

Simulations of ultra-light dark matter

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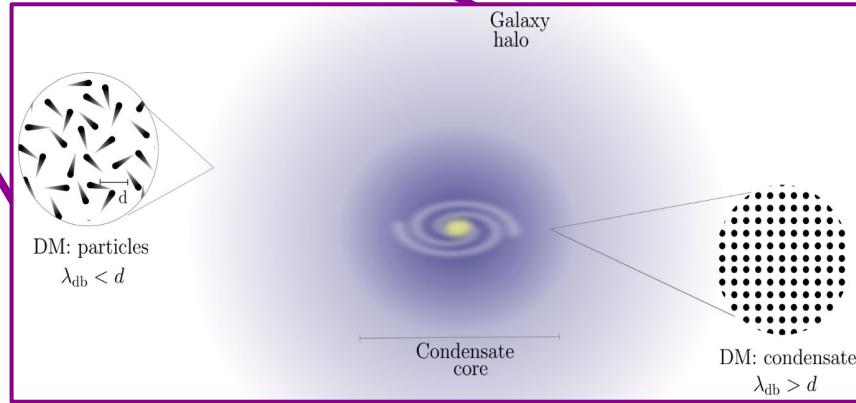
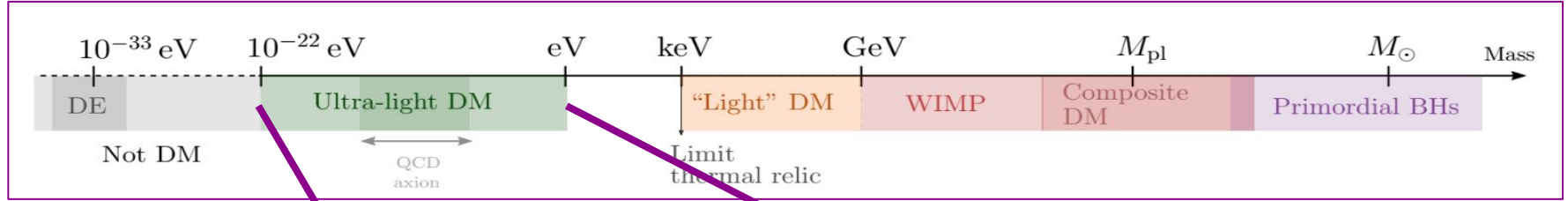
Co-funded by
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*General
description*

Dark Matter candidates

We have rich evidence for the existence of DM, but we don't know its basic properties, e.g. mass, yet



Ferreira '21

Inside virialized halos, ULDM forms cores that can be described as **BECs** - bosons occupying the same ground state, behaving as a single macroscopic quantum wave.

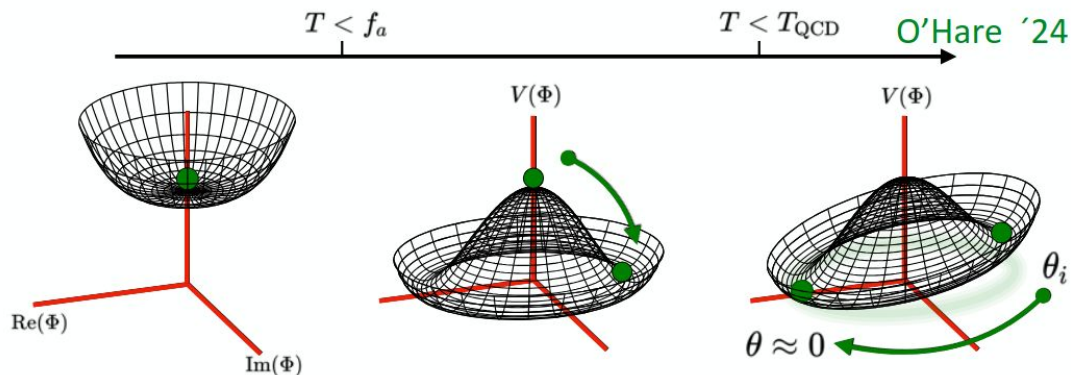
The simplest case: particle physics motivation

There are many theories that predict an ultralight scalar with mass m

- ❑ A well-known example is the QCD axion (Peccei, Quinn and Weinberg, ...)
- ❑ There are also many axion-like particles in string theories (Svrcek, Witten, ...)

For structure formation:

- classical (but waves)
- non-relativistic
- only gravitationally coupled



The simplest case: FDM

The Klein-Gordon equation in an expanding universe reads as

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0.$$

❖ *Freezing regime* $H \gg m$

$$3H\dot{\phi} \gg m^2\phi \quad \longrightarrow \quad \dot{\phi} \approx 0.$$

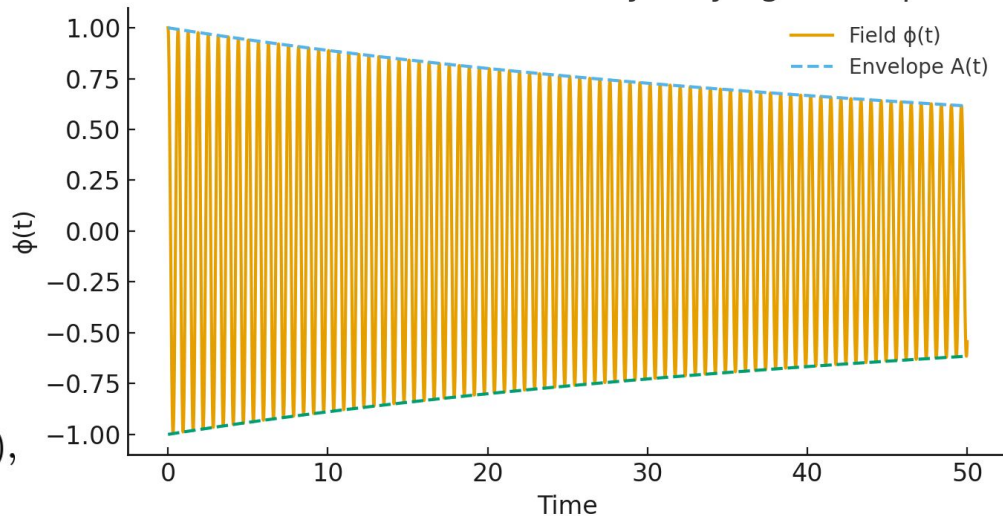
❖ *Late-time regime* $H \ll m$

$$\ddot{\phi} + m^2\phi \approx 0. \quad \longrightarrow \quad \phi(t) = A(t) \cos(mt),$$

With the average energy density

$$\langle \rho_\phi \rangle \propto a^{-3}.$$

Fast oscillations with slowly varying envelope



Separating the fast and slow oscillations

$$\phi(x, t) \approx \frac{\hbar}{m} \left[\psi(x, t) e^{-imc^2 t/\hbar} + \psi^*(x, t) e^{+imc^2 t/\hbar} \right]$$

The simplest case: Fuzzy Dark Matter

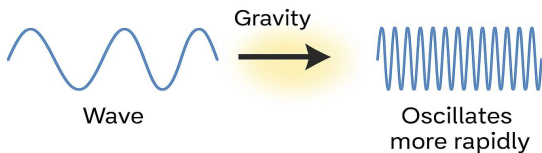
Such decomposition gives rise to the **Schrodinger-Poisson** system

Madelung transformation

$$\psi(\mathbf{x}, t) = \sqrt{\frac{\rho(\mathbf{x}, t)}{\rho_0}} e^{iS(\mathbf{x}, t)/\hbar}$$

$$\mathbf{v}(\mathbf{x}, t) = \frac{1}{m} \nabla S(\mathbf{x}, t)$$

$$\lambda \downarrow \Rightarrow k \uparrow \Rightarrow |\nabla \psi| \uparrow$$



$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi + m\Phi \psi,$$

$$\nabla^2 \Phi = 4\pi G \rho_0 (|\psi|^2 - 1).$$

Wave approach

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla \Phi - \nabla Q$$

$$Q(\mathbf{x}, t) = -\frac{\hbar^2}{2m^2} \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}}.$$

Fluid approach



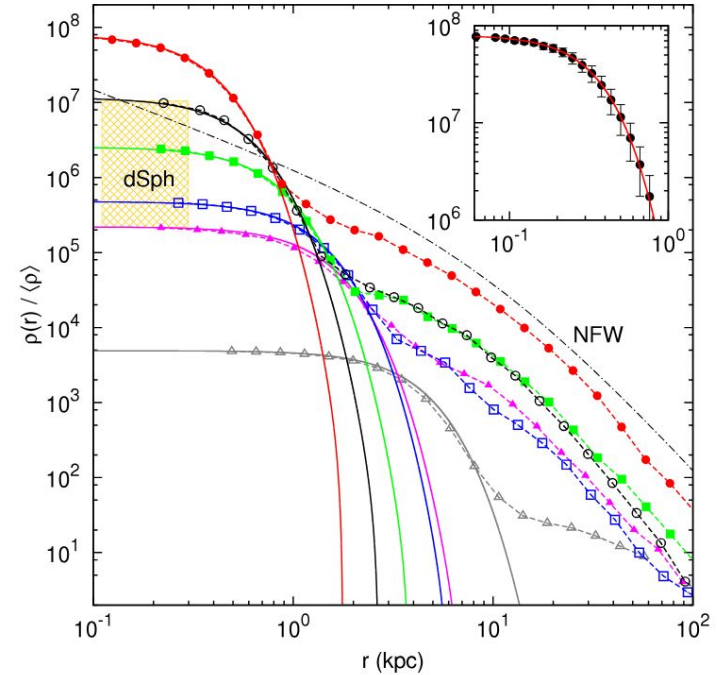
ULDM features: Core profile

The halos density profiles can be parametrized by a step function for the inner and outer regions

$$\rho_{\text{halo}}(r) = \Theta(r_{\epsilon} - r)\rho_{\text{sol}}(r) + \Theta(r - r_{\epsilon})\rho_{\text{NFW}}(r)$$

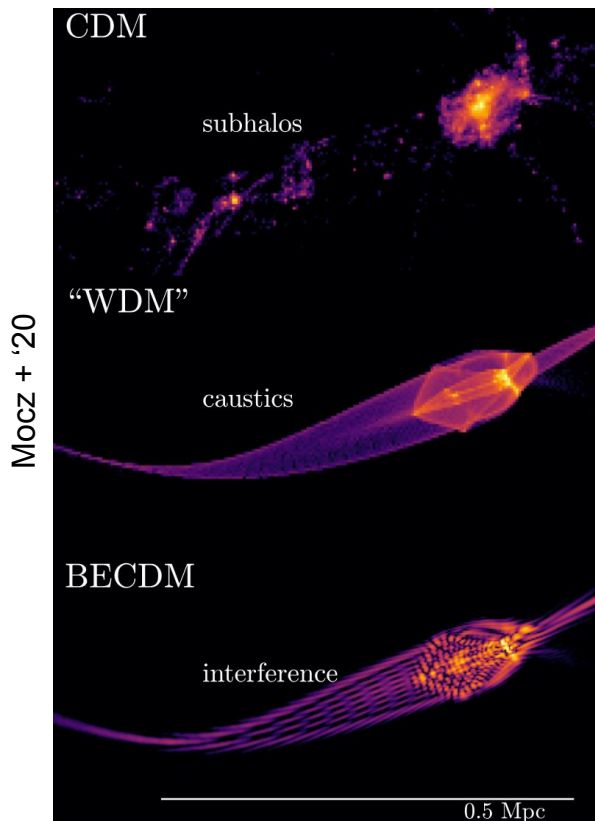
with a soliton and a NFW configuration

$$\rho_{\text{sol}}(r) = \frac{\rho_c}{\left[1 + \alpha(r/r_c)^2\right]^8},$$
$$\rho_{\text{NFW}}(r) = \frac{\rho_s}{(r/r_s) \left(1 + r/r_s\right)^2},$$



Schive+ '14

ULDM unique feature: Interference patterns



- ❑ ULDM field behaves as a coherent wave.
- ❑ These waves overlap and their phases add or cancel out.
- ❑ This behaviour changes over time.

$$\lambda_{\text{dB}} \sim \frac{h}{mv} \sim \text{kpc for } m \sim 10^{-22} \text{ eV.}$$

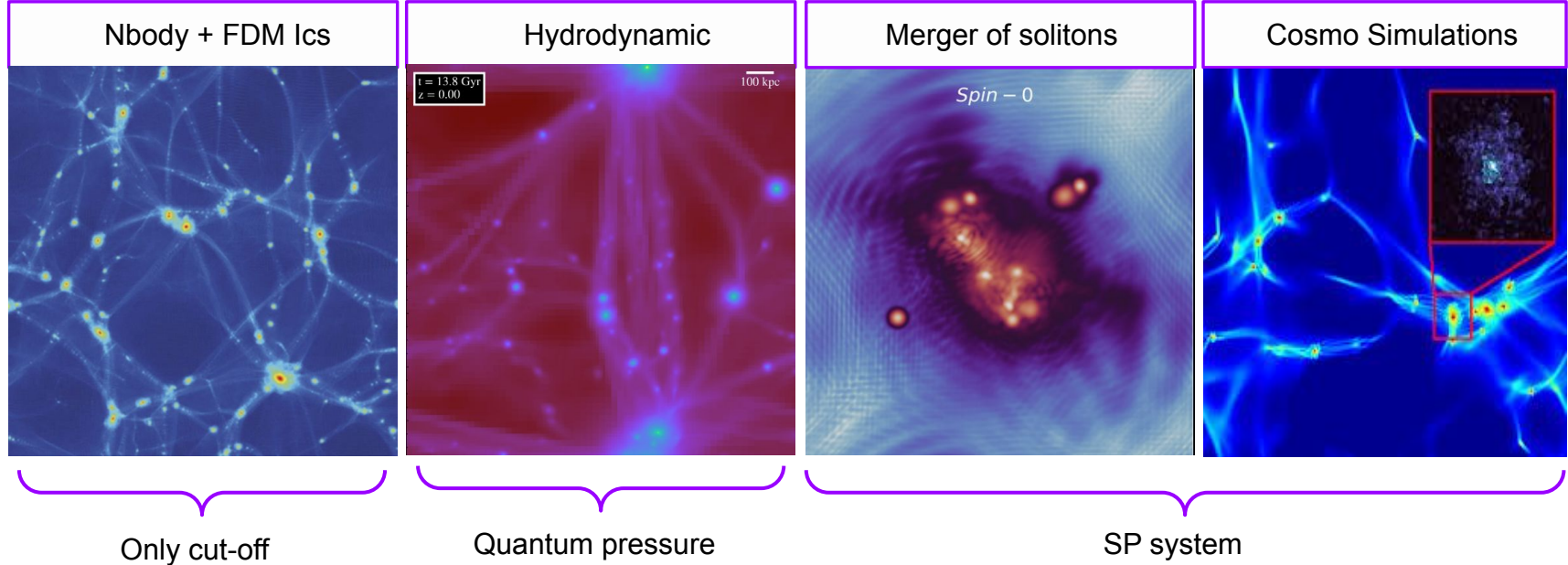
Interference patterns



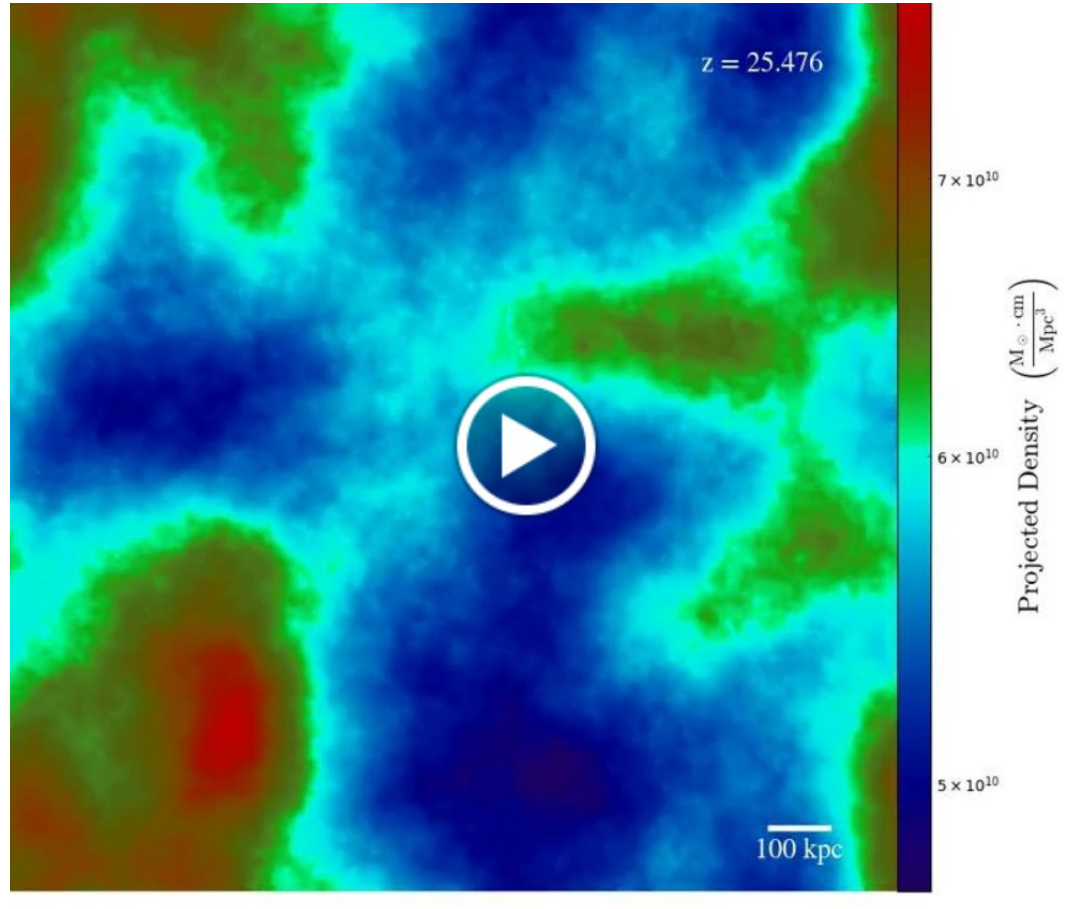
Granularities

(O(1) fluctuations of the density)

How do we study structure formation with ULDM?

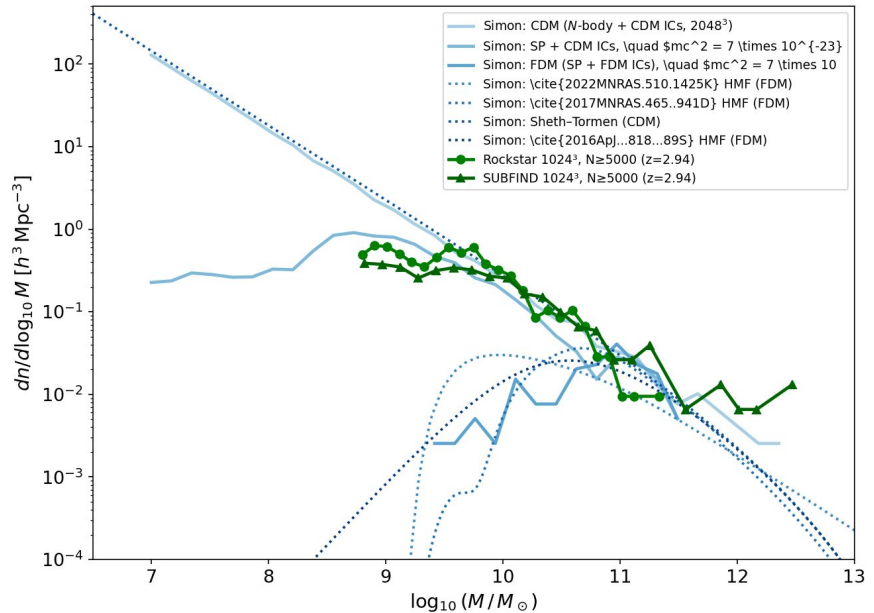
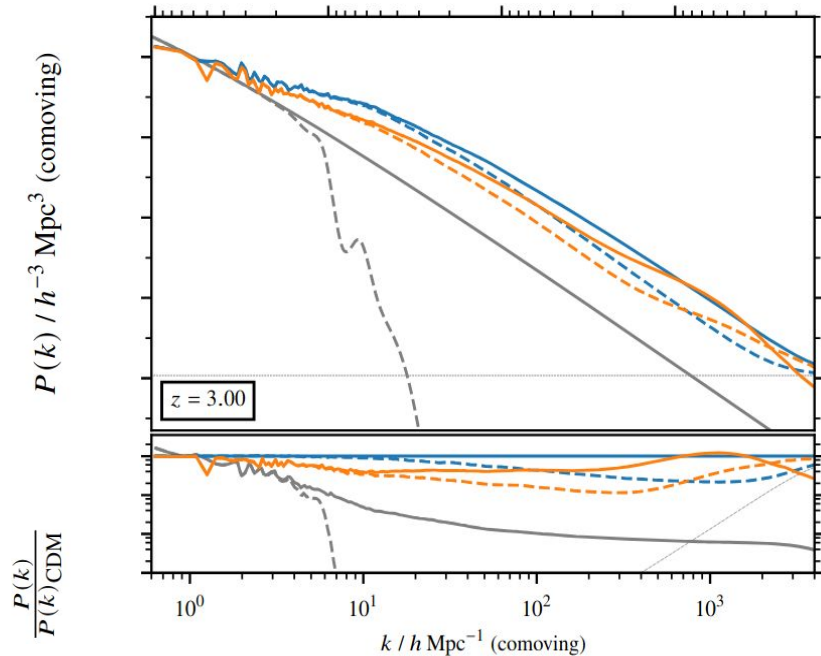


Example of cosmological simulations



ULDM features: Suppression of small-scales structures

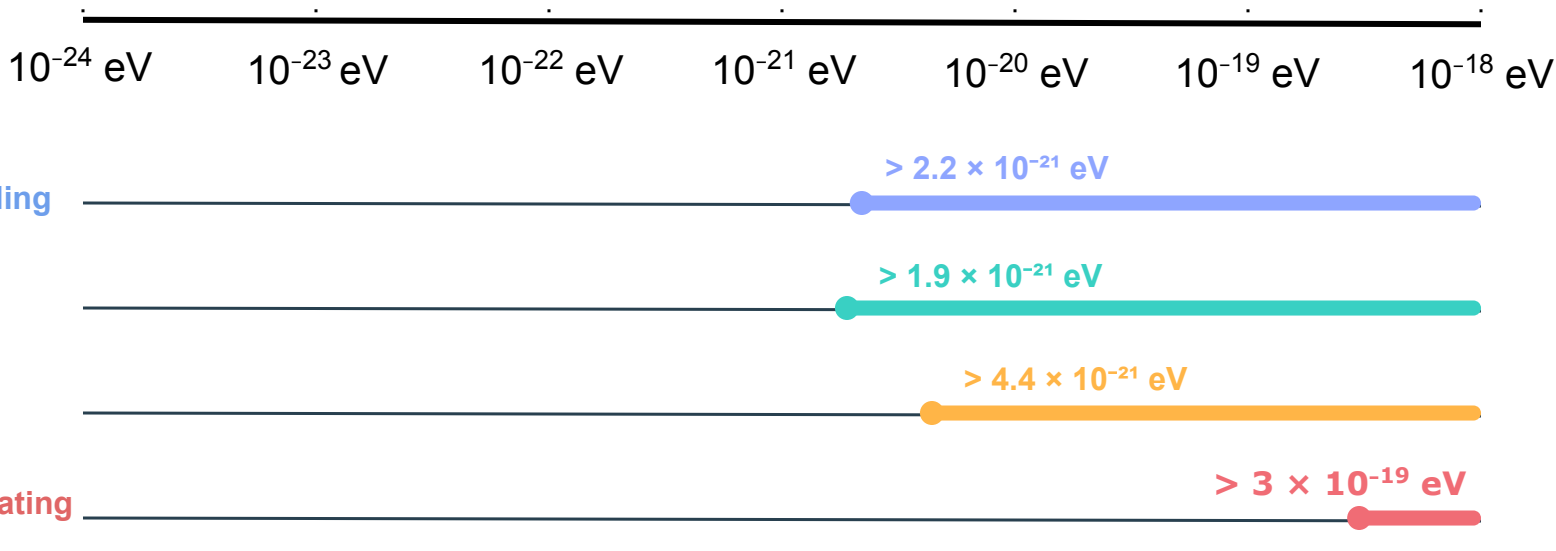
The MPK and the HMF also show a cut-off at small scales respect to CDM.



Work in progress

Current constraints on minimal fuzzy dark matter

Current bounds assume a single, free scalar field comprising all dark matter.



SI-SFDM
model

Self-interacting Scalar Field Dark Matter

With self-interactions, the potential is given by

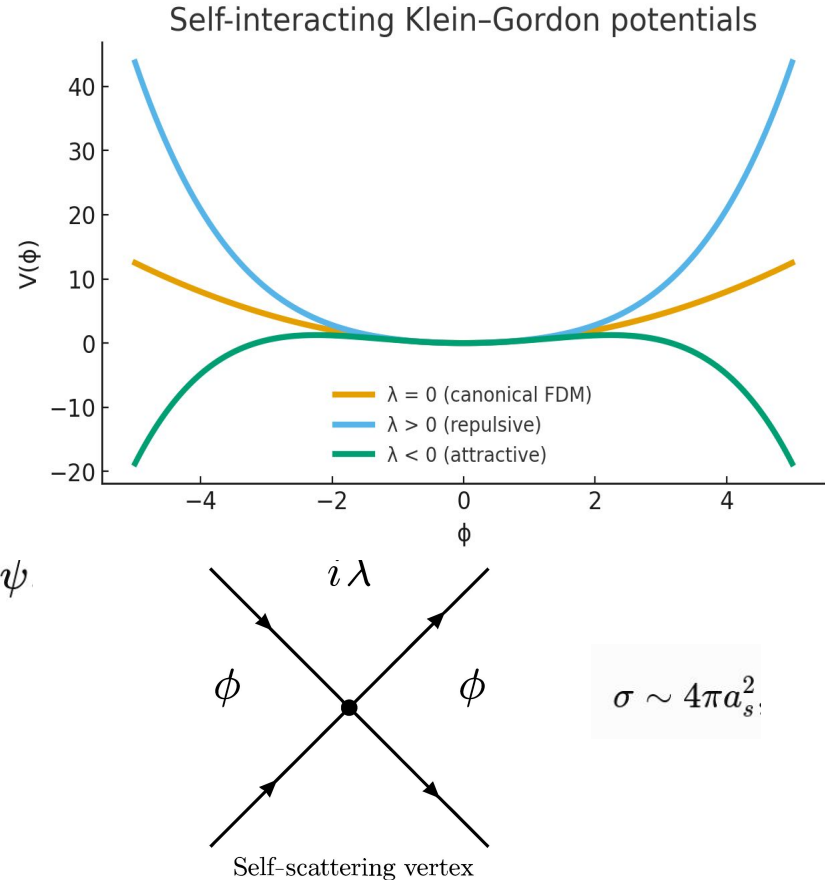
$$V(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4.$$

The quartic term represents a 4-point interaction.

In the non-relativistic regime, we have: $\frac{4\pi\hbar^2 a_s}{m} |\psi|^2 \psi$

a_s is the scattering length

$$\lambda = 32\pi\hbar^2 a_s m.$$

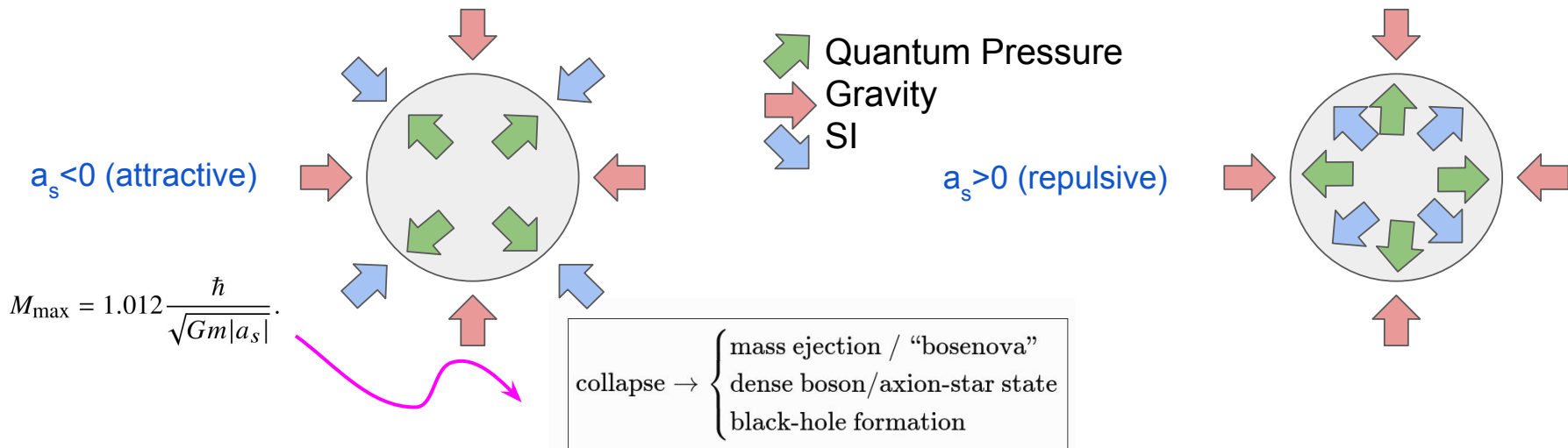


Gross-Pitaevskii-Poisson system

The evolution of the wave function in the non-relativistic regime is described by:

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2ma^2} \nabla^2 \psi + mV\psi + \frac{4\pi\hbar^2 \rho_{0m} a_s}{m^2 a^3} |\psi|^2 \psi$$

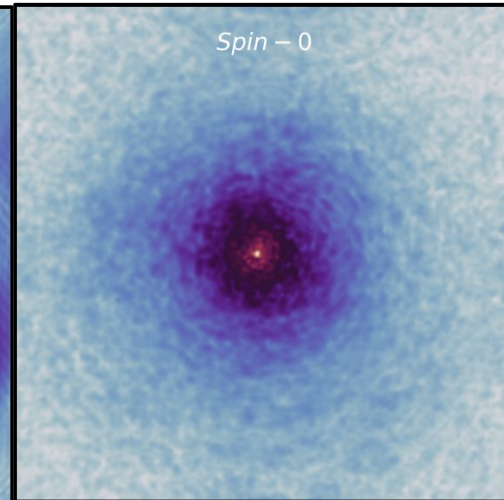
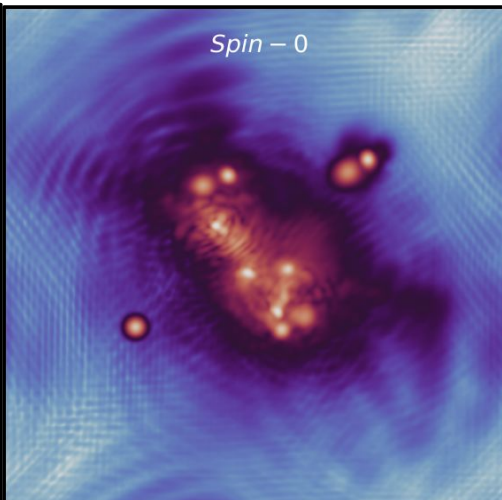
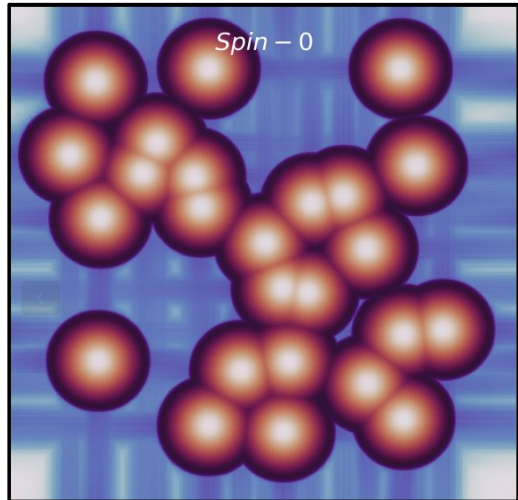
$$\nabla^2 V = \frac{4\pi G \rho_{0m}}{a} (|\psi|^2 - 1)$$



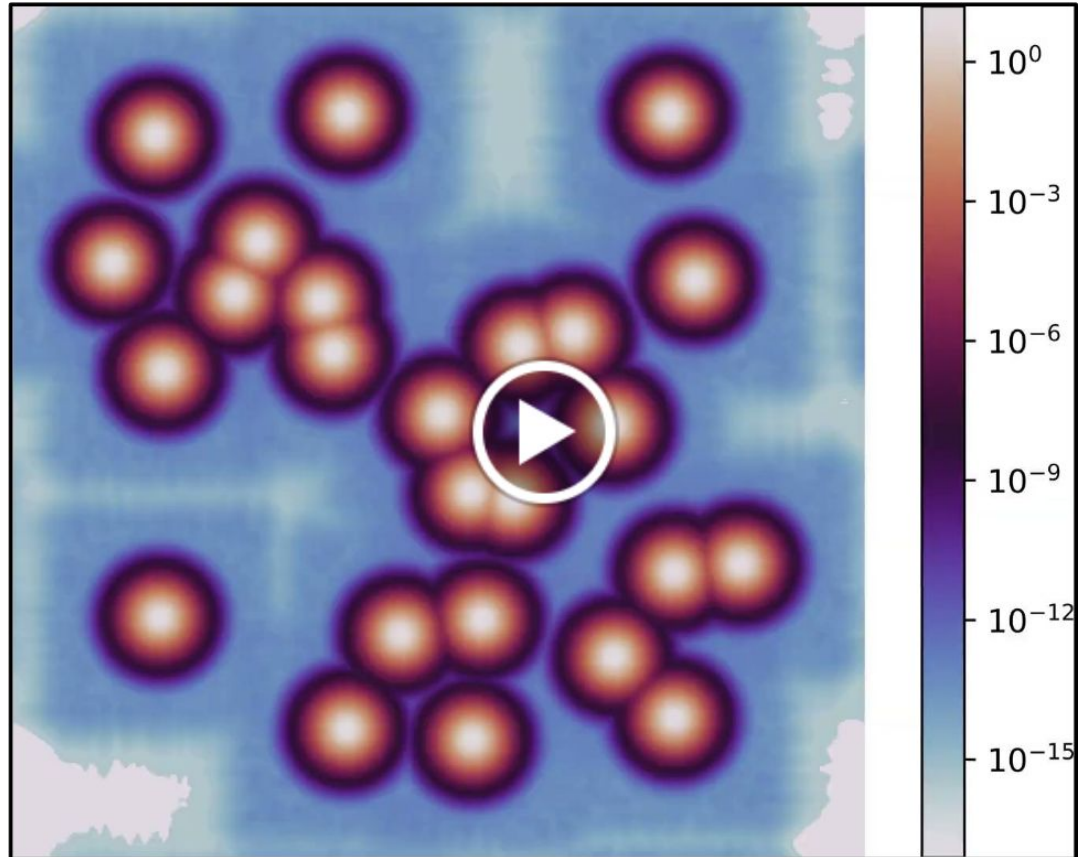
Idealised simulations of multiple solitons merger

Stationary equilibrium configuration are given by $\psi = e^{-i\omega t/\hbar}\psi_{\text{sol}}(r)$

leading to the spherical symmetry GPP system
$$\left\{ \begin{aligned} -\frac{\hbar^2}{2m} \frac{1}{r} \frac{\partial^2}{\partial r^2} (r\psi_{\text{sol}}) &= \left(\omega - m\Phi - \frac{4\pi\hbar^2\rho_0 a_s}{m^2} \psi_{\text{sol}}^2 \right) \psi_{\text{sol}} \\ \frac{1}{r} \frac{\partial^2}{\partial r^2} (r\Phi) &= 4\pi G\rho_0 \psi_{\text{sol}}^2. \end{aligned} \right.$$



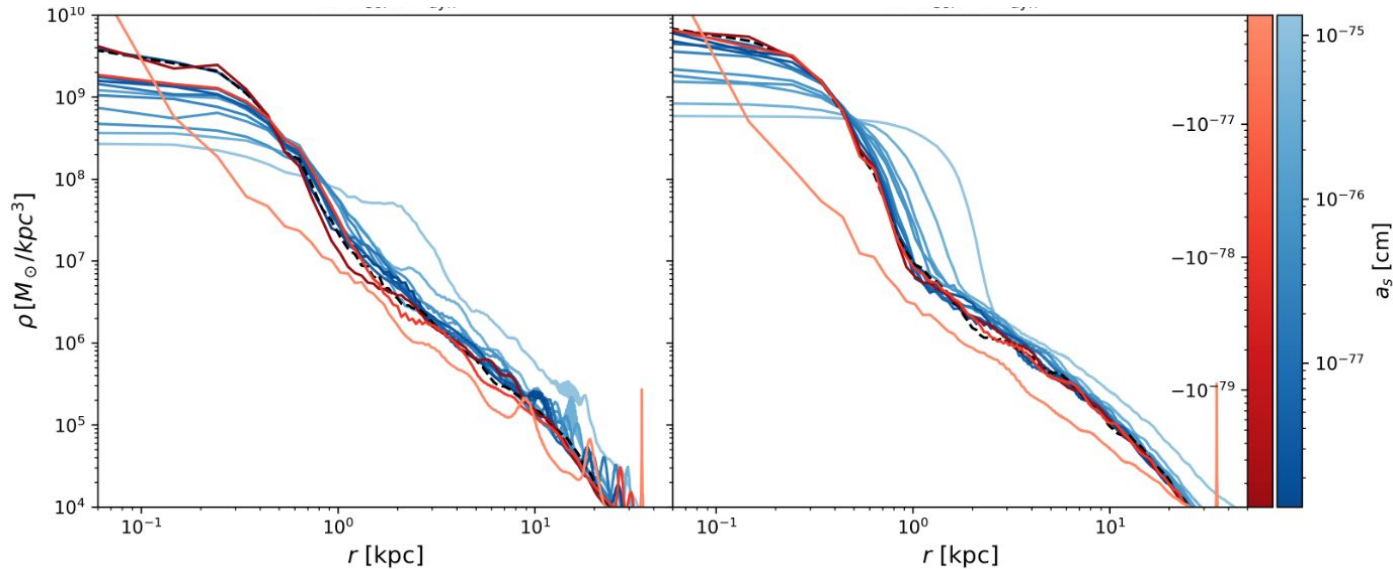
Soliton merge evolution



Evolution of the density profile

And the density profile of the final configuration is given by a core + NFW-tail

$$\rho_{\text{halo}}(r) = \Theta(r_{\epsilon} - r)\rho_{\text{sol}}(r) + \Theta(r - r_{\epsilon})\rho_{\text{NFW}}(r) \quad \left\{ \begin{array}{l} \rho_{\text{sol}}(r) = \frac{\rho_c}{[1 + \alpha(r/r_c)^2]^8}, \\ \rho_{\text{NFW}}(r) = \frac{\rho_s}{(r/r_s)(1 + r/r_s)^2} \end{array} \right.$$



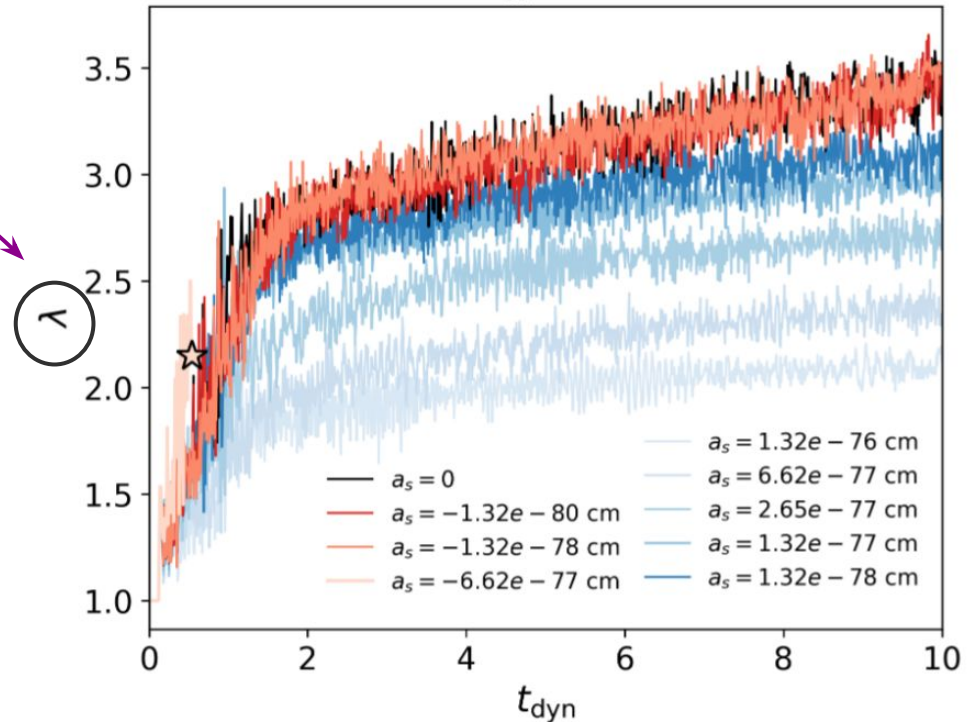
Evolution of the maximum density

The change of the central density over time shows a hierarchical behaviour: higher values for the attractive case and the opposite for the repulsive one.

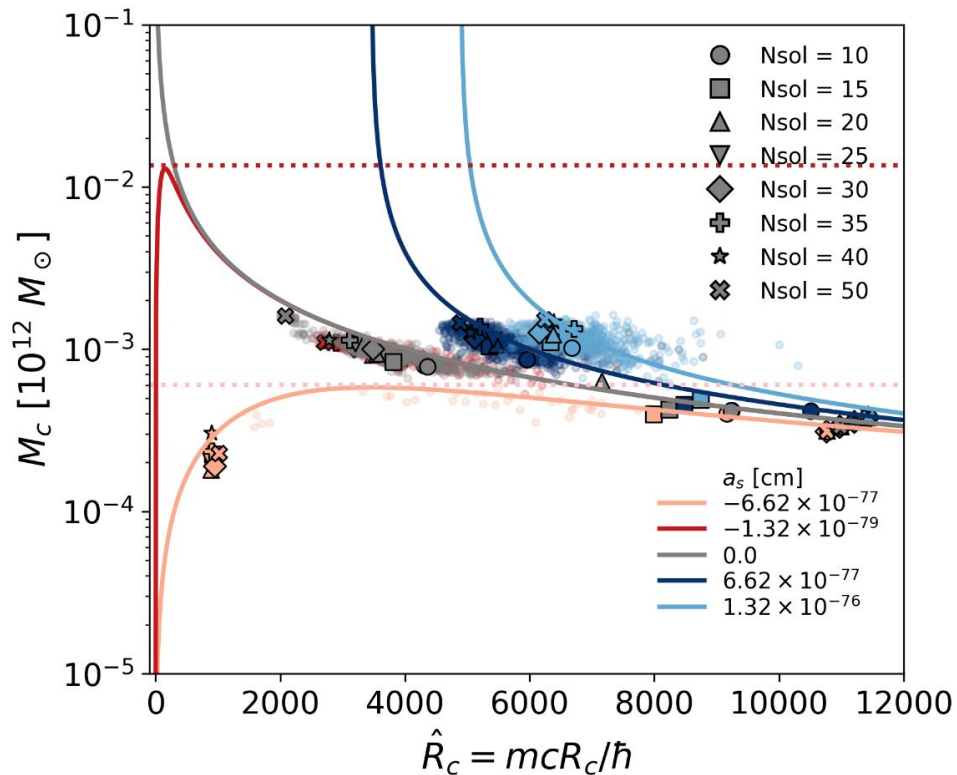
$$\lambda = \left(\frac{\rho(r=0, t)}{\rho(0, 0)} \right)^{1/4}$$

For the attractive case, there is a limit for the mass that the soliton can hold

$$M_{\max} = 1.012 \frac{\hbar}{\sqrt{Gm|a_s|}}.$$



Core-size relation: SFDM case



We compared the mass-size relation with the analytical prescription in Padilla +21:

$$M_c = 3\sqrt{2\pi} \frac{\hbar^2}{Gm^2 r_c} \left(\sqrt{2\pi} - \frac{6\hbar^2 a_s}{Gm^3 r_c^2} \right)^{-1}$$

They considered a Gaussian distribution

$$\rho_{\text{core}}(r) = \frac{M_c}{(\pi r_c^2)^{3/2}} e^{-r^2/r_c^2}$$

Core-halo relation: FDM case

Considering the invariant quantity

$$\Xi = \left(\frac{|E|}{M^3 (Gm/\hbar^2)} \right)$$

Under the transformations

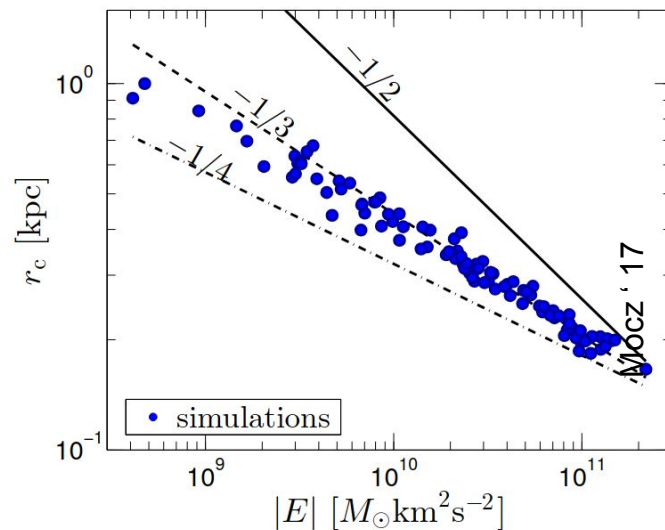
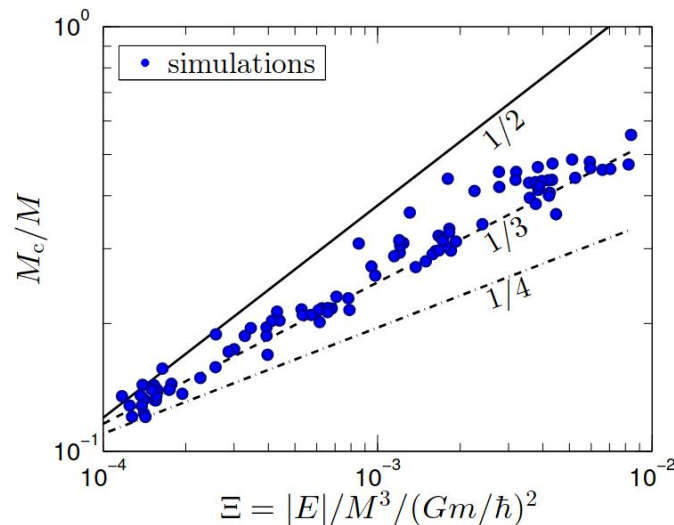
$$\{t, x, V, \psi, \rho\} \rightarrow \{\lambda^{-2}\hat{t}, \lambda^{-1}\hat{x}, \lambda^2\hat{V}, \lambda^2\hat{\psi}, \lambda^4\hat{\rho}\}$$

In Mocz +17, they found the following relations for the FDM case

$$\frac{M_c}{M} \sim \Xi^{1/3}$$

and

$$r_c \propto E^{-1/3}$$



Core-halo relation: SFDM case

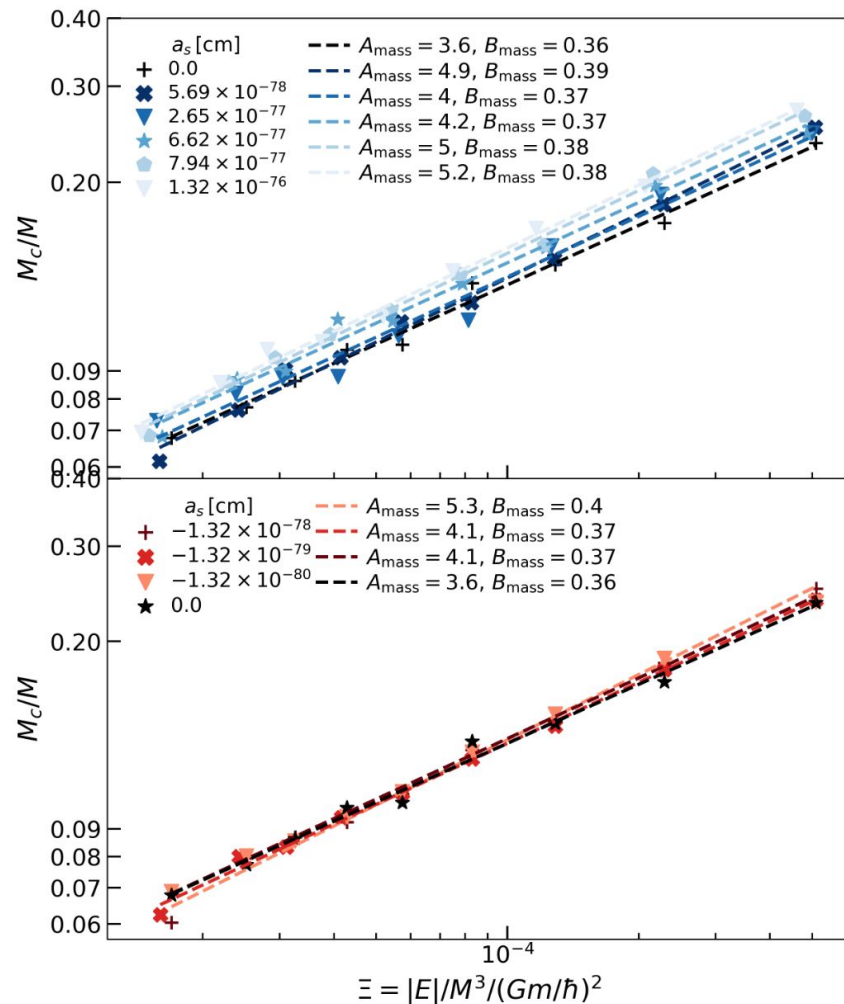
For FDM: $\frac{M_c}{M} \approx 3.64 \Xi^{0.362}$

$$\{t, x, \psi, \rho, a_s\} \rightarrow \{\lambda^{-2} \hat{t}, \lambda^{-1} \hat{x}, \lambda^2 \hat{\psi}, \lambda^4 \hat{\rho}, \lambda^{-2} \hat{a}_s\}$$

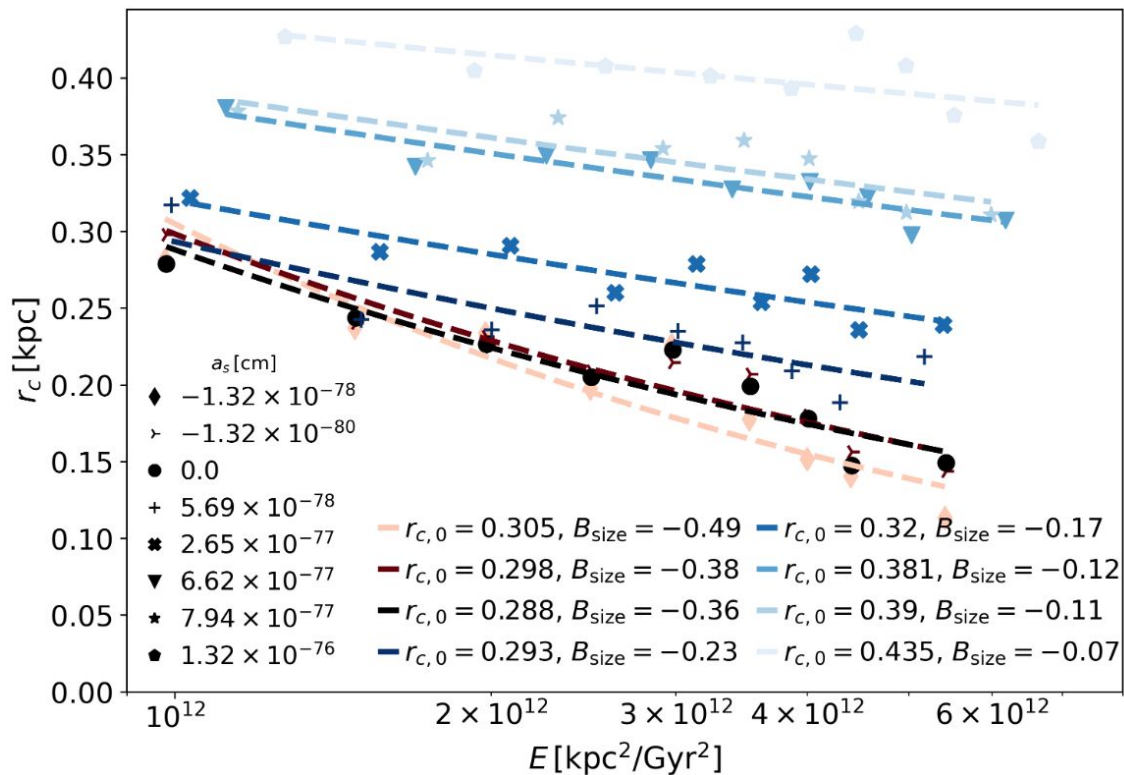
For SFDM, now we should consider:

$$\left. \begin{aligned} K &= \underbrace{\frac{\rho_0 \hbar^2}{2m^2} \int_V dV \nabla \sqrt{\psi \psi^*}}_{K_v} + \underbrace{\frac{\rho_0}{2} \int_V dV \psi \psi^* \mathbf{v}^2}_{K_\rho}, \\ W &= \frac{\rho_0}{2} \int_V dV \Phi \psi^* \psi, \\ U_{\text{SI}} &= \frac{2\pi \hbar^2 \rho_0^2 a_s}{m^3} \int_V dV (\psi^* \psi)^2. \end{aligned} \right\} E = K + W + U_{\text{SI}},$$

All a_s values show similar slopes!



Size-Energy relation: SFDM case



For the FDM case the relation is

$$r_c = 1.10 \times 10^4 E^{-0.381}$$

There is a clear dependency on a_s which contributes significantly in the internal pressure of the core

Time evolution

So far, all previous relations are shown at the final time of the simulation.

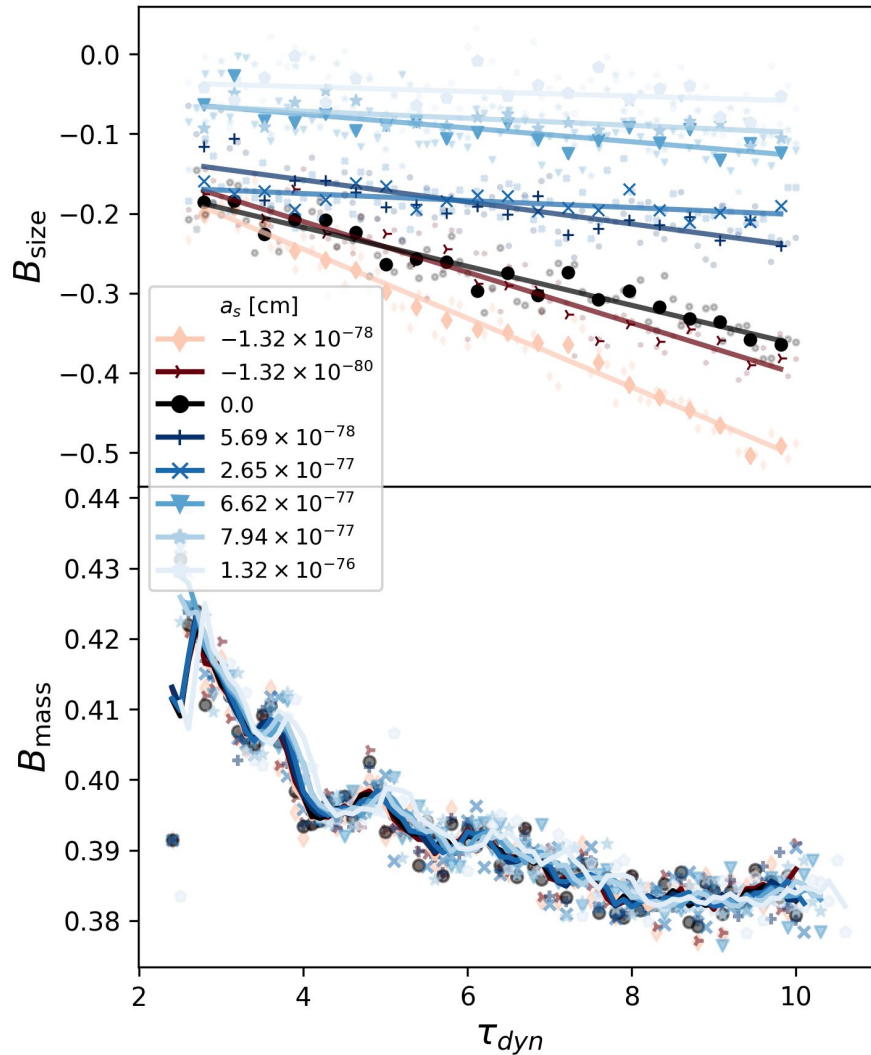
However, the slopes of the relations:

$$\frac{M_c}{M} = A_{\text{mass}} \Xi^{B_{\text{mass}}}$$

and

$$r_c = A_{\text{size}} E^{B_{\text{size}}}$$

also change over time.



Self-similar solutions in SF-ULDM

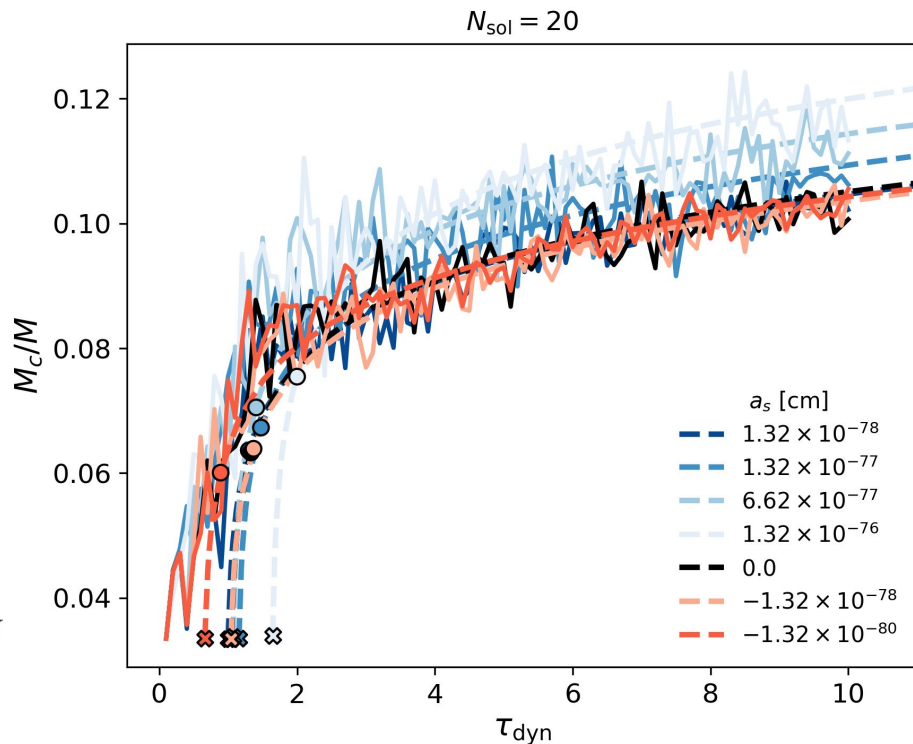
This can be rewritten for our case

$$\left(1 + \left(\frac{M_c}{M}\right)^3 \frac{1}{\epsilon^2}\right) \left(1 - \frac{Me}{M} - \frac{M_c}{M}\right)^{-5} \sim \frac{\tau - \tau_i}{\tau_{\text{dyn}}}$$

$$\epsilon^2 = \frac{|E|}{\gamma M^3 (Gm\hbar^2)} = \frac{\Xi}{\gamma}$$

First transition: $\left|\frac{E_c}{M_c}\right| = \left|\frac{E}{M}\right|$ with $M_c = \epsilon M$

Second transition: $E_c = E$ with $M_c = \epsilon^{2/3} M$



The repulsive cases promote the condensation of the core.

Summary

- ★ **The accretion of mass in the core is faster for repulsive interactions**



These interactions promote the condensation process even though the density reach lower values.

- ★ **The changes in the core-halo relation are less evident than the changes in the size-energy relation**



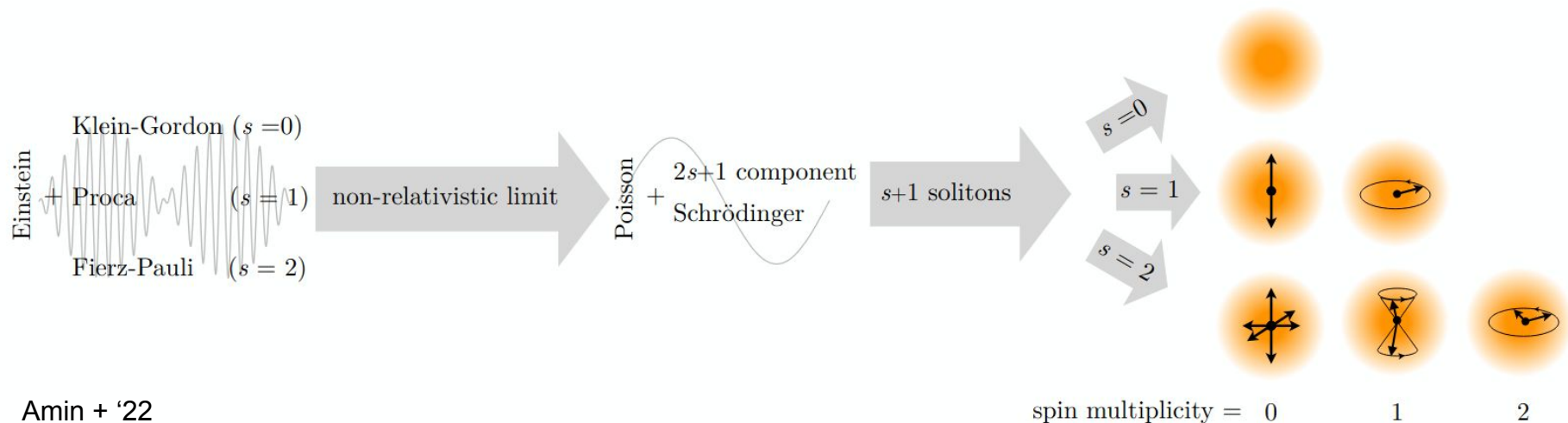
Mass accretion is more sensitive to the gravity force while the self-interactions play a crucial role in internal pressure balance.



*Spin- s
ULDM
model*

Phenomenology of spin s ULDM

ULDM can also admit higher spin values



Amin + '22

Evolution of spin-s ULDM

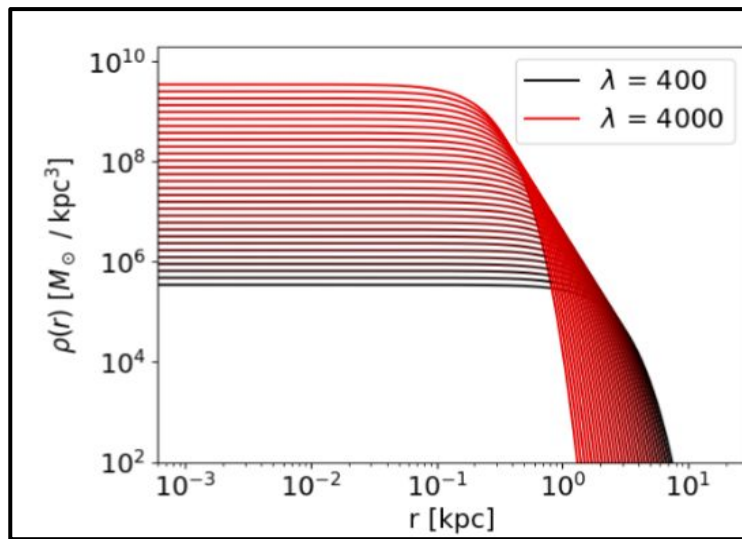
The evolution of the field is described by the Schrodinger-Poisson system

$$i\hbar \frac{\partial}{\partial t} \Psi = -\frac{\hbar^2}{2m_s} \nabla^2 \Psi + m_s \Phi \Psi,$$
$$\nabla^2 \Phi = 4\pi G \rho_0 (\text{Tr}[\Psi^\dagger \Psi] - 1),$$

Scalar
Vector
Tensor

For the **merger of solitons (ICs)** :

Solitons are the ground state solution of the time-independent SP system.



The ground state solution for higher spin

The wave function can be decomposed as

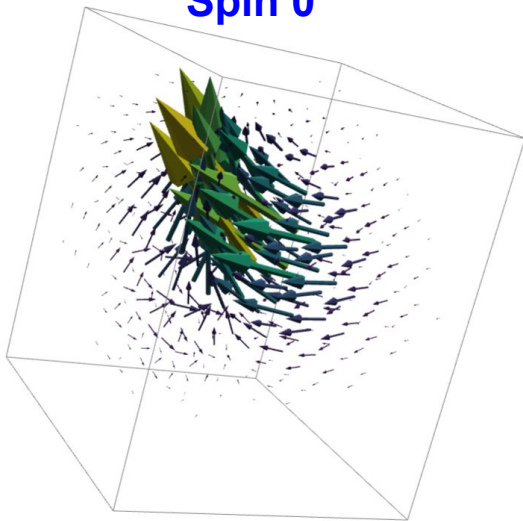
$$\Psi(t, \mathbf{x}) = \sum_p \psi_p(t, \mathbf{x}) \epsilon^{(p)}$$

$$\psi_p(t=0, \mathbf{x}) = \psi_{\text{sol}}(\mathbf{x}) c_p e^{-i\theta_p}$$

$$\sum_p c_p^2 = 1$$

$$e^{-i\theta_0}$$

Spin 0



$$\epsilon^{(\pm 1)} \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ \pm i \\ 0 \end{pmatrix}; \quad \epsilon^{(0)} \equiv \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Spin 1

$$\epsilon^{(\pm 2)} \equiv \frac{1}{2} \begin{pmatrix} 1 & \pm i & 0 \\ \pm i & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\epsilon^{(\pm 1)} \equiv \frac{1}{2} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & \pm i \\ 1 & \pm i & 0 \end{pmatrix},$$

$$\epsilon^{(0)} \equiv \frac{1}{\sqrt{6}} \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}.$$

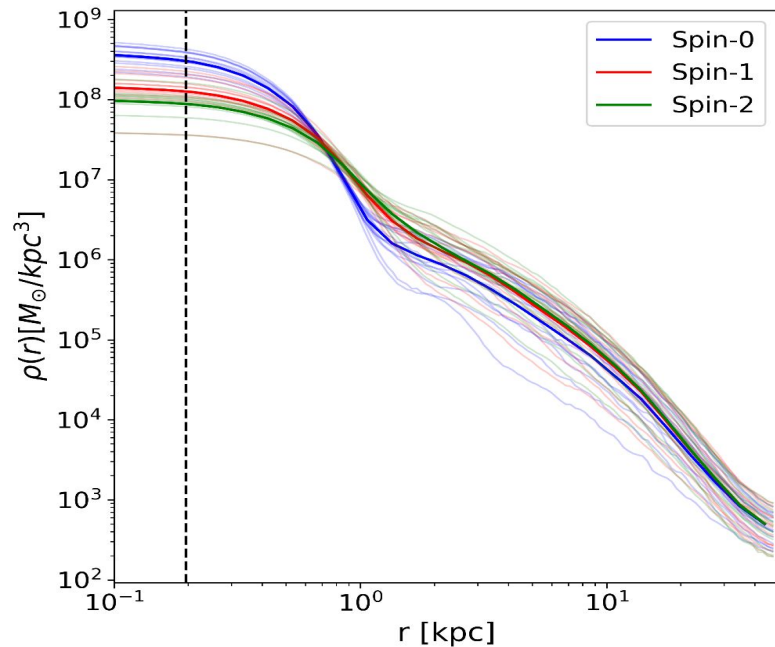
Spin 2

Comparison between models

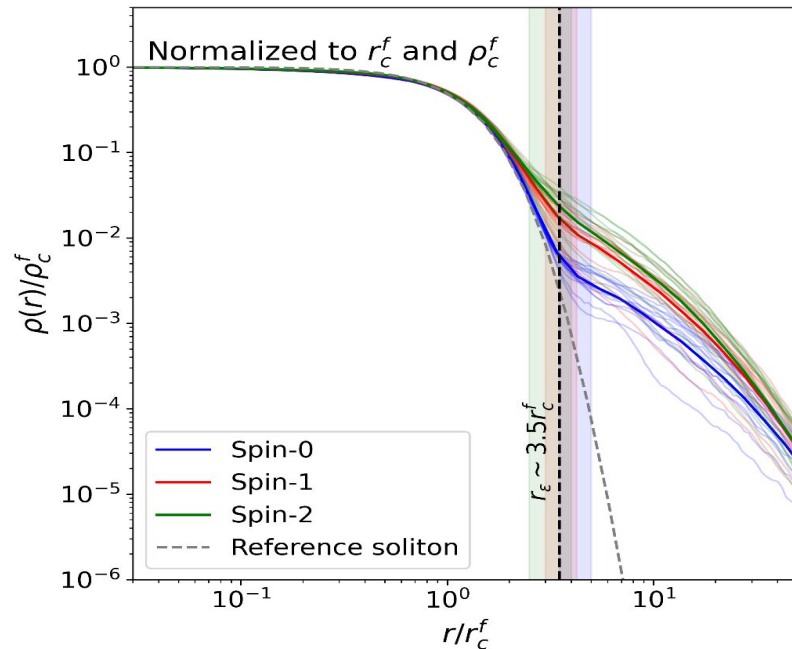


Spin-s ULDM halos: Density profiles

DM density profile as a function of the radius.



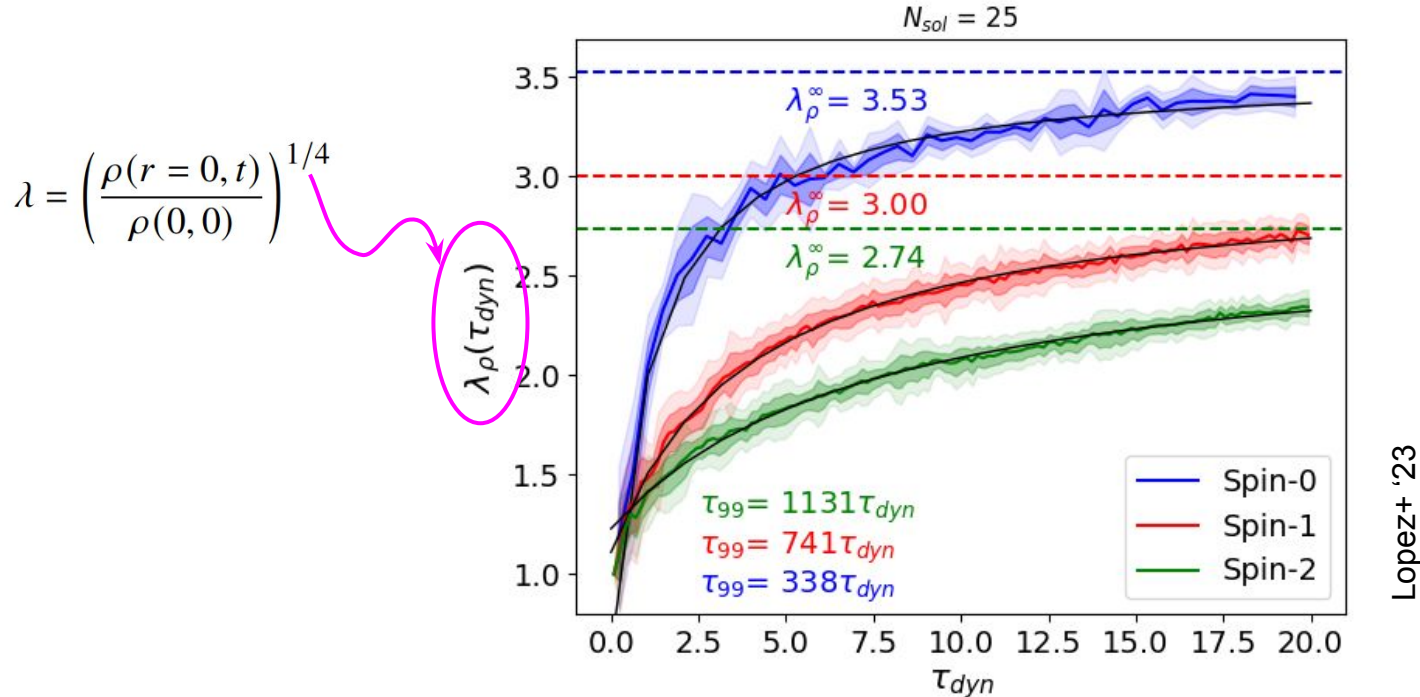
Normalized DM density profile.



Having more spin components reduces granular interference $\sim 1/(2s+1)^{1/2}$ and smooths the halo but more internal degrees of freedom, giving broader solitons

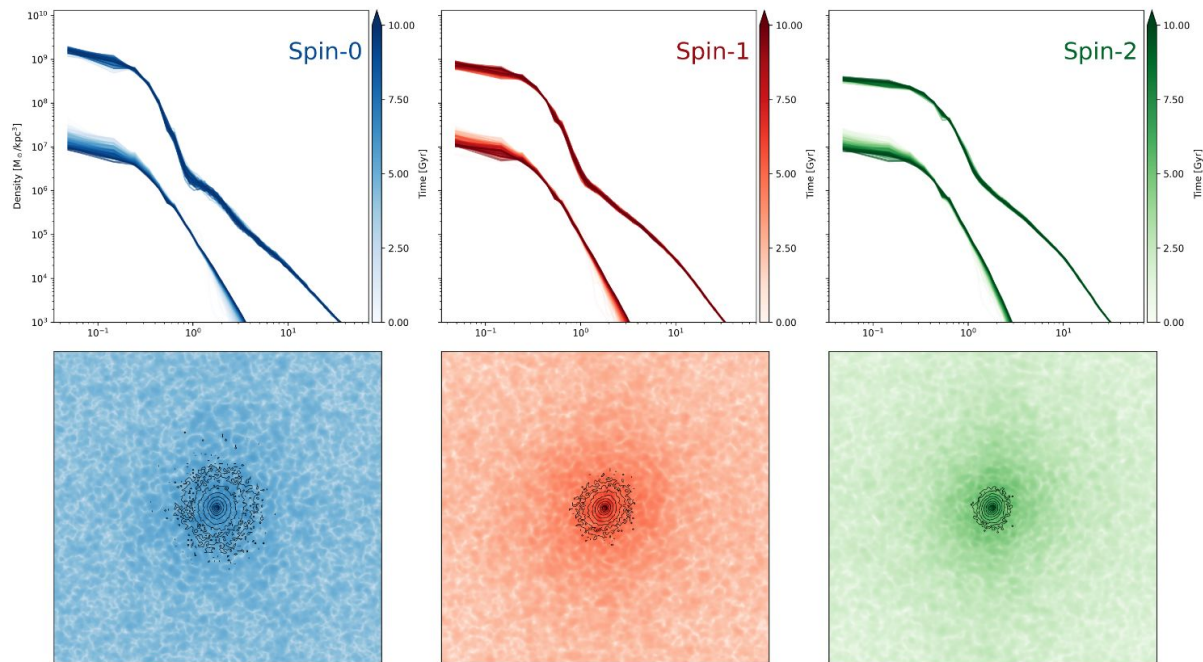
Density evolution

In this case, the evolution of the density shows an evident hierarchical behaviour.



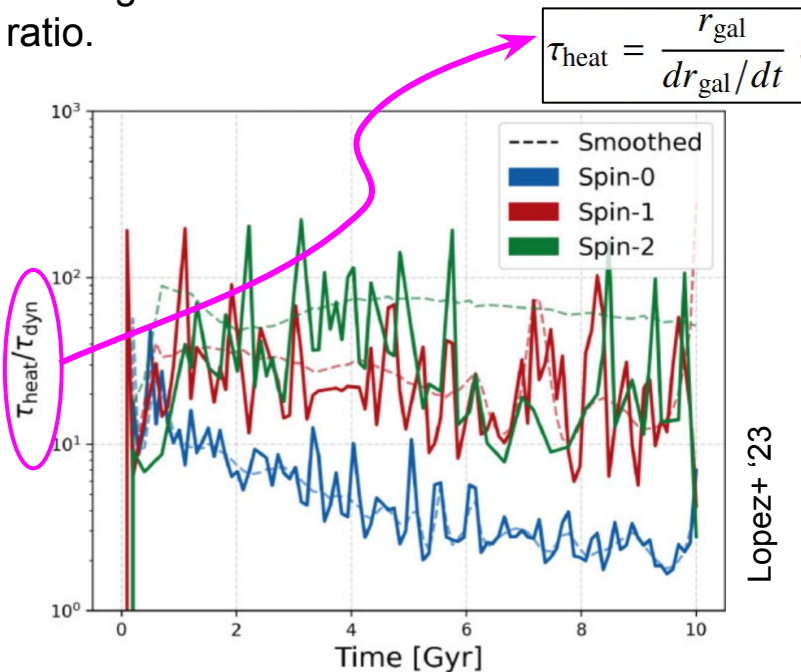
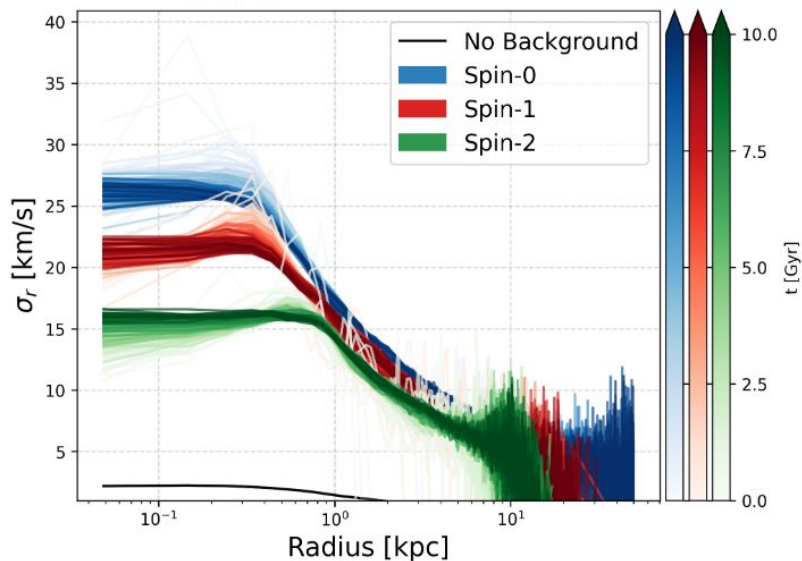
Dynamical heating

- ★ Granularities can heat up galactic systems modifying the velocity dispersion
- ★ Add N-body tracers (10^6) each representing a star and following a Hernquist profile
- ★ The system evolves all together following the gravitational potential



Velocity dispersion

- ★ DM-only CDM simulations historically predicted several massive subhalos that were too centrally dense to host the observed MW dwarfs $\longrightarrow$ stellar velocity dispersions larger than observed.
- ★ Spin 0 shows larger σ at smaller radii (perturbations more pronounced) in the core
- ★ A higher ratio indicates lower heating efficiency.
- ★ All models exceeded 1 indicating that the system reconfigures faster than it heats.
- ★ Halos with higher spin exhibit larger values of this ratio.



Summary

- ★ **Higher spin give rise to less dense core structures**



It could relax the problem in Veltmaat et al. (2020):
Spin-0 + baryons lead to cuspy profiles (Core-cusp problem)

- ★ **Higher spin leads to longer heating times up to an order of magnitude**



It could give an insight about the timing problem for dwarf galaxies

- ★ **Comparison with observations!**



*Mixed
dark matter*

Mixed Dark Matter

We consider two components of DM: CDM+FDM (Isocurvature).

The adiabatic component is given by $\mathcal{P}_{RR}(k) = A_s \left(\frac{k}{k_*} \right)^{n_s-1}$

The localized isocurvature matter power spectrum is parametrized by:

- Narrow band spectrum

$$\mathcal{P}_{SS}^{\text{NB}}(k) = A_s \beta_{\text{NB}} \delta(\ln k - \ln k_0).$$

- Broken power law spectrum

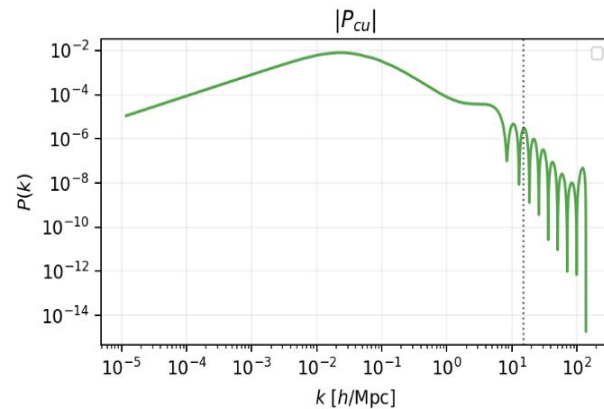
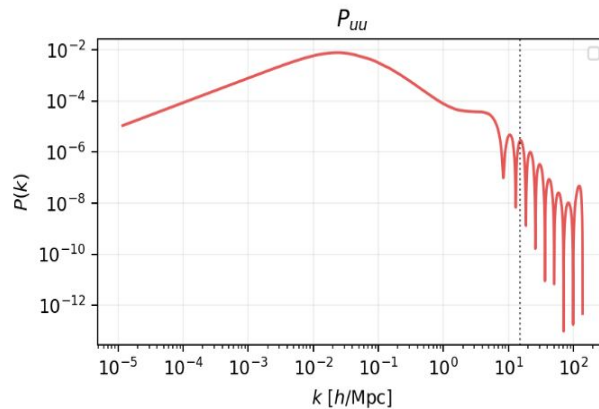
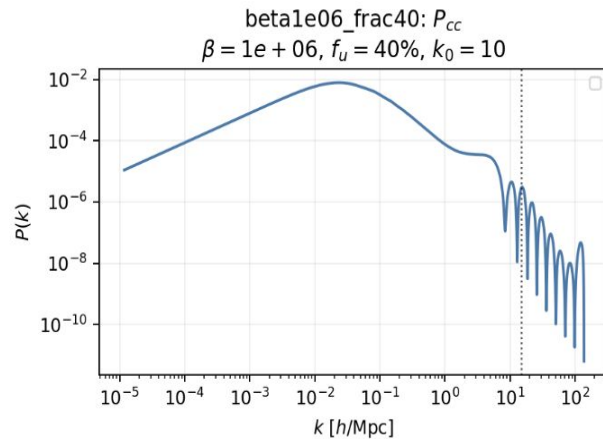
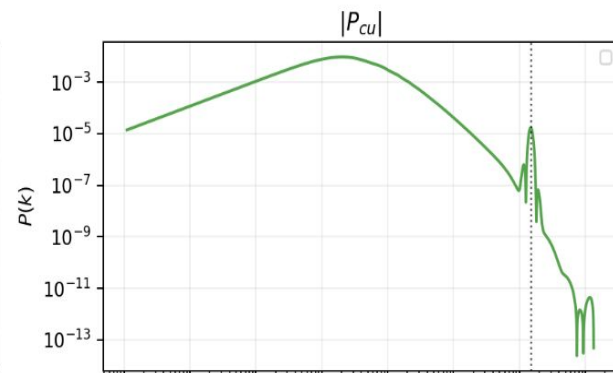
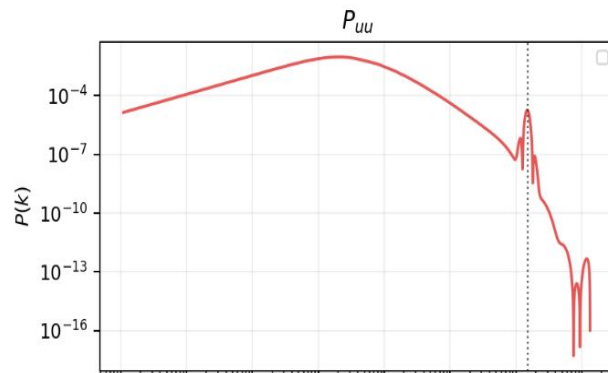
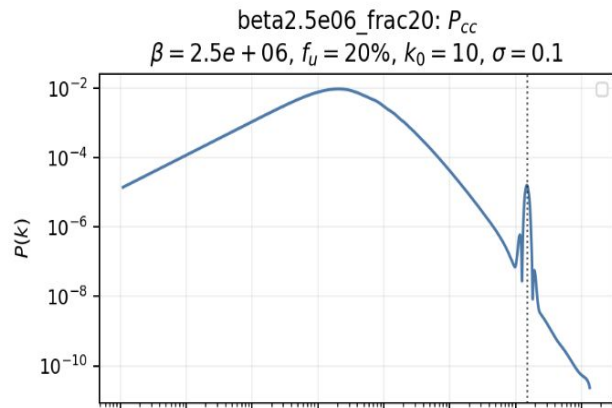
$$\mathcal{P}_{SS}^{\text{BPL}}(k) = A_s \beta_{\text{BPL}} \frac{1}{(k/k_0)^{-3} + (k/k_0)^p}.$$

CMB anisotropies, BAO,
Lya, CMB spectral distortions

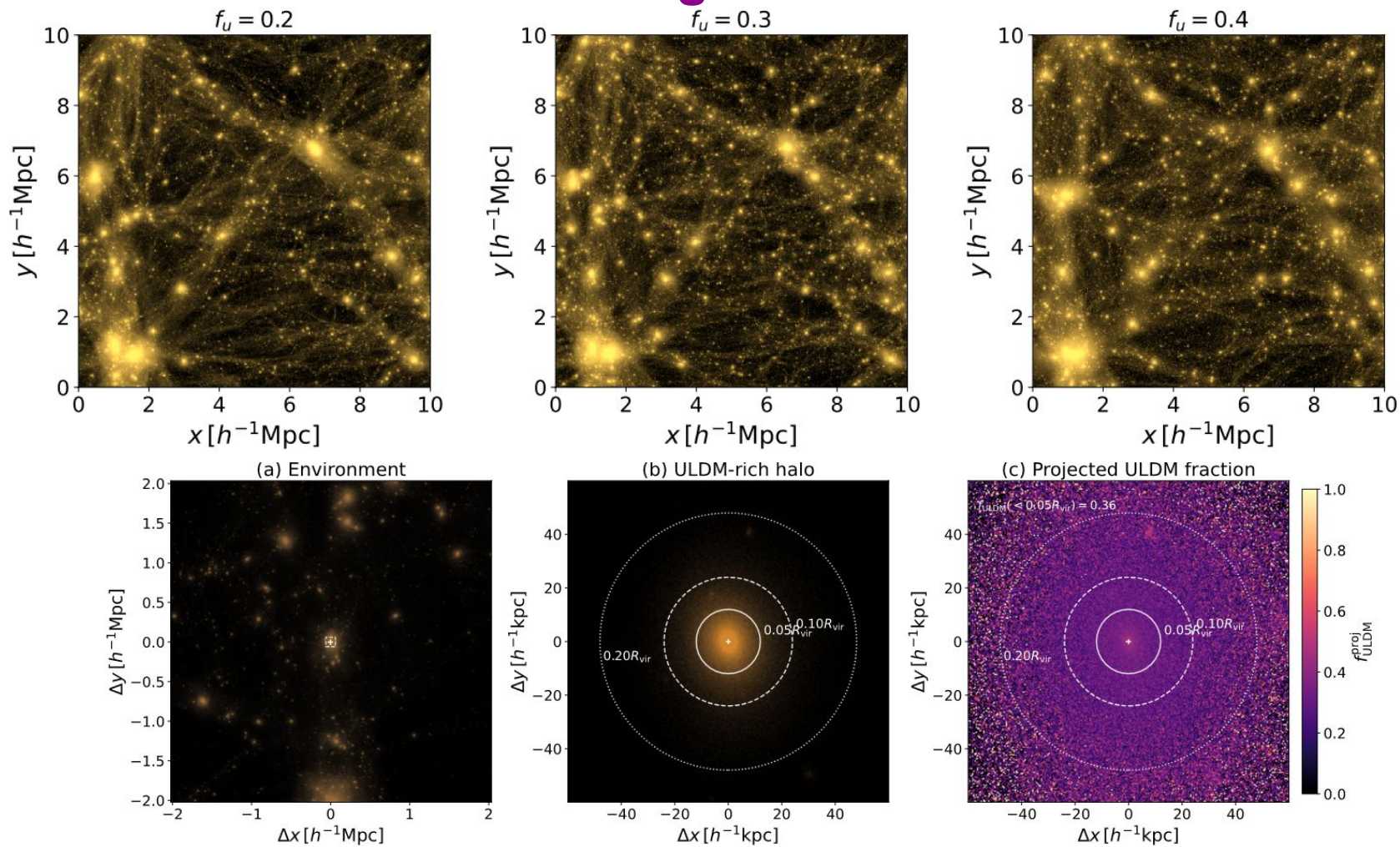
$$\beta_{\text{BPL}} \lesssim \mathcal{O}(10^3-10^6) r_m^{-2}, \quad 10 \text{ Mpc}^{-1} \lesssim k_0 \lesssim 100 \text{ Mpc}^{-1}$$

$$\beta_{\text{NB}} \lesssim \mathcal{O}(10^{11}-10^{12}) r_m^{-2}, \quad 10 \text{ Mpc}^{-1} \lesssim k_0 \lesssim 100 \text{ Mpc}^{-1}$$

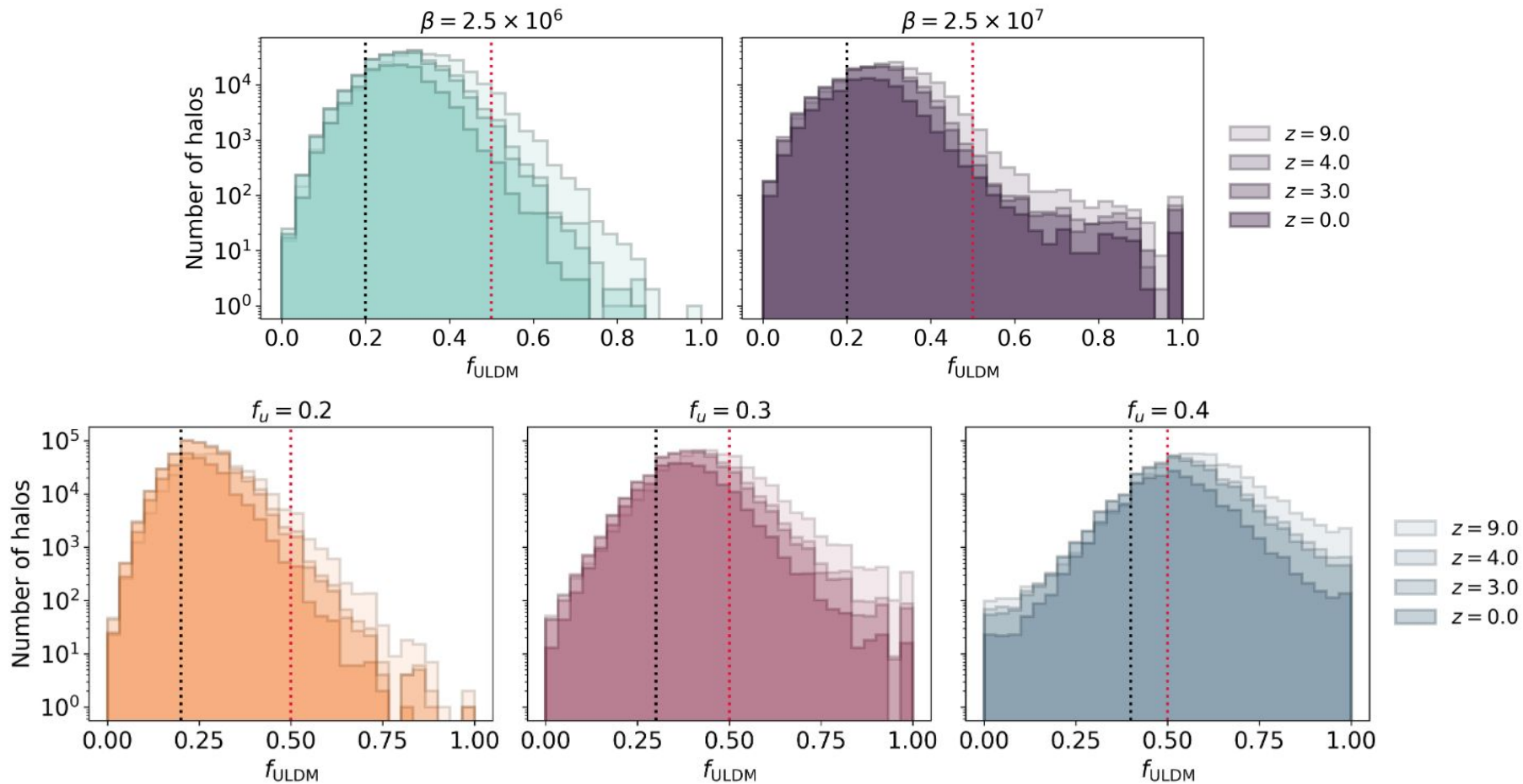
Localized P_{ss} spectrum



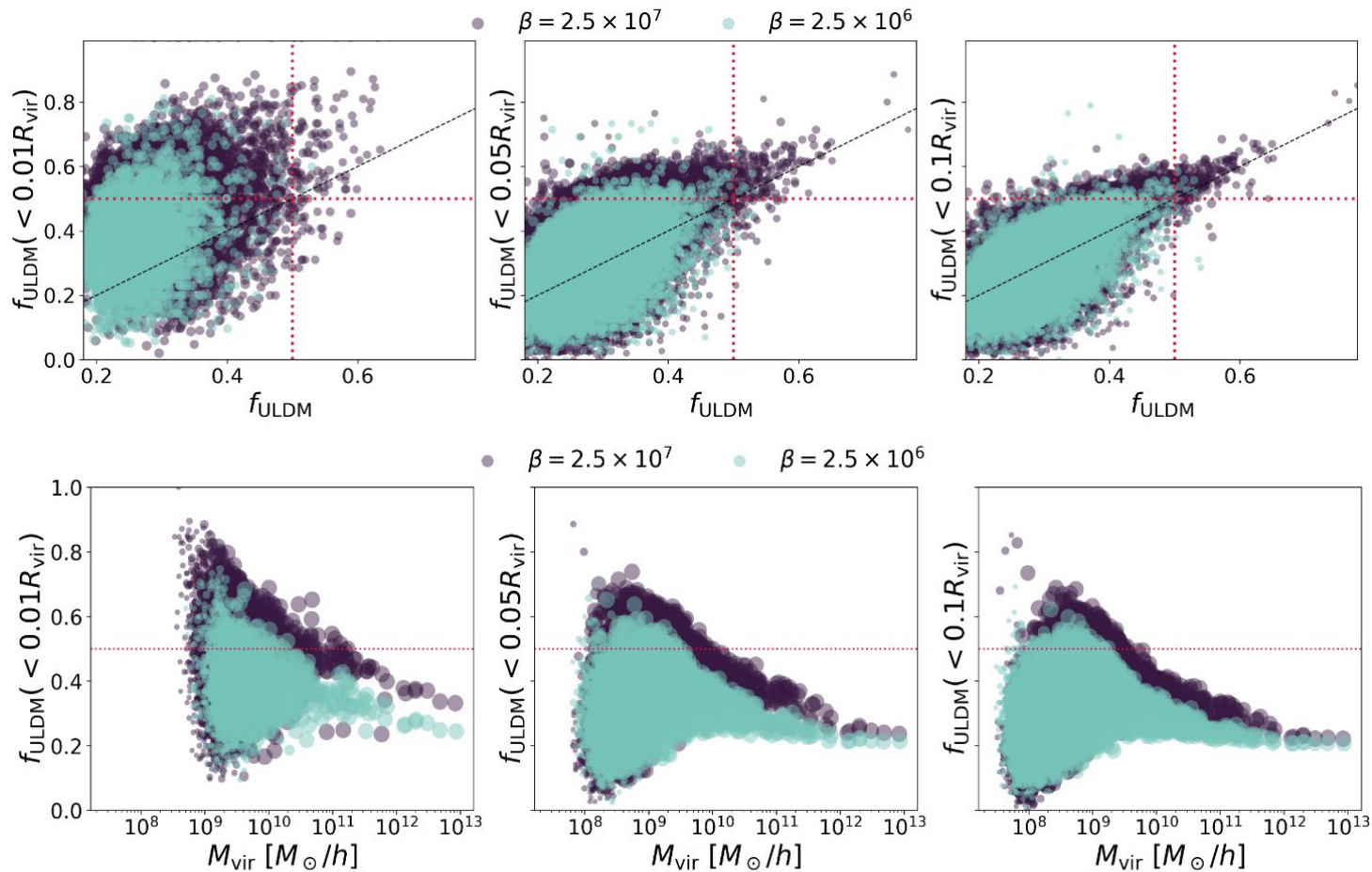
Cosmological evolution



Halo populations



Inner distribution of DM components



Summary

- ★ The ULDM-dominated region can extend beyond $0.01R_{\text{vir}}$
- ★ It contributes more than half of the central dark matter density,
- ★ This happens even when its cosmological fraction is below $O(10\%)$



This mechanism may produce either cusp-like or core-like central structures.

*Thank
you*

