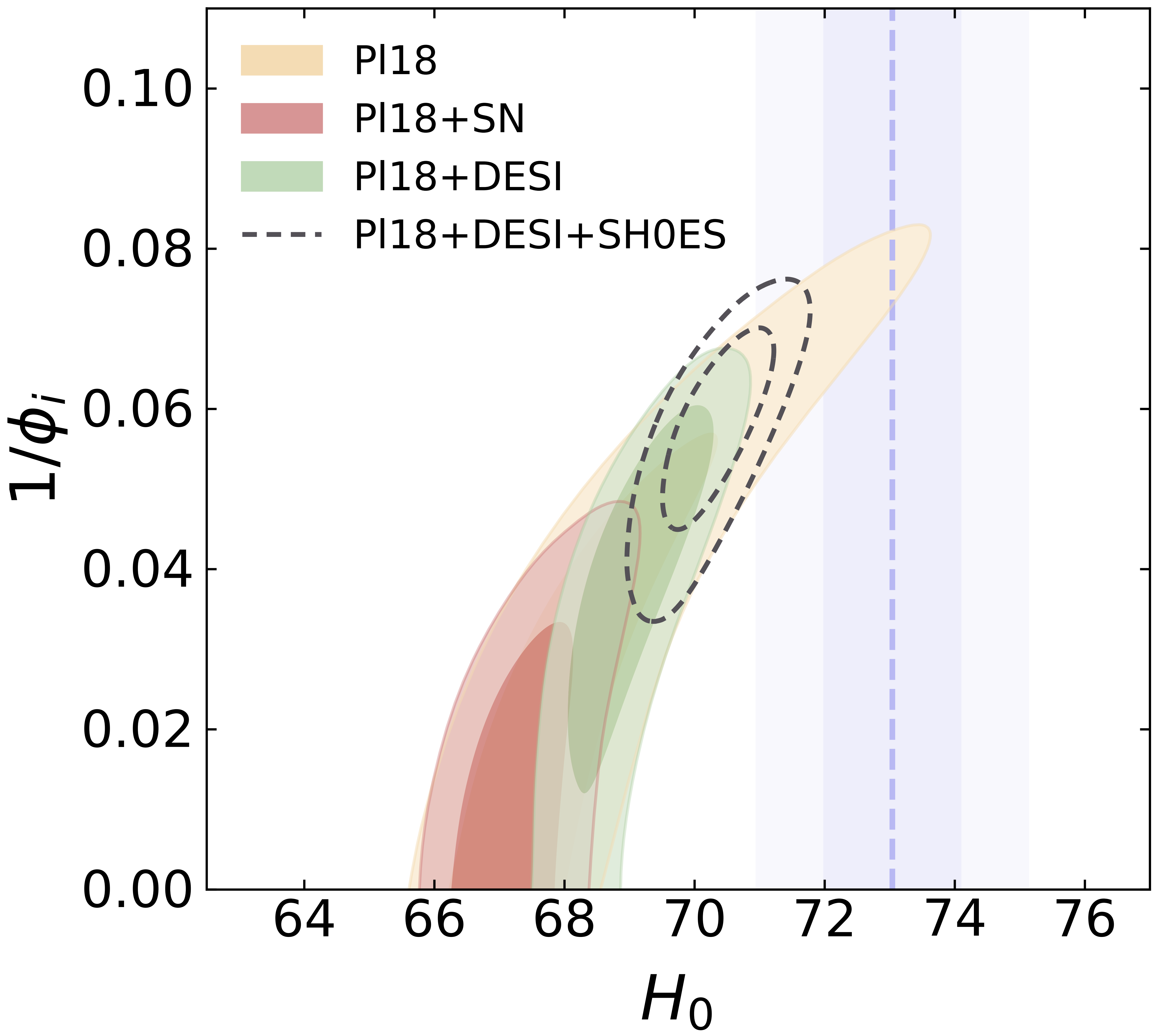


This model alleviates the Hubble tension but does not solve it. Note, we assumed adiabatic initial conditions, ignoring isocurvature modes in our analysis.

Additionally, this version of the model does not predict phantom behaviour.

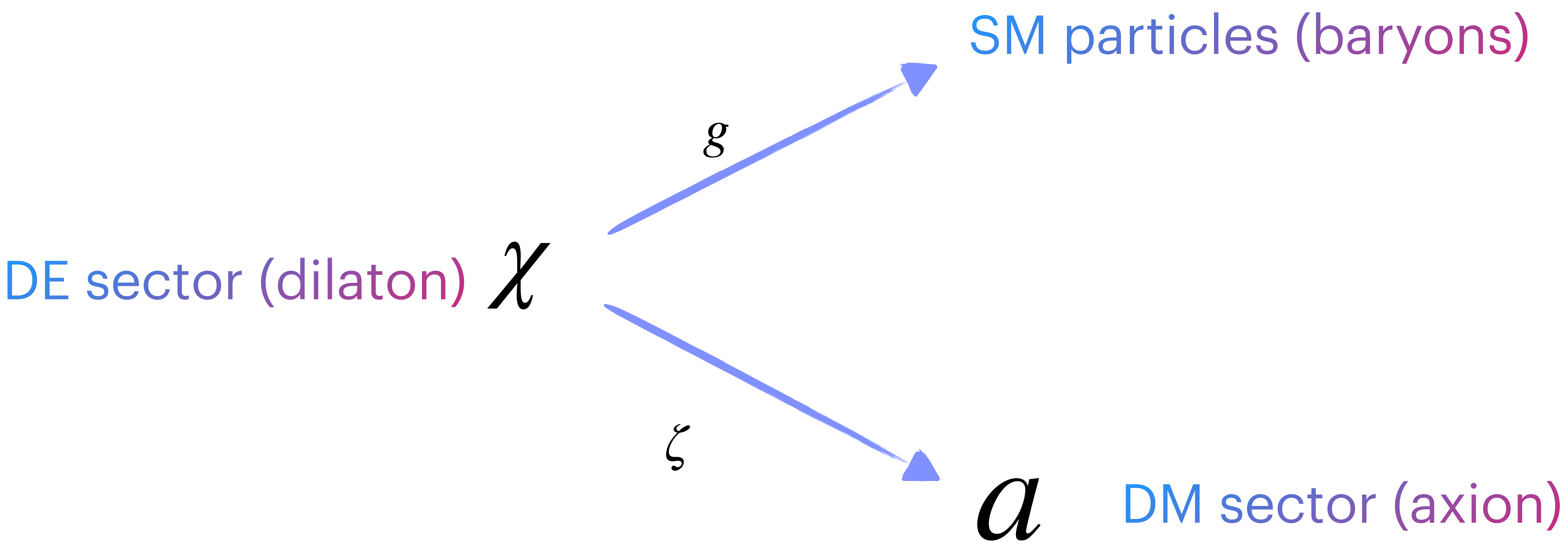
This model is only be a first step. Some issues with this model (and other models of this kind):

- We need field excursions larger than M_{Pl}. Flatness of potential in danger due to e.g. loop corrections.
- Need suppression of isocurvature perturbations. In current version of model this needs to be put in by hand.

There might be more. Nevertheless, this bottom up approach might be useful to learn about a possible origin of the dark sector.

3. A minimal axio-dilaton dark sector

The setup:



Smith, Mylova, van de Bruck, Brax, Burgess, Davis; JCAP 023 (2025)

3. A minimal axio-dilaton dark sector

The setup:

SM particles (baryons)

Coupling between dilaton χ and baryons $\longrightarrow$ $g < 0$

DE sector (dilaton) χ

Coupling between dilaton χ and axion (DM) a $\longrightarrow$ $\zeta > 0$

a DM sector (axion)

There is no other form of DM, just the axion.

Smith, Mylova, van de Bruck, Brax, Burgess, Davis; JCAP 023 (2025)

Effective action for the system:

$$S = -\int d^4x\sqrt{-g}\left(\frac{1}{2}M_p^2\left[R + \partial_\mu\chi\,\partial^\mu\chi + W^2(\chi)\partial_\mu a\ \partial^\mu a\right] + V(\chi, a)\right) + S_m(\Psi, \tilde{g})$$

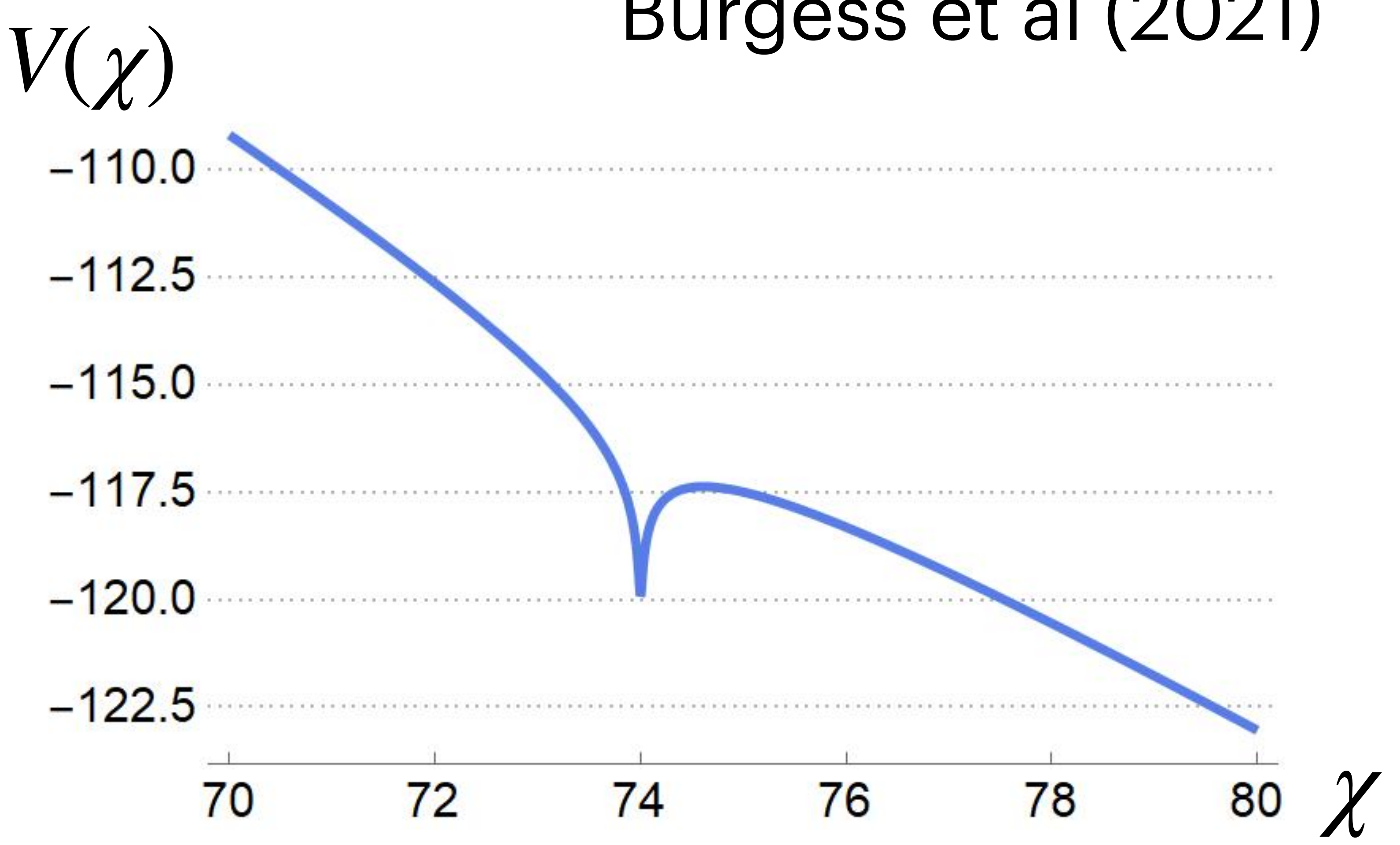
Potential:

$$V(\chi, a) = U(\chi)e^{-\lambda\chi} + V_{\rm ax}(a) \qquad U(\chi) = U_0\left[1 - u_1\chi + \frac{u_2}{2}\chi^2\right] \qquad V_{\rm ax}(a) \simeq \frac{1}{2}m_{\rm ax}^2 M_p^2\left(a - a_0\right)^2$$

Couplings:

$$W(\chi) = W_0\, e^{-\zeta\chi} \quad \text{(kinetic coupling)}$$

$$\tilde{g}_{\mu\nu} := C^2(\chi)\, g_{\mu\nu} \qquad C(\chi) = e^{g\chi} \quad \text{(conformal coupling)}$$



Again, a fluid-field description was developed (more involved that in the other models). Dilaton is kicked around by matter and axion coupling.

$$\chi'' + 2\mathscr{H}\chi' + \frac{a^2}{M_{\rm Pl}^2}V_{,\chi} = \frac{a^2}{M_{\rm Pl}^2}\left(-g\rho_b - \zeta\rho_{\rm ax}\right)$$

Axion oscillations around minimum is DM fluid. Scales like

$$\rho_{\rm ax} = \frac{\mathscr{A}m}{a^3} \qquad m := \frac{m_{\rm ax}}{W(\chi)}$$

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$$\rho_{\rm ax} = \frac{\mathscr{A}m}{a^3} \qquad m := \frac{m_{\rm ax}}{W(\chi)}$$

Scale factor, sorry!

Fifth forces in this model:

$$\delta_a'' + \left(\mathscr{H} - \frac{m'}{m}\right)\delta_a' + \frac{k^4}{4a^2m^2(\eta)}\delta_a = 4\pi a^2 G_\mathrm{N}\left[\left(1 + 2\left(\frac{W_{,\chi}}{W}\right)^2 \frac{k^2}{a^2k_\chi^2}\right)\bar{\rho}_a\delta_a + \left(1 - 2\frac{W_{,\chi}}{W}\,\mathbf{g}\frac{k^2}{a^2k_\chi^2}\right)\bar{\rho}_B\delta_B\right]$$

dilaton-axion:

$$G_{aa}(k) = G_\mathrm{N}\left[1 + 2\left(\frac{W_{,\chi}}{W}\right)^2 \frac{k^2}{a^2k_\chi^2}\right]$$

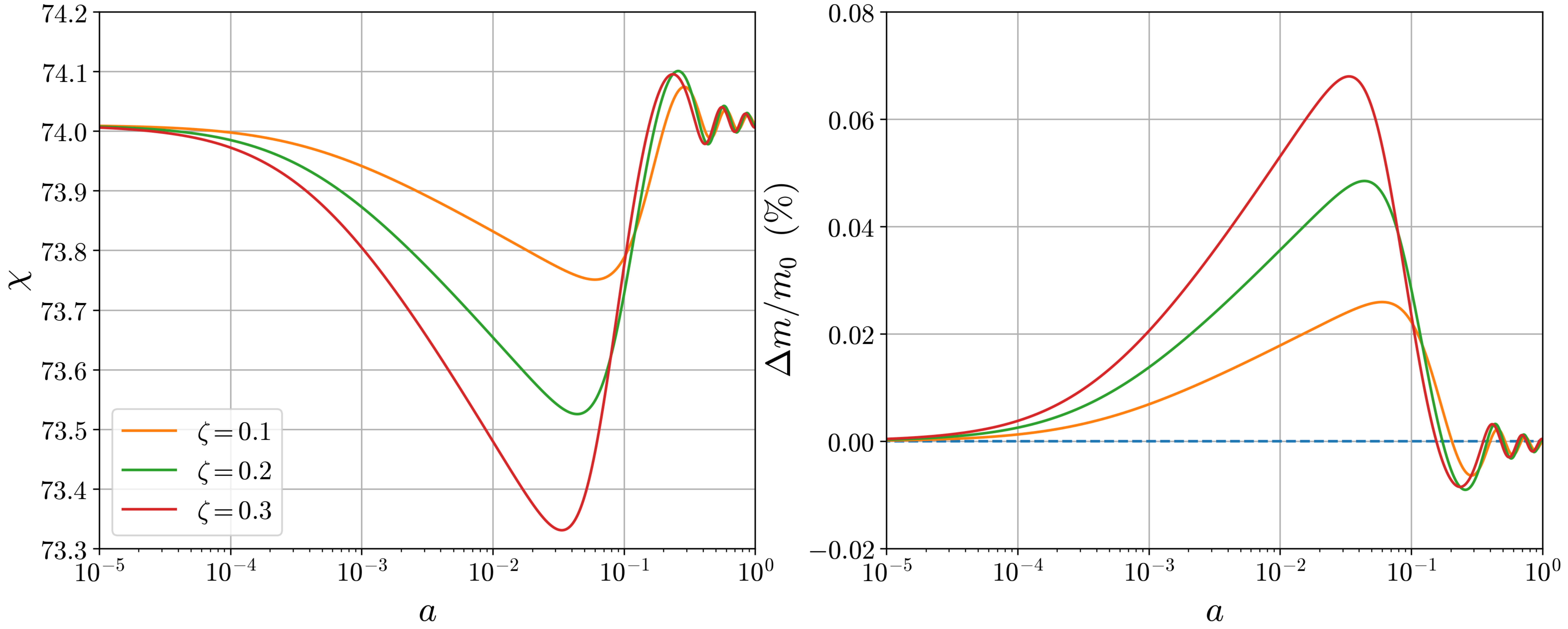
axion-baryon:

$$G_{aB}(k) = G_\mathrm{N}\left[1 - 2\,\frac{W_{,\chi}}{W}\,\mathbf{g}\,\frac{k^2}{a^2k_\chi^2}\right]$$

$$k_\chi^2 \equiv k^2 + a^2 m_{\chi,\mathrm{eff}}^2$$

+ baryon-baryon (determined by $\mathbf{g}$)!

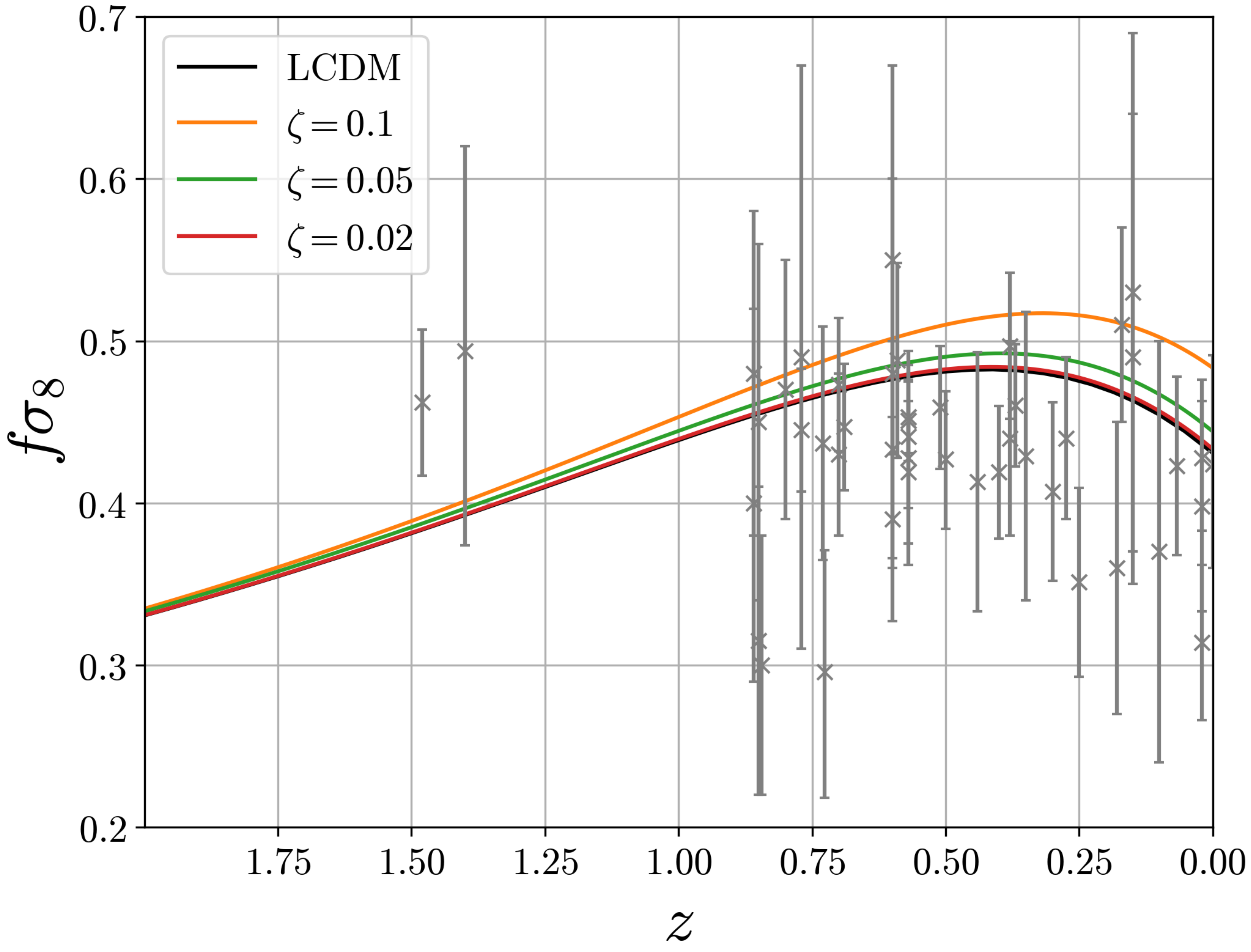
Dilaton evolution:



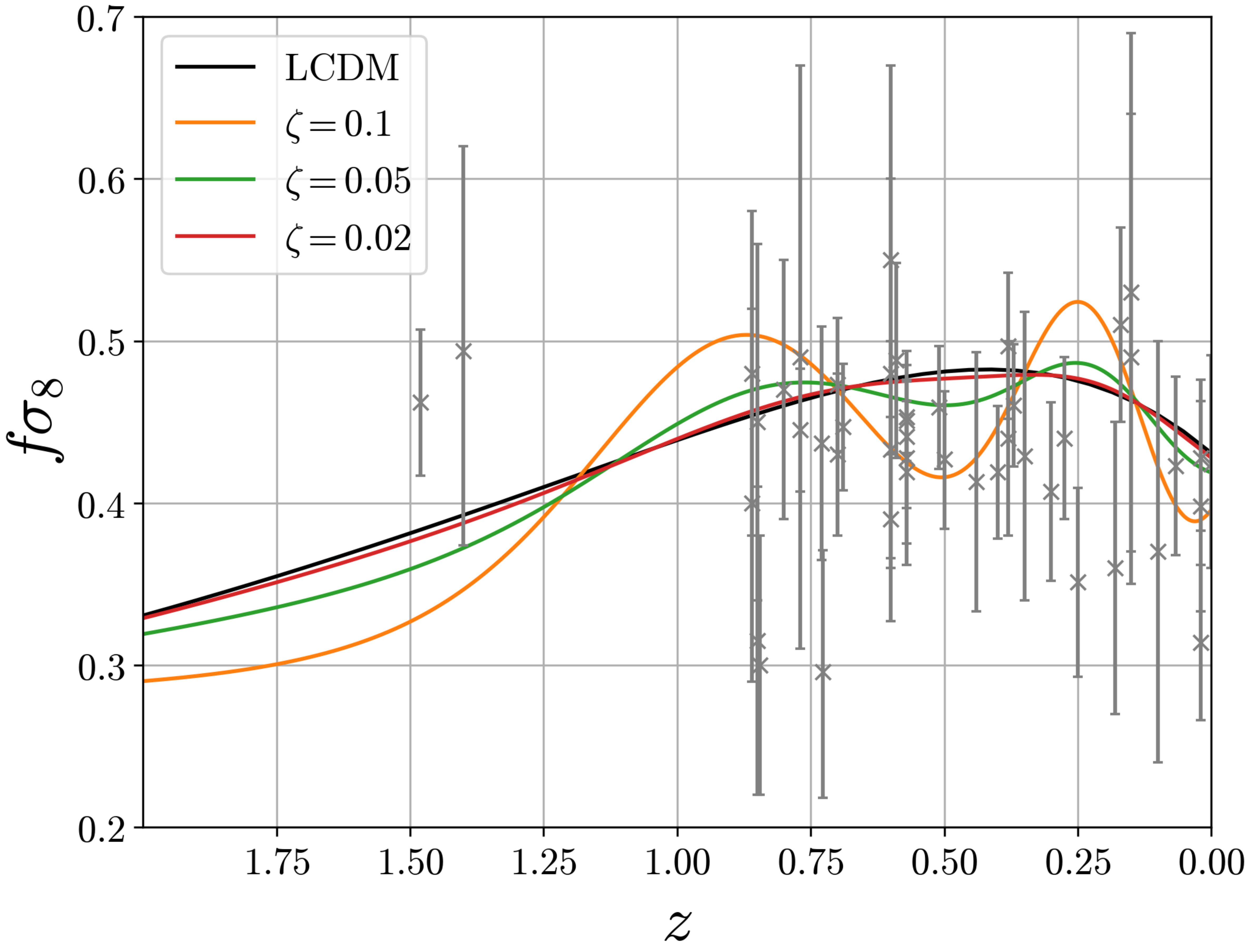
$$\mathbf{g} = -10^{-3}$$

Structure growth parameter

Pure exponential:

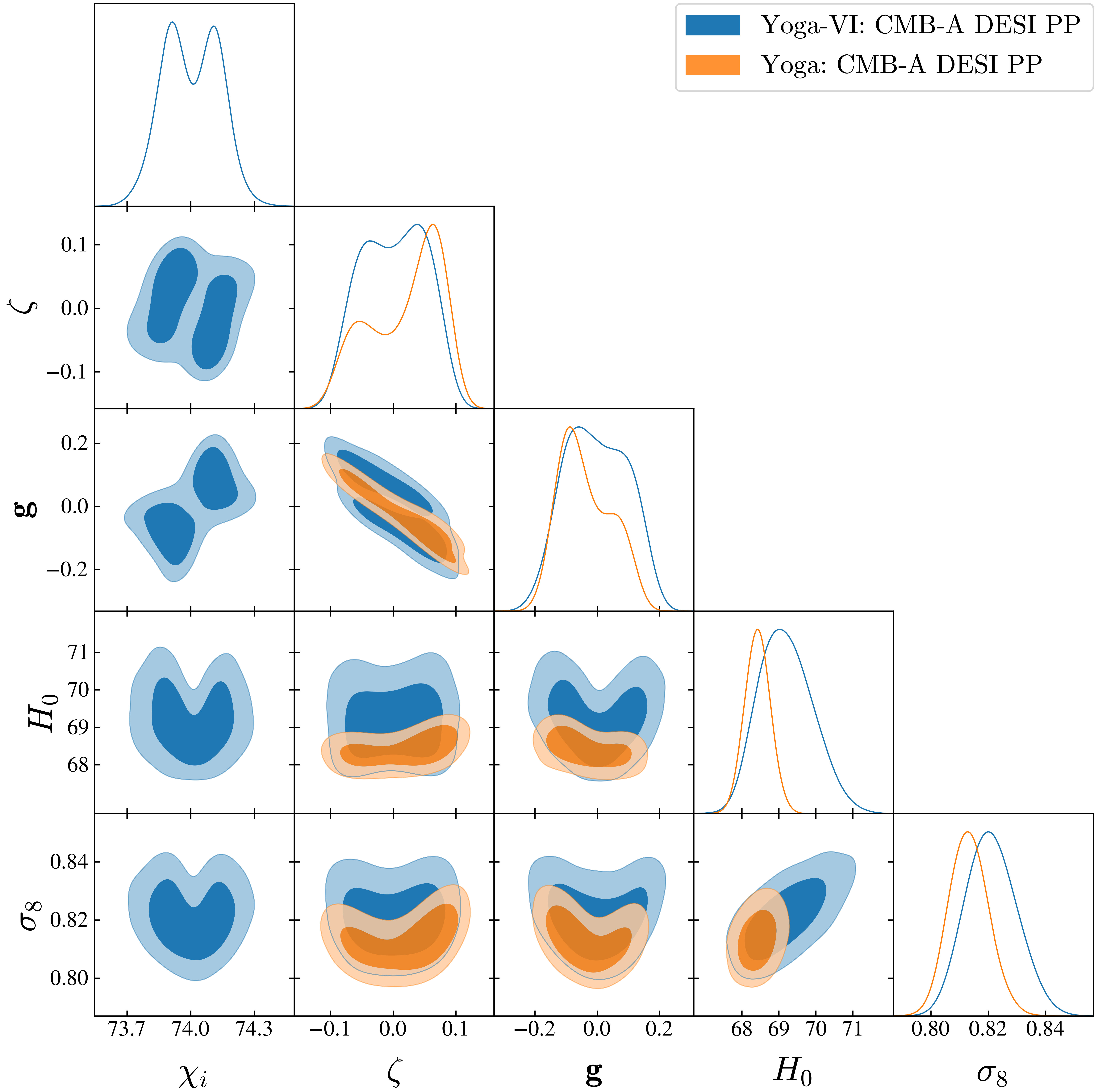


Exponential with local minimum:

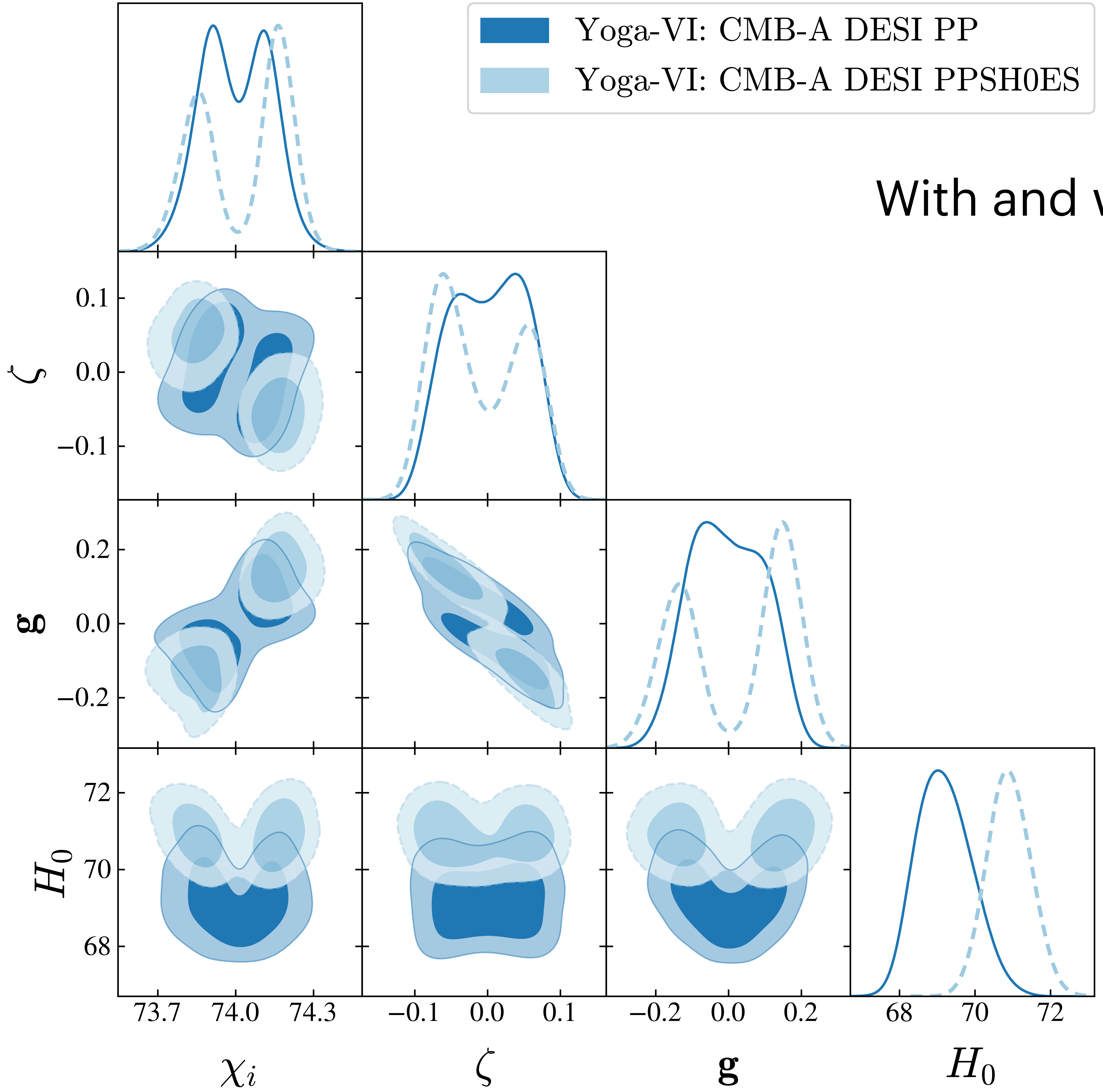


Constraints:

arXiv: 2512.13544



With and without SHOES prior

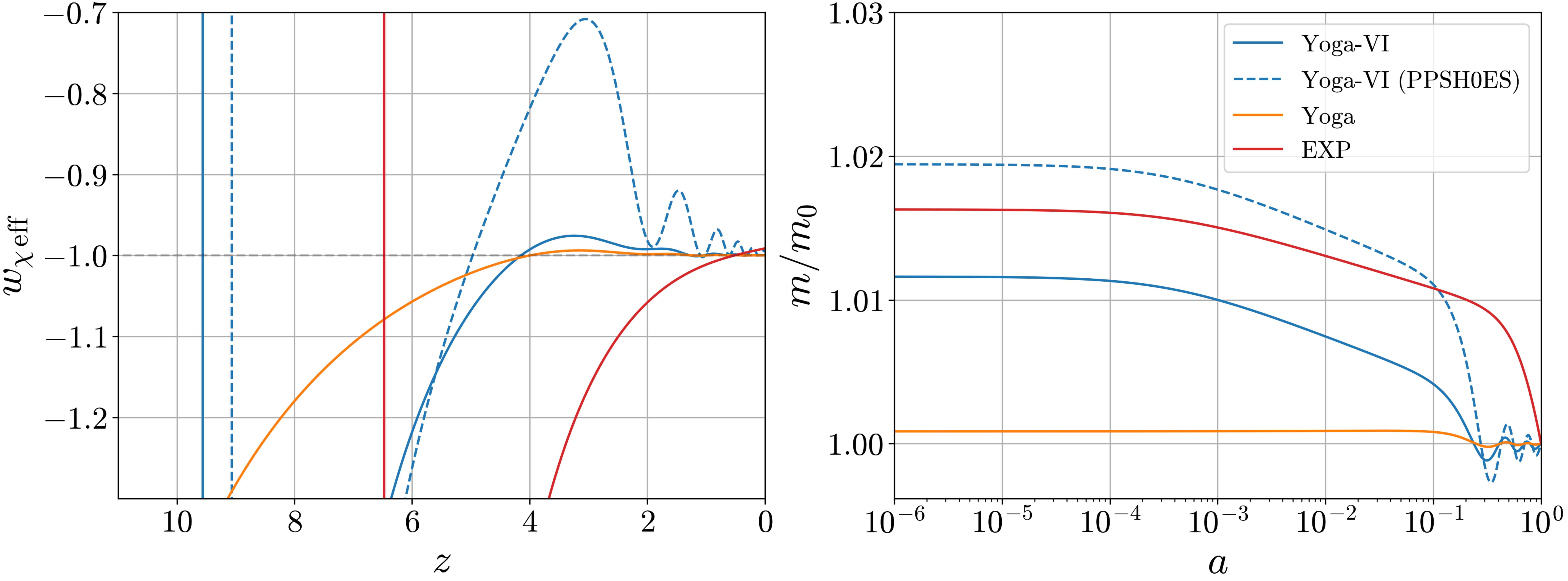


Good news: we can reduce Hubble tension!

Sting:

Need large(ish) dilaton coupling an hence a screening mechanism is needed to “hide” dilaton fifth force. Chameleon screening is not going to work but possibly BBQ screening (dilaton charge is reduced by energy minimisation). Works in principle, but has yet to be shown for the minimum model discussed above (see Anne’s talk for overview).

Effective equation of state and mass evolution



4. Interacting scalar field DM and DE: a time varying equation of state

In this part, we look at another type of interaction:

$$\mathcal{S} = \int d^4x\sqrt{-g}\left(\frac{M_{\rm Pl}^2}{2}\mathcal{R} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - U(\phi) - \frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - V(\chi)\right)$$

$$-\int d^4x\sqrt{-g}\left(\phi Q_0\left(\chi, X^{(\chi)}\right) + \frac{1}{2}\phi^2 Q_1\left(\chi, X^{(\chi)}\right) + \dots\right) + \mathcal{S}_{\rm SM}$$

χ is the **DE** field, whereas ϕ is the **DM** field. The Q_i functions are slowly evolving.

Poulot, Teixeira, van de Bruck, Nunes; JCAP 11 (2025) 005

Again, we assume a massive DM field

$$\ddot{\phi} + 3H\dot{\phi} + m_{\rm eff}^2(t)\phi = -\,Q_0(t)$$

$$m_{\rm eff}^2(t) \equiv m^2 + Q_1(t)$$

Field no longer oscillate around zero but solution has the form

$$\phi(t) = \phi_{\rm osc}(t) + A(t)$$

$$\phi_{\rm osc}(t) = \left(\frac{a_0}{a}\right)^{3/2}\left(\frac{m_0}{m_{\rm eff}}\right)^{1/2}\big(\phi_+ \sin(m_{\rm eff}t) + \phi_- \cos(m_{\rm eff}t)\big) \qquad A(t) \approx -\,\frac{Q_0(t)}{m_{\rm eff}^2(t)}$$

As a consequence, the pressure of DM no longer vanishes:

$$w_\phi \equiv \frac{\langle p_\phi \rangle}{\langle \rho_\phi \rangle} = \left(1 - \frac{m^2 A^2}{2\langle \rho_\phi \rangle}\right) \frac{m_{\text{eff}}^2 - m^2}{m_{\text{eff}}^2 + m^2} - \frac{m^2 A^2}{2\langle \rho_\phi \rangle}$$

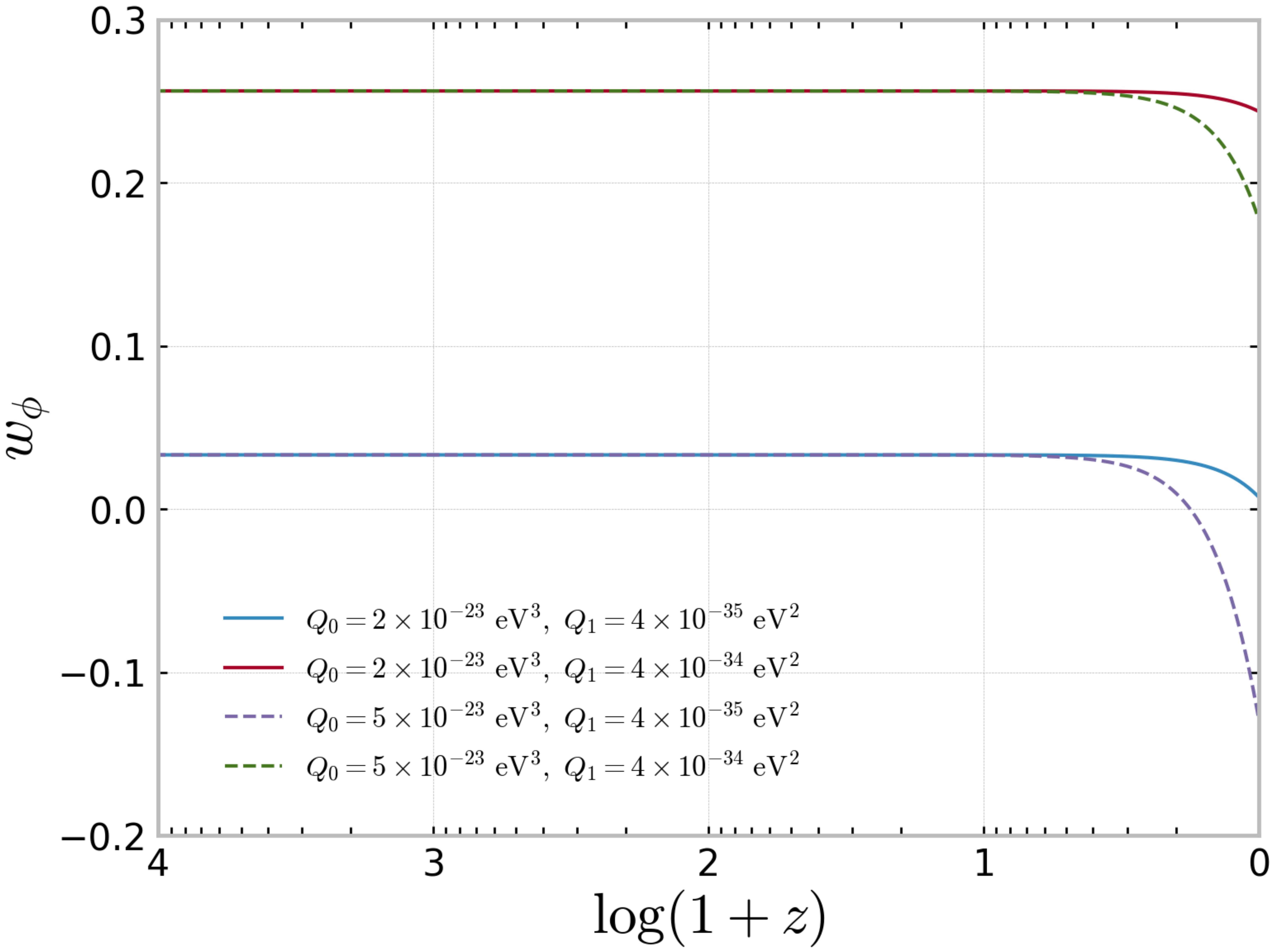
Even if Q_1 is zero, the eos is non-zero (and negative). The sound speed is

$$c_a^2 = \frac{\frac{1}{2}\frac{k^2}{a^2 m_{\text{eff}}^2} + \frac{Q_1}{m_{\text{eff}}^2}}{\frac{1}{2}\frac{k^2}{a^2 m_{\text{eff}}^2} + 2 - \frac{Q_1}{m_{\text{eff}}^2}} = \frac{\frac{1}{2}\frac{k^2}{a^2 m^2} + \frac{Q_1}{m^2}}{\frac{1}{2}\frac{k^2}{a^2 m^2} + 2 + \frac{Q_1}{m^2}}$$

Note that this expression is ***independent*** of Q_0 .

Such an interaction changes the physical properties of DM!

Evolution of equation of state (minimal model) $\quad Q_i =$ const.



Constraints

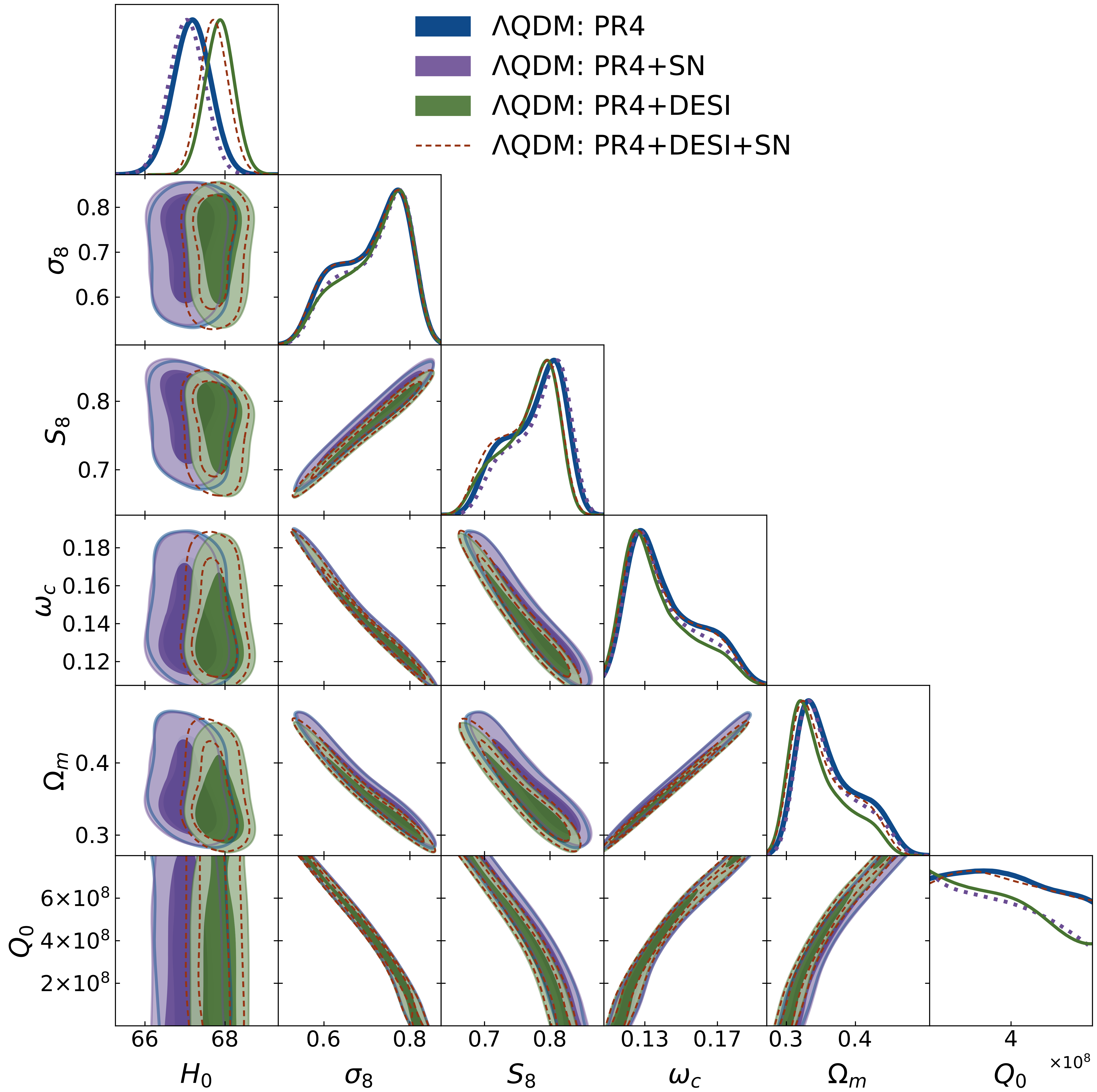
In preparation

$$\mathcal{S} = \int d^4x\sqrt{-g}\left(\frac{M_{\rm Pl}^2}{2}\mathcal{R} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - U(\phi) - \frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - V(\chi)\right)$$

$$-\int d^4x\sqrt{-g}\left(\phi Q_0\left(\chi, X^{(\chi)}\right) + \frac{1}{2}\phi^2 Q_1\left(\chi, X^{(\chi)}\right) + \ldots\right) + \mathcal{S}_{\rm SM}$$

We looked at the simplest model: constant DE, and

$$Q_0 = \text{const} \quad Q_1 \equiv 0$$



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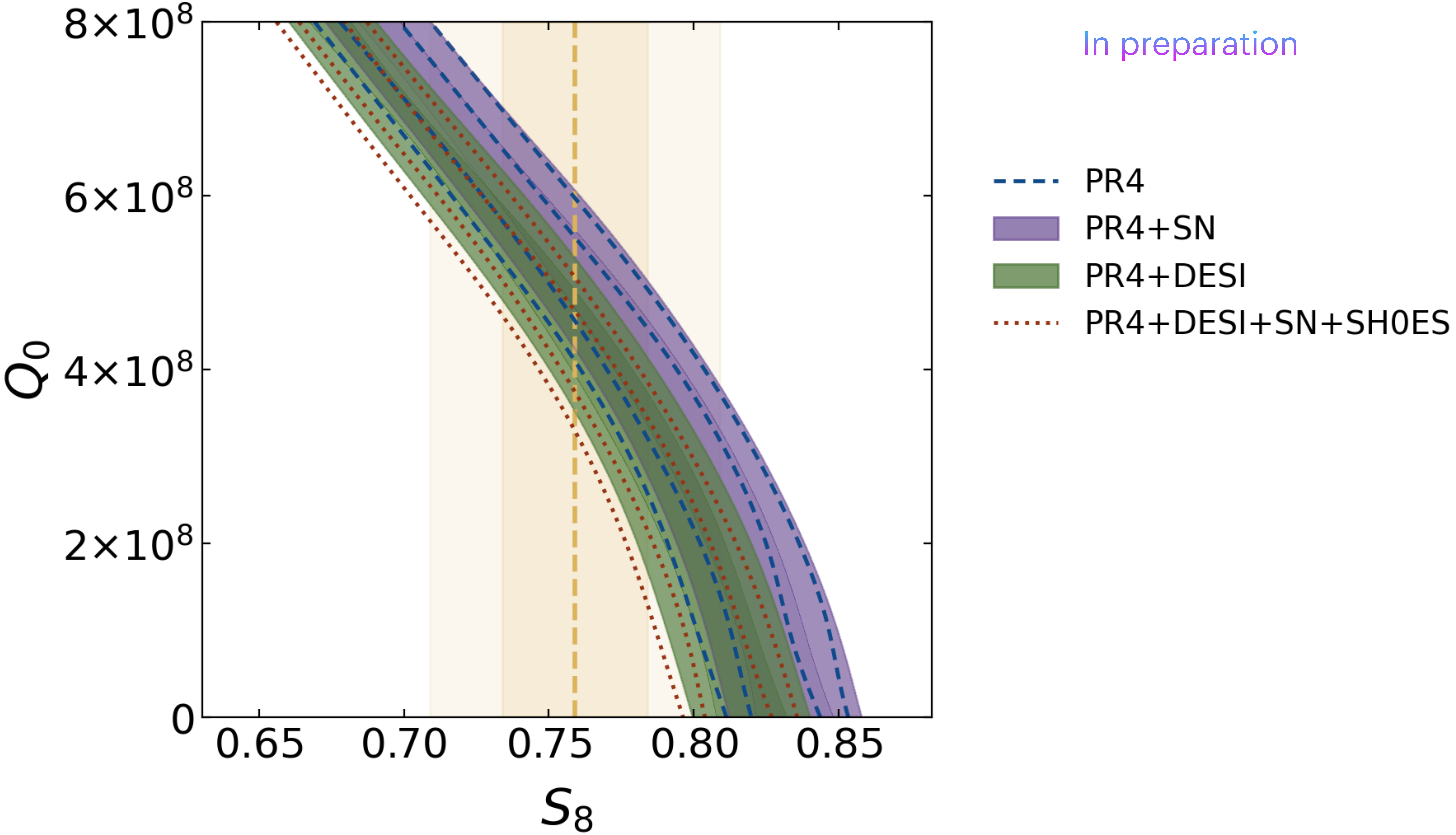

Intriguing: for reported hints of effects described here (but not statistically strong evidence!) see

M. Khurshudyan and E. Elizalde, arXiv:2402.08630;

M. Abedin, L. Escamilla, S. Pan, E. Di Valentino, W. Yang, arXiv: 2505.09470;

A. Ajith, U. Kumar, arXiv: 2607.18234.

If this evidence persists in future work, this model might be a first step in constructing a physical theory of what is going on.

5. Outlook

Scalar field model of DM interacting with DE are worth developing.

- For the hybrid model, it would be interesting to include isocurvature perturbations (although probably would not help with Hubble tension).
- For that model it would be good to extend it to tackle the problem of super-Planckian field excursions. Maybe such an extension deals with suppression of isocurvature perturbations as well?
- Adding neutrinos, curvature etc would be worth investigating as well.

5. Outlook

Possible extensions for minimal axio-dilaton model (well motivated, but then model is no longer minimal):

- Couplings of axion to baryons
- Axion is not the only DM component
- Minima in coupling function $W(\chi)$, e.g. $W(\chi) = 1 + \frac{(\chi - \chi_*)^2}{2\Lambda_\chi^2}$
- Screening of dilaton via axion
- Early dark energy can arise naturally in some models (see arXiv:2505.05450)

Axio-dilaton models have rich phenomenology (but price to be paid is that number of parameter grows).

5. Outlook

Interactions of the kind presented in the last model are genuinely new in the sense that we recognised that the physical properties of DM can change due to interactions with DE.

Model as it stands is an ugly duckling (to use Eric's words) but results may carry over to other models developed from fundamental physics. If anything, this model suggests to search for imprints of DM beyond simple CDM (EOS in particular, allows for negative values).

Thank you!