

Stochastic models of the cosmic web: clues for dark matter searches ?

Sandrine Codis
- CNRS / AIM -

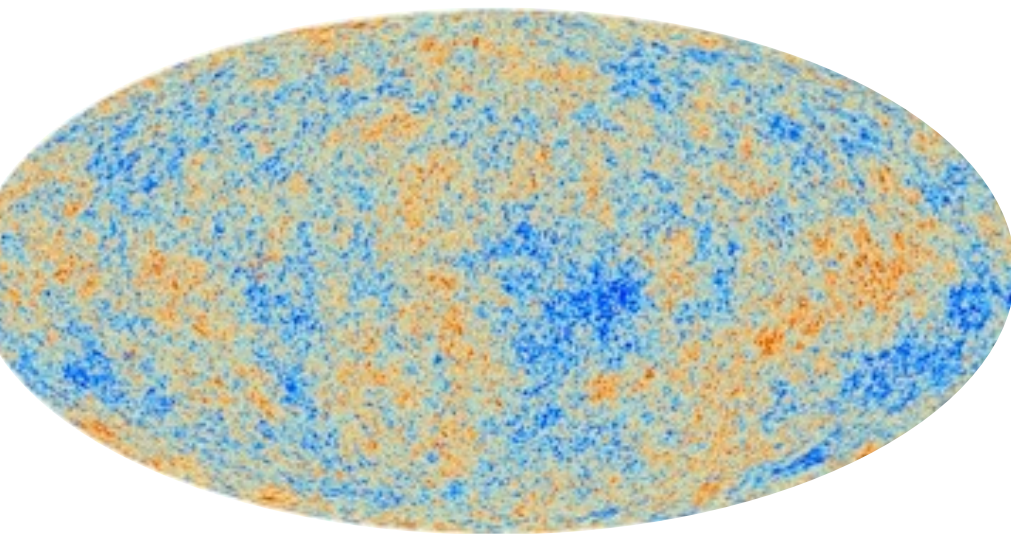


30/09/2026

News From The Dark 2026

How is the cosmic web woven?

Gaussian primordial fluctuations

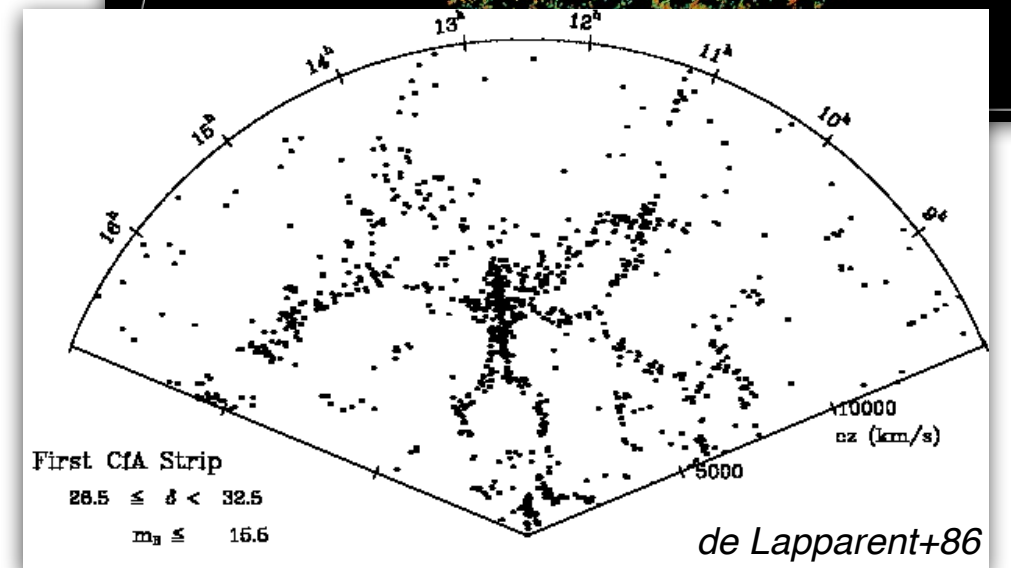
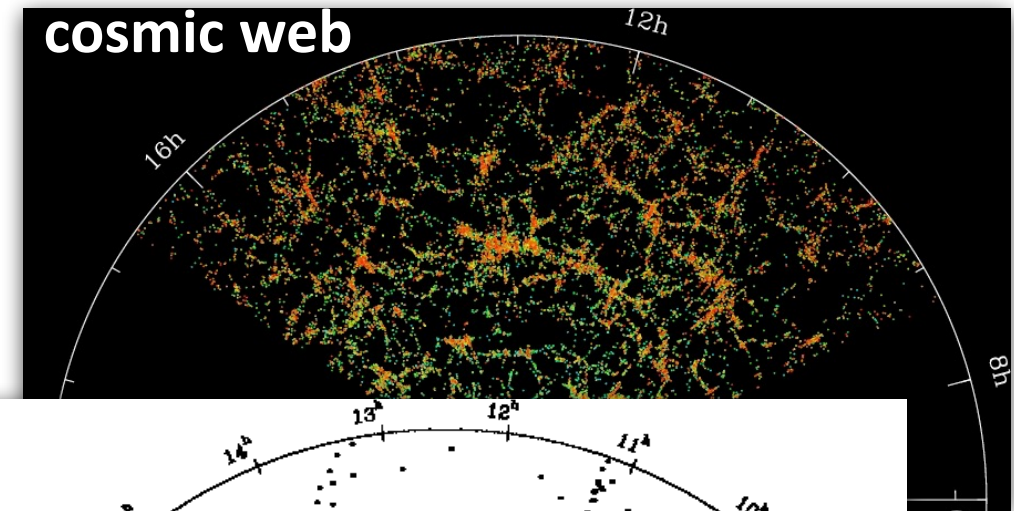


gravity

expansion



cosmic web



Vlasov-Poisson equations: dynamics of a self-gravitating collisionless fluid

Liouville theorem:

$$\left[\frac{\partial}{\partial t} + \frac{\mathbf{p}}{ma^2} \frac{\partial}{\partial \mathbf{x}} - m \nabla \phi \frac{\partial}{\partial \mathbf{p}} \right] f(\mathbf{x}, \mathbf{p}, t) = 0$$

Poisson equation:

$$\Delta \phi = 4\pi a^2 G(\rho - \bar{\rho})$$

These highly non-linear equations can be solved using numerical simulations or analytically in some specific regimes. Exact solutions are crucial to understand the details of structure formation.

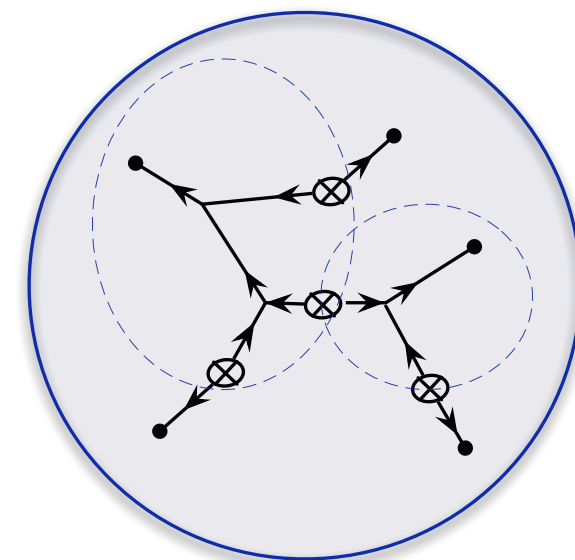
Before shell-crossing, moments > 2 can be neglected (velocity dispersion,...)

continuity equation:

$$\frac{\partial \delta}{\partial t} + \frac{1}{a} \nabla \cdot [(1 + \delta) \mathbf{u}] = 0$$

Euler equation:

$$\frac{\partial u_i}{\partial t} + \frac{\dot{a}}{a} u_i + \frac{u_j \partial_j u_i}{a} = -\frac{\partial_i \phi}{a} - \frac{\partial_j [\rho \delta_{ij}]}{\rho a}$$

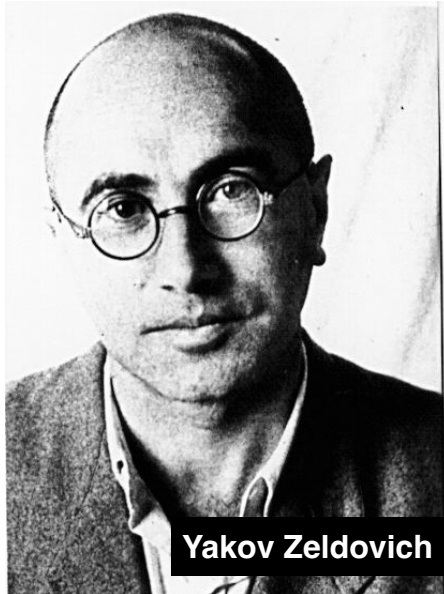


Stochastic models of the cosmic web



- ▶ Anisotropic cosmic web
- ▶ Galaxy alignments
- ▶ Cosmic shear nulling

Lagrangian dynamics: Zeldovich pancakes



$$\underset{\text{final position}}{\mathbf{x}} = \underset{\text{initial position}}{\mathbf{q}} + \underset{\text{displacement}}{\boldsymbol{\zeta}}$$

At linear order in the displacement, the Vlasov Poisson system reduces to

$$\nabla_{\mathbf{q}} \ddot{\boldsymbol{\zeta}} + 2H \nabla_{\mathbf{q}} \dot{\boldsymbol{\zeta}} = \frac{3}{2} \Omega H^2 \nabla_{\mathbf{q}} \boldsymbol{\zeta}$$

which has the same solution as the linear density contrast i.e

$$\zeta_{\text{ZA}} = D_+(t) \zeta_+(\mathbf{q})$$

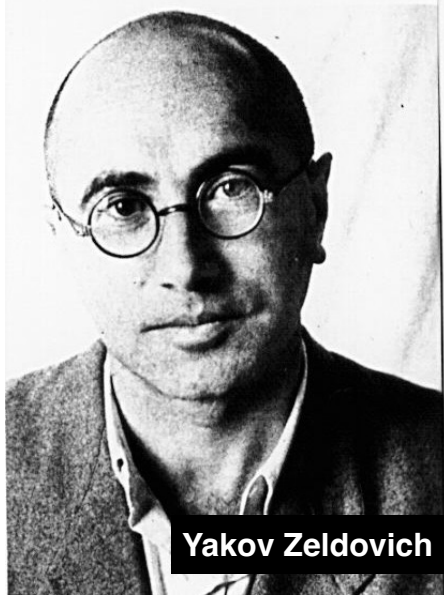
so that the density after a Zeldovich displacement reads:

$$\rho_{\text{ZA}}(\mathbf{q}, t) = \frac{\bar{\rho}}{\left| \prod_{i=1}^3 (1 - D_+(t) \lambda_i) \right|}$$

eigenvalues of the deformation tensor

Anisotropic collapse of structures and formation of *caustics*!
Walls form first followed by filaments and nodes.

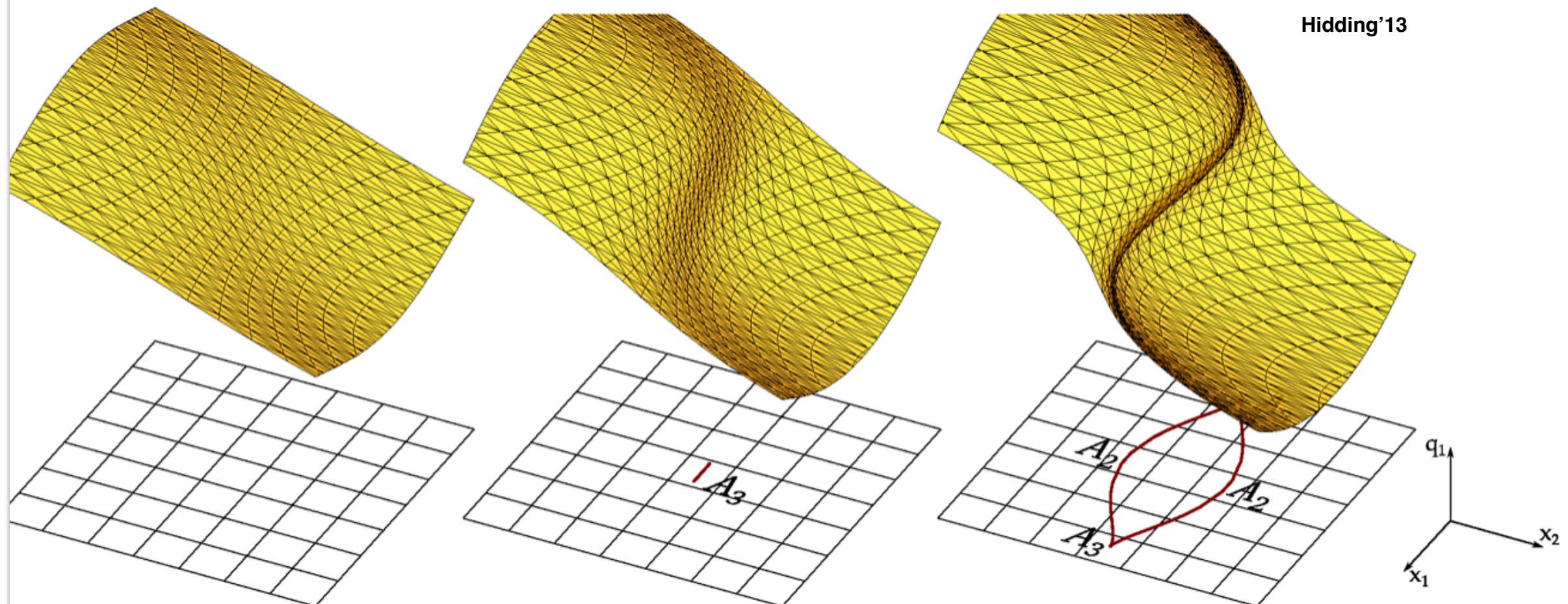
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The connected cosmic web



Dick Bond



Lev Kofman

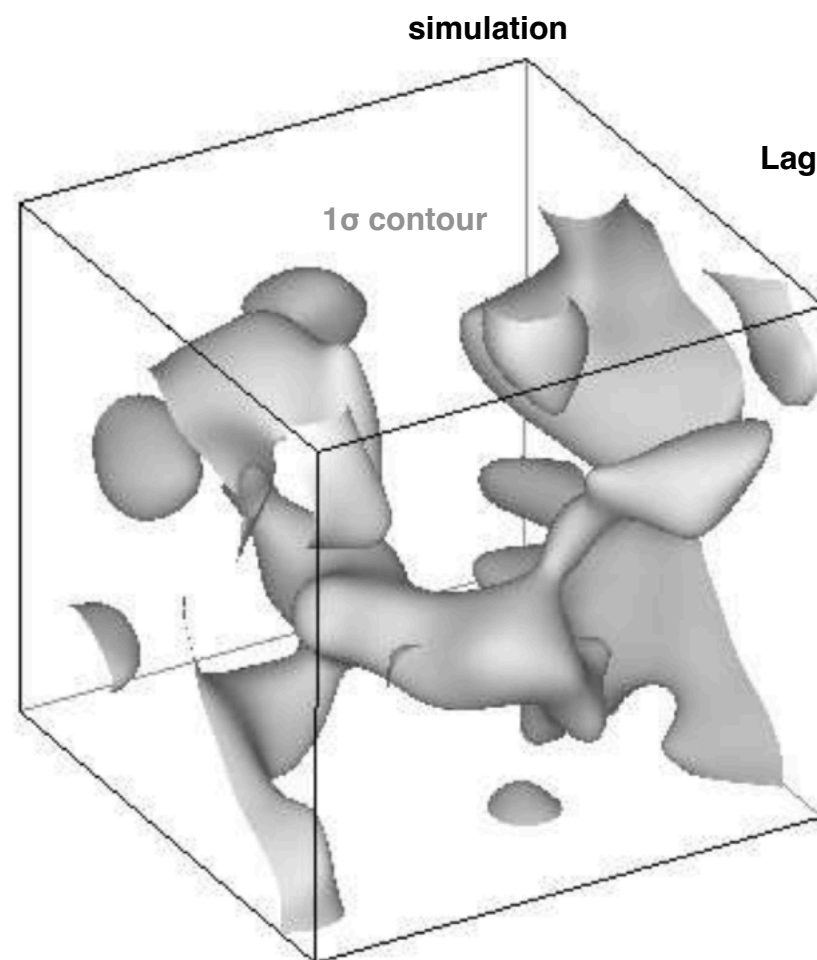


Dmitri Pogosyan

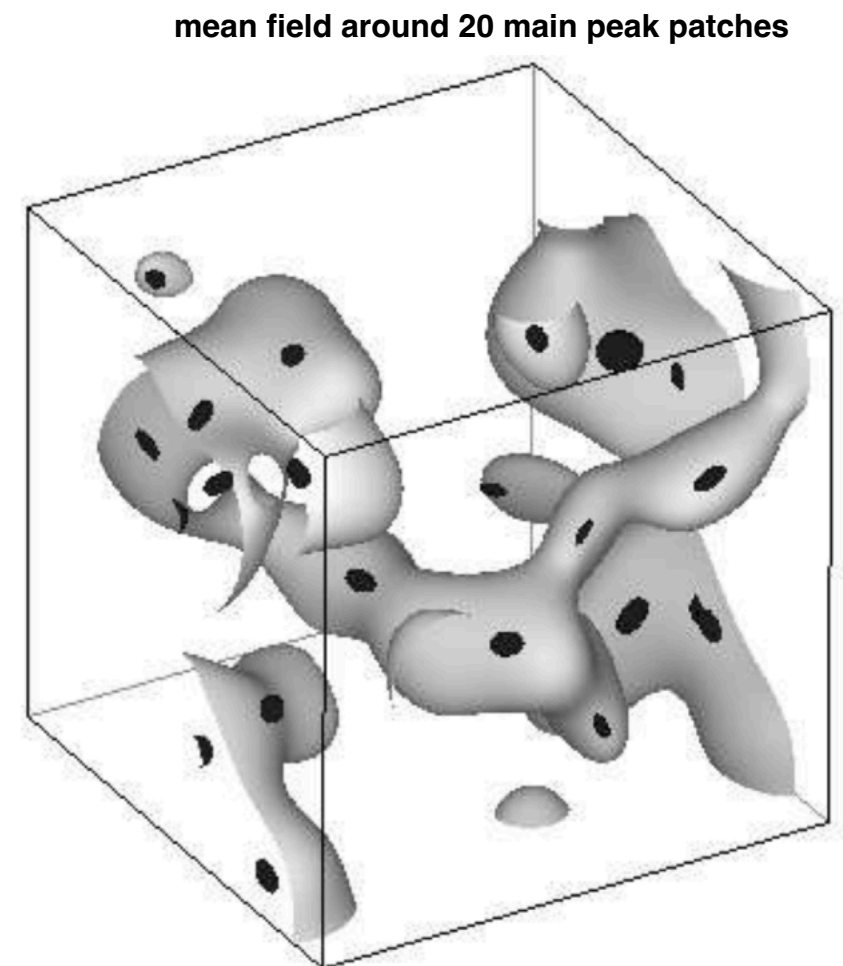
Bond, Kofman, Pogosyan 1996: first *understanding* of the origin of the cosmic web.

The seeds of walls, filaments and nodes lie in the asymmetries of the primordial Gaussian random field then amplified by gravitational instability.

Rare *peaks* in the ICs will become the nodes of the cosmic web i.e rich clusters. Their initial *shear* will set the preferred directions along which correlation bridges will connect them to other nodes.



Lagrangian space



Importance of peak & constrained random field theories

The connected cosmic web



Dick Bond



Lev Kofman

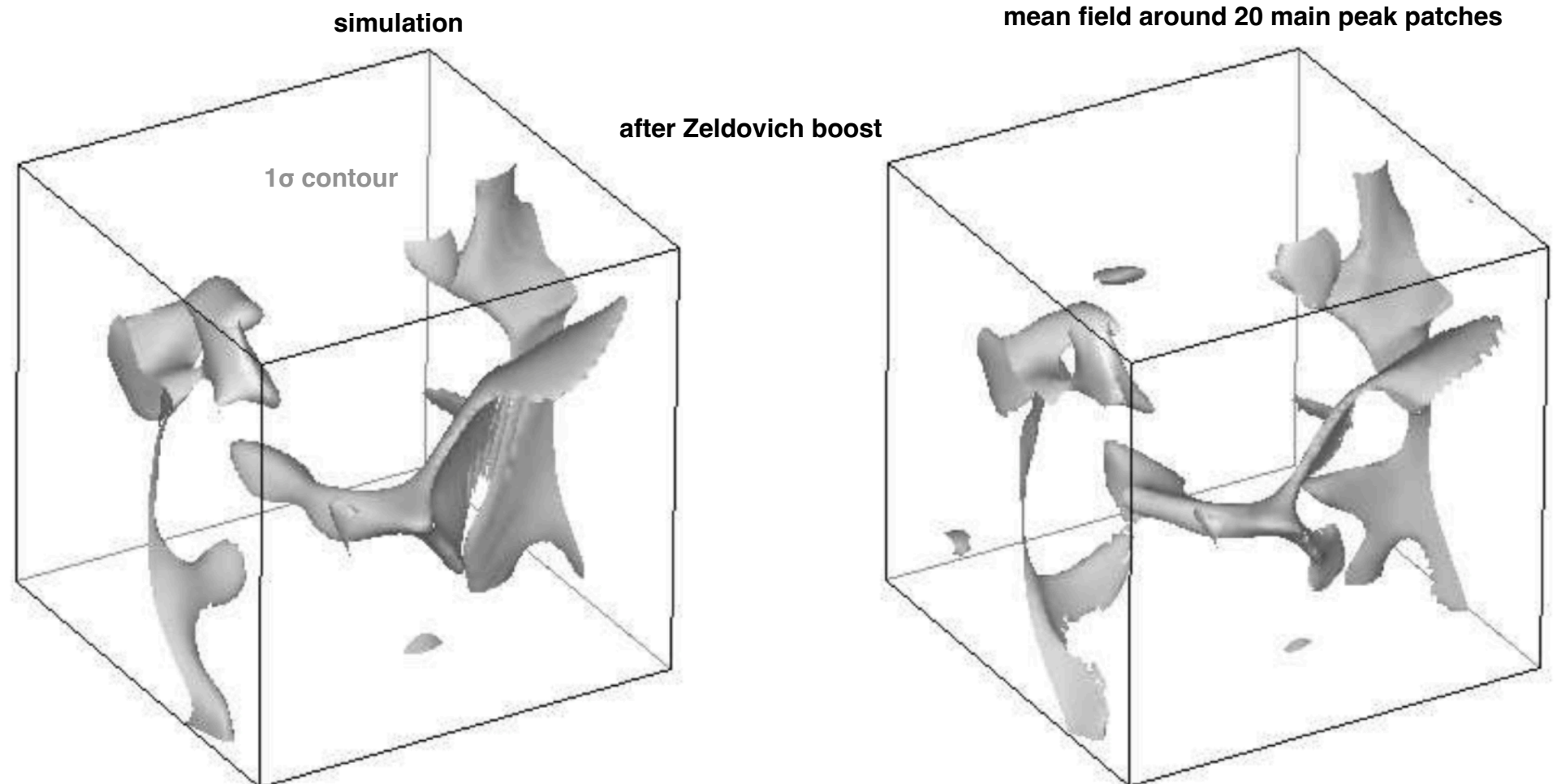


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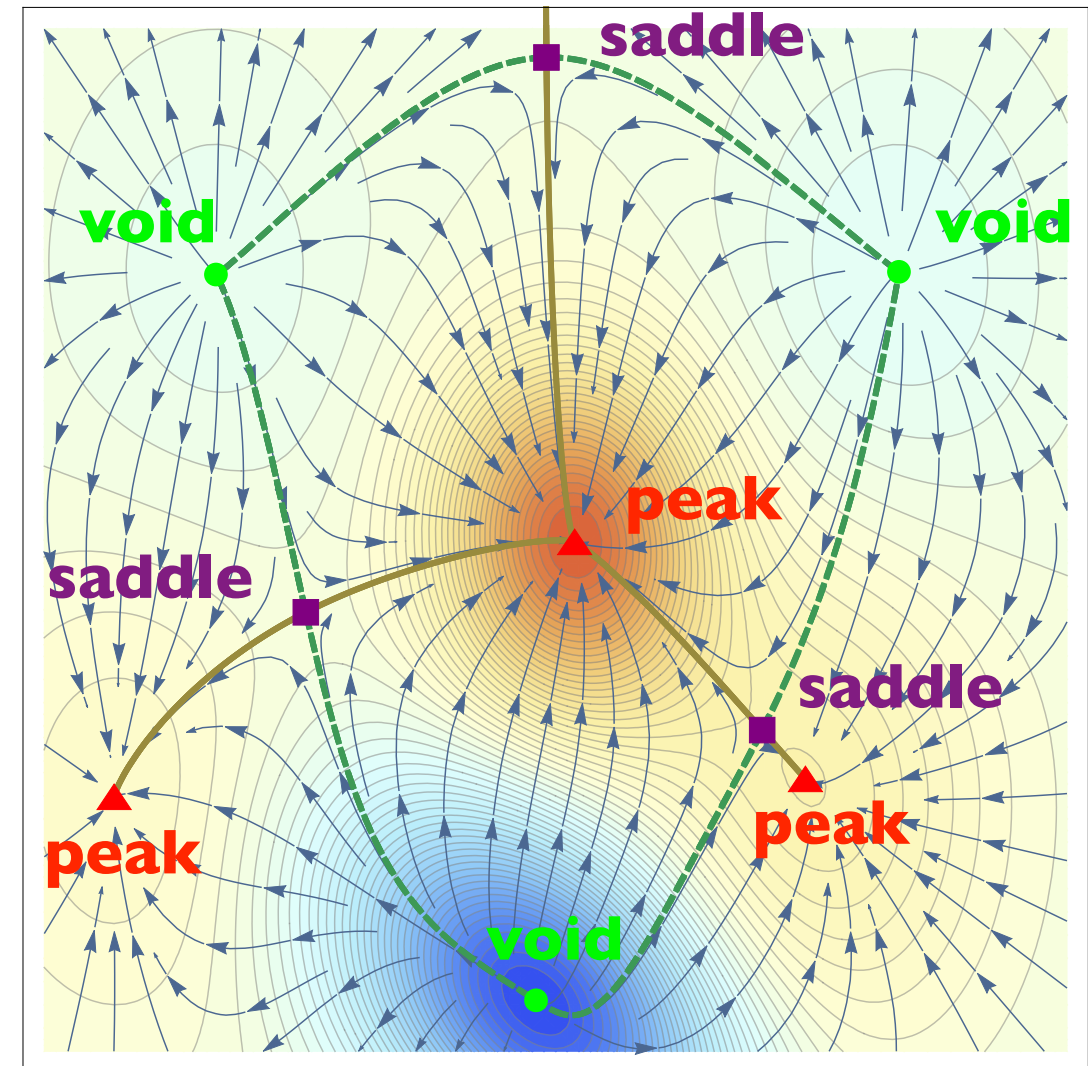


Importance of peak & constrained random field theories

The skeleton picture

Filaments are the field lines joining the maxima through saddle points.

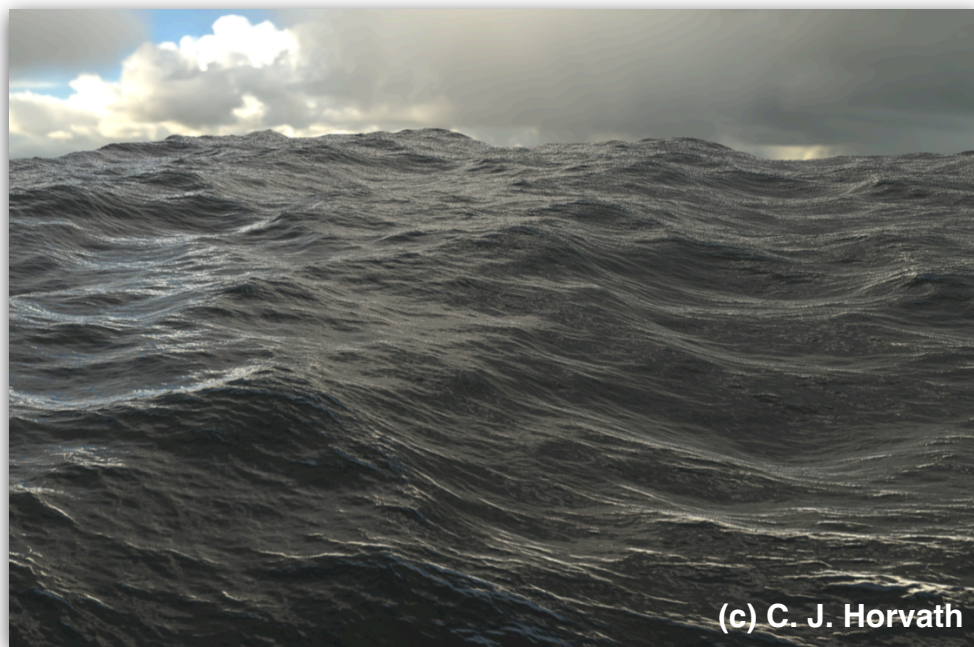
- cosmic web extractors (water-shedding, discrete topology, ...) *Sousbie+08, Sousbie+11, ...**
- local theory allowing for theoretical predictions for extrema counts, length of filaments, surface of voids, curvature ... which are very competitive cosmological probes! *BBKS, Pogosyan+09, Gay+12, SC+18 ...**



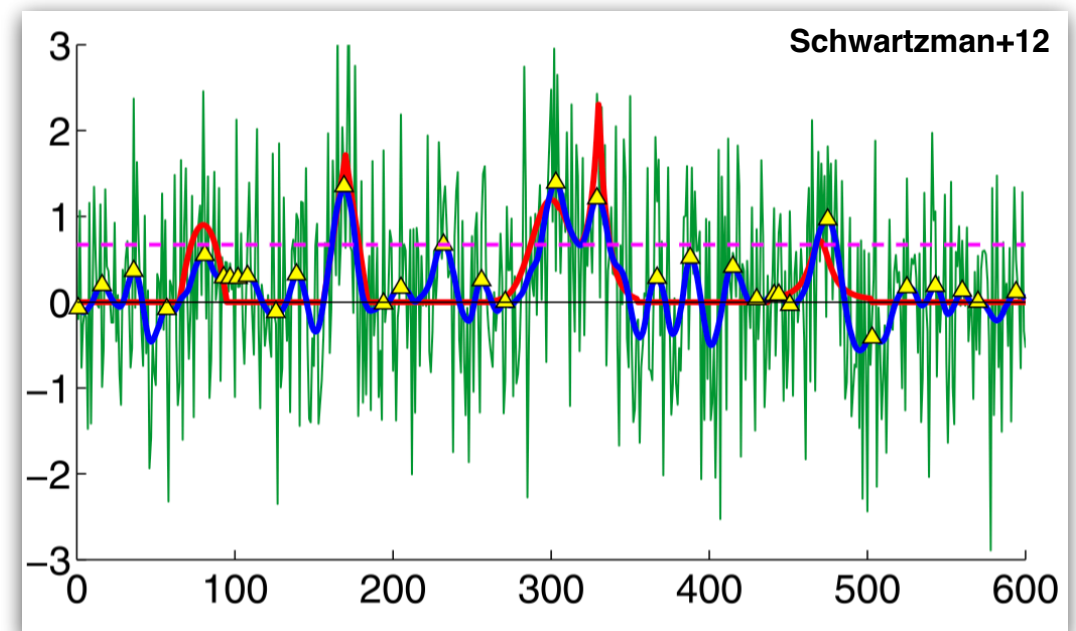
Importance of peak & constrained random field theories

Peak theory

1940's: Kac-Rice first studied the peaks in 1D signals, with important applications in communication theory and electronic signals



Applications to cosmology then follows with Doroshkevich (1970), Bardeen-Bond-Kaiser-Szalay (1986) and many others ...



1957: Longuet-Higgins extended this work to the 2D case in the context of ocean surface waves (width and shape of the crests, distance between troughs, ...)

Peak theory

Let us consider a field x and its first x_i and second x_{ij} derivatives.

The number density of peaks is:

$$n_{\text{peak}}(\vec{r}) = \sum_k \delta_D(\vec{r} - \vec{r}_{\text{peak } k})$$

A Taylor expansion of x_i around a peak k reads:

$$\nabla x_i(\vec{r}) = 0 + \sum_j x_{ij}(\vec{r}_{\text{peak } k}) \times (\vec{r} - \vec{r}_{\text{peak } k})_j$$

which can be inverted: $(\vec{r} - \vec{r}_{\text{peak } k}) = \mathcal{H}^{-1} \cdot \nabla \delta$

and leads to:

$$n_{\text{peak}}(\vec{r}) = \det \mathcal{H} \delta_D(\nabla \delta)$$

Eventually, the mean number density of peaks reads:

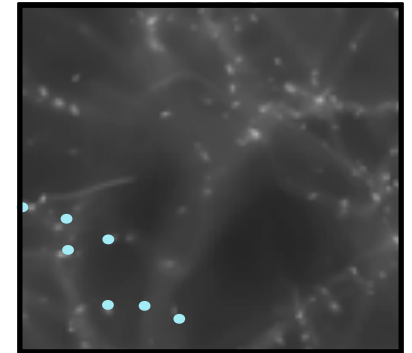
$$\langle n_{\text{peak}} \rangle = \int \frac{d^3 \vec{r}}{V} n_{\text{peak}}(\vec{r}) = \int dx d^3 x_i d^6 x_{ij} P(x, x_i, x_{ij}) |\det x_{ij}| \delta_D(x_i) \times \Theta(-\lambda_1)$$



ergodicity!

spatial average=ensemble average

peaks = point process



Peak theory: Gaussian predictions

If the field is Gaussian (large scales/early times), the total number density of critical points then reads

2D

$$\langle n_{\max} \rangle = \langle n_{\min} \rangle = \frac{1}{8\sqrt{3}\pi R_{\star}^2}$$
$$\langle n_{\text{sad}} \rangle = \frac{1}{4\sqrt{3}\pi R_{\star}^2}$$

3D

$$\langle n_{\max} \rangle = \langle n_{\min} \rangle = \frac{29\sqrt{15} - 18\sqrt{10}}{1800\pi^2 R_{\star}^3}$$
$$\langle n_{\text{sadf}} \rangle = \langle n_{\text{sadw}} \rangle = \frac{29\sqrt{15} + 18\sqrt{10}}{1800\pi^2 R_{\star}^3},$$

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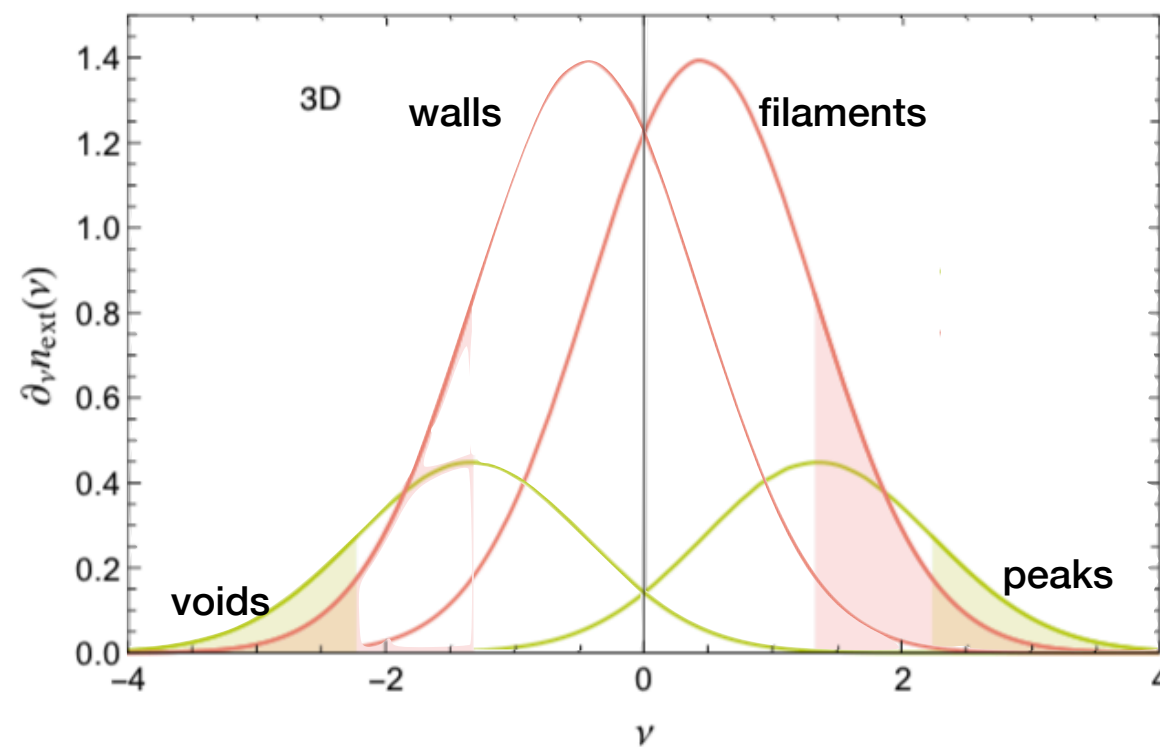
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And as a function of peak height (analytical in 2D, not in 3D) :



Peak theory: Non-Gaussian predictions

Guy+11

see also Pogosyan+00, SC+13

Gram-Charlier expansion (analogous to a Taylor expansion for PDF):

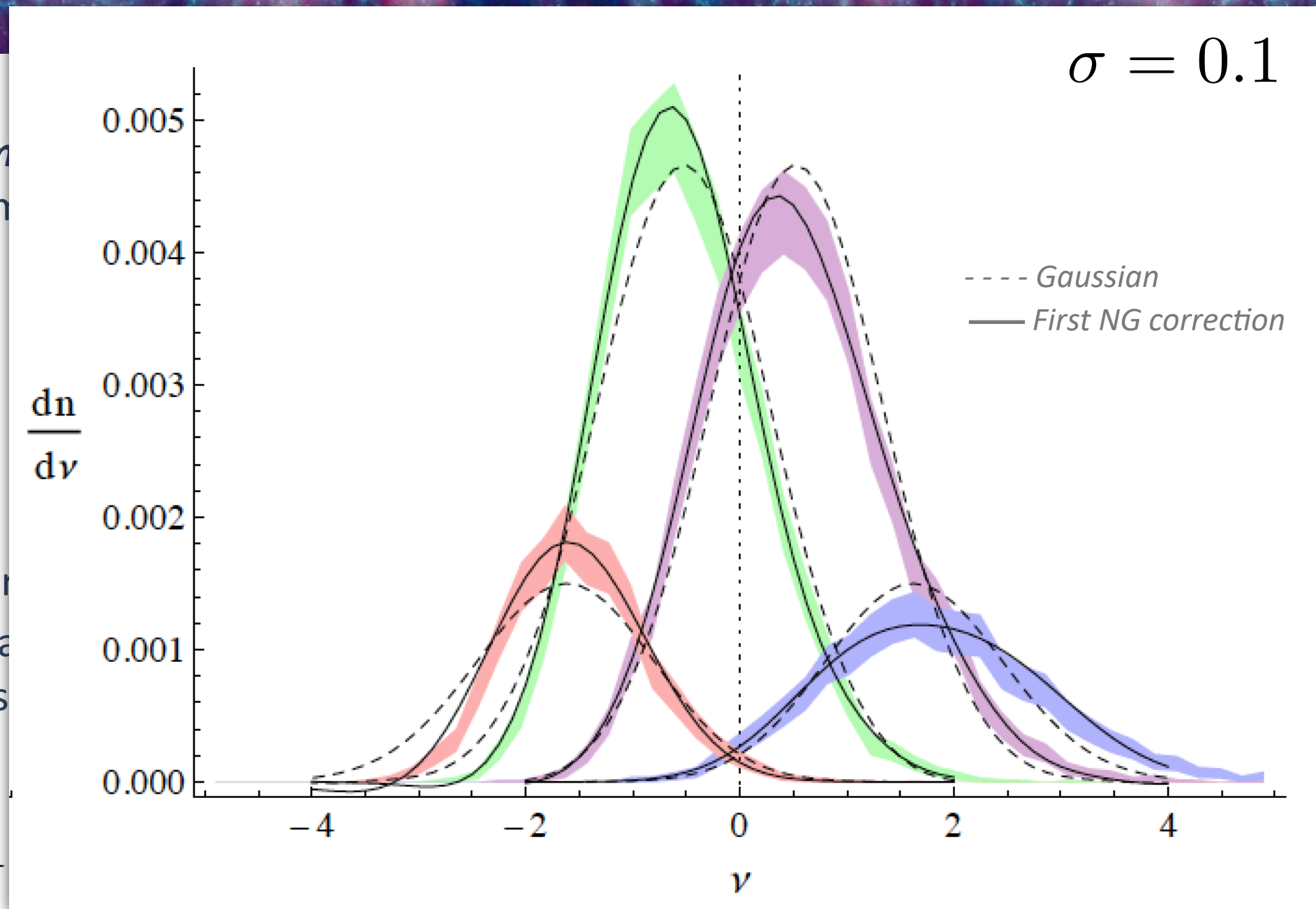
The moment expansion of the general PDF around a Gaussian G is an Hermite expansion:

$$P(x) = G(x) \left[1 + \sum_{n=3}^{\infty} \frac{1}{n!} \langle x^n \rangle_{GC} H_n(x) \right]$$

where Hermite polynomials are polynomials of order n in x,
and allows us to get predictions for number density of peaks to all order in non-Gaussianity once rotational invariants are used :

$$\begin{aligned} n_{\mp--} &= \frac{29\sqrt{15} \mp 18\sqrt{10}}{1800\pi^2 R_*^3} + \frac{5\sqrt{5}}{24\pi^2 \sqrt{6\pi} R_*^3} \left(\langle q^2 J_1 \rangle - \frac{8}{21} \langle J_1^3 \rangle + \frac{10}{21} \langle J_1 J_2 \rangle \right) \\ n_{++\pm} &= \frac{29\sqrt{15} \mp 18\sqrt{10}}{1800\pi^2 R_*^3} - \frac{5\sqrt{5}}{24\pi^2 \sqrt{6\pi} R_*^3} \left(\langle q^2 J_1 \rangle - \frac{8}{21} \langle J_1^3 \rangle + \frac{10}{21} \langle J_1 J_2 \rangle \right) \end{aligned}$$

Peak theory: Non-Gaussian predictions



Gram
The n

osyan+00, SC+13

expansion:

when
and a
Gaus

$n_{\mp-}$

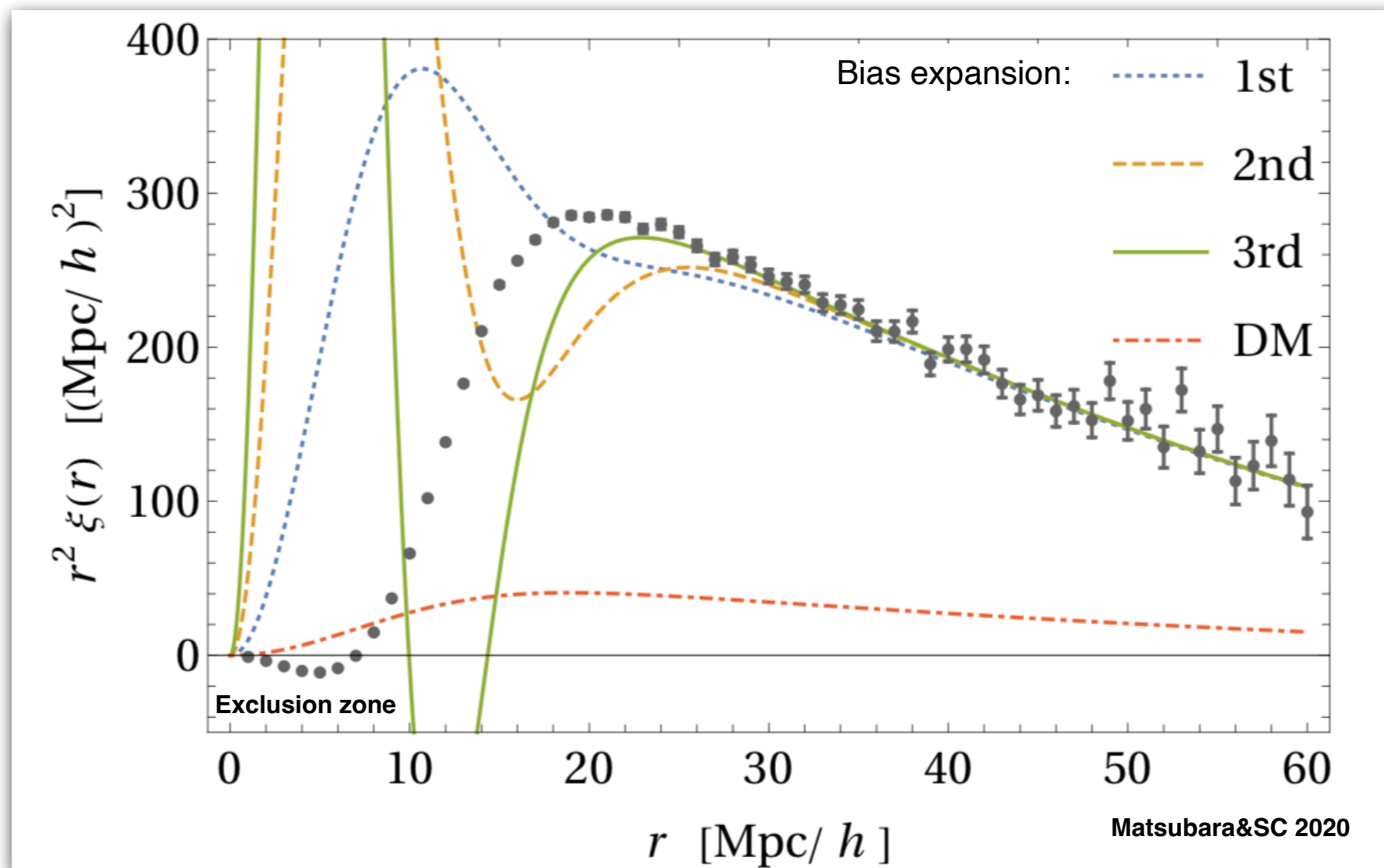
n-

$\frac{0}{1} \langle J_1 J_2 \rangle$

$$n_{++\pm} = \frac{29\sqrt{15} \mp 18\sqrt{10}}{1800\pi^2 R_*^3} - \frac{5\sqrt{5}}{24\pi^2 \sqrt{6\pi} R_*^3} \left(\langle q^2 J_1 \rangle - \frac{8}{21} \langle J_1^3 \rangle + \frac{10}{21} \langle J_1 J_2 \rangle \right)$$

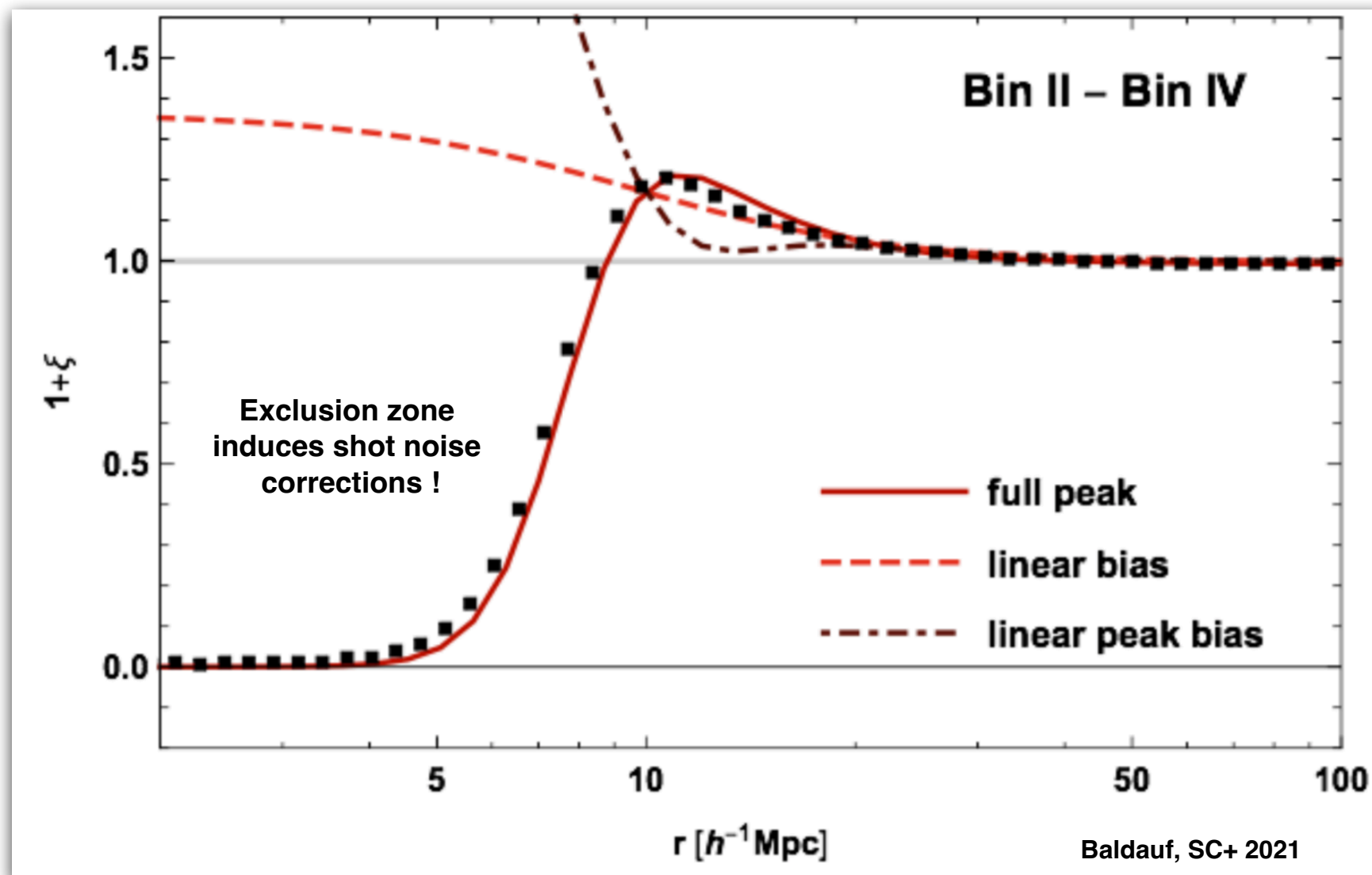
Peak theory: clustering (i.e 2pt stat)

Same ideas can be used to also predict the **clustering of peaks** by means of their 2 point correlation function (higher order statistics are also possible although not much investigated so far), *which can be analytically predicted in the large-separation regime with a **bias expansion***:



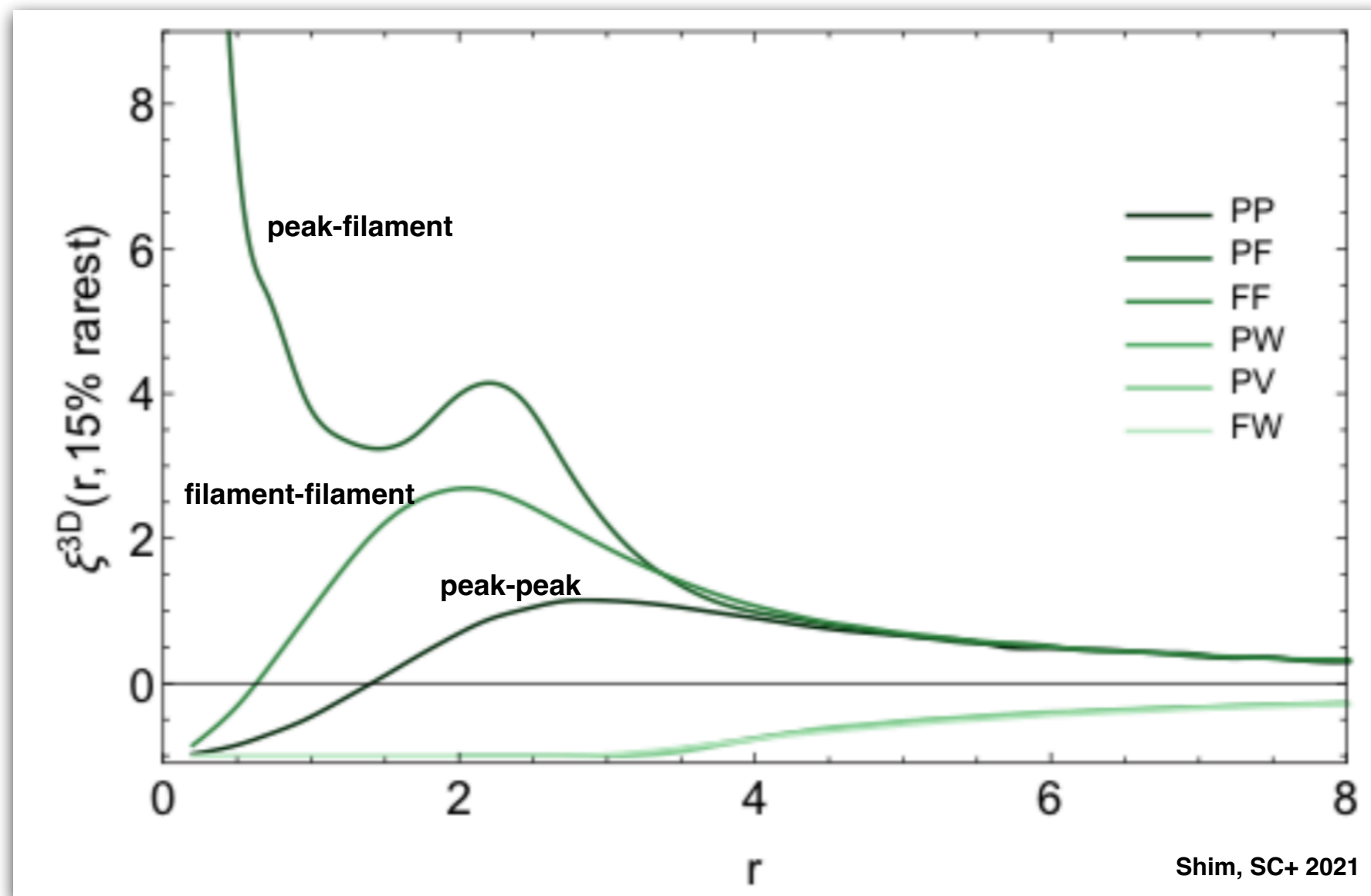
Peak theory as a proxy for haloes?

Massive haloes can be shown to be relatively accurately modelled by peak theory. At lower masses, departure arises (e.g Ludlow+11) due to tidal effects.



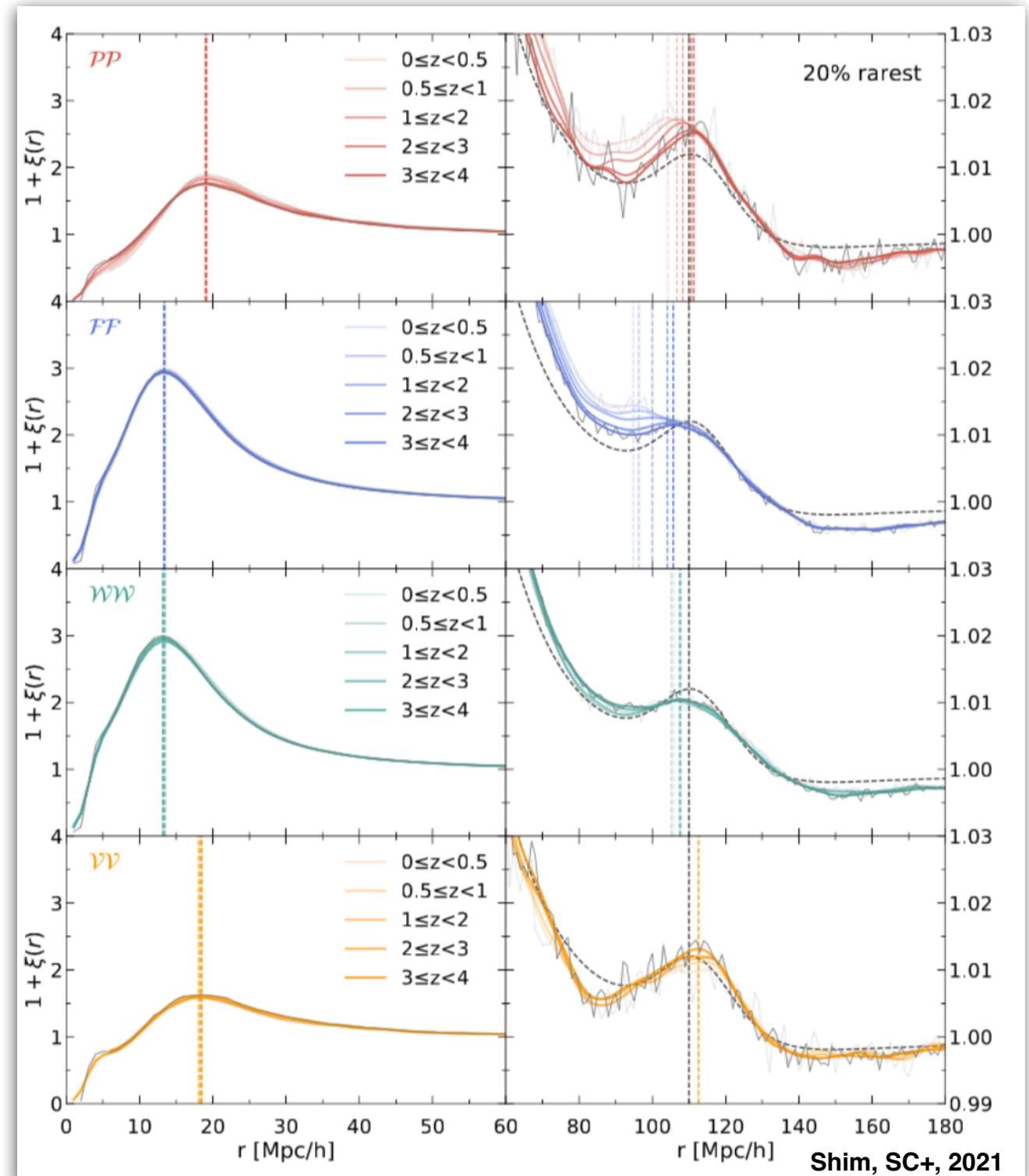
clustering of peaks (P), filaments (F), walls(W) and voids (V)

The clustering of all-type critical points can be similarly investigated in Gaussian random fields



clustering of peaks (P), filaments (F), walls(W) and voids (V)

- ➔ BAO are enhanced with highly biased tracers
- ➔ Some points are more robust wrt non-linearities/z-evolution
- ➔ One issue: S/N

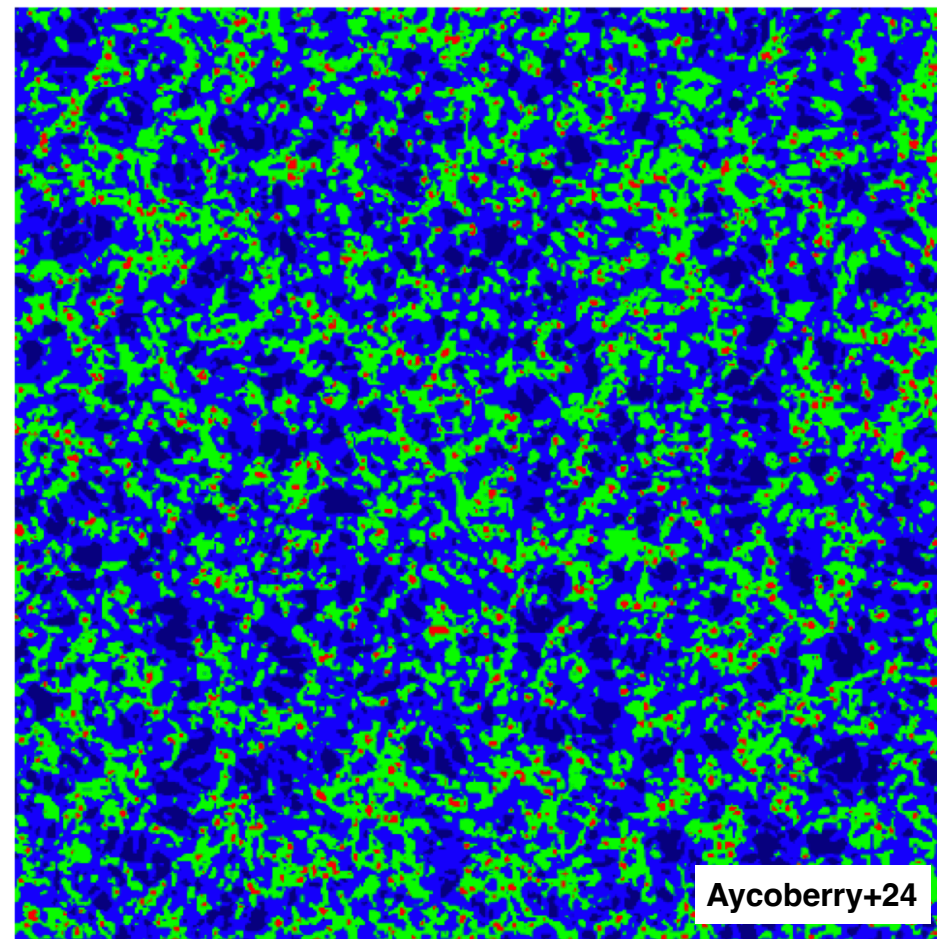
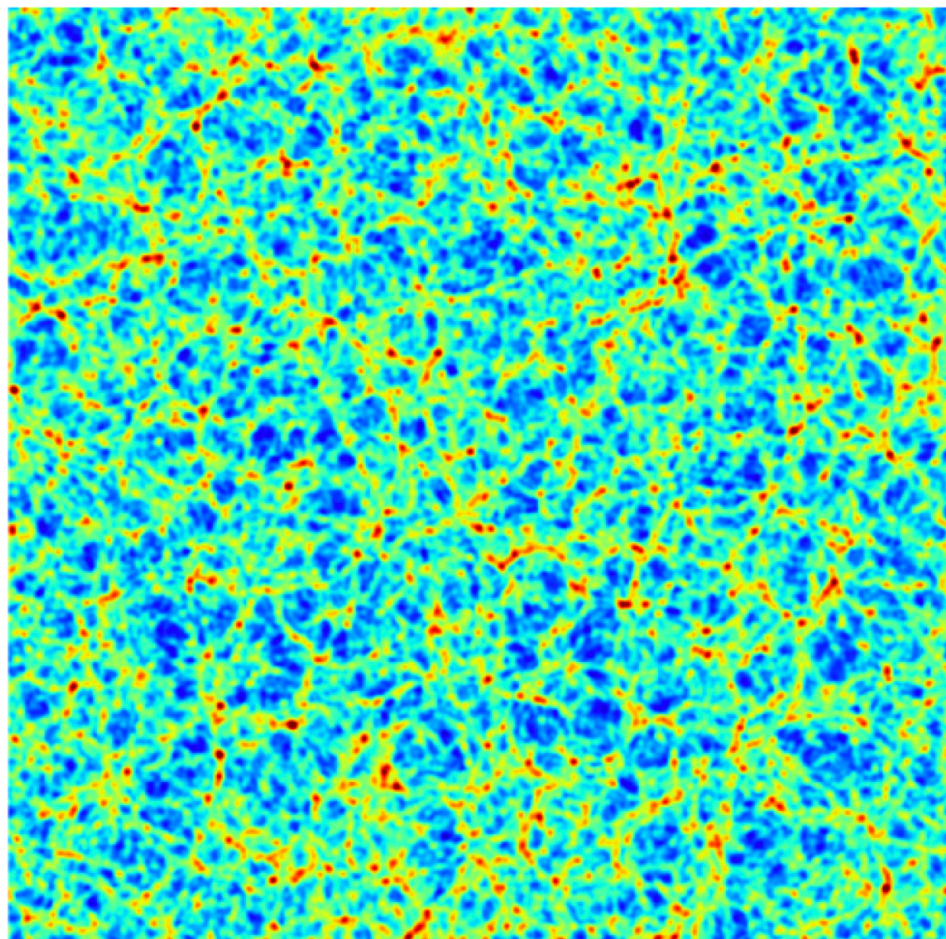
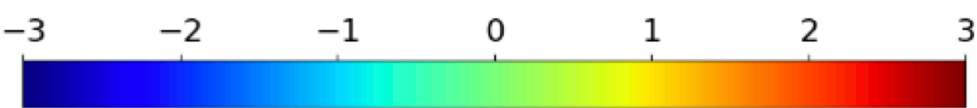


Environment-split statistics

Goal : Clustering in subregions defined by the Tweb



Anaïs Meunier
PhD student @ AIM
ERC COLIBRI



Void	$\lambda_1 < \lambda_2 < \lambda_3 < \lambda_{th} :$
------	--

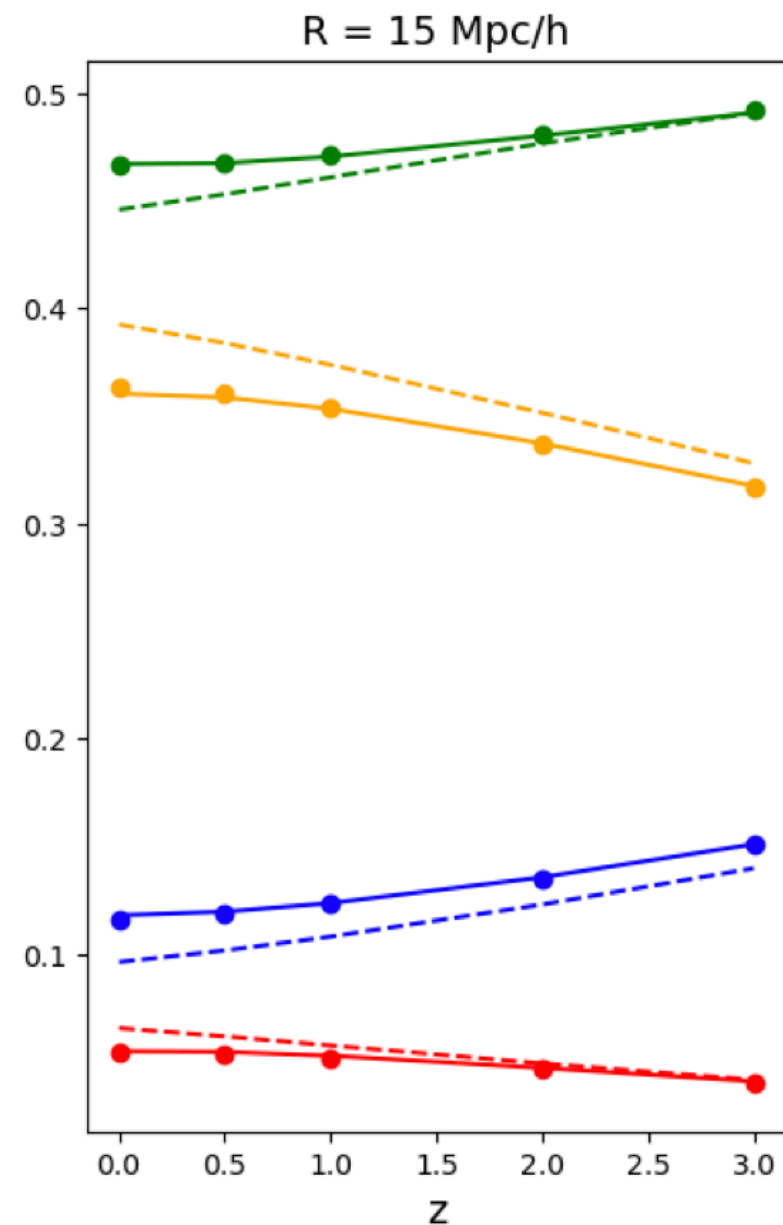
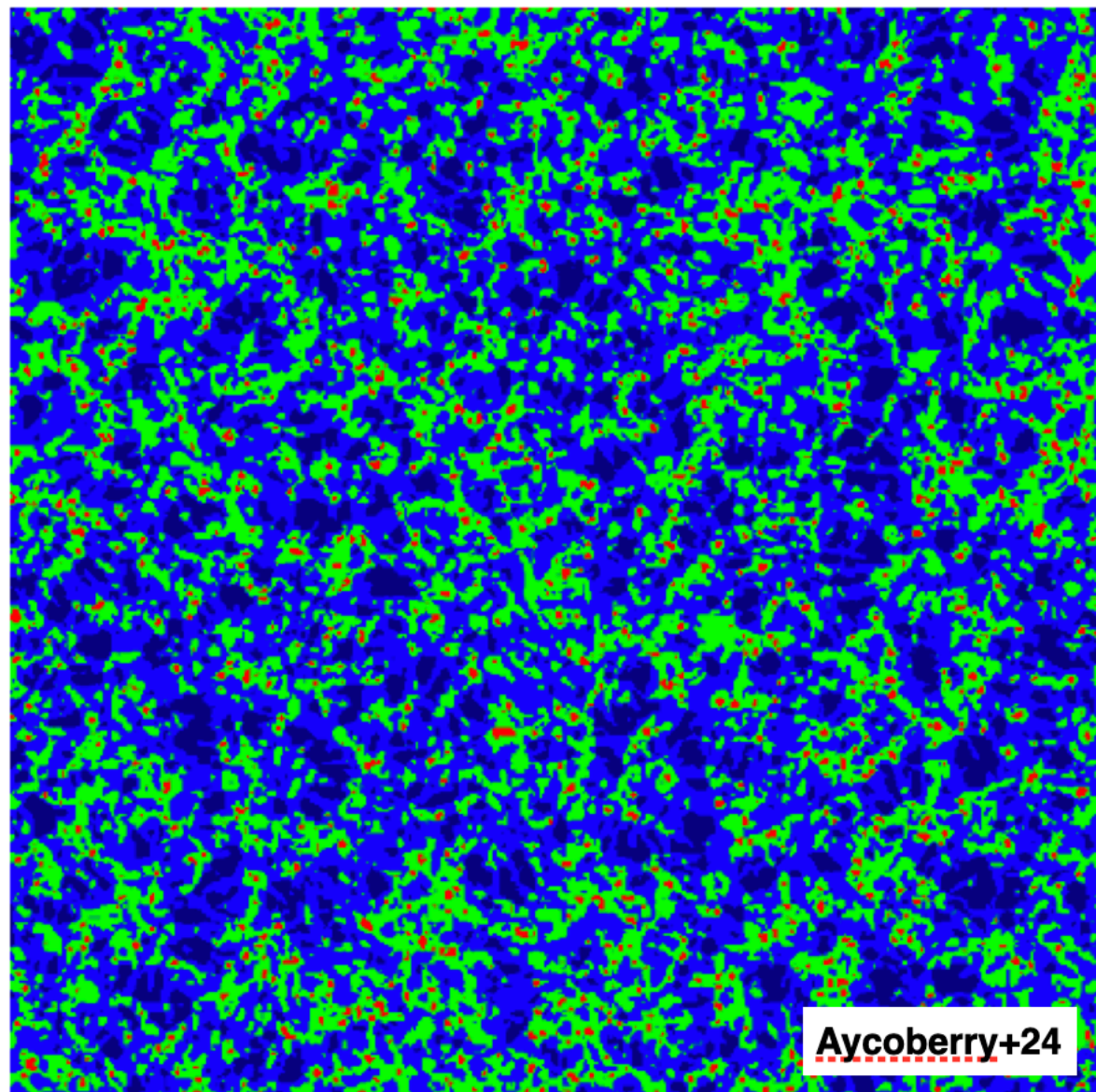
Wall	$\lambda_1 < \lambda_2 < \lambda_{th} < \lambda_3 :$
------	--

Filament	$\lambda_1 < \lambda_{th} < \lambda_2 < \lambda_3 :$
----------	--

Node	$\lambda_1 < \lambda_{th} < \lambda_2 < \lambda_3 :$
------	--

Environment-split statistics

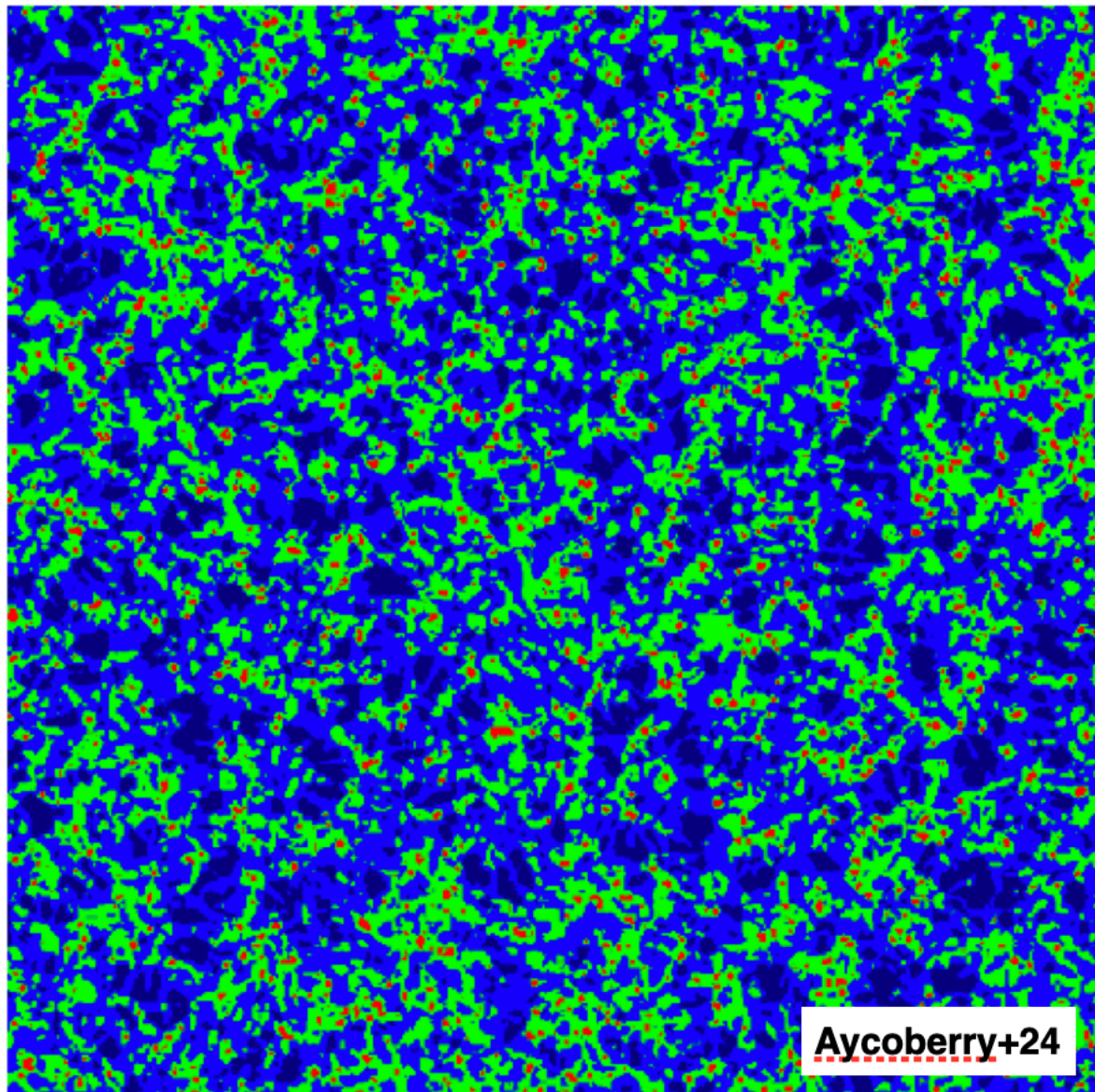
Abundance (one-point) of Tweb environments :



Emma Ayçoberry
Postdoc @ AIM
ERC COLIBRI

Environment-split statistics

Clustering of Tweb environments :



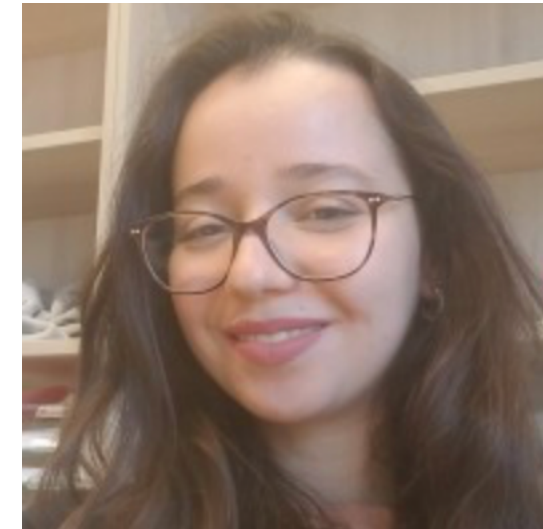
Anaïs Meunier
PhD student @ AIM
ERC COLIBRI

Environments

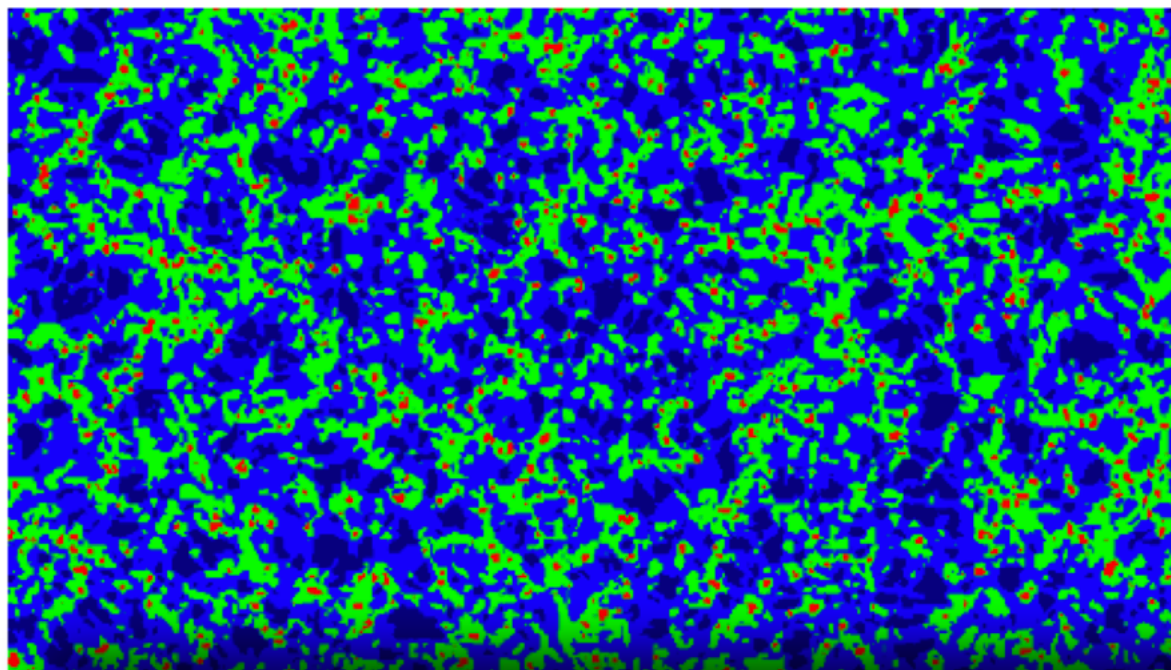
$$\xi_{\alpha\alpha'} = \left\langle \frac{\rho_{|\alpha} - \langle \rho \rangle_{\alpha}}{\langle \rho \rangle_{\alpha}} \frac{\rho'_{|\alpha'} - \langle \rho' \rangle_{\alpha'}}{\langle \rho' \rangle_{\alpha'}} \right\rangle ;$$

Environment-split statistics

Clustering of Tweb environments :



Anaïs Meunier
PhD student @ AIM
ERC COLIBRI



Environments

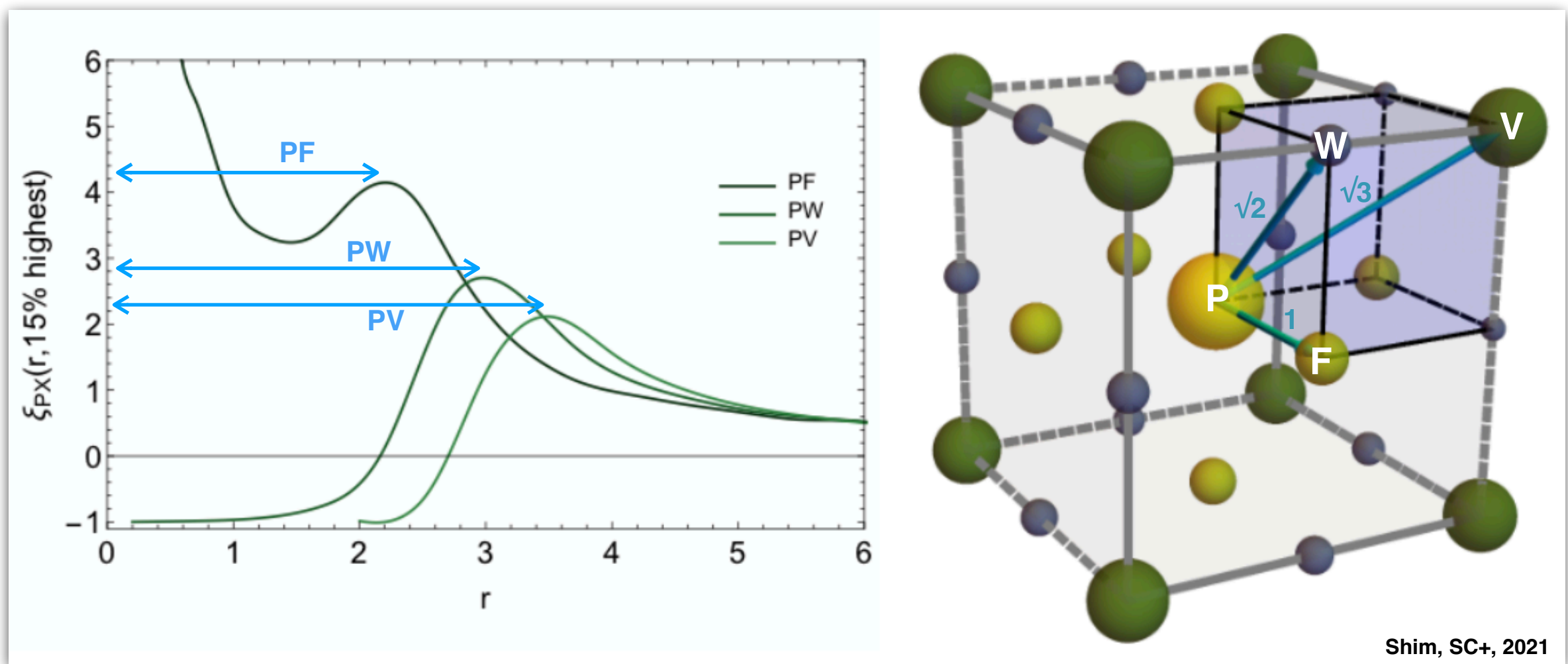
$$\xi_{VF}(r) = \frac{\xi \left(-3 \sqrt{3} \sigma^2 + \sigma^2 + 2 \sqrt{\frac{6}{\pi}} \sigma \right)}{6 \sqrt{\frac{6}{\pi}} \sigma - 9 \sqrt{3} + 9} - \frac{2 \left(\sqrt{\frac{6}{\pi}} \sigma (7 \xi^2 - 16 \xi \xi_{\chi\chi} - 28 \xi_{\Delta\pi}^2 + 64 \xi_{\chi\chi}^2) - 6 \xi^2 + 24 \xi_{\Delta\Delta} \xi_{\chi\chi} + 24 (\xi_{\Delta\pi}^2 - 4 \xi_{\chi\chi}^2) \right)}{9 \sqrt{3} (2 \sqrt{\frac{2}{\pi}} \sigma + \sqrt{3} - 3)} + \dots$$

Meunier+, in prep.

Aycoberry+24

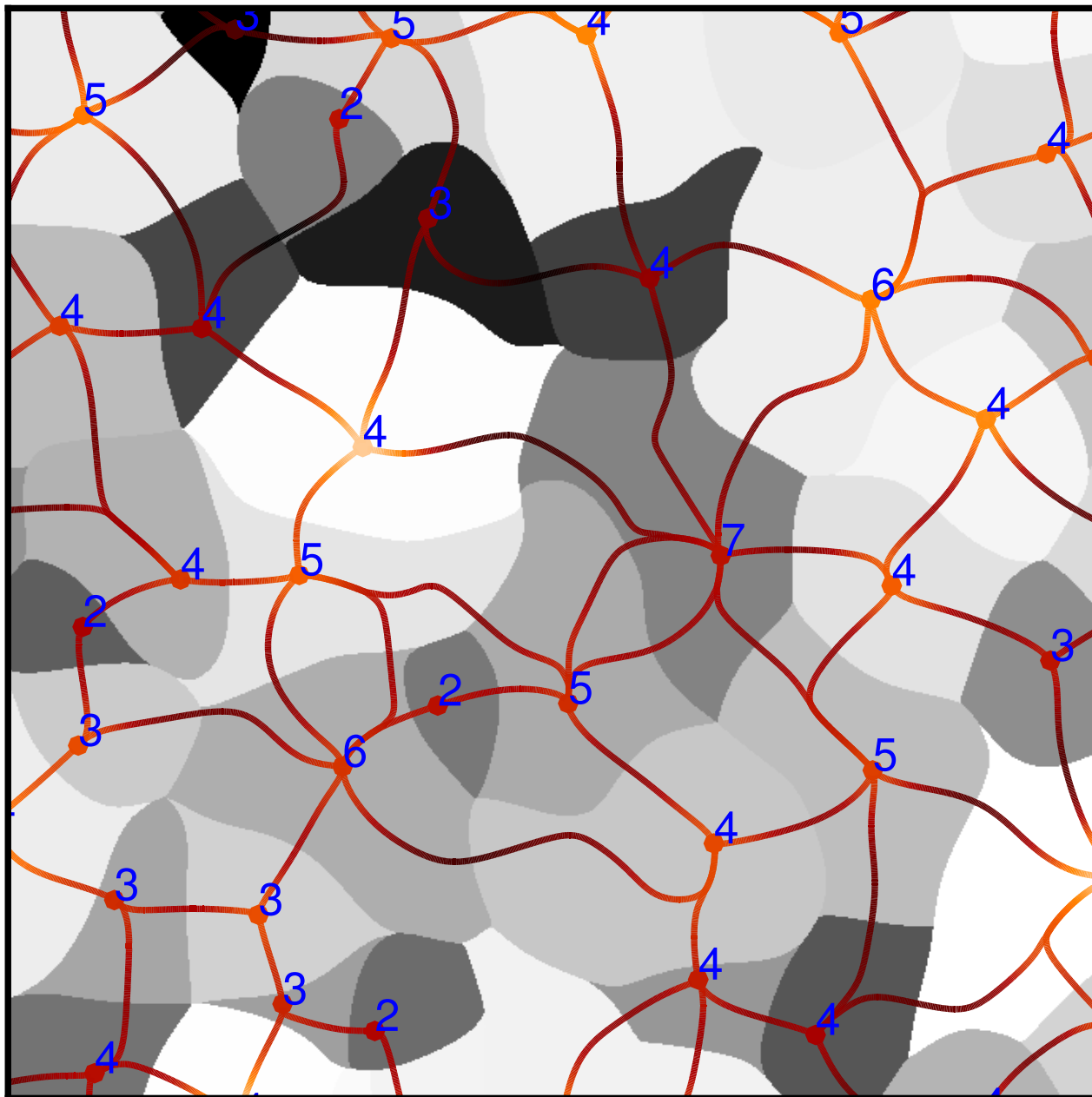
Cosmic crystal

The clustering of all-type critical points can be analytically predicted



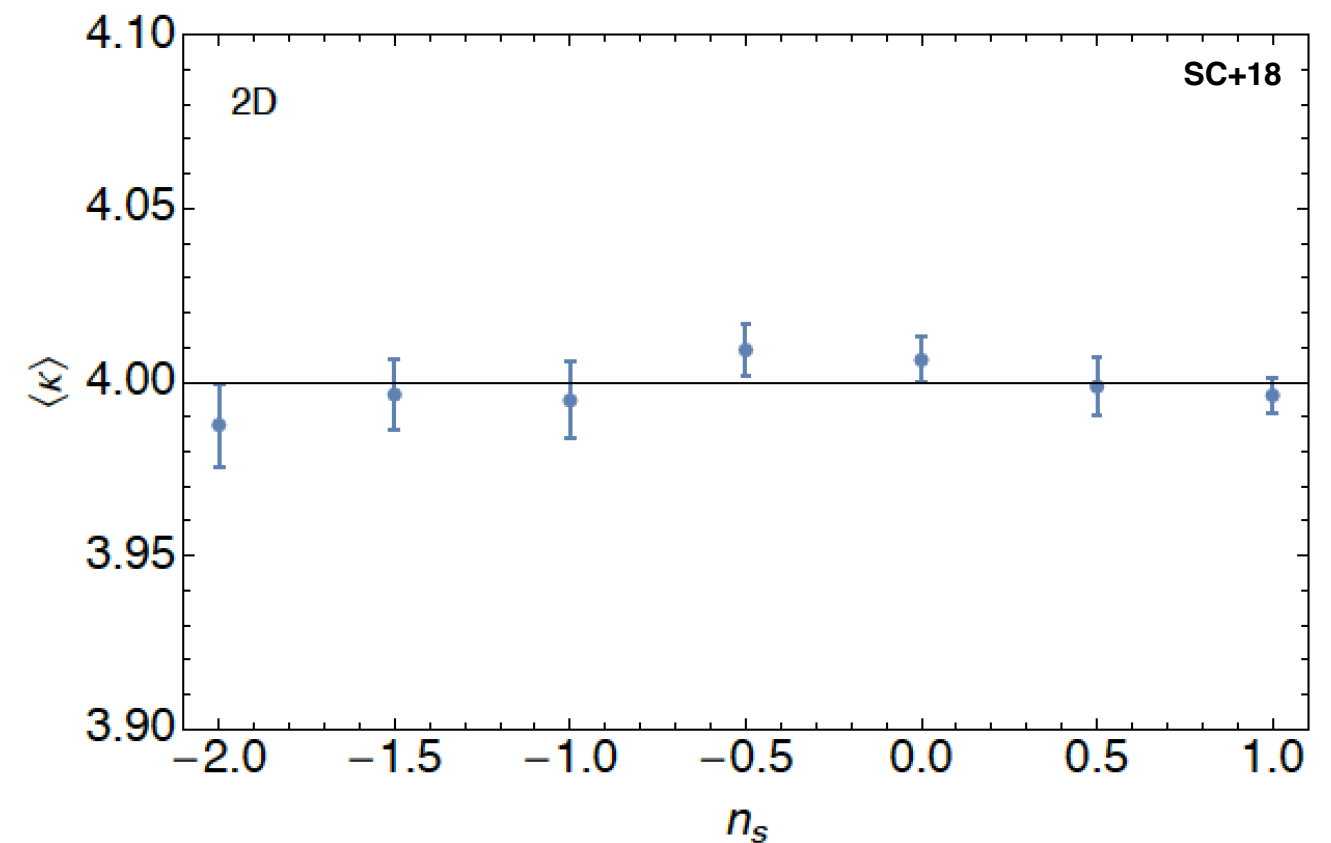
=> The clustering of filaments, walls and voids around a rare peak show a first layer of neighbours with characteristic separations reminiscent of a cubic lattice

Global connectivity for GRF



How many filaments connect to a node?

Number of connected saddles are measured using the **Disperse** skeleton algorithm (Sousbie+11) in GRF realisations.

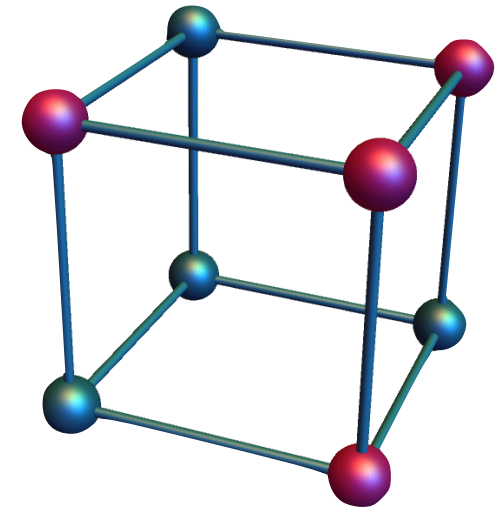


Can we predict the mean connectivity?

Global connectivity for GRF: theory

Because each filament goes through one and only one saddle pt, on average:

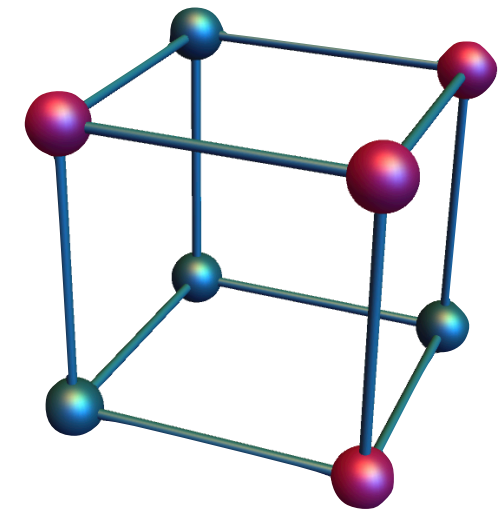
$$\begin{aligned}\langle \kappa \rangle &= \frac{2\bar{n}_{\text{sad}}}{\bar{n}_{\text{max}}} \\ &= 4 \quad \text{in 2D GRF} \\ &= \frac{2(1057 + 348\sqrt{6})}{625} \approx 6.11 \quad \text{in 3D GRF}\end{aligned}$$



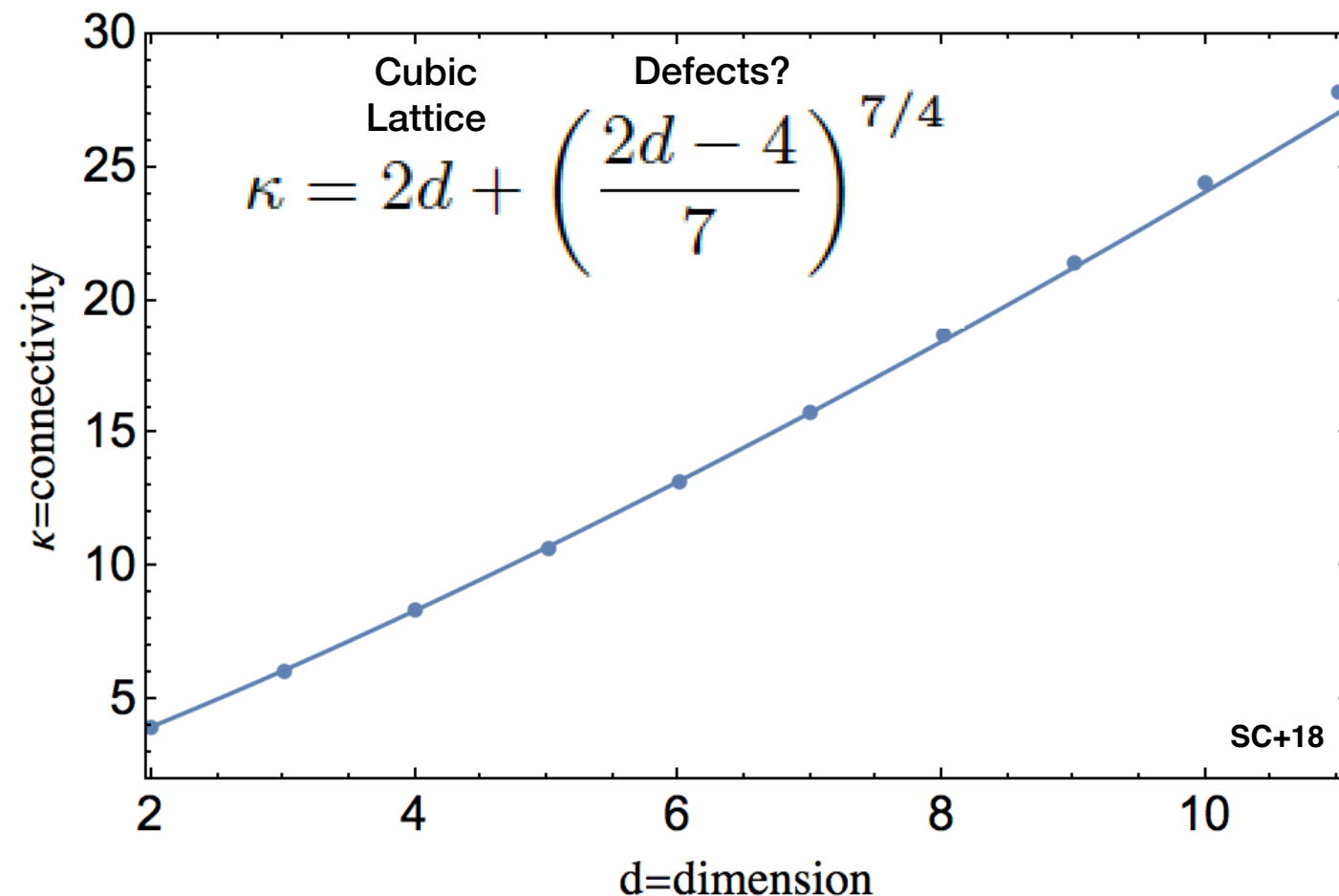
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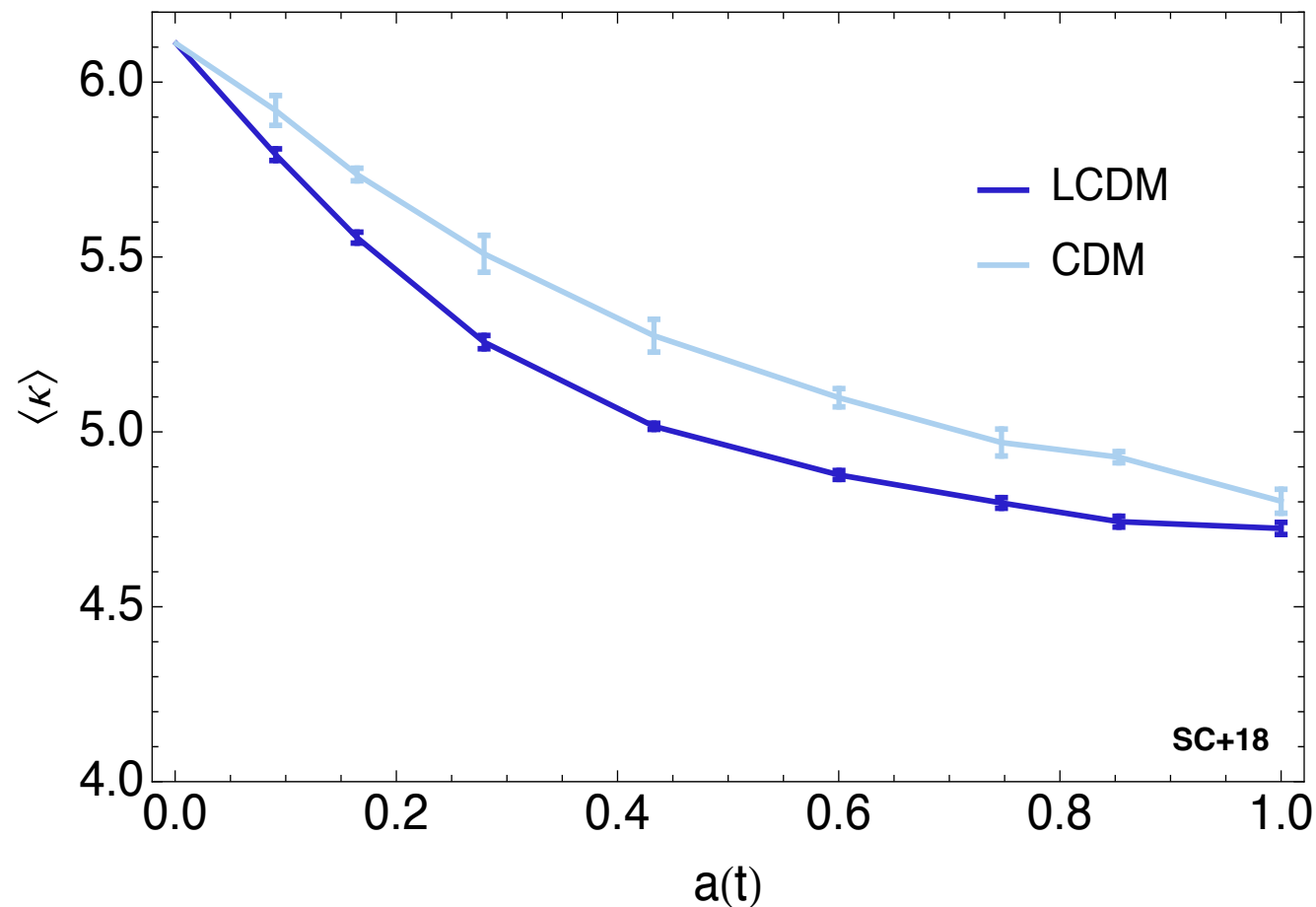
In d dimensions, we relied on numerical integrations:



Asymptotic result?

Global connectivity: evolution with cosmic time

- Measurement in cosmological simulations:



Filaments merge in a cosmology-dependent way!

- Predictions:

Using a Gram Charlier expansion, one can get prediction at arbitrary order in NG

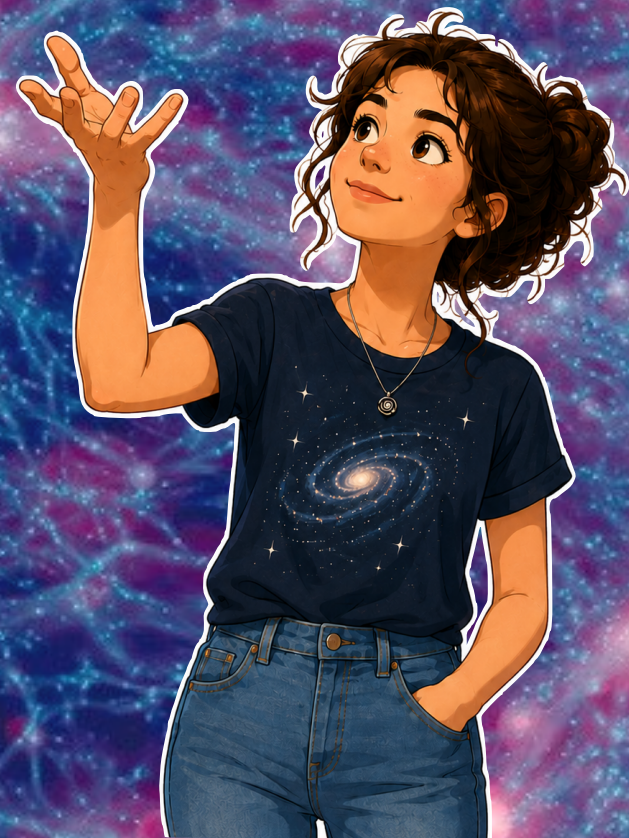
$$\langle \kappa \rangle = \kappa^G \left(1 + \sum_{i \geq 1} \kappa^{(i)} \sigma_0^i \right)$$

With

$$\kappa^G = 2 \times \frac{29\sqrt{3} + 18\sqrt{2}}{29\sqrt{3} - 18\sqrt{2}} \approx 6.11.$$

$$\kappa^{(1)} = \frac{4\sqrt{3}}{35\sqrt{\pi}\sigma_0} (8 \langle J_1^3 \rangle - 10 \langle J_1 J_2 \rangle - 21 \langle q^2 J_1 \rangle)$$

Stochastic models of the cosmic web

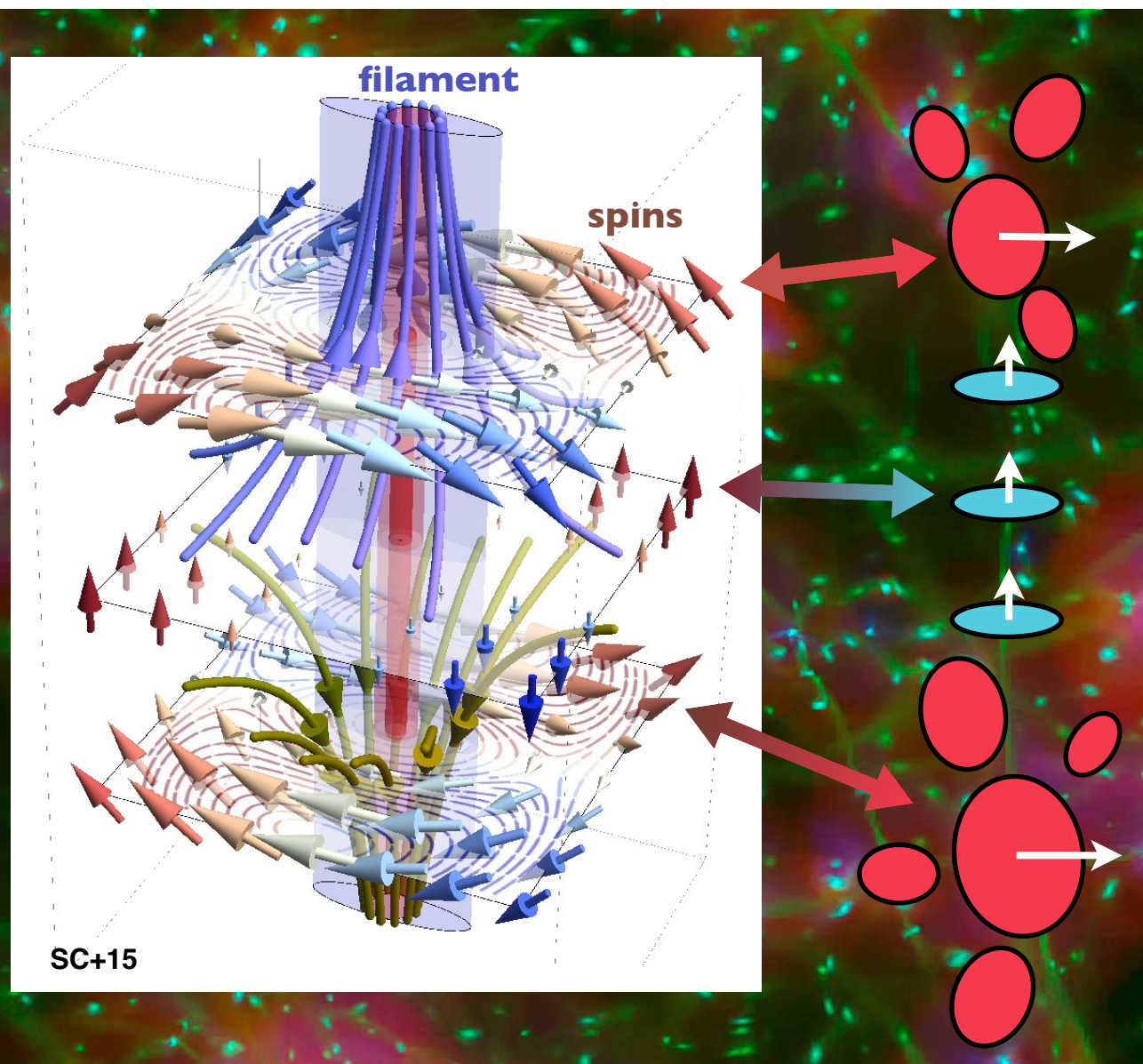


- Anisotropic cosmic web
- Galaxy alignments
- Cosmic shear nulling

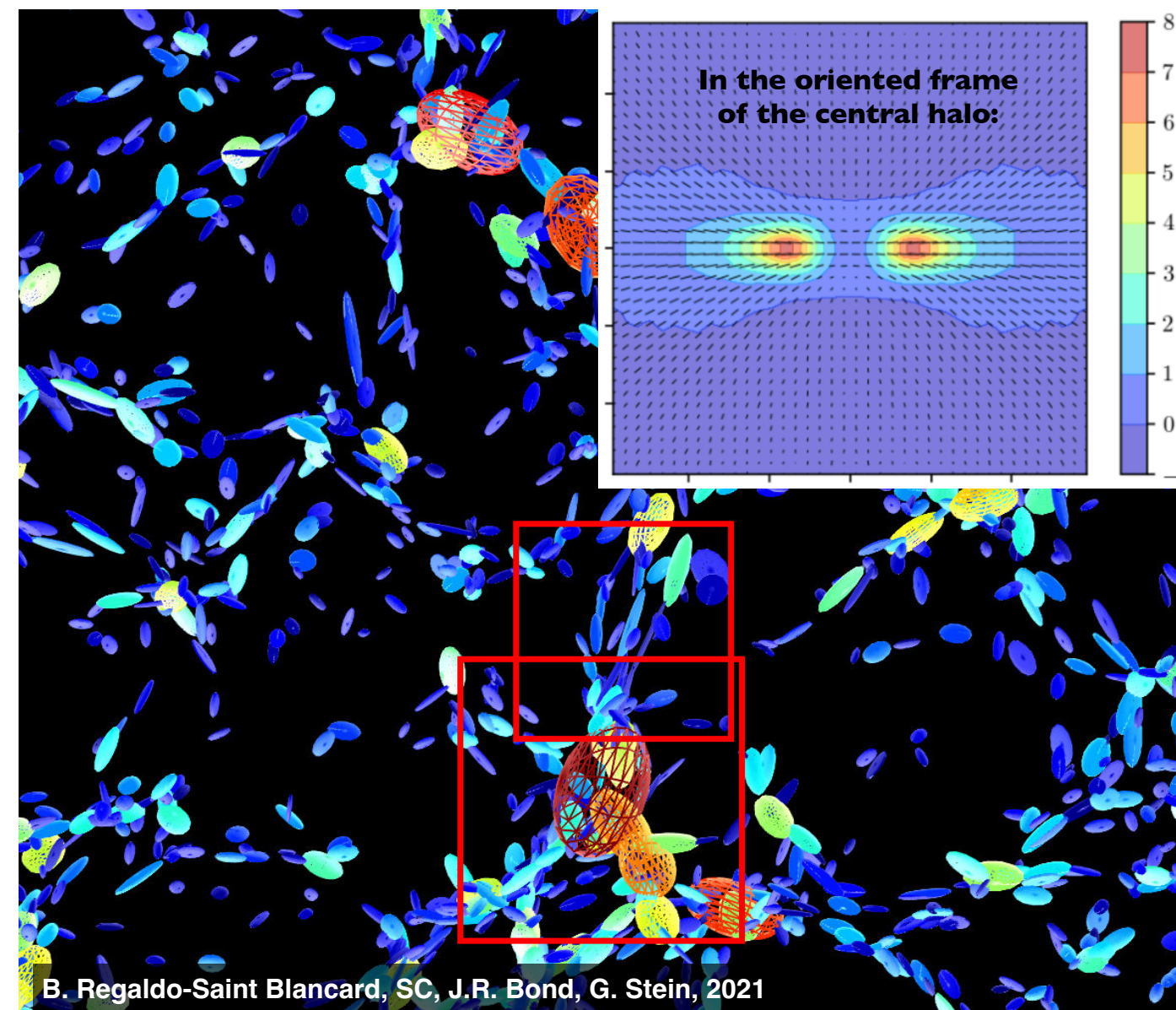
Cosmic web and galaxy formation

Many more aspects of the cosmic web environment can be studied, e.g to understand large-scale alignments of galaxies, their morphology, ...

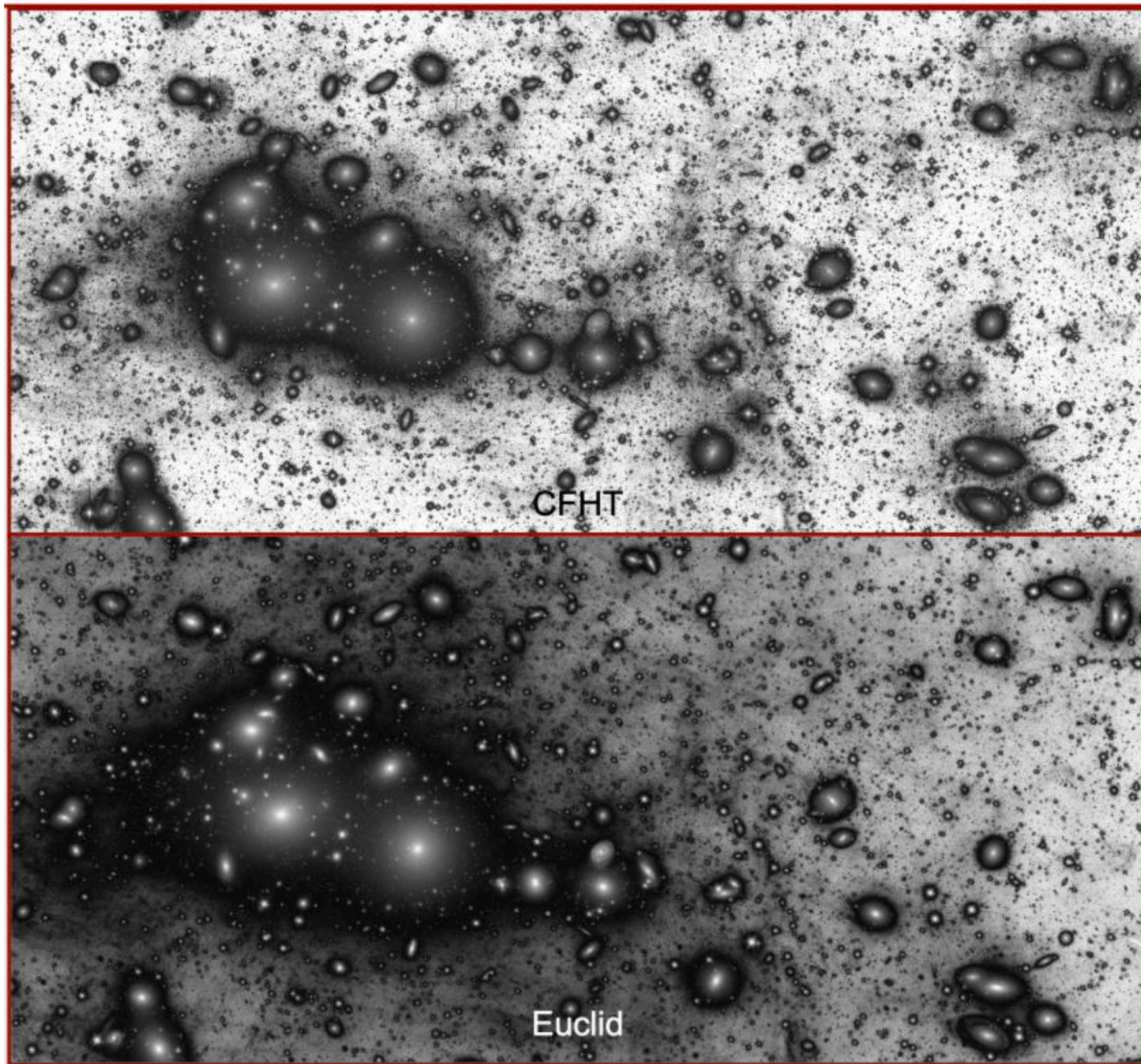
Anisotropic acquisition of spin by tidal torquing:



Orientation of proto-halos in the peak patch picture:



Galactic alignments in LSB observations



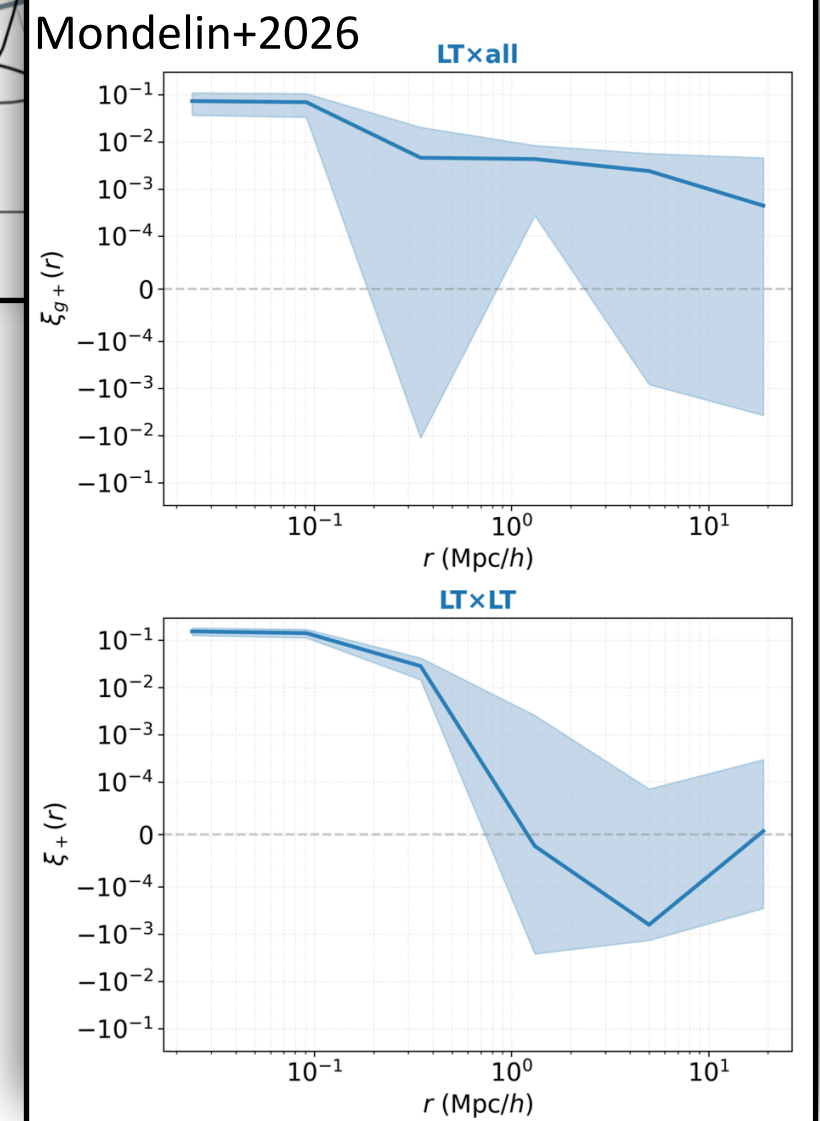
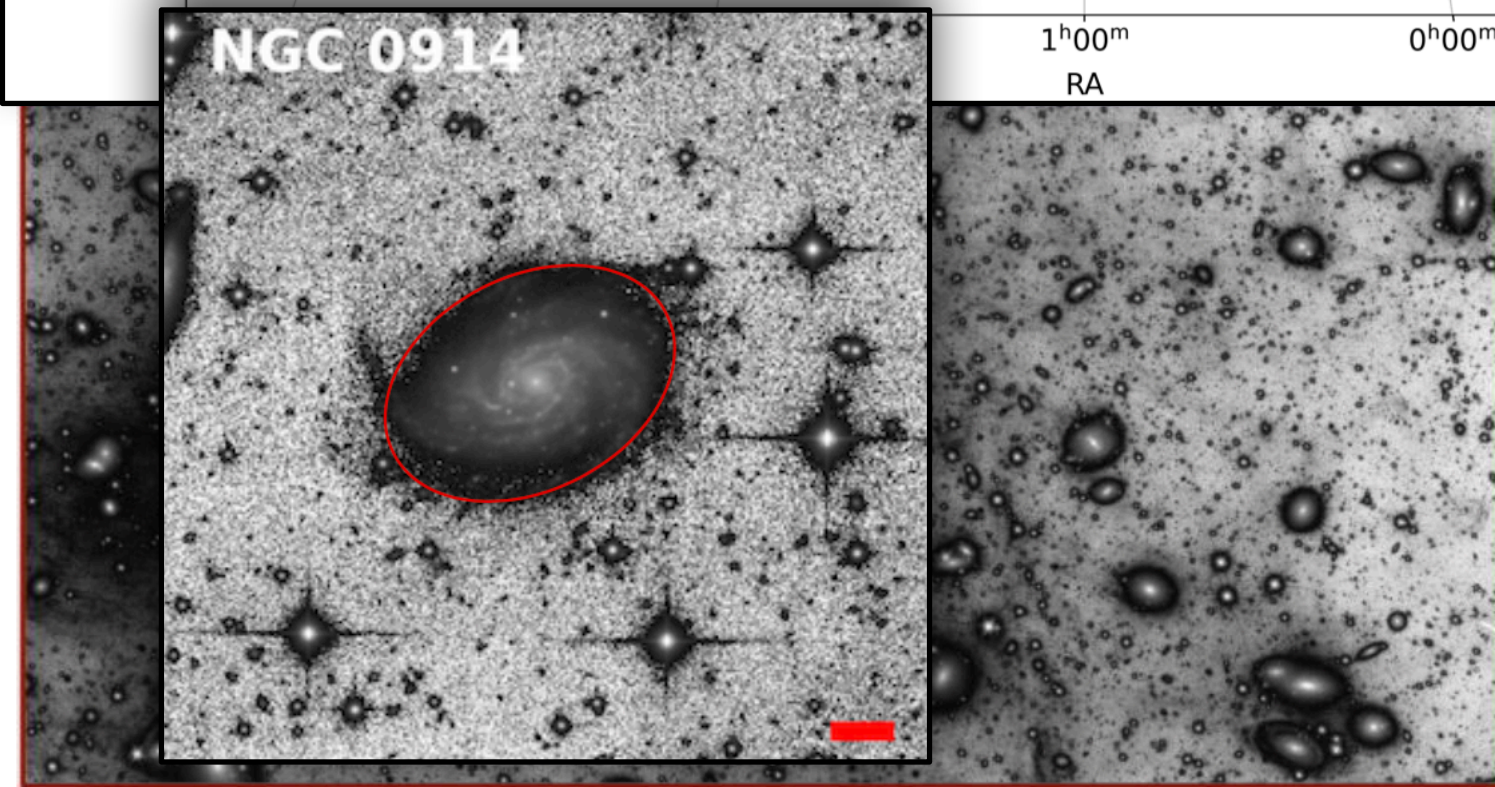
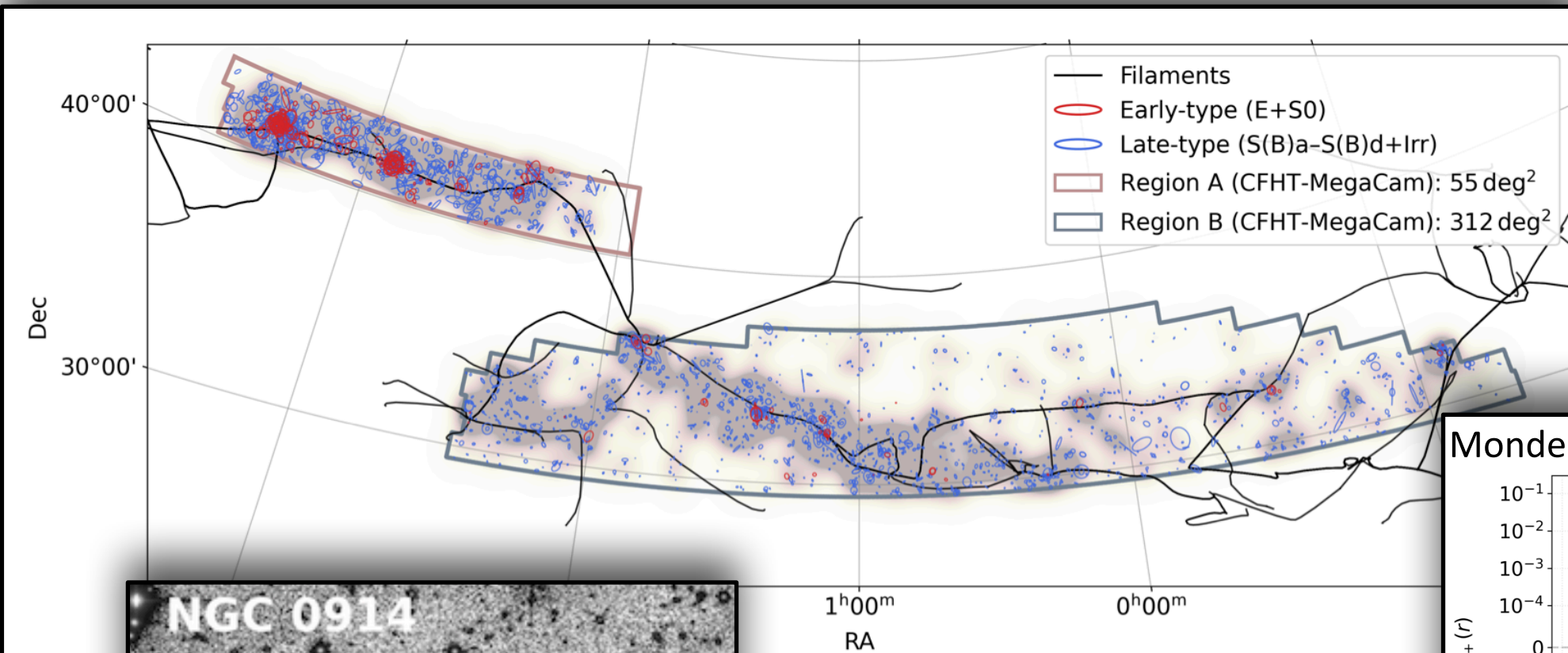
(c) Cuillandre, ESA



Maëlie Mondelin
PhD student @ AIM/CEA

- Allows for a detailed view on galactic profiles in external regions
- Sensitive to recent accretion

Galactic alignments in LSB observations



UNIONS galaxy alignments driven by blue spiral galaxies !

Conclusions

- ▶ We have models of the large-scale cosmic web,
- ▶ including its effect on galaxy alignments.
- ▶ How can we use it to constrain DM models ?

