

Entanglement at Quantum Criticality

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Various works with T. Dupic, Y. Ikhlef, J. Lin, A. Morin-Duchesne, A. Rossboth, A. Rotaru, E. Tonni

Jean Bernard is 80 : Still Irreducible

A primer on entanglement entropy

(Bipartite) Entanglement in a nutshell

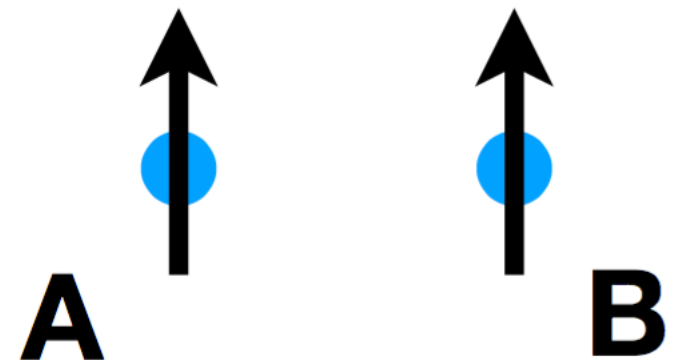
Given a quantum mechanical problem, with Hilbert space $\mathcal{H}$.

Consider a quantum state $|\Psi\rangle \in \mathcal{H}$, and a **bipartition** of the Hilbert space $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$

- if $|\Psi\rangle = |\Psi_A\rangle \otimes |\Psi_B\rangle$ (**product-state**) : there is **no entanglement**
- otherwise the degrees of freedom in A and B are said to be **entangled** in the state $|\Psi\rangle$.

Example : two spin 1/2 for which $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$

with $\mathcal{H}_A = \mathcal{H}_B = \mathbb{C}|\uparrow\rangle \oplus \mathbb{C}|\downarrow\rangle \simeq \mathbb{C}^2$



Then $|\uparrow\uparrow\rangle = |\uparrow\rangle \otimes |\uparrow\rangle$ is a pure state, while $\frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}$ is entangled.

But it's not always this easy :

$$\frac{|\uparrow\uparrow\rangle + |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle + |\downarrow\downarrow\rangle}{2} = \left(\frac{|\uparrow\rangle + |\downarrow\rangle}{\sqrt{2}} \right) \otimes \left(\frac{|\uparrow\rangle + |\downarrow\rangle}{\sqrt{2}} \right)$$

Schmidt decomposition and reduced density matrix

Schmidt decomposition of $|\Psi\rangle$ w.r.t. the bipartition $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$:

$$|\Psi\rangle = \sum_{j=1}^r \sqrt{p_j} |u_j\rangle \otimes |v_j\rangle, \quad p_j > 0, \quad \sum_{j=1}^r p_j = 1.$$

where $\langle u_i | u_j \rangle = \langle v_i | v_j \rangle = \delta_{ij}$

The reduced density matrix $\rho_A = \text{Tr}_{\mathcal{H}_B} (|\Psi\rangle\langle\Psi|)$ contains the information accessible from A :

$$\rho_A = \sum_{j=1}^r p_j |u_j\rangle\langle u_j|$$

Thus :

- $\{p_j\}$ is the (non-zero) spectrum of $\rho_A = \text{Tr}_{\mathcal{H}_B} (|\Psi\rangle\langle\Psi|)$
- The subsystem A is described in a statistical superposition :
A is in state $|u_j\rangle$ with probability p_j

How to quantify entanglement ?

The subsystem **A** is the statistical superposition of the states $|u_j\rangle$ with probability p_j .

to quantify entanglement : **Shannon entropy** from classical information theory

(Von Neumann) entanglement entropy
$$S = - \sum_j p_j \log p_j = - \text{Tr} (\rho_A \log \rho_A)$$

Some properties :

- $S = S(A) = S(B)$
- $S \geq 0$, with equality iff there is no entanglement
- S satisfies strong subadditivity : $S(A_1 \cup A_2) + S(A_1 \cap A_2) \leq S(A_1) + S(A_2)$

“Replica trick” :

Rényi entropy of order N

$$S_N = \frac{1}{1-N} \log \text{Tr} (\rho_A^N)$$

Entanglement in many-body quantum systems

Why care about entanglement entropy ?

- ★ Resource for quantum computation
- ★ Design of (classical) **new algorithms** to compute and store efficiently quantum many-body states (DMRG, MPS, tensor networks, ...)
- ★ Theoretical probe for quantum many body systems :
Can detect topological order, edge excitations, criticality, central charge, field content ...

Objective for theorists : compute the EE in the typical situation

- A and B are two spatial regions (partition of the underlying lattice/manifold)
- A and B contain a large number of degrees of freedom
- the total system is in its ground state.

Generic features of EE : Area law

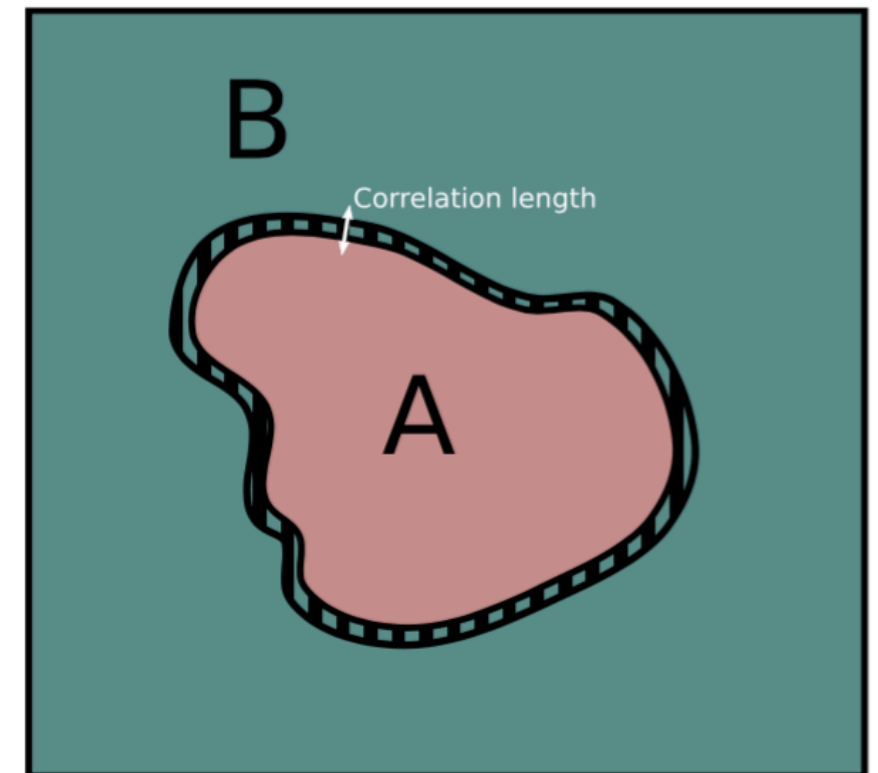
For *gapped* quantum systems

- spatial bipartition
- $|\Psi\rangle$ ground-state of a *local* Hamiltonian with a *spectral gap*
- finite correlation length ξ

Expectation : the leading asymptotic behavior of EE is governed by a **area/boundary law**

$$S_n \sim C_n \frac{\text{Vol}(\partial A)}{\xi^{d-1}}, \quad (\xi \rightarrow 0)$$

for regions A much larger than the correlation length ξ , and C_n some non-universal constant



Hastings 2007 : proof of area law in 1D (Von Neumann entropy). Implies the ability to approximate one-dimensional ground state by a matrix product state.

**At criticality the correlation length diverges $\xi \rightarrow \infty$
a logarithmic enhancement of the area law can happen**

- Free fermions in d dimensions [Gioev, Klich (05)]

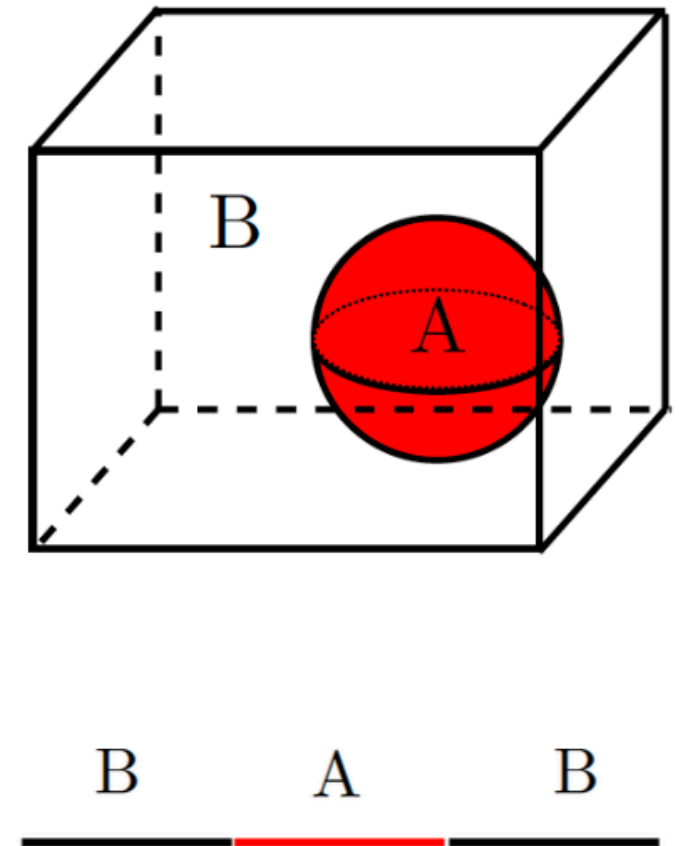
$$S_A \sim L^{d-1} \log L$$

where L is the linear size of A .

- One-dimensional critical systems ($T = 0$)
[Holzey, Larsen, Wilczek (94)] and [Cardy, Calabrese (04)]

$$S_A \sim \frac{c}{3} \log L$$

where c is the central charge (CFT)



Replica trick (for 1d systems)

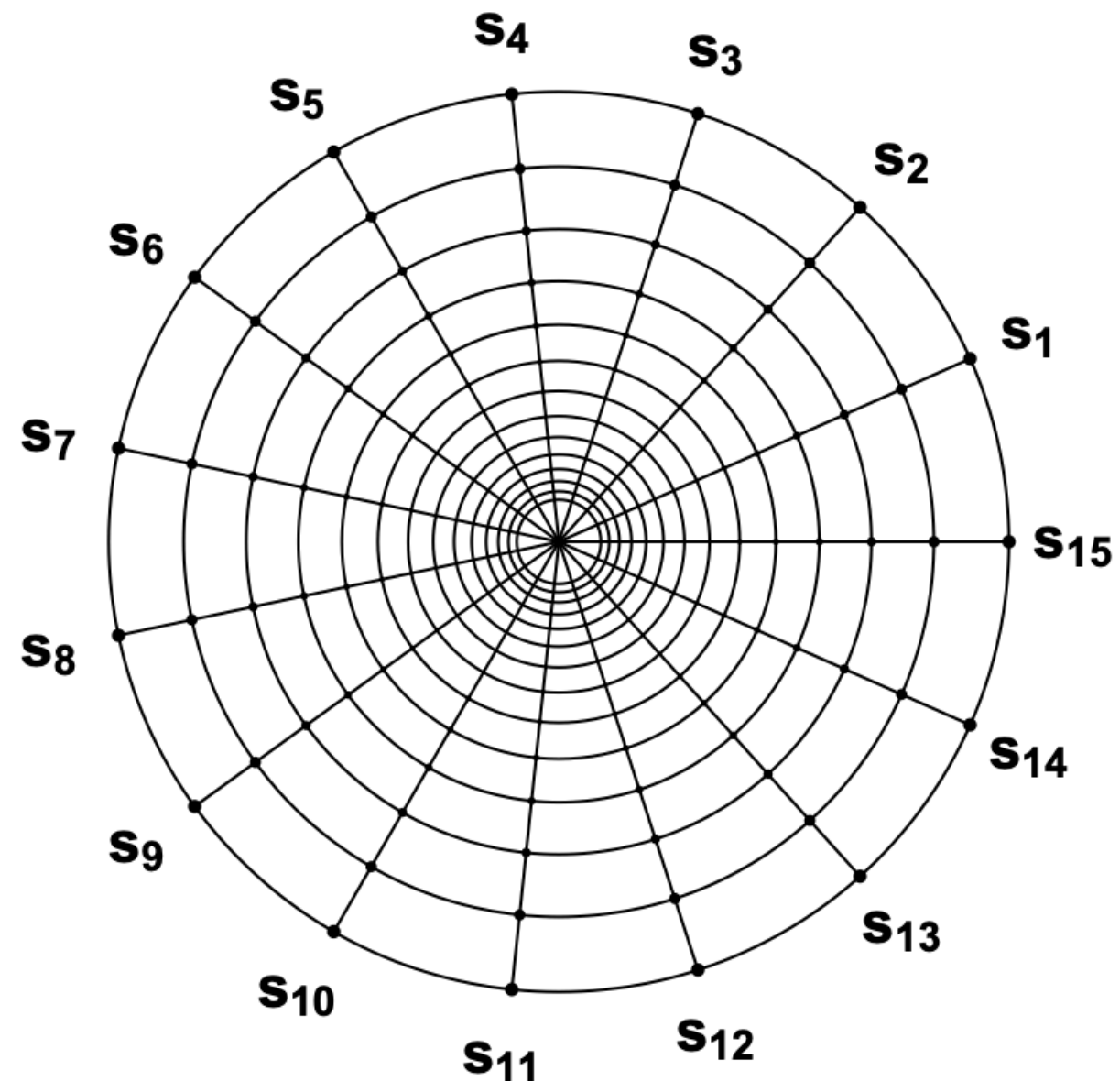
**from entanglement entropies to partition functions on
branched cover**

Euclidean path integral

states as Segal amplitudes (on the lattice)

$$|\Psi_0\rangle = \sum_{\mathbf{s}=(s_1, \dots, s_L)} Z_{\mathbf{s}} |s_1, \dots, s_L\rangle$$

where $Z_{\mathbf{s}}$ is the partition on the following lattice with outer boundary conditions $(s_1, \dots, s_L)$

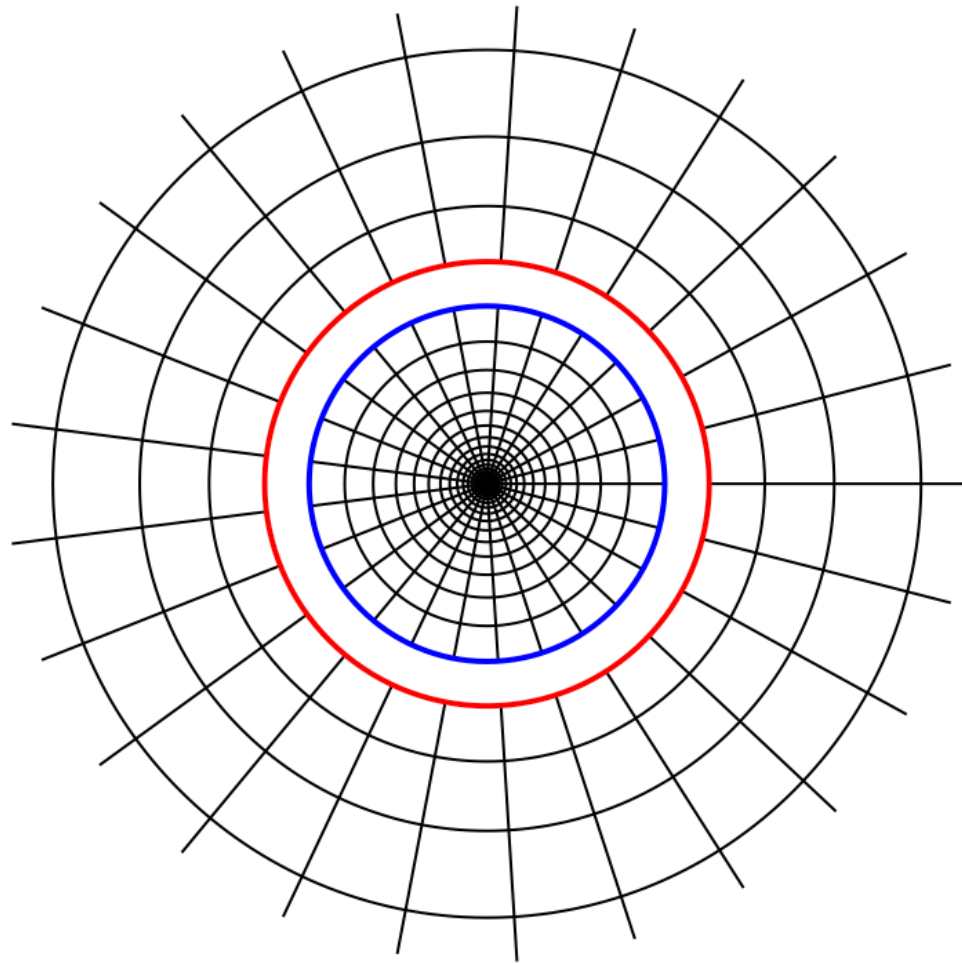


Euclidean path integral

reduced density matrix

projector onto the ground state

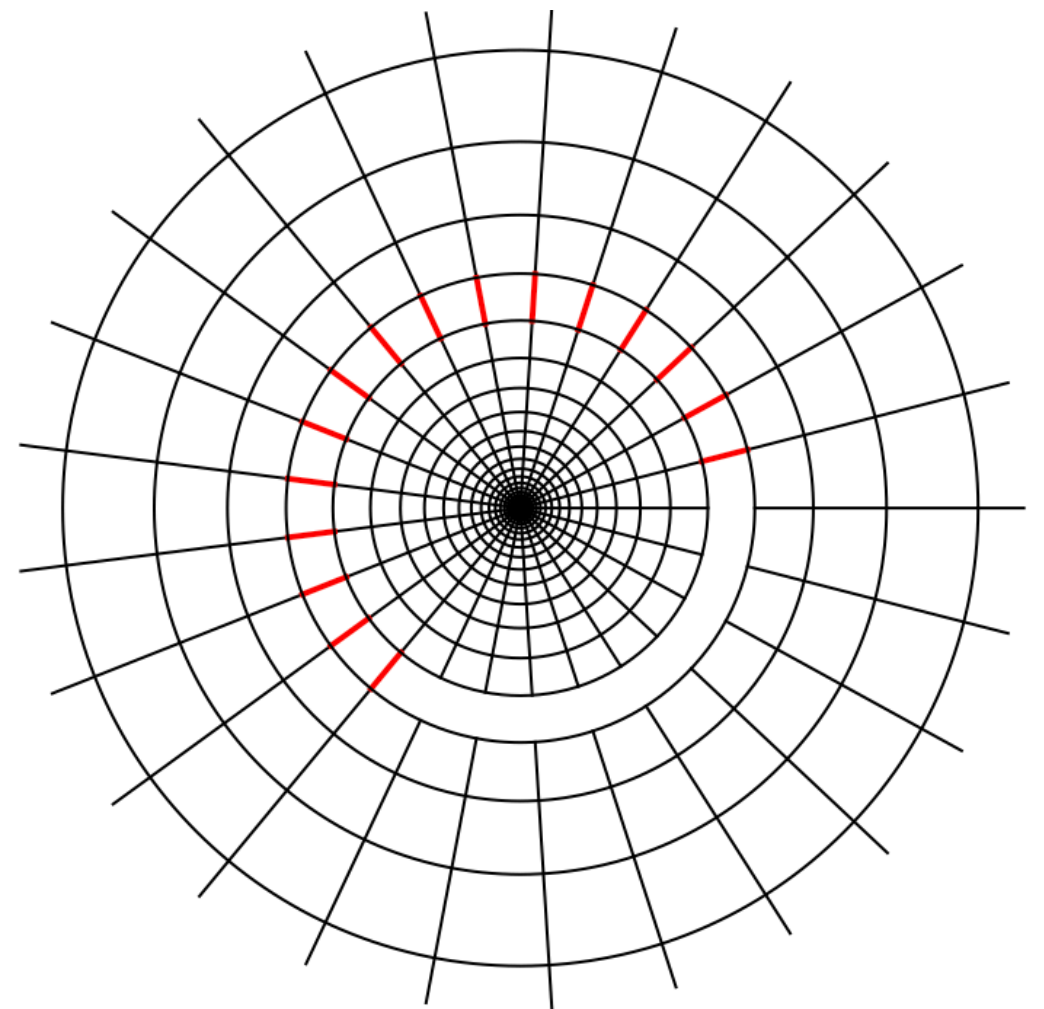
$$= |\Psi_0\rangle\langle\Psi_0|$$



Tracing over B yields

$$\rho_A = \text{Tr}_{\mathcal{H}_B} (|\Psi_0\rangle\langle\Psi_0|)$$

becomes a partition function on a surface with a slit

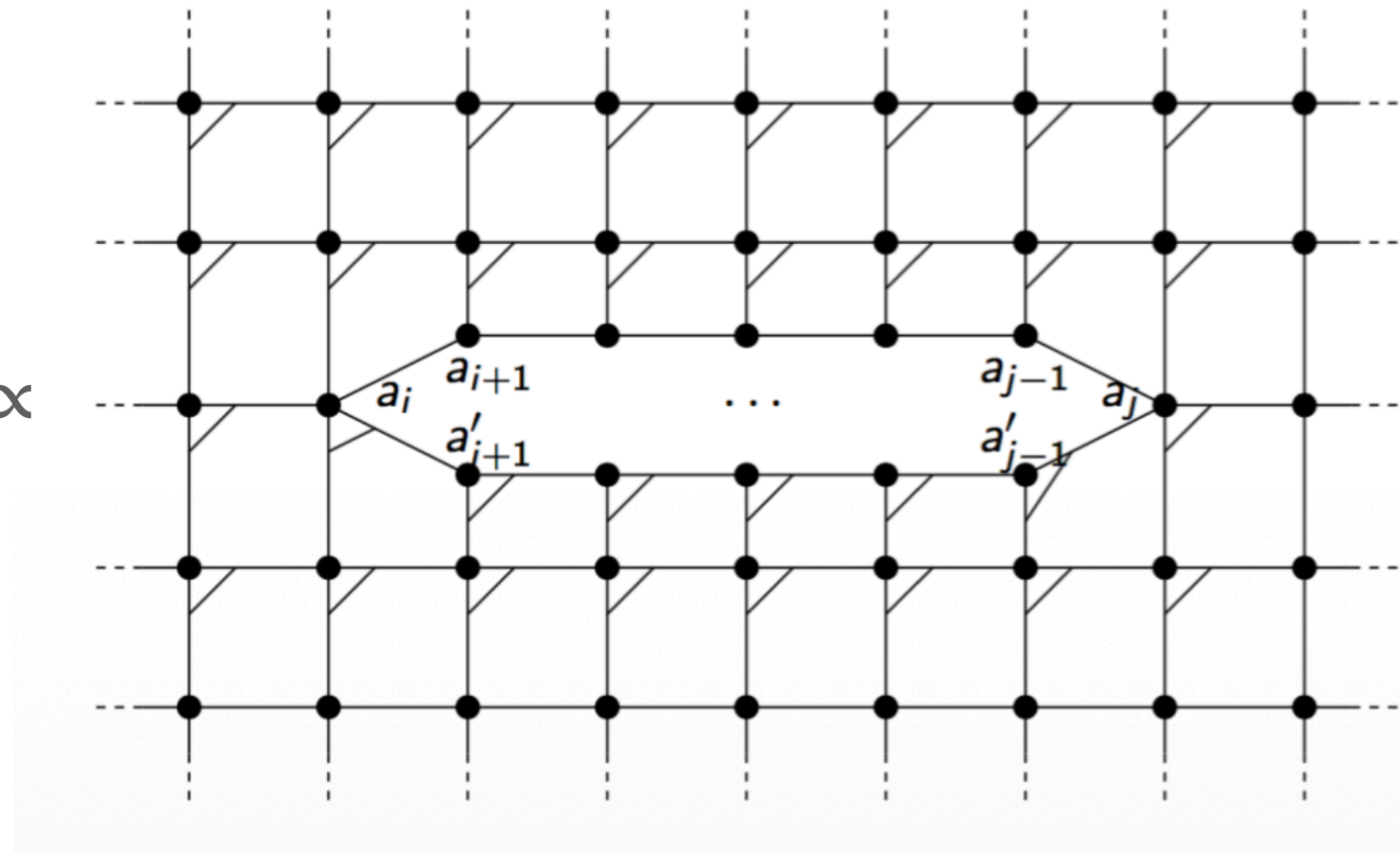


Euclidean path integral

reduced density matrix

Each matrix element of the reduced density matrix corresponds to a partition function

$$\left\langle a'_{i+1}, \dots, a'_{j+1} \left| \rho_A \right| a_{i+1}, \dots, a_{j+1} \right\rangle \propto$$



ρ_A is some kind of transfer matrix

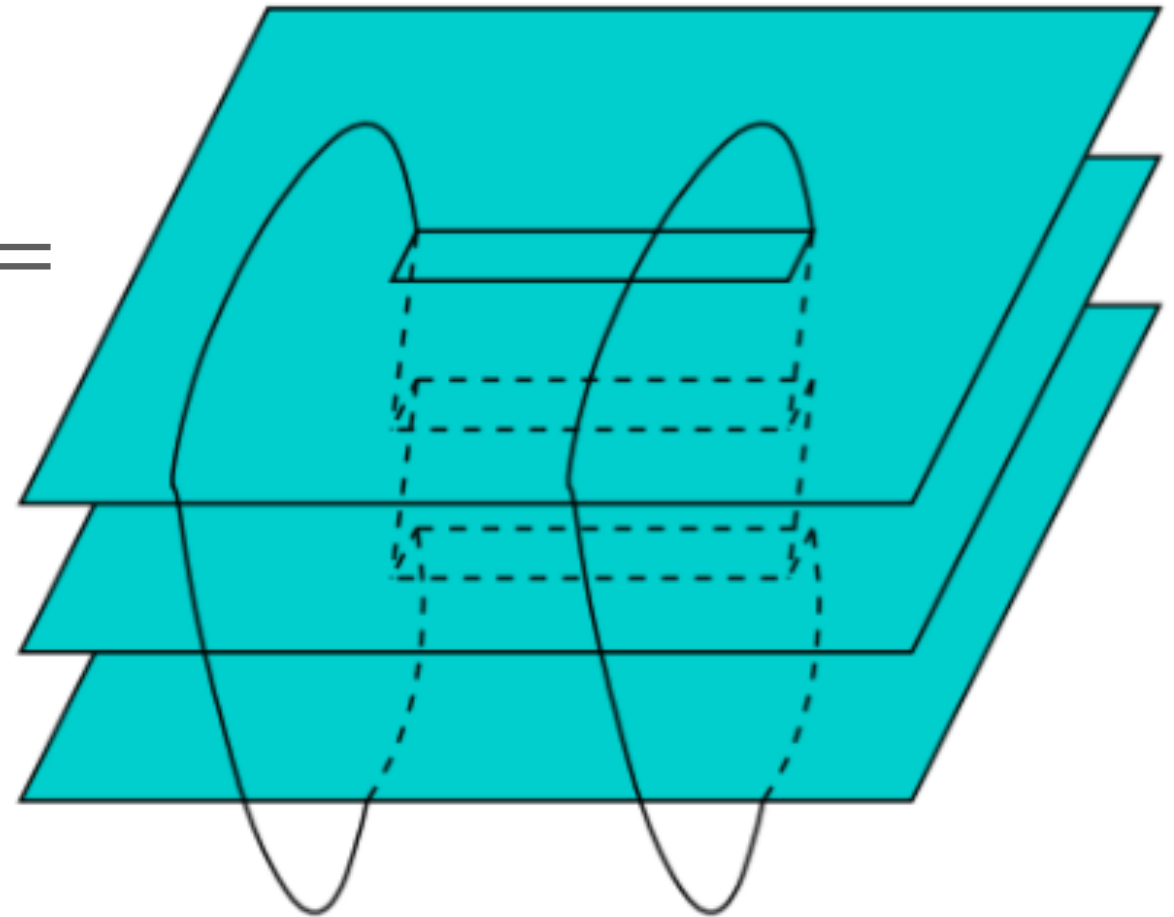
Euclidean path integral

Rényi entropies as partition functions on branched cover

$$\mathrm{Tr} \left(\rho_A^N \right) = \frac{Z \left(\Sigma_N \right)}{Z(\Sigma)^N}$$

where Σ_N is a N-sheeted
branched cover of $\Sigma = \mathbb{C}$
(or cylinder/torus/strip/annulus)

$\Sigma_N =$



Note : $Z(\Sigma_N)$ denote the partition
function of the **lattice model**, with
a lattice embedded in Σ_N

Scaling limit and CFT

Rényi entropies as partition functions on branched cover

Expected scaling of partition functions as the lattice spacing $a \rightarrow 0$

$$\log Z(\Sigma) \sim -f_0 \frac{\text{Area}(\Sigma)}{a^2} + \log Z_{\text{CFT}}(\Sigma) + o(1)$$

where f_0 (the free-energy density) is a non-universal (model dependent) number.

In ratios of the form

$$\frac{Z(\Sigma_N, f^*g)}{Z(\Sigma, g)^N}$$

where $f: \Sigma_N \rightarrow \Sigma$ is a branched cover (and Σ_N is equipped with the pull-back metric)

The non universal piece cancels out as $\text{Area}(\Sigma_N) = N \text{Area}(\Sigma)$.

$\Rightarrow$ entanglement entropy is universal !

and in the scaling limit one ends up with the ratio of CFT partition functions

Note : the subleading corrections (the $o(1)$ term) can also be computed

Conical Approach to Entanglement Entropy via Polyakov Formula

[Cardy, Peschel (1988)]

[Calabrese, Cardy (2004)]

[Estienne, Lin (2025)]

1. Partition functions on surfaces with conical singularities can be defined by excising a small disk of radius ϵ around each singularity and adding the appropriate counter-term.
2. One then notice that ratios of the form

$$R_N = e^{(1-N)S_N} = \frac{Z(\Sigma_N, f^*g)}{Z(\Sigma, g)^N}$$

(where $f : \Sigma_N \rightarrow \Sigma$ is a cyclic branched cover) behave very nicely under Weyl rescaling :

$$R_N[e^{2\sigma}g] = e^{-\sum_j \Delta_j \sigma(x_j)} R_N[g]$$

where $\{x_j\}$ are the branch points (on Σ), and the **critical exponent** is

$$\Delta_j = \frac{c}{12} \left(N - \frac{1}{N} \right)$$

This is the first hint that the ratio R_N behave like a correlation function

Simple case : A is an interval

Ground state entropy (on $\mathbb{R}$ or $\mathbb{S}^1$), A is a single interval of length ℓ

or finite temperature entropy (on $\mathbb{R}$)
single interval

branched cover of $\mathbb{C}P^1$, two branched points : this is easy since the cover Σ_N is a sphere

$$\frac{Z(\Sigma_N)}{Z(\Sigma)^N} = |\ell|^{-2\Delta_N} = |\ell|^{-\frac{c}{6}\left(N - \frac{1}{N}\right)}$$

leading to the ubiquitous formula :

$$S_N = \frac{c}{6} \left(1 + \frac{1}{N} \right) \log \ell$$

[Holzhey, Larsen, Wilczek (1994)]
[Vidal, Latorre, Rico, Kitaev (2003)]
[Calabrese Cardy (2004)]

Beyond this simple case

moduli is catching up

A = **two** intervals, N=2 entropy, zero T

The two-sheeted cover Σ_2 of $\mathbb{CP}^1$ with branch points at 0, x , 1 and ∞ :

$$w^2 = z(z-1)(z-x)$$



Replicated surface Σ_2 is a torus $\mathbb{C}/(\mathbb{Z} + \tau\mathbb{Z})$ with $\tau(x) = i \frac{{}_2F_1\left(\frac{1}{2}, \frac{1}{2}; 1; 1-x\right)}{{}_2F_1\left(\frac{1}{2}, \frac{1}{2}; 1; x\right)}$

and, taking care of the conical singularities, one obtains

$$\frac{Z(\Sigma_2)}{Z(\Sigma)^2} = \left| x(1-|x|) \right|^{-\frac{c}{4}} Z[\tau(x)] \quad \text{where} \quad Z[\tau] = \text{Tr} \left(q^{L_0 - \frac{c}{24}} \bar{q}^{\bar{L}_0 - \frac{c}{24}} \right)$$

Topological issues

Rényi entropies as partition functions on branched cover

Two-intervals Rényi N : the genus of Σ_N is $N - 1$

with period matrix (obtained by integrating holomorphic forms over homology cycles)

$$\tau_{rs}(x) = \frac{2i}{N} \sum_{k=1}^{N-1} e^{2\pi i \frac{k(r-s)}{N}} \sin\left(\pi \frac{k}{N}\right) \frac{{}_2F_1\left(\frac{k}{N}, 1 - \frac{k}{N}; 1; 1-x\right)}{{}_2F_1\left(\frac{k}{N}, 1 - \frac{k}{N}; 1; x\right)}$$

One can relate the entanglement entropy to partition functions on genus $N-1$ surfaces.

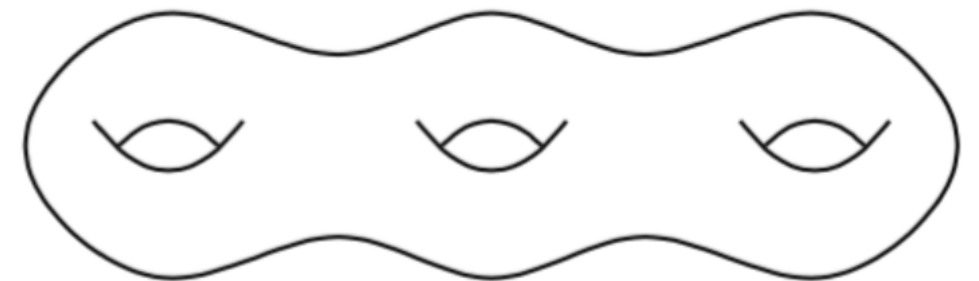
- for non-interacting theories (compact free field) [Calabrese Cardy Tonni, 2009].
- in general : **not very helpful**

Note : same problem for **A = single interval on $\mathbb{S}^1$**

@ finite temperature :

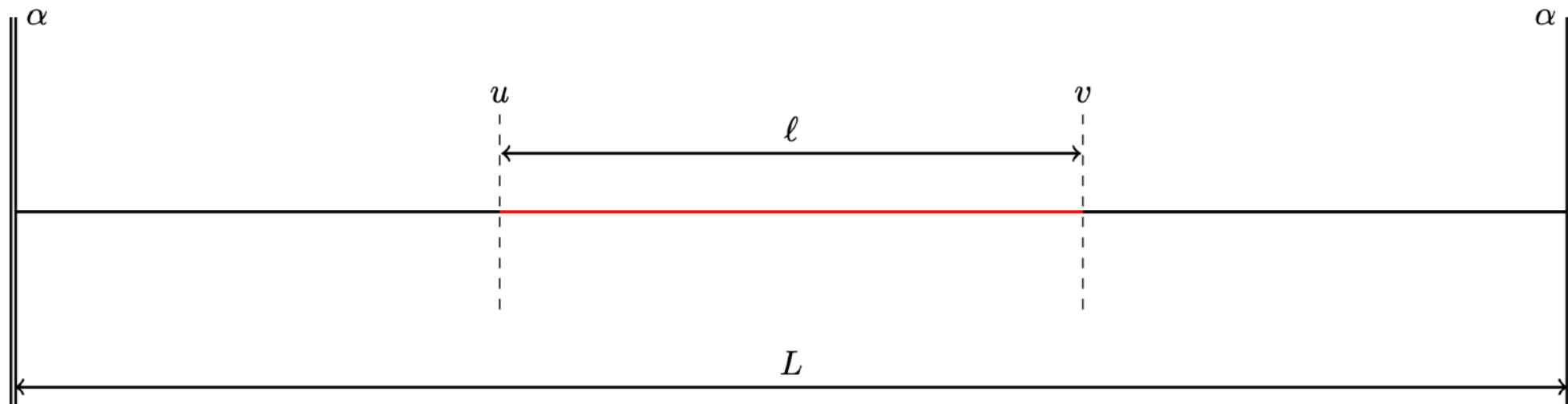
branched cover of a torus with two branch points :

genus Σ_N is N



Other physically relevant situations

Quantum system on $[0, L]$ with conformal boundary condition α



Σ_N is a branched cover of the unit disk (with boundary condition α), two branch points.

Topologically this is a sphere with N disks removed.

In particular for $N=2$ this is an **annulus**

[Estienne, Ikhlef, Rotaru (2021)]

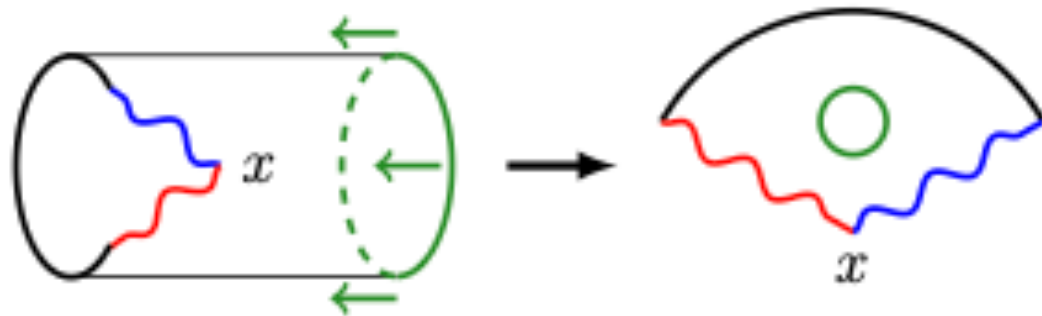
$$\frac{Z(\Sigma_2)}{Z(\Sigma)^2} = g_\alpha^{-2} 2^{-c/3} (x(1-x))^{-\frac{c}{24}} Z_{\alpha\alpha}(\tau)$$

where $Z_{\alpha\alpha}(\tau)$ is the annulus partition function with conformal boundary α on both

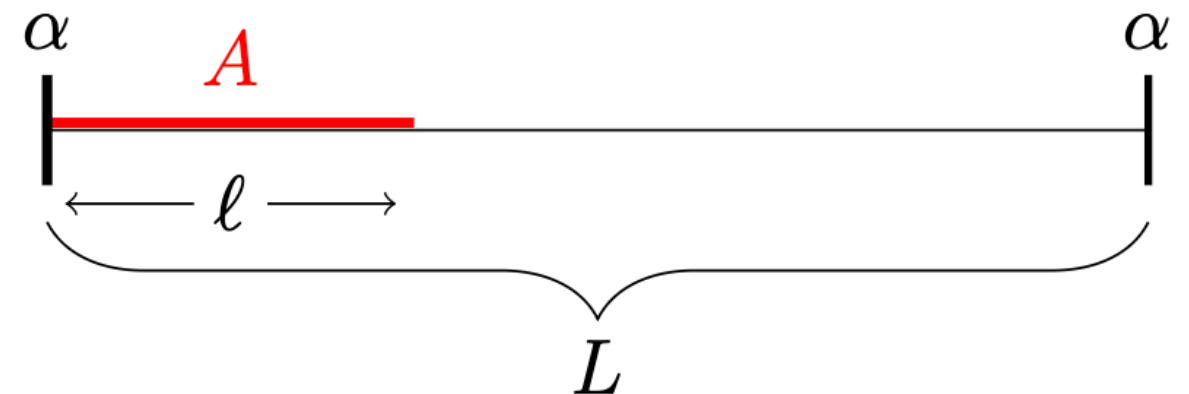
boundary components ; $\tau(x) = i \frac{{}_2F_1\left(\frac{1}{2}, \frac{1}{2}; 1; 1-x\right)}{{}_2F_1\left(\frac{1}{2}, \frac{1}{2}; 1; x\right)}$ and $x = \frac{\sin^2 \frac{\pi}{2L}(v-u)}{\sin^2 \frac{\pi}{2L}(v+u)}$

Quantum system on $[0, L]$ with conformal boundary condition α
 @ finite temperature

[Estienne, Ikhlef, Rossboth (2026)]



The reduced density matrix ρ_A

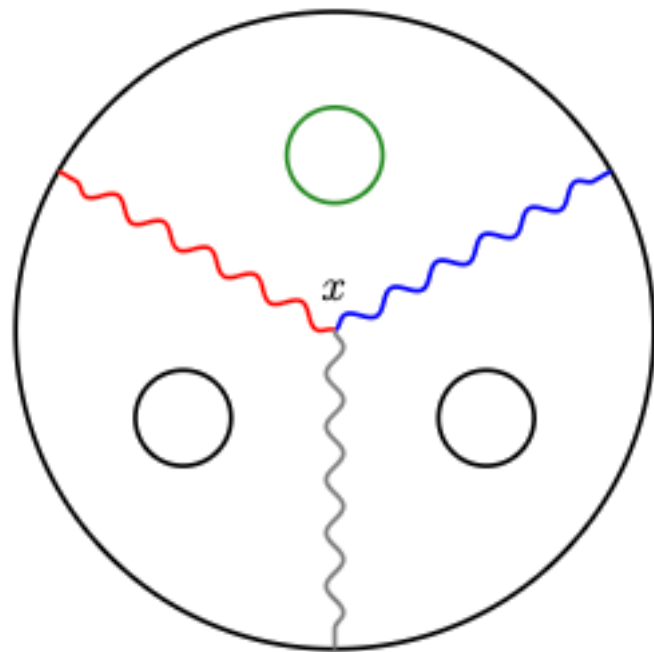


The schottky double has genus N (branched cover of the torus), with period matrix :

$$\tau_{rs} = \frac{1}{N} \sum_{k=1}^N \frac{B_k}{A_k} \cos \left(\frac{2\pi k(s-r)}{N} \right)$$

in terms of the periods $A_k = \oint_{\mathcal{A}} \eta_k$, $B_k = \oint_{\mathcal{B}} \eta_k$.

$$\eta_k = \frac{\vartheta_1(z - x_k)}{\vartheta_1(z - x)^{k/N} \vartheta_1(z + \bar{x})^{(N-k)/N}} dz$$



The replicated surface Σ_N

Replicated target space

from entanglement entropies to the cyclic orbifold

The cyclic orbifold picture

Revisiting the replica trick : **space-time** $\rightarrow$ **degrees of freedom**

Idea : push the lattice (which is embedded in a branched cover of say $\mathbb{CP}^1$) down to $\mathbb{CP}^1$

This is a regular square lattice, but

- each site has **N degrees of freedom**

$$S = (s_1, \dots, s_N)$$

- locally degrees of freedom coming from different layers do not interact
- except through a defect line (where the branch cut used to be)

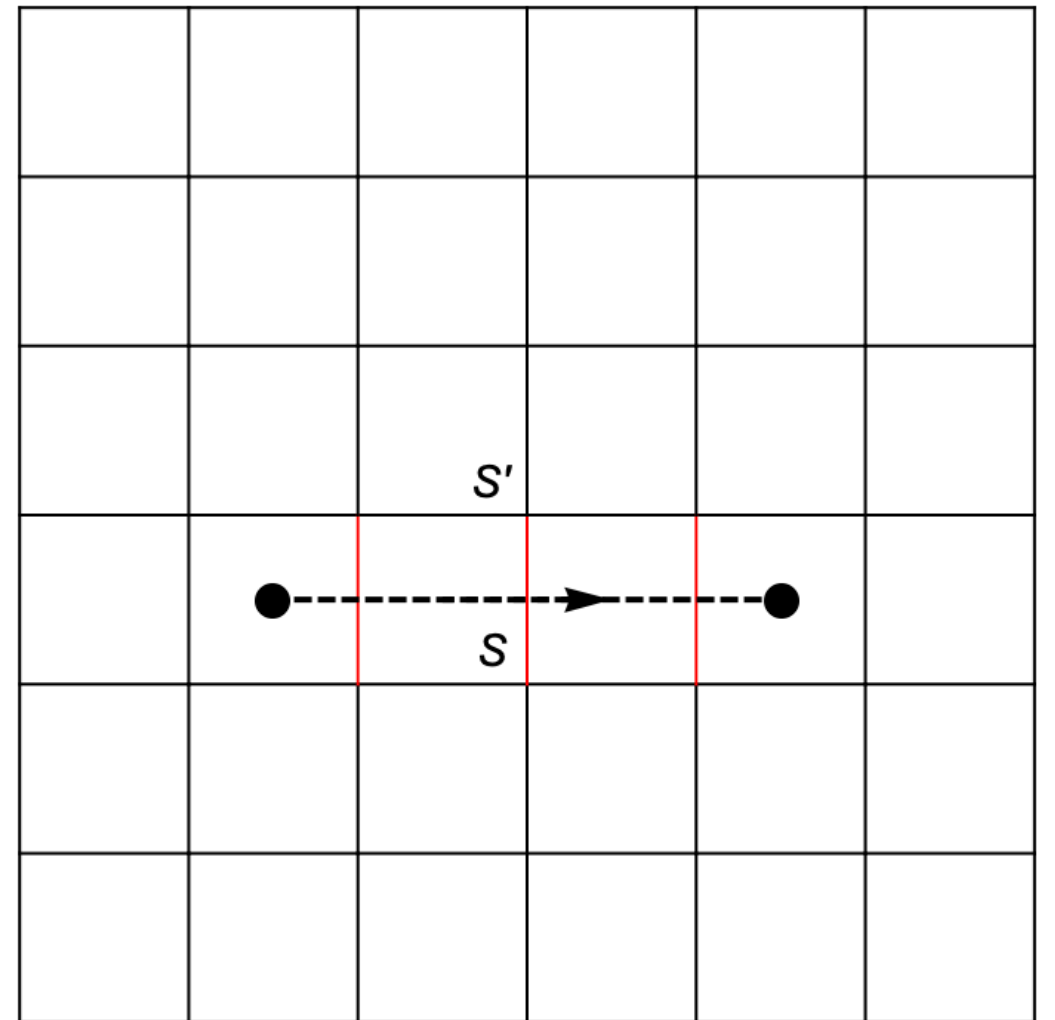
The edge interactions are

- for normal edges (black)

$$\exp \left(-J \sum_{a=1}^N s_a s'_a \right)$$

- for red edges

$$\exp \left(-J \sum_{a=1}^N s_a s'_{a+1} \right)$$

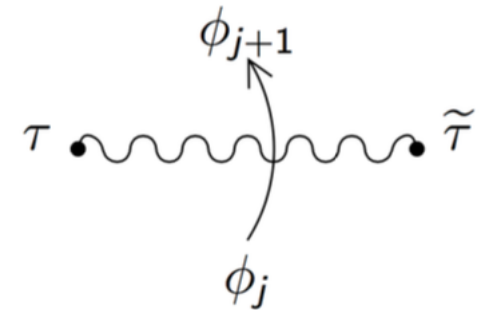


Analogous to the Kadanoff-Ceva construction of disorder operators in Ising.

The cyclic orbifold picture

Revisiting the replica trick : **space-time** $\rightarrow$ **degrees of freedom**

The inclusion of these defects requires the existence of **new local fields called twist fields** τ and $\tilde{\tau}$, located at the extremities of open-ended defect lines.



In turn, the state-operator correspondence implies the existence of twisted sectors, which describe states in radial quantization around a twist field, that is in the presence of a defect line along the radial/time direction.

In practice, we have traded **geometric defects** (conical singularities) for **algebraic defects** (twist operators) τ

The Rényi entropy ratio of partition functions becomes a correlation of twist fields

$$\frac{Z(\Sigma_N)}{Z(\Sigma)^N} = \left\langle \tau(u_1) \tilde{\tau}(v_1) \cdots \tau(u_p) \tilde{\tau}(v_p) \right\rangle_{\Sigma}$$

Hilbert space of the orbifold

the untwisted sector

[Crnković-Sotkov-Stanishkov (89)], [Klemm-Schmidt (90)], [Borisov-Halpern-Schweigert (98)]

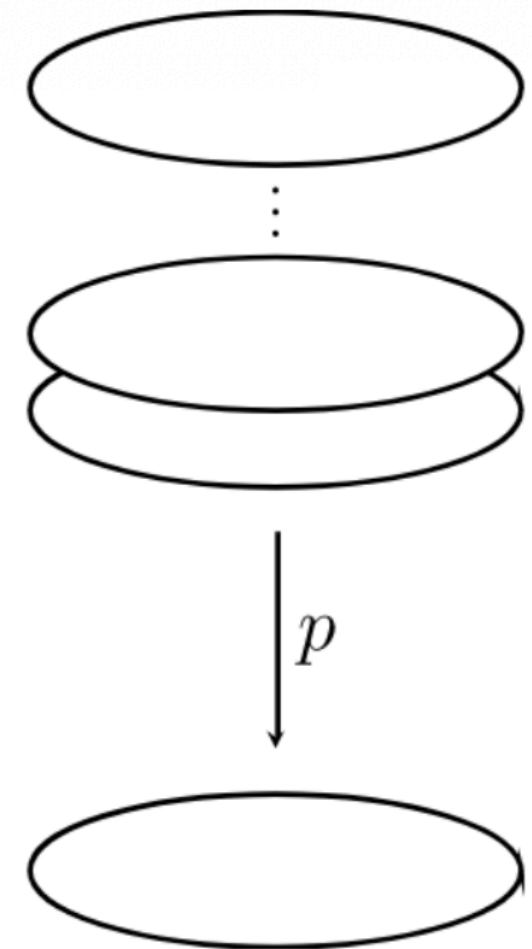
- N copies of the mother theory
- Fields $(\phi_1, \dots, \phi_N)$
- N stress-energy tensors $T^{(j)}(z)$

Defining

$$\hat{T}^{(r)}(z) = \sum_j e^{\frac{2i\pi jr}{N}} T^{(j)}(z)$$

we get the orbifold algebra

$$[\hat{L}_n^{(r)}, \hat{L}_m^{(s)}] = (n - m) \hat{L}_{n+m}^{(r+s)} + \frac{Nc}{12} n(n^2 - 1) \delta_{n+m,0} \delta_{r+s,0}$$



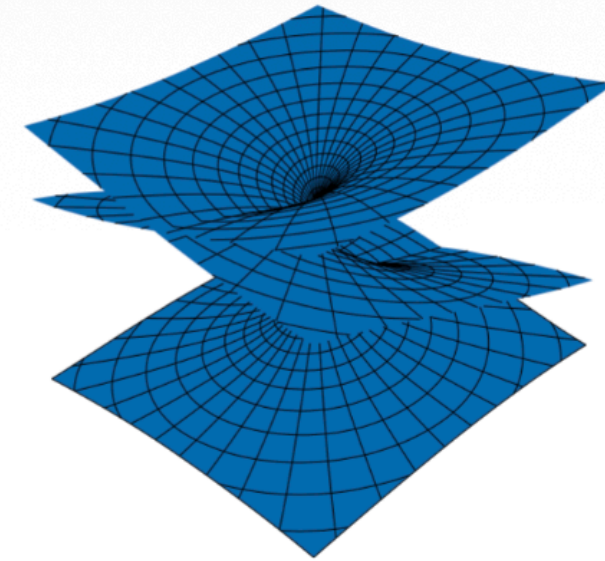
$\hat{T}^{(0)}(z)$: stress-energy tensor, $\hat{T}^{(r \neq 0)}(z)$: additional currents

The twisted sector

radial quantization around a conical singularity

Twisted sector : the $\mathbb{Z}_N$ orbifold is related to the mother theory through unfolding ($z = w^N$)

$$\begin{aligned}\hat{T}(z) &= \left(\frac{dz}{dw}\right)^{-2} \left[T(w) - \frac{c}{12} \{z, w\} \right] \\ &= \frac{1}{N} \sum_{u \in \mathbb{Z}/N} z^{-u-2} \hat{L}_u\end{aligned}$$



$$\boxed{\hat{L}_u = \frac{1}{N} L_{Nu} + \frac{c}{24} \left(N - \frac{1}{N} \right) \delta_{u,0}} \quad \Rightarrow \quad \hat{L}_0 = \frac{1}{N} L_0 + \frac{c}{24} \left(N - \frac{1}{N} \right)$$

$\hat{T}(z)$ splits into N currents $\hat{T}^{(r)}(z)$ with a definite monodromy

$$\hat{T}^{(r)}(z) = \sum_{u=r/N \bmod 1} z^{-u-2} \hat{L}_u, \quad T^{(r)}(e^{2i\pi} z) |\tau\rangle = e^{-\frac{2i\pi r}{N}} T^{(r)}(z) |\tau\rangle$$

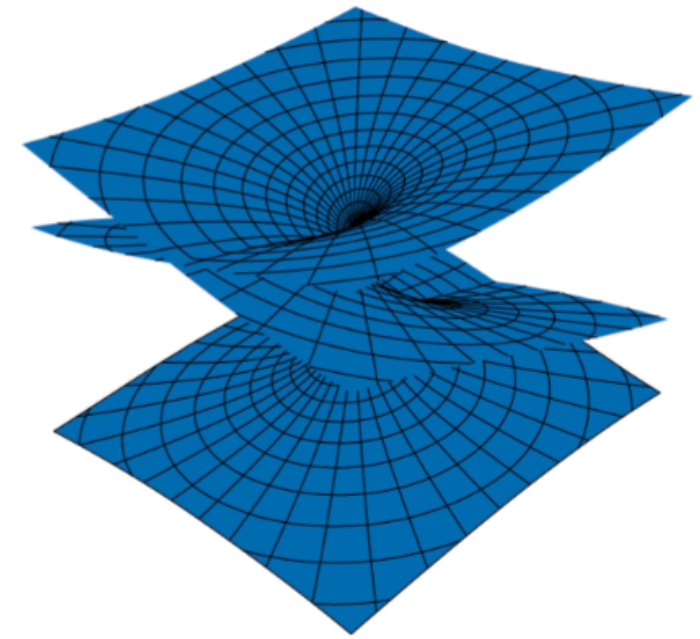
Induction procedure

[Borisov-Halpern-Schweigert (98)]

Start from the mother theory with state $|\phi_i\rangle$.
Then fold $\Rightarrow$ field $|\tau_i\rangle$ in the twisted sector

$$\hat{\Delta}_i = \frac{\Delta_i}{N} + \frac{c}{24} \left(N - \frac{1}{N} \right)$$

In particular $\hat{\Delta}_0 = \frac{c}{24} \left(N - \frac{1}{N} \right)$.



This induction also applies to null vectors

For instance if $(L_{-2} + gL_1^2) |\phi_i\rangle = 0$ then

$$\left(N\hat{L}_{-2/N} + gN^2\hat{L}_{-1/N}^2 \right) |\tau_i\rangle = 0$$

The $\mathbb{Z}_N$ cyclic orbifold is rational as long as the mother theory is !

Example : the Yang-Lee CFT

The Yang-Lee CFT is one of the simplest minimal models.

- contains only two primary fields 1 and ϕ
- null vectors of the vacuum :

$$L_{-1} |0\rangle = 0, \quad (3L_{-4} - 5L_{-2}^2) |0\rangle = 0$$

$\mathbb{Z}_2$ cyclic orbifold

10 primary fields $(1 \otimes 1)^{0/1}$, $1 \otimes \phi \pm \phi \otimes 1$, $(\phi \otimes \phi)^{0/1}$, τ and τ_ϕ .
Induction procedure :

$$L_{-1/2} |\tau\rangle = 0, \quad (3L_{-2} - 10L_{-1}^2) |\tau\rangle = 0$$

For instance $F(z, \bar{z}) = \langle \tau(\infty) \tau(z, \bar{z}) \tau(1) \tau(0) \rangle$ obeys

$$33F + 120z(z-1)(2z-1)\partial F + 400(z-1)^2 z^2 \partial^2 F = 0$$

Example : the Yang-Lee CFT

Hypergeometric equation $\Rightarrow$ two solutions :

$$F_1(z) = z^{33/60} (1-z)^{3/20} {}_2F_1 \left[\begin{matrix} 3/10, 7/10 \\ 3/5 \end{matrix}; 1-z \right]$$

$$F_2(z) = z^{3/20} (1-z)^{33/60} {}_2F_1 \left[\begin{matrix} 3/10, 7/10 \\ 7/5 \end{matrix}; 1-z \right]$$

Bootstrap

The monodromy imposes:

$$F(z, \bar{z}) = \sum_i X_i |F_i(z)|^2$$

$$z \rightarrow (1-z) \quad F_i \rightarrow \sum_j A_{i,j} F_j$$

$$\sum_i X_i A_{i,1} A_{i,2} = 0 \quad \Rightarrow \quad \frac{X_1}{X_2} = 2^{\frac{16}{5}}$$

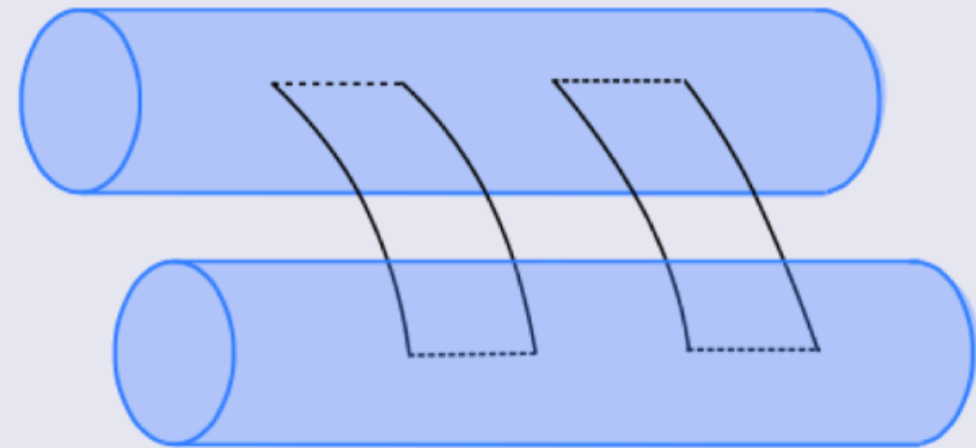
$$F(z, \bar{z}) \propto 2^{\frac{16}{5}} |F_1(z)|^2 + |F_2(z)|^2$$

Example : the Yang-Lee CFT

Mapping from the torus (τ) to the branched sphere (x)

Link between z and the modular parameter $q = e^{i\pi\tau}$:

$$z = \left(\frac{\theta_4(\tau)}{\theta_3(\tau)} \right)^4 = \prod_{n=1}^{\infty} \left(\frac{1 - q^{2n-1}}{1 + q^{2n-1}} \right)^8$$



Modular characters of the Yang-Lee primaries:

$$\chi_{3,1}(\tau) = 2^{2/15} F_1(z) \quad \chi_{4,1}(\tau) = 2^{-22/15} F_2(z)$$

$$Z(\tau) \propto \left(2^{16/5} |F_1(z)|^2 + |F_2(z)|^2 \right)$$

Partition function of the Yang-Lee model [Eguchi '88]

The torus partition function is recovered through a purely algebraic approach.

Example : Ising

Ising CFT

Null vectors of the identity:

$$L_{-1}\mathbb{1} = 0, \quad (108L_{-6} + 264L_{-4}L_{-2} - 93L_{-3}^2 - 64L_{-2}^3)\mathbb{1} = 0.$$

Ising orbifolded, $N = 2$

Induction procedure:

$$L_{-1/2}\tau_1 = 0 \quad (54L_{-3} + 264L_{-2}L_{-1} - 93L_{-3/2}^2 - 128L_{-1}^3)\tau_1 = 0$$

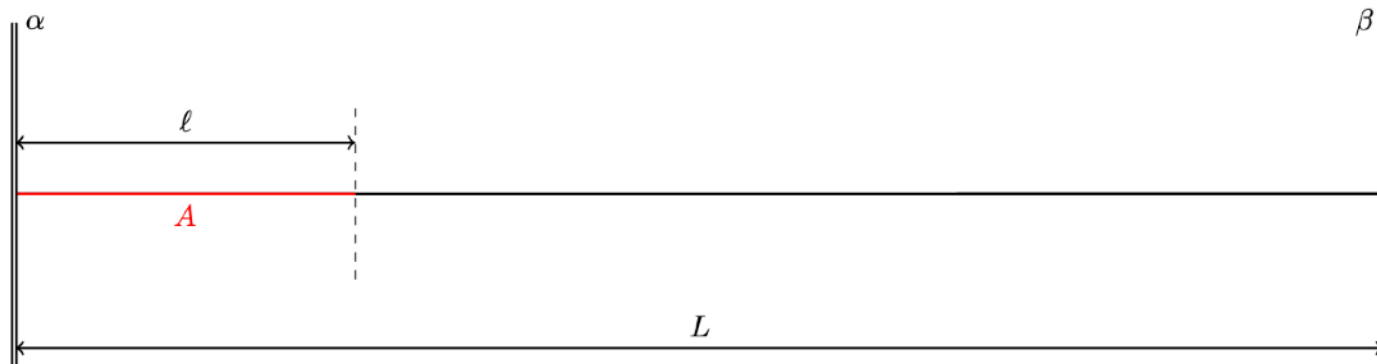
If $u(z) = \langle \tau_1(0)\tau_1(z)\tau_1(1)\tau_1(\infty) \rangle$:

$$128u^{(3)}(x) + \frac{3(16x(176xu''(x) + 5u'(x)) + 5u(x))}{32x^3} = 0$$

Solutions : characters of the Ising model on the torus

Another example : different b.c.

[Estienne, Ikhlef, Rotaru (2021)]



$$\frac{Z(\Sigma_N)}{Z(\Sigma)^N} = \langle \tau(x) \rangle_{\mathbb{H}}^{\alpha\beta}$$

One twist field in the bulk and two boundary condition changing operators $\Psi^{\alpha\beta}$.

- For $\alpha = \beta$, there is no BCCO, and the computation is straightforward.

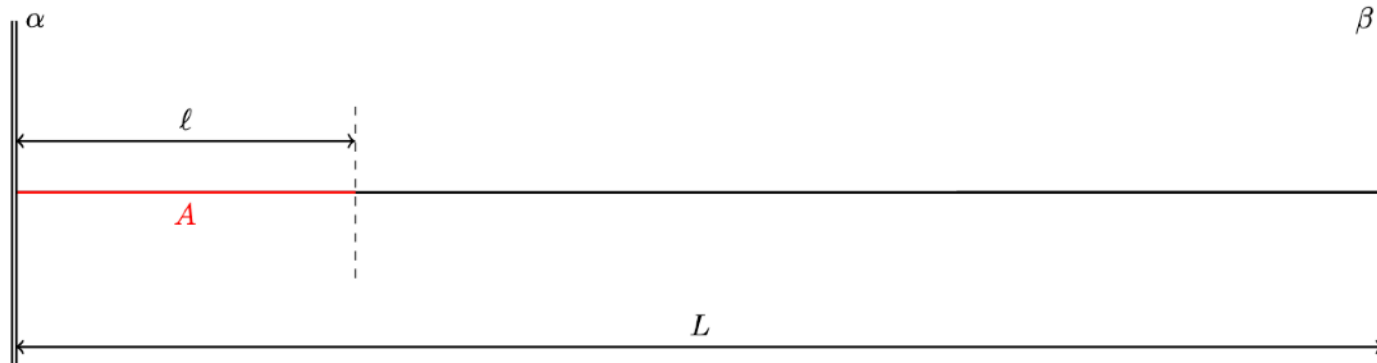
$$S_N^{(\alpha\alpha)} = \frac{c}{12} \frac{N+1}{N} \log \left(\frac{2L}{\pi} \sin \frac{\pi\ell}{L} \right) + \log g_\alpha$$

- For $\alpha \neq \beta$ the expression is generally unknown, but is of the form

$$S_N^{(\alpha\beta)} = \frac{c}{12} \frac{N+1}{N} \log \left(\frac{2L}{\pi} \sin \frac{\pi\ell}{L} \right) + \frac{1}{1-N} \log F_N^{(\alpha\beta)}(\ell/L)$$

where $F_N^{(\alpha\beta)}$ is a universal scaling function, interpolates between g_α^{1-N} and g_β^{1-N} .

Another example : different b.c.



$$\frac{Z(\Sigma_N)}{Z(\Sigma)^N} = \langle \tau(x) \rangle_{\mathbb{H}}^{\alpha\beta}$$

The BCCO in the orbifold is of the form $\Psi^{(\alpha\beta)} = \psi^{(\alpha\beta)} \otimes \cdots \otimes \psi^{(\alpha\beta)}$ where $\psi^{(\alpha\beta)}$ is a BCCO in the mother theory.

Assuming $\psi^{(\alpha\beta)}$ is degenerate at level 2 :

$$L_{-1}^2 \psi^{(\alpha\beta)} = g L_{-2} \psi^{(\alpha\beta)}$$

one can derive a ODE for the scaling function, at least for $N = 2$ and $N = 3$. For $N = 2$ the solution is simple, in terms of ${}_2F_1$ hypergeometric functions. For $N = 3$

$$4AB(6A + B + 3)(1 - 2\zeta)F(\zeta) + 2[9AB - C\zeta(1 - \zeta)]F'(\zeta) \\ - 9(4A + B - 2)(1 - 2\zeta)\zeta(1 - \zeta)F''(\zeta) + 18\zeta^2(1 - \zeta)^2F'''(\zeta) = 0,$$

$$A = g - 1, \quad B = 6g - 5, \quad C = 36A^2 + 2B^2 + 30AB - 18A - 3B.$$

Conclusion

Entanglement entropy in CFT sits at the intersection of symmetry, geometry, and the algebraic structure of quantum field theory.

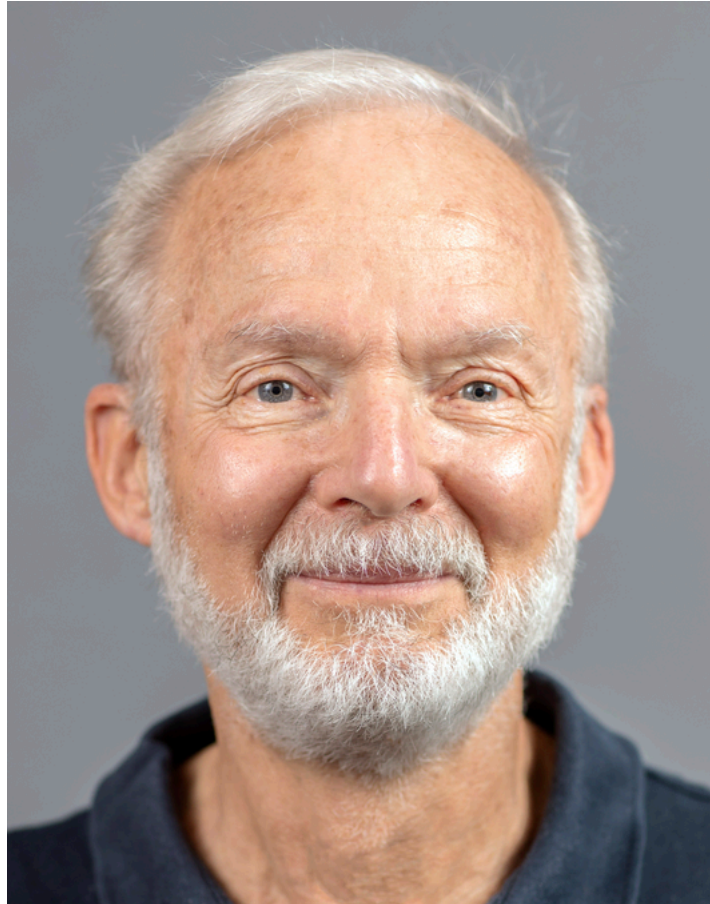
This is topic in the footsteps of Jean-Bernard 's work: extracting universal quantum information from conformal symmetry, geometry, and representation theory.

CFT is a powerful framework to compute entanglement entropy in 1D critical systems, but beyond a few simple cases the computations remain difficult. Two complementary approaches

(i) **the geometric approach** $\frac{Z(\Sigma_N, f^*g)}{Z(\Sigma, g)^N}$ partition function on branched cover.

(ii) **the algebraic approach** (cyclic orbifold). In the rational case, there is hope to derive ODEs for the conformal block.

Perhaps the reduced density matrix can be understood directly within the framework of Segal's amplitude (entanglement Hamiltonian)



80 years of critical thinking
... but not necessarily a conformal character.