

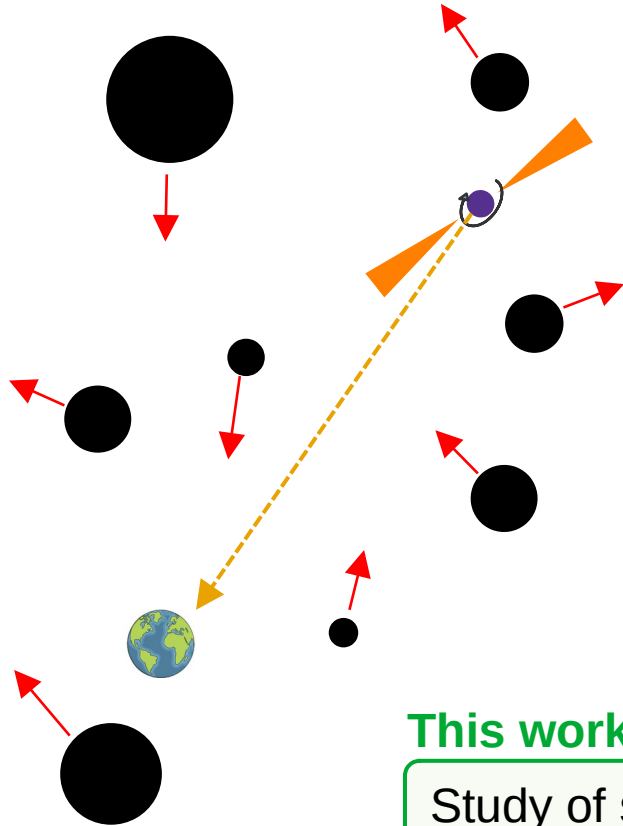
Dark subhalos population effect on pulsar timing

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From invisible subhalos to pulsar timing



A method to detect Dark Matter subhalos in the Milky Way:

- Dark **subhalos** are moving → **time-varying** gravitational perturbations
- **Pulsars** are extremely **sensitive** to these perturbations → pulsar timing arrays (PTA) as a probe of GW

Idea: Repurpose PTA data to search for subhalo-induced perturbations

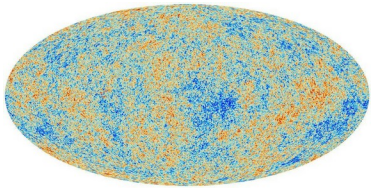
This work:

Study of subhalos **population effects** on a **single** pulsar

Why DM at small scales?

Nature of Dark Matter and Primordial Fluctuation

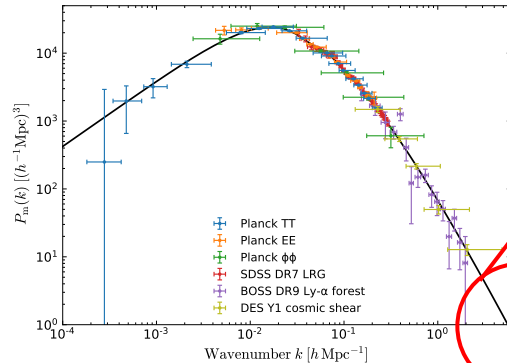
Strongly constrained at cosmological scales



[Planck 2018]



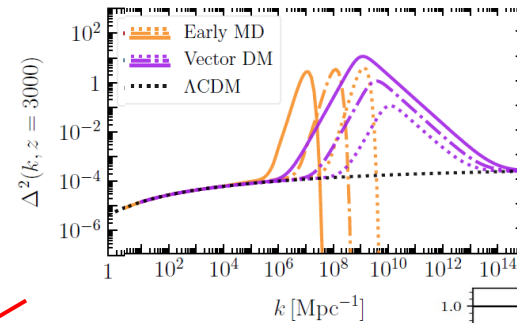
[NASA, ESA, CSA, STSCI 2022]



[ESA and the Planck collaboration 2018]

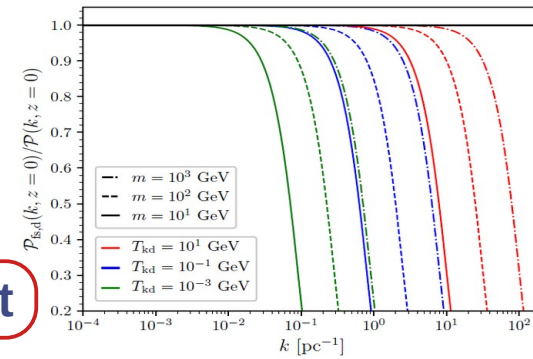
Little to no constraints at small scales

Power spectrum at recombination



[Lee, Mitridate, Trickle, Zurek 2020]

[Facchinetti 2021]

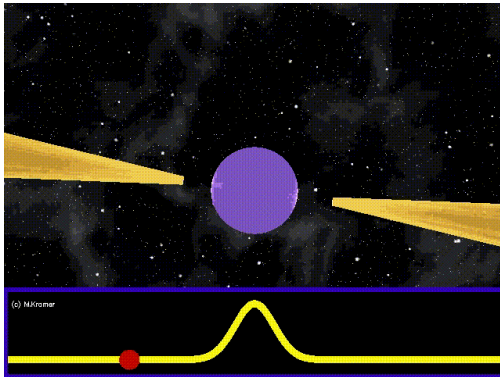


dark matter granularity at small scales is model-dependent

Pulsar Timing Arrays (PTAs)

Pulsar

Magnetised **rotating** neutron star acting as a **stable clock**



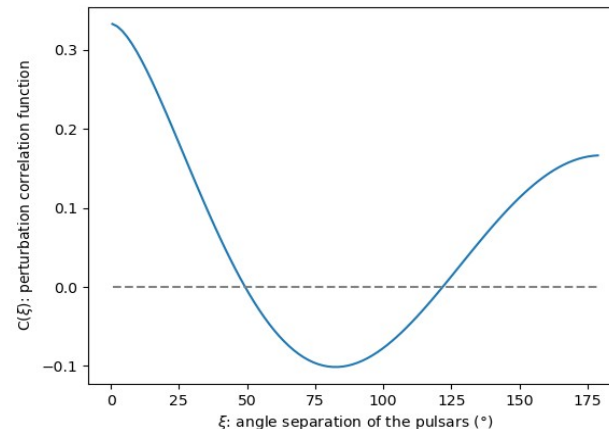
[Michael Kramer (JBCA, U. Manchester)]

Millisecond pulsars
are the most precise

Pulsar array

Angular **correlation** of timing perturbation:

- Galaxy-sized **detector** of GW
- Collab: NANOGrav, EPTA...

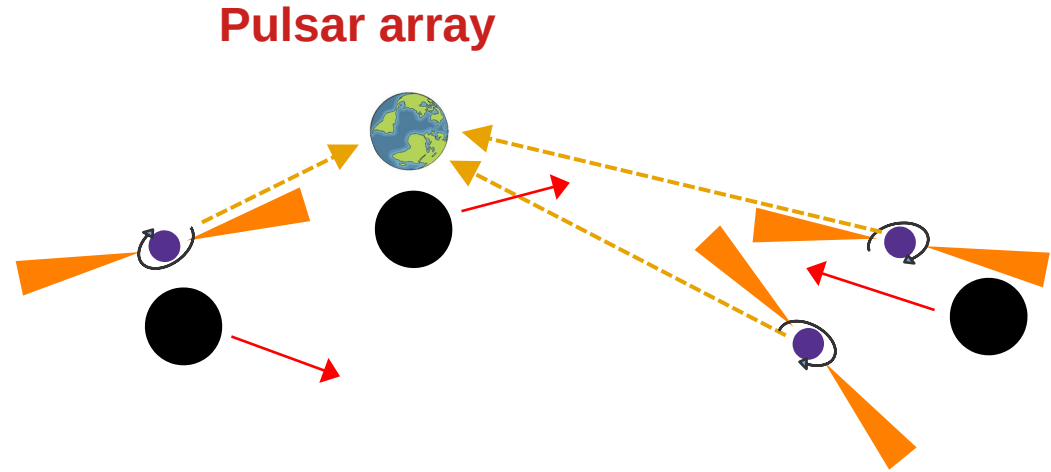


signature of stochastic
GW background:
Hellings-Downs curve

Pulsar Timing Arrays (PTAs)

Signature of perturbation sources in correlated signal

- Pulsars **close** to each other are affected by the **same halos**
- Halos **close** to the Earth affect **all pulsars**



Halo population signal in the **background** of the GW correlation signature

The only known gravitational probe capable of detecting fuzzy or very small halos

State of the art

Single close encounter

One halo (or PBH) passing close to the Earth or a pulsar
→ **deterministic, transient** signal

*[Siegel, Hertzberg & Fry 2007]
[Seto & Cooray 2007]
[Kashiyama & Seto 2012]
[Clark et al. 2016]*

Population of halos

Many halos → **stochastic** Doppler & Shapiro signal, **predictions** for PTA spectra & correlations

*[Baghran, Afshordi & Zurek 2011]
[Dror, Ramani, Trickle & Zurek 2019]
[Ramani, Trickle & Zurek 2020]
[Lee, Mitridate, Trickle & Zurek 2021]
[Cherukupalli, Lee, Berghaus & Zurek 2026]*

Searches in PTA data

First constraints from **real data**: NANOGrav 15-yr search for new physics (PBHs, DM substructure)

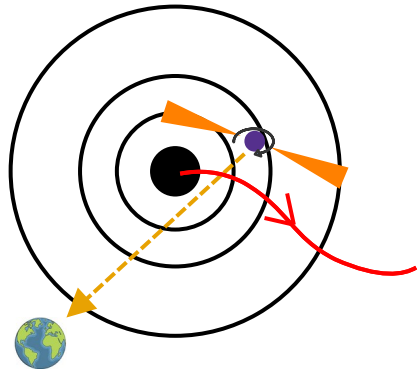
[NANOGrav, Afzal et al. 2023]

This work: general formulae + realistic Λ CDM population effects with timing fit

Pulsar perturbation by one halo

Einstein effect

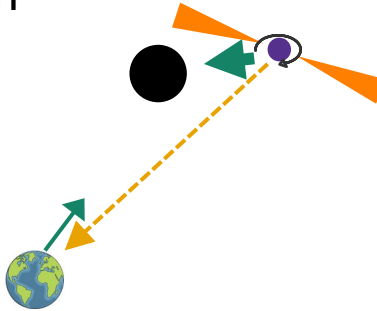
From the difference in the **gravitational field** at the pulsar and at the Earth



$$\left(\frac{\delta\nu}{\nu}\right)_E = -(\Phi_s - \Phi_o)$$

Doppler effect

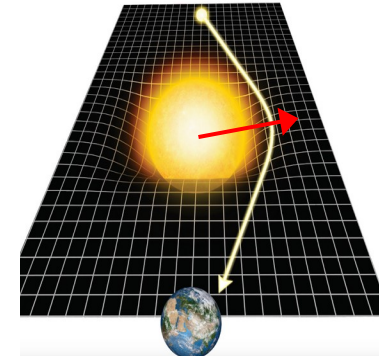
From the **relative movement** between the pulsar and the Earth created by the halo attraction



$$\left(\frac{\delta\nu}{\nu}\right)_D = -\vec{n} \cdot (\vec{v}_s - \vec{v}_o)$$

Integrated Shapiro effect

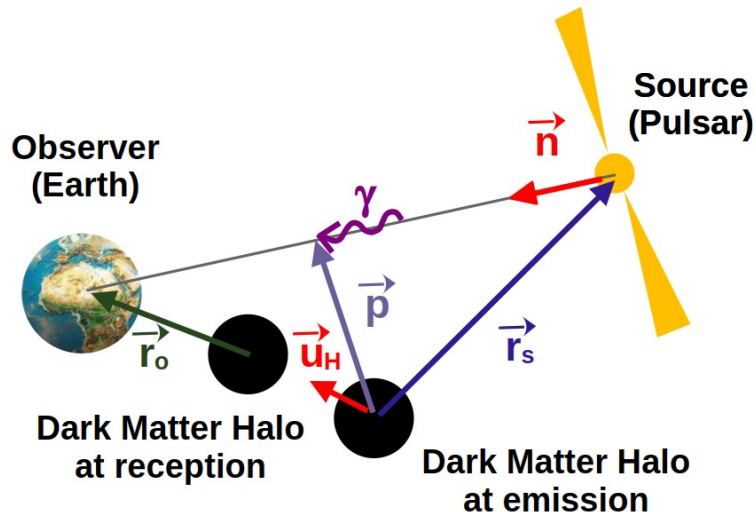
From the **variation** of the photons **travel time**



$$\left(\frac{\delta\nu}{\nu}\right)_S = -2 \int_0^D \vec{u}_H \cdot \vec{\nabla} \Phi dz$$

Determination of the most general analytic formulae possible

Analytic formulae



Non-linear dependence on the **direction** of:

- Halo velocity
- Halo position wrt the Earth
- Halo position wrt the pulsar

Exact solution for:

- Point-mass halos
- Constant velocity

$$\left(\frac{\delta\nu}{\nu}\right)_E = -(\Phi_s - \Phi_o) = \frac{\tilde{R}_s}{2} \left(\frac{1}{\|\vec{r}_s\|} - \frac{1}{\|\vec{r}_o\|} \right)$$

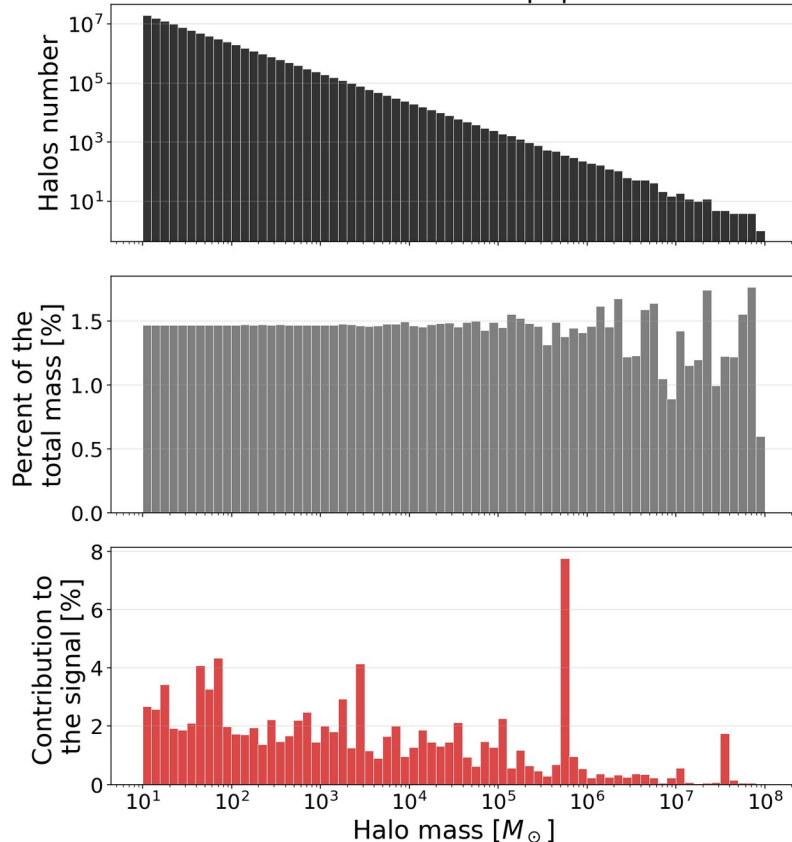
$$\left(\frac{\delta\nu}{\nu}\right)_S = -2 \int_0^D \vec{u}_H \cdot \vec{\nabla} \Phi dz = \tilde{R}_s \left[\frac{(\vec{n} - \vec{u}_H) \cdot (\vec{r}_o \wedge (\vec{u}_H \wedge \vec{p}))}{\|(\vec{n} - \vec{u}_H) \wedge \vec{r}_o\|^2 \|\vec{p}\|} \right]_{\vec{p}=\vec{r}_o}^{\vec{p}=\vec{r}_s}$$

$$\left(\frac{\delta\nu}{\nu}\right)_D = -\vec{n} \cdot (\vec{v}_s - \vec{v}_o) = \frac{\tilde{R}_s}{2} \vec{n} \cdot \left[\frac{\vec{r}_s(t_i)(\vec{u}_H \cdot \vec{r}_s(t)) - \vec{u}_H(\vec{r}_s(t_i) \cdot \vec{r}_s(t))}{((\vec{u}_H \cdot \vec{r}_s(t_i))^2 - u_H^2(\vec{r}_s(t_i))^2) \|\vec{r}_s(t)\|} \right]_{t=t_i}^{t=t_r} - \frac{\vec{r}_o(t_i)(\vec{u}_H \cdot \vec{r}_o(t)) - \vec{u}_H(\vec{r}_o(t_i) \cdot \vec{r}_o(t))}{((\vec{u}_H \cdot \vec{r}_o(t_i))^2 - u_H^2(\vec{r}_o(t_i))^2) \|\vec{r}_o(t)\|} \Big|_{t=t_i}$$

General formulae match literature's results within relevant limits

Halo population: Λ CDM mass function

Statistics of the halo population



Model:

- Earth-pulsar distance: 1kpc
- 15 years of observation
- Constant DM density
- Isotropic & maxwellian velocity distribution

Λ CDM subhalo mass function:

$$\frac{\partial n}{\partial m} = m^{-2}$$

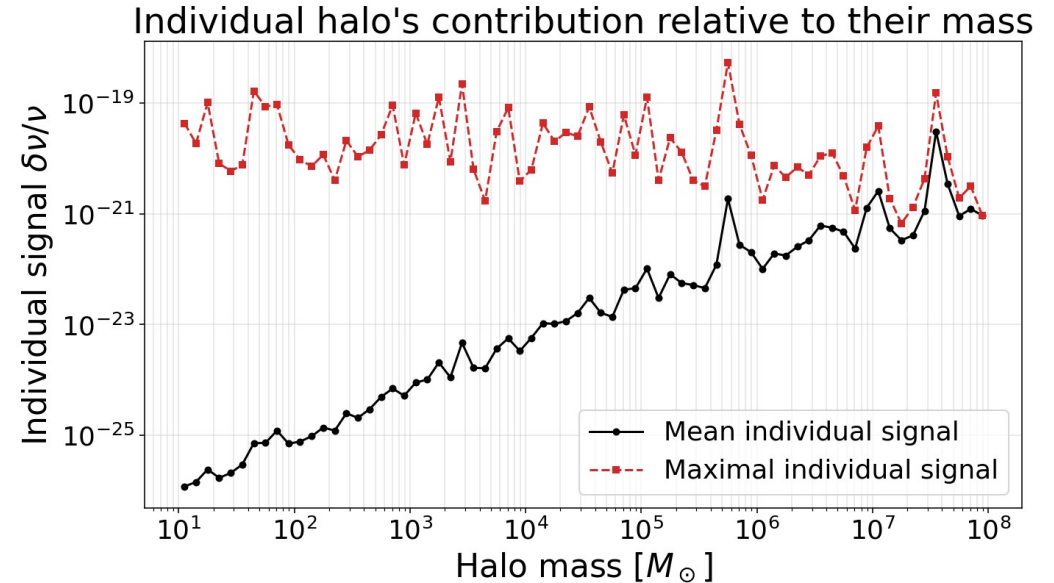
- Number of halos drops steeply with mass
- Same total mass within all mass bins
- Small halos contribute more than big halos

**All masses contribute:
the full population is needed**

Halo population effect

Individual halo contribution vs mass:

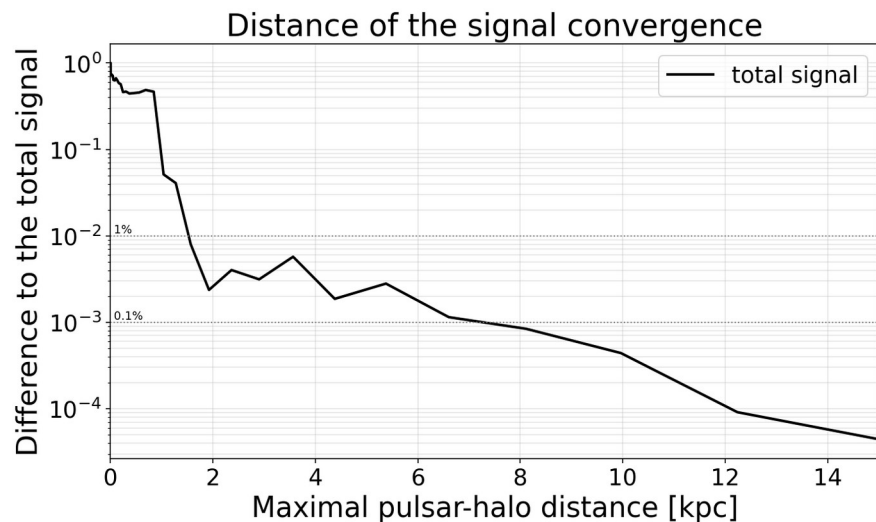
- **Mean** individual signal **grows** with mass
- **Maximal** individual signal **similar** for each mass bin
- Light halos: **weak** on average, but very numerous → statistically, some get very **close**
- Max. individual signal set by **closest halos** in every bin



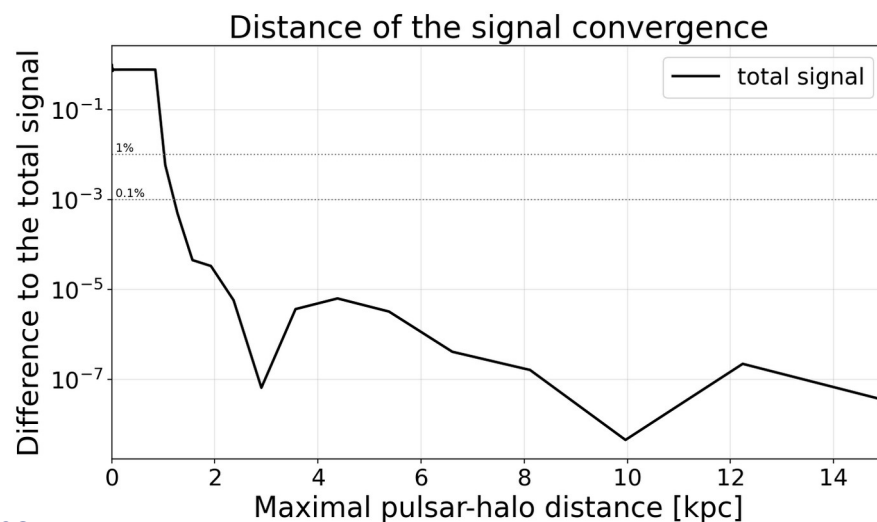
Maximum effects set by the geometry, not the mass

Mean Field Approximation

Evolution of the **perturbation** as a function of a **cutoff distance**



Λ CDM mass function



Monochromatic mass function
($10^1 M_{\odot}$)

$D_{\text{pulsar}} \approx 1 \text{ kpc}$

Stabilisation of the signal at very high distance from the line-of-sight

Observed signal

Measured pulsar phase Halos perturbation

$$\phi(t) = \phi_0 + \nu_0 \left(t + \int_0^t \frac{\delta\nu}{\nu_0} dt' \right) + \frac{1}{2} \dot{\nu}_0 t^2$$

Pulsar intrinsic frequency

Pulsar intrinsic spin-down

Quadratic fit

Measured pulsar frequency

$$\phi_{\text{fit}}(t) = \phi_{\text{ifit}} + \nu_{\text{fit}} t + \frac{1}{2} \dot{\nu}_{\text{fit}} t^2$$

Measured pulsar spin-down

Ratio perturbation/phase $\sim 10^{-12}$

Our signal:

$$s(t) = \phi(t) - \phi_{\text{fit}}(t)$$

Ratio signal/original perturbation $\sim 10^{-9}$

Degeneracy:

- ν_0 and $\dot{\nu}_0$ **unknown**, measured by a fit
- **Impossible** to separate quadratic halo effect from **pulsar spin** and spin-down
- **Degenerate** parts of the signal get **absorbed** in the fit

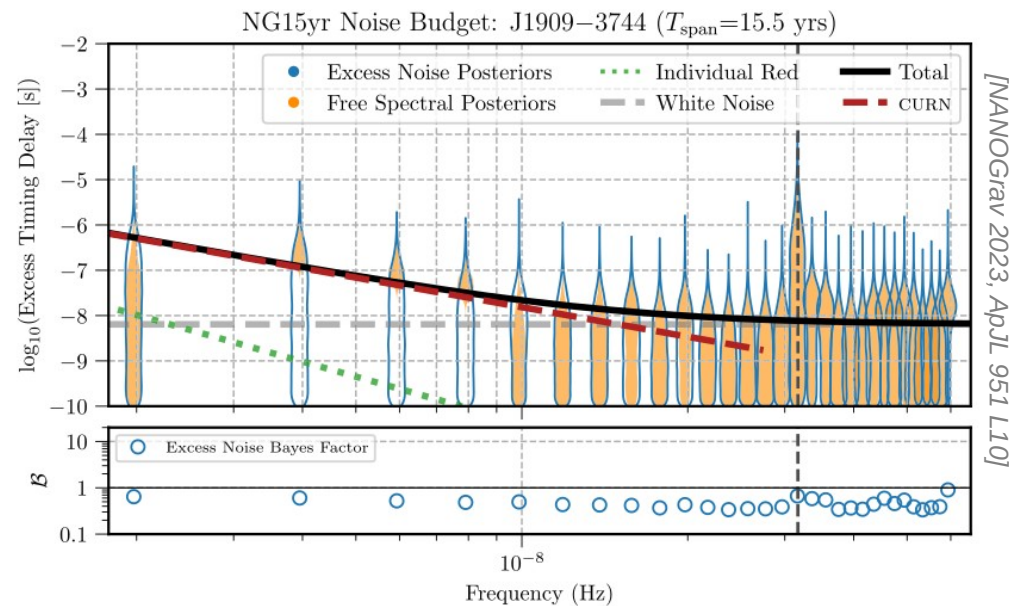
Only the residual is observable

Signal analysis

Signal analysis with **tools of PTAs** for GW detection, for **comparison** purpose:

- **Spectral** analysis (Power Spectrum Density)
- Power-law **fit** with model used in GW detection
- **Direct comparison** with real data

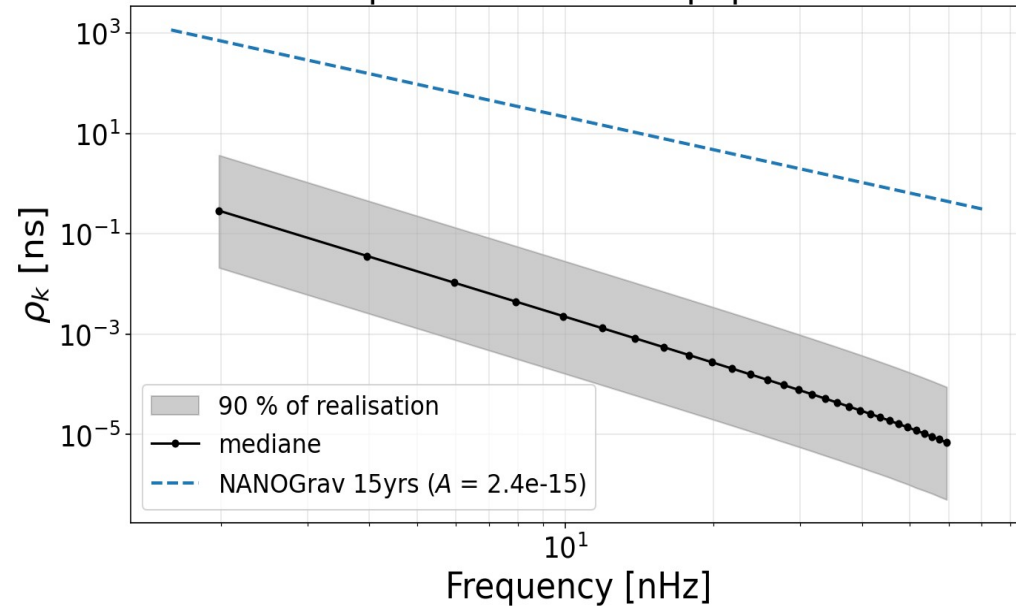
Example of real data of one pulsar by NANOGrav



Results of the simulations

Power spectrum

Free spectrum on 1000 populations



Free spectrum over 1000 Λ CDM population realisations:

- A **red** (low-frequency) power law
- **Large variance** between statistically similar populations
- -3 to -4 orders of magnitude below the GW signal: **background** signal

Pulsar arrays with correlations needed to detect the signal

Spectral analysis

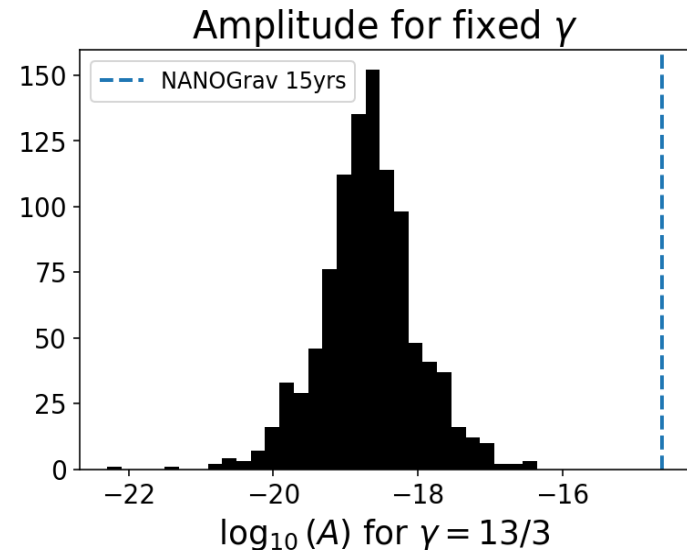
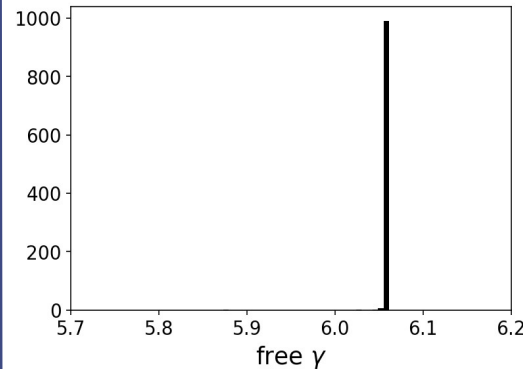
Model:

$$\text{PSD}(f) = \frac{A^2}{12\pi^2} \left(\frac{f}{f_{\text{yr}}} \right)^{-\gamma} f_{\text{yr}}^{-3}$$

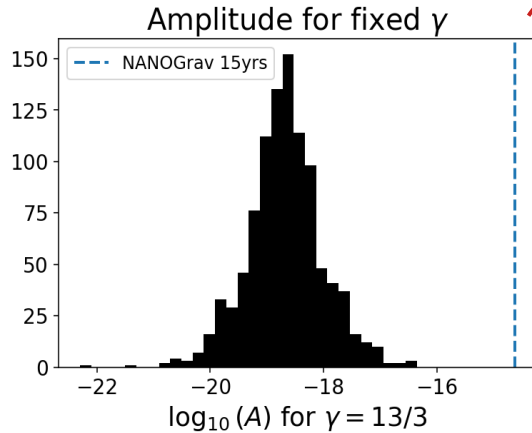
- **Spectral index** γ fixed by the timing model (data processing): **identical** for all realisation
- Large **variance** of the amplitude

All the physics (model dependence) is in the amplitude

Simulation of **1000** realisations of a Λ CDM population:



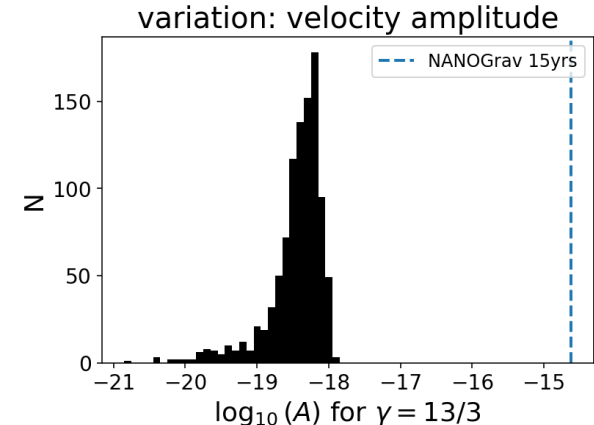
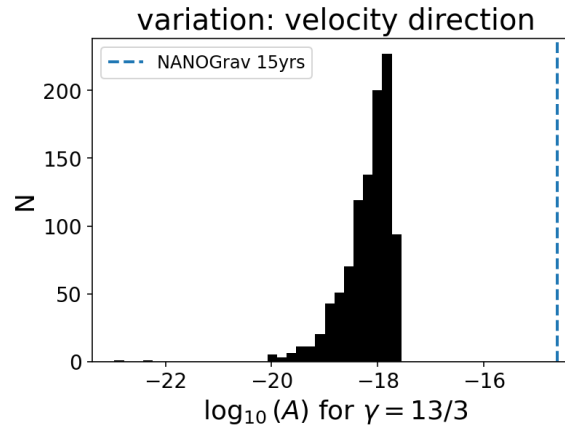
Sources of variability



~2 order of magnitude
between **statistically
similar** populations

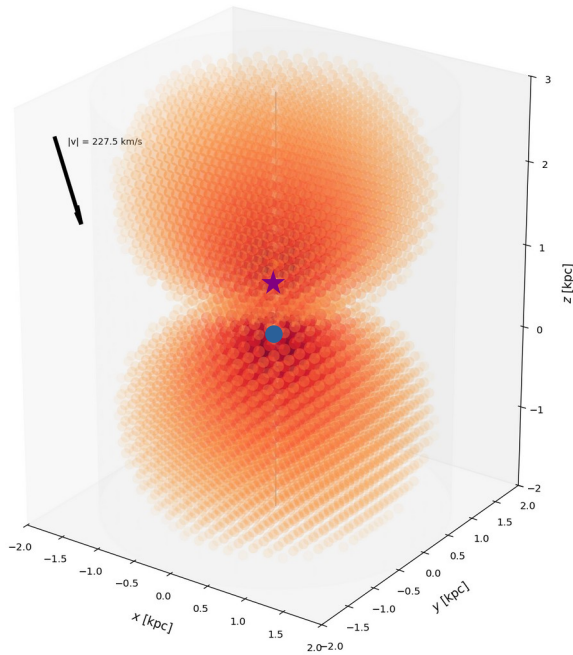
Same mass function, same density, same velocity distribution, but **different realisations**

Now **same realisation** but **one** random parameter:
The halo **velocity** impacts heavily the amplitude variance



Impact of removing degeneracies

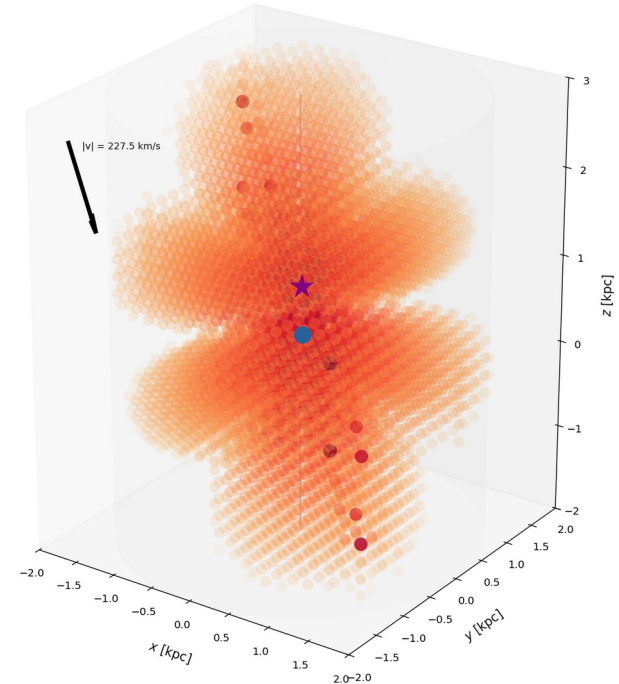
Raw effect



RMS signal of **one halo** vs its position:

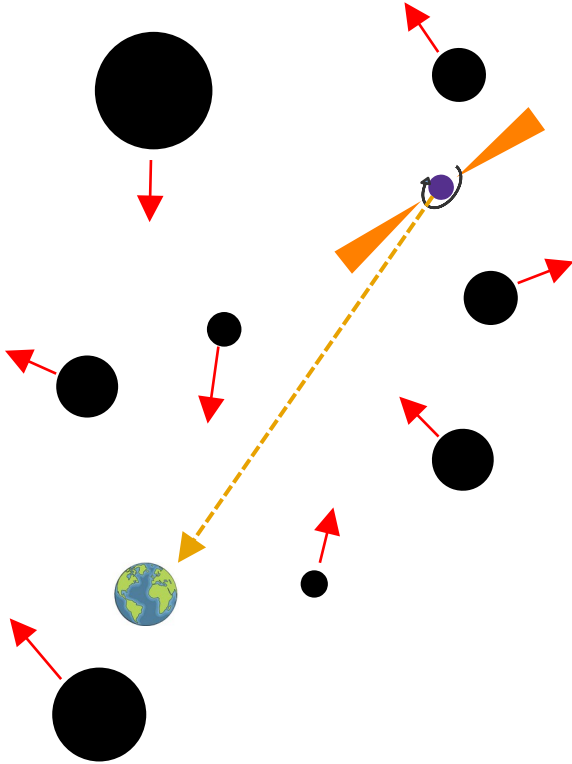
- Before fit: set by the **distance** to the Earth or the pulsar
- After fit: **preferred directions**, strong dependence on the **velocity** direction

After quadratic fit



After the fit, the velocity direction matters as much as the distance

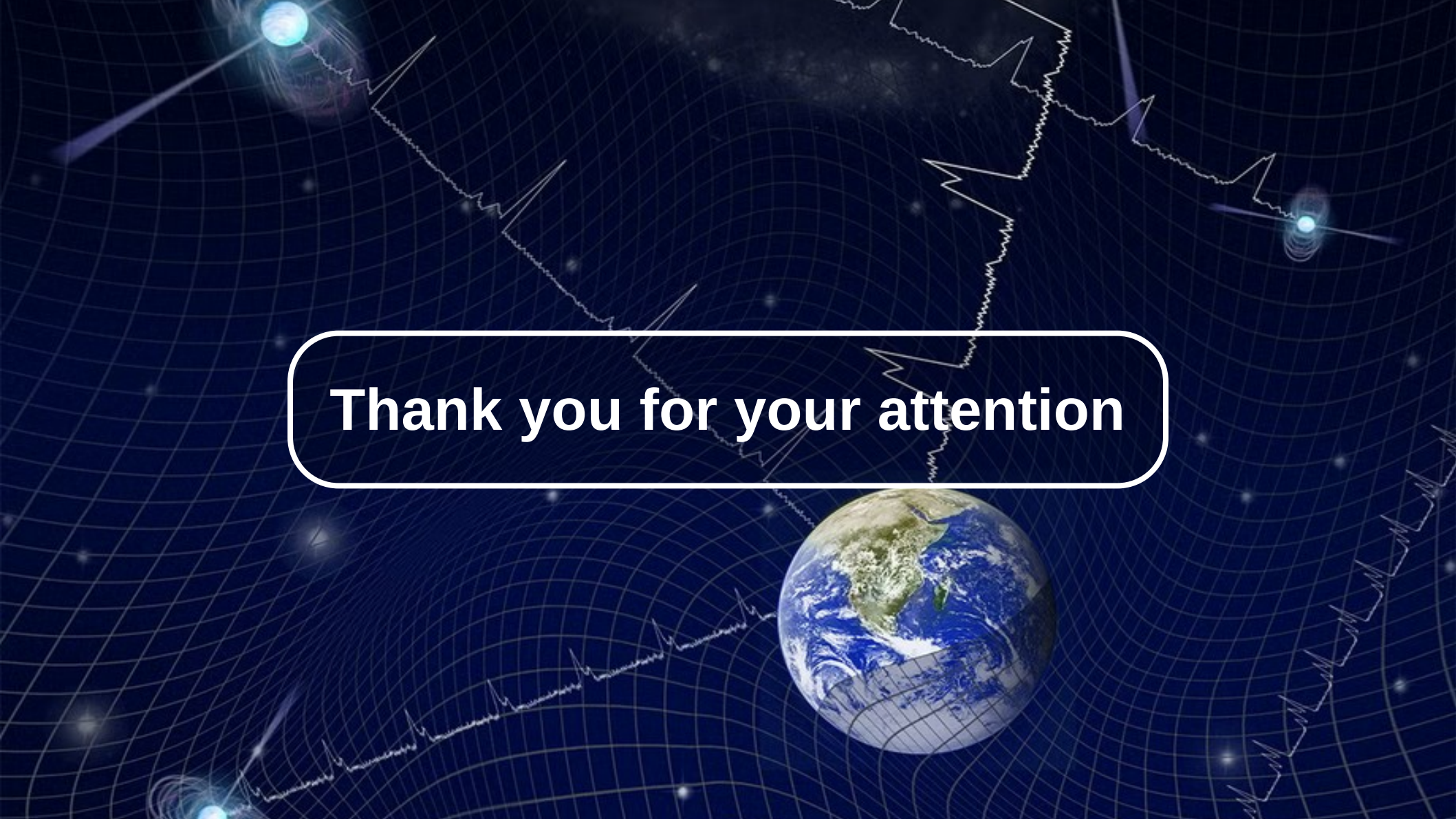
Conclusion



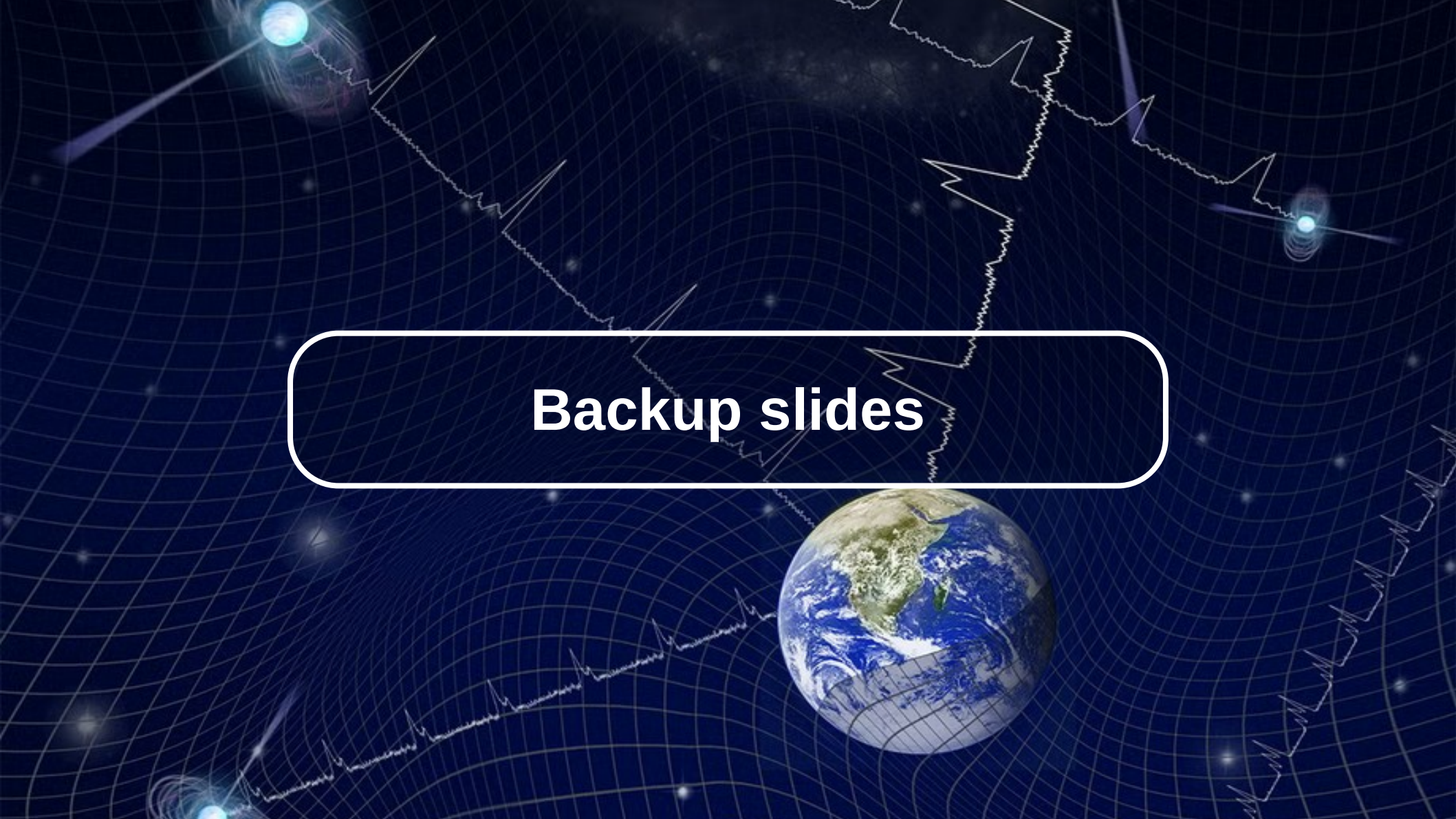
- Dark matter subhalos generate a signal on pulsar timing through **population effect**
- Real data analysis **must** be considered to understand the **actual observable**
- Contributing halos can be up to **~ 5 kpc away** from the line of sight
- Halo's **contribution** not only depends on mass & distance but heavily on **velocity direction**
- **All physics** is in the power spectrum **amplitude**

Next work:

PTAs **angular correlation** of the halo signal
Specificity of signal for **non Λ CDM** subhalos **models**

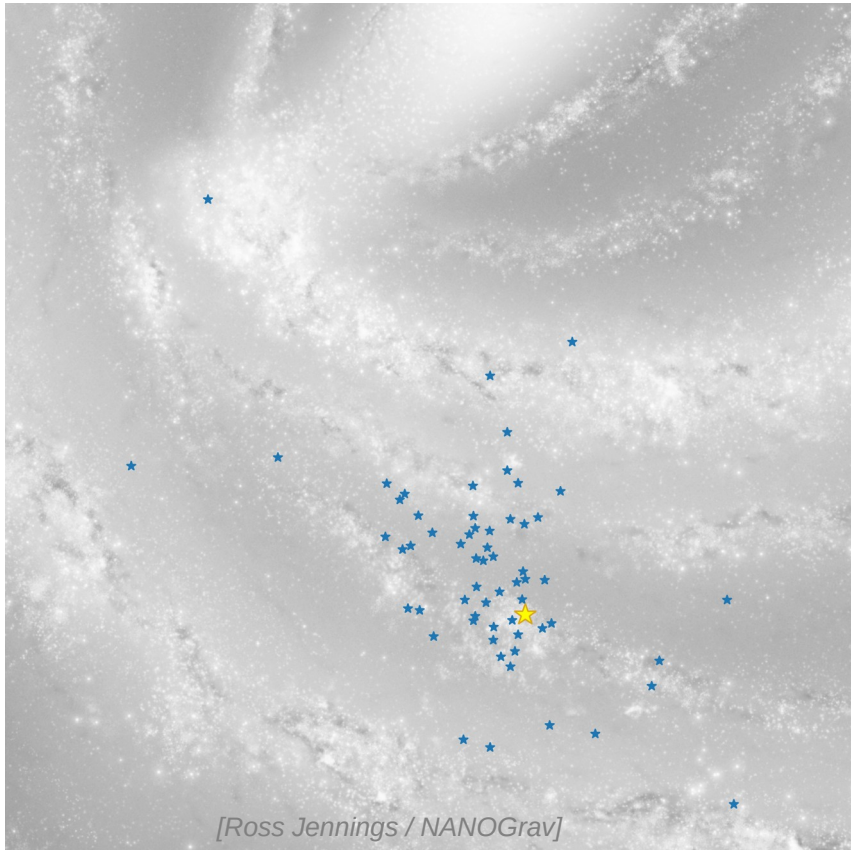
The background is a dark blue space-themed image. It features a white grid pattern that curves and warps, suggesting gravitational fields or spacetime curvature. Several bright blue stars or distant galaxies are visible, some with long, thin white trails. A realistic image of the Earth, showing continents and oceans, is positioned in the lower right quadrant. A white, rounded rectangular box with a thin white border is centered in the image, containing the text "Thank you for your attention" in a bold, white, sans-serif font.

Thank you for your attention

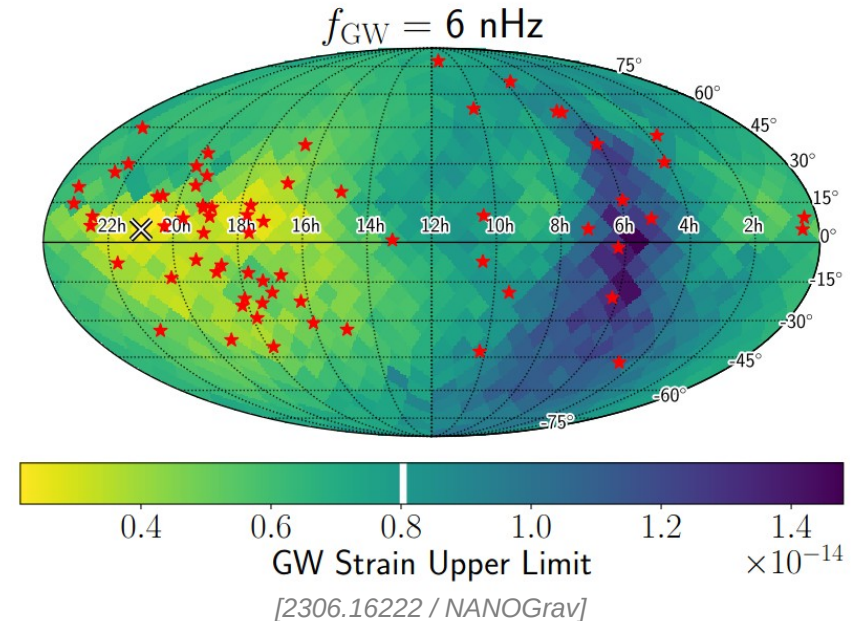


Backup slides

Pulsar Population



Earth-Pulsar distance: 0,1 to 5 kpc
Typical distance: 1 to 1,5 kpc



Sources of signal perturbation

Our hypothesis:

Distinctive spatial correlation **signature**

→ halo perturbations **distinguishable** from all other sources (including GWs)

Noise source	Origin	Time correlations	Frequency dependence or correlations	Spatial correlations
Pulse jitter	Pulsar	No (white)	Yes (correlated)	No
Spin noise	Pulsar	Yes (red)	No (achromatic)	No
Orbital variations	Pulsar system	Yes (red)	No (achromatic)	No
DM variations	ISM	Yes (red)	Yes (v^{-2})	No
Diffraction scintillation	ISM	No (white)	Yes (v^{-x} , $x \approx 4$)	No
Solar wind	Solar system	Yes (red)	Yes (v^{-2})	Yes (solar elongation)
Solar system ephemeris	Solar system	Yes (red)	No (achromatic)	Yes (dipolar)
Polarization calibration	Telescope	Either	Yes (correlated)	No
Clock corrections	Telescope	Yes (red)	No (achromatic)	Yes (monopolar)
RFI	Telescope	No (white)	No (uncorrelated)	No
Gravitational waves	GW sources	Yes (red)	No (achromatic)	Yes (quadrupolar)

[Ross Jennings, U. of Pittsburgh PTA workshop, Nov. 30, 2023]