Influence of matter on large-scale dynamics of light scalars

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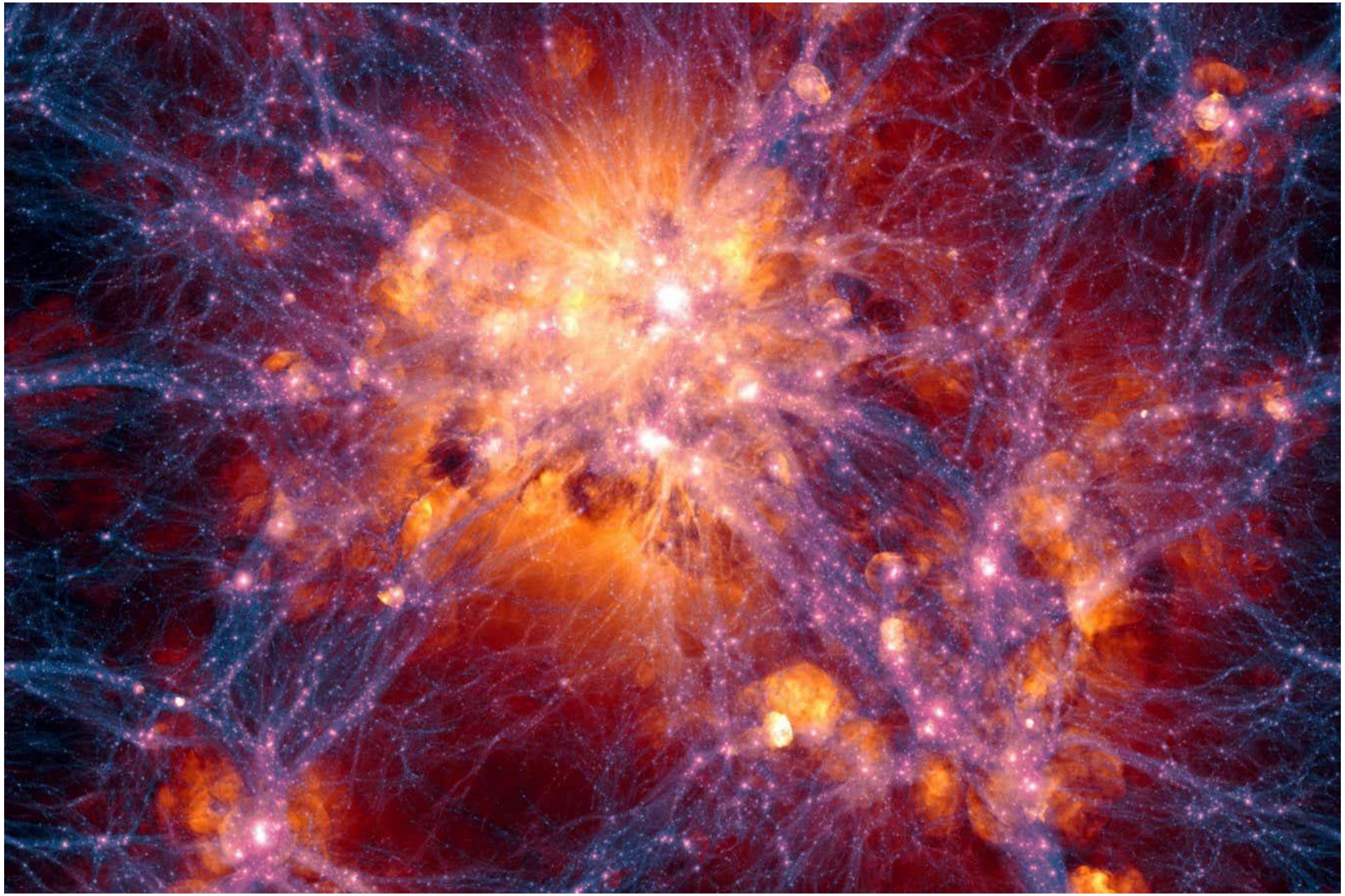
Light scalars coupled to matter propagate through the cosmic web



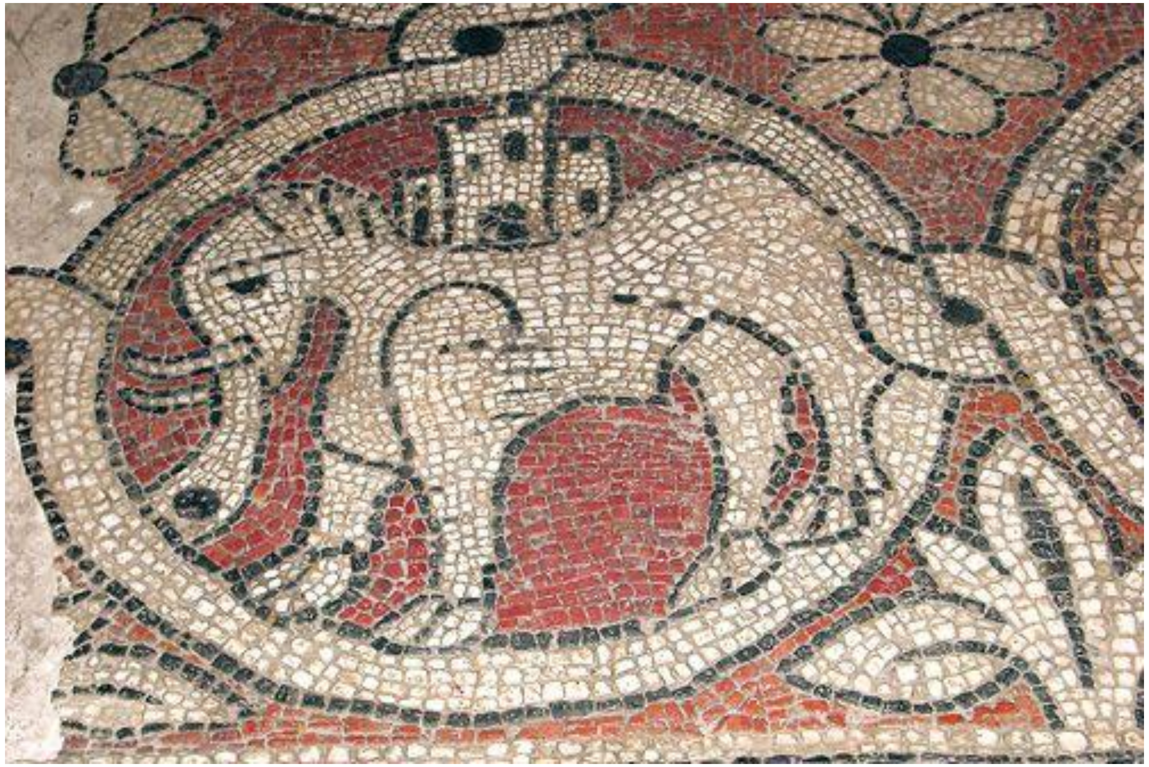
The cosmic web is relatively recent so could this **influence the dynamics** of light scalars?

Could this be related to the **emergence of dark energy**?

Could this help understand the **coincidence problem**?

In the early 10th century, monks in Provence had the faint memory of Hannibal's elephants but knew that goats certainly existed…

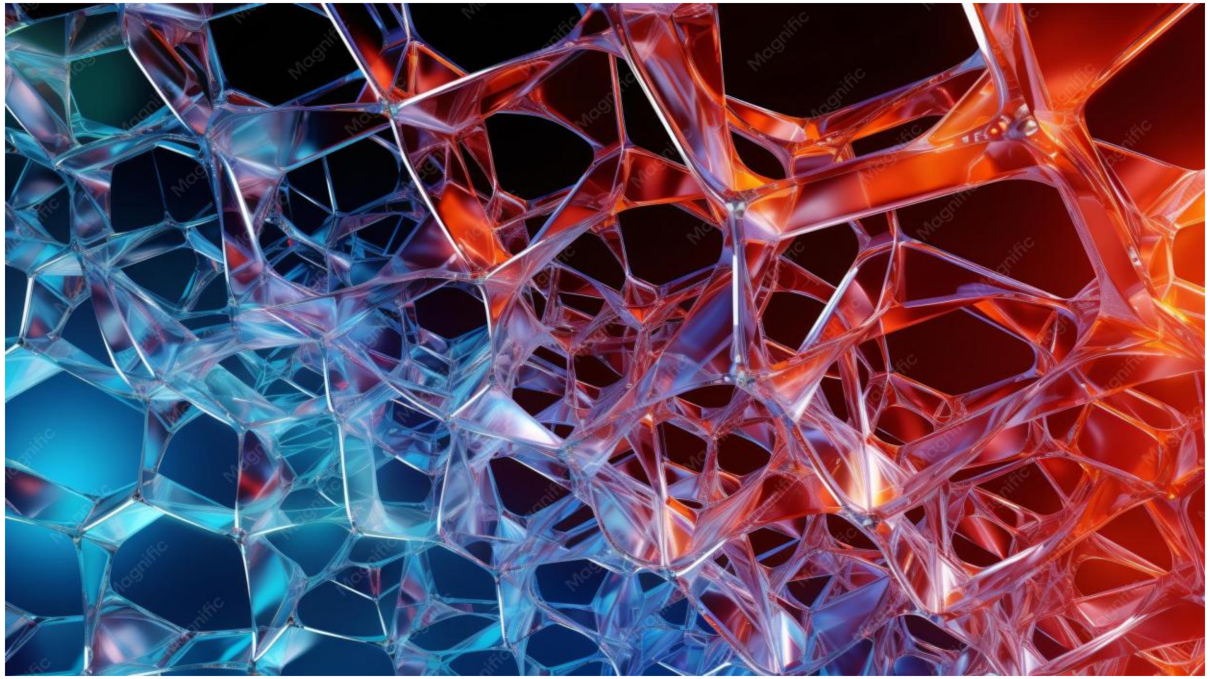
Saint-André de Rosans abbey



Ganagobie abbey

Mixing the old with the new

Photons interact with conduction electrons in matter.



On scales much larger than the lattice size, one can integrate out the heavy electrons to obtain an effective action for the massless photons.

$$\vec{\nabla}\vec{B} - \partial_0 \vec{E} = \dot{\vec{j}}_{\mathrm{ind}}$$

$$\vec{p} = ne\vec{x}$$

Integrating out is tantamount to solving Newton’s law

$$m\ddot{\vec{x}} = -e\vec{E}$$

The polarisability depends both on the coupling e of photons to matter and the matter number density n.

$$\vec{E} = -\vec{\nabla}\Phi_c$$

The displacement field (in Fourier space):

$$\vec{D} = \vec{E} + \vec{p} = \epsilon(\omega)\vec{E}$$

$$\epsilon(\omega) = 1 - \frac{\omega_{\text{pl}}^2}{\omega^2}$$

The **effective** Maxwell equation:

$$\epsilon(\omega)\omega^2\vec{E} + \vec{k}^2\vec{E} = 0$$

Photons become effectively **massive**:

$$(\omega^2 - \vec{k}^2 - \omega_{\text{pl}}^2)\vec{E} = 0$$

$$\omega_{\text{pl}}^2 = \frac{ne^2}{m}$$

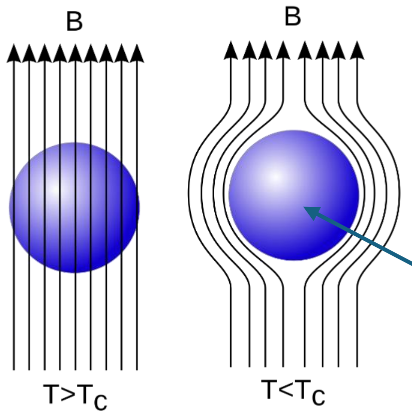
The photon mass depends on the electron number density and the coupling strength.

Two physical consequences:

Weak coupling-small density: $v_k = \frac{k}{\omega} \leq 1$ Photons travel less fast

Strong coupling-large density: Screening!

On large scales $k \ll \omega_{\rm Pl}$



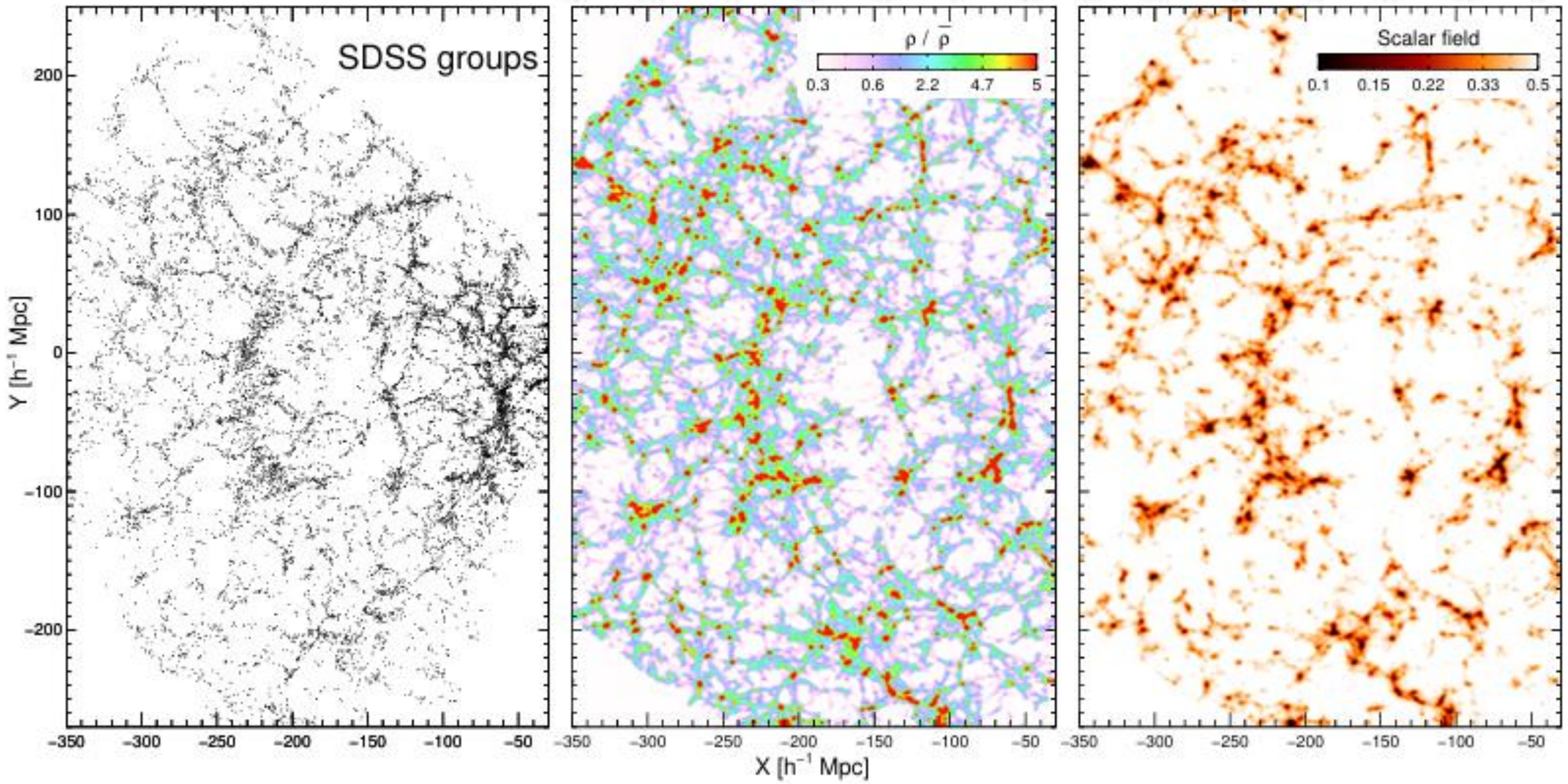
$\Delta \vec{B} = \omega_{\rm pl}^2 \vec{B}$ $\longrightarrow$ **Meissner effect**

$\vec{B} = 0$

Penetration length: $L = \frac{1}{\omega_{\rm pl}}$

Thin shell

In cosmology, light scalar fields are ubiquitous and travel through a cosmic web of matter. Generically they are coupled to matter. We will *integrate out the heavy dark matter* on scales less than a few Mpcs and study the effective field theory of light fields on large scales resulting from the interaction with dark matter on short distances.



What is the effective description of light fields in the cosmic web?

The matter distribution and its screening map (f(R) theory)

The low energy description

On large scales, GR can be seen as the lowest order effective action involving the metric up to two derivatives:

$$S_{\mathrm{Einstein-Hilbert}} = \frac{m_{\mathrm{Pl}}^2}{2} \int d^4x \sqrt{-g} (R + \dots)$$

Incorporating a single scalar field, the effective action up to second order in derivatives:

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi G_N} - \frac{(\partial\phi)^2}{2} - V(\phi) + \mathcal{L}_m(\psi_i, \tilde{g}_{\mu\nu}) \right)$$

$$\tilde{g}_{\mu\nu} = A^2(\phi) g_{\mu\nu}$$

The scalars in matter obey a modified Klein-Gordon equation:

$B(\phi)$

$$\Box\phi = \frac{\partial V_{\text{eff}}}{\partial\phi}$$

$$V_{\text{eff}}(\varphi) = V(\varphi) + \left(\frac{A(\varphi)}{A(\varphi_c)} - 1\right)\bar{J}$$

We will assume that the scalar field is nearly massless in vacuum and behaves like a “test” field.

$$V \ll \bar{J}$$

The matter distribution comprises a time-dependent background and random fluctuations:

$$\bar{J} = \bar{J}_0 + j$$

Gaussian distribution.

Power spectra:

4-vector

$$\langle j(x) j(y) \rangle = P(x, y)$$

When time and space translation are respected: $P(x, y) = P(x - y)$

For Markovian process: $P(x, y) = P(\vec{x} - \vec{y}) \delta(x_0 - y_0)$

In cosmology time translation invariance is broken so only a mixed representation:

$$\langle j(\vec{k}, t) j(\vec{k}', t') \rangle = P(k, t, t') (2\pi)^3 \delta^3(\vec{k} + \vec{k}')$$

isotropy

Integrating out dark matter:

All short distance fluctuations are integrated out $k \gtrsim \frac{1}{L}$

Interested in **average values** so pretend that we are in a quantum field theory and eventually only pick up the **classical approximation**:

$$e^{i\Gamma(\varphi)} = \int \mathcal{D}\phi_0 e^{iS(\phi)}$$

$$\phi = \phi_0 + \varphi$$

Large distance

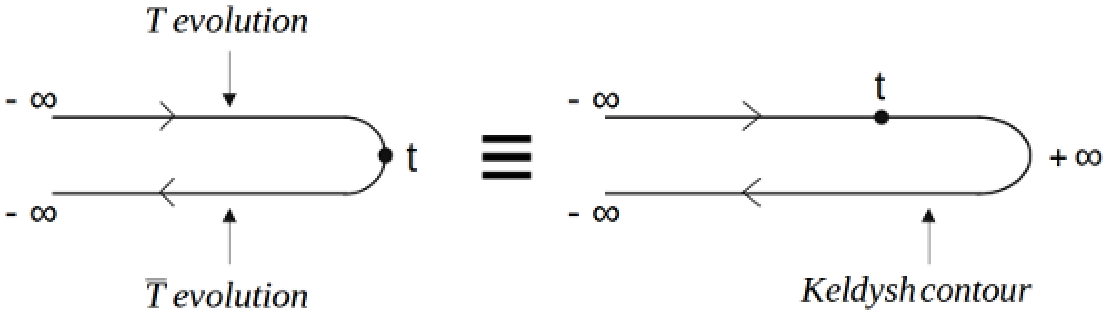
$$\Gamma(\varphi)$$

Effective action

$$\frac{\delta\Gamma}{\delta\varphi} = 0$$

Effective equations of motion

One obstacle: use the **Schwinger-Keldysh** formulation to calculate averages



$$\langle in|F(\phi)|in\rangle = \langle in|\bar{U}F(\phi_I)U|in\rangle$$

Need to double the number of fields, integrate over them with the action:

$$S(\phi_1, \phi_2) = S(\phi_1) - S(\phi_2)$$

Eventually the classical configuration is obtained by changing basis:

$$\phi_+ = \frac{\phi_1 + \phi_2}{2}, \ \phi_- = \phi_1 - \phi_2$$

$$\frac{\partial\Gamma(\phi_+, \phi_-)}{\partial\phi_-}|_{\phi_-=0} = 0$$

In practice after integrating out matter:

$$\Gamma(\varphi_1, \varphi_2) \equiv \Gamma(\varphi_1, \varphi_2, \bar{\phi}_{0,1}, \bar{\phi}_{0,2})$$

Solution to Klein-Gordon with only short distance sources.

Using ergodicity, the average over many patches of size L in the Universe reproduces the averaged behaviour of the effective action

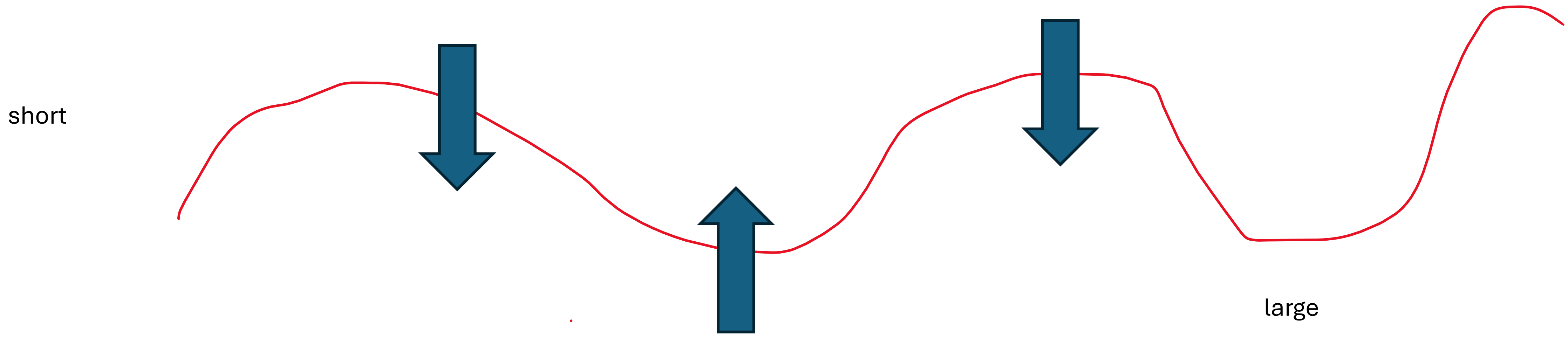
$$e^{i\Omega(\varphi_1,\varphi_2)} = \langle e^{i\Gamma(\varphi_1,\varphi_2))} \rangle$$

Grand potential

Average over short distance dark matter fluctuations.

The random noise:

This does not quite work like this as the random fluctuations on short scales become a Langevin noise on large scales:



The continuous exchange of information between short and large scales makes the Universe an

Open system

Works the same for inflation or GW

The noise characterising the openness is such that:

$$E(e^{i\Omega(\varphi_1,\varphi_2,\xi_{1,2})}) = \langle e^{i\Gamma(\varphi_1,\varphi_2)}\rangle$$

Noise average

This is constructed as follows:

$$\ln\langle e^{i\Gamma}\rangle = \sum_{n=0}^{\infty}\frac{i^n}{n!}C_n(\Gamma)$$

Only the odd terms are imaginary and construct an action.

The even terms define the generating functional for the noise

$$\ln E(e^{\int d^4x\sqrt{-g}(\xi_2\varphi_2-\xi_1\varphi_1)}) = \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}C_{2n}(\Gamma)$$

The grand potential is then:

$$\Omega = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} C_{2n+1}(\Gamma) + \int d^4x \sqrt{-g}(\xi_2 \varphi_2 - \xi_1 \varphi_1)$$

Separating the cumulant as:

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} C_{2n+1}(\Gamma) = \mathcal{S}(\varphi_1) - \mathcal{S}(\varphi_2) + R(\varphi_1, \varphi_2)$$

The large-scale Klein-Gordon equation becomes:

$$\frac{\delta \mathcal{S}}{\delta \varphi} = \xi_+ + \left(\frac{\delta R}{\delta \varphi_1} - \frac{\delta R}{\delta \varphi_2}\right)_{\phi=\phi_{1,2}}$$

Radiation-reaction occurs when dissipation into small scales or radiation to infinity happen. This is the case of GW which can be treated with the same method.

Langevin noise

Radiation-reaction force

The classical equations of motion:

As we are treating a classical theory:

$$\Gamma(\varphi_{1,2}, \bar{\phi}_{0,1,2}) = S(\varphi_1, \bar{\phi}_{1,0}) - S(\varphi_2, \bar{\phi}_{2,0})$$

No mixing between the two fields

$$\Gamma = \sum_{n=0}^{\infty} \frac{1}{2^{2n}(2n+1)!} \int \left(\prod_{p=1}^{2n+1} d^4x_p \varphi_-(x_p)\right) \frac{\delta S}{\delta\varphi_-(x_1)\ldots\delta\varphi_-(x_{2p+1})}\Big|_{\varphi+\bar{\phi}_0}$$

$$R = 0$$

Odd in ϕ_-

No radiation-reaction

The only cumulant which matters in the equations of motions is the first one:

$$\Omega = \langle \Gamma \rangle + \int d^4x\sqrt{-g}(\xi_2\varphi_2 - \xi_1\varphi_1)$$

Naïve result! Averaging the effective action

Langevin noise

As the even cumulants of the one-particle irreducible Γ involves even powers of ϕ_-

$$E(\xi(x_1)\ldots\xi(x_{2p+1})) = 0 \longrightarrow E(\xi) = 0$$

Acts only at the perturbative level

$$j_> \to \xi$$

The noise has no background effect

The only cumulant which matters in the equations of motions is the first one:

$$\Omega = \langle \Gamma \rangle + \int d^4x \sqrt{-g}(\xi_2 \varphi_2 - \xi_1 \varphi_1)$$

Naïve result! Averaging
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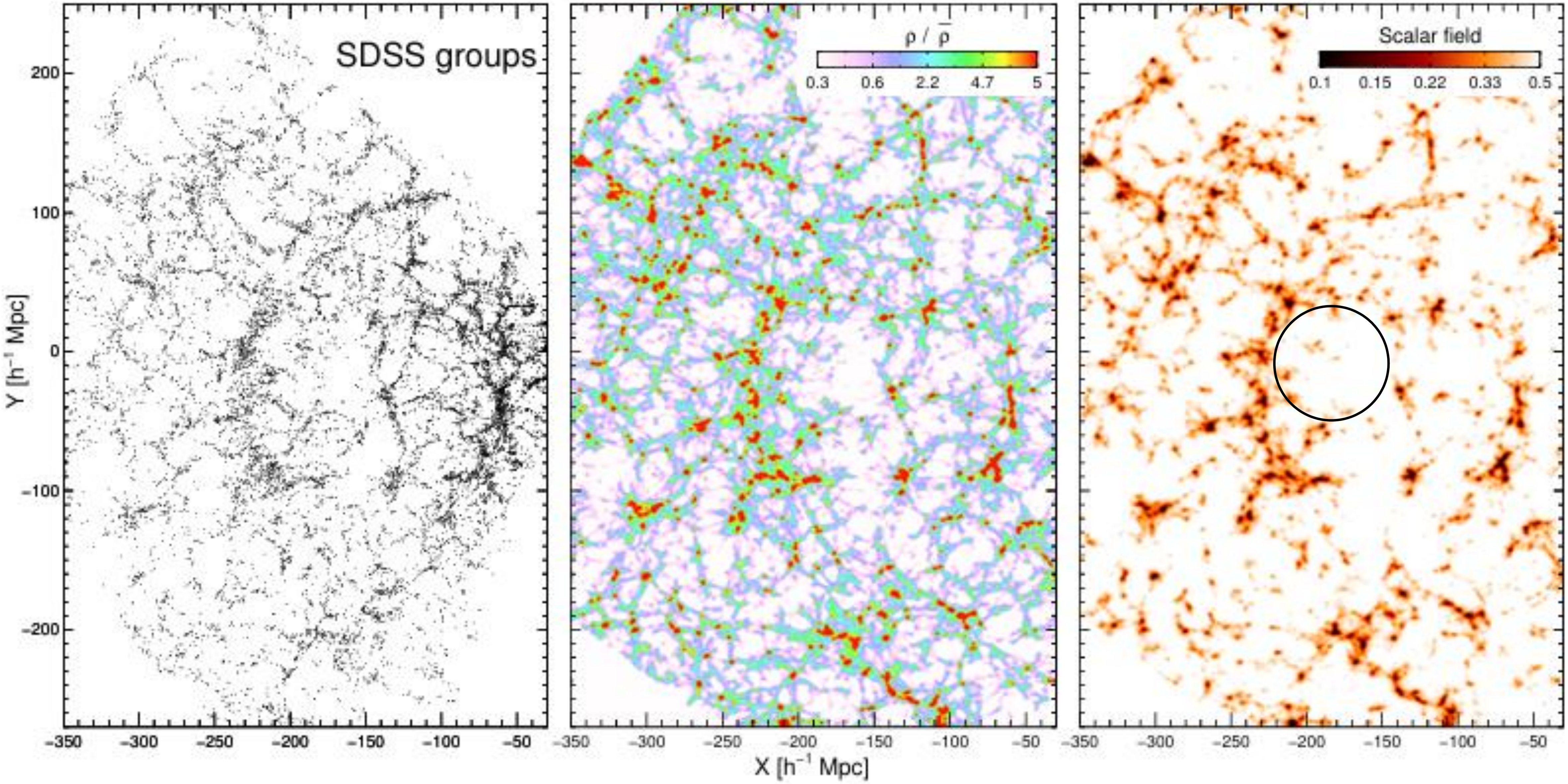
Langevin noise

We can apply this technology to two drastically opposite situations:

Unscreened cosmology

Totally screened cosmology

Unscreened cosmology



In an unscreened cosmological setting, the background cosmological field penetrates all structures:

$$\phi = \phi_0 + \varphi$$

Solve the local equations of motions in the background of the cosmological field

The solution to the Klein-Gordon equation can be obtained by iteration. The leading order is:

The source is:

$$\mathcal{J}(\vec{p},\omega) = \frac{\beta}{m_{\rm Pl}} j_>(\vec{p},\omega)$$

We assume that all couplings and potentials involves a single coupling scale $\frac{m_{\rm Pl}}{\beta}$

Time- translation:

$$\bar{\phi}_0(\vec{p},\omega) = -\frac{i\beta}{m_{\rm Pl}}\frac{j(\vec{p},\omega)}{2\omega}(2\pi)\delta(|\omega|-\omega(p))1_{|\vec{p}|\geq\frac{1}{L}}$$

$$\omega(p) = \sqrt{\vec{p}^2 + m_\phi^2}$$

$$m^2(\phi) = (\frac{\beta}{m_{\rm Pl}})^2(j_> B_2(\varphi) + V_2(\varphi))$$

In the cosmological case when time translation is broken, the field induced by small distance fluctuations is:

$$\phi_0(\vec{p},t) = \frac{\beta}{m_{\rm Pl}} \frac{1_{|\vec{p}|\geq \frac{1}{L}}}{\omega(p)} \int_{-\infty}^{t} \sin(\omega(p)(t-t'))j(\vec{p},t')dt'$$

Only on short scales

Depends on the matter history

The deterministic dynamics on large scales depends on the average over short scale:

$$\langle\Gamma\rangle = \langle\Gamma^{(0)}\rangle - \frac{\beta}{m_{\rm Pl}} \int d^4x\sqrt{-g}\langle\bar{\phi}_0\bar{J}\rangle(B_1(\varphi_+ + \frac{\varphi_-}{2}) - B_1(\varphi_+ - \frac{\varphi_-}{2}))$$

Bare action on large scales

$$B(\varphi+\phi_0) = \sum_{n=0}^{\infty} \frac{1}{n!}(\frac{\beta}{m_{\rm Pl}})^n B_n(\varphi)\phi_0^n$$

The leading correction comes from **quadratic** couplings.

The leading correction comes from the average:

$$A(L) = W_< * \langle \bar{\phi}_0 \bar{J} \rangle$$

Window function projecting on large scales

Correlation function between integrated out field and small scale matter fluctuations

In the time-translation invariant case:

$$A(L) = \frac{\beta}{m_{\rm Pl}} \int_{|\vec{p}| \geq \frac{1}{L}} \frac{\Im(P(|\vec{p}|, \omega(p))}{\omega(p)}$$

Vanishes in the Markovian case or when the fluctuation-dissipation theorem stands.

In the cosmological case, the new coefficient is non-vanishing:

$$A(L) = \frac{\beta}{m_{\rm Pl}} \int_{|\vec{k}|\geq \frac{1}{L}} \frac{d^3k}{(2\pi)^3\omega(k)} \int_{-\infty}^{t} \sin(\omega(k)(t-t'))P(\vec{k},t,t')dt'$$

This is sensitive to the coupling, the power spectrum of matter and depends on the whole history of matter.

With this, we can write the Klein-Gordon equation:

$$\mathcal{B}_2(\varphi) = \frac{dB_1(\varphi)}{d\varphi}$$

$$\Box\varphi = W_< \star (\partial_\phi(V(\varphi) + j_< B(\varphi)) + (\frac{\beta}{m_{\rm Pl}})^2\langle\bar{\phi}_0\bar{J}\rangle\mathcal{B}_2(\varphi) + \xi)$$

Projects on large scales

Usual driving terms corresponding to the effective potential on large scales

Small scale correction

Noise

Cosmological background:

$$\ddot{\bar{\varphi}} + 3H\dot{\bar{\varphi}} + \partial_\phi V_{\text{eff}}(\bar{\varphi}) + (\frac{\beta}{m_{\text{Pl}}})^2 A(L)\mathcal{B}_2(\bar{\varphi}) = 0$$

New operator due to fluctuations on short scales

This correction can modify the background evolution!

see in a minute

Cosmological perturbations:

The Langevin noise is connected to the random fluctuations on short scales

$$\delta\varphi = \frac{\frac{\beta}{m_{\text{Pl}}} B_1(\bar{\varphi}) j_<(\vec{p}, \omega) + \xi}{\omega^2 - \vec{p}^2 - M^2} 1_{|\vec{p}| \leq \frac{1}{L}}$$

Mass of the scalar

How to deduce the properties of the noise?

The n-point functions of the noise are calculated from the cumulants

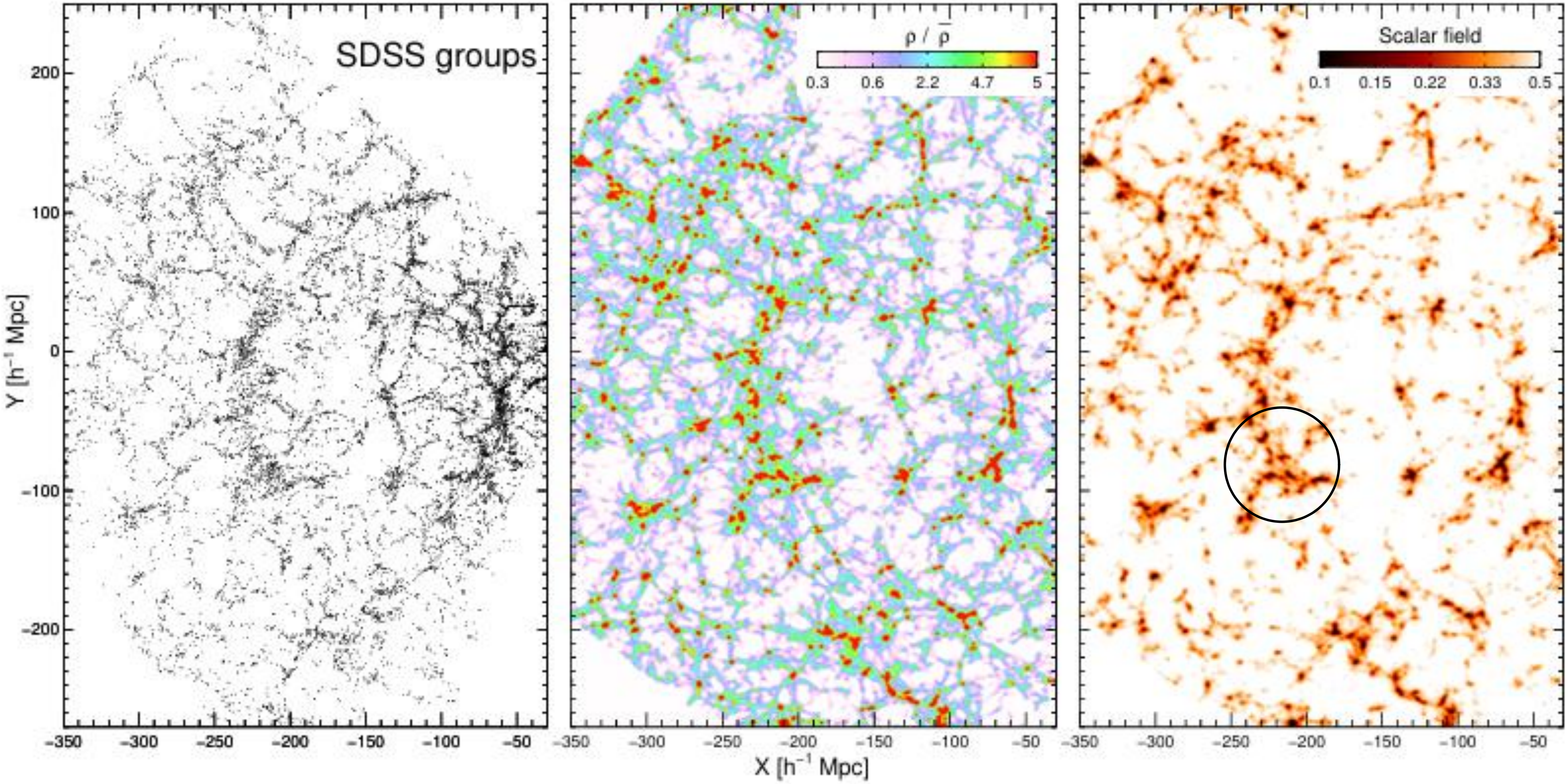
$$C_2(\Gamma) \to \langle \xi\xi \rangle$$

At leading order the noise is Gaussian.

$$\xi = \frac{\beta}{m_{\mathrm{Pl}}} B_1(\bar{\varphi}) j_>$$

This is nothing **but linear response theory**. Hence no effect of the noise on large scales at leading order.

Totally screened cosmology



Can only deal with a very rigid Universe where all small scales are screened.

$$\phi = \phi_0 + \varphi$$

Dominates the background

$$j_> \partial_\phi B(\bar{\phi}_0) + \partial_\phi V(\bar{\phi}_0) = 0$$

The field is pinned locally and reacts to the local matter density.

The effect on the cosmological background is drastic. The Klein- Gordon equation becomes:

$$\Box\varphi = W_< \star (\frac{\beta}{m_{\rm Pl}}\langle B_1(\bar\phi_0)\rangle j_< + (\frac{\beta}{m_{\rm Pl}})^2\langle B_2(\bar\phi_0)J + V_2(\bar\phi_0)\rangle\varphi + \xi)$$

Effect coupling on large scale obtained by averaging over small scales. The cosmological field is a forced oscillator

Mass term

The cosmological field is a forced oscillator

$$\ddot{\bar\varphi} + 3H\dot{\bar\varphi} + M^2\bar\varphi = -\frac{\beta}{m_{\rm Pl}}\langle B_1(\bar\phi_0)\rangle J_0$$

The field is stuck as screening requires M>>H !

$$\frac{\beta}{m_{\rm Pl}}\delta\bar\varphi \sim \frac{\beta^2 H^2}{M^2} \ll 1$$

The cosmological perturbations show the **Meissner screening effect**:

$$\delta\varphi = -\frac{1}{4\pi}\int_{|\vec{x}-\vec{y}|\geq L} d^3y e^{-M|\vec{x}-\vec{y}|}\left(\frac{\beta}{m_{\rm Pl}}\langle B_1(\bar{\phi}_0)\rangle j_<(\vec{y},t) + \xi(\vec{y},t)\right)$$

Exponentially suppressed as ML >> 1 corresponding to screening of all substructures

In the completely screened case, the **large-scale field drifts slowly with exponentially small perturbations**.

Let's get physical

Quadratic coupling

$$B_1(\varphi) = 1 + \frac{\beta}{m_{\rm Pl}}\mathcal{B}_2\varphi$$

Coming back to **unscreened** case.

$$V(\phi) = \frac{1}{2}m_0^2\phi^2$$

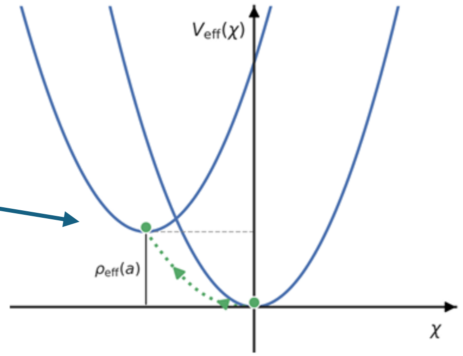
The effect of small scales:

$$m_\phi^2 = m_0^2 + \frac{\beta^2}{\mathrm{m}_{\rm Pl}^2}\mathcal{B}_2\rho \qquad \hat{\varphi} = -\frac{\beta\rho_0}{m_{\rm Pl}m_\phi^2}$$

$$\delta V = \frac{\beta}{m_{\rm Pl}}A(L)B_1(\varphi)$$

Two main effects:

$$\delta\hat{\varphi} = -\frac{\beta^2}{m_{\rm Pl}^2 m_\phi^2}\mathcal{B}_2 A(L)$$



$$V_0 = -\frac{1}{2}m_\phi^2\hat{\varphi}^2 + \frac{\beta}{m_{\rm Pl}}A(L)$$

Dynamical

Contribution from small scales

Integrating small-scale dynamics results in a "cosmological" constant due to the short-scale fluctuations of matter:

Size of overdensity

$$A(L) \sim \frac{\beta}{m_{\rm Pl}} \rho^2 L^2$$

"galactic density"

The induced "vacuum" energy due to small scales:

$$\boxed{V_0 \sim (\frac{\beta}{m_{\rm Pl}})^2 \rho^2 L^2} \longrightarrow \boxed{V_0 \sim \rho_{\rm now}}$$

$L \sim \mathrm{Mpc}$ $\beta \sim 10^{-2}$ $\rho \sim 10^6 \rho_{\rm now}$

Coincidence?

Summary

Integrating out dark matter fluctuations on short scales could influence the dynamics of light field

Need **numerical** simulations

What happens for dark energy fields (non-test) ?

What happens when partially screened?

Otranto cathedral