

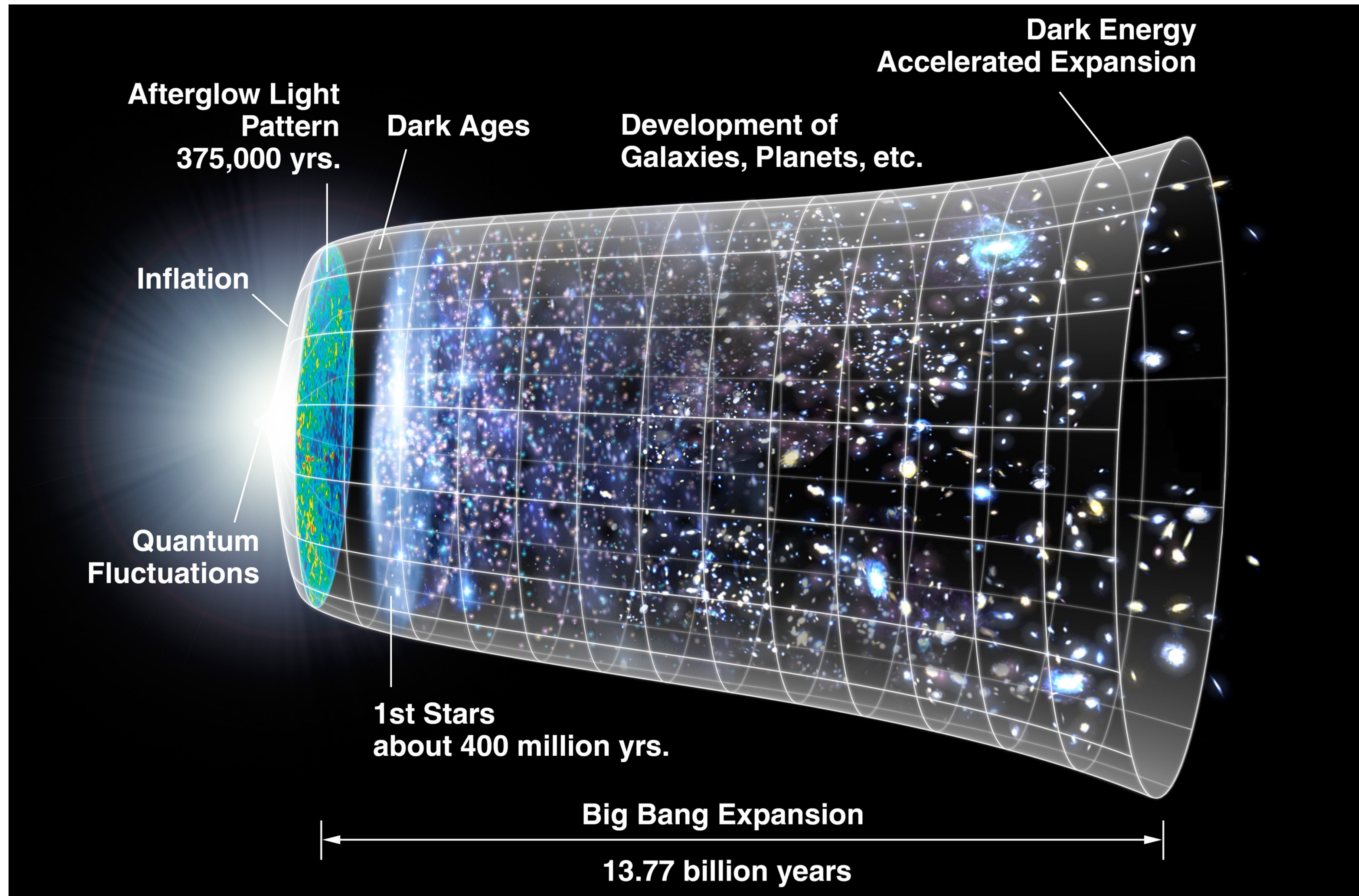


Model Independent Reconstruction of the Cosmological Evolution at Low Redshifts and Possible Theoretical Explanation

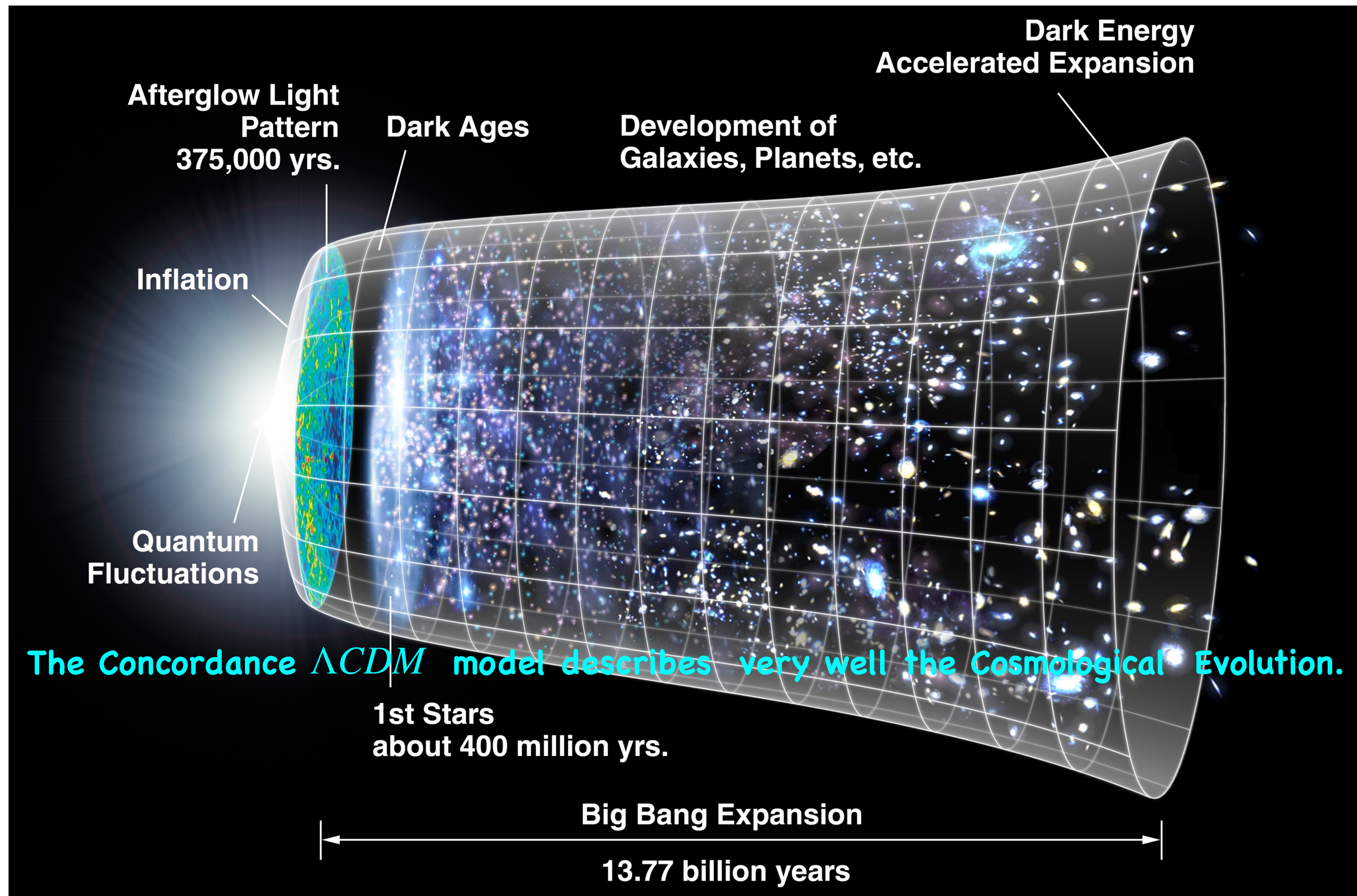
Anjan A Sen
Centre For Theoretical Physics
Jamia Millia Islamia (Central University)
New Delhi, India

GGI workshop on
Exploring New Frontiers in Cosmology
6-31 July 2026

Cosmological Evolution of the Universe



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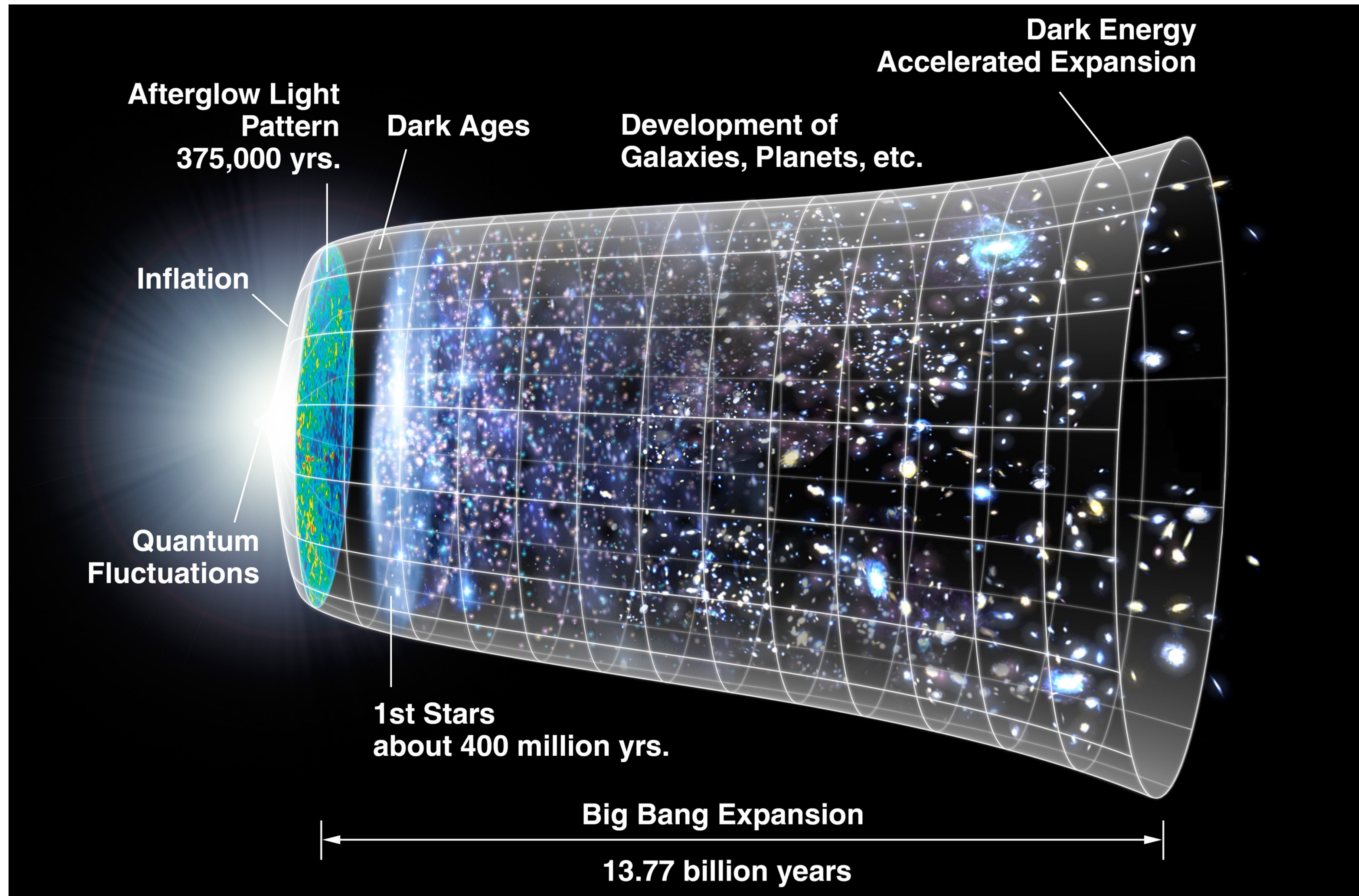


Motivation & The Big Picture



Why do we need to go beyond Λ CDM?

Cosmological Evolution of the Universe



H0DN Global Collaboration

Casertano, S. et al. (2026) Astronomy & Astrophysics

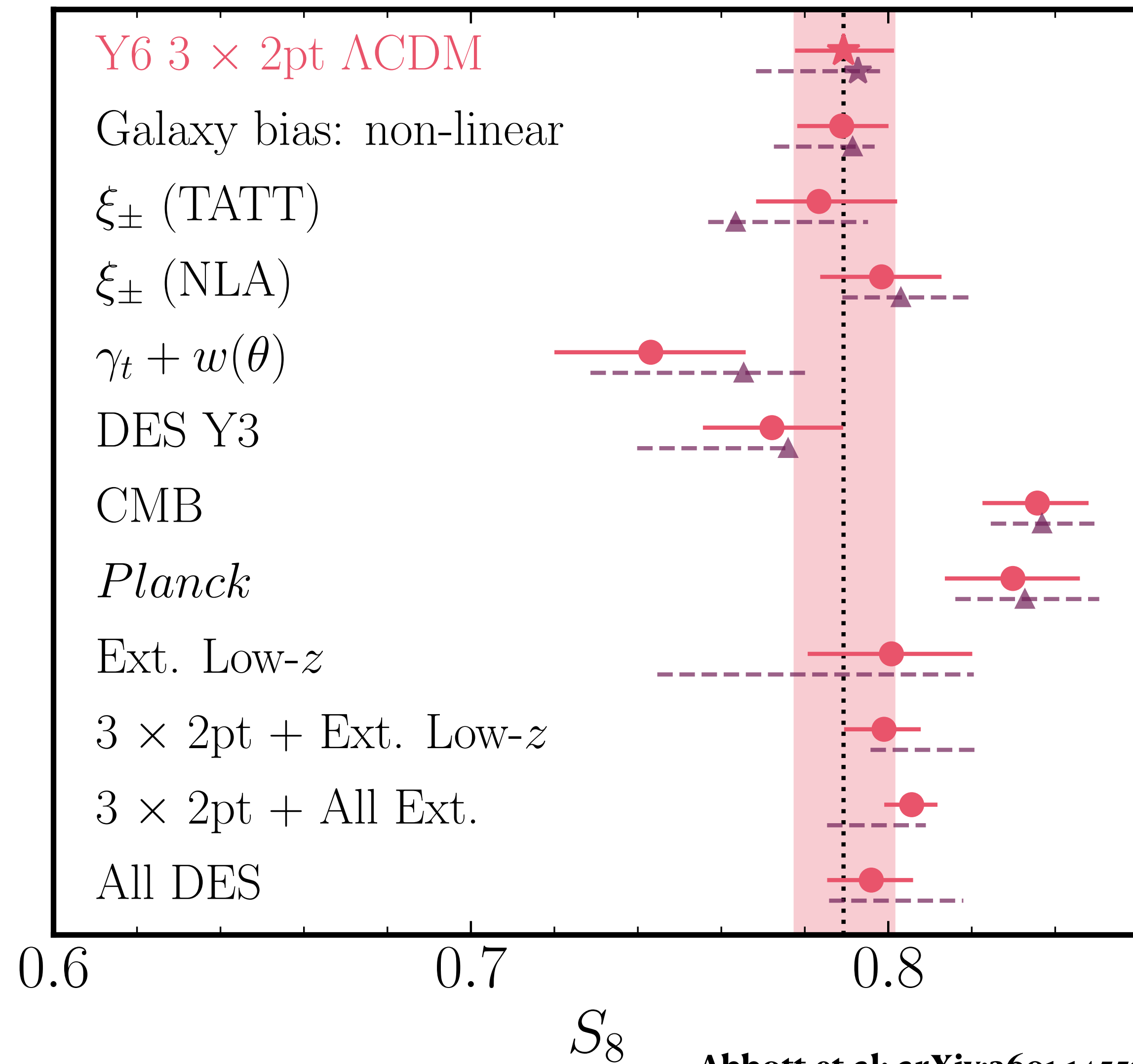


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$$H_0 = 73.5 \pm 0.81 \text{ km/s/Mpc}$$

In Tension with Planck Λ CDM value ~ 70

Latest DES Results and S_8 Tension



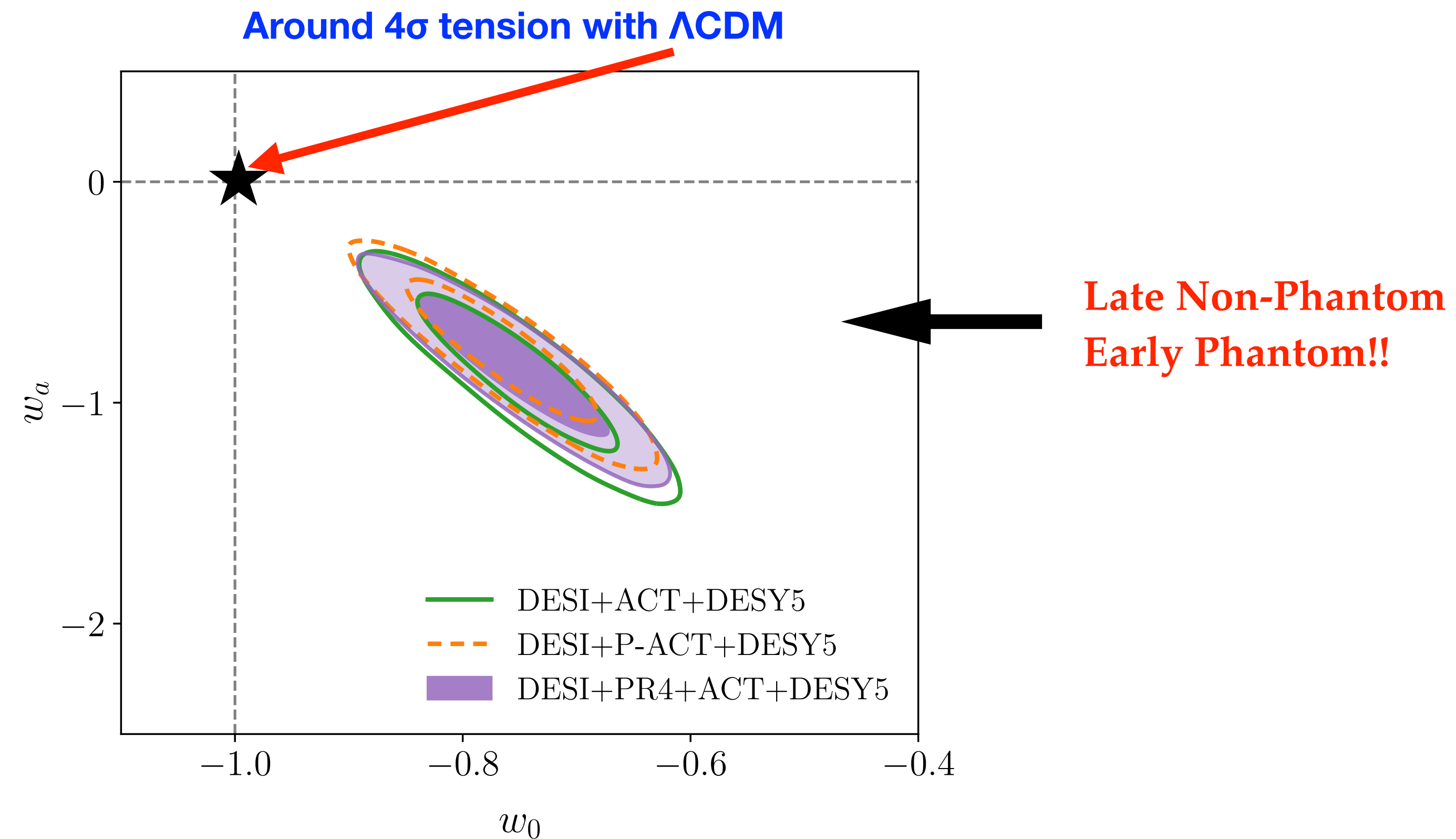
$$S_8 = 0.789 \pm 0.012$$

Around 2.8σ Tension with Planck ΛCDM

Latest Status for DE after DESI-DR2

Garcia-Quintero et al: [arXiv:2504.18464](#)

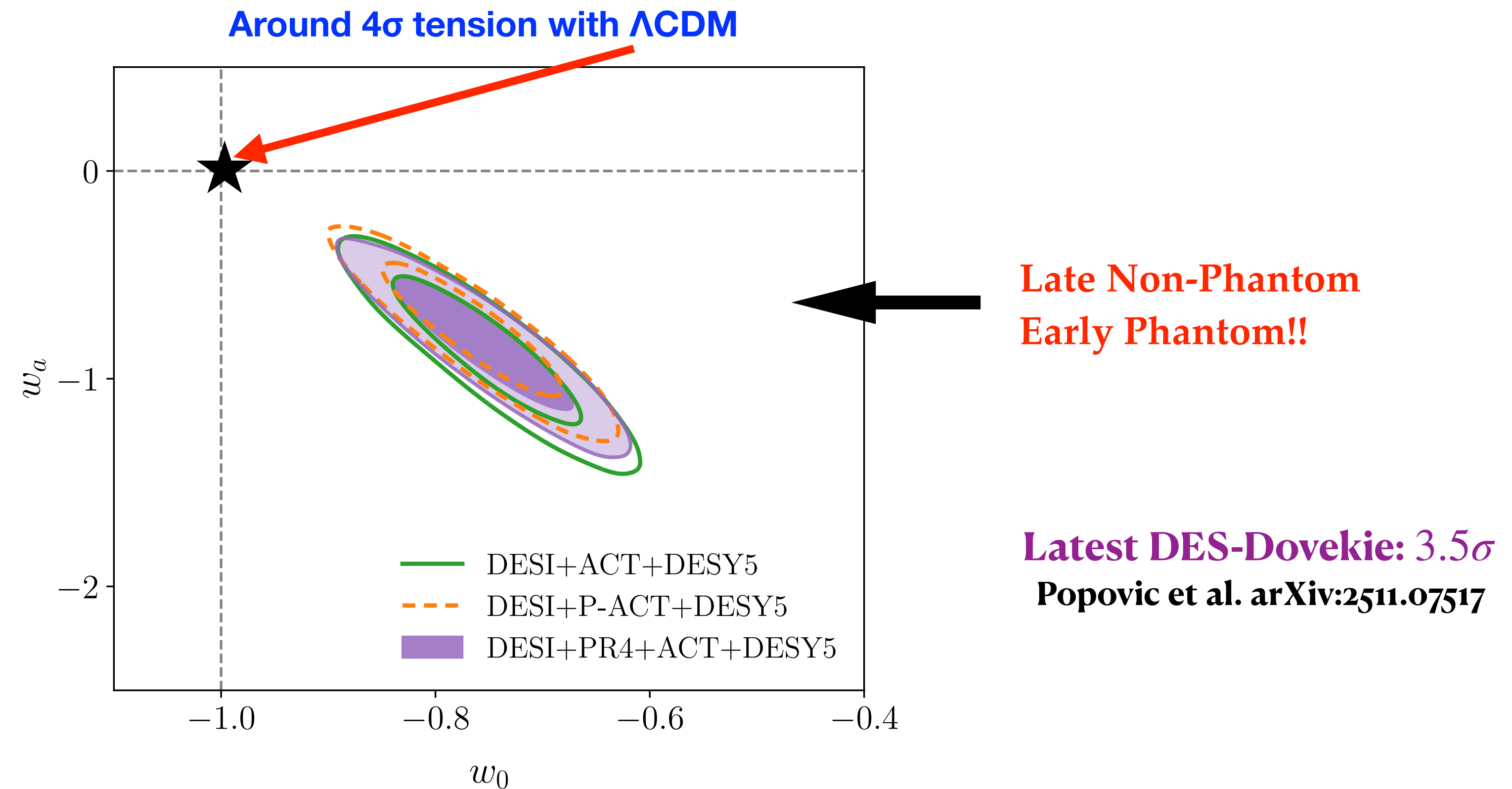
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- ▶ Our approach: Reconstruct cosmological distances directly from low redshift BAO + SN data
- ▶ No assumption on DE sector
- ▶ Does not need any information for Ω_{m0}
- ▶ Gives Model Independent Low-Redshift Expansion history

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Expanding FRW Universe

Background Cosmology is governed by Spatially-Flat FRW Metric

$$ds^2 = dt^2 - a^2(t)[dr^2 + r^2 d\Omega^2]$$

Various Distances: **Comoving Distance:** $d_M(z) = c \int_t^{t_0} \frac{dt}{a(t)} = c \int_0^z \frac{dz'}{H(z')}$ $H = \frac{\dot{a}}{a}$ **Hubble Parameter**

Angular Diameter Distance: $d_A(z) = \frac{d_M(z)}{1+z}$ **BAO, CMB Angular Scale**

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**We can reconstruct $d_m(z)$ directly from Low Redshift data.
And subsequently reconstruct $H(z)$.**

No DE Model is assumed.

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Also this method can not reconstruct if the cosmological evolution has phenomena like Sign-Switching Cosmological Constant.

Reconstructing Comoving Distance

Gupta Choudhury, Mukherjee, Di Valentino and AAS, [arXiv: 2607.13009](#)

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- We place six knots at $z = [0.350, 0.512, 0.782, 1.236, 1.626, 2.33]$ in addition, we have $D_M(0) = 0$ & $D_M'(0) = 1$

Characteristic Redshifts and Hubble Parameter

- There are seven (07) characteristics redshifts:

- $d'_A(z_1) = d_L(z_1); \quad d'_A(z_2) = d_M(z_2); \quad d'_A(z_3) = d_A(z_3)$

- $d'_M(z_4) = d_L(z_4); \quad d'_M(z_5) = d_M(z_5); \quad d'_L(z_6) = d_L(z_6)$

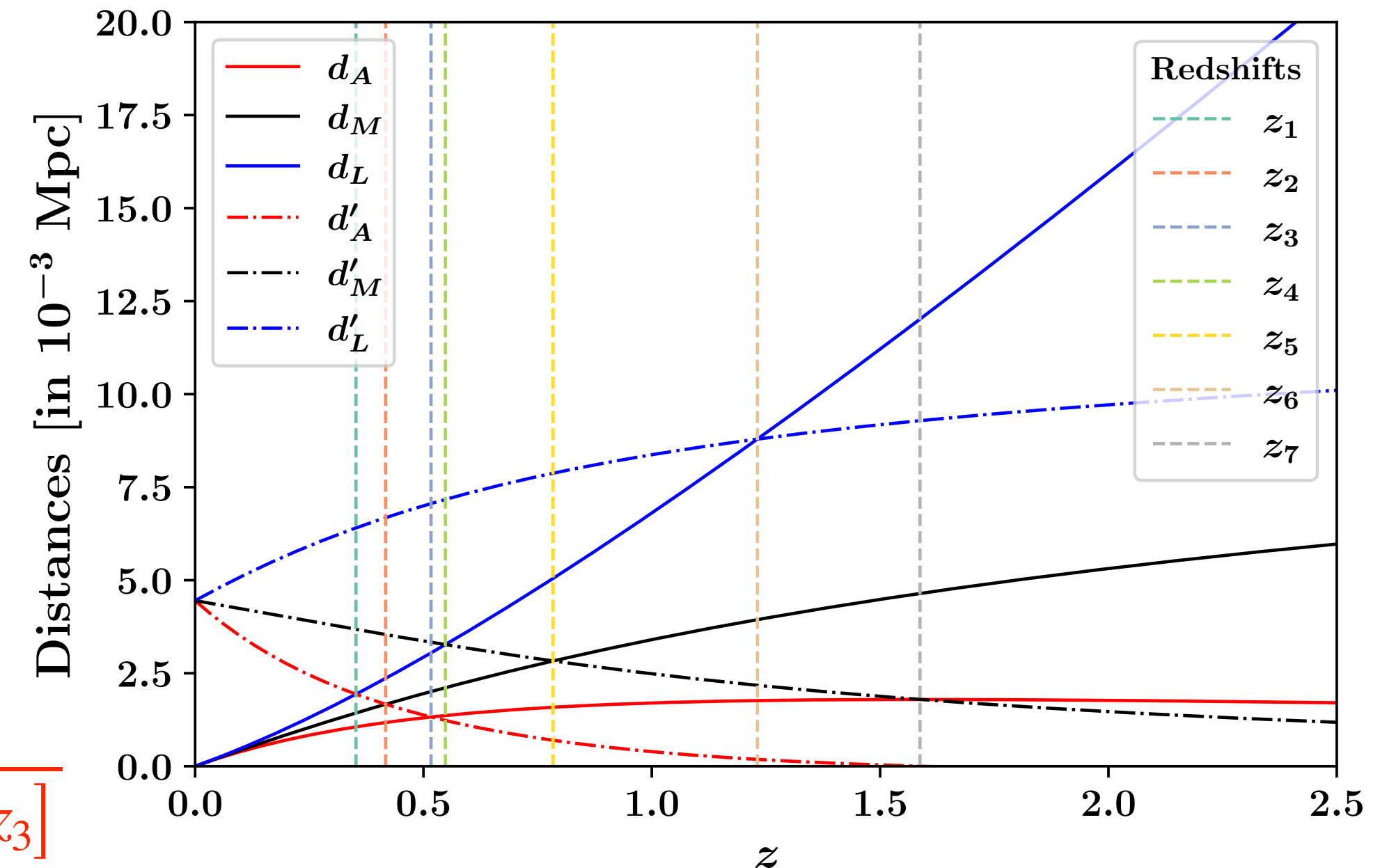
- $d'_A(z_7) = 0$

- This allows us to measure $H(z)$ at these characteristics redshifts:

- $H(z_1) = \frac{c}{d_A(z_1)[(1+z_1)^3+1]}; \quad H(z_2) = \frac{c}{d_A(z_2)[(1+z_1)^2+1]}; \quad H(z_3) = \frac{c}{d_A(z_3)[2+z_3]}$

- $H(z_4) = \frac{c}{d_A(z_4)[(1+z_4)^2]}; \quad H(z_5) = \frac{c}{d_A(z_5)[1+z_1]}; \quad H(z_6) = \frac{c}{d_A(z_6)z_6}$

- $H(z_7) = \frac{c}{d_A(z_7)}$



Avoiding Fully The Derivatives of the Distances

$$D'_A(z_1) = D_L(z_1) \implies D'_M(z_1) = \frac{D_M(z_1)}{1+z_1} [1 + (1+z_1)^3],$$

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$$Z_5(z) = \exp(-z) D_M(z),$$

$$Z_6(z) = (1+z) \exp(-z) D_M(z),$$

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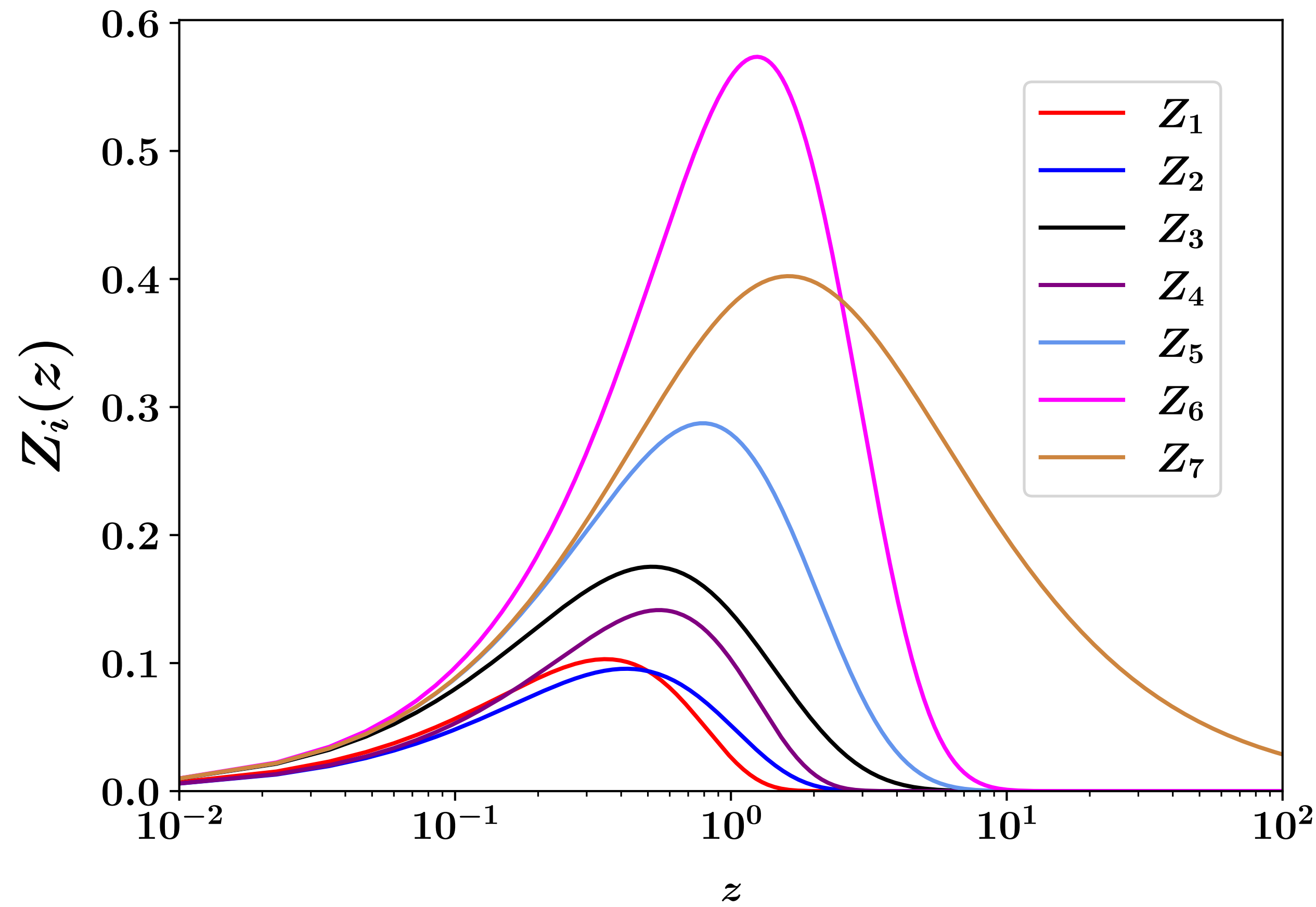
$$Z_7(z) = \frac{D_M(z)}{1+z}.$$

Using reconstructed $D_M(z)$, we need to reconstruct $Z_i(z)$

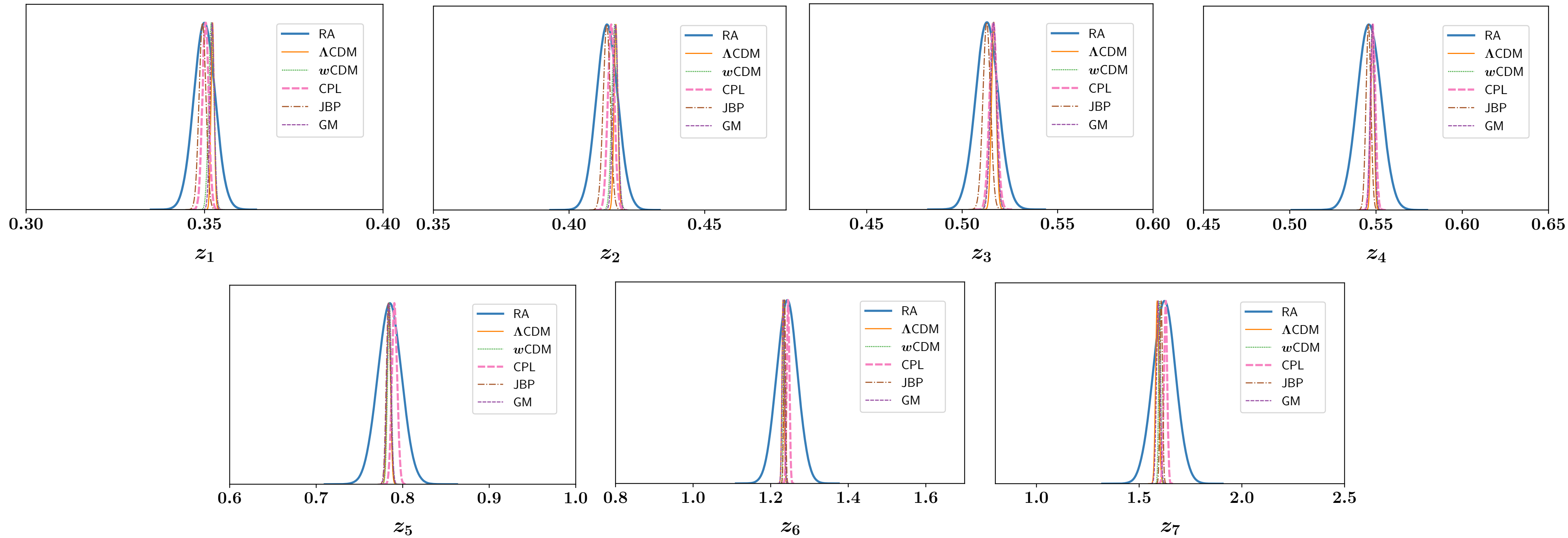
And find their maximum to get the redshifts z_i .

We can use non-derivative extrema finder like Golden Search Algorithm

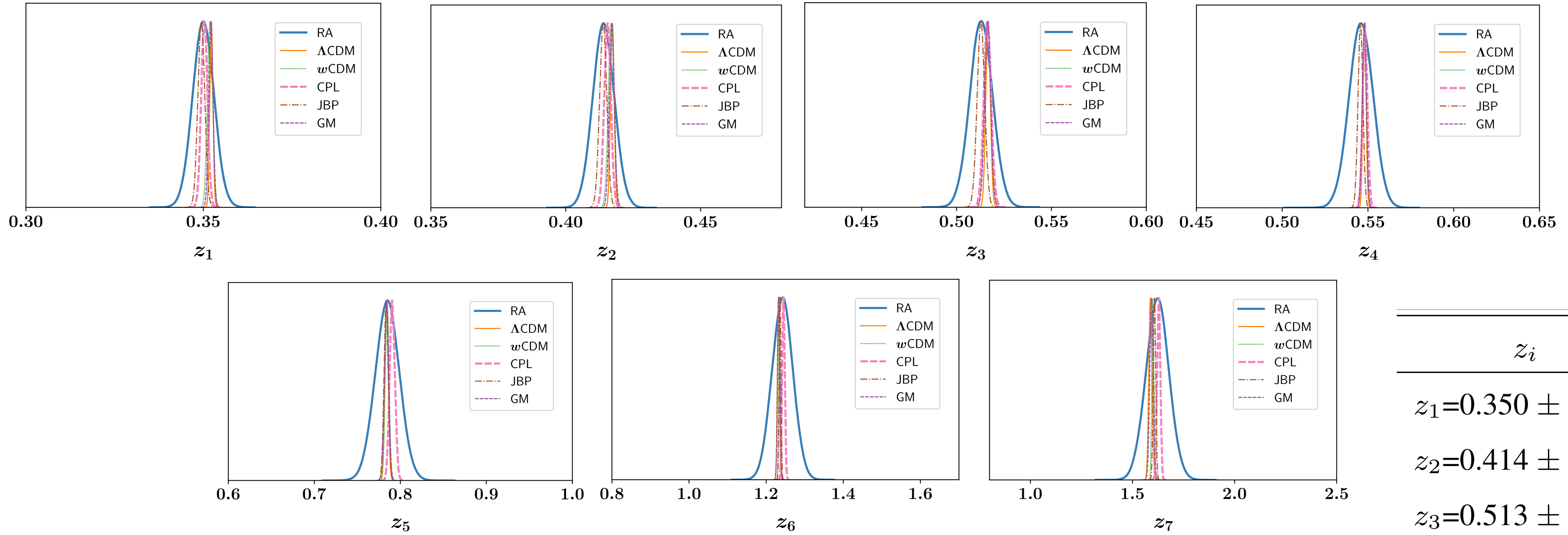
For Planck-2018 Λ CDM model



Knot Redshifts



Knot Redshifts



$$Z_i$$

$$z_1=0.350 \pm 0.003$$

$$z_2=0.414 \pm 0.004$$

$$z_3=0.513 \pm 0.006$$

$$z_4=0.546 \pm 0.007$$

$$z_5=0.785 \pm 0.014$$

$$z_6=1.242 \pm 0.027$$

$$z_7=1.622 \pm 0.056$$

Reconstructing Comoving Distance

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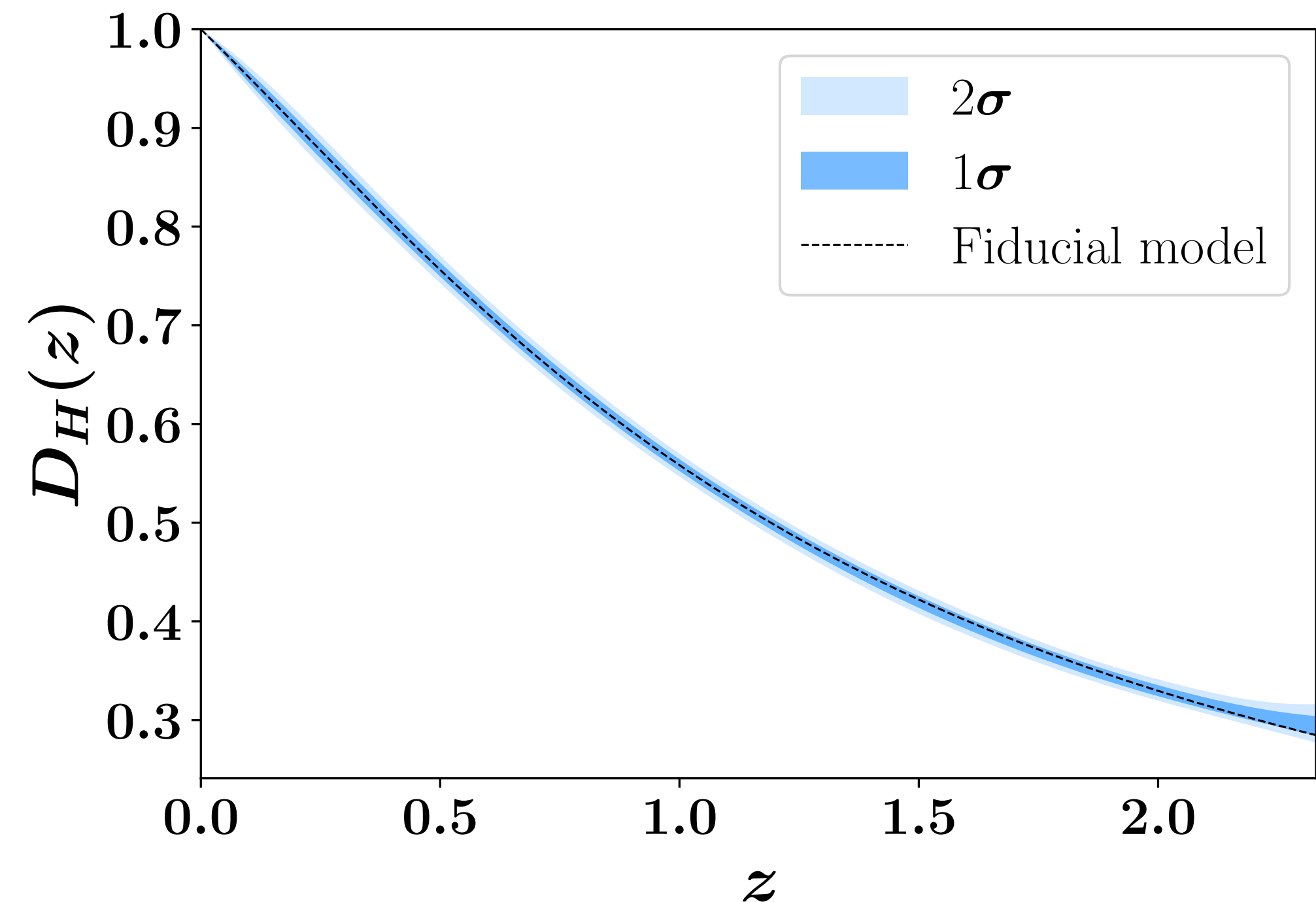
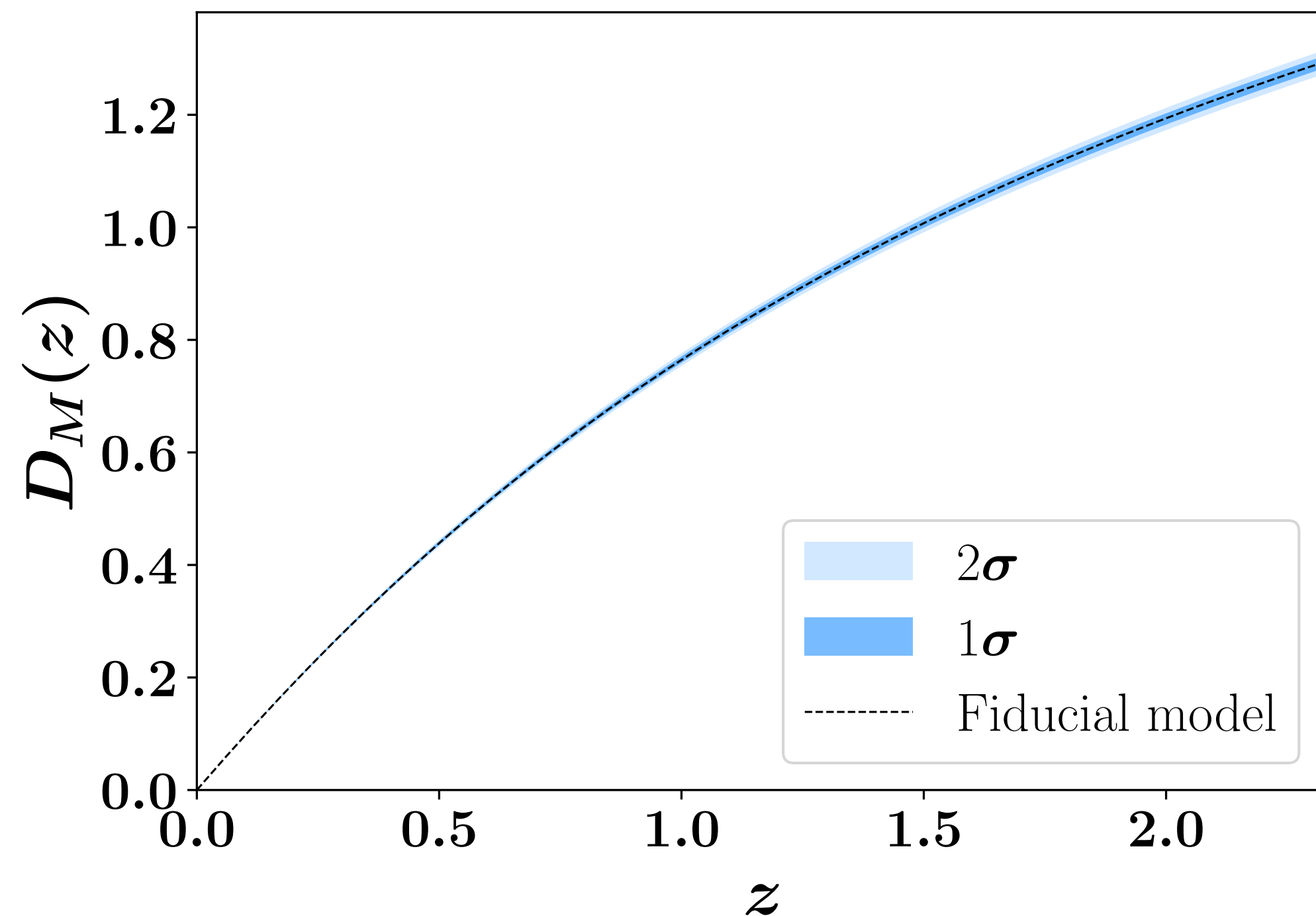
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- We call this reconstruction **“RA”**.
- When we impose the condition involving RE : $P_T < 0 \longrightarrow 2\frac{D''_M}{D'^3_M} + \frac{3}{D'^2_M} > 0$ in our reconstruction algorithm, we call it **“REIRA”**.

Mock Validation without Noise

- We first validate our reconstruction methodology using mock datasets.
- We adopt the best-fit Planck 2018 Λ CDM cosmology as the fiducial model and generate DESI-DR2- and DES-like mock observations using the original covariance matrices of the corresponding datasets.
- We first perform a noiseless mock analysis by directly using the fiducial model predictions.



Mock Validation (With Noise)

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Next, we generate 500 independent mock realizations of the DESI-DR2 and DES datasets using their corresponding covariance matrices.

The reconstruction analysis is then repeated for each realization.

For each parameter θ , we compute the posterior mean $\hat{\theta}_i$ and posterior standard deviation σ_i from the i-th realization.

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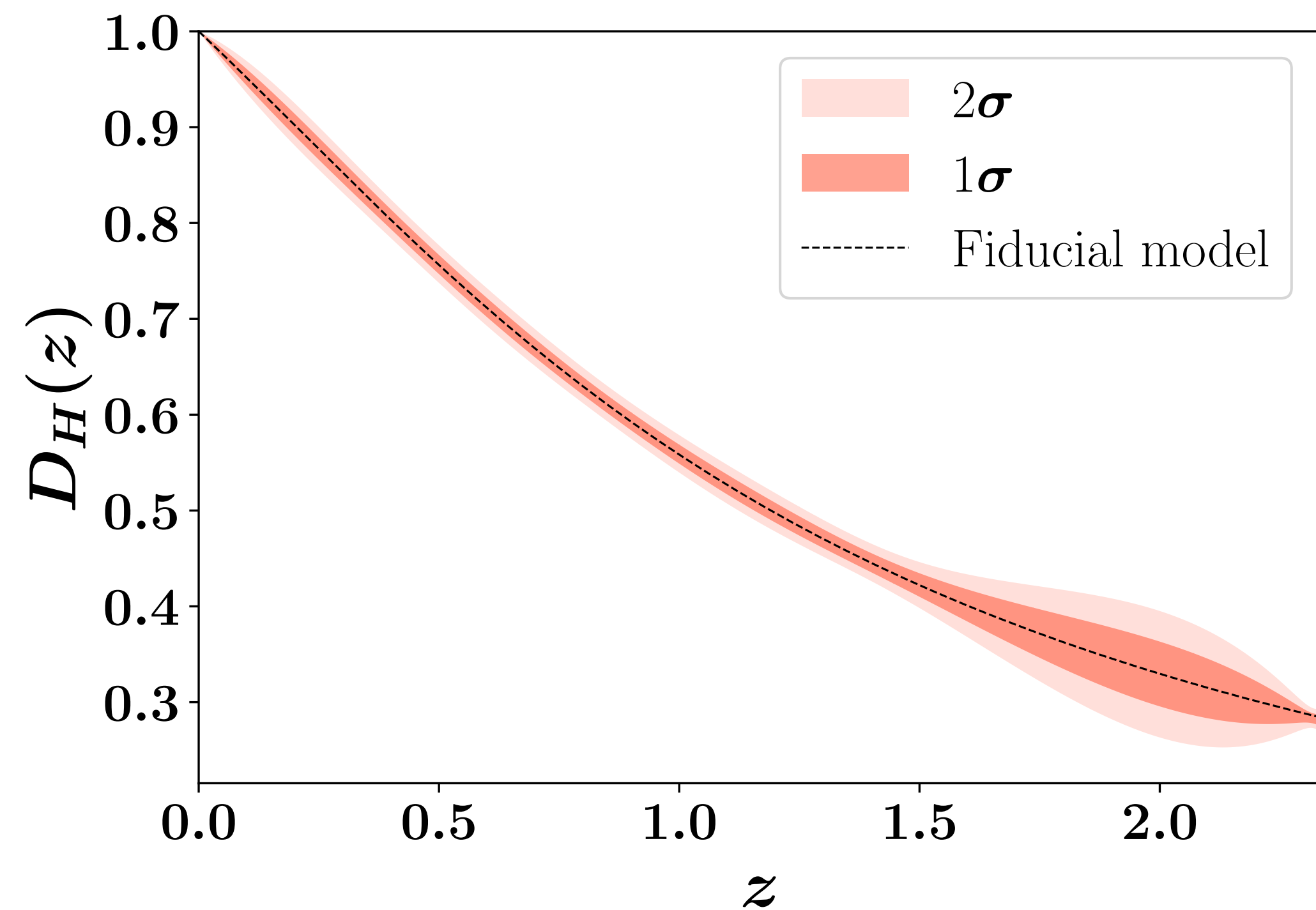
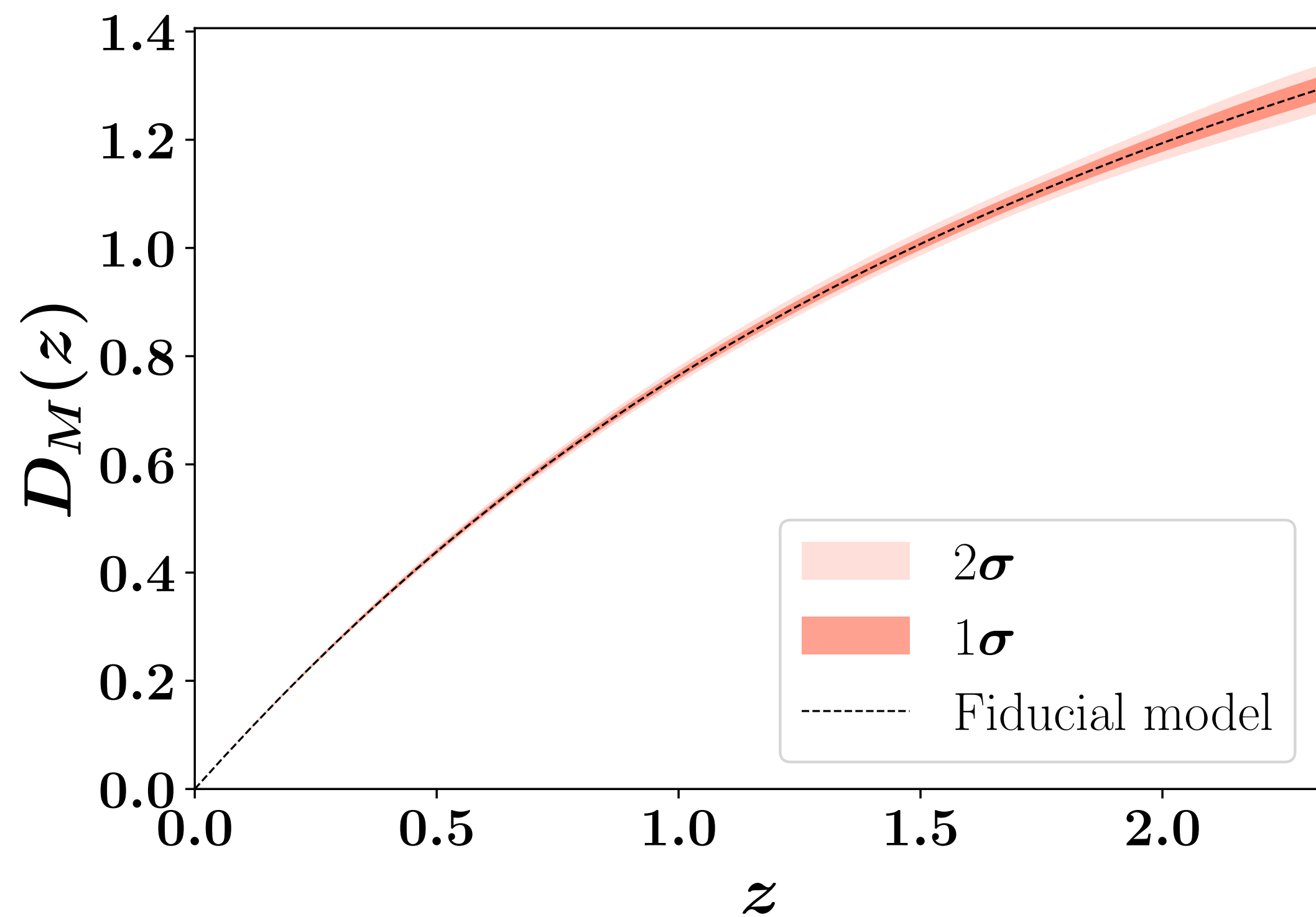
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Coverage Test: Coverage quantifies the fraction of mock realizations for which the fiducial value lies inside the reconstructed 68 % and 95 % credible intervals. The expected coverage fractions are 0.68 and 0.95, respectively.

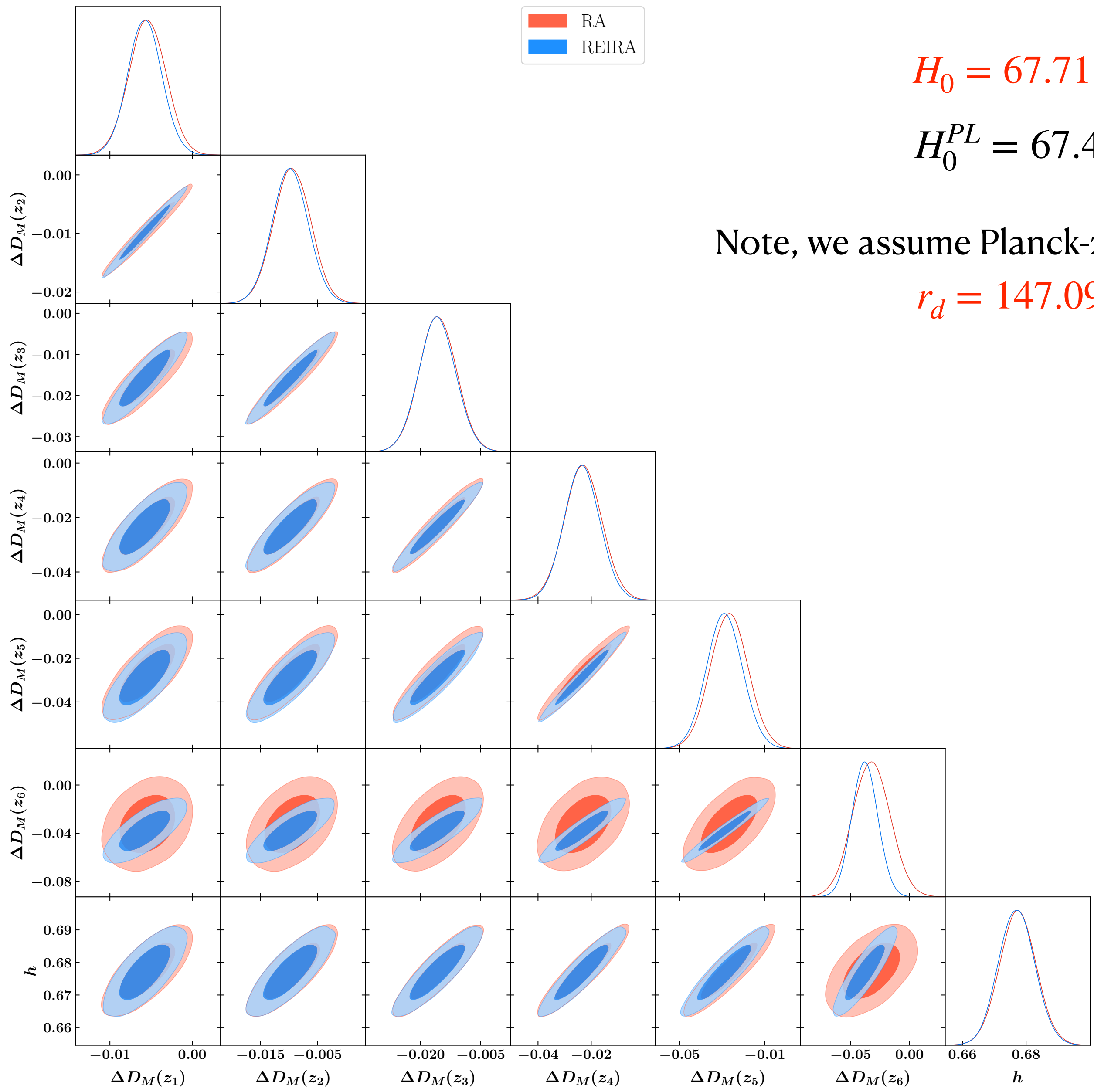
Mock Validation with noise



Mock Validation (With Noise)

Parameter	$(\langle \hat{\theta} \rangle - \theta_{\text{fid}}) / \sigma_{\text{mocks}}$	$\sigma_{\text{mocks}} / \langle \sigma_{\text{post}} \rangle$	Pull Mean	Pull Width	68% Coverage	95% Coverage
$\Delta D_M(z_1)$	0.072	0.948	0.060	0.947	0.672	0.966
$\Delta D_M(z_2)$	0.080	0.938	0.067	0.937	0.704	0.966
$\Delta D_M(z_3)$	0.079	0.943	0.068	0.941	0.700	0.960
$\Delta D_M(z_4)$	0.079	0.953	0.067	0.951	0.708	0.956
$\Delta D_M(z_5)$	0.073	0.953	0.062	0.951	0.718	0.960
$\Delta D_M(z_6)$	0.014	0.988	0.009	0.989	0.670	0.954
h	0.015	0.941	0.006	0.939	0.696	0.970

Reconstruction Results



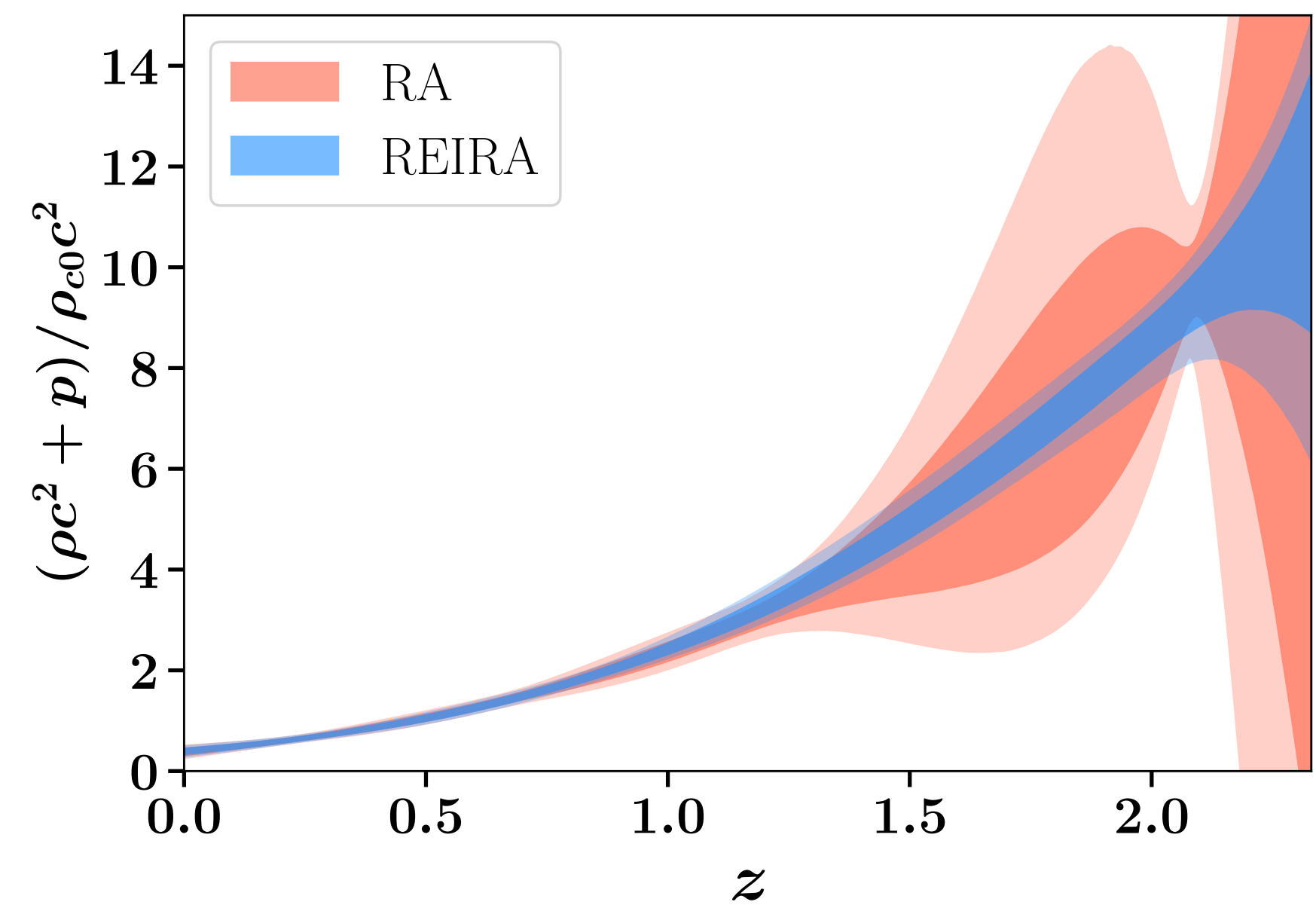
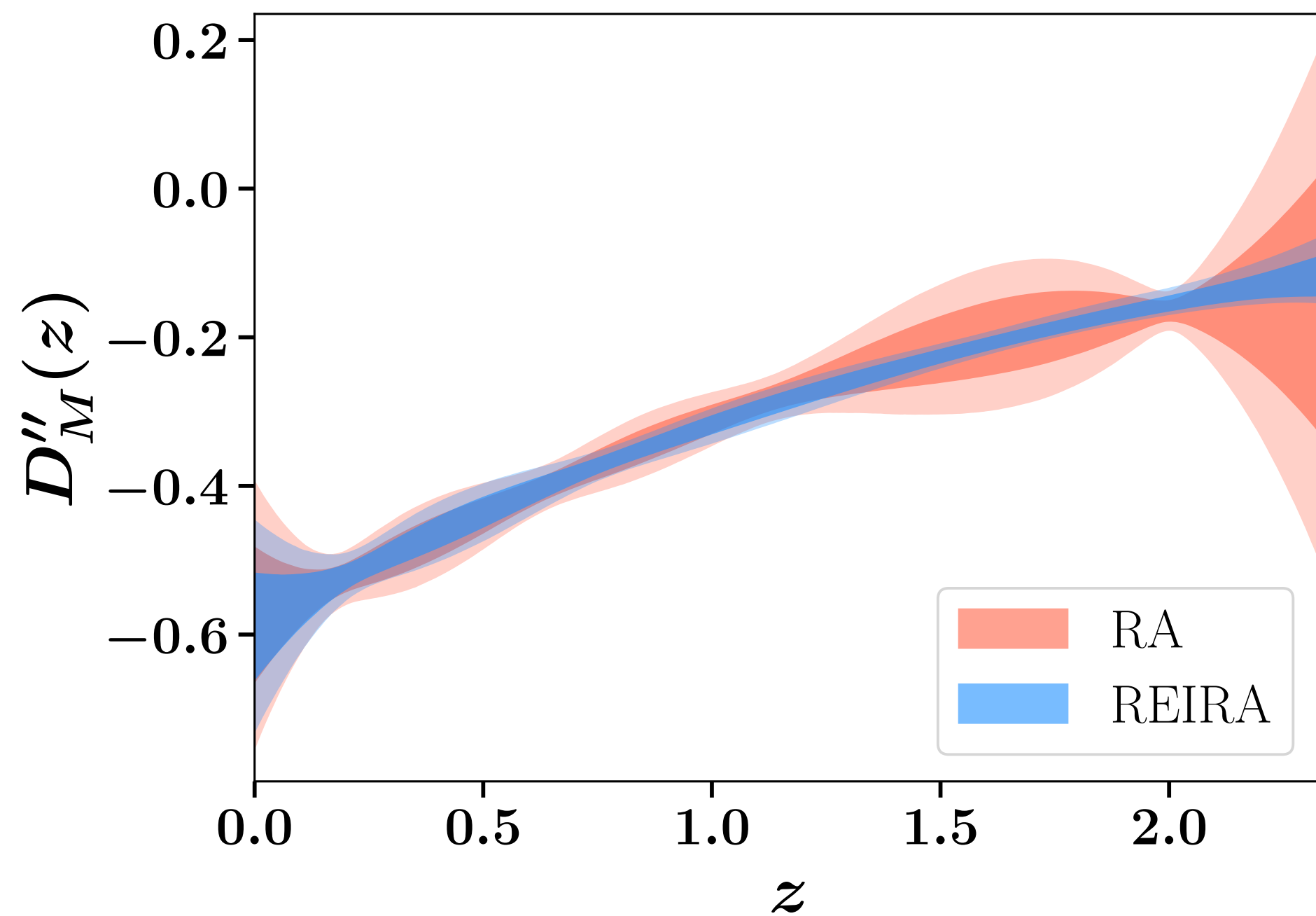
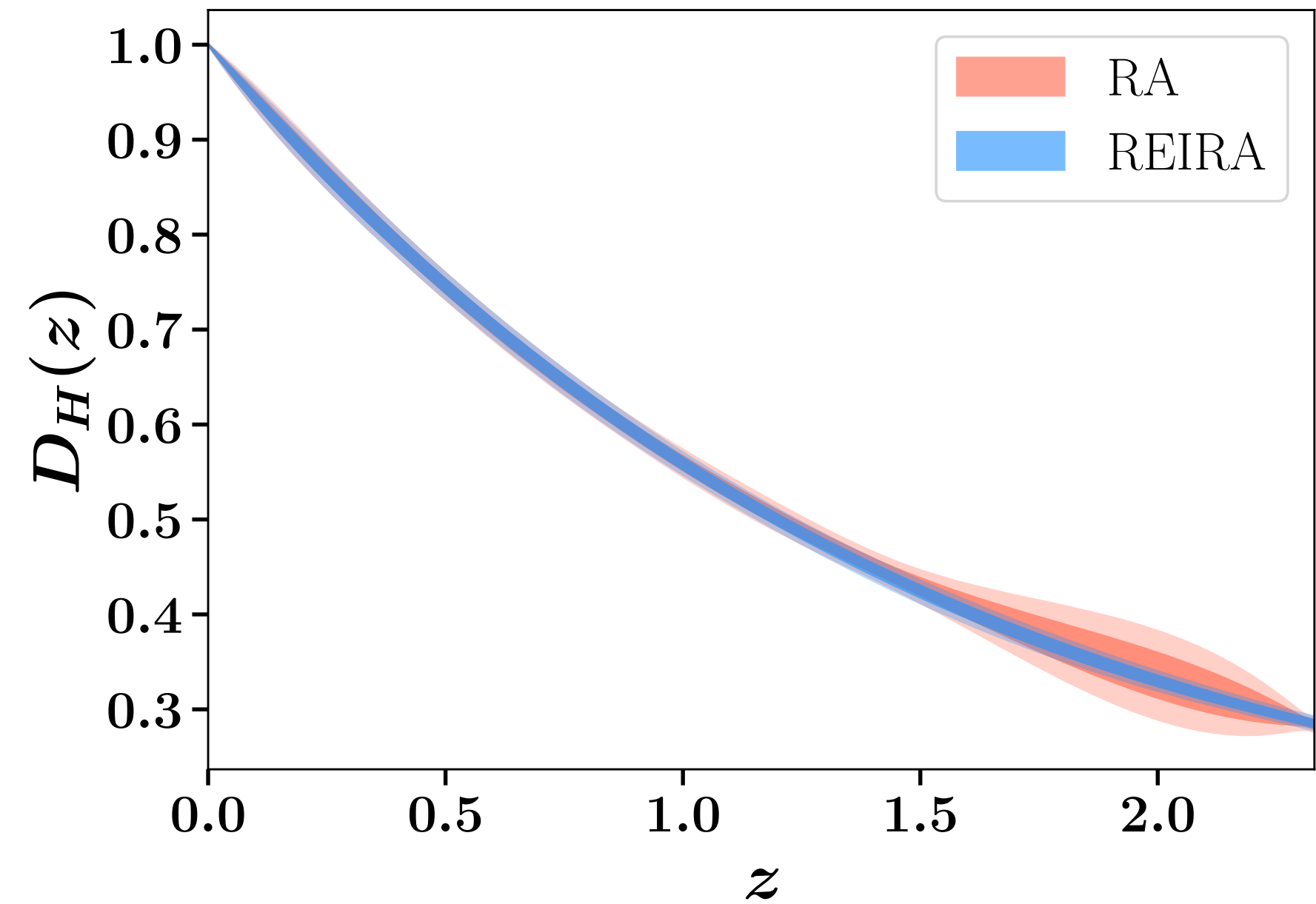
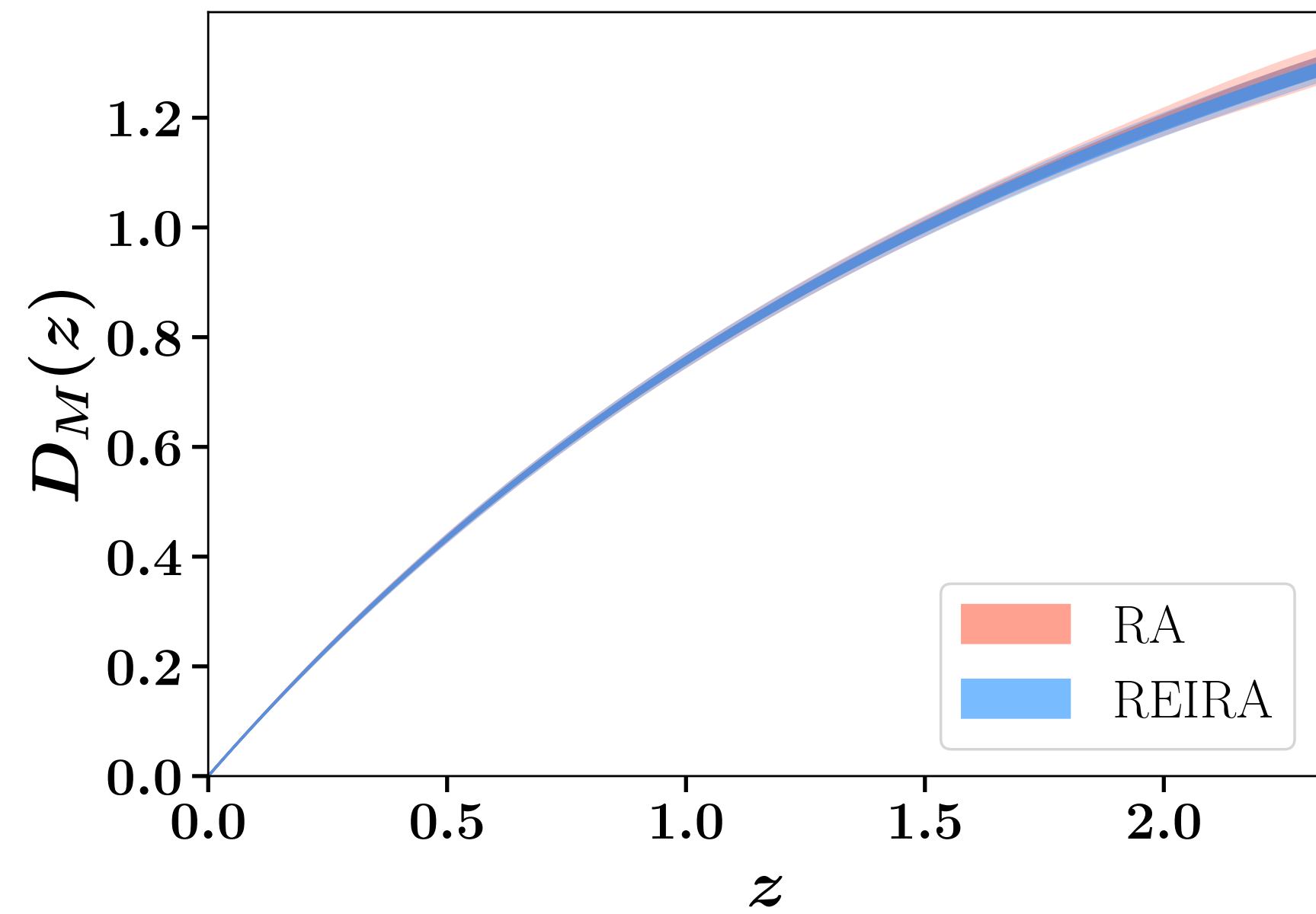
$H_0 = 67.71 \pm 0.57 \text{ km/s/Mpc}$

$H_0^{PL} = 67.4 \pm 0.5 \text{ km/s/Mpc}$

Note, we assume Planck-2018 Baseline Λ CDM prior

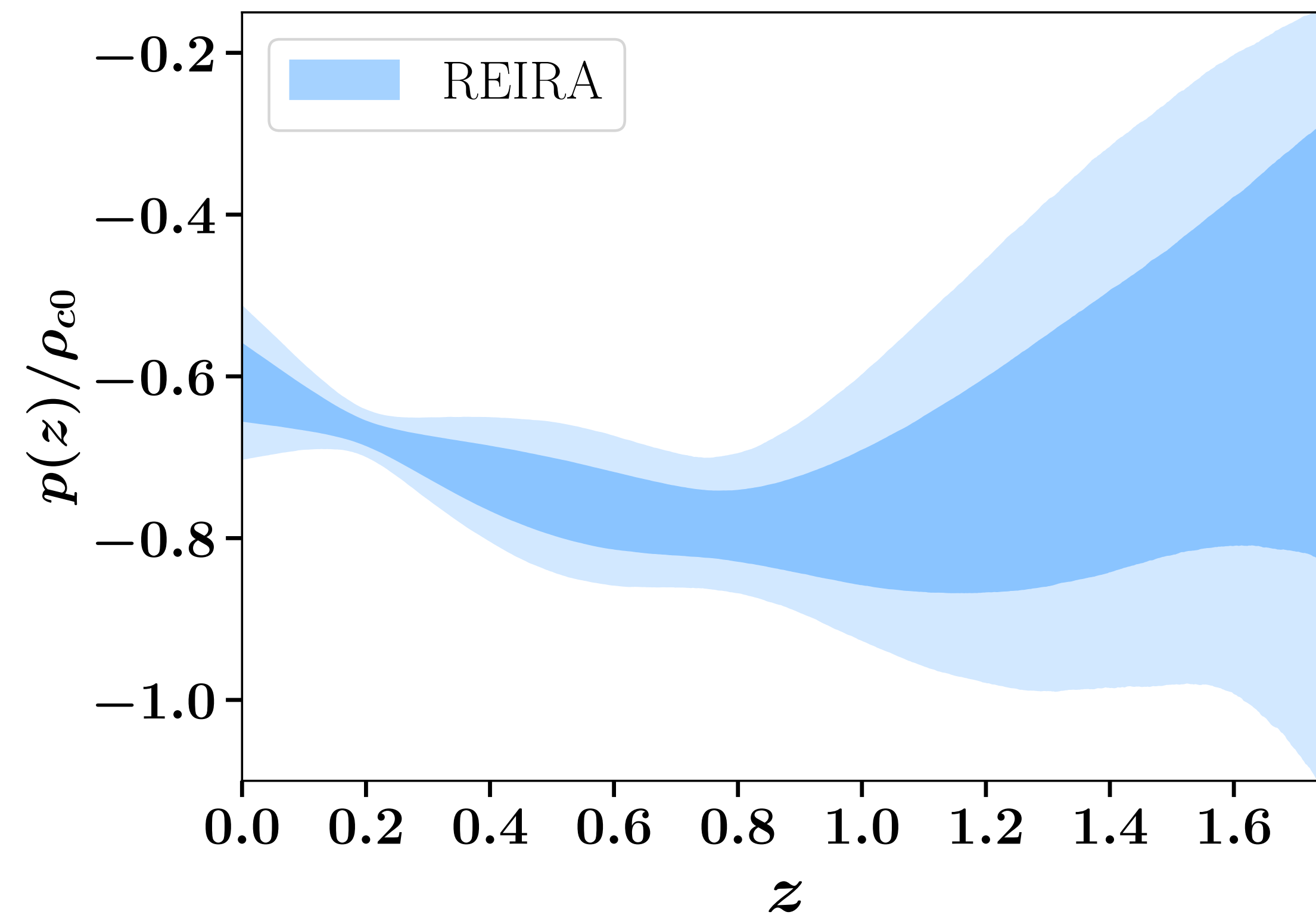
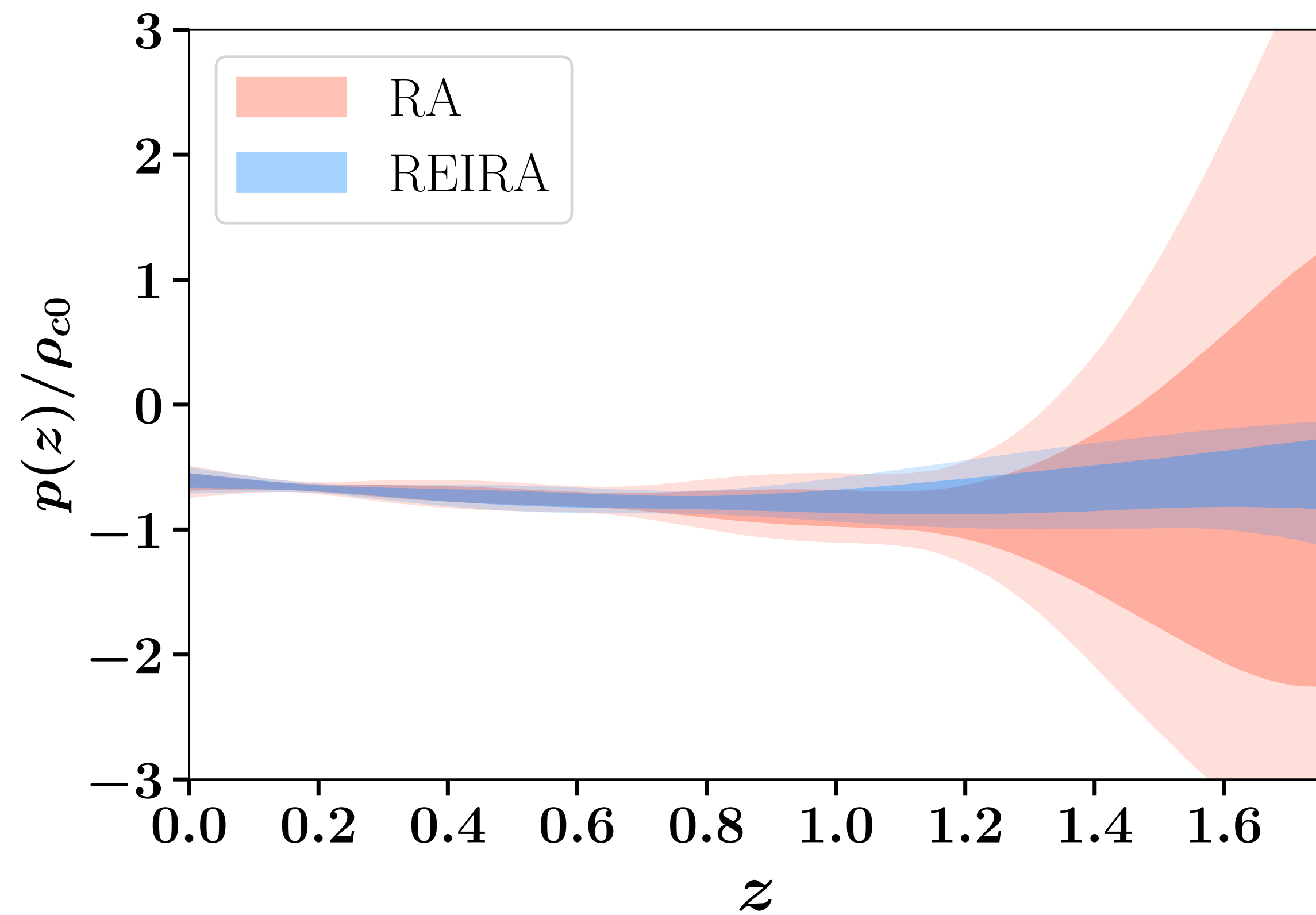
$r_d = 147.09 \pm 0.27 \text{ Mpc}$

Reconstruction Results

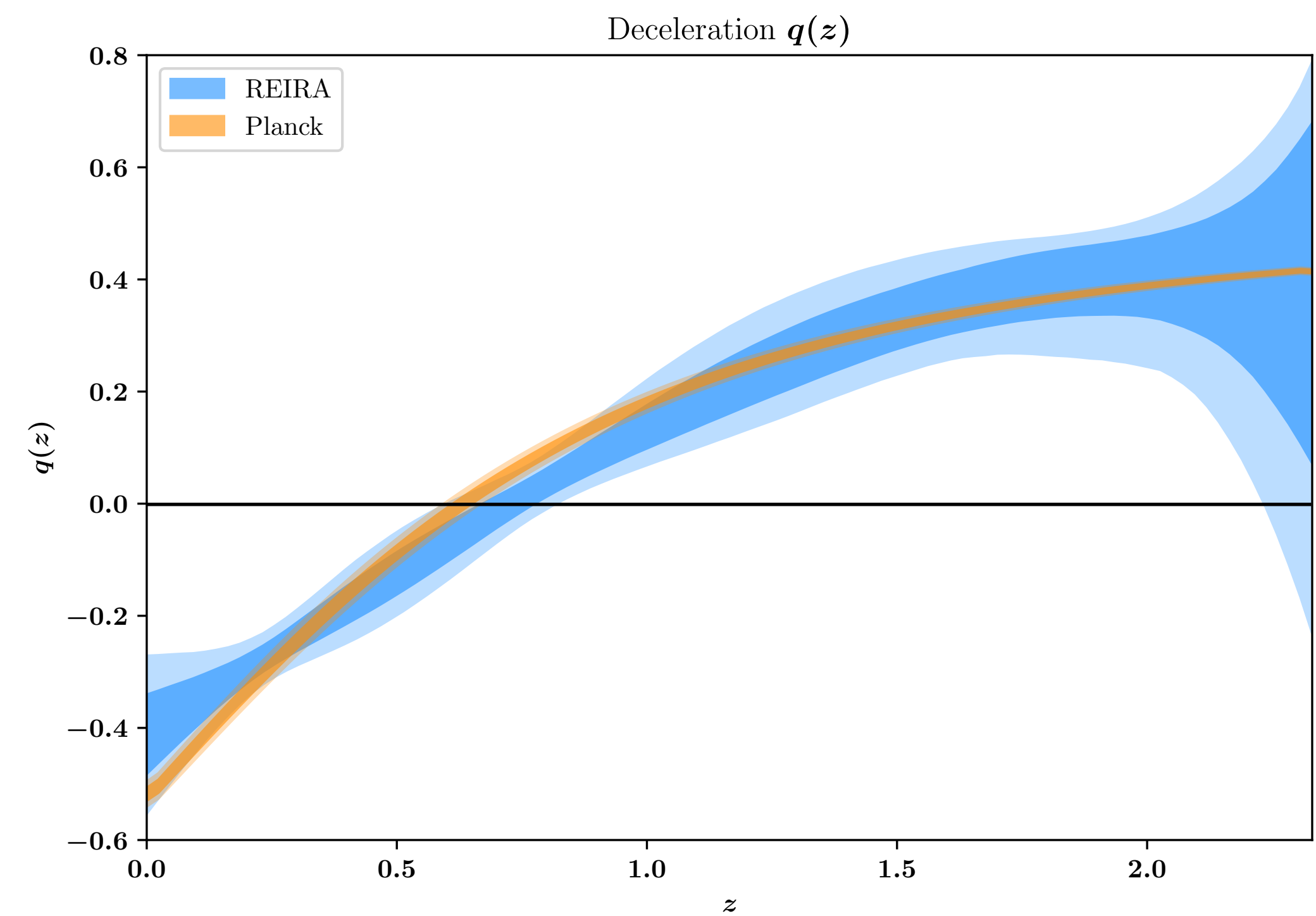
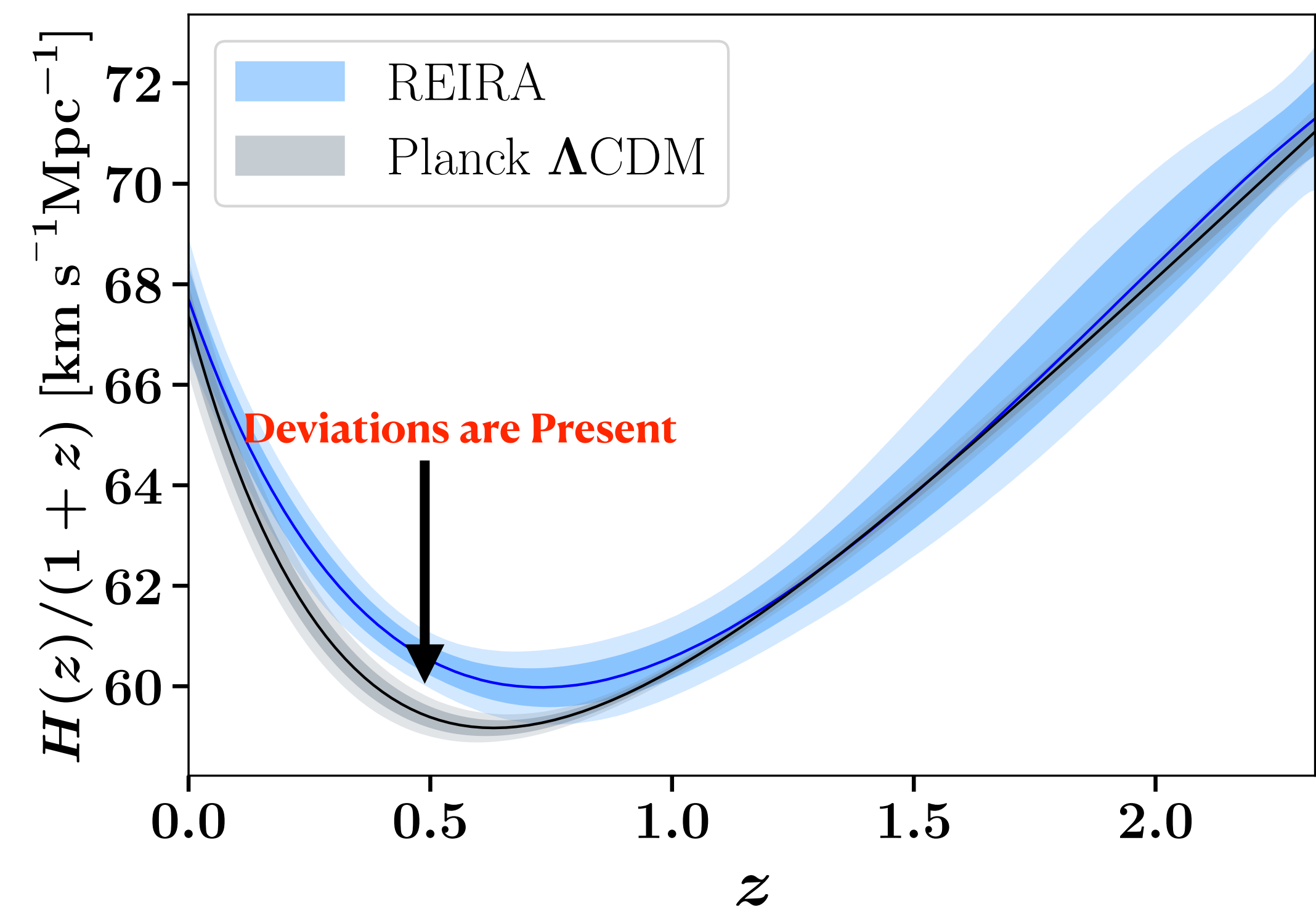


No Violation of NEC, $R^{\mu\nu}k_\mu k_\nu > 0$

Reconstruction Results



Expansion Rate $H(z)$ and Acceleration Rate $q(z)$

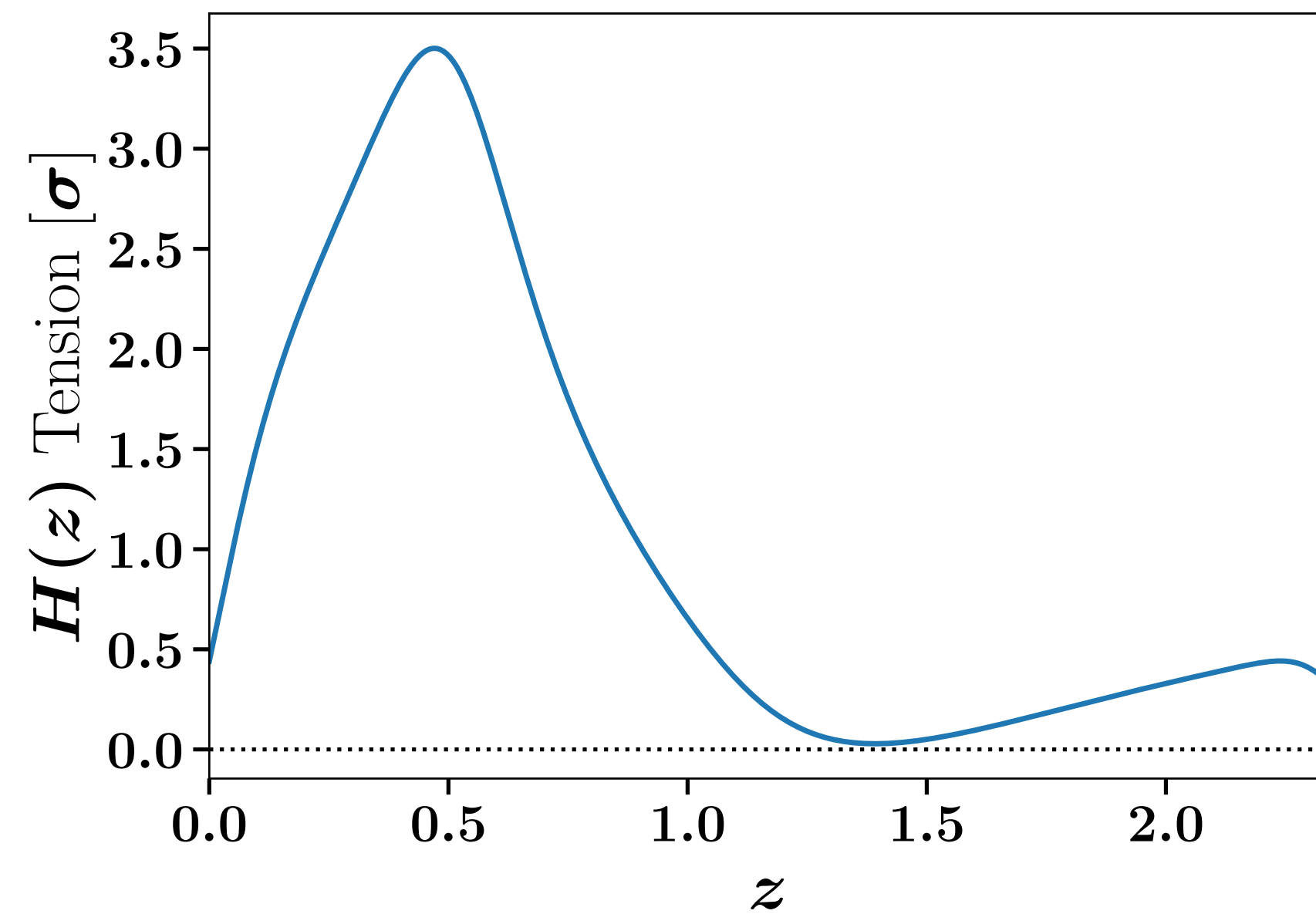


Quantify The Deviation in $H(z)$

We define the Tension as between reconstructed $H(z)$ and $H_{P18}(z)$ as : $T \equiv \frac{H_{\text{REIRA}} - H_{\text{P18}}}{\sqrt{\sigma_{\text{REIRA}}^2 + \sigma_{\text{P18}}^2}}$ and calculate it at every z .

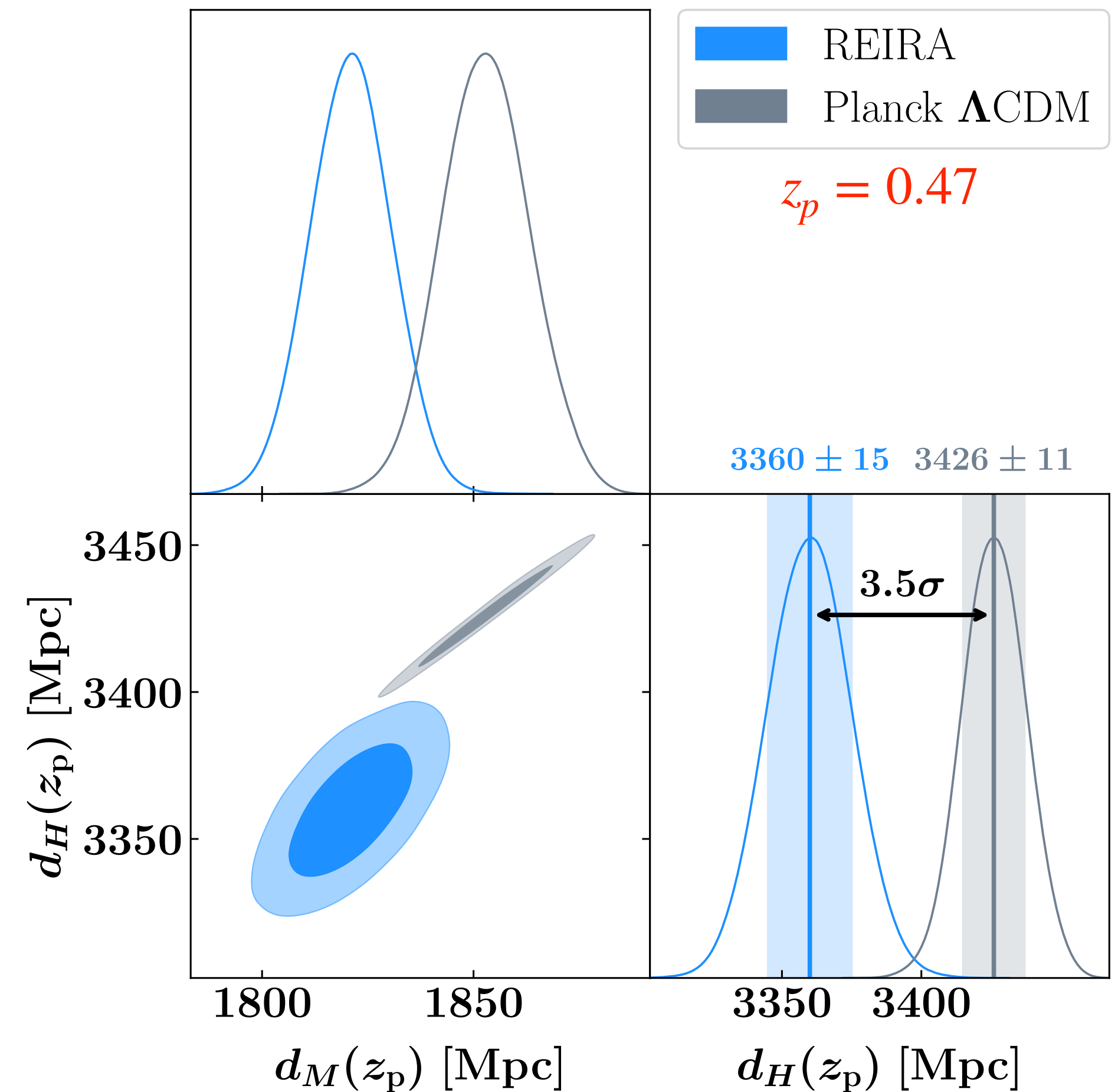
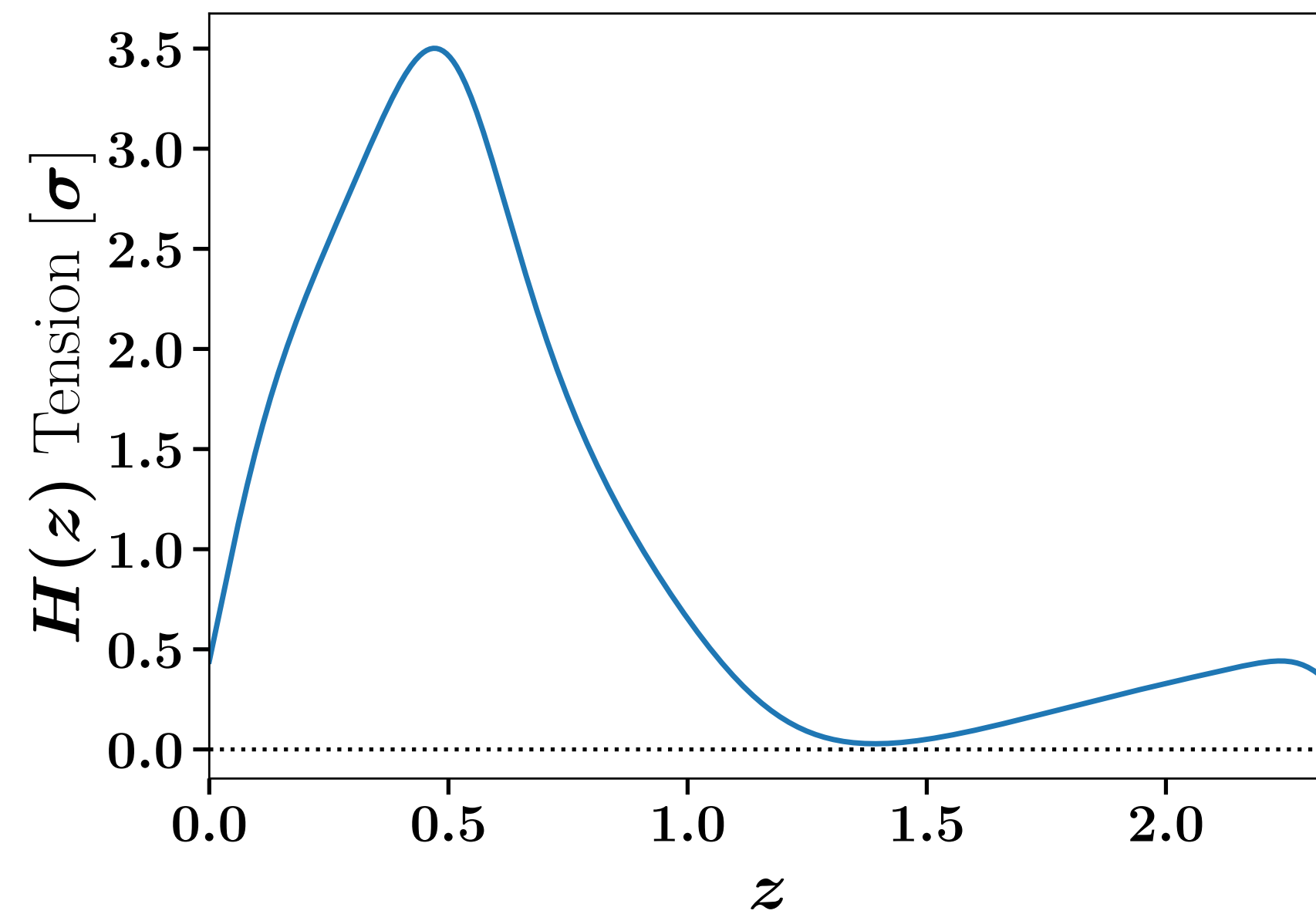
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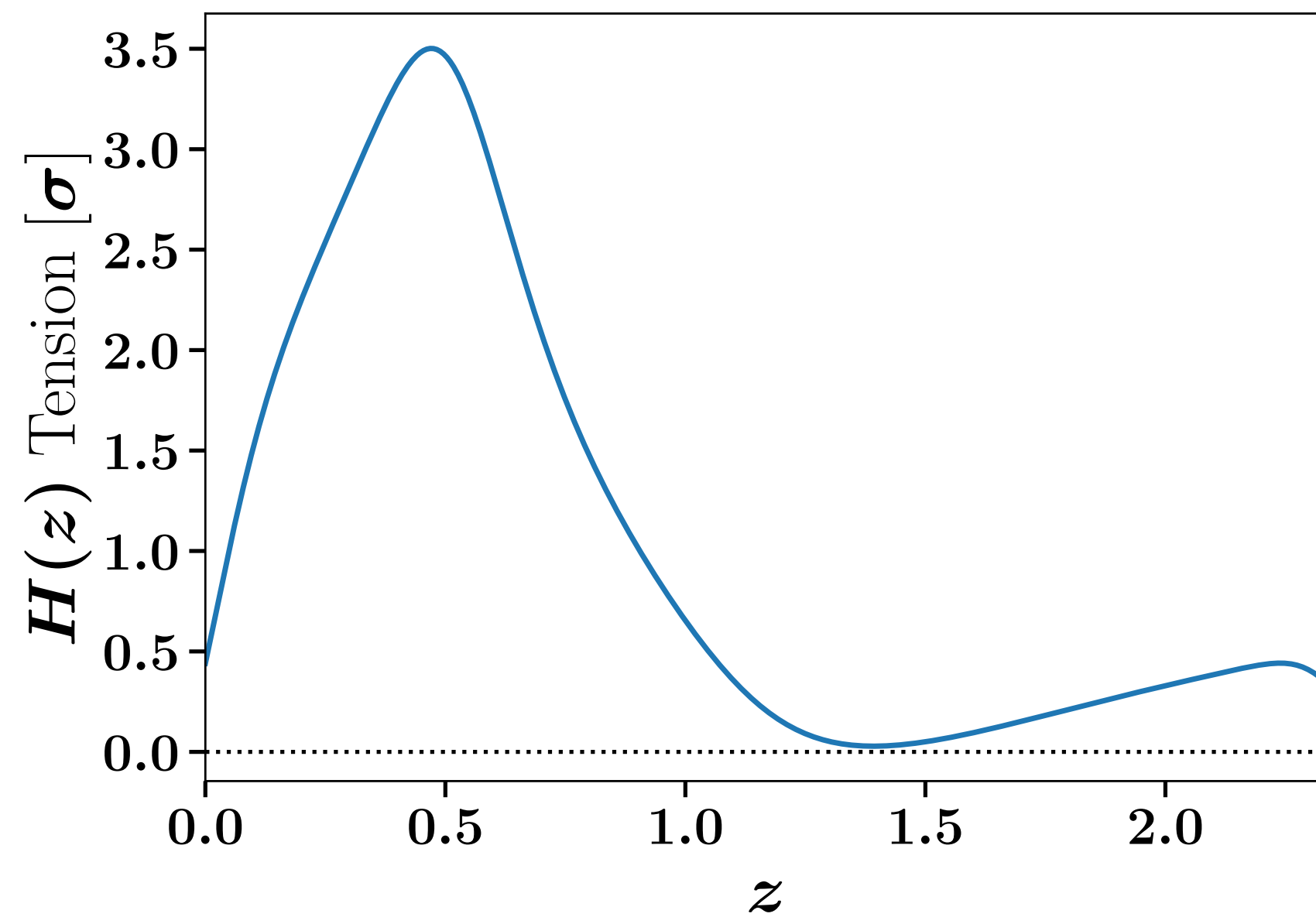
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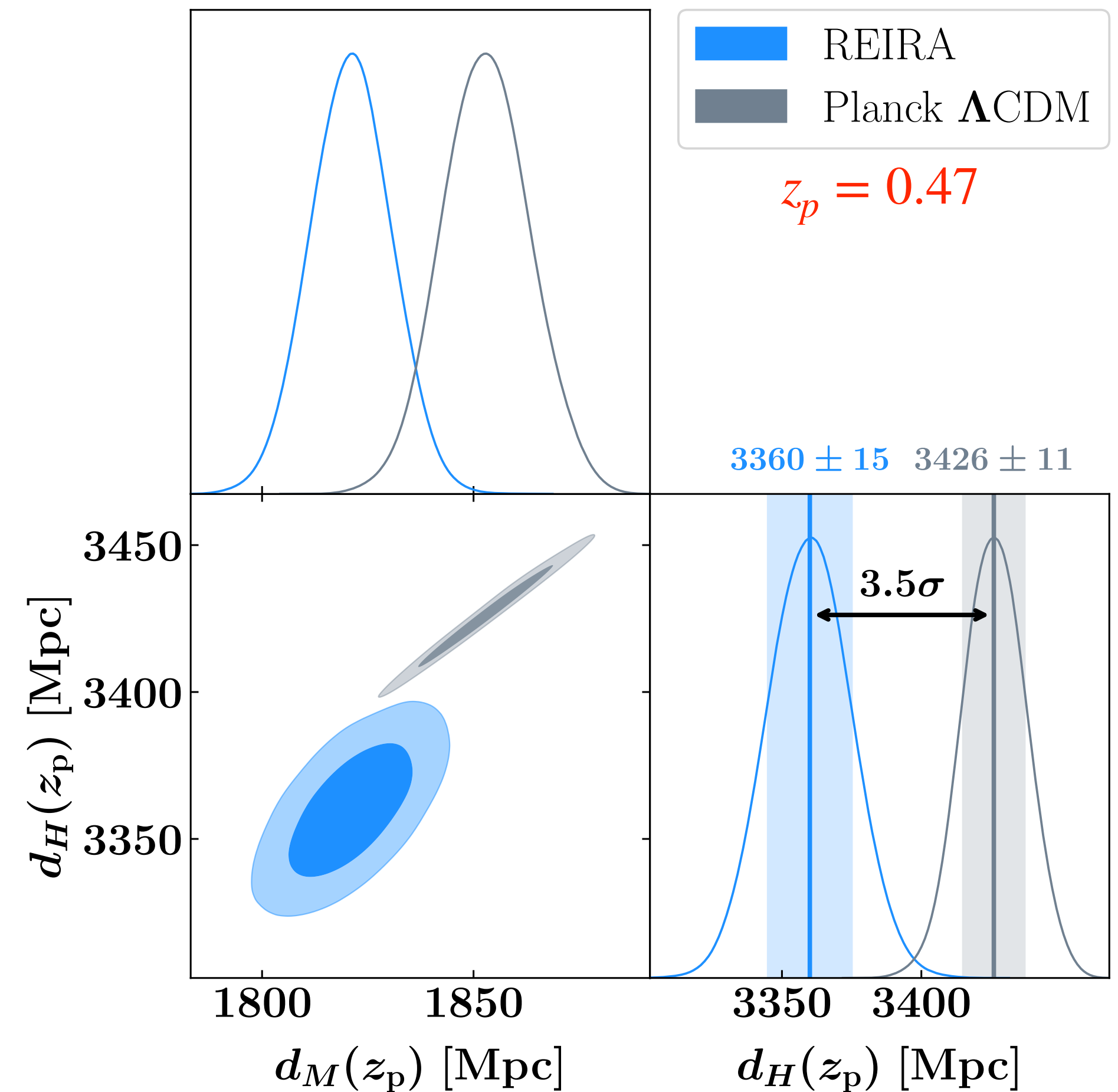


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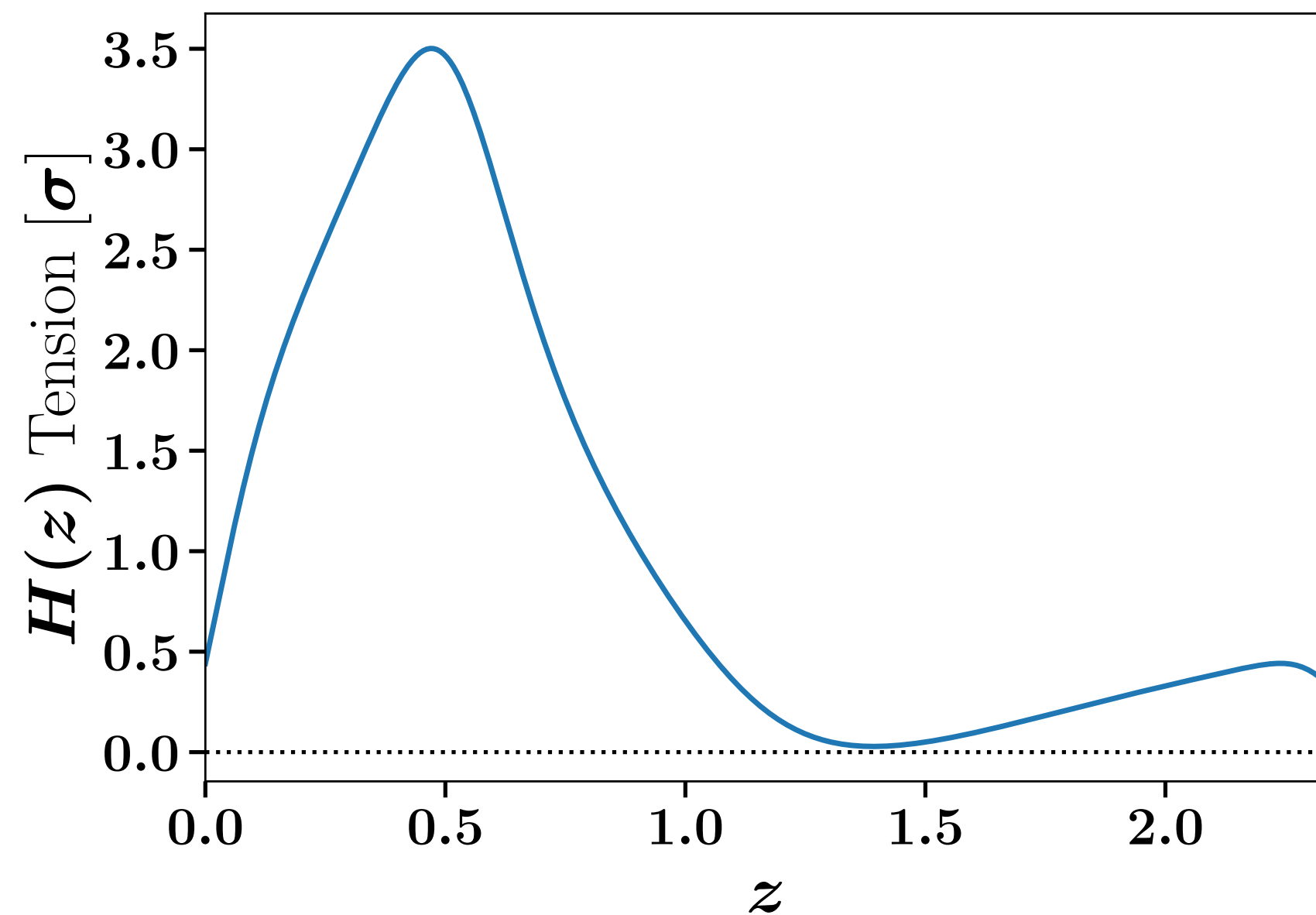


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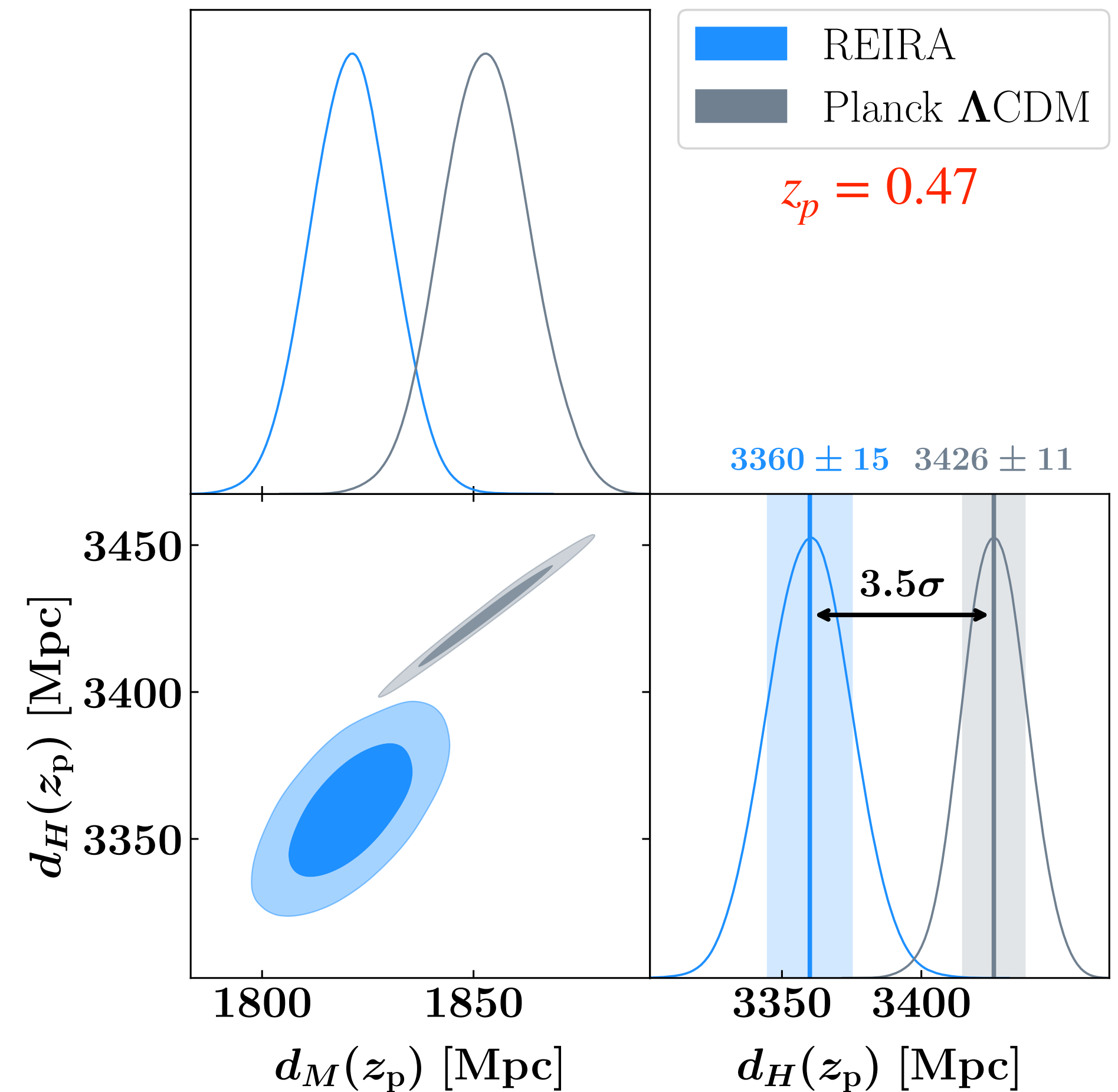
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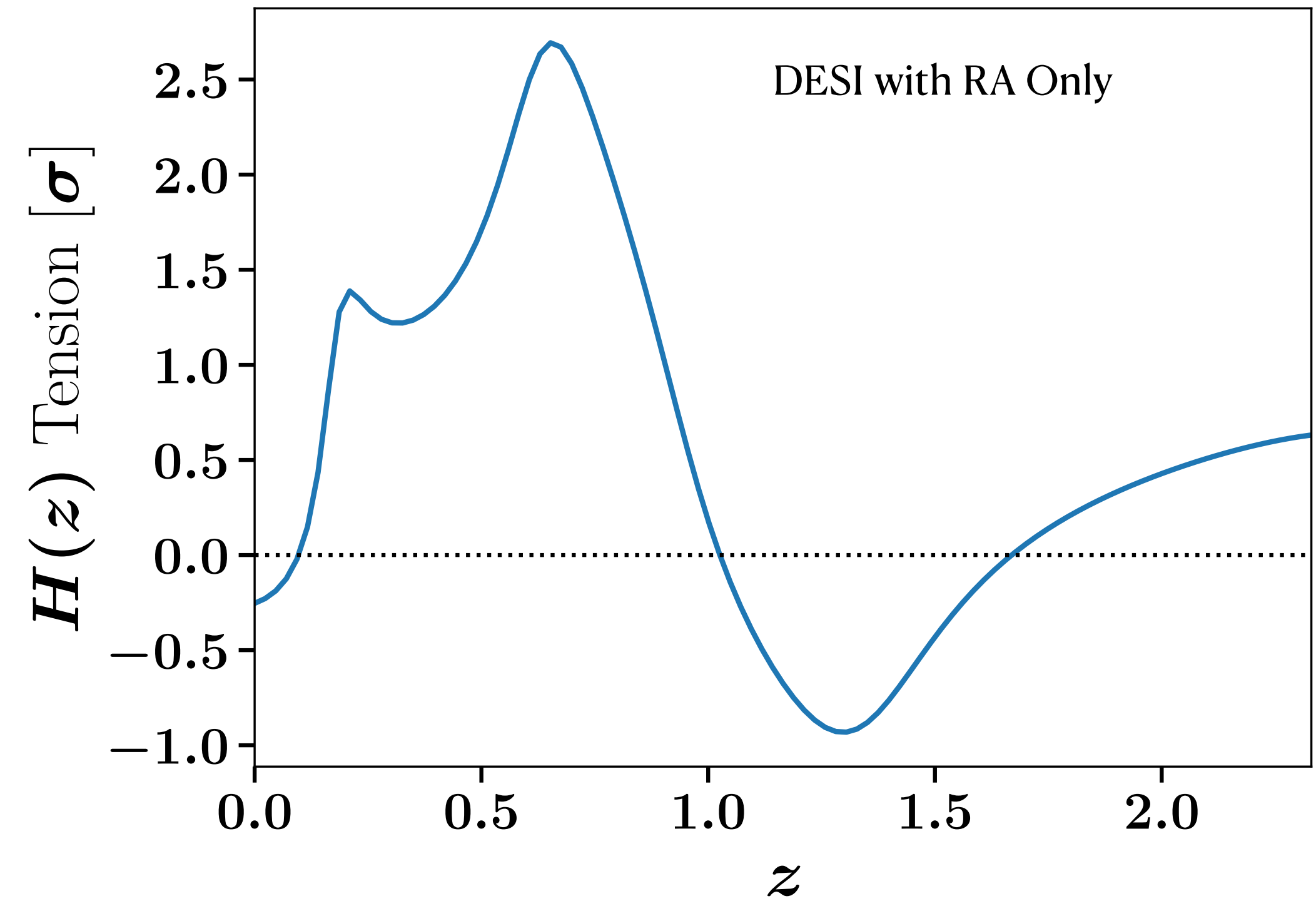
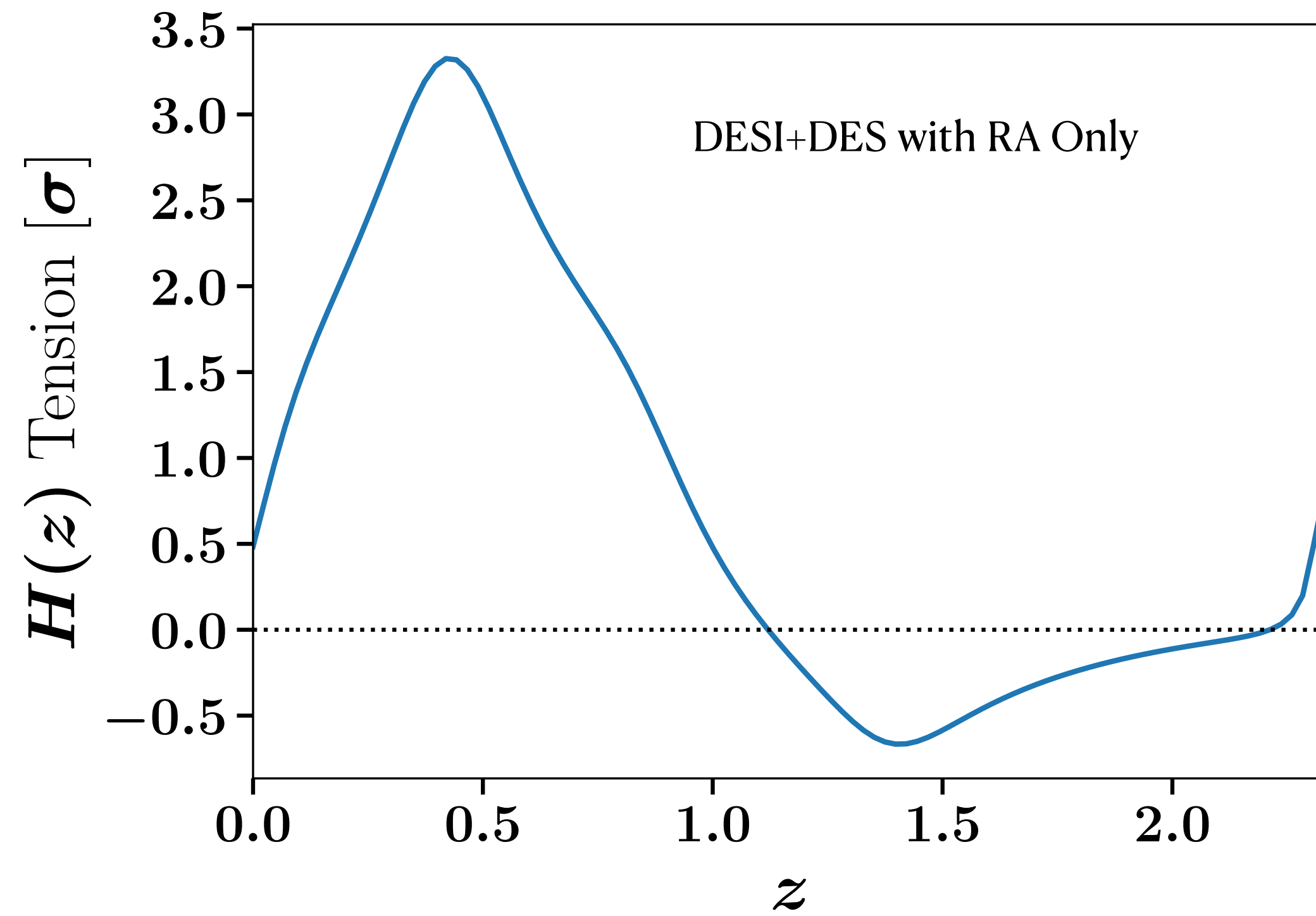


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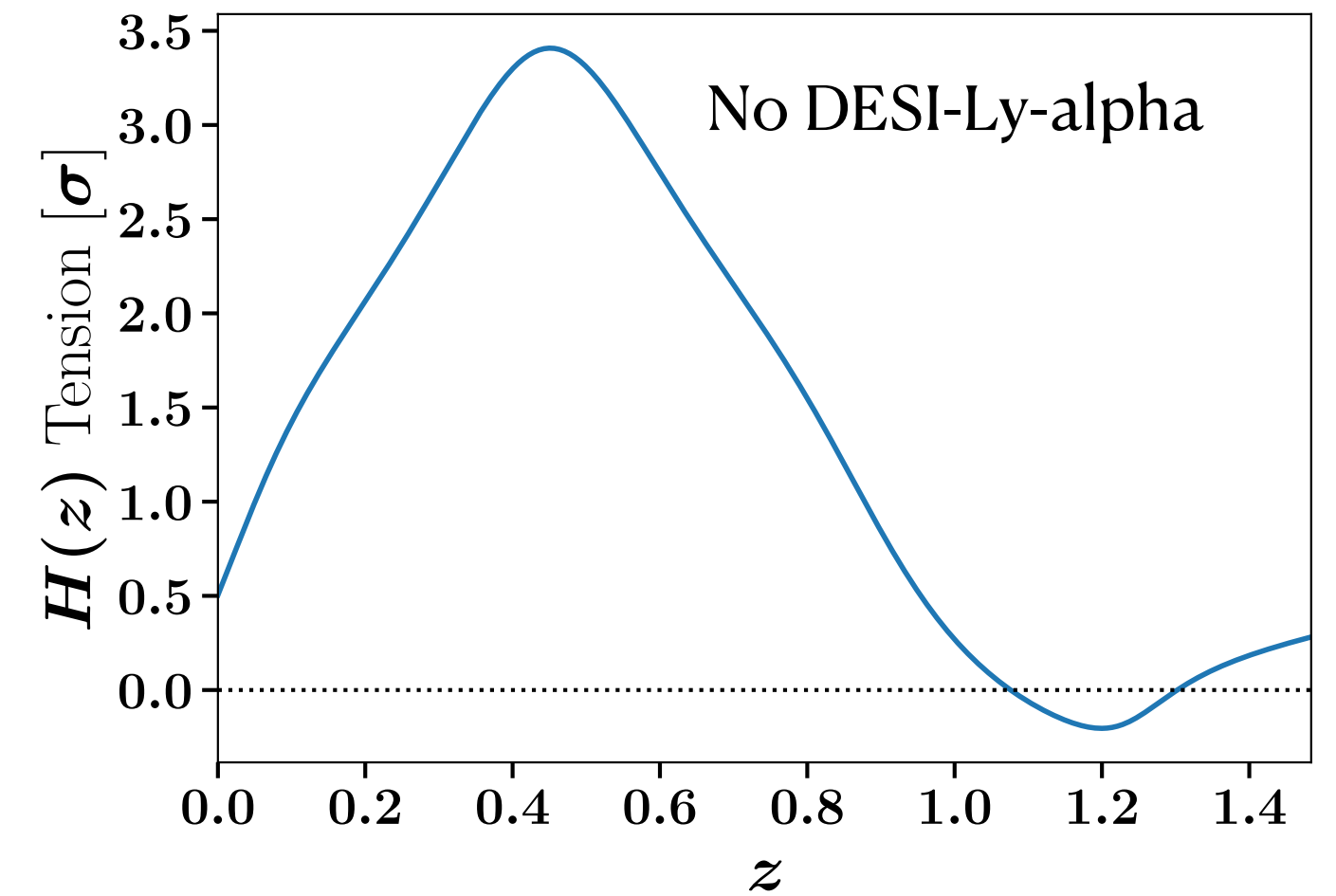
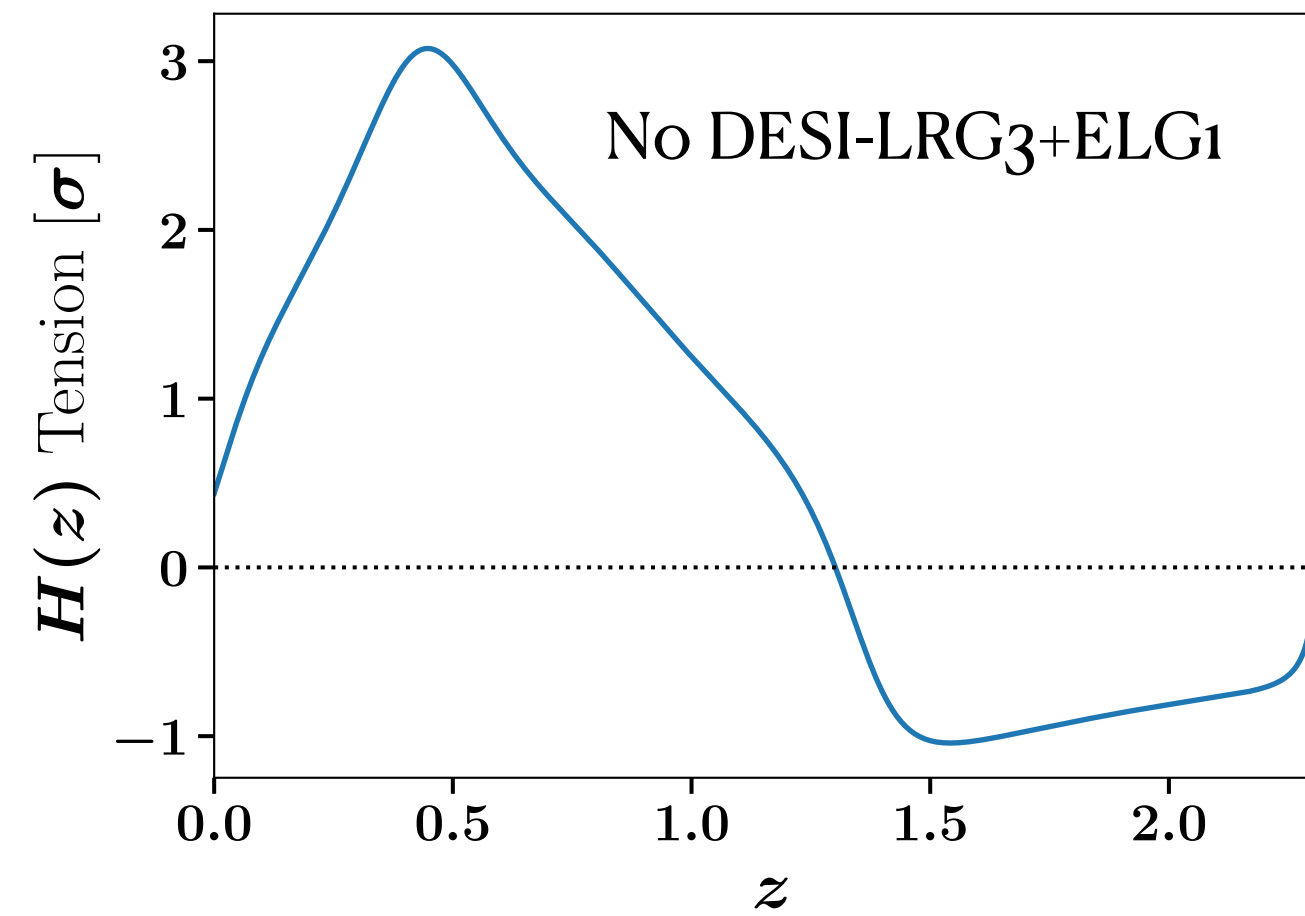
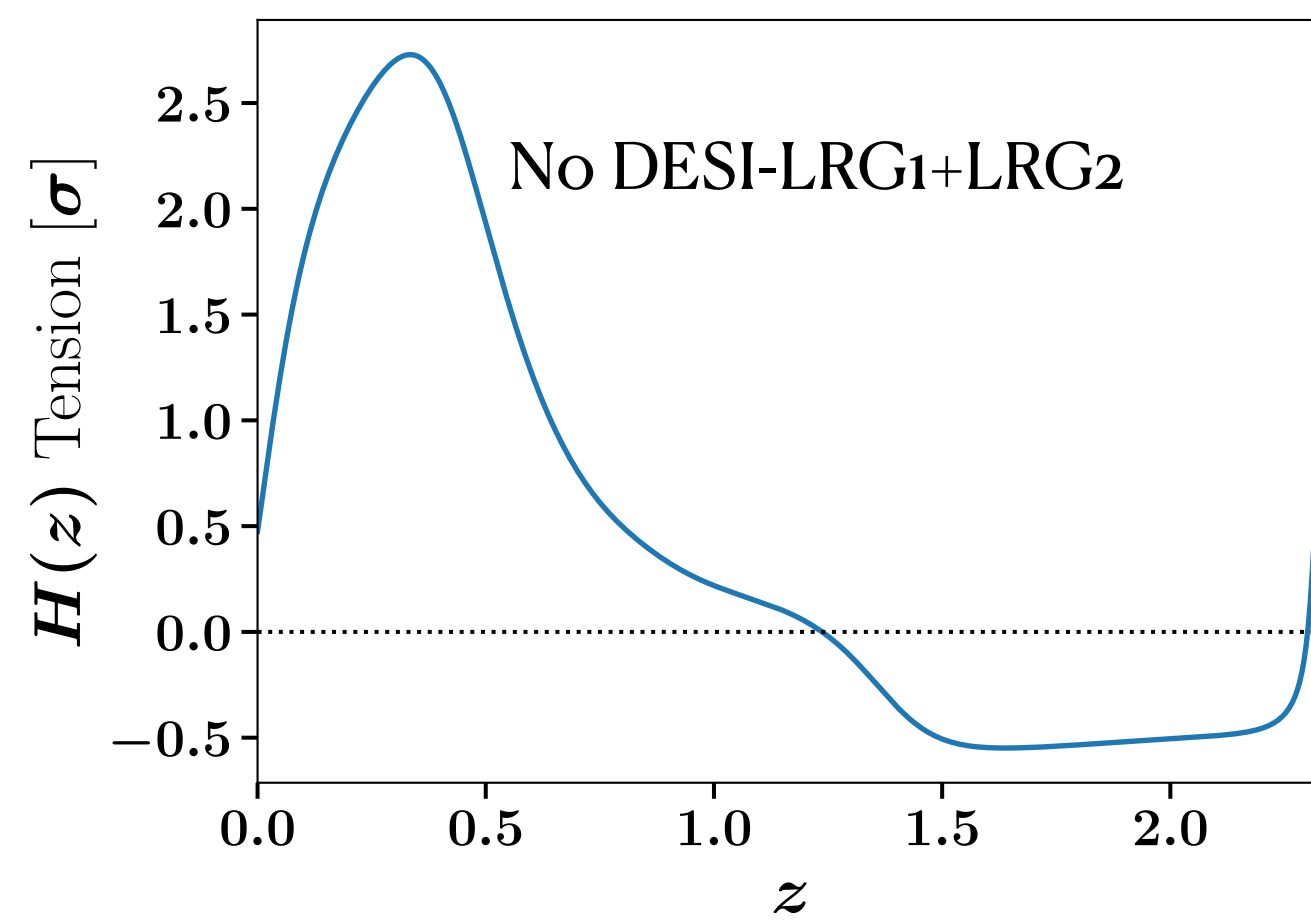
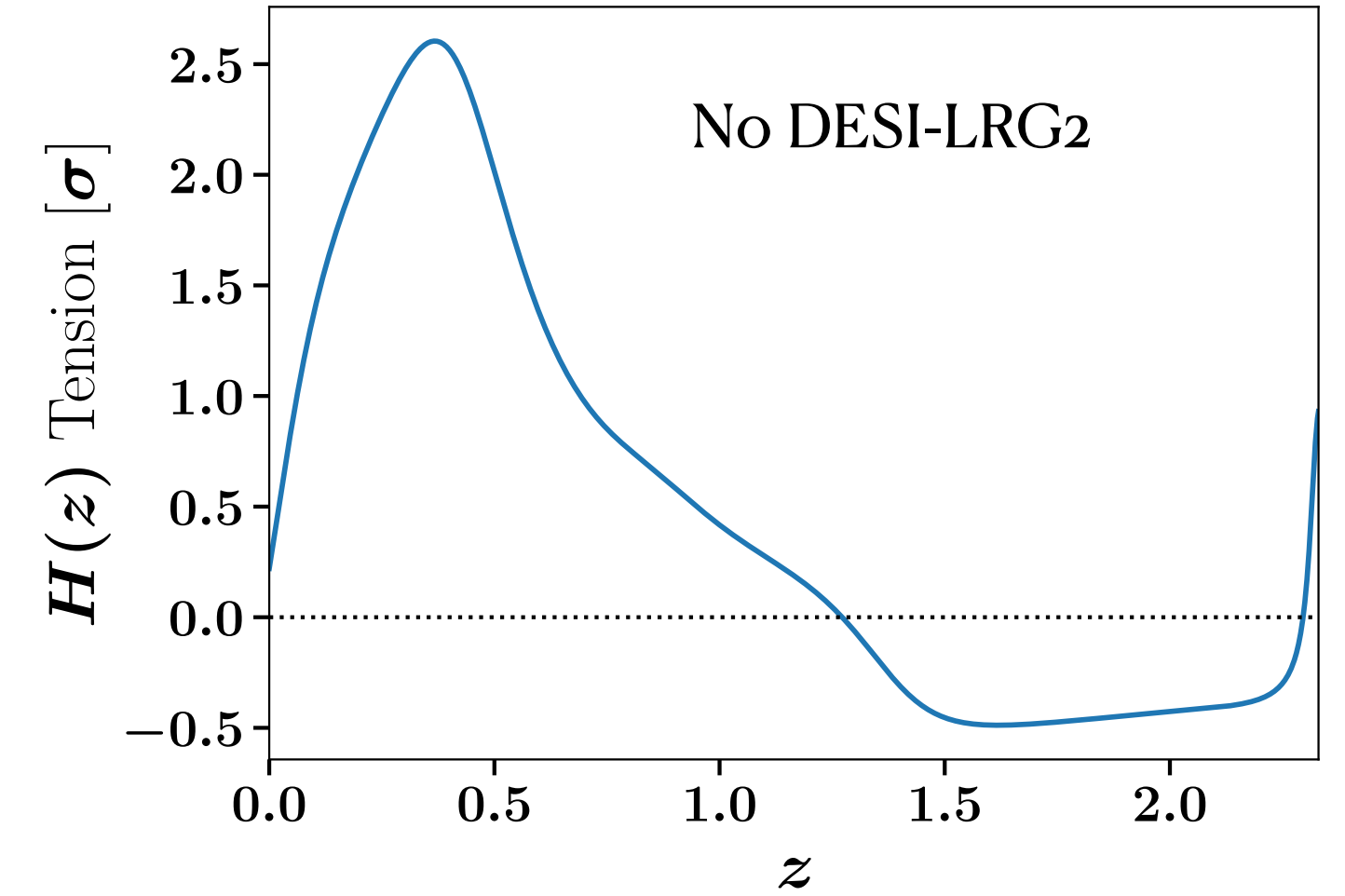
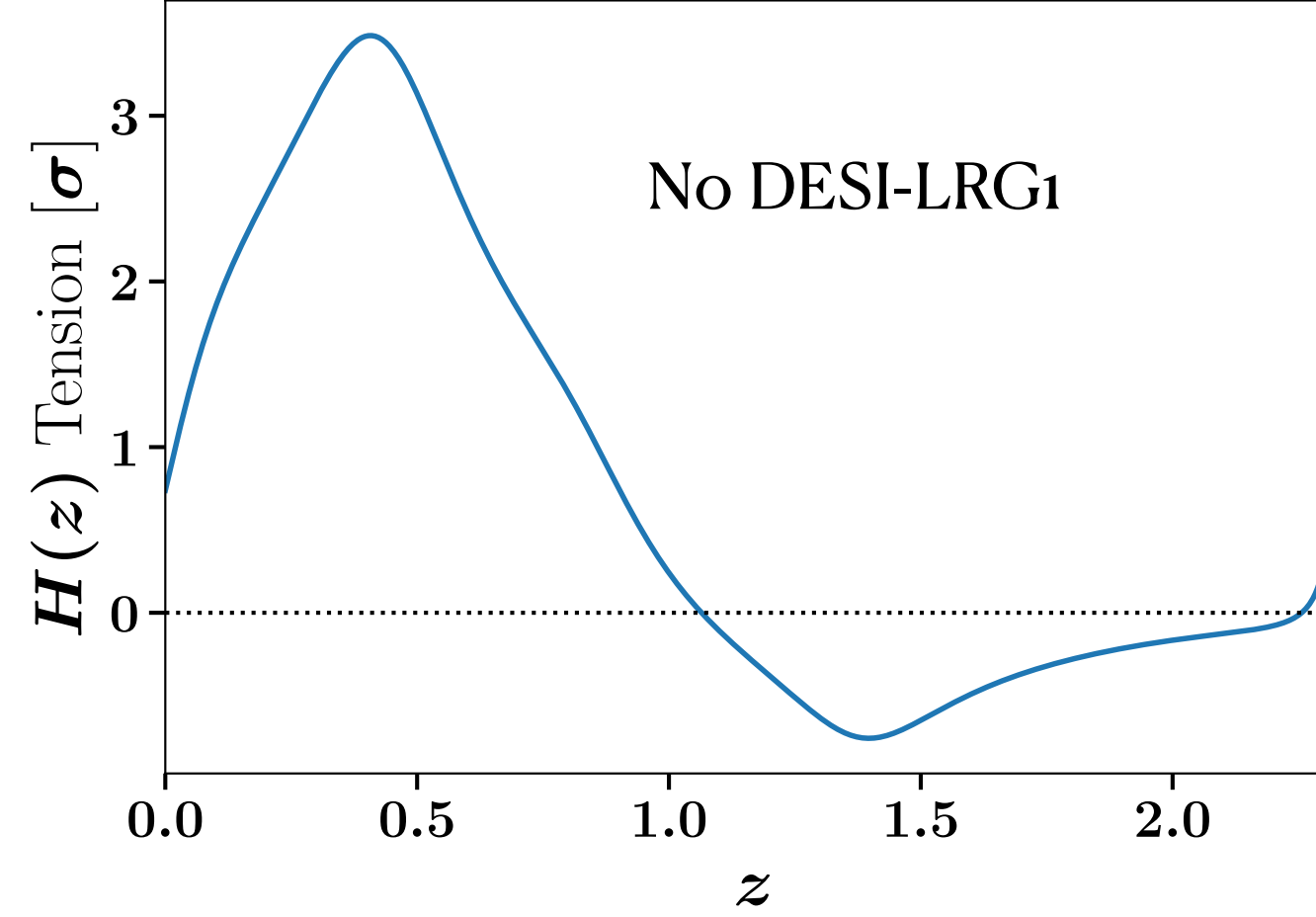
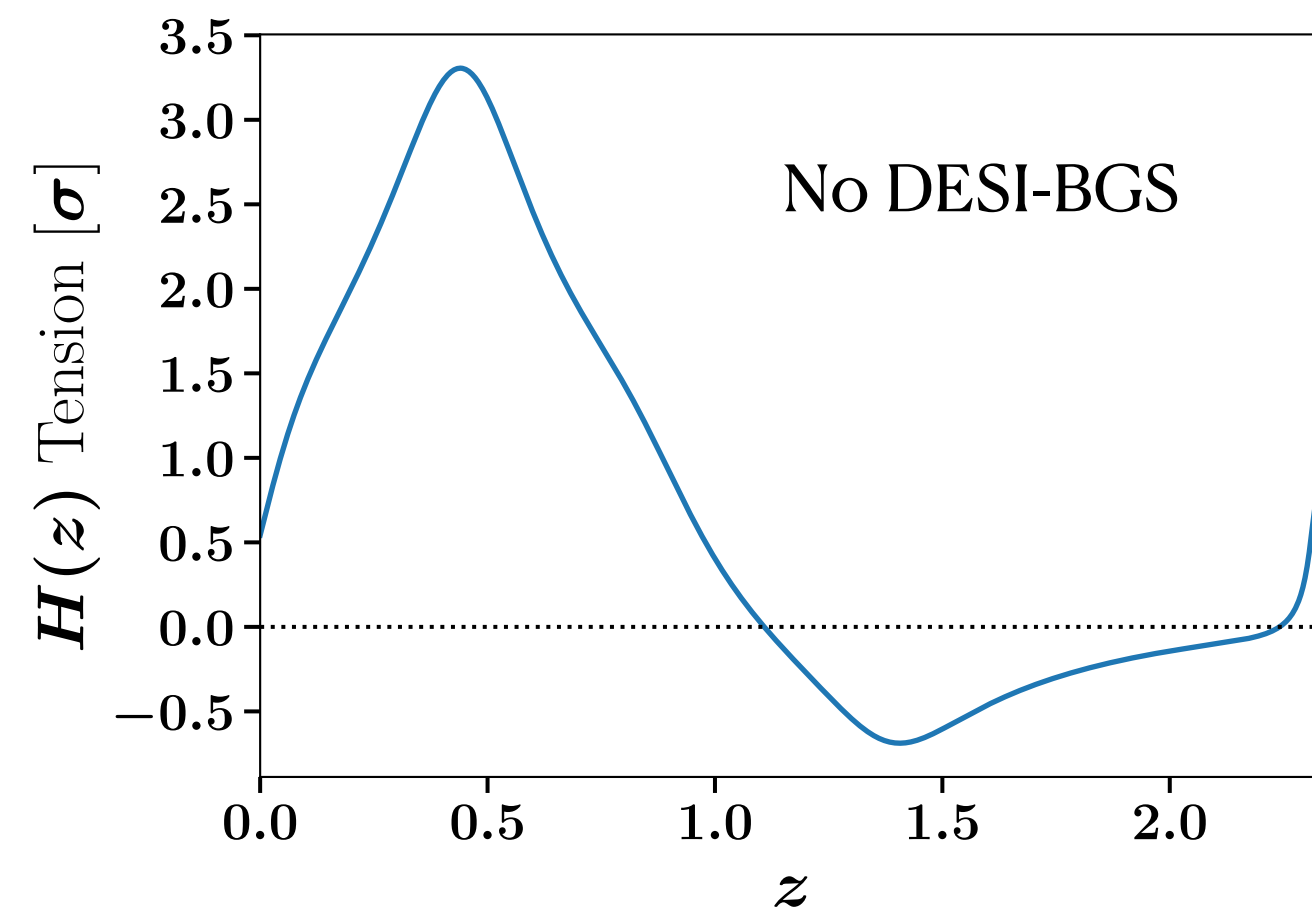
This is independent of any dark energy model or Ω_{m0} or H_0 -Tension.



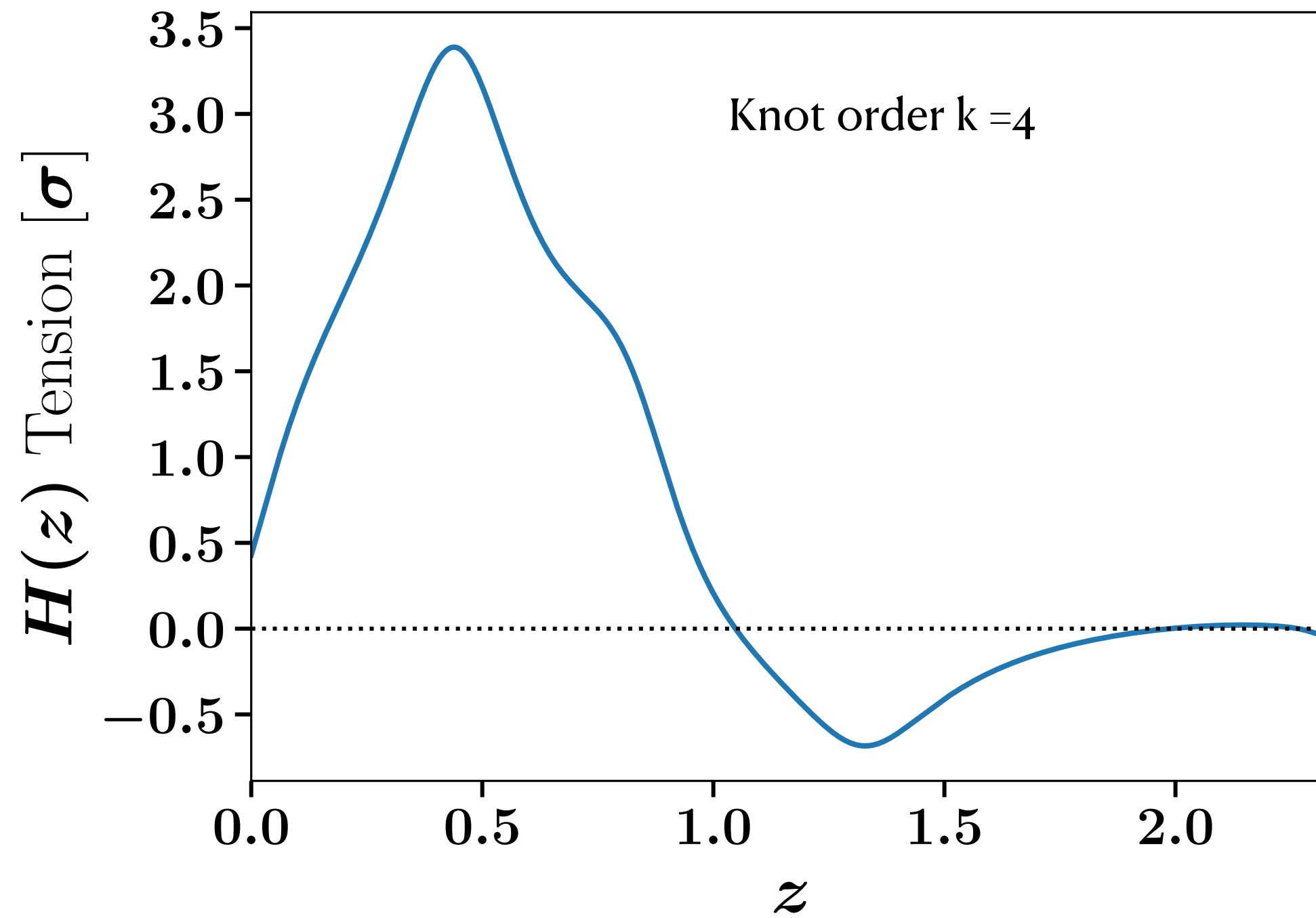
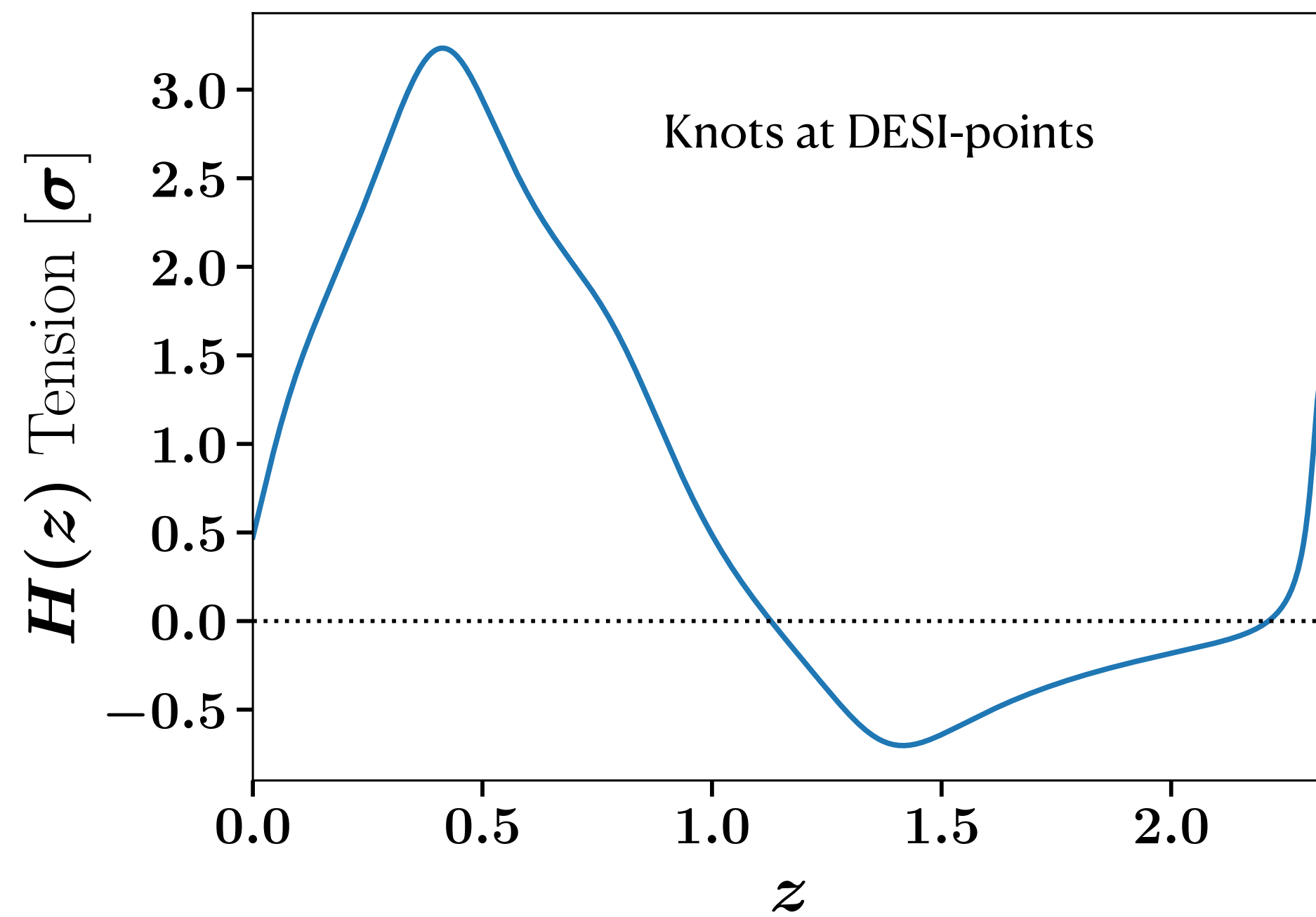
Robustness of this Localized Deviation



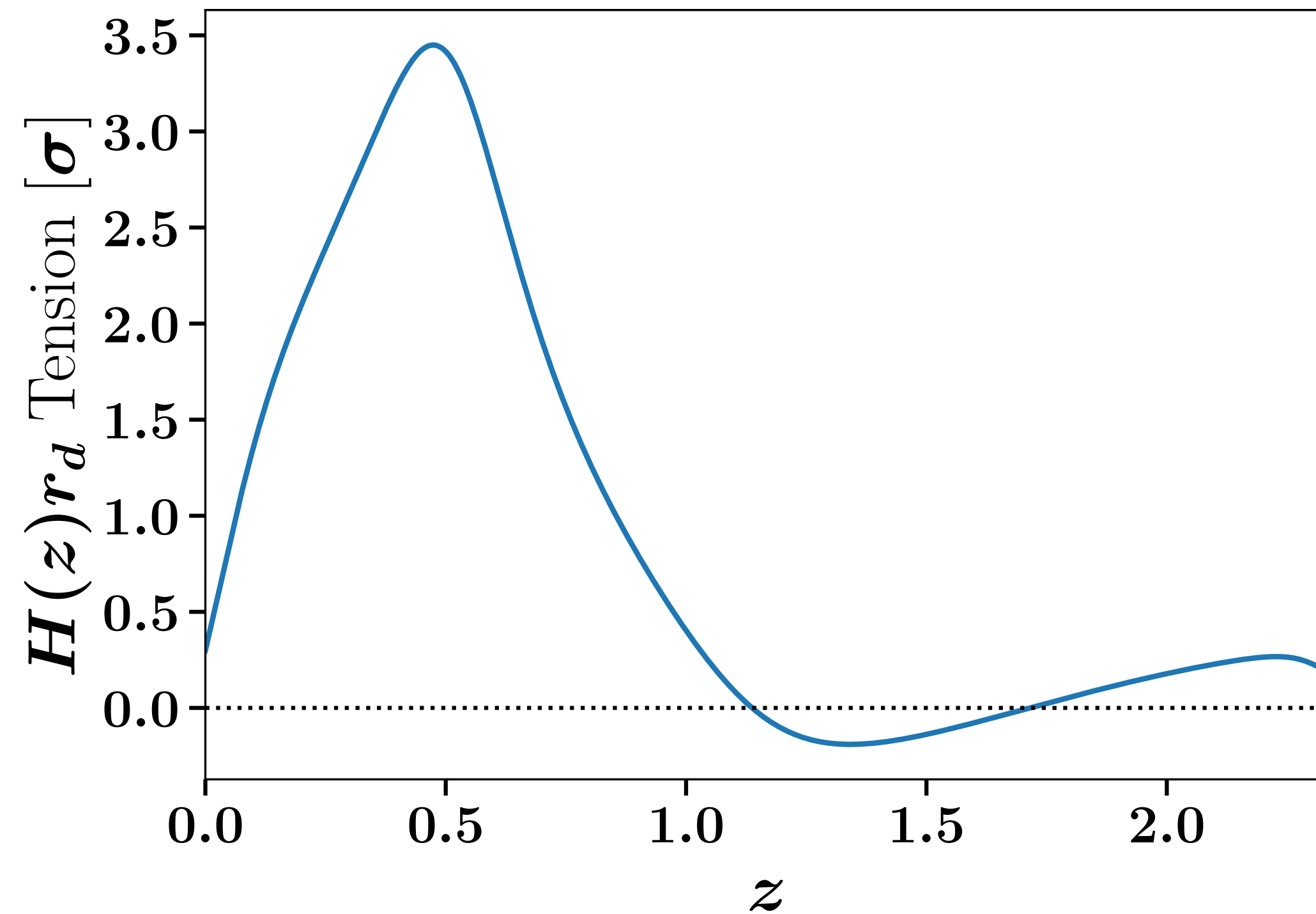
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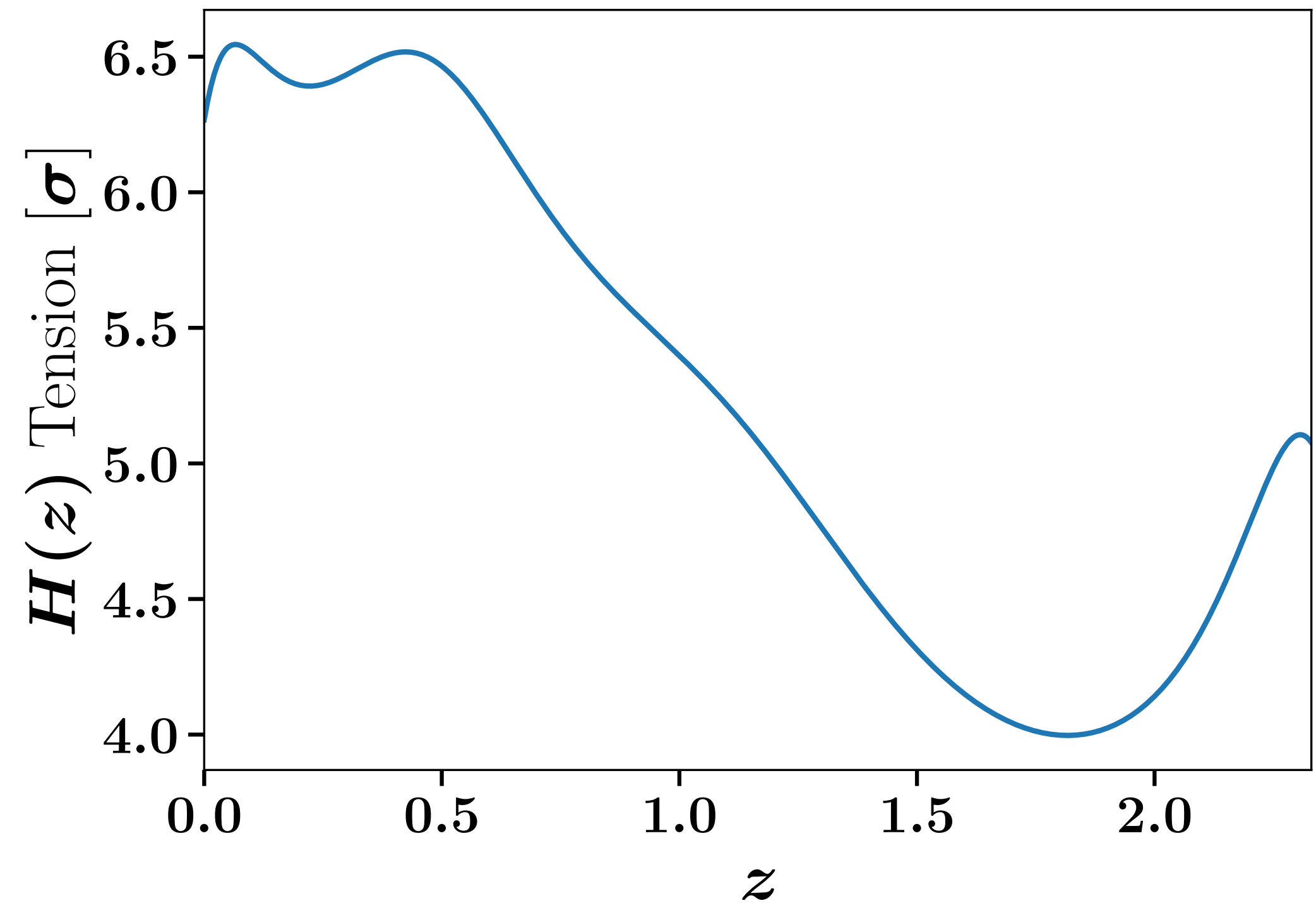
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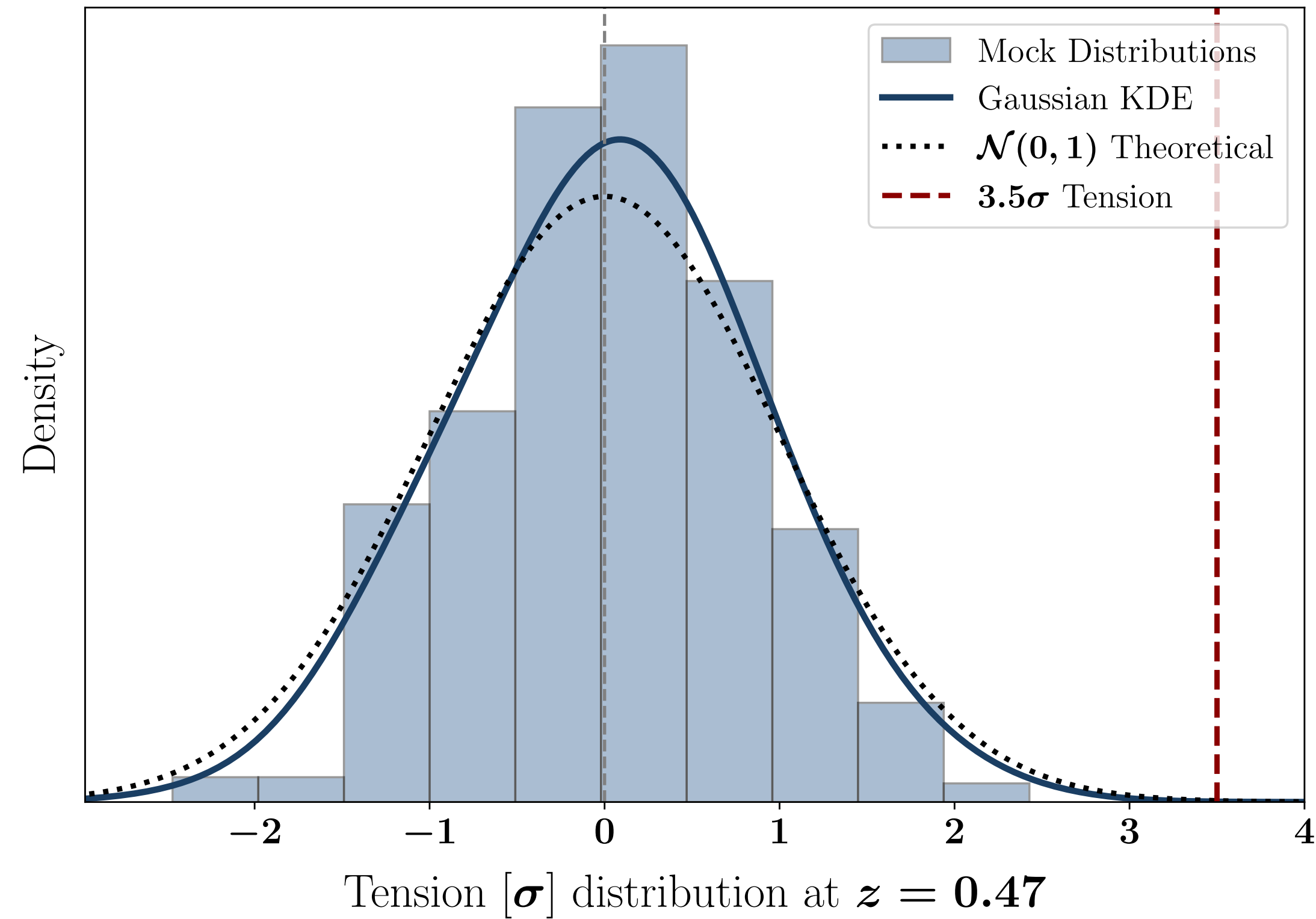


No r_d Planck – Λ CDM Prior. Instead we sampled hr_d

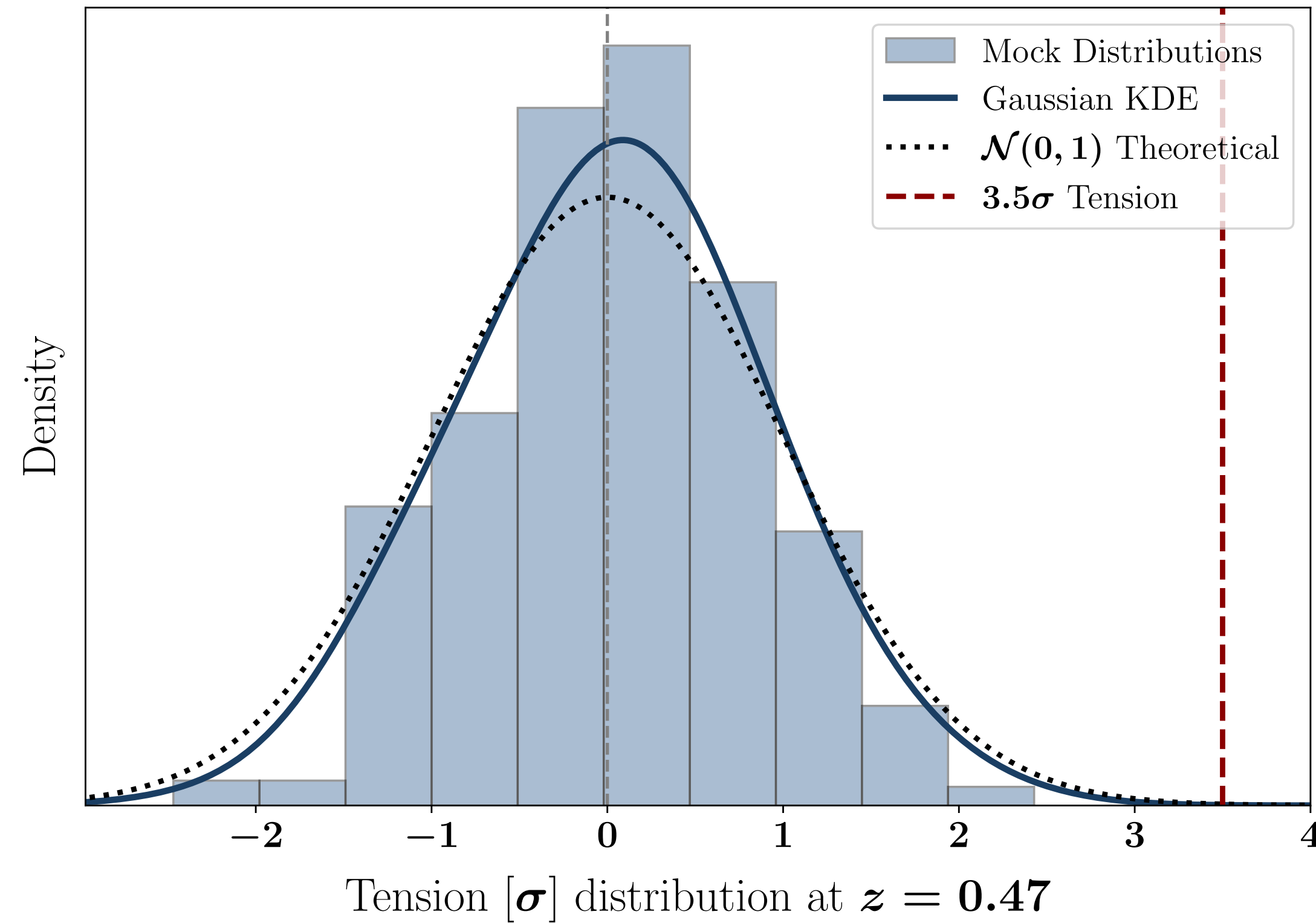


Instead of r_d Planck – Λ CDM Prior, we use HoDN prior on H_0

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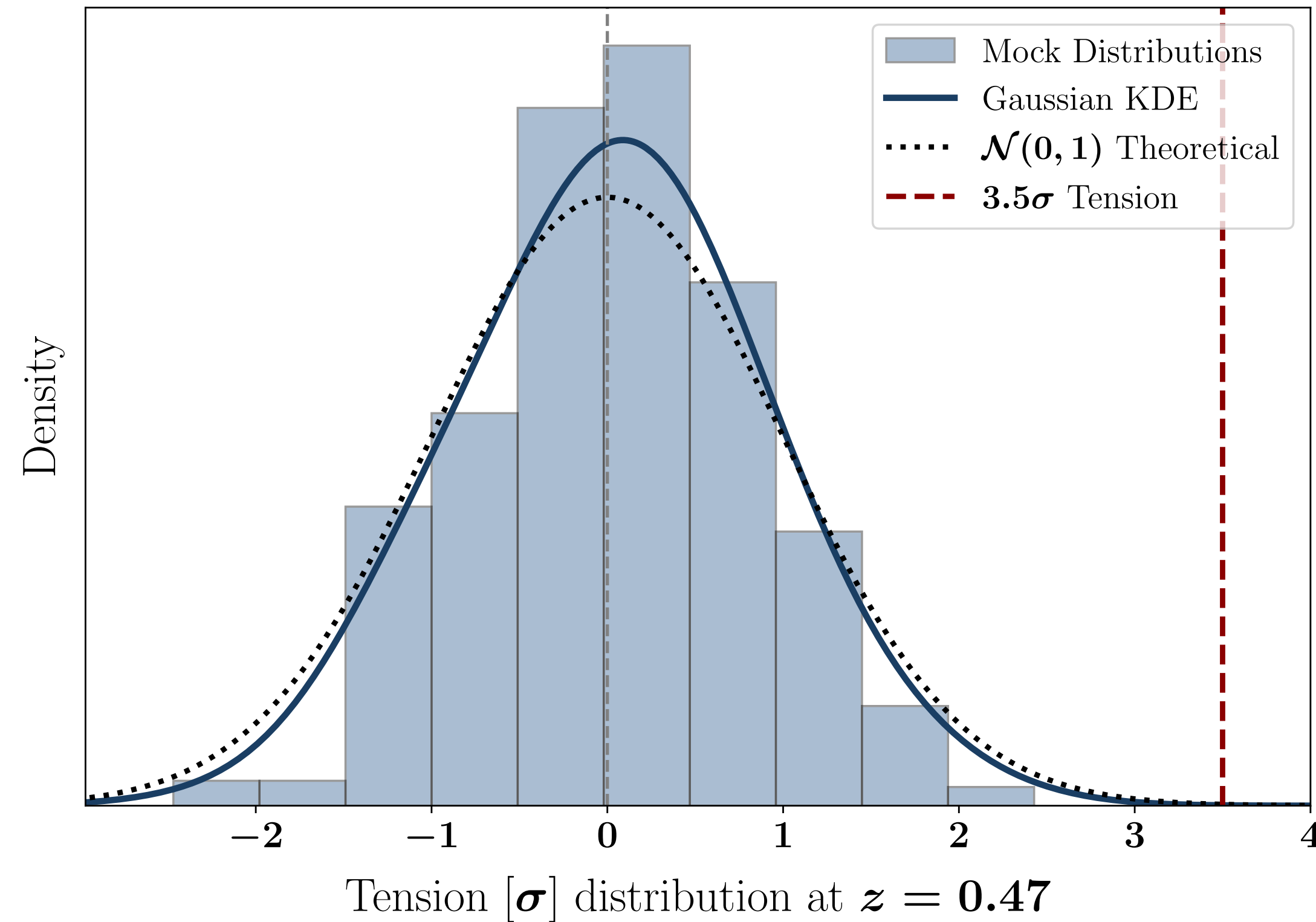


Global Significance:

Empirical $P < 0.002$

Gaussian two-tail P-value: 4.65×10^{-4}

Robustness of this Localized Deviation



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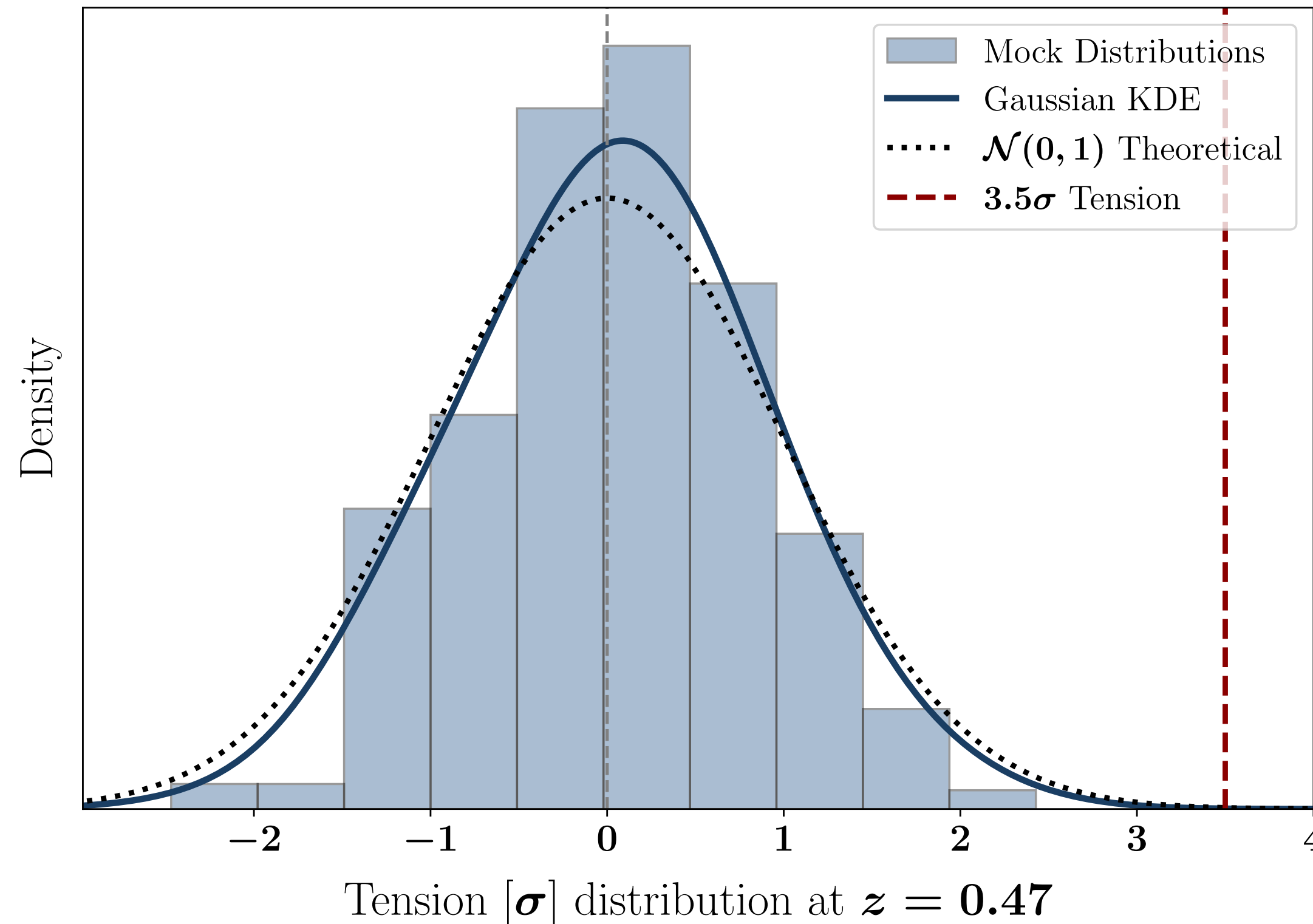
Any bias (the centre is at 0, not shifted)

Excess variance (the KDE is not broader than $\mathcal{N}(0,1)$)

Heavy tails (the mock distribution doesn't overshoot $\mathcal{N}(0,1)$ at $\pm 2\sigma$ or beyond)

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The 3.5σ tension from the real data cannot be attributed to any of the usual suspects — it is not a spline artefact, not a boundary effect, not an underestimated uncertainty from the MCMC, not a non-Gaussian tail in the reconstruction.

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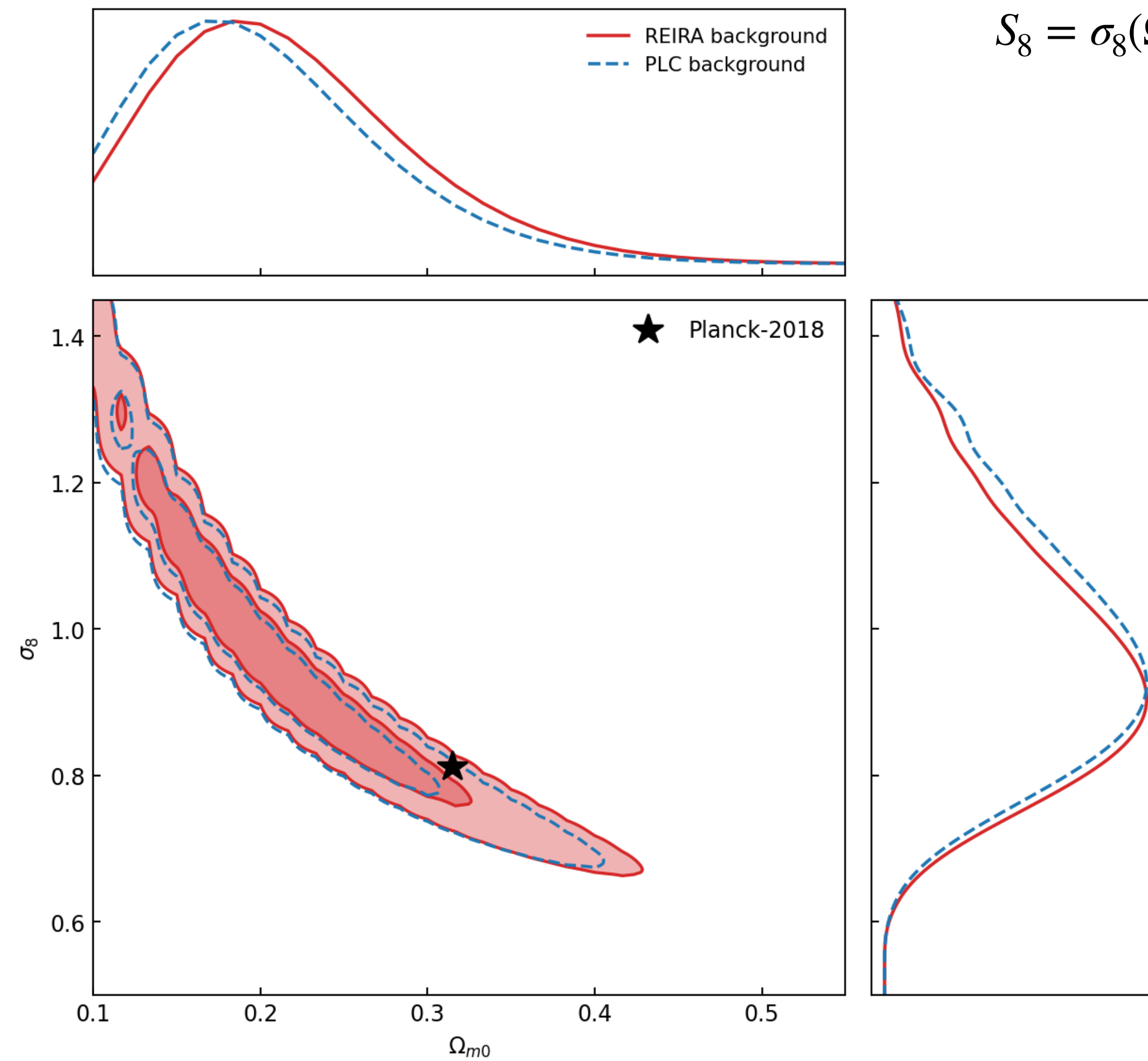
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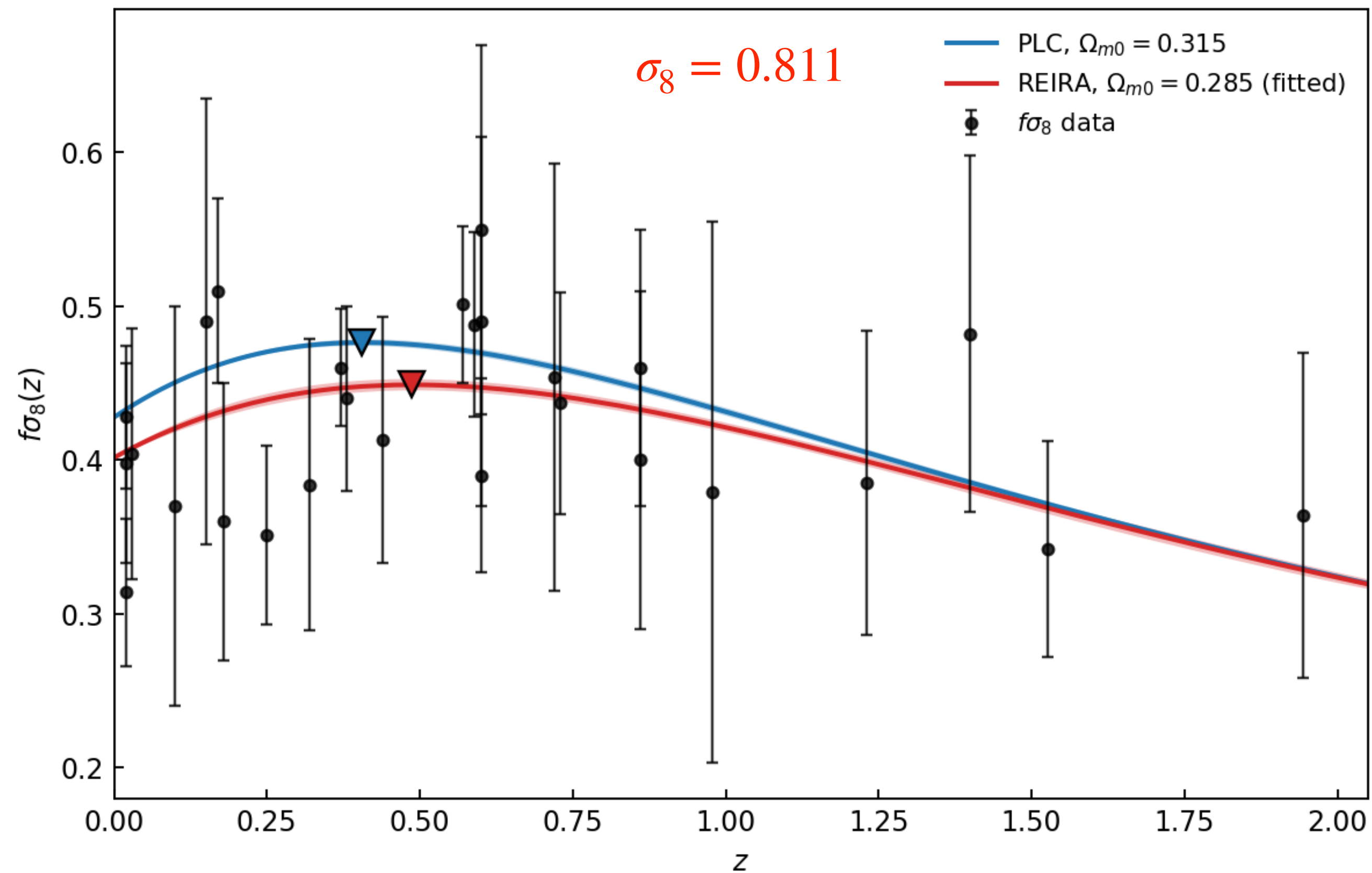
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We also need the $\sigma_8(0)$ information. But we can constrain $S_8 = \sigma_8(\Omega_{m0}/0.3)^{1/2}$

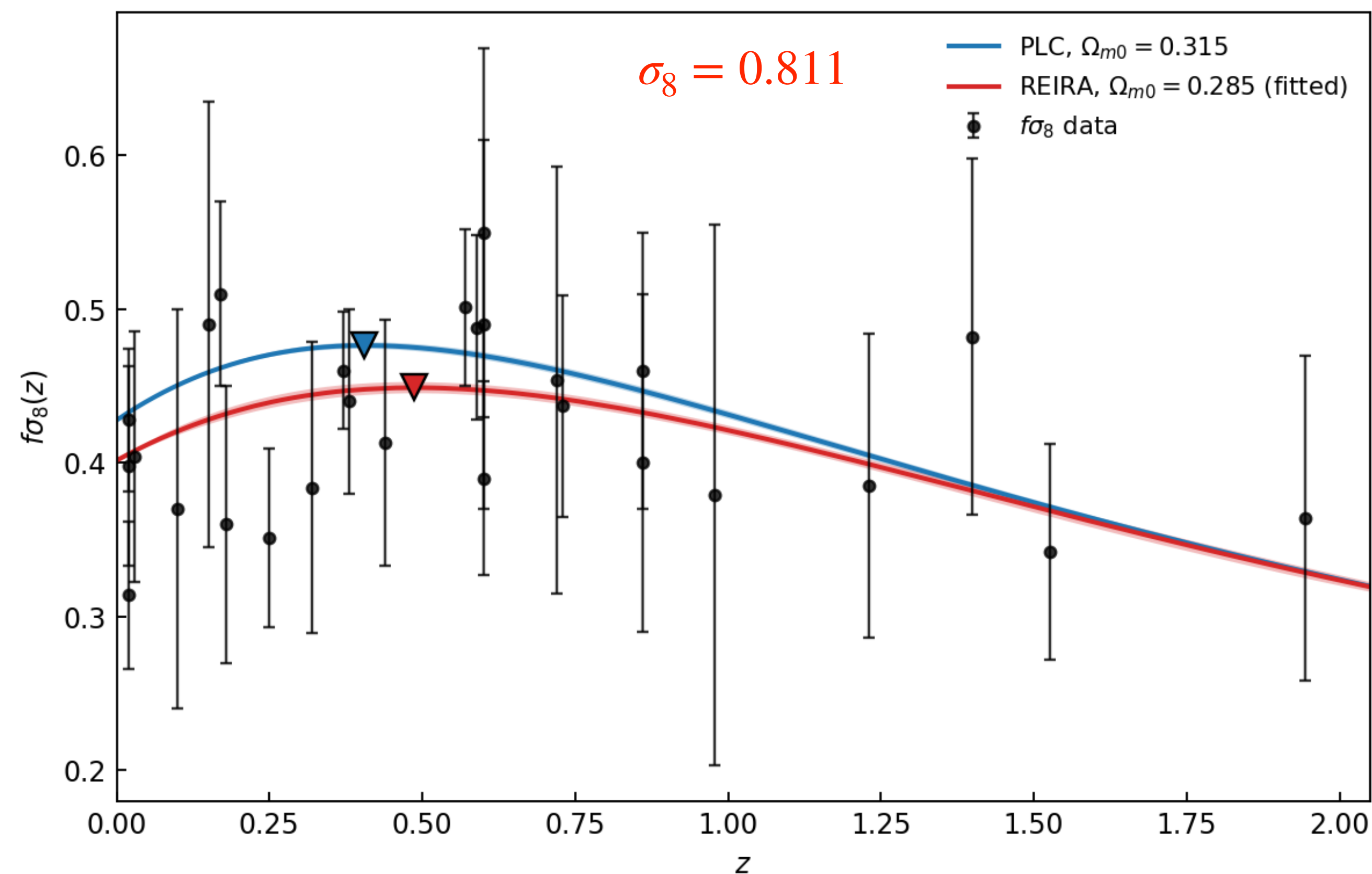
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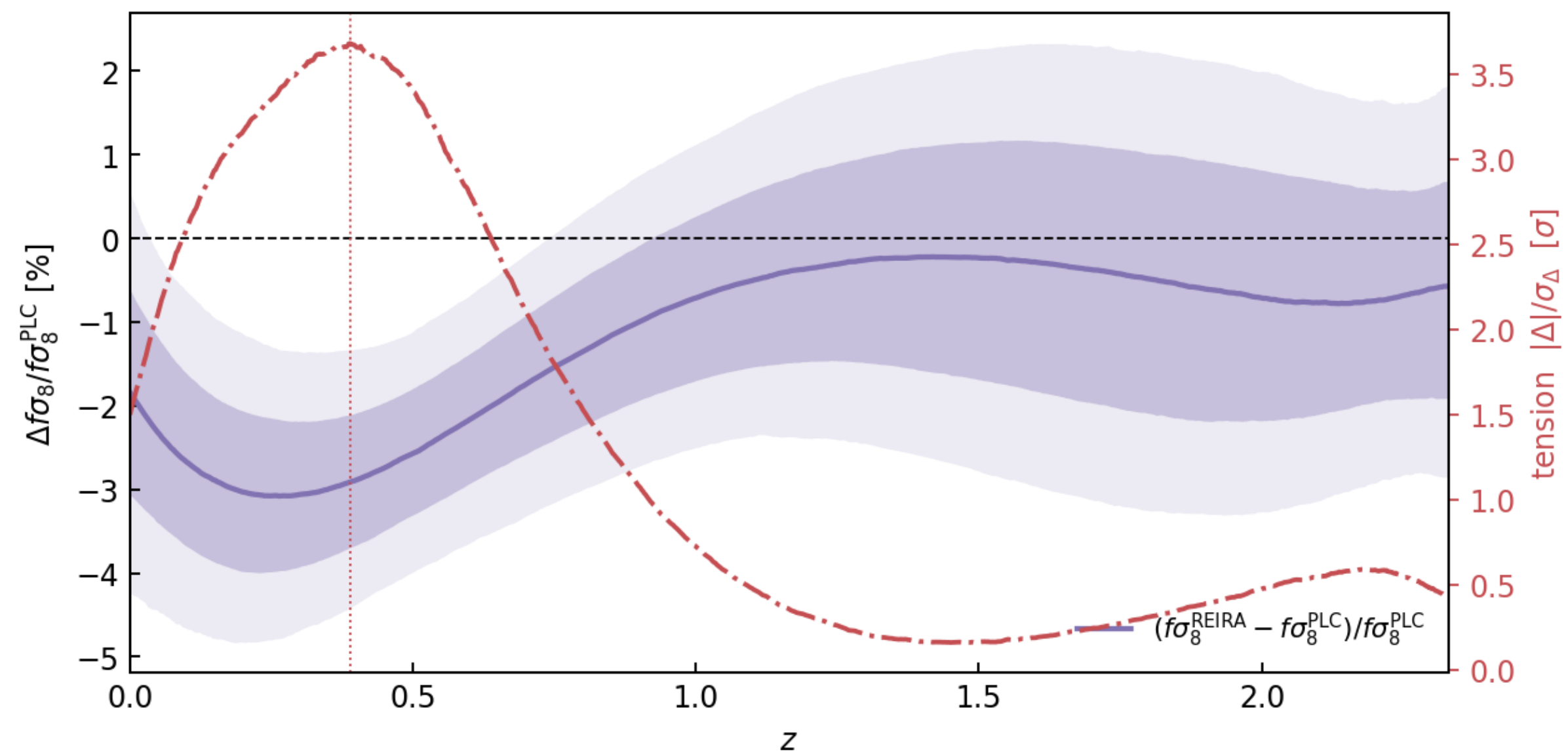
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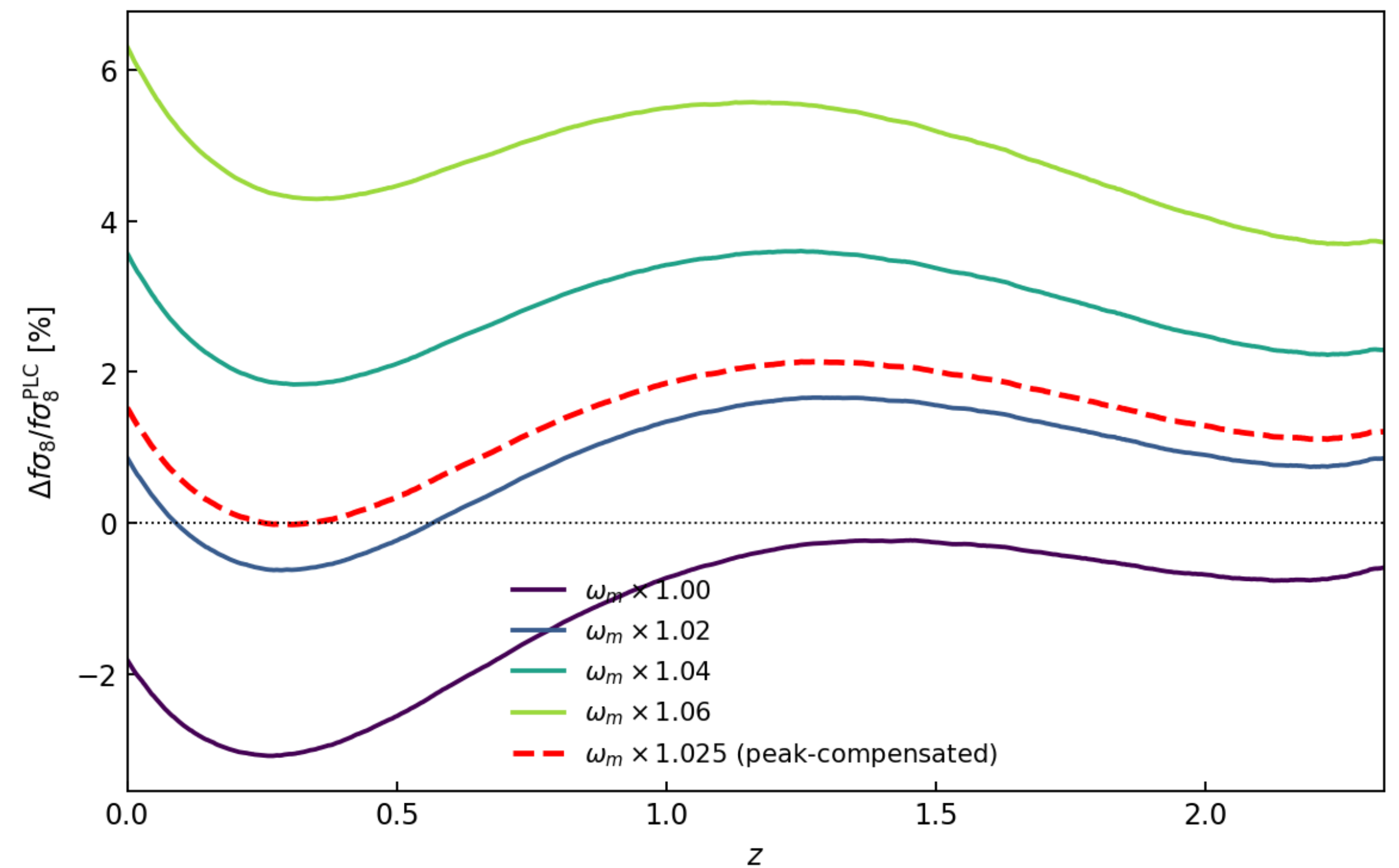
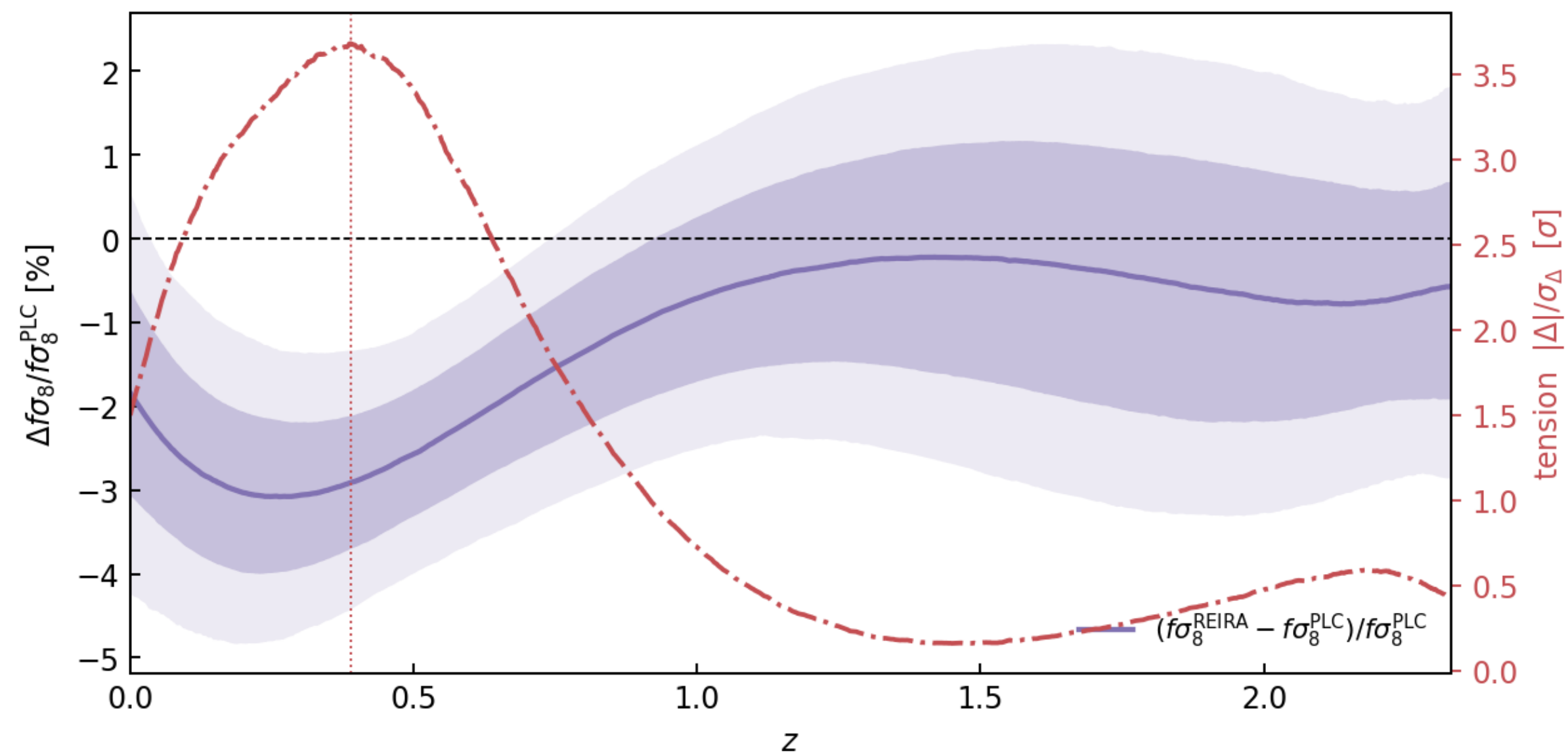
Model		z_{pk}	f_{pk}	$\Omega_m(a_{\text{pk}})$
Λ CDM	$\Omega_{m0} = 0.25$	0.569	0.729	0.563
Λ CDM	$\Omega_{m0} = 0.315$	0.405	0.728	0.561
Λ CDM	$\Omega_{m0} = 0.40$	0.245	0.729	0.562
REIRA	$\omega_m \times 0.975$	0.459	0.719 ± 0.010	0.549
REIRA	$\omega_m \times 1.000$	0.447	0.726 ± 0.010	0.558
REIRA	$\omega_m \times 1.025$	0.438	0.732 ± 0.010	0.566

Further Constraining Using Growth Data

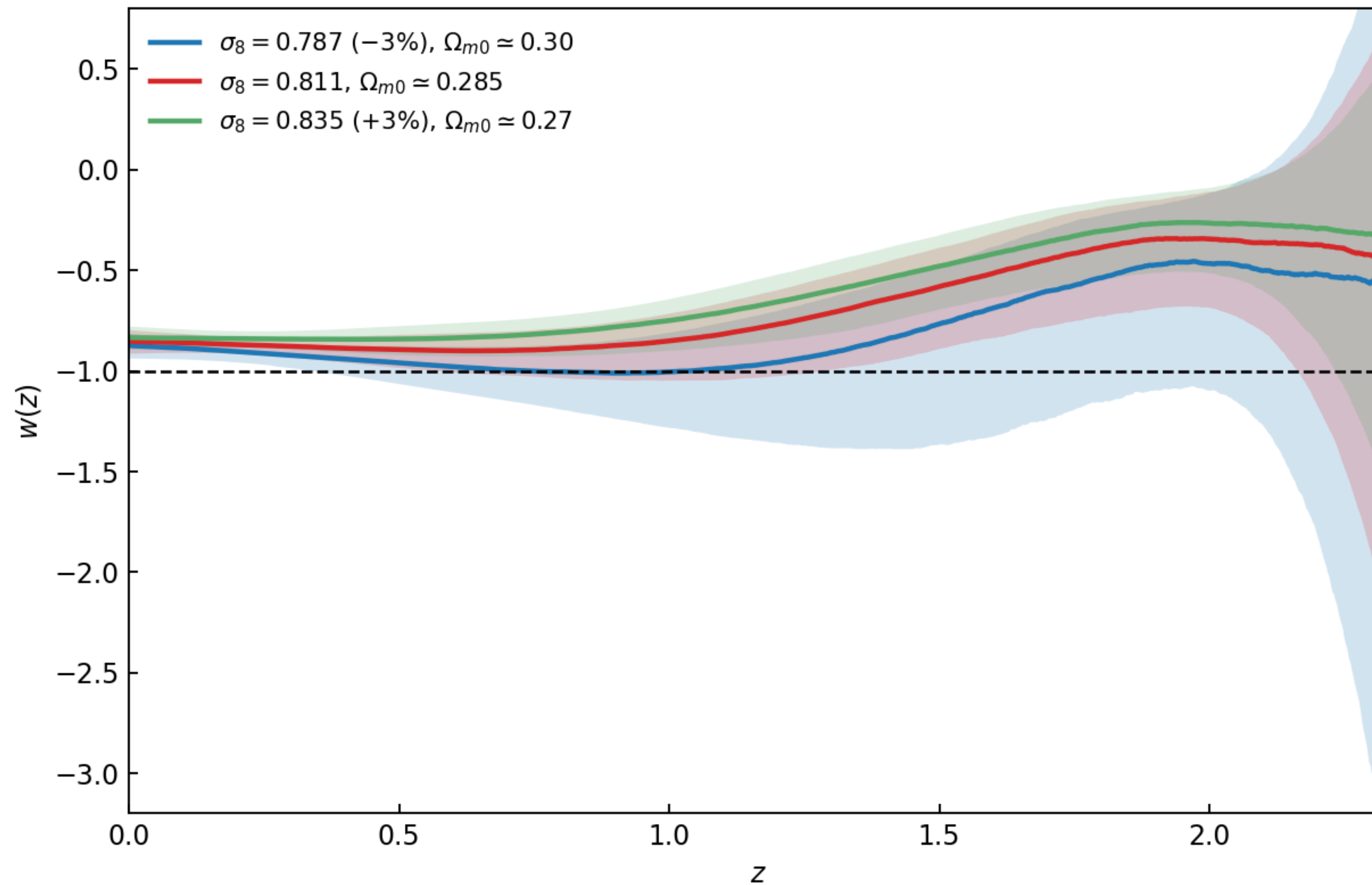
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Possible Dark Energy Behaviour



Essentially the σ_8 normalization plays an important role in deriving DE behaviour.

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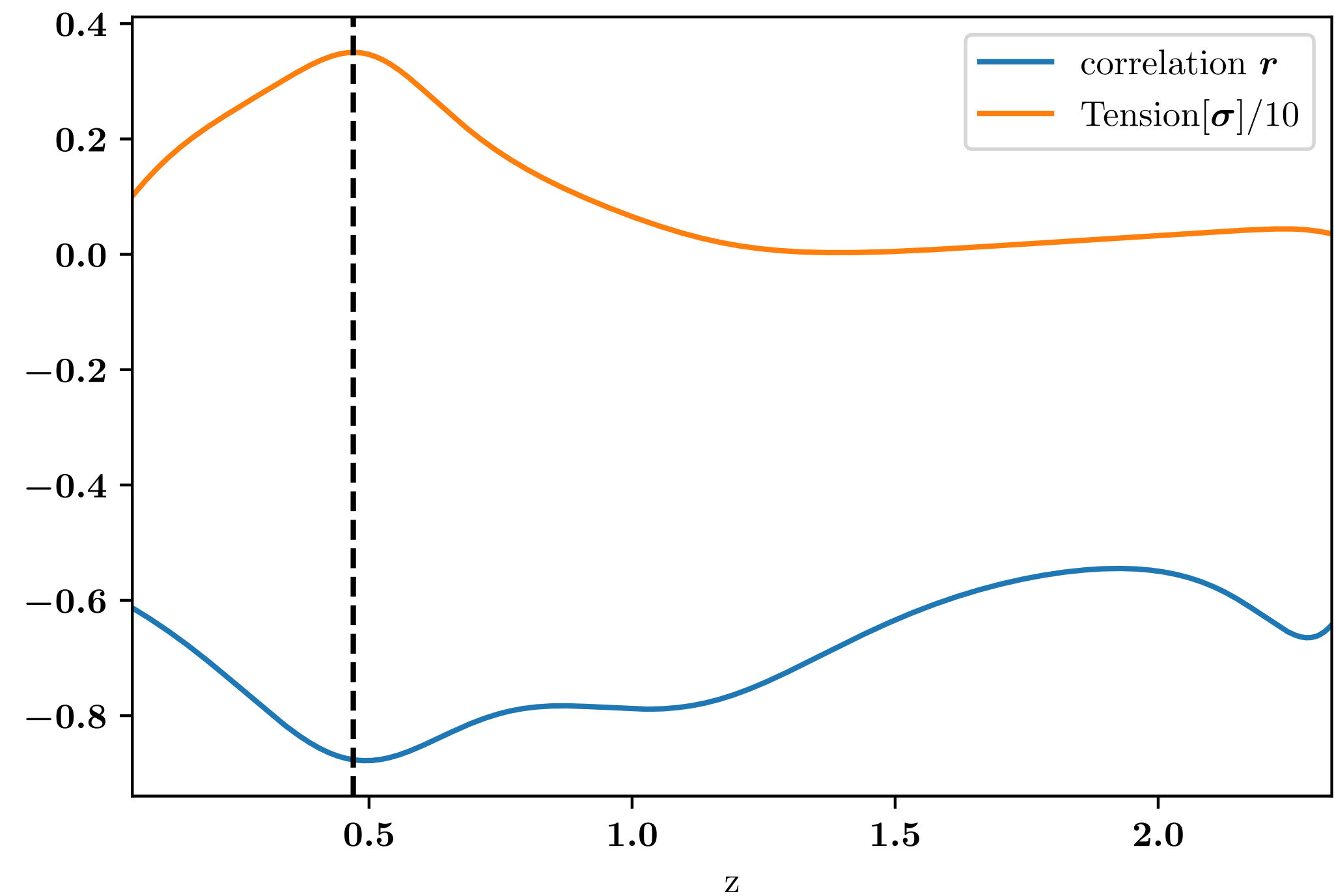
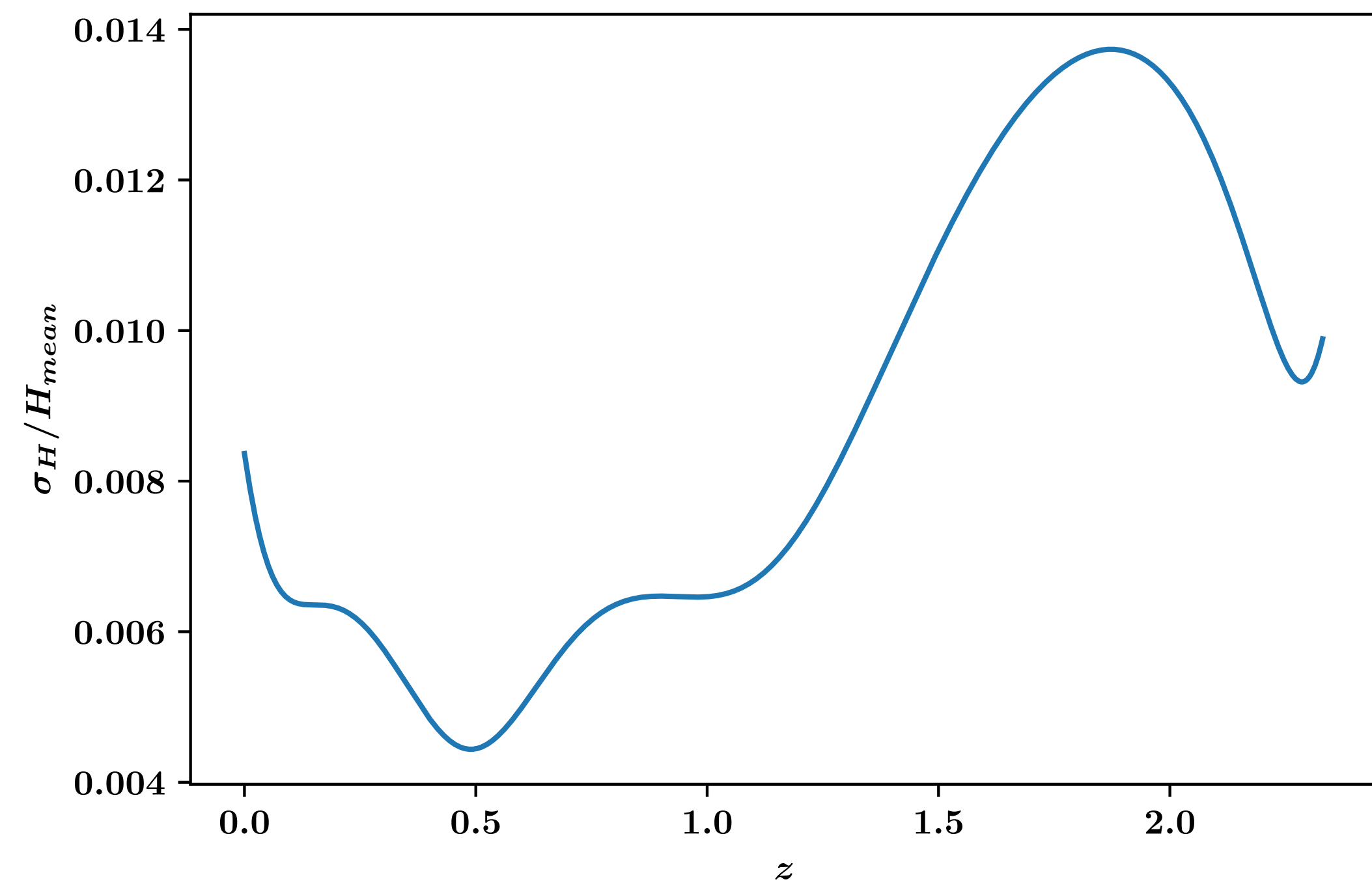
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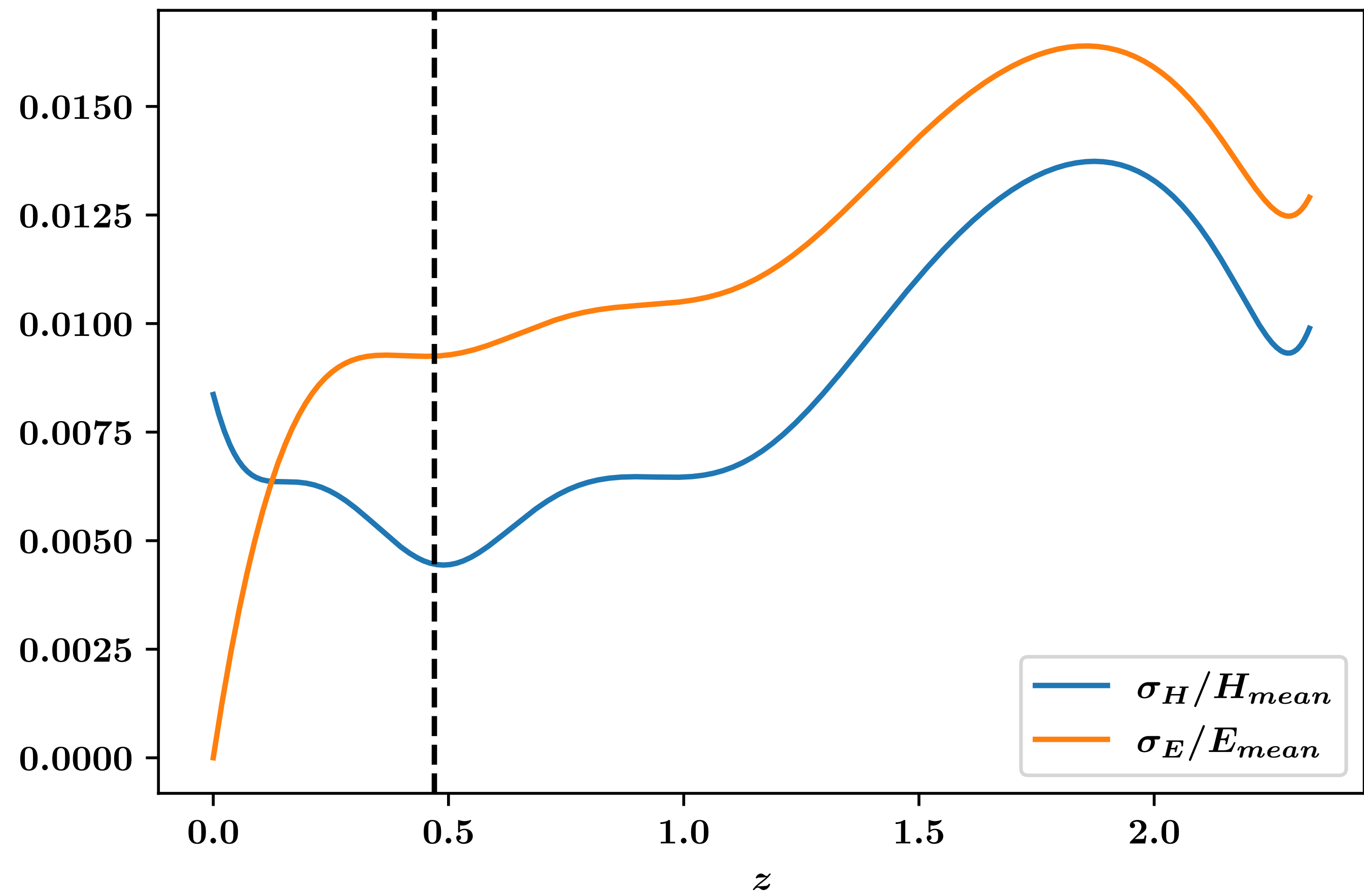
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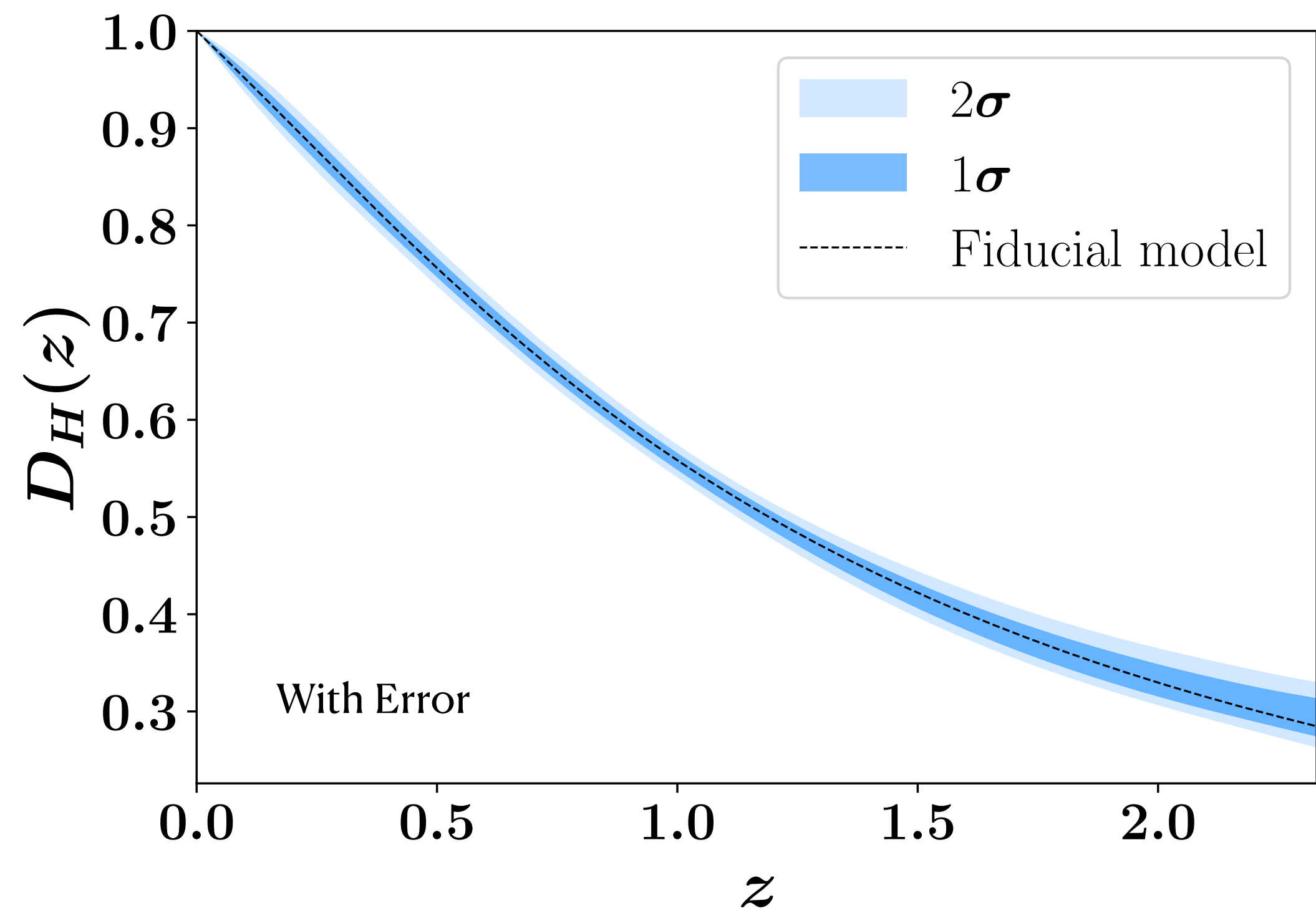
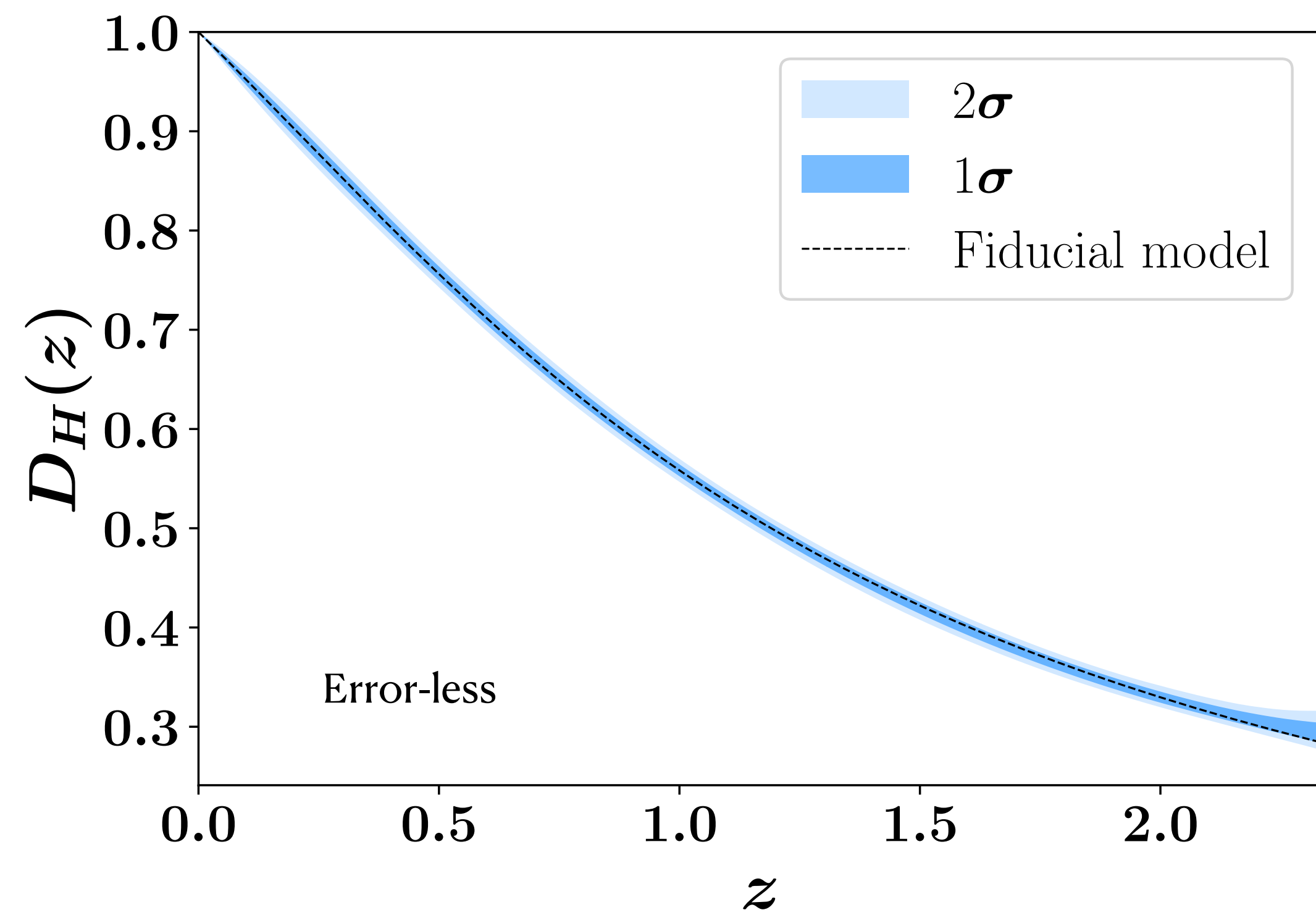
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Thank You For Your Attention





Mock Validation



21cm Linear Power Spectrum

$$P_{21}(z, k, \mu) = \bar{T}_b^2(z) [b_{HI}(z) + f(z)\mu^2]^2 P_m(k, z)$$

Mean Brightness Temperature: $\bar{T}_b(z) = 180 \frac{H_0(1+z)}{H(z)} \Omega_{HI}(z) h \text{ mK}$ $\Omega_{HI}(z) = \frac{\rho_{HI}(z)}{\rho_{c0}}$

Growth Factor: $f = \frac{d \log \delta}{d \log a}$ Cosine of the angle between $\mathbf{k}$ and the line-of-sight: $\mu = \frac{k_{\parallel}}{k}$

Linear Bias: b_{HI} Linear Matter Power Spectrum: $P_m(k, z)$

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For Galaxies: we can measure amplitude of power spectrum: $\propto (b_g \sigma_8)^2$ and from RSD, we can measure $\frac{f}{b_g}$, and combining these two, we can get $f\sigma_8$ which is bias free. But we can not get separately the growth factor f . Remember for Λ CDM, $f = \Omega_m(z)^{6/11}$ which is test for validity of Λ CDM model.

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For 21cm power spectrum, we need to know $b_{HI} = (\frac{P_{21}}{P_m})^{1/2}$. Observation of the P_{21} , can give us the measurement of the ratio $\beta = \frac{f}{b_{HI}}$ using the monopole to quadropole ratio of the of the P_{21} , but to know b_{HI} , we need to know f separately.