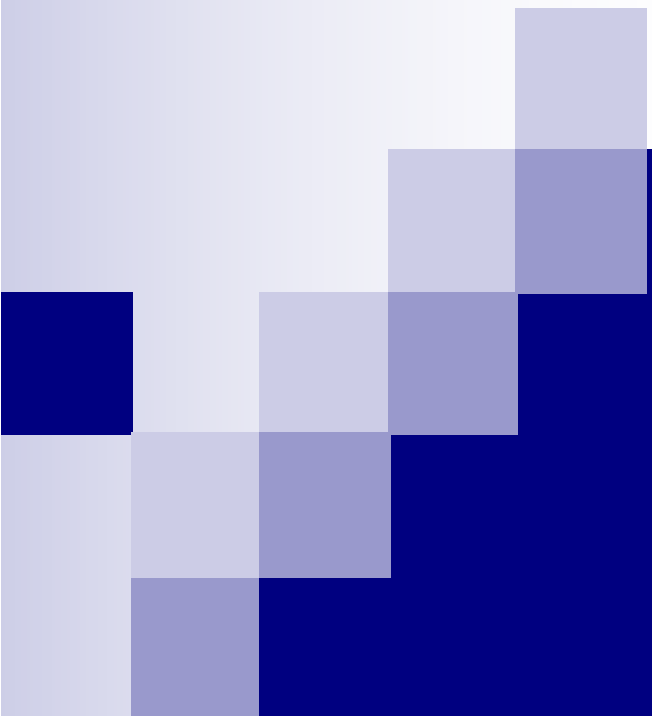




MOND Part I: Modified gravity on small scales

Benoit Famaey

CNRS - Observatoire astronomique de Strasbourg

Is dark «matter» inevitably matter?

■ Generalized Dark Matter » (Hu 1998; Kopp et al. 2016):

consider a very general fluid with non-zero pressure and anisotropic stress, such that

$$w_g(a) = \bar{p}/\bar{\rho},$$

with a sound speed that can depend on scale $c_s^2(\mathbf{k}, a)$

and a « viscosity » $c_{vis}^2(\mathbf{k}, a)$

This can typically describe the evolution of a k-essence scalar field

$$\mathcal{L}_\phi \propto \sqrt{-g} f(X) \quad \text{where } X \propto \nabla^\nu \phi \nabla_\nu \phi$$

... describes a « dark matter-like » fluid that naturally arises for instance in relativistic MOND theories of modified gravity, but what does one mean by « matter » ?

Is dark «matter» inevitably matter?

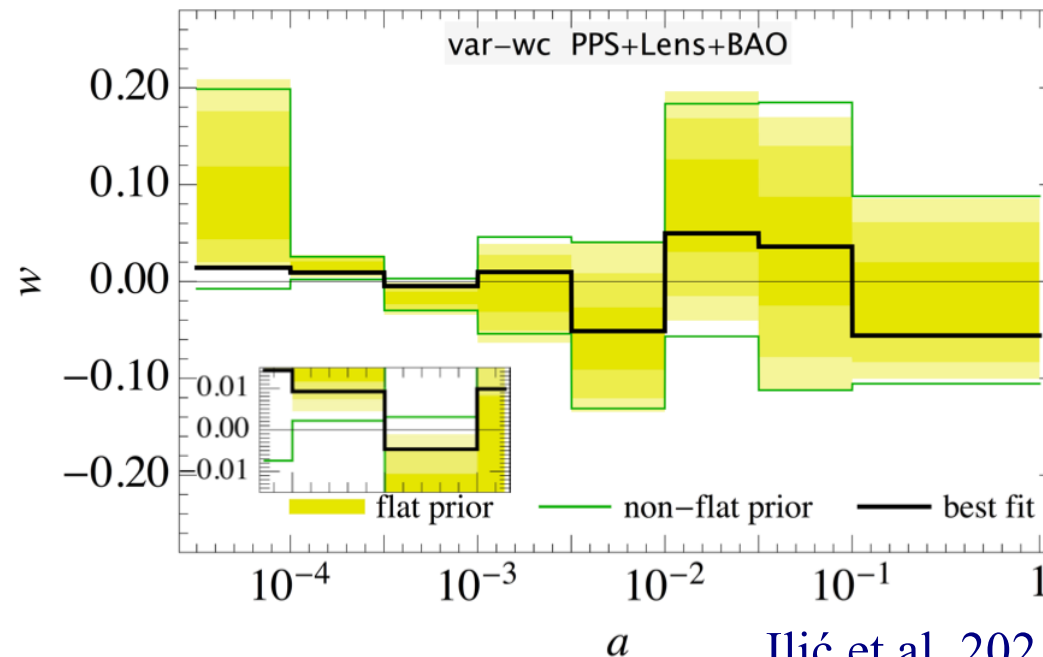
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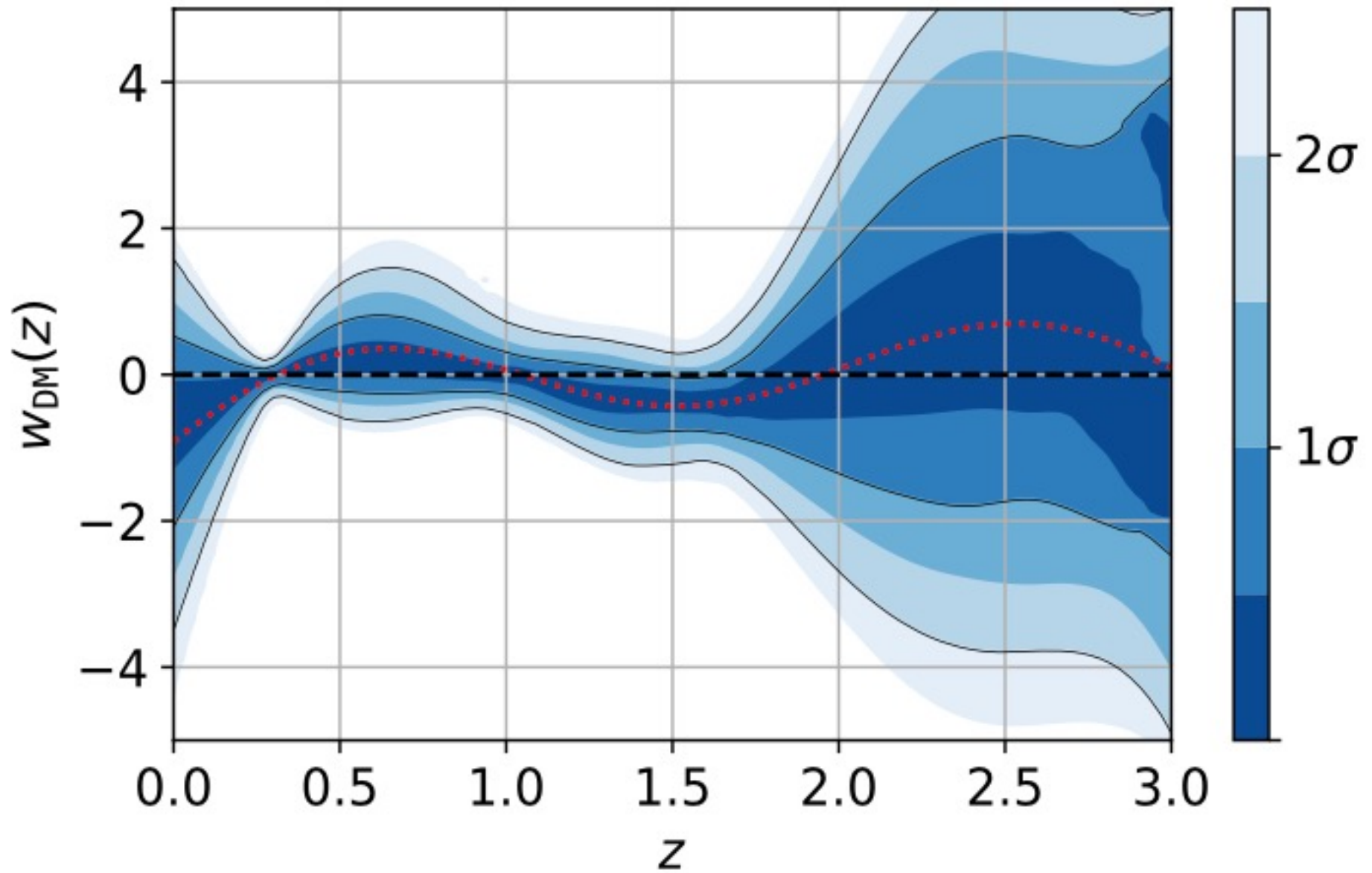
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CC+DESI-DR2



Back to galaxies : the « NO DM » hypothesis

$$\begin{array}{ll} g = g_N & \text{if } g \gg a_0 \\ g = (g_N a_0)^{1/2} & \text{if } g \ll a_0 \end{array}$$

MOND
Milgrom 1983

$$a_0 \sim 10^{-10} \text{ m/s}^2 \sim c^2 \sqrt{\Lambda}$$

$$a_N = \mu(a/a_0) a$$

or

for $x \ll 1$, one should have $\mu(x) \rightarrow x$ or $v(x) \rightarrow 1/\sqrt{x}$.

$$a = v(a_N/a_0) a_N,$$

Back to galaxies : the « NO DM » hypothesis

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Newtonian Lagrangian density in the non-relativistic limit : $\mathcal{L} = -\rho \left(\Phi - \frac{1}{2} \vec{v} \cdot \vec{v} \right) - \frac{1}{8\pi G} \mathcal{L}_N$

$$\mathcal{L}_N = \nabla \Phi \cdot \nabla \Phi \equiv (\nabla \Phi)^2 \rightarrow \mathcal{L}_{\text{AQUAL}} = a_0^2 \mathcal{F}((\nabla \Phi)^2 / a_0^2).$$

$$\nabla \cdot \left(\frac{\partial \mathcal{L}_{\text{AQUAL}}}{\partial (\nabla \Phi)} \right) = \frac{\partial \mathcal{L}_{\text{AQUAL}}}{\partial \Phi}$$

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$$\mathcal{F}(Y) \rightarrow Y \text{ for } Y \gg 1 \text{ and } \mathcal{F}(Y) \rightarrow \frac{2}{3} Y^{3/2} \text{ for } Y \ll 1$$

$$\mathcal{F}'(Y) = \mu(\sqrt{Y}) \rightarrow \nabla \cdot (\mu(|\nabla \Phi|/a_0) \nabla \Phi) = 4\pi G \rho$$

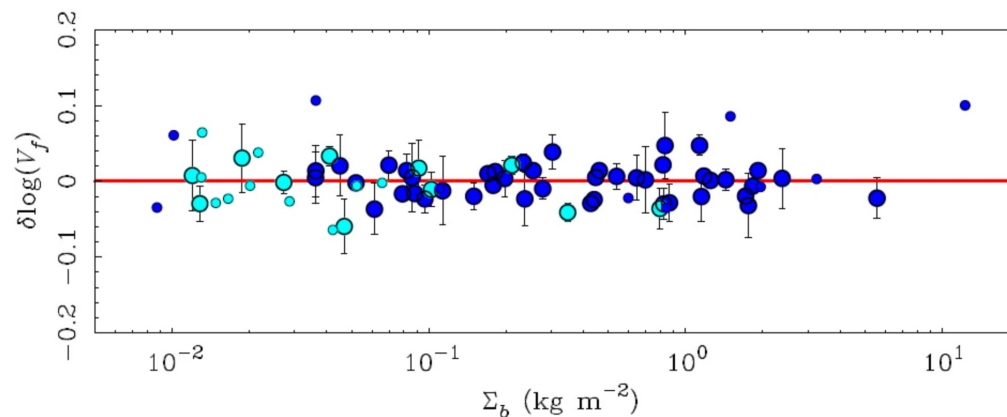
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MOND
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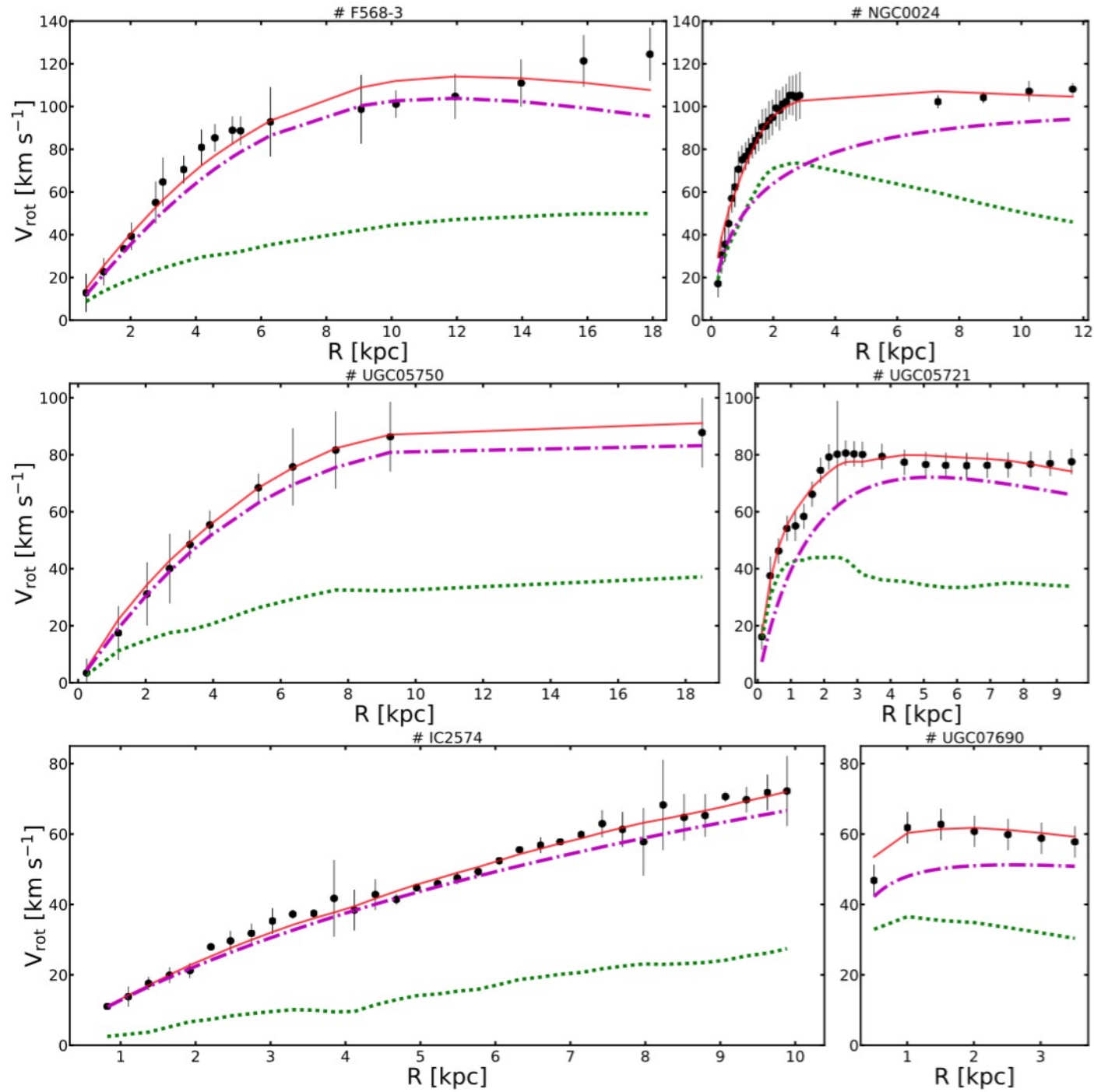
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Assume fixed factor between the total mass and the baryonic one at fixed baryonic mass M_b , then within a given R , expect $V^4 \sim M_b^2/R^2 \sim M_b \Sigma_b$ where Σ_b is the surface density
In MOND, what is predicted is $V^4 \sim M_b$, with no dependence on size or Σ_b



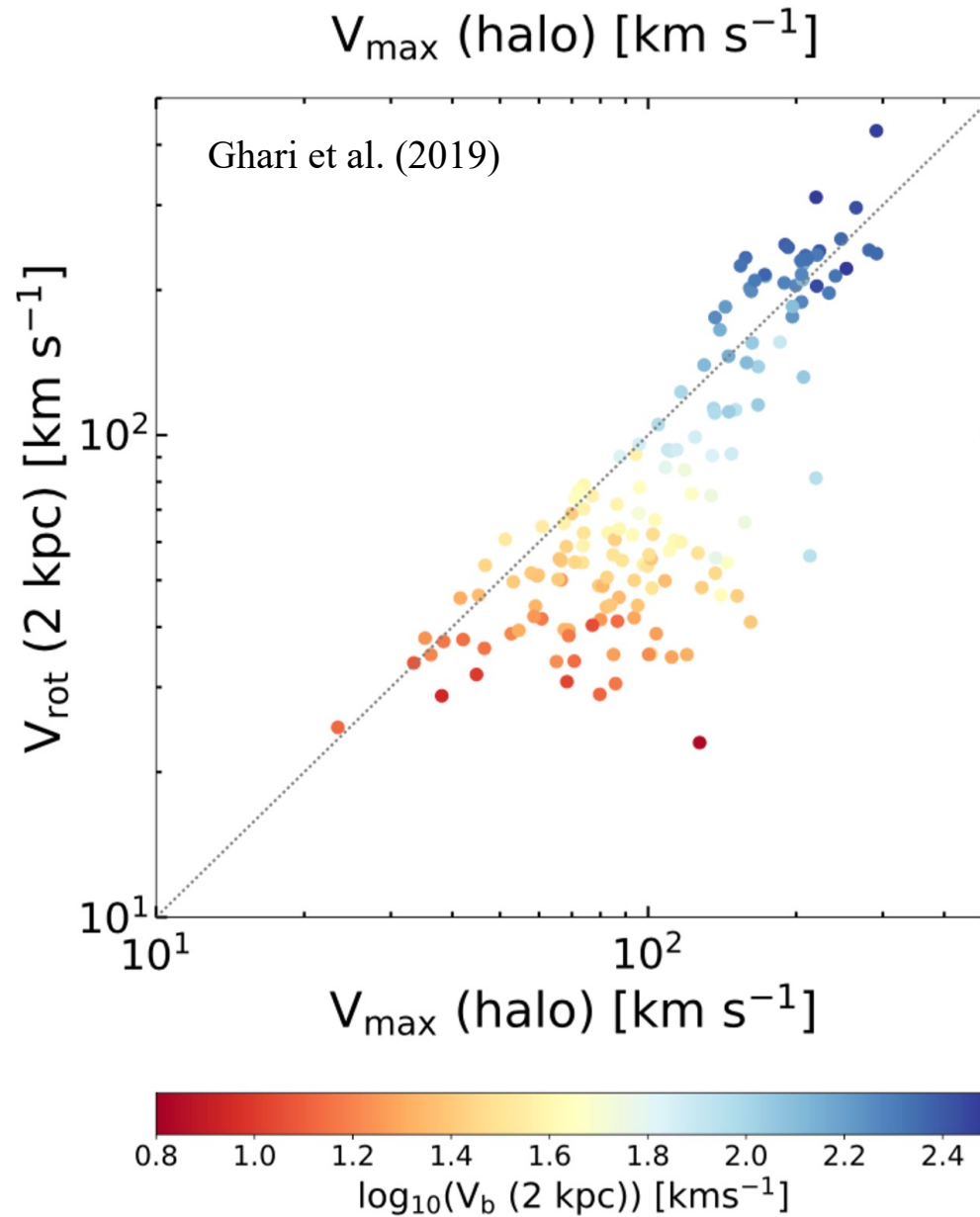
“We predict a correlation between the value of the average surface density of a galaxy and the steepness with which the rotational velocity rises to its asymptotic value.”

Milgrom (1983)

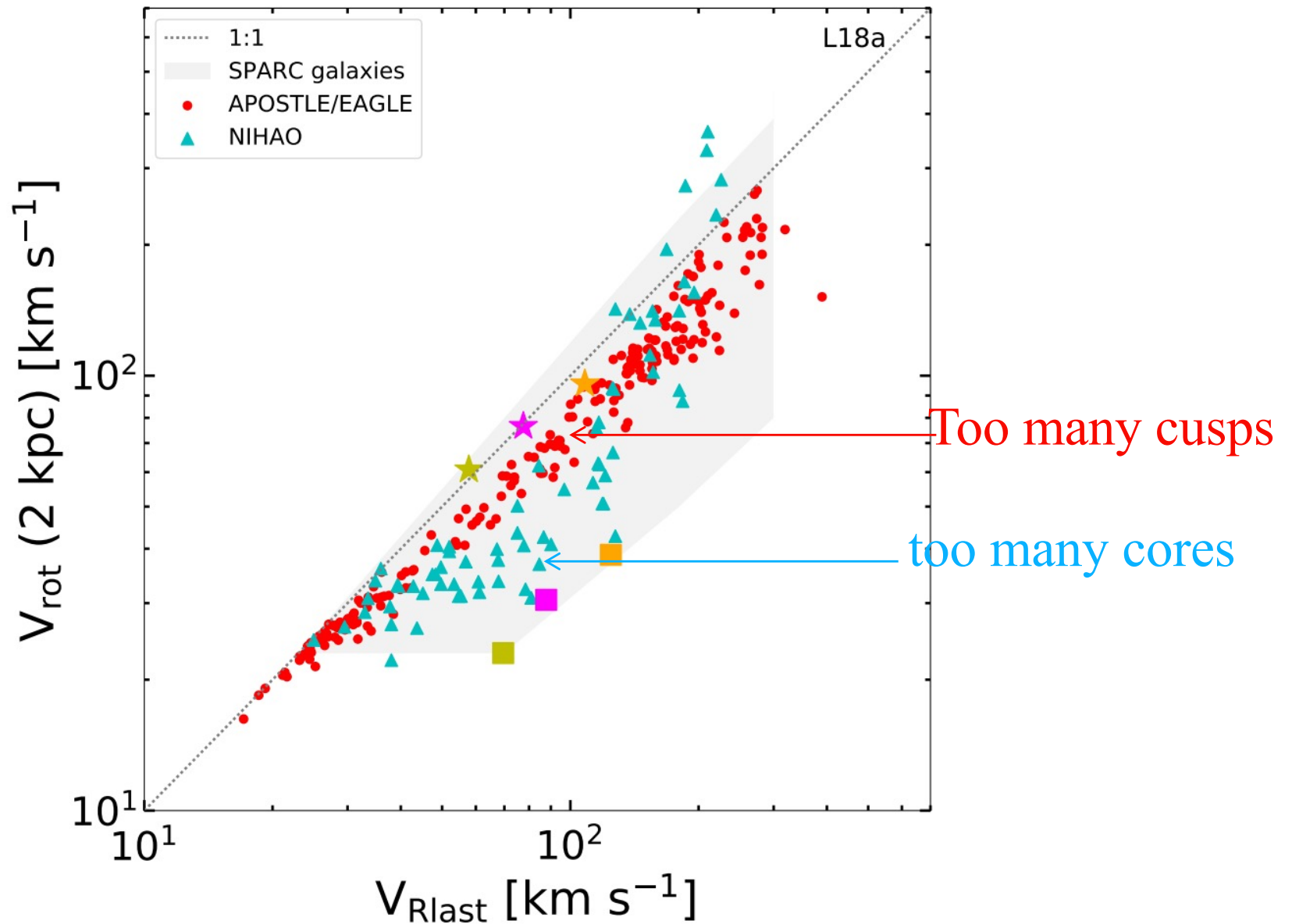


Ghari, Famaey, et al.
(2019)

Diversity of rotation curves



NOT explained within cold dark matter simulations



The rotation curves shapes of late-type dwarf galaxies

R. A. Swaters^{1,2,★}, R. Sancisi^{3,4}, T. S. van Albada³, and J. M. van der Hulst³ (2009)

HI observations for a sample of 62 galaxies [...] procedure takes the rotation curve shape, the HI distribution, the inclination, and the size of the beam into account, and makes it possible to correct for the effects of beam smearing.

In spiral galaxies and even in the central regions of late-type dwarf galaxies, the shape of the central distribution of light and the inner rise of the rotation curve are related. This implies that galaxies with stronger central concentrations of light also have higher central mass densities, and it suggests that the luminous mass dominates the gravitational potential in the central regions, even in low surface brightness dwarf galaxies (NB: dominated by... dark matter?!)



MOND

$$g = g_N$$

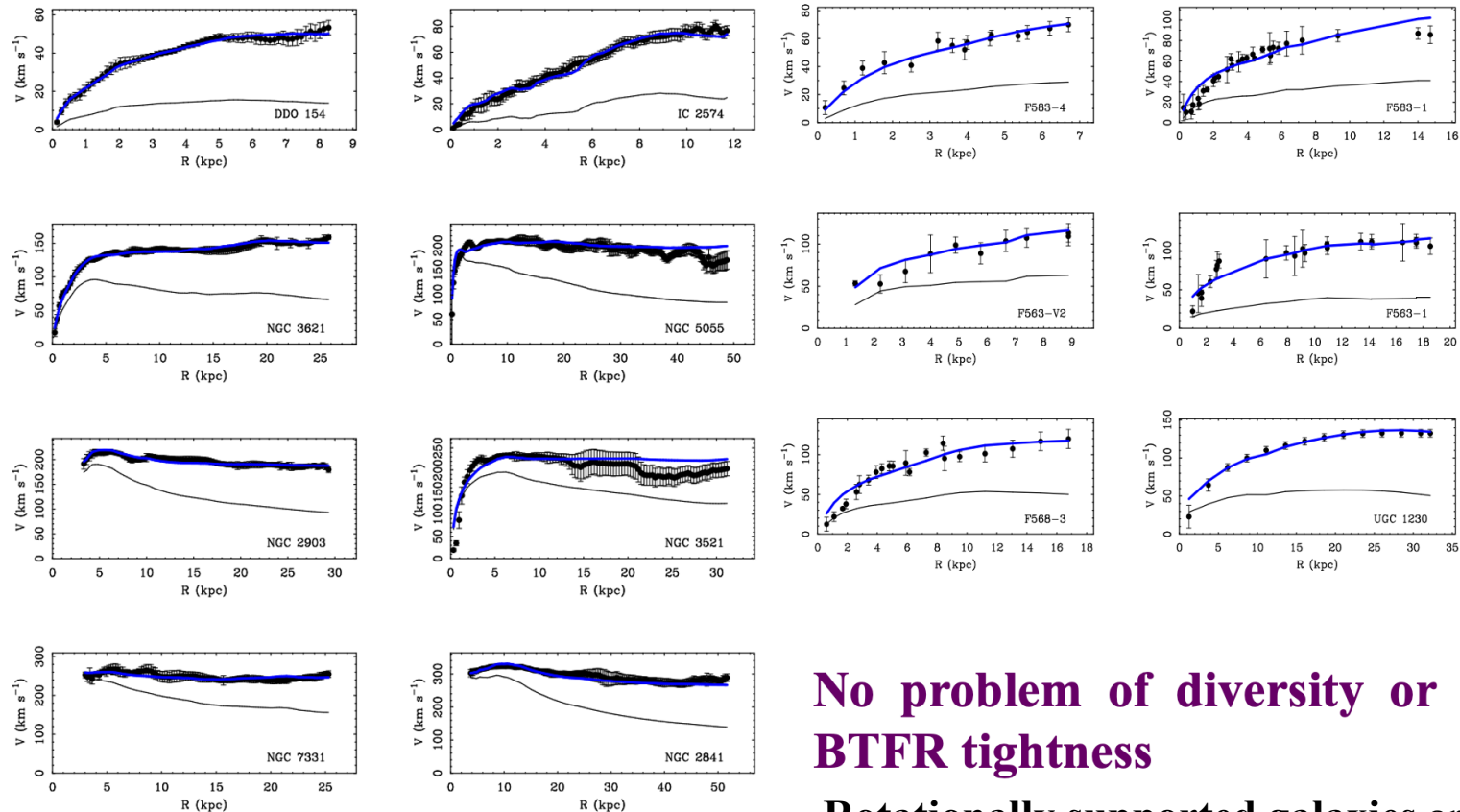
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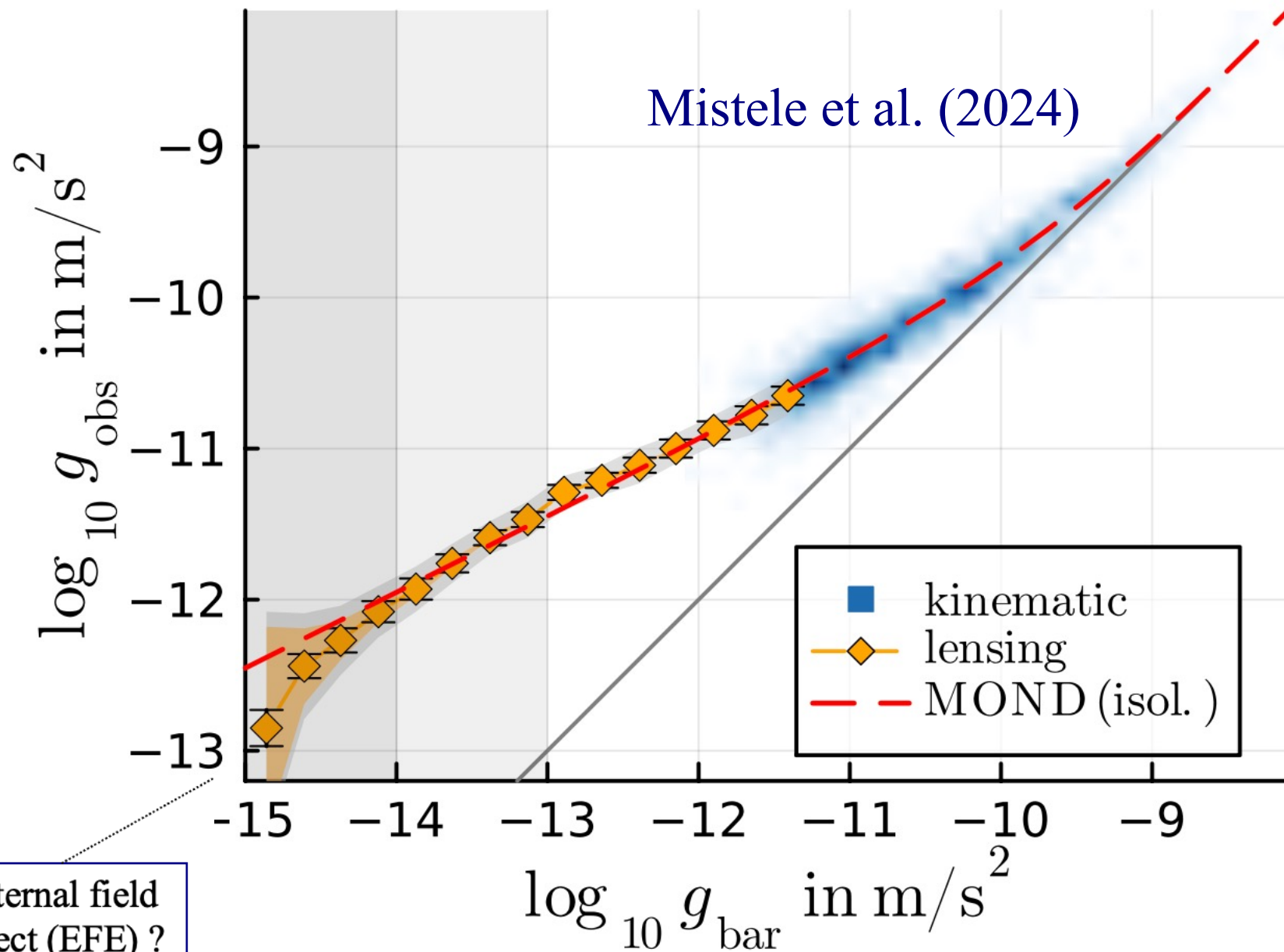


**No problem of diversity or
BTFR tightness**

Rotationally supported galaxies are all “MONDian”
also valid for rotationally supported “UDGs”

Famaey & McGaugh (2012)

Weak lensing



Relativistic MOND

MOND action: $S_{\text{grav BM}} \equiv - \int \frac{a_0^2 F(|\nabla\Phi|^2/a_0^2)}{8\pi G} d^3x dt,$

⇒ Add a k-essence-like scalar + a vector field for lensing (goes back to TeVeS, [Bekenstein 2004](#), now generalized), but can be recasted as a pure « khronon » **scalar-tensor theory** as in [Blanchet & Skordis \(2024\)](#)

$$\begin{aligned} \mathcal{Q} = A^\mu \nabla_\mu \phi &\longrightarrow \dot{\phi} \\ \mathcal{Y} = \mathcal{Q}^2 + (\nabla\phi)^2 &\longrightarrow |\vec{\nabla}\phi|^2 \end{aligned}$$

$$\mathcal{F} = -2\mathcal{K}_2 (\mathcal{Q} - \mathcal{Q}_0)^2 + (2 - K_B)\mathcal{Y} + \frac{2(2 - K_B)}{3a_0} \mathcal{Y}^{3/2} + \dots$$

“dust” cosmology

Mixing

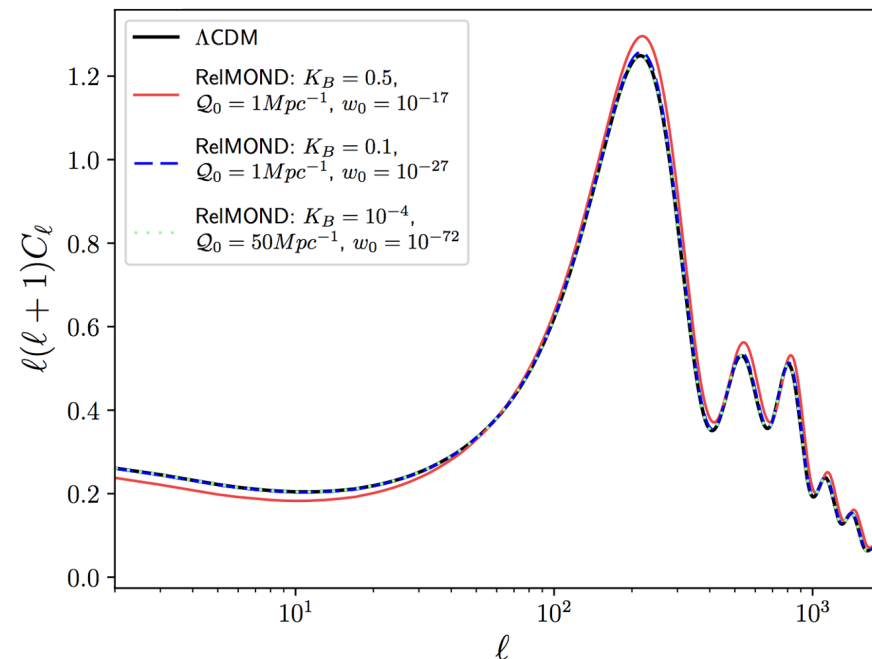
MOND

Bonus: GW and light speeds are equal

Relativistic MOND

Perturbations: $\phi(t, x) = \mathcal{Q}_0 t + \varphi(t, x)$

=> mimicks LCDM to linear order, but pressure contrast (**Generalized DM !!**)



Skordis & Zlosnik

Static weak field limit: $\vec{\nabla} \cdot (\mathcal{M} \vec{\nabla} \Phi) + \tilde{\mu}^2 \Phi = 4\pi G_N \rho_b$

=> Almost MOND but oscillations of the potential at large radii (problem ?)

Conclusions on MOND

MOND is rather successful at predicting the dynamics of **galaxies**, especially rotationally-supported ones: the question is **why** does it make successful predictions?

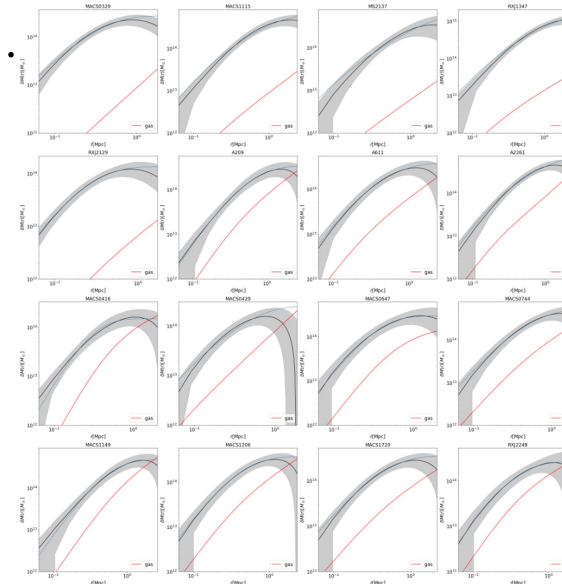
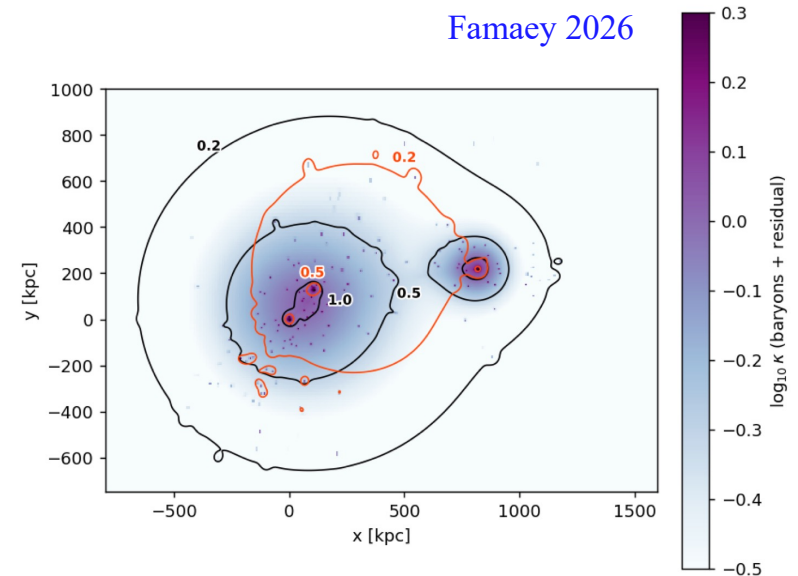
Main problems with the modified gravity approach:

- CMB ok, but at the price of oscillations around galaxies?
- Clusters! MOND **does not work** (without unseen baryons). But see what happens with relativistic MOND? Bullet cluster always complicated to get..

Note: in MOND, missing mass follows almost exactly the ICM at 100-300 kpc from the center

=> Missing baryons ?

Famaey 2026



$$\rho_{\text{res}}(r) = \eta \rho_{\text{gas}}(r) \exp(-r/r_{\text{cut}})$$

Famaey, Pizzuti,
Saltas (2025)

Conclusions on MOND

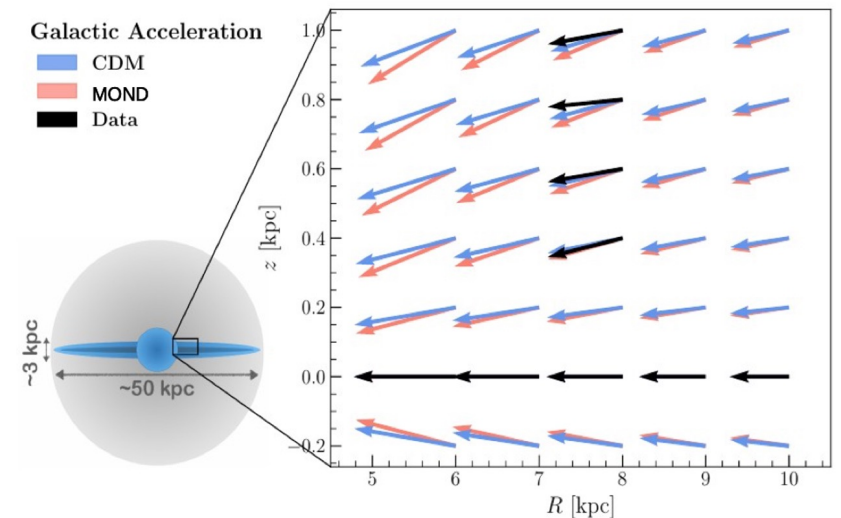
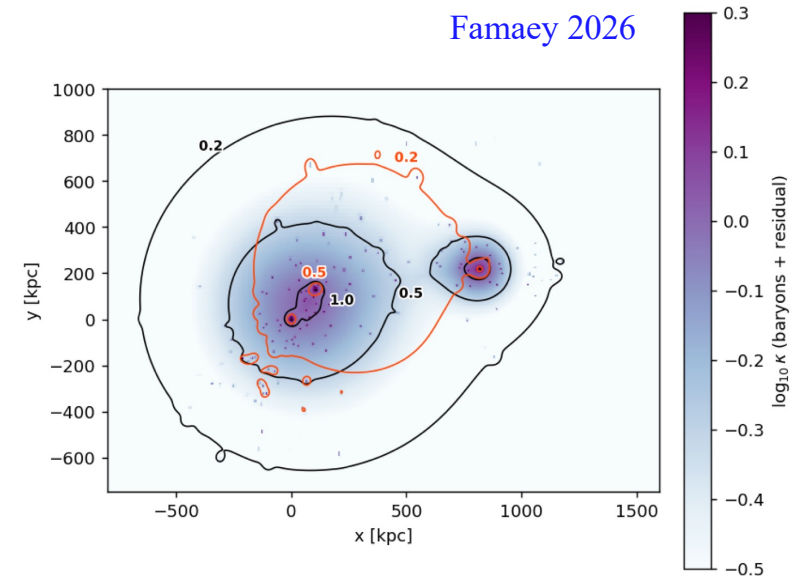
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- Modified gravity MOND fails in Milky Way disk

=> The « phantom » density needs to be **rounder** (no dark disk)

+ question of **dynamical friction** in the MW :
can the bar slow down in MOND ?

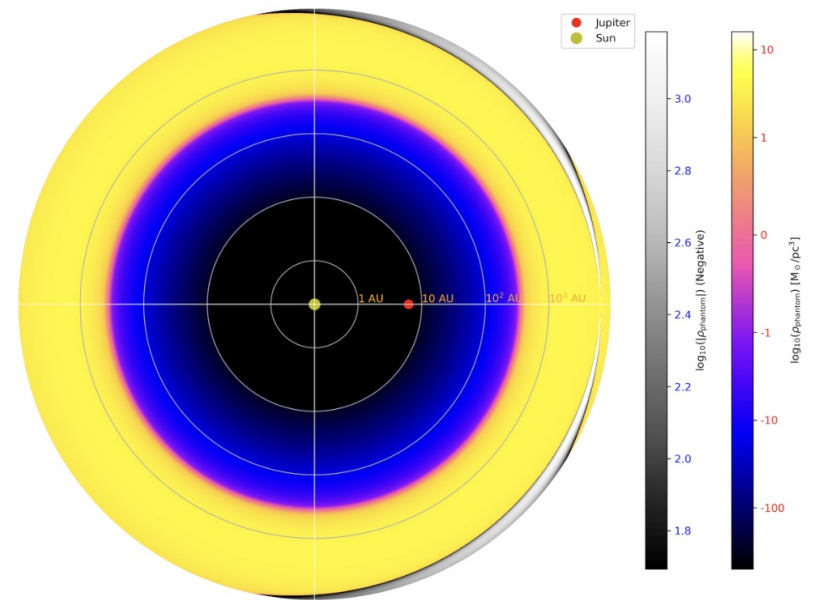


Conclusions on MOND

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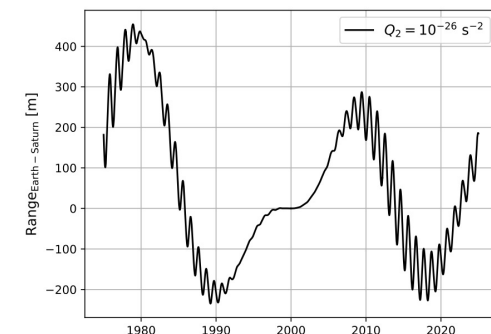
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- Modified gravity MOND fails in Milky Way disk
- Also fails in the Solar System (quadrupole well constrained at Saturn by Cassini, needs screening, that should also screen any effect in wide binaries)



$$\mathbf{a}_p = -\nabla\Phi_p = Q_2 \left[(\hat{\mathbf{e}} \cdot \mathbf{x}) \hat{\mathbf{e}} - \frac{\mathbf{x}}{3} \right]$$

$$|Q_2| \sim 10^{-26} \text{ s}^{-2} \quad |\mathbf{a}_p| \sim 10^{-4} a_0$$



13 years of Cassini data: [Park et al. 2026](#) $\Rightarrow Q_2 = (1.6 \pm 1.8) \times 10^{-27} \text{ s}^{-2} \quad (1\sigma)$

Generalizations with more dimensioned constants

$$\mathcal{L}_{\text{AQUAL}} = a_0^2 \mathcal{F}((\nabla\Phi)^2/a_0^2)$$

$$\mathcal{L}_{\text{QUMOND}} = 2\nabla\Phi \cdot \nabla\Phi_{\text{N}} - a_0^2 \mathcal{Q}((\nabla\Phi_{\text{N}})^2/a_0^2)$$

$$\mathcal{L}_{\text{EMOND}} = \Lambda(\Phi) \mathcal{F}((\nabla\Phi)^2/\Lambda(\Phi)),$$

$$\mathcal{L}_{\text{GQUMOND}} = 2\nabla\Phi \cdot \nabla\Phi_{\text{N}} - a_0^2 \mathcal{P}(\Phi_{\text{N}}, \nabla\Phi_{\text{N}}, \nabla^2\Phi_{\text{N}}, \dots, \nabla^n\Phi_{\text{N}})$$

$$\mathcal{L}_{\text{TriMOND}} = 2\nabla\Phi \cdot \nabla\Phi_{\text{N}} - a_0^2 \mathcal{P}((\nabla\Phi_{\text{N}})^2/a_0^2, (\nabla\varphi)^2/a_0^2, (2\nabla\Phi_{\text{N}} \cdot \nabla\varphi)/a_0^2)$$