A quick guide to fractionally charged particles of the Standard Model

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Electroweak Interactions and Unified Theories, Recontres de Moriond

The Standard Model Group

- We know that the Standard Model gauge group is one of the 4 choices:

$$G_p = \frac{SU(3)_c \times SU(2)_L \times U(1)_Y}{\mathrm{Z}_p}, \;\; p \in \{1, 2, 3, 6\}$$

with $\mathrm{Z}_p = \{1, ..., \xi^{p-1}\}$ and $\xi = e^{2\pi i Q_Y}\, e^{2\pi i \frac{n_c}{3}}\, e^{\pi i n_L}$.

$$G_p : \;\; \xi^{6/p} R = R, \;\; R \in G_{SM}$$

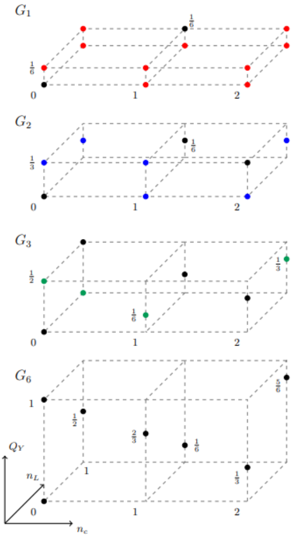
eg. for p=6, $G_6 : \xi R = R \Rightarrow \frac{n_c}{3} + \frac{n_L}{2} + Q_Y = \mathbb{Z}.$

- The invariance of SM representations under the action of Z_p leads to quantisation conditions for hyperchage.

D. Tong [1705.01853]

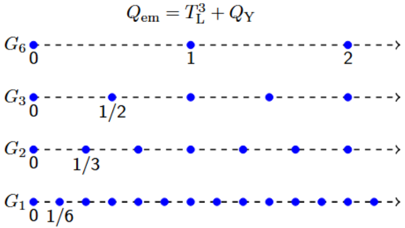
J. Hucks [Phys. Rev. D 43 (1991) 2709]

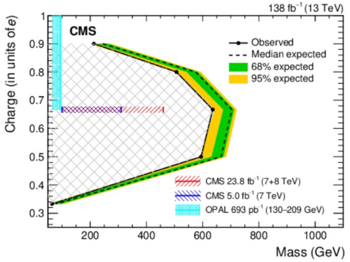
Constituent blocks of allowed hypercharge



Electromagnetic charge spectrum

- Notice periodicity of $p/6$ for G_p.





R. Alonso, **DD**, M. West [2404.03438]

S. Koren, A. Martin [2406.17850]

Embedding the whole SM group

$SU(5)$

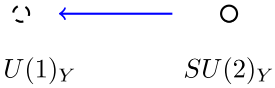
$SU(3)_c \times SU(2)_L \times U(1)_Y$

- The Georgi-Glashow model of $SU(5)$ combines q's and ℓ's into a single irrep.

$$\bar{\mathbf{5}} = \begin{pmatrix} \bar{d}_r \\ \bar{d}_b \\ \bar{d}_g \\ e \\ -\nu_e \end{pmatrix} \; ; \; \mathbf{10} = \begin{pmatrix} 0 & \bar{u}_g & -\bar{u}_b & u_r & d_r \\ -\bar{u}_g & 0 & \bar{u}_r & u_b & d_b \\ \bar{u}_b & -\bar{u}_r & 0 & u_g & d_g \\ -u_r & -u_b & -u_g & 0 & \bar{e} \\ -d_r & -d_b & -d_g & -\bar{e} & 0 \end{pmatrix}$$

- The hypercharge generator is structured as $\begin{pmatrix} SU(3)_c & \\ & SU(2)_L \end{pmatrix} \in SU(5)$.
- $SU(5)$ leads to G_6, Pati-Salam to G_3 and Trinification to G_2.

Embedding hypercharge



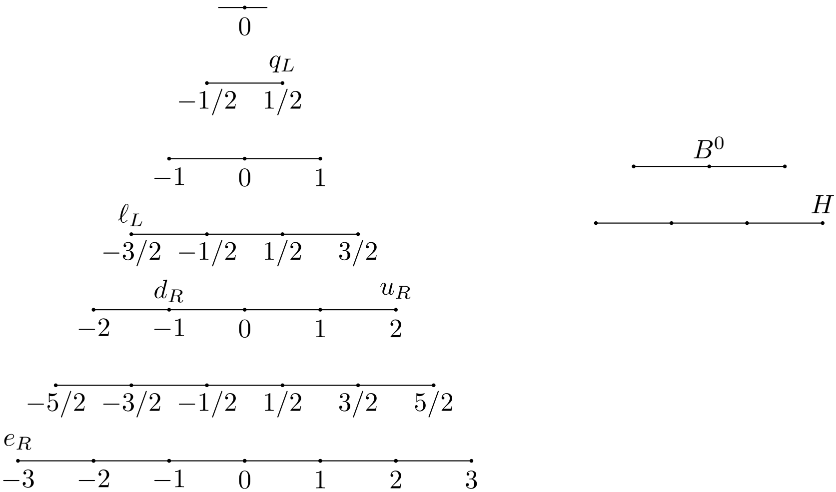
- $SU(2)_Y \ni U(1)_Y$ is the smallest non-abelian gauge group that can further break into $U(1)_{em}$.
- The hypercharge operator is rescaled as

$$Q_{em} = T_L^3 + Q_Y = T_L^3 + \frac{T_Y^3}{3}.$$

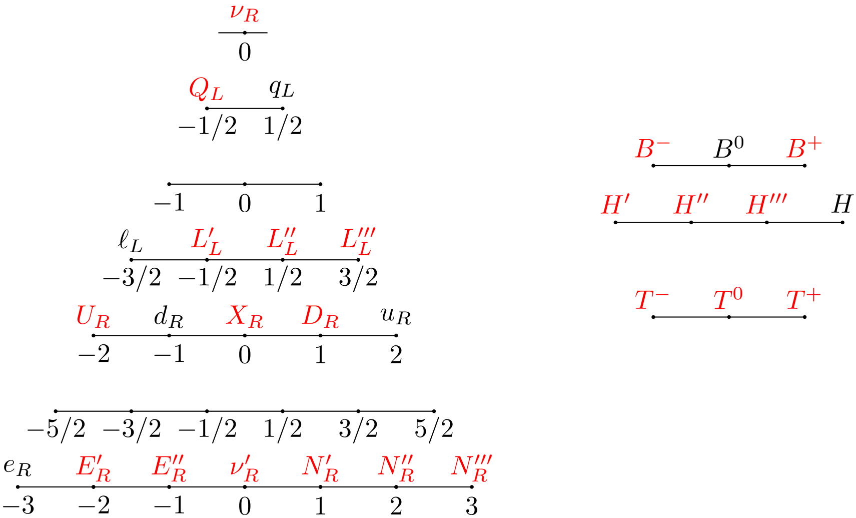
- Anomaly free (not only we embedded SM fermions but added others so anomalies cancel).

R. Alonso, **DD**, Y. Ha, V. Khoze, arXiv [2507.01777]

Matter content of $SU(3) \times SU(2)^2$ model

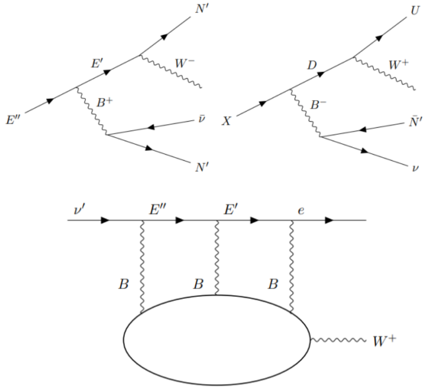


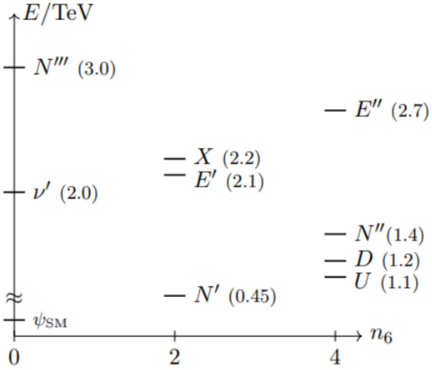
Matter content of $SU(3) \times SU(2)^2$ model



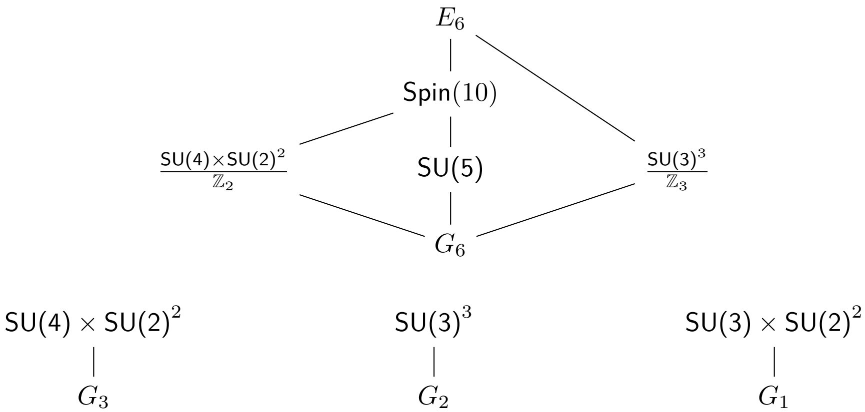
BSM masses and decays

Cannot decay to solely SM particles!





GUT maps to SM



Conclusion

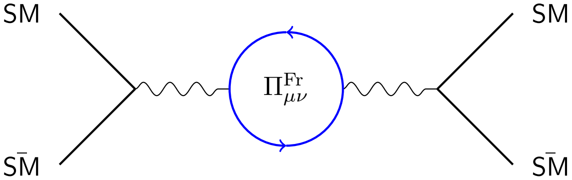
- Detection of fractionally charged particles would showcase the actual Standard Model Group.
- More fundamentally, they may help us understand charge quantisation.
- Probing the discrete symmetries of the Standard Model, we can learn more about BSM theories (eg. GUTs).

Thank you for listening!

Extra material

How do these fractional states couple to SM?

- Due to their fractional nature these new fields cannot couple linearly to SM.

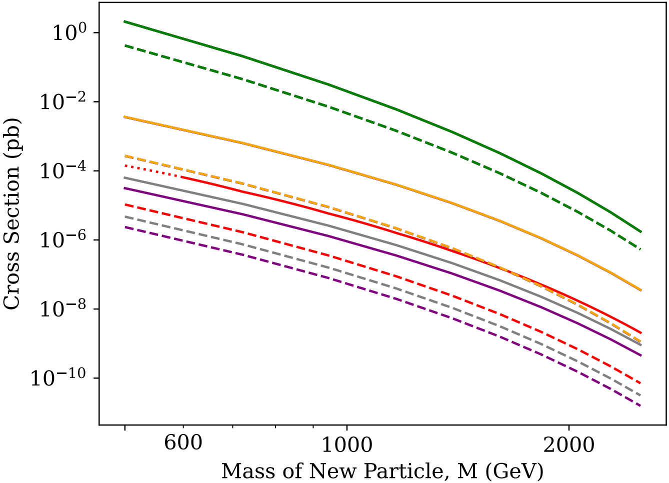


$$\sigma_{q\bar{q}\to \mathsf{Fr}\bar{\mathsf{Fr}}} = \frac{g^4}{s}\mathrm{I}(q)\mathrm{I}(R)\,\mathsf{Im}(i\Pi^{\mu\nu}_{Fr})\,[p_{2\mu}p_{1\nu} + p_{2\nu}p_{1\mu} - g_{\mu\nu}(p_2\cdot p_1 + m_q^2)]\mathrm{d}(\mathsf{Ad})$$

$$\sum_a T_a(R)T_a(R) \equiv C(R)\mathbb{1} \quad (1)$$

$$C(R)\mathrm{d}(R) = \mathrm{I}(R)\mathrm{d}(\mathrm{Ad}) \quad (2)$$

Hadronic cross section varying with mass



Applications in SMEFT with fractional states

SMEFT is an effective field theory that describes SM interactions with higher dimension operators.

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{d=6}} + \ldots \tag{3}$$

$$\mathcal{L}_{d=6} \supset -\frac{2\delta}{v^2}\left(a_c J_c^b J_c^b + a_L J_L^I J_L^I + a_Y J_Y J_Y\right) \tag{4}$$

with b the colour index, I the isospin, the coefficients a_c, a_Y

$$\delta = \frac{a_s \mathrm{d}_L \mathrm{d}_c v^2}{(4\pi)^2 240 M^2}\,, \qquad a_c = \frac{\mathrm{I}_c g_c^4}{\mathrm{d}_c}\,, \tag{5}$$

$$a_L = \frac{\mathrm{I}_L g^4}{\mathrm{d}_L}\,, \qquad a_Y = Q_Y^2 (g_Y)^4\,. \tag{6}$$

and the $SU(2)_W, U(1)_Y$ Higgs currents

$$\begin{aligned} J_{L,\mu,a} &= i(H^\dagger T_{L,a} D_\mu H - (D_\mu H)^\dagger T_{L,a} H) \\ J_{Y,\mu} &= iQ_Y(H^\dagger D_\mu H - (D_\mu H)^\dagger H). \end{aligned} \tag{7}$$

BSM electroweak precision observables

The ratio of a_L and a_Y takes a discrete set of values for G_p that can be used to infer the quantum numbers of the new particle.
It cannot be determined by a single observable as δ is a free parameter but it can be determined by the correlation between two observables.
Let us define our input scheme:

$$M_W^2 = \frac{g^2 v^2}{4}\left(1 - a_L \delta\right) , \tag{8}$$

$$M_Z^2 = \frac{g^2 v^2}{4 c_w^2}\left(1 - a_L \delta - a_Y \delta\right) , \tag{9}$$

$$G_F = \frac{1}{\sqrt{2} v^2} , \tag{10}$$

$$s_w \equiv \frac{\sqrt{4\pi\alpha_{em}}}{2 M_W (\sqrt{2} G_F)^{1/2}} = s_w (1 + \frac{1}{2} a_L \delta) . \tag{11}$$

ρ parameters

We can then substitute these in the expression for two other observables that maximise the range of angles for the correlation of the Wilson coefficients,

$$\rho_{\Gamma 3} \equiv \frac{1}{6} \frac{M_Z^3 \Gamma_W}{M_W^3 \Gamma_Z^{\rm inv}} = (1 + \delta a_Y)\,, \tag{12}$$

$$\rho_{\Gamma 5} \equiv \frac{1}{6} \frac{(1-\bar{s}_w^2) M_Z^5 \Gamma_W}{M_W^5 \Gamma_Z^{\rm inv}} = (1 - \frac{1}{2} 2\bar{t}_w^2 \delta a_L)\,. \tag{13}$$