On bulk reconstruction in Lorentzian AdS and its flat space limit

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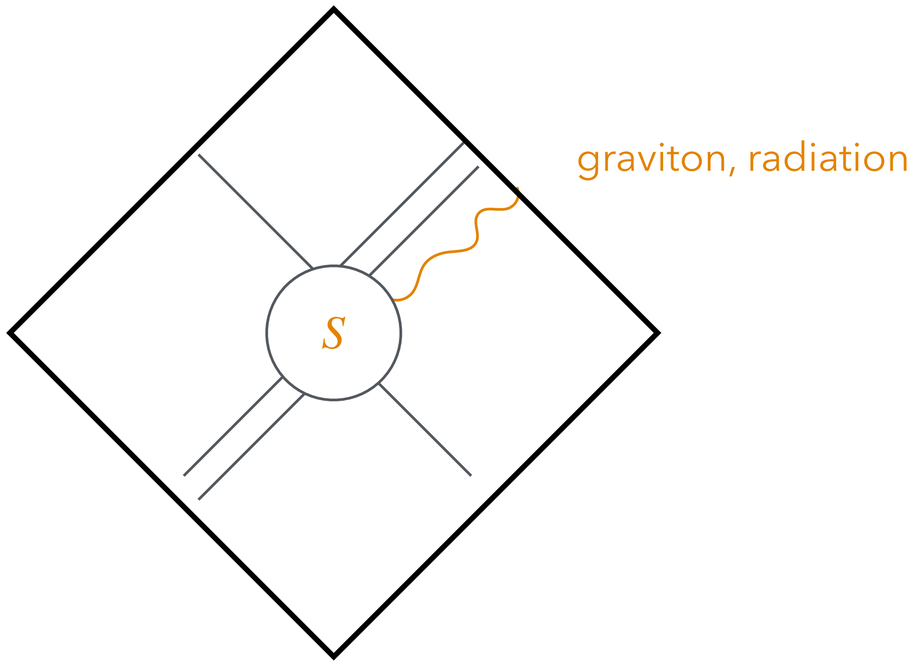
Based on 2512.13541, 2605.16641 w. Núria Navarro

ENS summer institute, Paris 2026

Motivation

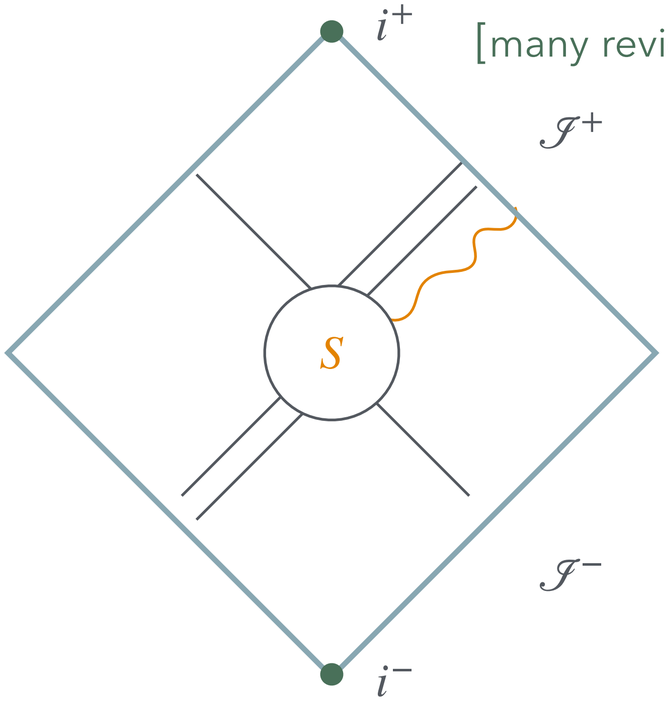
Gravity in asymptotically flat spacetimes



Motivation

Gravity in 4d asymptotically flat spacetimes, long wavelength sector

$\simeq$ 3d Carrollian/2d celestial CFT



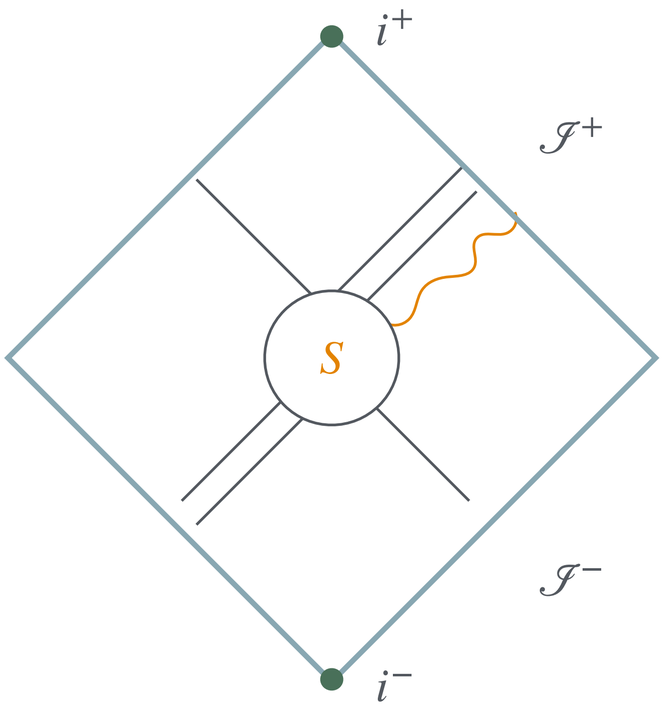
[many reviews '17-'26: Bagchi et. al.; Donnay; Ruzziconi; Strominger; A.R.; Pasterski; SAGEX,….]

[Andrea's talk]

Motivation

Gravity in asymptotically flat spacetimes

$\simeq$ Carrollian/celestial CFT



- Asymptotic symmetries ~ global symmetries [cf. AdS/CFT]

(d+1)-dimensional Poincare algebra $\simeq$ d-dimensional conformal Carroll algebra

[conformal symmetries of d-dimensional null surfaces]

[Duval, Gibbons, Horvathy '14]

+ infinite dimensional enhancements (BMS_{d+1})

Motivation

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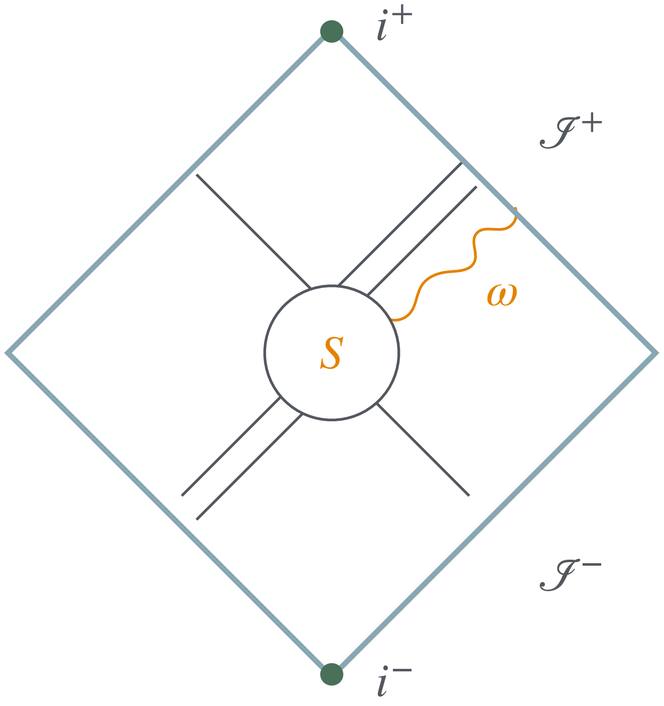


- Asymptotic symmetries ~ global symmetries [cf. AdS/CFT]
- Classical soft theorems ~ asymptotic symmetries

$$A_{n+1} \sim \sum_{k=0}^{\infty} \omega^{k-1} S^{(k)} A_n + \cdots$$

– $k = 0$ [Weinberg '60s] ~ supertranslation symmetries [BMS]

– $k = 1$ [Cachazo-Strominger '14] ~ superrotation symmetries [Barnich, Troessaert '10]

– $k > 1$ [Guevara et. al. '19-'21] ~ radial expansion of gravitational constraints [Freidel, Pranzetti, A.R. '22]

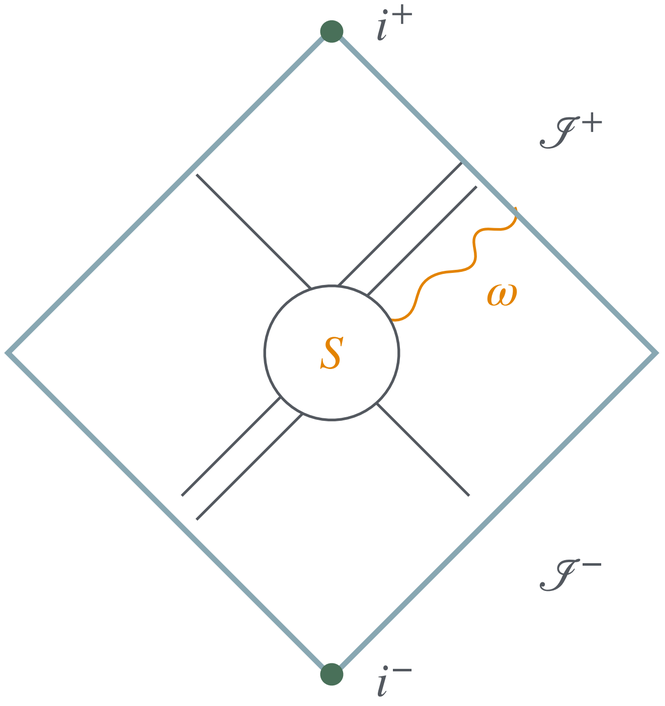
???

[Strominger, He, Kapec, Mitra '14; + Cachazo, Campiglia, Freidel, Guevara, Himwich, Laddha, Pasterski, Pate, Pranzetti, Puhm, A.R., Yuan….]

Motivation

Gravity in asymptotically flat spacetimes

$\simeq$ Carrollian/celestial CFT



- Asymptotic symmetries ~ global symmetries [cf. AdS/CFT]
- Soft theorems ~ CFT Ward identities

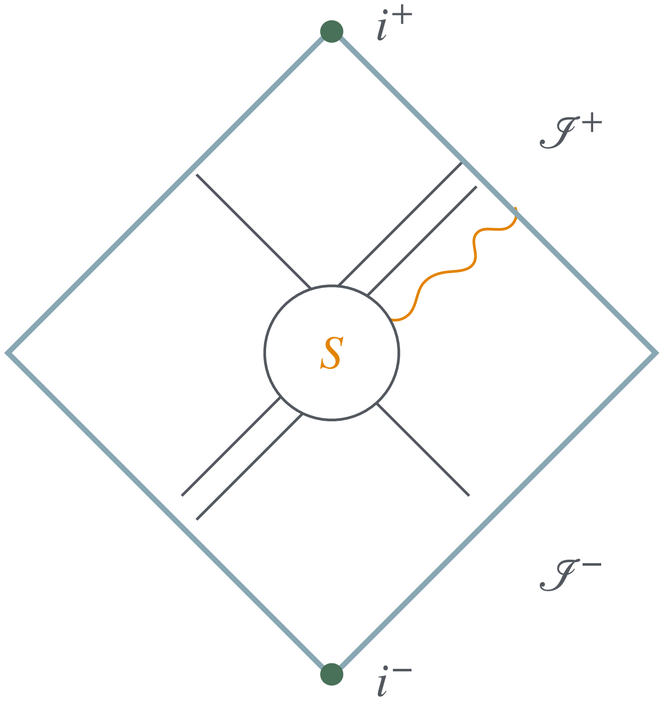
$$A_{n+1} \sim \sum_{k=0}^{\infty} \omega^{k-1} S^{(k)} A_n + \cdots \to \langle J_k(x) O(x_1) \cdots O(x_n) \rangle$$

- Low energy graviton modes ~ currents in 2d CFT
  - leading ~ weight-shifting operators
  - subleading ~ stress tensor
  - (sub)^{n}leading ~ infinite tower of higher spin symmetries

Motivation

Gravity in asymptotically flat spacetimes

$\simeq$ Carrollian/celestial CFT



Implications for dynamics (beyond self-dual sector)?

- radiation/infrared divergences
- energy correlators in gravity
- quantum aspects of scattering amplitudes
- holography

[Gabai, Sever '16; Kapec, Perry, A.R., Strominger; Akhouri, Choi;…….; Bekaert, Donnay, Herfray; Chicherin, Korchemsky, Sokatchev, Zhiboedov; Alday, Strominger '26]

Motivation

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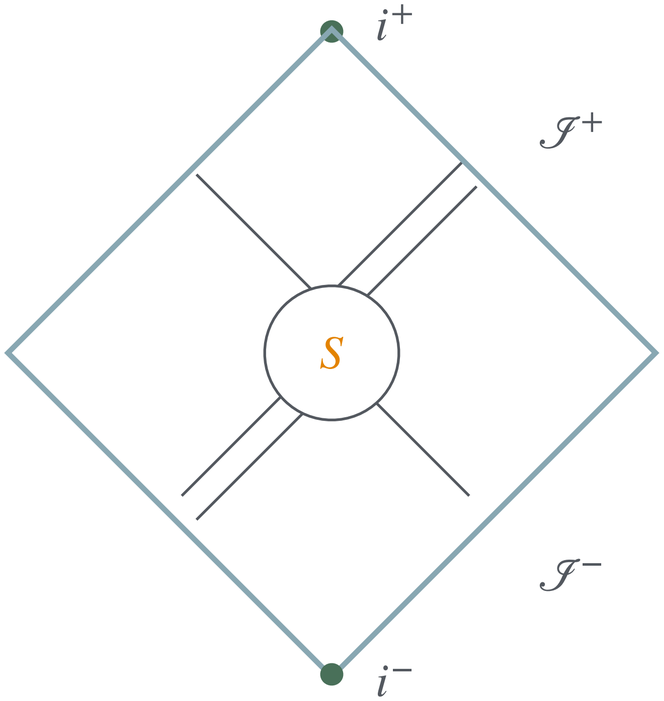
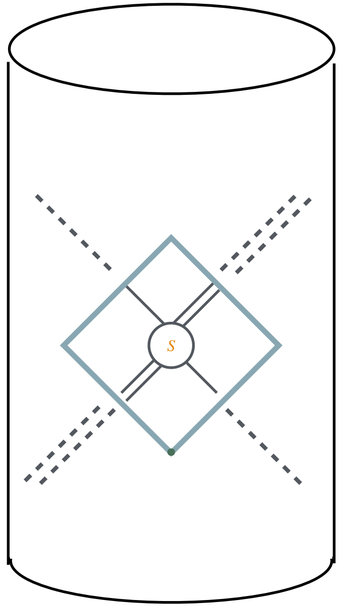


$$\text{"}\Lambda \to 0\text{"}$$

[Polchinski; Susskind; Giddings, Penedones; Komatsu et.al.; Hijano; Skenderis et. al., ….]

AdS/CFT



$$\langle \mathcal{O}_1 \cdots \mathcal{O}_n \rangle$$

Motivation

Gravity in asymptotically flat spacetimes

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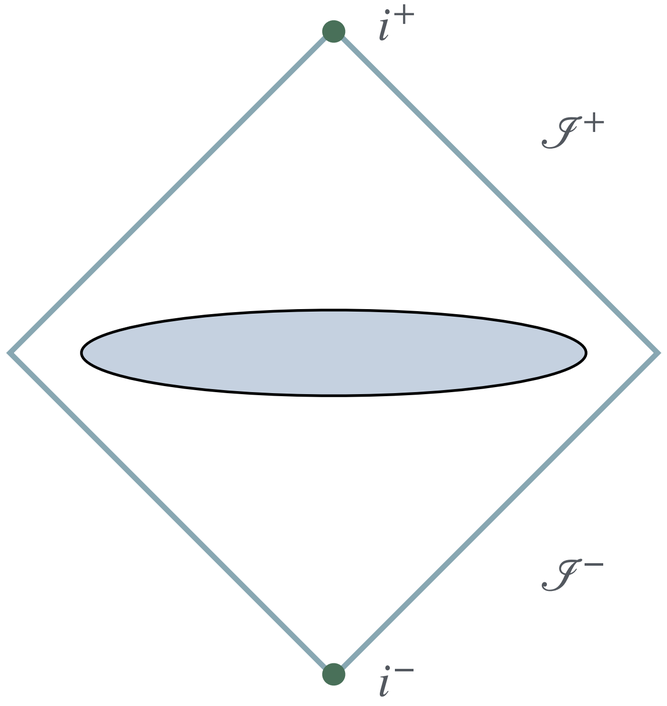

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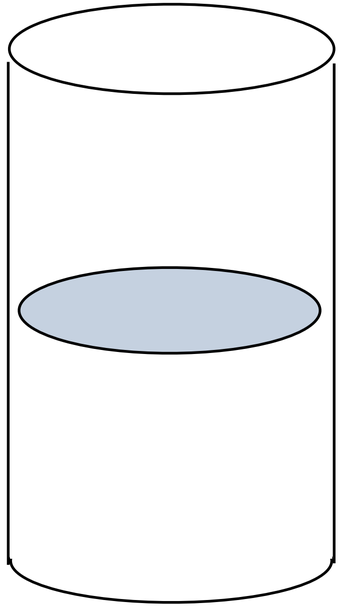

CFT_3

$$t \to ct, \; c \to 0$$

CCFT

flat space kinematics emerges in limit

[Duval, Gibbons '14; de Gioia, A.R. '22-'25; Bagchi et. al., ….]

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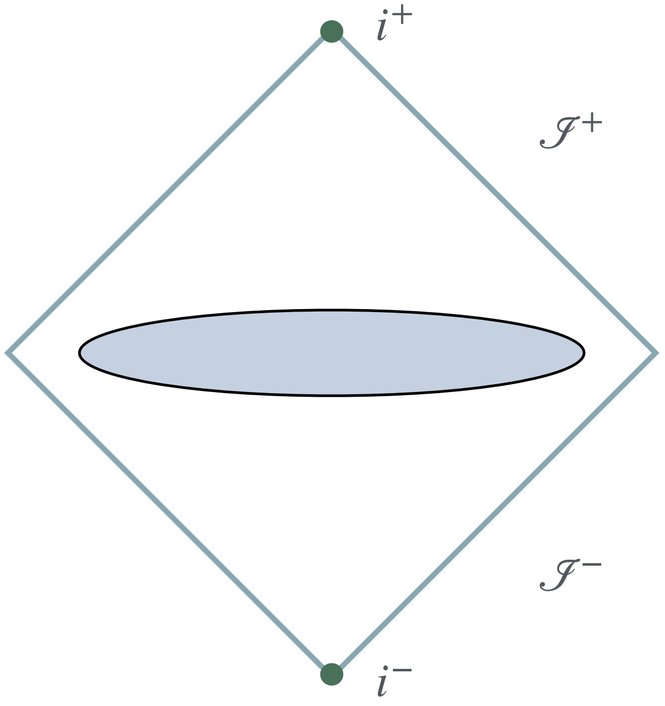

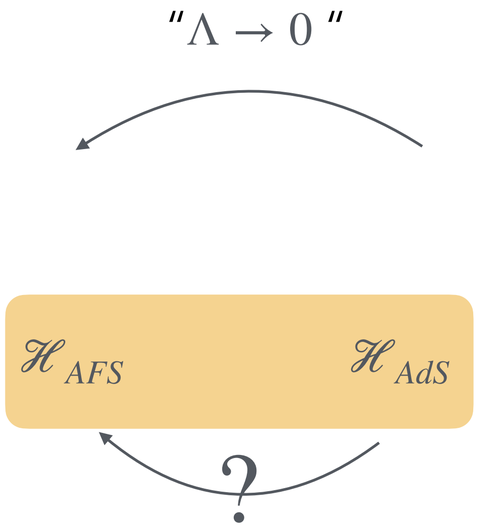

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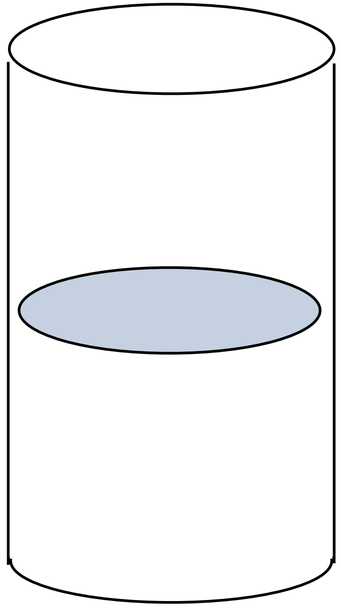

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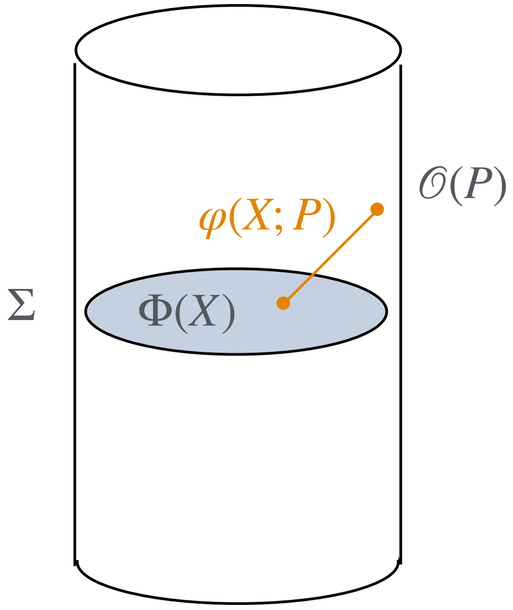
This talk in a nutshell



Free scalar in (3+1)d AdS

What are the "wavefunctions" $\varphi(X;P)$?

- Quantize the normalizable mode
- Orthogonality properties wrt. **bulk** inner product
- Different bases of in/out scattering states in flat space limit(s):
  - plane waves
  - ``Carrollian'' wavefunctions [Poincare covariant, $\varphi \sim G_{\Delta}^{(A).T.O.}(X;P)$]
  - ``celestial"/conformal primary wavefunctions

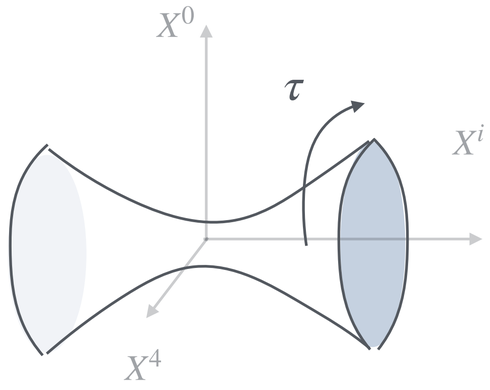
    [so(3,1) covariant, $\varphi \sim$ new wavefunctions preserving $so(3,1) \subset so(3,2)$]

[Witten; Gubser, Klebanov, Polyakov; Harlow, Stanford; Hamilton, Kabat, Lifschytz, Lowe; Fitzpatrick, Kaplan;….; Berenstein, Simon '25]

AdS basics

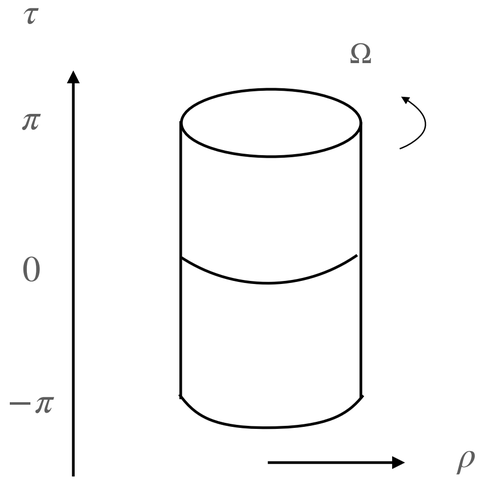
Lorentzian AdS_4 is realized as a hyperbolic surface in $\mathbb{R}^{2,3}$:

$$\eta_{AB}X^A X^B \equiv -(X^0)^2 + (\vec{X})^2 - (X^4)^2 = -\ell^2 .$$



Global coordinates:

$$X^0 = \ell \frac{\sin\tau}{\cos\rho}, \quad X^4 = \ell \frac{\cos\tau}{\cos\rho}, \quad X^i = \ell \tan\rho \Omega_i, \quad \Omega_i \cdot \Omega_i = 1$$



AdS_4 isometries are Lorentz transformations in $\mathbb{R}^{2,3}$:

$$M_{AB} = -i\left(X_A\partial_B - X_B\partial_A\right) \qquad \left\{ \quad D = i\frac{\partial}{\partial\tau}, \quad M_{ij}, \quad K_i \pm P_i \quad \right\}$$

AdS basics: free scalar

Representation of so(3,2): $(\Box_{AdS} - m^2)\Phi = 0, \quad m^2 = \Delta(\Delta - 3)$

Casimir of so(3,2)

Standard basis elements labelled by eigenvalues of Cartan subalgebra:

Casimir of so(3) ⊂ so(3,2)

$$D\psi^{\pm}_{n\ell m} = \pm\,\omega_n \psi^{\pm}_{n\ell m}, \quad L_0 \psi^{\pm}_{n\ell m} = m\psi^{\pm}_{n\ell m}, \qquad \Box_{S^2}\psi^{\pm}_{n\ell m} = -\,\ell(\ell+1)\psi^{\pm}_{n\ell m}$$

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Solutions: $\psi^{+}_{n\ell m}(\tau,\rho,\Omega) = e^{-i\omega_n \tau} f_{n\ell}(\rho) Y_{\ell J}(\Omega), \qquad \psi^{-}_{n\ell m} = (\psi^{+}_{n\ell m})^*$

Normalizability + regularity at origin $\Longrightarrow$

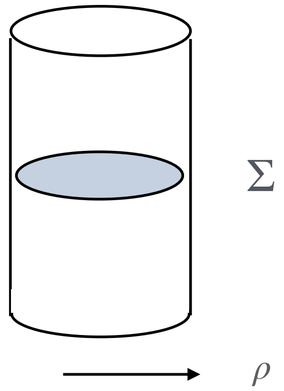
$$\begin{cases} f_{n\ell}(\rho) = \sin^{\ell}\rho \cos^{\Delta}\rho F_{n\ell}(\rho), \quad F_{n\ell}(\rho) = {}_2F_1\left(-n, \ell+\Delta+n; \frac{d}{2}+\ell; \sin^2\rho\right) \\ \omega_n = \Delta + 2n + \ell \end{cases}$$

AdS basics: bulk state space of free field

- Equip solution space with inner product $\Longrightarrow$ decomposition into positive/negative energy (irreps)

$\psi^{+}_{n\ell m}(\tau,\rho,\Omega)$ are orthogonal with respect to the KG inner product:

$$\langle \Phi_1, \Phi_2 \rangle \equiv i \int_{\Sigma} d\Sigma^{\mu} \left(\Phi_1^{\dagger} \partial_{\mu} \Phi_2 - \Phi_2 \partial_{\mu} \Phi_1^{\dagger} \right)$$



$$\langle \psi^{\pm}_{n\ell m}, \psi^{\pm}_{k\ell' m'} \rangle = \pm N^{d}_{n\ell} \delta_{nk} \delta_{\ell\ell'} \delta_{mm'} \qquad N^{d}_{n\ell} = \frac{\Gamma(n+1)\Gamma(\ell+\frac{3}{2})^2}{\Gamma(n+\ell+\frac{3}{2})} \frac{\Gamma(\Delta+n-\frac{1}{2})}{\Gamma(\Delta+\ell+n)} .$$

$$\Phi(\tau,\rho,\Omega) = \sum_{n\ell m} \left(\psi^{-}_{n\ell m}(\tau,\rho,\Omega) a^{\dagger}_{n\ell m} + \psi^{+}_{n\ell m}(\tau,\rho,\Omega) a_{n\ell m} \right)$$

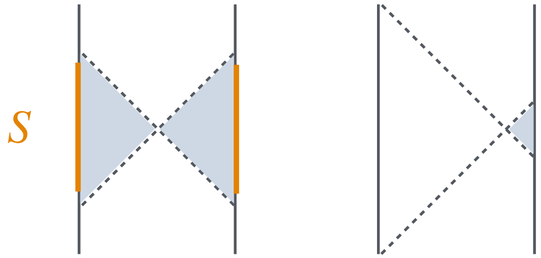
$\propto \mathcal{O}_{\Delta}(\tau,\Omega)$ CFT primary at $\rho = \frac{\pi}{2}$

HKLL proposal/bulk reconstruction

Bulk - boundary map: $\Phi(\tau,\rho,\Omega) \to \mathcal{O}_\Delta(\tau,\Omega)$ $\equiv \lim_{\epsilon\to 0}\lim_{\rho\to\frac{\pi}{2}-\epsilon} \epsilon^{-\Delta}\sum_{n\ell m}\left(\psi^-_{n\ell m}(\tau,\rho,\Omega)a_{n\ell m} + \psi^+_{n\ell m}(\tau,\rho,\Omega)a^\dagger_{n\ell m}\right)$

What about inverse?

Boundary orthogonality of wavefunctions $\Longrightarrow$ $a_{n\ell m} \propto \int_S d\tau d\Omega\, \widehat{\psi}^+_{n\ell m}(\tau,\Omega)\mathcal{O}_\Delta(\tau,\Omega)$



$$\Phi(\tau,\rho,\Omega) = \int_S d\tau' d\Omega' \underbrace{K(\tau,\rho,\Omega;\tau',\Omega')}_{\propto G_{3-\Delta}(X;P)\Theta((X-P)^2)}\mathcal{O}_\Delta(\tau',\Omega')$$

HKLL Kernel

HKLL proposal/bulk reconstruction

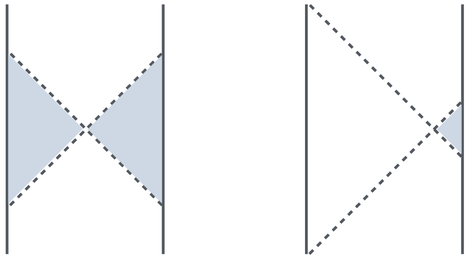
- Only non-vanishing in ``entanglement wedge''

$(\Box_{AdS} - m^2)G(x, x') = \delta(x - x')$

- K can also be obtained as boundary limit of bulk-to-bulk propagator subject to boundary condition on horizon

Boundary orthogonality of wavefunctions $\Longrightarrow$ $a_{n\ell m} \propto \int_S d\tau d\Omega\, \widehat{\psi}^{+}_{n\ell m}(\tau, \Omega)\mathcal{O}_\Delta(\tau, \Omega)$

entanglement wedge



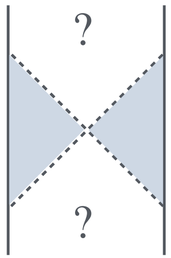
$$\Phi(\tau, \rho, \Omega) = \int_S d\tau' d\Omega' K(\underbrace{\tau, \rho, \Omega; \tau', \Omega'})\mathcal{O}_\Delta(\tau', \Omega')$$

$$\propto G_{3-\Delta}(X; P)\Theta((X - P)^2)$$ HKLL Kernel

[Hamilton, Kabat, Lifschytz, Lowe; Almheiri, Dong, Harlow, Hayden]

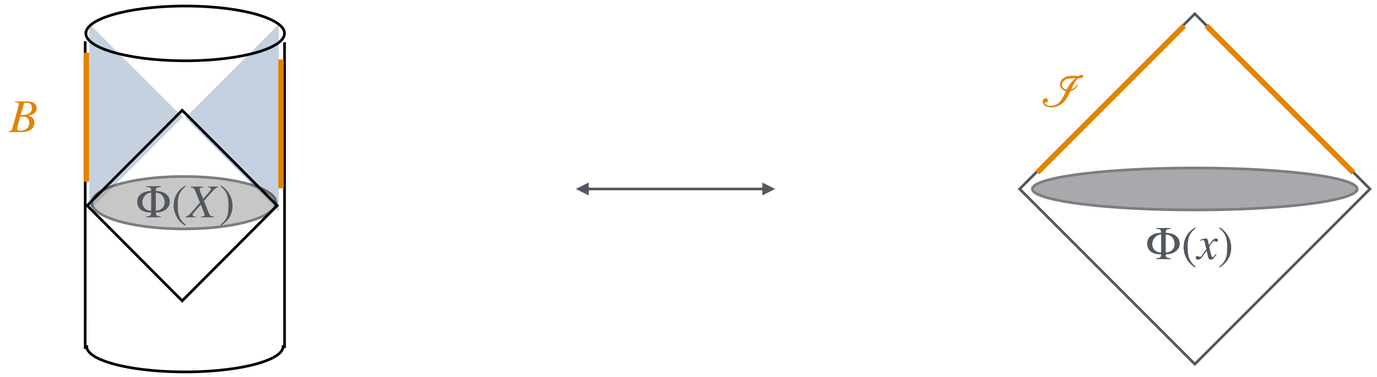
Towards a generalization of HKLL

- What about beyond the entanglement wedge?



- Is there a basis of positive/negative energy wavefunctions labelled by points on boundary?

- Direct contact to flat space reconstruction:



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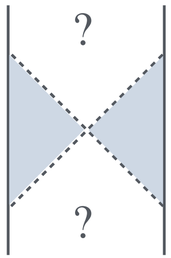
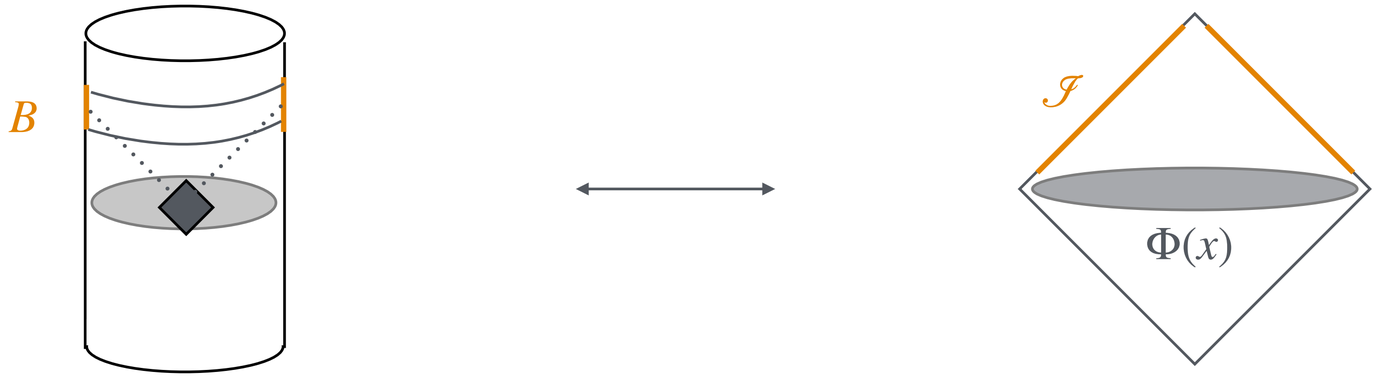


- Is there a basis of positive/negative energy wavefunctions labelled by points on boundary?

- Direct contact to flat space reconstruction: bulk flat space limit = contraction of so(3,2) to 4d Poincare



Wavefunctions from bulk-boundary propagators

so(3,2) covariance $\Longrightarrow$ wavefunctions constructed from AdS bulk-to-boundary propagators

$$G_{\Delta}^{\text{T.O.}}(X;P) = \frac{1}{(-X\cdot P + i\epsilon)^{\Delta}} \quad \xleftrightarrow{\text{shadow}} \quad G_{3-\Delta}^{\text{T.O.}}(X;P) = \frac{1}{(-X\cdot P + i\epsilon)^{3-\Delta}}$$

- Unlike HKLL kernel, both contain normalizable and non-normalizable modes

Guess: $$\Phi(\tau,\rho,\Omega) = \int_B d\tau_p d\Omega_p \left(\alpha G_{\Delta}(\tau,\rho,\Omega;\tau_p,\Omega_p)\widetilde{\mathcal{O}}_{3-\Delta}(\tau_p,\Omega_p) + \beta G_{3-\Delta}(\tau,\rho,\Omega;\tau_p,\Omega_p)\mathcal{O}_{\Delta}(\tau_p,\Omega_p)\right)$$

and choose α, β so that the non-normalizable mode cancels.

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and choose α, β so that the non-normalizable mode cancels.

which time-ordering?

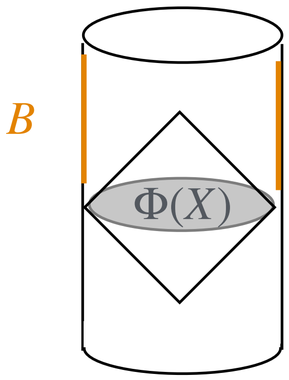
Lorentzian shadow?

Hermiticity?

New bulk reconstruction formulae

$$\Phi(\tau,\rho,\Omega) = \int_B d\tau_p d\Omega_p \left(\alpha G_\Delta^{\text{A.T.O}}(\tau,\rho,\Omega;\tau_p,\Omega_p)\widetilde{\mathcal{O}}^{+}_{3-\Delta}(\tau_p,\Omega_p) + \beta G_{3-\Delta}^{\text{T.O.}}(\tau,\rho,\Omega;\tau_p,\Omega_p)\mathcal{O}_\Delta(\tau_p,\Omega_p) \right)$$

defined with appropriate $i\epsilon$



Features:

- By construction solves KG eq. and is well-defined beyond entanglement wedge
- B such that components are positive/negative energy: mode expansions and bulk inner products
- Shadow $\Longrightarrow$ boundary non-locality: equivalent representations for $\Delta \to 3-\Delta$ in either term

since modes of $\widetilde{\mathcal{O}}_{3-\Delta}$ are just linear combinations of modes of $\mathcal{O}_\Delta$

Mode expansion of bulk-boundary propagator

- Decompose $G_\Delta^{\text{T.O.}}(X;P) = \dfrac{1}{(-X\cdot P + i\epsilon)^\Delta} = \dfrac{(\cos\rho)^\Delta}{\Big(\cos(\tau-\tau_p) - \sin\rho(\Omega\cdot\Omega_p) + i\epsilon\Big)^\Delta}$ into spherical harmonics:

$$G_\Delta^{\text{T.O.}}(X;P) = \underbrace{G_\Delta^+(X;P)}\Theta(\tau-\tau_p) + G_\Delta^-(X;P)\Theta(\tau_p-\tau)$$

Superposition of positive energy wavefunctions

$$G_\Delta^+(X;P) = \sum_{\ell=0}^{\infty}\frac{2\ell+1}{2}c_\ell(\tau-\tau_p,\rho)P_\ell(\Omega\cdot\Omega_p),$$

$$c_\ell(\tau-\tau_p,\rho) = \sum_{k=0}^{\infty}\sqrt{\pi}(-1)^k\frac{2^\Delta\Gamma(\Delta+\ell+k)}{k!\Gamma(\Delta)\Gamma(\frac{3}{2}+\ell)}(\cos\rho)^\Delta(\sin\rho)^\ell e^{-i(\tau-\tau_p)(\Delta+\ell+2k)}{}_2F_1\left(-k,\Delta+\ell+k;\frac{3}{2}+\ell;\sin^2\rho\right)$$

Mode expansion of bulk-boundary propagator

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Superposition of positive energy wavefunctions

- Unlike $\Psi_{n\ell m}(X)$, $G_\Delta(X;P)$ don't diagonalize D (for fixed, arbitrary P)

- For $\tau > \tau_p$, $G_\Delta^{\text{T.O.}} \subset$ positive energy subspace, while $G_\Delta^{\text{A.T.O.}} \subset$ negative energy subspace of solutions to eom

Bulk inner products?

Bulk inner product

- Follows from mode decomposition and the KG inner product of the standard wavefunctions:

$$\langle G_\Delta^\pm(X;P_1), G_\Delta^\pm(X;P_2)\rangle_{\tau=\tau_0} = \pm\, 2^{\Delta+1}\pi^{3/2}\frac{\Gamma(\Delta-\frac{1}{2})}{\Gamma(\Delta)}\underbrace{G_\Delta^\pm(P_1,P_2)}_{\text{CFT 2-point function}} \quad \Longrightarrow$$

$$\langle G_\Delta^{\text{T.O.}}(X;P_1), G_\Delta^{\text{T.O.}}(X;P_2)\rangle_{\tau=\tau_0} = 2^{\Delta+1}\pi^{3/2}\frac{\Gamma(\Delta-\frac{1}{2})}{\Gamma(\Delta)} \times \left(G_\Delta^+(P_1,P_2)\Theta(\tau_0-\tau_1)\Theta(\tau_0-\tau_2) - G_\Delta^-(P_1,P_2)\Theta(\tau_1-\tau_0)\Theta(\tau_2-\tau_0)\right)$$

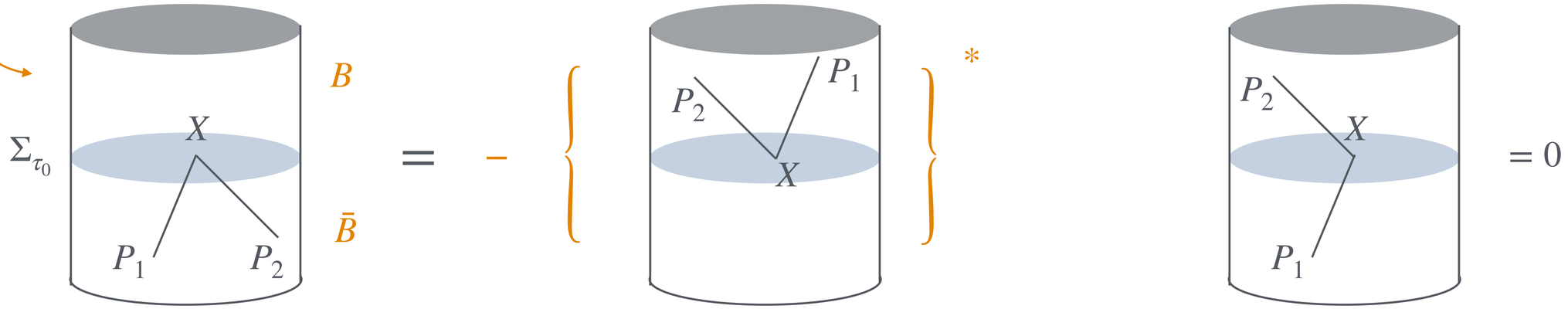
Time-ordered or anti-time-ordered CFT 2-pt function depending on sign of τ_{12}

Bulk inner product

- Follows from mode decomposition and the KG inner product of the standard wavefunctions:

$$\langle G_\Delta^\pm(X;P_1), G_\Delta^\pm(X;P_2)\rangle_{\tau=\tau_0} = \pm\, 2^{\Delta+1}\pi^{3/2}\frac{\Gamma(\Delta-\frac{1}{2})}{\Gamma(\Delta)}\underbrace{G_\Delta^\pm(P_1,P_2)}_{\text{CFT 2-point function}} \quad \Longrightarrow$$

$$\langle G_\Delta^{\text{T.O.}}(X;P_1), G_\Delta^{\text{T.O.}}(X;P_2)\rangle_{\tau=\tau_0} = 2^{\Delta+1}\pi^{3/2}\frac{\Gamma(\Delta-\frac{1}{2})}{\Gamma(\Delta)} \times \left(G_\Delta^+(P_1,P_2)\Theta(\tau_0-\tau_1)\Theta(\tau_0-\tau_2) - G_\Delta^-(P_1,P_2)\Theta(\tau_1-\tau_0)\Theta(\tau_2-\tau_0)\right)$$



Shadows

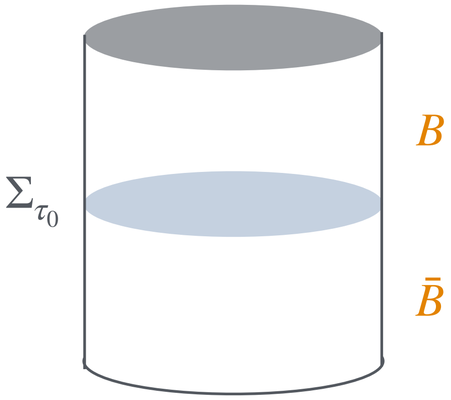
- Usually defined in Euclidean CFT: maps primary of dimension Δ to primary of $d-\Delta$

- Bulk-to-boundary propagators transform as:

$$X^2 = -\ell^2$$

$$\int_B \frac{d^3P'}{(-P\cdot P' - i\epsilon)^{3-\Delta}} G_\Delta^{\text{T.O.}}(X;P') = e^{i\pi\left(\Delta - \frac{3}{2}\right)} \pi^{3/2} 2^{\Delta+1} \cos\pi\Delta \frac{\Gamma(\Delta - \frac{3}{2})}{\Gamma(\Delta)} G_{3-\Delta}^{\text{A.T.O.}}(X;P), \quad P \in \bar{B}$$

Shadow kernel

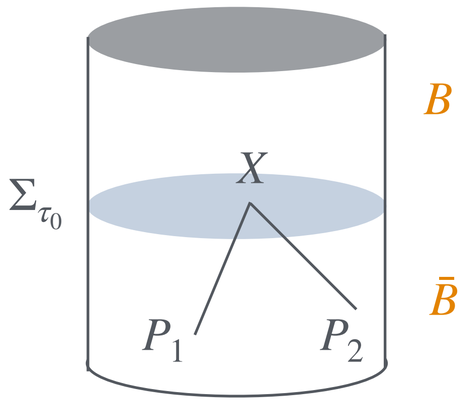


$$\int_B d^3P' \frac{1}{(-P\cdot P' + i\epsilon)^{3-\Delta}} G_\Delta^{\text{T.O.}}(X;P') = 0, \quad P \in \bar{B}$$

- Boundary integration region defined with respect to Σ: points in B mapped to $\bar{B}$ and vice versa

Bulk reconstruction, revisited

- Similarly $G^{\text{T.O.}}_{3-\Delta}(X,P)$ and $G^{\text{A.T.O.}}_{3-\Delta}(X,P)$ belong to the positive and negative energy subspaces provided that $P \in \bar{B}$ and vice-versa for $P \in B$



- We obtain a bulk reconstruction formula:

$$\Phi(\tau,\rho,\Omega) = \sum_{n\ell m}\left(\psi^{-}_{n\ell m}(\tau,\rho,\Omega)a^{\dagger}_{n\ell m} + \psi^{+}_{n\ell m}(\tau,\rho,\Omega)a_{n\ell m}\right) \equiv \Phi^{-}(\tau,\rho,\Omega) + \Phi^{+}(\tau,\rho,\Omega)$$

$$= \alpha\int_{\bar{B}} d\tau_p d\Omega_p G^{\text{A.T.O.}}_{3-\Delta}(\tau,\rho,\Omega;\tau_p,\Omega_p)\mathcal{O}_{\Delta}(\tau_p,\Omega_p) + \alpha^*\int_{\bar{B}} d\tau_p d\Omega_p G^{\text{T.O.}}_{3-\Delta}(\tau,\rho,\Omega;\tau_p,\Omega_p)\mathcal{O}_{\Delta}(\tau_p,\Omega_p)$$

$\Delta\tau = \pi$

Bulk reconstruction, revisited

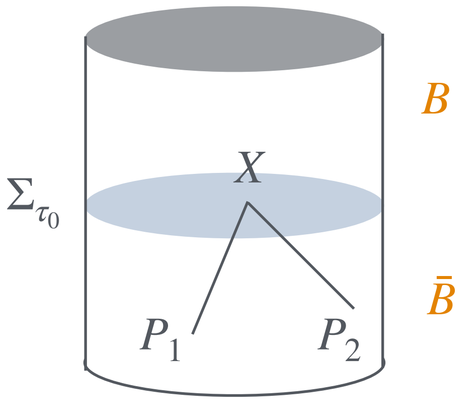
$$\Phi(\tau,\rho,\Omega) = \alpha \int_{\bar{B}} d\tau_p d\Omega_p G^{\text{A.T.O.}}_{3-\Delta}(\tau,\rho,\Omega;\tau_p,\Omega_p)\mathcal{O}_\Delta(\tau_p,\Omega_p) + \alpha^* \int_{\bar{B}} d\tau_p d\Omega_p G^{\text{T.O.}}_{3-\Delta}(\tau,\rho,\Omega;\tau_p,\Omega_p)\mathcal{O}_\Delta(\tau_p,\Omega_p)$$

- Naively, the near-boundary limit gives the shadow operator with non-normalizable fall-off

- In fact, consistency with the extrapolate dictionary follows from:

$$\widetilde{\mathcal{O}}^{-}_{3-\Delta}(P) \sim -\epsilon^{2\Delta-3} e^{i\pi(3-2\Delta)} \frac{2^{3-2\Delta}\Gamma(\Delta)\Gamma(\frac{3}{2}-\Delta)}{\Gamma(3-\Delta)\Gamma(\Delta-\frac{3}{2})} \sum_{n,\ell,m} \widehat{\psi}^{-}_{n\ell m}(\tau,\Omega) a^\dagger_{n\ell m}$$

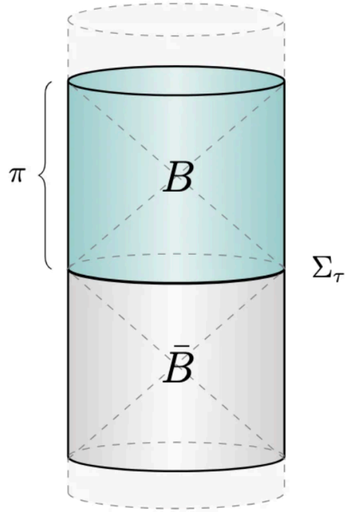
cancels against $\epsilon^{3-\Delta}$ from $G_{3-\Delta}$ to give normalizable fall-off!



Equivalent reconstruction formulae

$$\Phi(\tau,\rho,\Omega) = \alpha \int_B d\tau_p d\Omega_p G^{\text{A.T.O.}}_{3-\Delta}(\tau,\rho,\Omega;\tau_p,\Omega_p)\mathcal{O}_\Delta(\tau_p,\Omega_p) + \alpha^* \int_B d\tau_p d\Omega_p G^{\text{T.O.}}_{3-\Delta}(\tau,\rho,\Omega;\tau_p,\Omega_p)\mathcal{O}_\Delta(\tau_p,\Omega_p)$$

$$= \alpha \int_B d\tau_p d\Omega_p G^{\text{A.T.O.}}_{\Delta}(\tau,\rho,\Omega;\tau_p,\Omega_p)\widetilde{\mathcal{O}}^{+}_{3-\Delta}(\tau_p,\Omega_p) + \alpha^* \int_B d\tau_p d\Omega_p G^{\text{T.O.}}_{\Delta}(\tau,\rho,\Omega;\tau_p,\Omega_p)\widetilde{\mathcal{O}}^{-}_{3-\Delta}(\tau_p,\Omega_p)$$



	$G^{\text{T.O.}}_{\Delta}$	$G^{\text{A.T.O.}}_{\Delta}$
$B\ (\tau_p > \tau)$	ψ^-	ψ^+
$\bar{B}\ (\tau > \tau_p)$	ψ^+	ψ^-

Equivalent reconstruction formulae

$$\Phi(\tau,\rho,\Omega) = \alpha \int_{\bar{B}} d\tau_p d\Omega_p G^{\text{T.O.}}_{3-\Delta}(\tau,\rho,\Omega;\tau_p,\Omega_p)\mathcal{O}_\Delta(\tau_p,\Omega_p) + \alpha^* \int_{\bar{B}} d\tau_p d\Omega_p G^{\text{A.T.O.}}_{3-\Delta}(\tau,\rho,\Omega;\tau_p,\Omega_p)\mathcal{O}_\Delta(\tau_p,\Omega_p)$$

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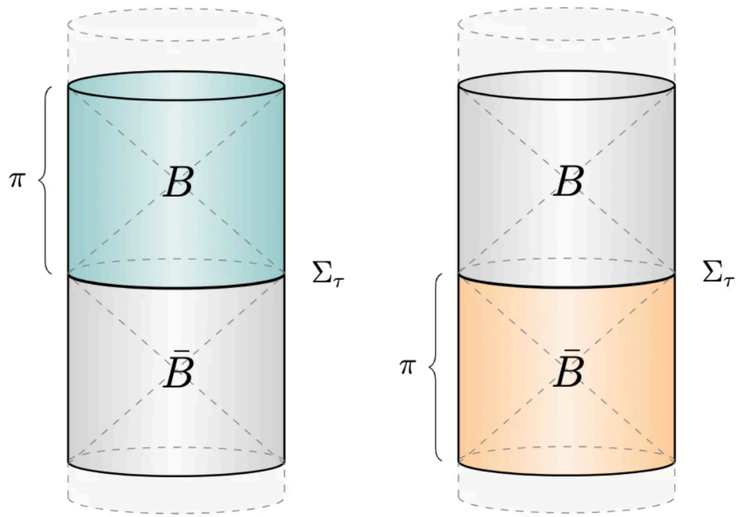


	$G^{\text{T.O.}}_{\Delta}$	$G^{\text{A.T.O.}}_{\Delta}$
$B\ (\tau_p > \tau)$	ψ^-	ψ^+
$\bar{B}\ (\tau > \tau_p)$	ψ^+	ψ^-

Aside: Reconstruction from a boundary cut

- In Minkowski space, free bulk fields can be reconstructed in terms of their value at a boundary ``cut'':

$$\Phi(u, r, \Omega) \propto \int d\Omega_p \partial_u \phi(u_p, \Omega_p)\,|_{u_p = -\hat{q}\cdot x}$$

- Our formulas imply an AdS generalization of this fact for integer Δ:

$$\Phi(\tau, r, \Omega) \propto \int d\Omega_p \left(\frac{1}{r}\Omega_p \cdot \partial_\Omega\right)^{\Delta-1} \frac{1}{f'(\tau_i)} r_p^2 \left(-\partial_{\tau_p} + \frac{\ell^2 + r_p^2}{\ell}\partial_{r_p}\right) \Phi(\tau_p, r_p, \Omega_p)\,|_{\tau_p = \tau_\mp, r_p = r_0}$$

where $\tau_p = \tau_\pm$ are solutions to $e^{i(\tau - \tau_p)} = \dfrac{i(\Omega \cdot \Omega_p r \pm \sqrt{r^2(\Omega \cdot \Omega_p)^2 - r^2 - \ell^2})}{\ell + ir}$

- AdS formula reduce to flat space Kirchhoff-Adhemar formula at $\ell \to \infty$ and for $\Delta = 1$

Flat from AdS I: plane waves

(Bulk) flat space limit: $\tau = t\ell^{-1}, \quad \rho = r\ell^{-1}, \quad \ell \to \infty$ with $\Delta \to \infty$

$$G^{+}_{\Delta}(X;P) = \sum_{n=0}^{\infty}\sum_{\ell=0}^{\infty} 2^{\Delta} e^{-i\Delta(\tau-\tau_p)} e^{-\Delta(1-2i\tau)e^{2i\tau_p}} \frac{\Delta^{2n} e^{2in\tau_p}}{n!} \pi\sqrt{2\ell+1} \frac{(\Delta e^{i\tau_p})^{\ell}}{\Gamma(\frac{3}{2}+\ell+n)} \rho^{2n+\ell} Y_{\ell 0}(\Omega\cdot\Omega_p)$$

- resums into a plane wave using its definition in terms of spherical Bessel functions

$$\sum_{\ell=0}^{\infty} \sqrt{4\pi(2\ell+1)}\, i^{\ell} j_{\ell}(-2imre^{i\tau_p}) Y_{\ell 0}(\Omega\cdot\Omega_p) = e^{-ir\mathbf{p}\cdot\Omega}$$

$$j_{\ell}(x) = \sqrt{\frac{\pi}{2x}} \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\Gamma(n+\ell+\frac{3}{2})} \left(\frac{x}{2}\right)^{2n+\ell+\frac{1}{2}}$$

- depending on time ordering, we obtain positive or negative frequency plane waves

$$G^{\pm}_{\Delta}(X;P) \propto e^{\pm ip\cdot x} \quad \Longrightarrow \quad \Phi_{AdS}(X) \to \# \int \frac{d^d\vec{p}}{2\omega(2\pi)^d} \left(e^{-ip\cdot x} a^{\dagger}(\vec{p}) + e^{ip\cdot x} a(\vec{p})\right)$$

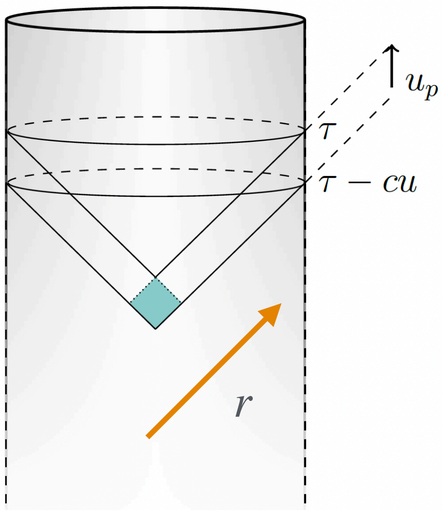
Flat from AdS II: Carrollian wavefunctions

$$\Phi(X) = \int_B d^3P G_\Delta^{\text{T.O.}}(P;X)\widetilde{\mathcal{O}}^-_{3-\Delta}(P) + \int_B d\tau_p d\Omega_p G_\Delta^{\text{A.T.O.}}(P;X)\widetilde{\mathcal{O}}^+_{3-\Delta}(P)$$

fix Δ

- Null foliation: $X^0 = \cos\tau r + \sin\tau\ell, \quad X^i = r\Omega^i, \quad X^4 = -\sin\tau r + \cos\tau\ell$

$$G_\Delta^{\text{T.O.}}(P;X) = \frac{1}{\left(r\cos(\tau-\tau_p) + \ell\sin(\tau-\tau_p) - r\Omega\cdot\Omega_p + i\epsilon\right)^\Delta}$$



- For infinitesimal B, $\tau_p = \tau - c(u-u_p)$ and $\ell \to \infty$:

$$\Phi(\tau, r, \Omega) = \ell^{-1}\int_{u_0+u}^\infty du_p \int d\Omega_p \frac{1}{(u-u_p+r(1-\Omega\cdot\Omega_p)+i\epsilon)^\Delta}\widetilde{\mathcal{O}}^-_{3-\Delta}(\tau-\ell^{-1}(u-u_p),\Omega_p) + h.c.$$

Carrollian positive energy wavefunction, boost eigenstate

$$\int_{-\infty}^\infty dt_p e^{\pm i\omega t_p}\frac{1}{(t-t_p-r\Omega\cdot\Omega_p\pm i\epsilon)^\Delta} = \frac{2\pi(\mp i)^\Delta}{\Gamma(\Delta)}\omega^{\Delta-1}e^{\mp i\omega\hat{q}_p\cdot x}$$

[see review by Bagchi et. al. '25]

$\Delta = 1$

Flat from AdS II: Carrollian wavefunctions

$$\Phi(X) = \int_B d^3P G_\Delta^{\text{T.O.}}(P;X)\widetilde{\mathcal{O}}^-_{3-\Delta}(P) + \int_B d\tau_p d\Omega_p G_\Delta^{\text{A.T.O.}}(P;X)\widetilde{\mathcal{O}}^+_{3-\Delta}(P)$$

fix Δ

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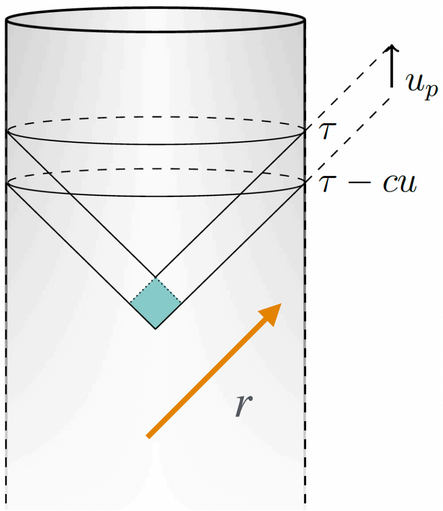
$$G_\Delta^{\text{T.O.}}(P;X) = \frac{1}{\left(r\cos(\tau-\tau_p) + \ell\sin(\tau-\tau_p) - r\Omega\cdot\Omega_p + i\epsilon\right)^\Delta}$$



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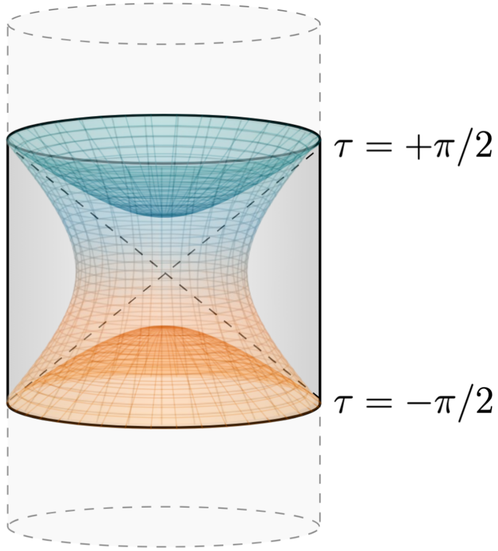
Carrollian primary identified with 3d CFT shadow!

so(3,1) covariant wavefunctions in 4d AdS



- Foliate AdS_4 with Euclidean AdS_3 and dS_3 surfaces

EAdS radius α: $$-(X^0)^2 + \sum_{i=1}^{3} (X^i)^2 = -\alpha^2, \quad X^4 = \pm\sqrt{\ell^2 - \alpha^2}$$

$$|\alpha| \in [0, \ell]$$

AdS_4 metric: $$ds^2 = -\ell^2 \frac{d\alpha^2}{\ell^2 - \alpha^2} + \alpha^2 ds^2_{H_3}$$

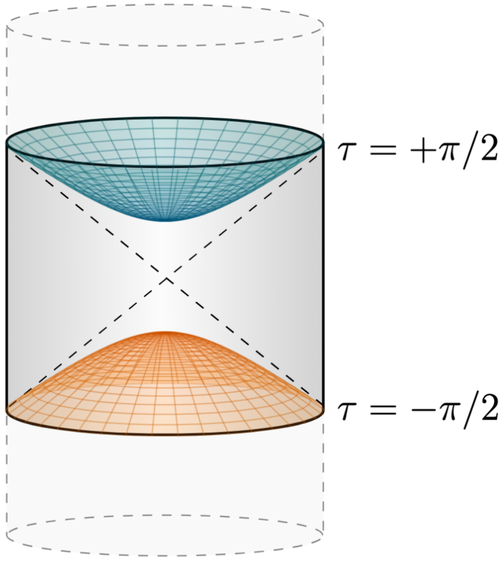
Set $\ell = 1$

$$\Box_{AdS_4} = \alpha^{-2}\Box_{H_3} + \alpha^{-1}\left((4\alpha^2 - 3)\partial_\alpha + \alpha(\alpha^2 - 1)\partial_\alpha^2\right)$$

Look for wavefunctions that diagonalize $\Box_{H_3}$: $$\Psi_\Delta(\alpha, \hat{x}; \delta, \hat{q}) = f_{\Delta,\delta}(\alpha) G^\delta_{H_3}(\hat{x}; \hat{q}), \qquad \Box_{H_3} G^\delta_{H_3}(\hat{x}; \hat{q}) = \delta(\delta - 2) G^\delta_{H_3}(\hat{x}; \hat{q})$$

S^2 at $\tau = \dfrac{\pi}{2}$

so(3,1) covariant wavefunctions in 4d AdS



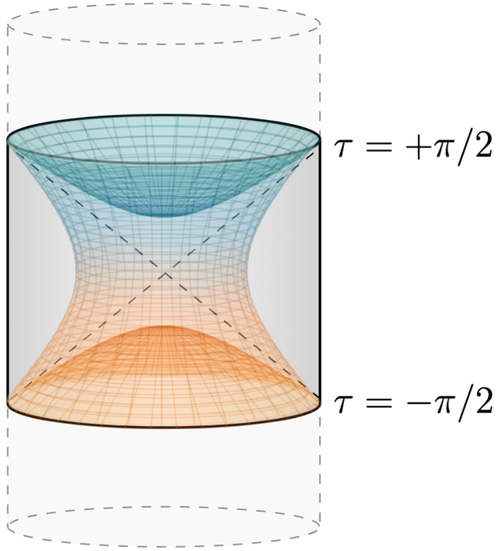
$$\left[\alpha^{-2}\delta(\delta-2)+\alpha^{-1}\left((4\alpha^2-3)\partial_\alpha+\alpha(\alpha^2-1)\partial_\alpha^2\right)\right]f_{\Delta,\delta}(\alpha)=\Delta(\Delta-3)f_{\Delta,\delta}(\alpha)$$

has solution

$$f(\alpha)=c_1 i^{\delta}\alpha^{\delta-2}{}_2F_1\left[\frac{1}{2}(1+\delta-\Delta),\frac{1}{2}(\delta-2+\Delta),\delta,\alpha^2\right]$$
$$+c_2 i^{-\delta}\alpha^{-\delta}{}_2F_1\left[\frac{1}{2}(3-\delta-\Delta),\frac{1}{2}(\Delta-\delta),2-\delta,\alpha^2\right]$$

Regularity at the origin and agreement with flat space limit $\Longrightarrow$ $$\frac{c_1 2^{\delta-1}\Gamma(\delta)}{c_2 2^{1-\delta}\Gamma(2-\delta)}=-1$$

so(3,1) covariant wavefunctions in 4d AdS



Completeness follows from Sturm-Liouville theory with inner product

$$\langle f, g \rangle = \int_0^1 d\alpha \alpha^{-2} \mu(\alpha) f(\alpha) g(\alpha)$$

provided that $\delta = 1 + i\lambda$ [EAdS$_3$ principal series]

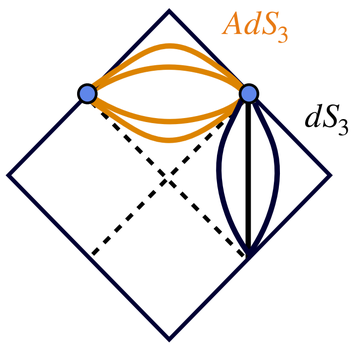
dS slicing by setting $\alpha \to i\alpha$

Decomposition of so(3,2) rep. into so(3,1) reps

$$\Phi(\alpha, x) \propto \int_{\mathbb{R}} d\lambda \int d^2\hat{q} \hat{f}(\alpha) G^{1+i\lambda}_{H_3}(\hat{q}; x) \mathcal{O}_{1-i\lambda}(\hat{q}) + \int_{\mathbb{R}} d\lambda \int d^2\hat{q} \hat{f}(i\alpha) G^{1+i\lambda}_{dS_3}(\hat{q}; x) \mathcal{O}_{1-i\lambda}(\hat{q})$$

Kinematics is the same as of celestial CFT operators. What about dynamics?

Flat from AdS III: Conformal primary wavefunctions



Flat space limit: $\Delta \to \infty, \quad \frac{\Delta}{\ell}$ fixed

Restoring ℓ and using the series representation of the hypergeometric function:

$$ {}_2F_1\left(\frac{1+\delta-\Delta}{2}, \frac{\delta-2+\Delta}{2}, \delta, \frac{\alpha^2}{\ell^2}\right) = \sum_{n=0}^{\infty} \frac{\Gamma(\frac{\delta+1-\Delta}{2}+n)\Gamma(\frac{\delta-2+\Delta}{2}+n)\Gamma(\delta)}{\Gamma(\delta+n)\Gamma(\frac{\delta+1-\Delta}{2})\Gamma(\frac{\delta-2+\Delta}{2})} \frac{\alpha^{2n}}{n!\ell^{2n}} \longrightarrow \Gamma(\delta)\left(\frac{2}{i\alpha}\right)^{\delta-1} I_{\delta-1}(i\alpha) $$

$$ \lim_{\ell\sim\Delta\to\infty} f_{\Delta,\delta}(\alpha) = c_1(i\alpha)^{-1}2^{\delta-1}\Gamma(\delta)I_{\delta-1}(i\alpha) + c_2 2^{1-\delta}(i\alpha)^{-1}\Gamma(2-\delta)I_{1-\delta}(i\alpha) \propto \alpha^{-1}K_{\delta-1}(i\alpha) $$

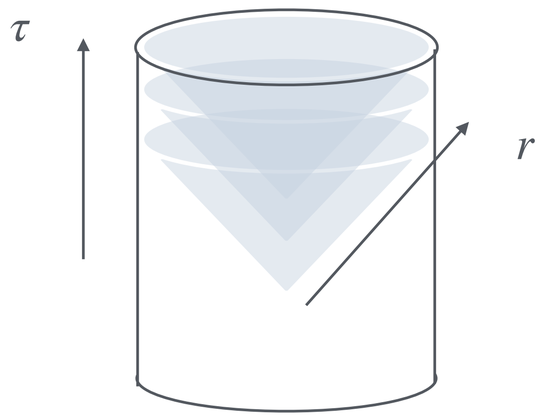
- Multiplied with the EAdS$_3$ propagator, we recover the conformal primary wavefunction of a massive particle in flat space

[Pasterksi, Shao, Strominger '16; Sleight, Taronna '21-'26,…]

so(3,1) covariant wavefunctions in 4d AdS

- Construction of so(3,1) covariant wavefunctions misses points on the lightcone through the origin

- Foliate AdS_4 with lightcones through the origin



- so(3,1) covariant solutions to the AdS_4 KG equation:

$$\Psi_{\Delta,\delta}(X;\hat{q}) \propto {}_2F_1\left[\frac{1}{2}(\Delta-\delta),\frac{1}{2}(\Delta-\delta+1);\Delta-\frac{1}{2};\frac{1}{(\ell\cos\tau - r\sin\tau)^2}\right]\frac{(\ell^2\cos\tau-\ell r\sin\tau)^{\delta-\Delta}}{(r\cos\tau+\ell\sin\tau-r\Omega\cdot\Omega_q)^{\delta}}$$

- They become massless conformal primary wavefunctions in the flat space limit ($\ell \to \infty$, fixed Δ)

Summary & outlook

- Revisited the canonical quantization of a free scalar in AdS_4
- Positive and negative energy subspaces spanned by wavefunctions labelled by points on boundary

  so(3,2) covariant bulk-to-boundary propagators with specified time orderings

  which become plane waves ($\Delta \to \infty$) or Carrollian wavefunctions (Δ fixed) in the flat limit

- Constructed wavefunctions labelled by points on codimension-1 surface on the boundary and so(3,1) "boost" weight which become conformal primary wavefunctions in the flat limit

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Could the decomposition of (A)dS/CFT observables in this basis reveal infinite ``symmetry'' algebras?

[Himwich, Pate '25, Strominger, Wei '26]

Thank you!