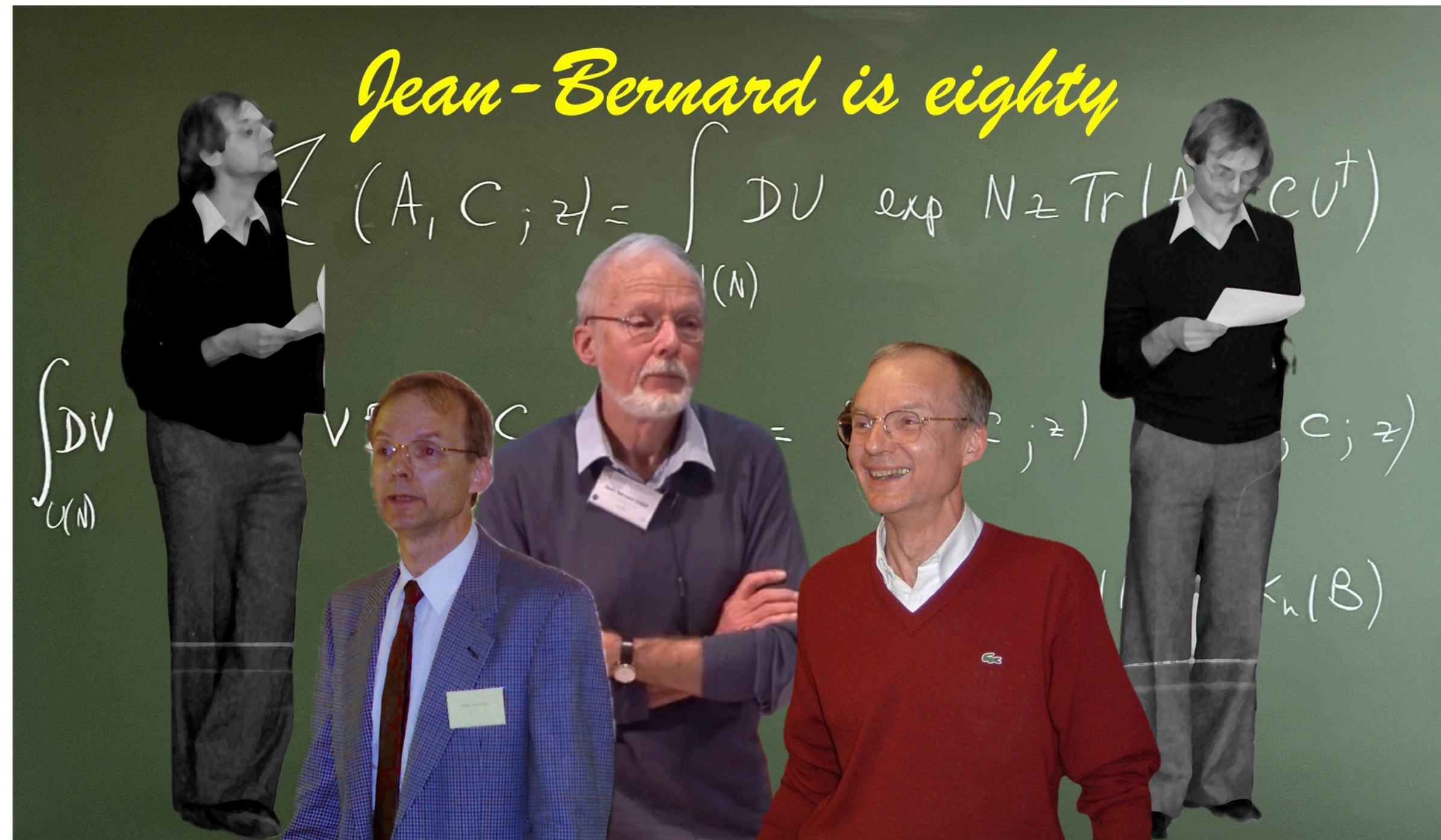


Jean-Bernard and his defects



Costas Bachas (*LPENS*)

Paris, 24-25 September 2026

Like Andrea, I first "met" JB on the cover of the famous Itzykson-Zuber book on *Quantum Field Theory*. This was circa 1980 and the first english edition of the book had just come out.

It is hard to overestimate the importance of this book for the training of an entire generation of theorists. The available textbooks at that time (Bjorken-Drell, Bogoliubov-Shirkov, Landau-Lifshitz, Schwinger) dated from the 60s or before, much had happened in between and one had to learn it from the Les Houches 75, Coleman's Erice lectures or articles. This was great but challenging reading for a beginning grad student.

So the Itzykson-Zuber book was a true relief, and it became a bedside companion.

So when I arrived in France I considered JB as one of my teachers, which means with respects but also apprehension. After some time I started to know JB better, especially after 2004 when we became neighbours in LPTHE-LPTENS

- DEA de Physique Théorique: JB was teaching **group theory**, & I was teaching his book ;
- FRIF: JB led it with his typical generosity and commitment; it made our shared LPTHE-LPTENS scientific life much much better for many years
- Joint seminars: to which JB & later Denis brought his broad knowledge and sharp questions

Over all these years I truly came to appreciate JB for his high standards, low profile, gentleness and generosity.

If he has any real defects I could not localize them -- they must be topological.

So thank you very much JB and stay young for many many more years.

Everyone knows JB's classic work on the **planar limit** of matrix models, and the celebrated **ADE classification** of modular invariant partition functions.

Less known but equally important was his 2000 paper with [Valentina Petkova](#) .
Let me try to put this work in context :

Magnetic impurities and Wilson lines played a key role in the early development of QFT.



Kondo RG flow



*order parameter
for confinement*

Nowadays defects have moved to center-stage.

But it was a niche subject in the '90s, mostly pursued by 2D-cond-mat practitioners

[Cardy](#); [Kane-Fisher](#); [Affleck-Ludwig-Oshikawa](#); [Saleur](#); [Fendley](#) . . .

These cond-mat defects were integrable or conformal; those in [Petkova, Zuber 0011021](#) had more symmetry, they were topological (they commuted with $\mathcal{A} \times \overline{\mathcal{A}}$ of a RCFT₂)

P+Z were the first to study topological twists that could be consistently inserted in the torus partition function, and showed that they obey the Verlinde algebra (whence they are called Verlinde lines, see also [Ocneanu; Coquereaux-Schieber](#))

Identified as *fine-tuned* local defects in $\left\{ \begin{array}{l} \text{Ising: } \textit{special values of marginal couplings} \\ \text{Frohlich, Fuchs, Runkel, Schweigert 0404051} \\ \text{WZW: } \textit{fixed point of Kondo flows} \\ \text{ } \textit{\in enveloping algebra of Kac-Moody} \end{array} \right.$

[CB, Gaberdiel 0411067; Alekseev, Monnier 0702174](#)

Witten '88 had earlier explained how at low energies there can be life between the empty theory and a CFT: *it is described by a TQFT*. Topological defects probe such topological order. Their study has since expanded much beyond the original scope:

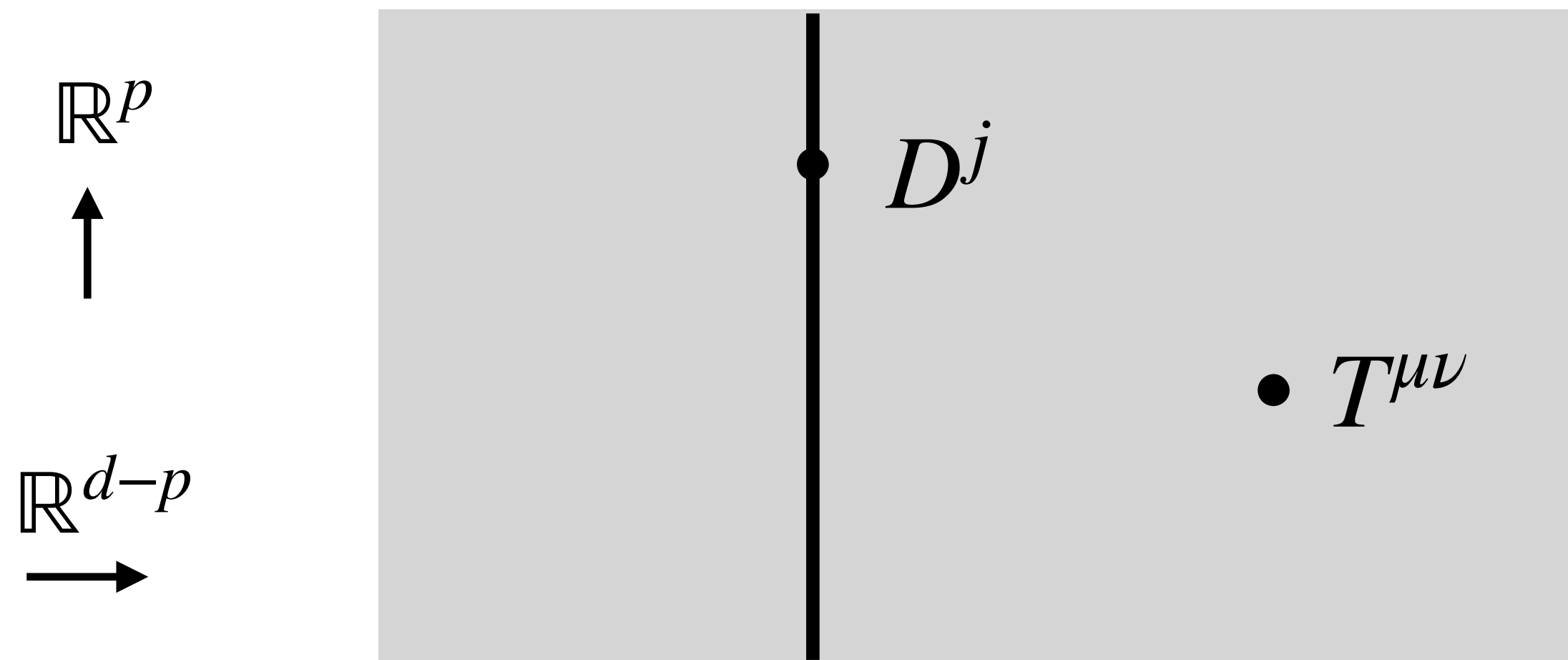
- Generalized & non-invertible symmetries in $d > 2$ Gaiotto, Kapustin, Seiberg, Willet 1412.5148
- Topological lines as logical q-bits Kitaev's surface codes; . . .
- Fusion categories Fuchs, Schweigert; . . .

. . . .

(one more example) of how good ideas can take on a life of their own

I'm afraid I have nothing more to say today about topological defects.
For something more recent let me discuss in the rest of this talk

A curious superconformal-defect identity



Defect CFTs contain two universal operators

- the (bulk) **energy-momentum tensor** $T^{\mu\nu}$
- the (defect) **displacement operator** D^j

For a planar defect, $\langle T^{\mu\nu} \rangle$ and $\langle D^j D^k \rangle$ are fixed by symmetry modulo two parameters

$$\langle T^{\alpha\beta}(x) \rangle = a_T \left(\frac{d-p-1}{d} \right) \frac{\eta^{\alpha\beta}}{|x_\perp|^d}$$

$$\langle T^{ij}(x) \rangle = a_T \left[\frac{x^i x^j}{|x_\perp|^{d+2}} - \left(\frac{p+1}{d} \right) \frac{\delta^{ij}}{|x_\perp|^d} \right]$$

$$\langle D^j(x) D^k(y) \rangle = \frac{C_D}{|x-y|^{p+1}}$$

a_T : energy density

[defined for $d-p>1$]

C_D : displacement norm

[defined for $p>0$]

The broken Ward identity $\partial_\mu T^{\mu j} = \delta^{(d-p)}(x_\perp) D^j$ fixes the scaling dimension $\Delta_D = p + 1$ but gives invariant meaning to the *normalization* of the displacement operator.

This conformal data controls many physical quantities, including defect Weyl anomalies, entanglement entropy, energy transmission coefficients etc. No time here.

$$\text{unitarity} \implies C_D \geq 0 \quad , \quad \text{positive E} \implies a_T \leq 0$$

Otherwise these two parameters are, in general, free & independent

But **supersymmetry** relates them:

$$\frac{C_D}{a_T} = - \frac{2(d-1)(p+2)\Gamma(p+1)}{d \pi^{p-d/2} \Gamma(\frac{p}{2} + 1) \Gamma(\frac{d-p}{2})} \quad \text{✿}$$

First noticed for line defects ($p = 1$) in $d = 4$ by [Lewkowycz, Maldacena 1312.5682](#)

& proved by [Bianchi, Lemos, Meineri 1805.0411](#)

Conjectured for all (p, d) by interpolating special cases in [Bianchi, Lemos 1911.05082](#)

Proof completed (*using classification of superconformal defects & Ward identities*) by

[CB, Bianchi, Chen 2501.13197](#)

defect	$(d, \mathcal{N})$	superalgebra	p -embedding
line	(3,2)	$\mathfrak{osp}(2 4; \mathbb{R})$	$\mathfrak{su}(1, 1 1) \oplus \mathfrak{u}(1)_c$
	(4,2)	$\mathfrak{su}(2, 2 2)$	$\mathfrak{osp}(4^* 2)$
	(5,1)	$F(4; 2)$	$D(2, 1; 2; 0) \oplus \mathfrak{su}(2)_c$
surface	(4,1)	$\mathfrak{su}(2, 2 1)$	$\mathfrak{su}(1, 1 1) \oplus \mathfrak{su}(1, 1)_c \oplus \mathfrak{u}(1)_c$
	(5,1)	$F(4; 2)$	$D(2, 1; 2; 0) \oplus \mathfrak{so}(2, 1)_c$
	(6,1)	$\mathfrak{osp}(8^* 2)$	$\mathfrak{osp}(4^* 2) \oplus \mathfrak{so}(2, 1)_c \oplus \mathfrak{so}(3)_c$
$p = 3$	(5,1)	$F(4; 2)$	$\mathfrak{osp}(2 4; \mathbb{R}) \oplus \mathfrak{u}(1)_c$
$p = 4$	(6,1)	$\mathfrak{osp}(8^* 2)$	$\mathfrak{su}(2, 2 1) \oplus \mathfrak{u}(1)_c$

Table 1. The minimal supersymmetric DCFTs discussed in the paper. The second column gives the smallest $\mathcal{N}$ that a d -dimensional SCFT must have to admit p -dimensional superconformal defects. The corresponding bulk superalgebras and maximal p -embeddings are given in the third and fourth columns.

Minimally supersymmetric DCFTs in $d > 2$ dimensions & $\text{codim} > 1$

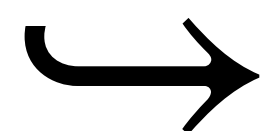
The rest of the proof is technical and tedious. A key ingredient is the correlator $\langle T^{\mu\nu} D^j \rangle$ which has no new free parameters besides a_T and C_D

Billo, Gonçalves, Lauria, Meineri 1601.02883

I will skip the details of the proof, and explain in the rest of this talk the implications of this identity in two different contexts:

- Radiation from an accelerated charge
- Tension of the dual holographic brane

Let us start with a simple example of line defect



A simple line defect + radiation

$$\mathcal{N} = 2 \text{ Maxwell CFT}_4$$

Bulk: A free vector, two scalars (plus fermions)

$$T_{\mu\nu}^{(s)} = \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} \eta_{\mu\nu} |\partial \phi|^2 + \frac{1}{6} (\eta^{\mu\nu} \square - \partial^\mu \partial^\nu) \phi^2$$

$$T_{\mu\nu}^{(v)} = F_\mu^\rho F_{\nu\rho} - \frac{1}{4} \eta_{\mu\nu} |F|^2$$

Line defect: $W = P \exp \left[\int ds (ie A_\mu \dot{Y}^\mu + g |\dot{Y}| \phi) \right]$

Describes a heavy external particle with worldline $Y^\mu(s)$, charge e and coupling g to a scalar.

A static defect is conformally invariant. It is **superconformal** iff $|e| = |g|$

The defect sources classical fields $\phi_{\text{class}} = \frac{g}{4\pi |x_{\perp}|}$ and $F_{\text{class}}^{0j} = \frac{ex^j}{4\pi |x_{\perp}|^3}$

Inserting in $T^{\mu\nu}$ gives

$$a_T^{(s)} = -\frac{g^2}{48\pi^2} \quad ; \quad a_T^{(v)} = -\frac{e^2}{16\pi^2}$$

Varying $Y^j(s)$ gives the displacement field $D^j = eF^{0j} + g \partial^j \phi \Big|_{\text{defect}}$ from which we get

$$C_D^{(s)} = \frac{g^2}{2\pi^2} \quad ; \quad C_D^{(v)} = \frac{e^2}{\pi^2}$$

In general $\frac{C_D}{a_T}$ depends on $\left| \frac{e}{g} \right|$; for susy defects it obeys the $p = 1, d = 4$ identity

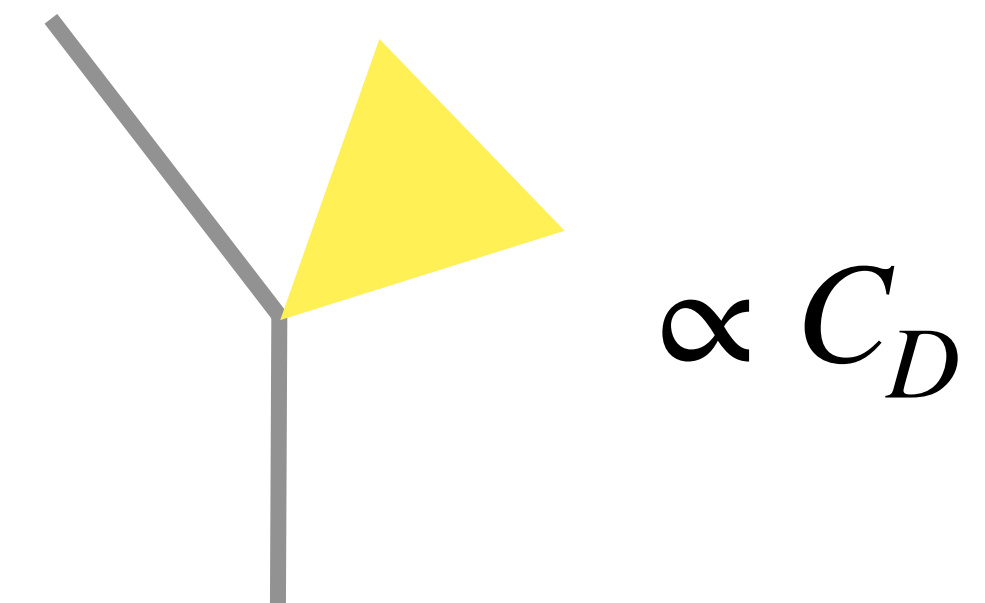
$C_D = -18a_T$



The displacement norm is related to the *Bremsstrahlung function* $E_{\text{rad}} = 2\pi B \int dt \dot{u}^2$

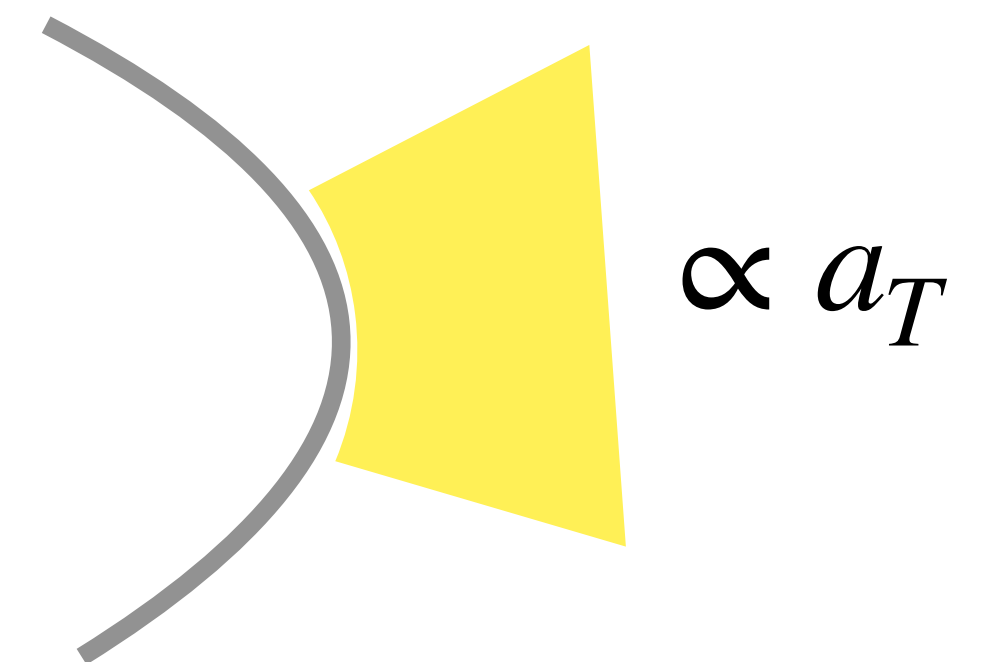
$$B = \frac{C_D}{12} \quad \left(= \frac{e^2}{12\pi^2} \text{ for Maxwell} \right)$$

Correa, Henn, Maldacena, Sever 1202.4455



But one may also compute the radiated energy by measuring $\langle T^{\mu\nu} \rangle_{\text{acc}}$
for a uniformly-accelerating charge, which is *conformal* to a static one

Fiol, Garolera, Lewkowycz 1202.5292



The two results disagree as seen by the fact that $\frac{C_D}{a_T}$ is not universal

Where is the catch ? For N=4 SYM M+L noticed that one can reconcile the results by modifying $T^{\mu\nu}$ so as to remove the singular stress $\langle T^{ij} \rangle$:

$$T^{\mu\nu} \longrightarrow \tilde{T}^{\mu\nu} = T^{\mu\nu} + \xi (\eta^{\mu\nu} \square - \partial^\mu \partial^\nu) \mathcal{O}$$

conserved but
not traceless

Here $\mathcal{O}$ is the scalar in the em-supermultiplet with dimension $\Delta_{\mathcal{O}} = d - 2$

But the recipe is not necessary for Maxwell, where the calculations agree even though $\langle T^{ij} \rangle \neq 0$ & does not work for the free-scalar theory, where $\langle \tilde{T}^{ij} \rangle$ can be made to vanish with $\mathcal{O} = \phi^2$ and $\xi = -\frac{1}{24}$, and yet the calculations disagree.

So the source of the discrepancy must be elsewhere.

In the free-scalar-vector CFT the choice that removes the total stress is $\xi = -\frac{g^2 + 3e^2}{24g^2}$
 Supersymmetry implies $\xi = -\frac{1}{6}$ so the subtraction *also removes the improvement term*
 in the scalar em-tensor. This term *violates the Null Energy Condition* & gives a $\propto \dot{a}^j$
 radiation contribution that does not integrate out for uniformly-accelerating motion

Fiol, Martinez-Montoya 1907.08161

This can be generalized to any superconformal line defect. The problematic term is the leading lightcone singularity of the correlator

$$\langle T^{0i}(x) D^j(y) \rangle = \frac{(x^0 - y^0)}{|x_\perp|^3 (x - y)^6} \left(-\mathfrak{b}_3 \delta^{ij} |x_\perp|^2 + (\mathfrak{b}_3 - 2\mathfrak{b}_1) x^i x^j + 4\mathfrak{b}_1 \frac{x^i x^j |x_\perp|^2}{(x - y)^2} \right)$$

$$\text{with } \mathfrak{b}_3 = \frac{8}{\pi} a_T, \quad \mathfrak{b}_1 = -\frac{3}{2\pi} (C_D + 16a_T)$$

Modifying the em-tensor so that $\langle \tilde{T}^{ij} \rangle = 0$ makes the coefficient of this term $\propto C_D + 18a_T$, so it vanishes by the superconformal identity. **Thus one subtraction kills both birds.**

CB, Bianchi, Chen 2501.13197

The argument extends to any p but with one caveat: for $\mathcal{N} = 2$ CFT₃ and $\mathcal{N} = 1$ CFT₄ the em-tensor multiplet has no scalar.

An old question: Can the DCFT be the limit of a consistent theory with *dynamical defects*?

infinite mass?
finite AdS throat?

A priori not { Problems of the Abraham-Lorentz-Dirac equation (runaway, Poincaré stress, . . .)
Collapse of bound states → Quantum Revolution → Second quantization

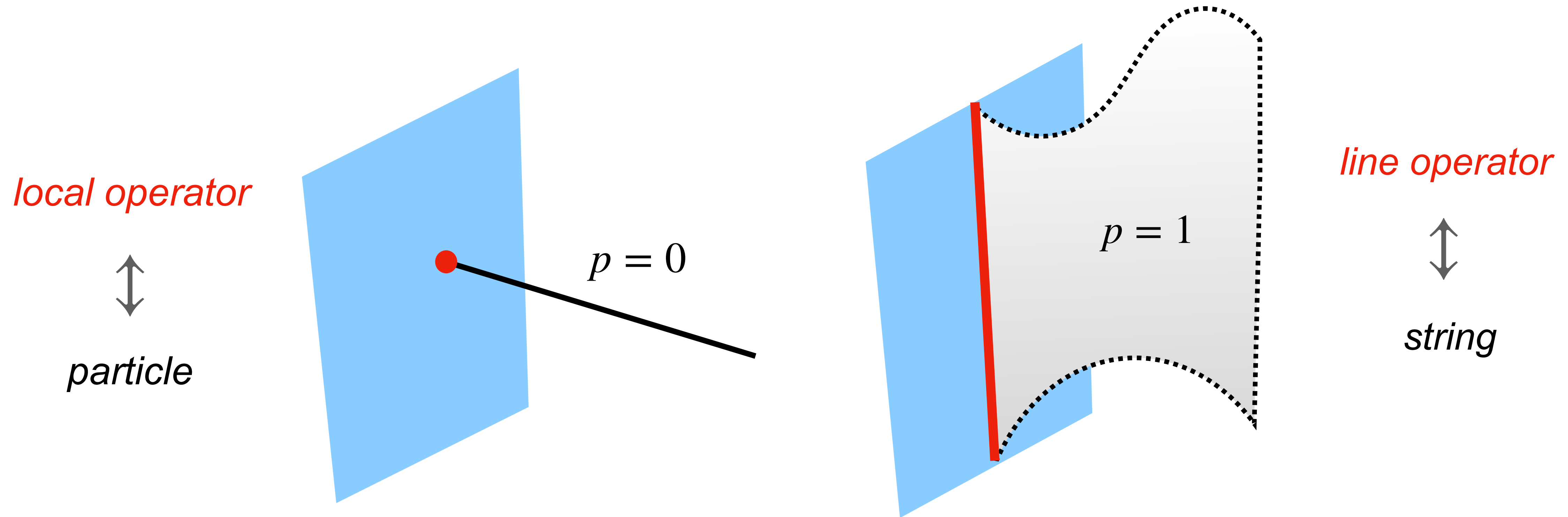
If Maxwell's theory was supersymmetric, some of these problems would not have occurred:

no static force, no stress

Is there a consistent super ALD equation ?

The tension of holographic branes

In **AdS/CFT** a p -dimensional defect is the imprint of the worldvolume of a p -brane on the boundary of AdS_{d+1}



Question: How does the brane tension σ translate to DCFT data?

Consider the known case $p = 0$: particle $\leftrightarrow$ local operator $\mathcal{O}$

For a *free* scalar particle Maldacena '97

$$\Delta_{\mathcal{O}} = \sqrt{m_0^2 + \left(\frac{d}{2}\right)^2} + \frac{d}{2} = m_0 \left[1 + \frac{d}{2m_0} + \dots \right] = m_0 + \boxed{O\left(\frac{\lambda_{\text{Compton}}}{L_{\text{AdS}}}\right)}$$

q-particle in a box

Turning on gravitational self-interactions gives new corrections even for $m_0 \gg 1$

$$\Delta_{\mathcal{O}} = m_0 \left[1 + \# G_N m_0 + \# (G_N m_0)^2 + \dots \right] = m_0 + \boxed{O\left(\frac{r_{\text{Schwarzschild}}}{L_{\text{AdS}}}\right)}$$

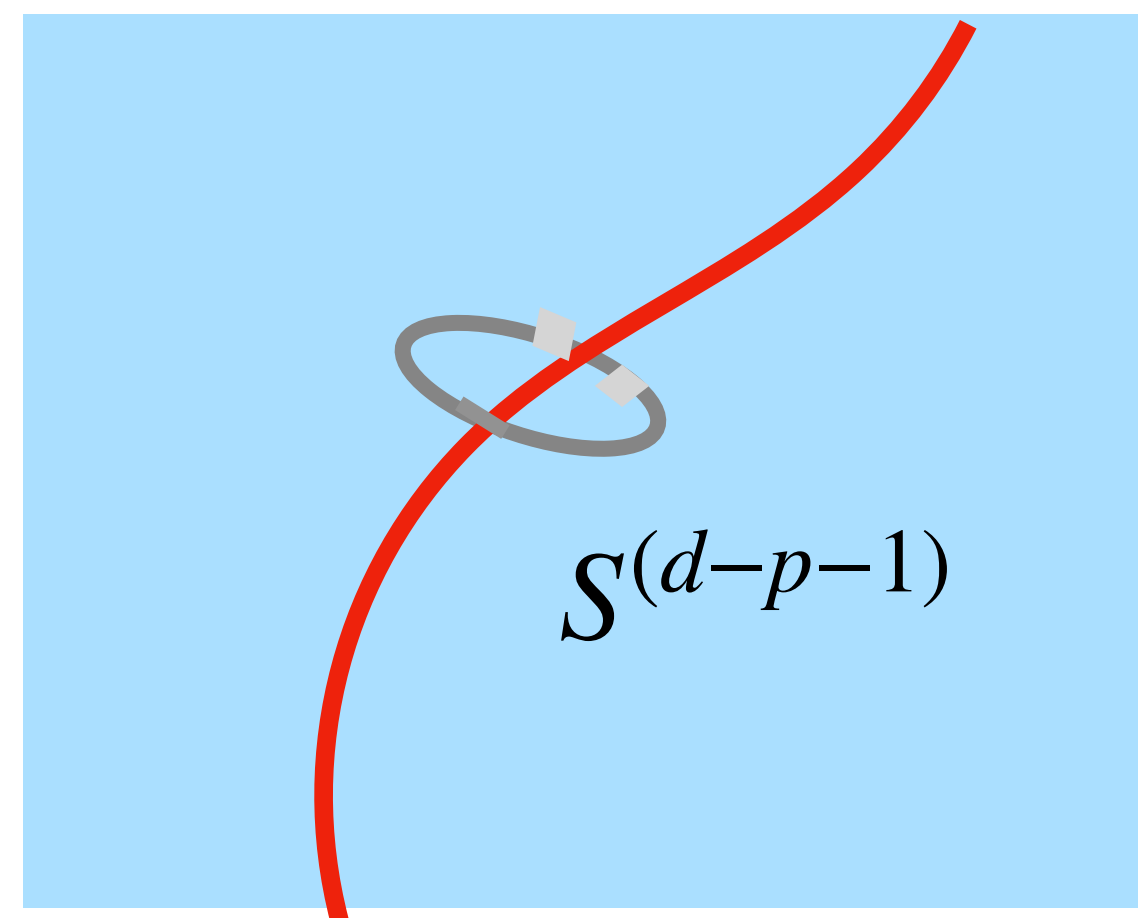
black hole in a box

Thus $\Delta_{\mathcal{O}} \simeq m_0$ only in the classical probe-particle limit. More generally m_0 has no invariant meaning. The invariant ADM-like mass is the CFT energy in radial qnz. $\Delta_{\mathcal{O}}$

The ADM mass is given by the asymptotic behaviour of the graviton $\langle h_{\mu\nu} \rangle \leftrightarrow \langle T_{\mu\nu} \rangle_{\text{CFT}}$

Indeed
$$m_{\text{ADM}} = \Delta_{\mathcal{O}} = \frac{2\pi^{d/2}}{\Gamma(d/2)} a_T$$

For $p > 0$ one may by analogy define an *invariant gravitational tension* from the dilatation current at the boundary of AdS

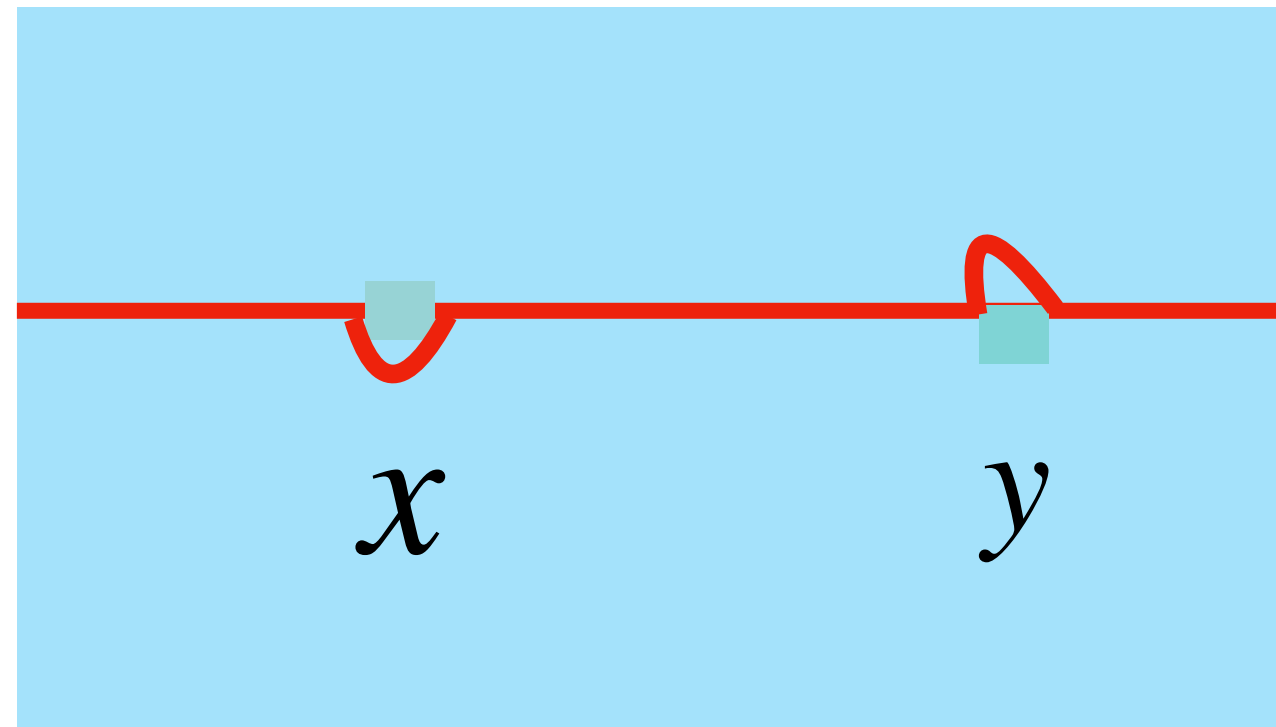


$$\sigma_{\text{gr}} \propto \oint ds^j x^\mu \langle T_{\mu j} \rangle$$

$$\Rightarrow \boxed{\sigma_{\text{gr}} = \gamma_{\text{gr}} a_T}$$

?

But for extended defects tension ("stiffness") can be also defined, without reference to gravity, from the 2-point function of the transverse displacement (*think of a violin string*)



invariant stiffness/inertial tension

$$\sigma_{\text{in}} = \gamma_{\text{in}} C_D$$

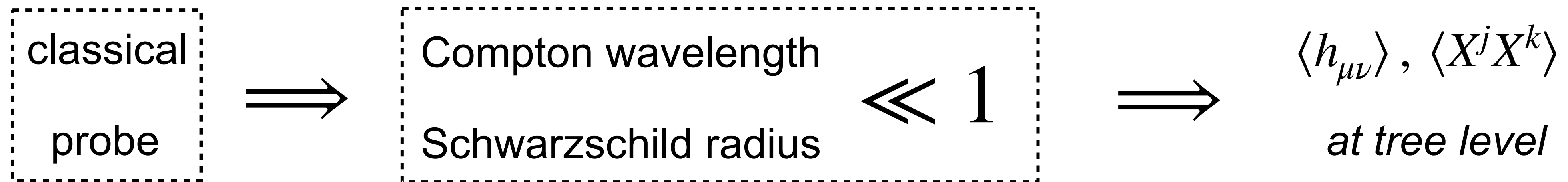


The proportionality constants γ_{gr} , γ_{in} can be determined in the classical probe limit with the result [CB, Chen 2404.14998](#)

$$\gamma_{\text{gr}} = -\frac{2(d-1)\pi^{(d-p)/2}}{d\Gamma(\frac{d-p}{2})}, \quad \gamma_{\text{in}} = \frac{\pi^{p/2}\Gamma(\frac{p}{2}+1)}{(p+2)\Gamma(p+1)}$$

Treating the branes as classical probes allows a perturbative calculation of C_D and a_T (from Witten diagrams)

$$S_{\text{eff}} = \frac{1}{16\pi G_N} \int \sqrt{g} (R + \Lambda) + \sigma_0 \int d^{p+1} \tau \sqrt{\det (g_{MN} \partial_a X^M \partial_b X^N)}$$



The tricky part is identifying the brane field $\sim X^j$ dual to the displacement D^j .

For this compute $\langle T^{\mu\nu} D^j \rangle$ and check the Ward identities that fix normalizations

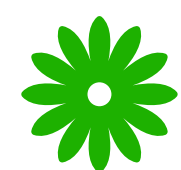
$$\langle T^{\mu\nu} \int D^j \rangle \sim \partial^j \langle T^{\mu\nu} \rangle \quad \text{and} \quad \langle D^j \int \partial_\mu T^{\mu k} \rangle \sim \langle D^j D^k \rangle$$

skip ?

To summarize: In asymptotically AdS gravity invariant quantities are defined from boundary (CFT) data. For extended branes of codimension >1 there are two definitions of the **invariant tension**

$$\sigma_{\text{gr}} = -\frac{2(d-1)\pi^{(d-p)/2}}{d\Gamma(\frac{d-p}{2})} a_T, \quad \sigma_{\text{in}} = \frac{\pi^{p/2}\Gamma(\frac{p}{2}+1)}{(p+2)\Gamma(p+1)} C_D$$

These agree in the classical probe limit, but receive different corrections in general. Nevertheless, for supersymmetric branes



$$\frac{C_D}{a_T} = -\frac{2(d-1)(p+2)\Gamma(p+1)}{d\pi^{p-d/2}\Gamma(\frac{p}{2}+1)\Gamma(\frac{d-p}{2})}$$



$$\sigma_{\text{gr}} = \sigma_{\text{in}}$$

An example: Supersymmetric Wilson lines in $\mathcal{N} = 4$ SYM . The relevant CFT data is known exactly from susy localization for any 't Hooft coupling λ and all N :

$$C_D = -18a_T = \frac{6}{\pi^2} \lambda \partial_\lambda \log \left(\frac{1}{N} e^{\lambda/8N} L_{N-1}^1 \left(-\frac{\lambda}{4N} \right) \right)$$

modified Laguerre

Erickson, Semenoff, Zarembo 0003055
Drukker, Gross 0010274
Pestun 0906.0638

Expanding:

$$\sigma_{\text{gr}} = \sigma_{\text{in}} = \frac{\sqrt{\lambda}}{2\pi} \left[1 + O\left(\frac{1}{\sqrt{\lambda}}\right) + O\left(\frac{\sqrt{\lambda}}{N}\right) + O\left(\frac{\sqrt{\lambda}}{N^2}\right) \right]$$

σ_0^F

Compton

D – string
Compton

BH radius

[Surprise: **No black string regime of Einstein gravity**]

Note that usual BPS equations relate mass/tension to charge.

This one relates two different notions of tension.

All elementary M-theory branes are supersymmetric; could this be some type of generalized equivalence principle ?

Joyeux anniversaire

Jean Bernard !