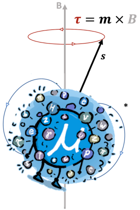
Muon $g-2$: a window on new physics?

Laurent Lellouch

CNRS & Aix-Marseille U.

[Budapest-Marseille-Wuppertal [BMW] + Davier-Malaescu-Zhang [DMZ] collaboration]

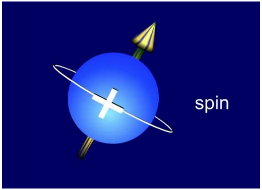


2407.10913 $\longrightarrow$ This work
Nature 593 (2021) $\longrightarrow$ BMW '20
PRL 121 (2018) 022002 (Editors' Selection) $\longrightarrow$ BMW '17
Aoyama et al., Phys. Rep. 887 (2020) 1-166 $\longrightarrow$ WP '20
Aliberti et al., 2505.21476 $\longrightarrow$ WP '25

Charged lepton magnetic moments

Muons are tiny magnets

A massive elementary particle w/ electric charge and spin behaves like a tiny magnet



[← Silver Swan)]

Magnetic moment of the muon

$$\vec{\mu}_\mu = \pm g_\mu \frac{e}{2m_\mu}\vec{S}$$

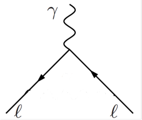
g_μ = Landé factor

Crucial point:

- g_μ can be **measured** & **calculated** very, very ... precisely
- measurement = SM prediction ?
  → Yes: another victory for the SM
  → No: we have uncovered new fundamental physics

Early history: the electron

- 1928 : Dirac's new theory predicts the existence of the positron and



$$g_e|_{\text{Dirac}} = 2$$

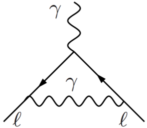
"That was really an unexpected bonus for me" (P.A.M. Dirac)

- 1934 : Kinsler & Houston confirm $g_e = 2$, w/ permil precision by studying spectrum of neon atom

- 1947 : Nafe, Nelson & Rabi, then Kusch & Foley measure hyperfine structure of hydrogen and deuterium, showing that $g_e > 2$ by 0.1%

  $\rightarrow$ there is a problem w/ Dirac!

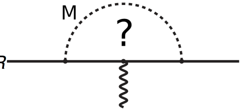
- 1947 : Schwinger understands very quickly that Dirac's theory neglects **quantum fluctuations** and manages to compute them to obtain the **"anomalous"** contribution



$$a_e = \frac{g_e - 2}{2} = \frac{\alpha}{2\pi} = 0.00116\ldots$$

$\rightarrow$ birth of QED and relativistic quantum field theory

Why are a_ℓ special?



$$\ell_R \quad \ell_L \quad \longrightarrow \quad \mathcal{L}_{\text{eff}} = -\frac{Qe}{2}\frac{a_\ell}{2m_\ell} F^{\mu\nu}[\bar{\ell}_L \sigma_{\mu\nu} \ell_R] + \text{hc}$$

- $a_{e,\mu}$ are rigorous predictions of the SM that can be measured very precisely $\Rightarrow$ excellent tests of SM
- Loop induced $\Rightarrow$ sensitive to new dofs that may be too heavy or too weakly coupled to be produced directly
- Flavor and CP conserving, chirality flipping $\Rightarrow$ probes mass generating mechanism and complementary to: EDMs, FCNCs (e.g. s and b decays), LHC direct searches, ...
- Chirality flipping $\Rightarrow$ generic contribution of particle w/ $M \gg m_\ell$

$$a_\ell^{\text{M}} = C\left(\frac{\Delta_{LR}}{m_\ell}\right)\left(\frac{m_\ell}{M}\right)^2$$

- In EW theory, $M = M_W$, chirality flipping from Yukawa, i.e.

$$\Delta_{LR} = m_\ell \qquad \text{and} \qquad C \sim \frac{\alpha}{4\pi \sin^2\theta_W}$$

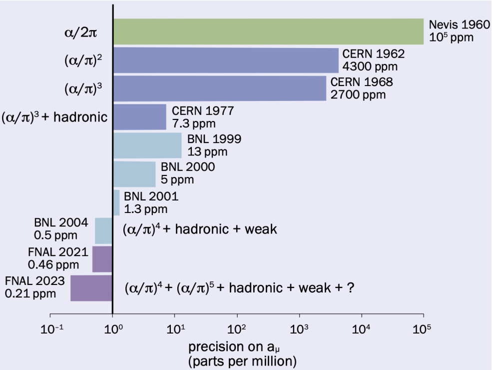
- In BSM, can have enhancements: e.g. SUSY $M = M_{\text{SUSY}}$ and $C \sim \alpha/\left(4\pi \sin^2\theta_W\right)$ & $\Delta_{LR} = (\mu/M_{\text{SUSY}}) \times \tan\beta \times m_\ell$; or radiative m_ℓ model, $\Delta_{LR} \simeq m_\ell$, $C \sim 1$ and $M = M_{\text{N}\Phi}$

Why is a_μ special?

$m_e : m_\mu : m_\tau = 0.0005 : 0.106 : 1.777\,\text{GeV}$ $\tau_e : \tau_\mu : \tau_\tau =$ "∞" $: 2. \cdot 10^{-6} : 3. \cdot 10^{-13}\,\text{s}$

- a_μ is $(m_\mu/m_e)^2 \sim 4.3 \times 10^4$ times more sensitive to new Φ than a_e
- a_τ is even more sensitive to new Φ, but measurement limited by short lifetime
- τ_μ small but manageable

$\rightarrow$ focus on the muon



Big question:

$a_\mu^{\text{exp}} = a_\mu^{\text{SM}}$?

Standard model calculation of a_μ

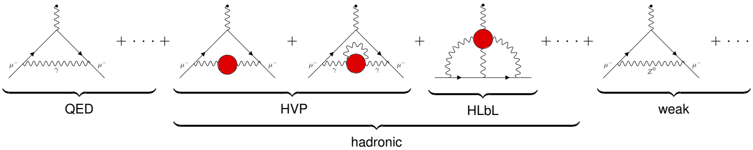
[Aoyama et al '20 = WP '20, Aliberti et al '25 = WP '25]

$$\begin{aligned} a_\mu^{\rm SM} &= a_\mu^{\rm QED} + a_\mu^{\rm had} + a_\mu^{\rm EW} \\ &= O\left(\frac{\alpha}{2\pi}\right) + O\left(\left(\frac{\alpha}{\pi}\right)^2\left(\frac{m_\mu}{M_\rho}\right)^2\right) + O\left(\left(\frac{\alpha}{16\pi\sin^2\theta_W}\right)\left(\frac{m_\mu}{M_W}\right)^2\right) \\ &= O\left(10^{-3}\right) + O\left(10^{-7}\right) + O\left(10^{-9}\right) \end{aligned}$$

$a_\mu^{\rm SM}$: standard model prediction, compiled in WP '25

To match experimental precision $\rightarrow$ all three interactions and all SM particles

$a_\mu^{\rm SM} =$



SM contribution	$a_\mu^{\rm contrib.} \times 10^{10}$
QED [5 loops]	11658471.88 ± 0.02
EW [2 loops]	15.44 ± 0.04
HVP LO [lattice]	713.2 ± 6.1
HVP HO [pheno]	-8.72 ± 1.3
HLbL Tot. [lattice+data+pheno]	11.3 ± 1.0
SM [0.53 ppm]	11659203.3 ± 6.2
Exp [0.124 ppm]	11659207.15 ± 1.45

$\text{HVP} \sim a_\mu^{\rm SM}/(2 \times 10^4) \sim 65 \times \text{HLbL}$

$\sigma^2_{\rm HVP}/\sigma^2_{a_\mu} \sim 97\%$

$\sigma^2_{\rm HLbL}/\sigma^2_{a_\mu} \sim 3\%$

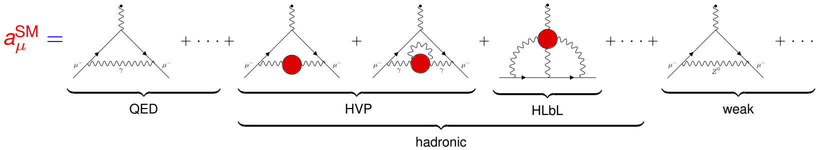
Hadronic contributions involve low-energy, nonperturbative QCD:

- Data-driven: unitarity, analyticity, short-distance pQCD and data
  $\rightarrow$ dominated WP '20 averages
  $\rightarrow$ still very important for HLbL
  $\rightarrow$ making a comeback for HVP [$\rightarrow$ Leonard Polat]
- Lattice: massively-parallel numerical simulations of QCD
  $\rightarrow$ on a par w/ data-driven for HLbL
  $\rightarrow$ currently determines HVP

[intro to lattice $\rightarrow$ Alejandro Vaquero, Felix Erben]

a_μ^{SM}: standard model prediction, compiled in WP '25

To match experimental precision $\rightarrow$ all three interactions and all SM particles



SM contribution	$a_\mu^{\text{contrib.}} \times 10^{10}$
QED [5 loops]	11658471.88 ± 0.02
EW [2 loops]	15.44 ± 0.04
HVP LO [This work]	715.1 ± 3.4
HVP HO [pheno]	-8.72 ± 1.3
HLbL Tot. [lattice+data+pheno]	11.3 ± 1.0
SM [0.31 ppm]	11659205.2 ± 3.6
Exp [0.124 ppm]	11659207.15 ± 1.45

HVP$\sim a_\mu^{SM}/(2\times 10^4)\sim 65\times$HLbL

$\sigma^2_{\text{HVP}}/\sigma^2_{a_\mu}\sim 89\%$

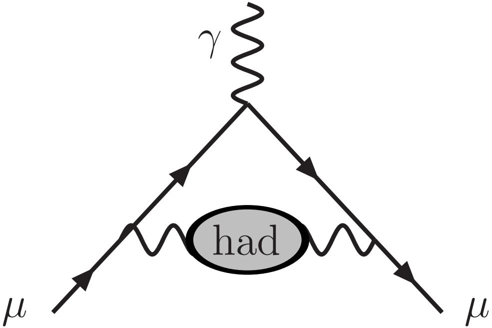
$\sigma^2_{\text{HLbL}}/\sigma^2_{a_\mu}\sim 8\%$

Hadronic contributions involve low-energy, nonperturbative QCD:

- Data-driven: unitarity, analyticity, short-distance pQCD and data
  $\rightarrow$ dominated WP '20 averages
  $\rightarrow$ still very important for HLbL
  $\rightarrow$ making a comeback for HVP [$\rightarrow$ Leonard Polat]
- Lattice: massively-parallel numerical simulations of QCD
  $\rightarrow$ on a par w/ data-driven for HLbL
  $\rightarrow$ currently determines HVP

[intro to lattice $\rightarrow$ Alejandro Vaquero, Felix Erben]

Hybrid calculation of $a_\mu^{\text{LO-HVP}}$: taking the best from both approaches



All quantities related to a_μ will be given in units of 10^{-10}

$a_\mu^{\text{LO-HVP}}$ from LQCD: introduction

Compute on $T \times L^3$ Euclidean-time lattice w/ spacing a [Bernecker et al '11]

$$C_L(t) = \frac{a^3}{3} \sum_{i=1}^{3} \sum_{\vec{x}} \langle J_i(x) J_i(0) \rangle$$

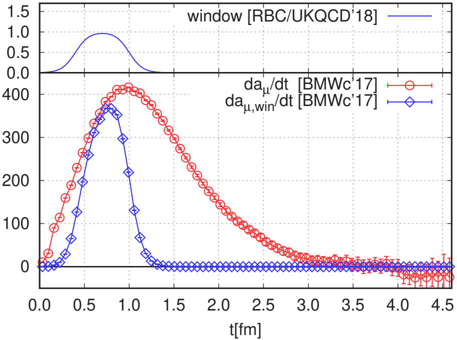
w/ $J_\mu = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s + \frac{2}{3}\bar{c}\gamma_\mu c + \cdots$

Decompose

$$C_L(t) = C_L^{ud}(t) + C_L^s(t) + C_L^c(t) + C_L^{\text{disc}}(t) + C_L^{\text{QED}}(t) + C_L^{\text{SIB}}(t)$$

Then get

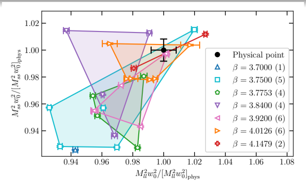
$$a_{\mu,f}^{\text{LO-HVP}} = \lim_{\substack{a \to 0 \\ L,T \to \infty}} \left(\frac{\alpha}{\pi}\right)^2 \left(\frac{a}{m_\mu^2}\right) \sum_{t=0}^{T/2} K(tm_\mu)\, \mathrm{Re}C_L^f(t)$$



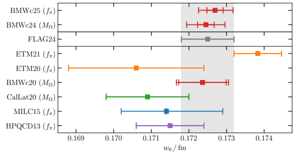
Define "windows" [RBC/UKQCD '18]

$K(\tau) \to W(\tau; \tau_i, \tau_f, \bar{\Delta})K(\tau)$

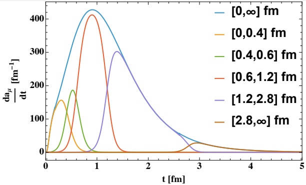
Strategy for improvement over BMW '20



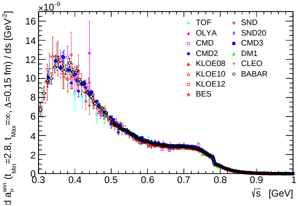
New finer ("Monster") lattices:
$a = 0.064$ fm [$96^3 \times 144$] $\longrightarrow$
$a = 0.048$ fm [$128^3 \times 192$]
$\rightarrow$ distance to continuum limit $\div 1.8$



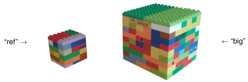
Significantly improve tuning of QCD parameters
$\rightarrow$ (scale uncertainty)^2 $\div 2.1$



Break up calculation into set of 5 windows
$\rightarrow$ optimize calculation in small distance scale intervals



Use $e^+e^- \rightarrow$ hadrons (and τ) data for long-distance 2.8 fm $\rightarrow \infty$ contribution [1fm $\rightarrow \infty$ proposed in RBC/UKQCD '18]



Dedicated simulations on huge $L = 11$ fm lattice to correct finite-V effects

[ericwilliambarnum.wordpress.com]

Calculation fully blinded

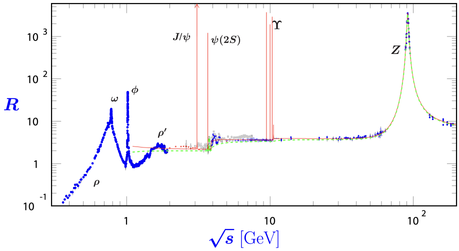
Re-evaluated all QED and SIB corrections

Windows from R-ratio

[Berneke '11, RBC '18]

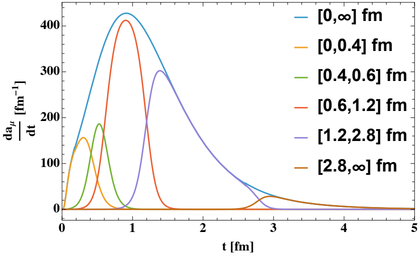
$$a_{\mu,win}^{\text{LO-HVP}}(t_i,t_f)=\left(\frac{\alpha}{\pi}\right)^2\int_0^\infty \frac{dt}{m_\mu^2}K(tm_\mu)W(t;t_i,t_f)C(t)$$

$$R(s)\equiv\frac{\sigma(e^+e^-\to\text{had})}{\sigma(e^+e^-\to\mu^+\mu^-)}\quad\longrightarrow\quad C(t)=\frac{1}{24\pi^2}\int_{s_{\text{th}}}^\infty ds\sqrt{s}\,R(s)\,e^{-|t|\sqrt{s}}$$



[PDG '18 compilation]

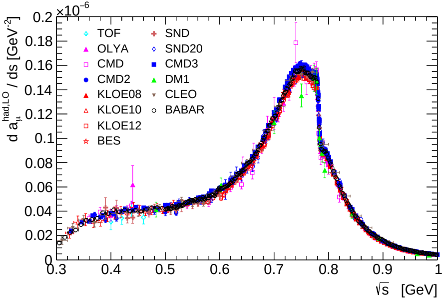
$\longrightarrow$

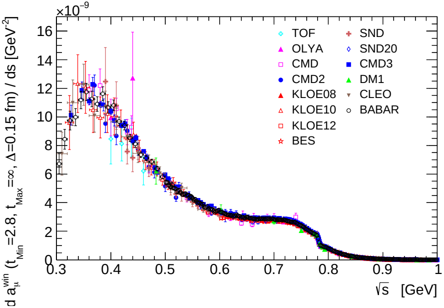


- Compute 4 windows from $0\to 2.8\,\text{fm}$ on lattice
- Obtain small contribution from $2.8\,\text{fm}\to\infty$ using $e^+e^-\to$ hadrons data

Data-driven tail: 2.8 fm $\rightarrow \infty$

$a_\mu^{\text{LO-HVP}}$ $\qquad$ $a_{\mu,28-\infty} \equiv a_\mu^{\text{LO-HVP}}|_{\text{tail}}$

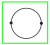


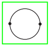


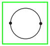
- Over 70% of tail comes from below ρ peak
- All datasets agree well in that range
- Good agreement with lattice for $a_{\mu,28-35}$
- Contributes only 4% to $a_\mu^{\text{LO-HVP}}$
- 96% comes lattice computation
- Helps reduce many of our uncertainties

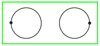
Summary of contributions to $a_\mu^{\text{LO-HVP}}$: 2020

Isospin symmetric

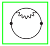
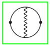

connected light
633.7(2.1)(4.2)

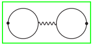
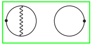

connected strange
53.393(89)(68)


connected charm
14.6(0)(1)

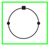

disconnected
-13.36(1.18)(1.36)

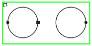
QED isospin-breaking: valence



connected -1.23(40)(31)

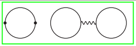
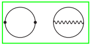


disconnected -0.55(15)(10)

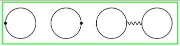
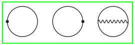
Strong isospin-breaking


connected
6.60(63)(53)


disconnected
-4.67(54)(69)

QED isospin-breaking: sea



connected 0.37(21)(24)

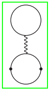


disconnected -0.040(33)(21)

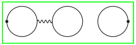
Etc.

bottom; higher order; perturbative

0.11(4)

QED isospin-breaking: mixed


connected -0.0093(86)(95)


disconnected 0.011(24)(14)

Finite-size effects

isospin-symmetric
18.7(2.5)

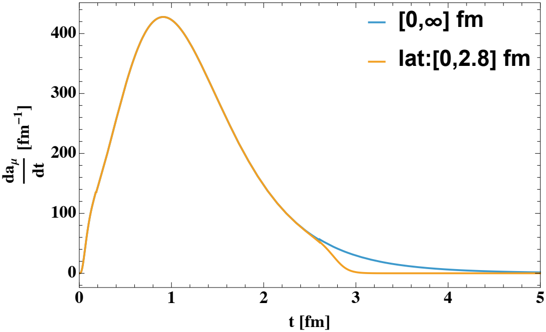
isospin-breaking
0.0(0.1)

$$10^{10}\times a_\mu^{\text{LO-HVP}} = 707.5(2.3)_{\text{stat}}(5.0)_{\text{sys}}[5.5]_{\text{tot}}$$

$a_\mu^{\text{LO-HVP}}$ to 0.77%

HVP contribution result

Re-computed all lattice HVP contributions [This work]



$$a_\mu^{\text{LO-HVP}} = a_{\mu,\text{0-2.8 fm}}^{\text{LO-HVP}}\big|_{\text{lat}} + a_{\mu,\text{2.8-}\infty\text{ fm}}^{\text{LO-HVP}}\big|_{e^+e^-,\tau}$$

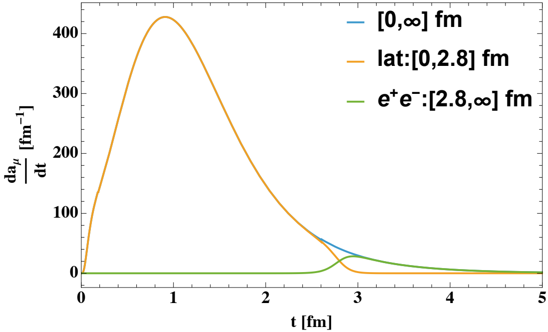
$$= 687.51(3.36) + 27.59(52)$$

$$= 96\% + 4\%$$

$$= 715.1(3.4) \quad [0.48\%]$$

HVP contribution result

Re-computed all lattice HVP contributions [This work]



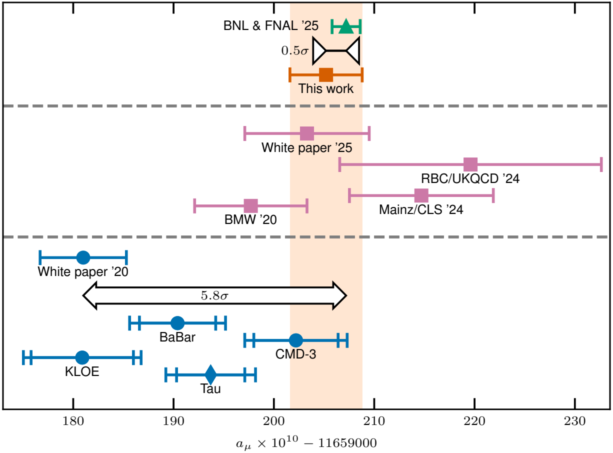
$$
\begin{aligned}
a_\mu^{\text{LO-HVP}} &= a_{\mu,0\text{-}2.8\,\text{fm}}^{\text{LO-HVP}}\Big|_{\text{lat}} + a_{\mu,2.8\text{-}\infty\,\text{fm}}^{\text{LO-HVP}}\Big|_{e^+e^-,\tau} \\
&= 687.51(3.36) + 27.59(52) \\
&= 96\% + 4\% \\
&= 715.1(3.4) \quad [0.48\%]
\end{aligned}
$$

Experiment vs SM: March 2026

$$a_\mu|_{\text{expt}} = (11659207.15 \pm 1.45) \times 10^{-10}$$

$$a_\mu|_{\text{WP25}} = (11659203.3 \pm 6.2) \times 10^{-10}$$

$$a_\mu|_{\text{This work+WP25}} = (11659205.1 \pm 3.4) \times 10^{-10}$$



Conclusions

- New "hybrid" calculation (lattice $+ e^+e^- \to$ hadrons) of $a_\mu^{\text{LO-HVP}}$ to 0.48%
- Decades of perturbative QED & EW calculations, combined w/ fully nonperturbative QCD (+QED) and data-driven ones, predict g_μ to 0.36 ppb !
- Decades of measurement give g_μ to 0.14 ppb !
- With our result for $a_\mu^{\text{LO-HVP}}$, SM confirmed to 0.38 ppb
- Stringent, single test of the complete SM (all particles & interactions)
- Lattice calculation of $0 \to 2.8\,\text{fm}$ window $> 96\%$ of total
- $e^+e^- \to$ hadrons evaluation of $2.8 \to \infty\,\text{fm}$ window $\leq 4\%$ of total
- Fully blinded analysis
- $\sim$ 10 years of lattice work and billions of supercomputer core-hours
- Error reduction vs BMW '20:
  - $\sim 37\%$ from finer ensembles
  - additional $\sim 22\%$ from $e^+e^- \to$ hadrons data for tail
- Lattice calculation agrees w/ others in windows: $0 \to 0.4\,\text{fm}$, $0.4 \to 1.0\,\text{fm}$ & $1.5 \to 1.9\,\text{fm}$, $1.0\,\text{fm} \to \infty$

Conclusions

- Still need $\times 2$ improvement to match experiment !
- Significant work to understand tensions among e^+e^- measurements [BaBar '23, Colangelo et al, ...] and w/ lattice [ETM '22, BMW-DMZ '23, ...]
- BMW also has complete calculation of $a_\mu^{\rm HLbL}$ [BMW '25]
- Eagerly await
  - Lattice results for complete $a_\mu^{\text{LO-HVP}}$ by Fermilab/HPQCD/MILC expected soon
  - Next generation of 100% lattice calculations w/ precision ∼BMW '24
  - New BABAR $e^+e^- \to$ hadrons analysis [→ Leonard Polat]
  - New KLOE analysis
  - New BES III, BELLE-II, CMD-3, SND-2 data and analysis
  - J-PARC entirely new method for a_μ measurement
  - MUonE @ CERN for spacelike HVP
- $\alpha(M_Z^2)$ for EWPO tests of SM @ FCC-ee
- ...