Progress in lattice simulations for two Higgs doublet models

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Based on:

- **G. Catumba**, A. Hiraguchi, W.-S. Hou, K. Jansen, Y.-J. Kao, C.-J. D. Lin, A. Ramos, and M. Sarkar, *Lattice investigation of custodial two-Higgs-doublet model at weak quartic couplings*, **JHEP 10** (**2025**) **214**, arXiv:2507.07759.

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Excelencia Severo Ochoa

Higgs sector in the Standard Model

The Good

- Mechanism to provide masses to fundamental particles.
- Validated experimentally

- "Higgs physics" will be precision physics
- Challenge the SM

The Ugly

- High sensitivity to new physics

$$m_R^2 \approx m_0^2 + \mathcal{O}(\Lambda_{BSM}^2)$$

- Big motivator in model building
- Driven many BMS searches

The Bad

- **Trivial** $\Longrightarrow$ Non renormalizable
- No strong first-order phase transition in the early Universe (prerequisite for baryogenesis)

- Needs understanding beyond perturbationb theory
- **Lattice simulations**

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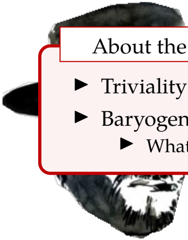

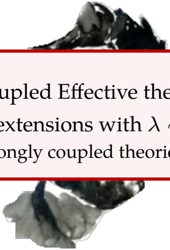

About the Bad

- Triviality in SM $\Longrightarrow$ weakly coupled Effective theory (i.e. “Landau Pole” is very far)
- Baryogenesis: Multiple Higgs extensions with $\lambda \sim \mathcal{O}(1)$
  - What about triviality with strongly coupled theories?

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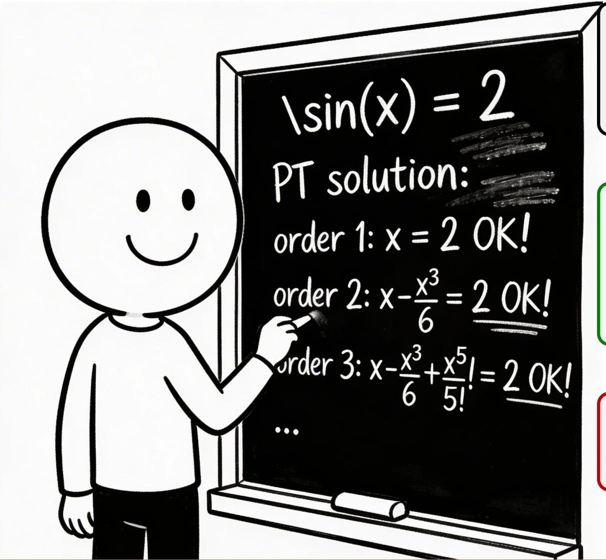
Beyond Perturbation Theory: Lattice field theory



"Nature does not use perturbation theory, so what happens order-by-order in perturbation theory is irrelevant."

Aneesh Manohar - *Les Houches lectures*

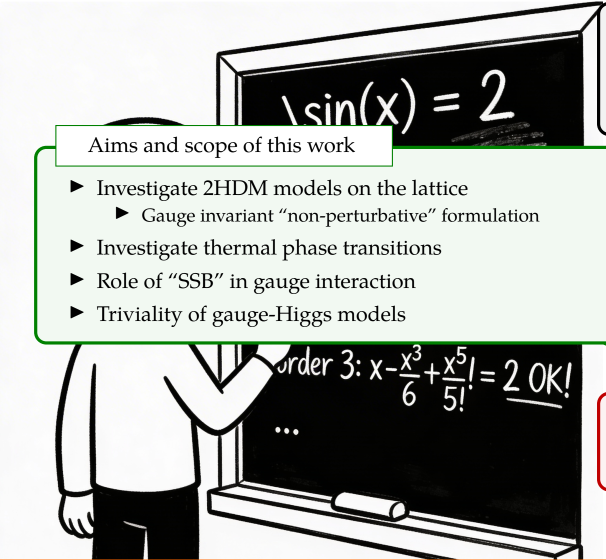
Lattice formulation

- Gauge invariant formulation of Scalar-Higgs theories.
- Setup applicable beyond perturbation theory
- Investigate early universe phase transition

Limitations

- No chiral gauge theory (conceptual)
- No vector fermions

Beyond Perturbation Theory: Lattice field theory



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Aims and scope of this work

- Investigate 2HDM models on the lattice
  - Gauge invariant "non-perturbative" formulation
- Investigate thermal phase transitions
- Role of "SSB" in gauge interaction
- Triviality of gauge-Higgs models

Limitations

- No chiral gauge theory (conceptual)
- No vector fermions

2HDM model

$$\mathcal{L}_{2\mathrm{HDM}} = -\frac{1}{2g^2}[G_{\mu\nu}G_{\mu\nu}] + (D_\mu\Phi_1)^\dagger(D_\mu\Phi_1) + (D_\mu\Phi_2)^\dagger(D_\mu\Phi_2) + V_{2\mathrm{HDM}}$$

$$\begin{aligned}V_{2\mathrm{HDM}} =& \mu_{11}^2(\Phi_1^\dagger\Phi_1) + \mu_{22}^2(\Phi_2^\dagger\Phi_2) + {\mu_{12}}^2\Re(\Phi_1^\dagger\Phi_2)\\ &+ \eta_1(\Phi_1^\dagger\Phi_1)^2 + \eta_2(\Phi_2^\dagger\Phi_2) + \eta_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \eta_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)\\ &+ \eta_5\Re(\Phi_1^\dagger\Phi_2)^2 + \Re(\Phi_1^\dagger\Phi_2)\left[\eta_6(\Phi_1^\dagger\Phi_1) + \eta_7(\Phi_2^\dagger\Phi_2)\right]\end{aligned}$$

- Choose $\eta_4 = \eta_5$ and $\mu_{12} = \eta_6 = \eta_7 = 0$
- $SU(2)$ custodial symmetry $\Longrightarrow$ Same as SM
- $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetric: $\Phi_1 \to -\Phi_1/\Phi_2 \to -\Phi_2$

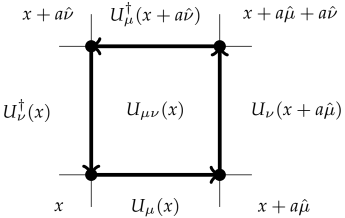
2HDM Model on the Lattice

$$S_{2HDM}^{O(4)} = \sum_x \sum_{n=1}^{2} \left\{ \sum_\mu -2\kappa_n \left(\hat{\Phi}_n^\dagger(x) U_\mu(x) \hat{\Phi}_n(x+\hat{\mu})\right) + \left(\hat{\Phi}_n^\dagger \hat{\Phi}_n\right) + \hat{\eta}_n \left[\left(\hat{\Phi}_n^\dagger \hat{\Phi}_n\right) - 1\right]^2 \right\} +$$
$$\hat{\eta}_3 \left(\hat{\Phi}_1^\dagger \hat{\Phi}_1\right)\left(\hat{\Phi}_2^\dagger \hat{\Phi}_2\right) + \hat{\eta}_4 \left(\hat{\Phi}_1^\dagger \hat{\Phi}_2\right)^2 + \frac{\beta}{4} \sum_x \sum_{\mu,\nu} \Re(1 - U_{\mu\nu}(x))$$

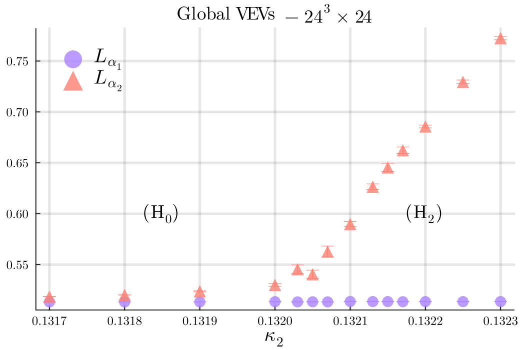
- Gauge field $U_\mu(x)$: link between adjacent lattice points
- Scalar field lives at the nodes $\hat{\Phi}_i(x)$
- Scalar masses given by κ: $a^2 m^2 = \frac{1-2\eta-8\kappa}{\kappa}$
- Elitzur theorem: No breaking of gauge symmetries **[S. Elitzur. Phys.Rev.D Vol.12 Num.12 1975]**
- Gauge invariant characterization of Higgs/Confining phases with $\alpha_s(x) = \hat{\Phi}_n(x)/\rho_n$ and $\rho_n^2 = \frac{1}{V}\sum_x \mathrm{Det}[\hat{\Phi}_n(x)]$

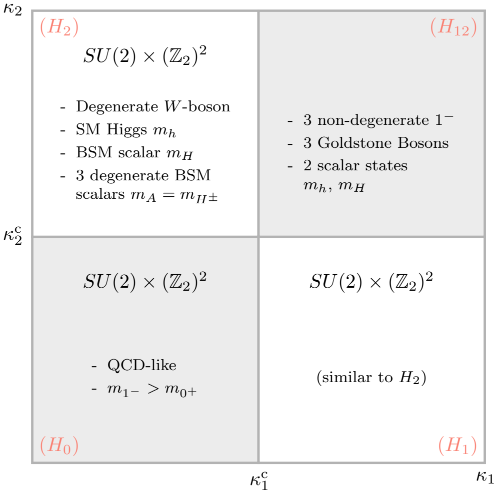
$$L^a_{\alpha_{nm}} = \frac{1}{8V} \sum_{x,\mu} \alpha_n^\dagger(x) U_\mu(x) \alpha_m(x+\hat{\mu}) \theta^a$$

use $\theta_0 = 1_{2\times2}, \quad \theta_i = i\sigma_i$



Phase structure and spectrum of 2HDM





- Scanning κ_i separates confinning/Higgs regions
- Three Higgs-like phases, one confinning phase
- $H_1(H_2)$ phases of the "inert" models

Phase structure and spectrum of 2HDM

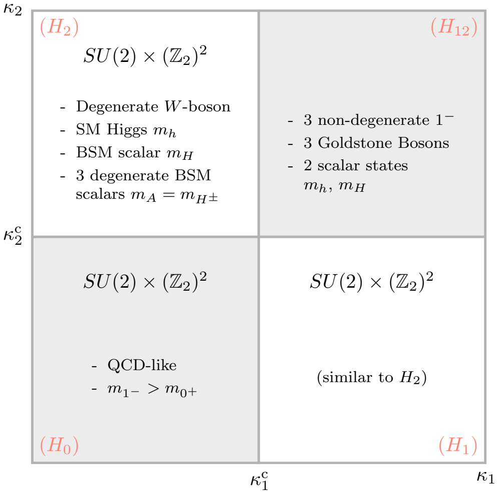
Analysis of spectrum (gauge invariant)

- Study two-point functions of suitable interpolating operators

$$\langle O(x)O(y)\rangle \xrightarrow{x_0\to\infty} e^{-Mx_0}\,.$$

- Extract masses (m_h, m_W, m_{BSM})

	Gauge-invariant	
PT	Continuum	Lattice
Φ_1	$\mathrm{Tr}[\Phi_1^\dagger\Phi_1(x)]$	$\mathrm{Tr}[\Phi_1^\dagger\Phi_1(x)]$
W_μ^a	$\mathrm{Tr}[\sigma^a\Phi_1^\dagger D_\mu\Phi_1(x)]$	$\mathrm{Tr}[\sigma^a\Phi_1^\dagger(x+\mu)U_\mu(x)\Phi_1(x)]$
$\Phi_H(x)$	$\mathrm{Tr}[\theta_0\Phi_1^\dagger\Phi_2(x)]$	$\mathrm{Tr}[\theta_0\Phi_1^\dagger\Phi_2(x)]$
$\Phi_A(x)$	$\mathrm{Tr}[\theta_i\Phi_1^\dagger\Phi_2(x)]$	$\mathrm{Tr}[\theta_i\Phi_1^\dagger\Phi_2(x)]$



Line of constant SM physics: How to embed the SM?

Tune lattice parameters κ_2, η_2 to satisfy

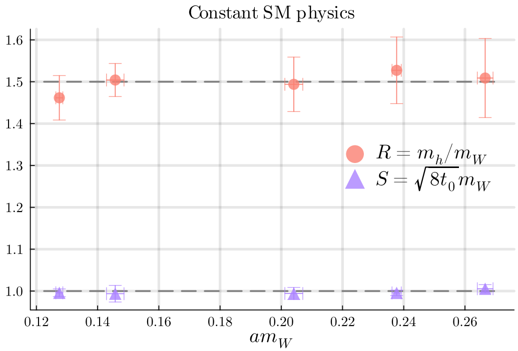
$$R = \left(\frac{m_h}{m_W}\right)_{\text{latt}} = \left(\frac{m_h}{m_W}\right)_{\text{phys}} = 1.5$$

$$S = \left(\frac{m_W}{\mu_0}\right)_{\text{latt}} = \left(\frac{m_W}{\mu_0}\right)_{\text{phys}} = 1.0,$$

$$g^2(\mu_0) = 0.5$$

- Conditions to be satisfied for several values of the lattice spacing a
- Gauge invariant, non-perturbative coupling definition?
- Define cutoff (lattice spacing a) by

$$\Lambda^{-1} = a \equiv \frac{am_W}{m_W}$$



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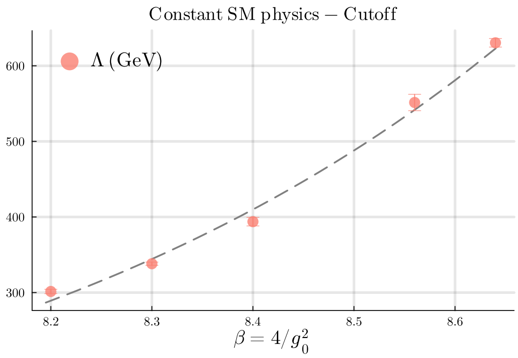

Gauge invariant non-perturbative coupling definitions

- Add extra (flow) time coordinate $t(\neq x_0)$ with $[t] = \text{length}^2$. Define gauge field $B_\mu(t,x)$

$$\begin{aligned} \frac{dB_\mu(x,t)}{dt} &= D_\nu G_{\nu\mu}(x,t); \quad \left(\sim -\frac{\delta S_{\text{YM}}[B]}{\delta B_\mu}\right) \\ G_{\nu\mu}(x,t) &= D_\nu B_\mu(x,t) - D_\nu B_\mu(x,t)\,, \end{aligned}$$

with initial condition $B_\mu(t,x)|_{t=0} = A_\mu(x)$

- Composite gauge invariant operators are renormalized observables defined at a scale $\mu = 1/\sqrt{8t}$ **[M. Lüscher '10; M. Lüscher, P. Weisz '11]**.
- Dimensionless observable *that depends on a scale!*

$$t^2\langle E(t)\rangle = -\frac{t^2}{2}\text{Tr}\langle G_{\mu\nu}(x,t)G_{\mu\nu}(x,t)\rangle$$

Is a coupling!

$$g^2(\mu) = \frac{128\pi^2}{9}t^2\langle E(t)\rangle\Big|_{\mu=1/\sqrt{8t}}$$

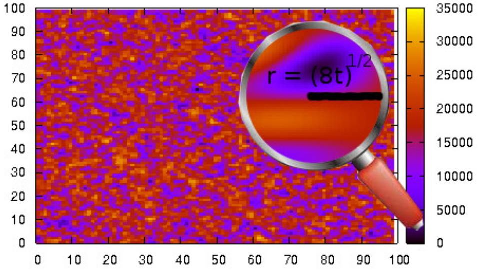

Figure: Smearing the world with a resolution $\sim \sqrt{8t}$

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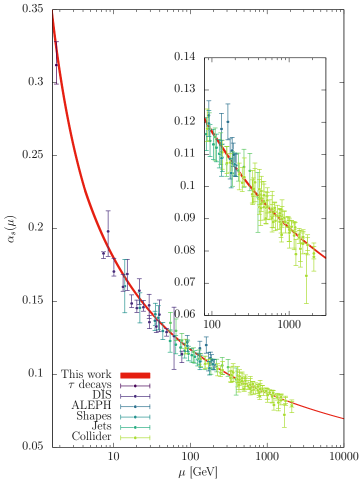


Figure: Precision QCD coupling determination **[ALPHA '25]**

Gauge coupling in the Higgs phase

- For energies $E < M_W$: Yukawa-like screening

$$V_Y(r) \propto \frac{1}{4\pi r} e^{-mr}$$

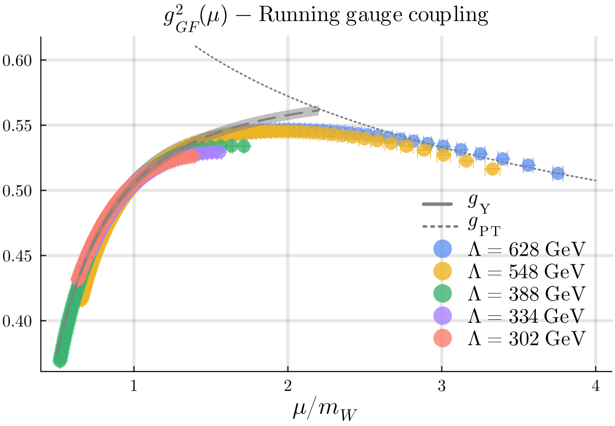
- For energies $E > M_W$: Massless running

$$\beta(g) = -\frac{b_0 g^3}{16\pi^2} + \dots$$

with

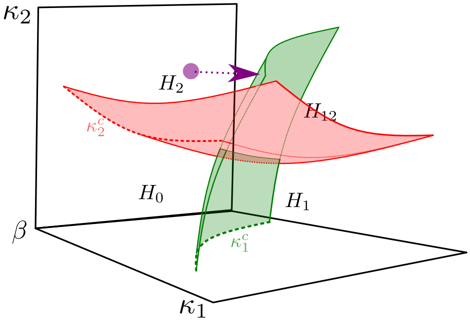
$$b_0 = \frac{11N - n_s}{3}$$

- Little cutoff dependence (i.e. "continuum limit")



BSM spectrum

- LCP defined with $\{\beta, \kappa_2, \eta_2\}$
- Probe spectrum as a function of κ_1
- Light scalar BSM masses
- Little cutoff dependence
- Pushing $\kappa_1 \to \kappa_{1,c}$: Goldstone
- Could there be light (undiscovered) scalars? **[A. Crivellin talk]**



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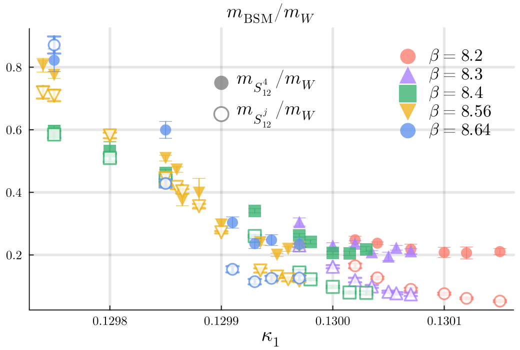


Higgs models at finite temperature

- QFT at finite temperature: Euclidean field theory with (anti-)periodic BCs in time.
- Temperature T given by inverse temporal extent
- $L_{\alpha_{22}}$ signals Higgs mechanism

$$L_{\alpha_{nm}} = \frac{1}{8V} \sum_{x,\mu} \alpha_n^\dagger(x) U_\mu(x) \alpha_m(x+\hat{\mu})$$

- Multiplicative renormalization: Look at

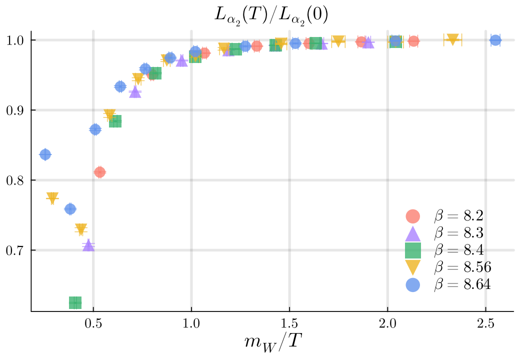
$$L_{\alpha_{22}}(T)/L_{\alpha_{22}}(0)$$

$T_c \approx 0.5 M_W$

- Or its susceptibility

$$\chi(L) = L^3 \left(\langle L^2_{\alpha_{22}} \rangle - \langle L_{\alpha_{22}} \rangle^2 \right)$$

- $\chi(L)$ volume independent $\Longrightarrow$ **CROSSOVER**



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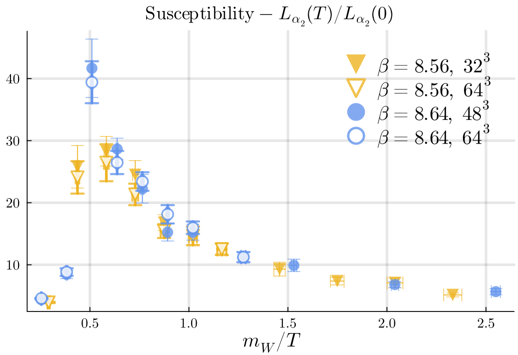

Conclusions

Research program: Lattice gauge-scalar models

- Fully gauge invariant study of 2HDM on the lattice
- First steps: study at (all) weak couplings
  - Constant line of SM physics
  - Screening of gauge coupling in Higgs phase
  - BSM spectrum
  - Thermal phase transition/crossover

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MANY THANKS FOR YOUR ATTENTION