Detector Amplitudes

how Positivity Meets the Real World

Brando Bellazzini

arXiv:2512.13780
with J. Berman, G. Isabella, F. Riva, M. Romano, F. Sciotti

ENS-Summer Institute 2026, July 1st

EFT Pillars

Principles + Symmetries + light d.o.f.

EFT Pillars

Principles + *Symmetries* + *light d.o.f.*

QM

$P^2 \geq 0 \; P^0 \geq 0$

Poincaré

EFT Pillars

Principles + *Symmetries* + *light d.o.f.*

QM

$P^2 \geq 0 \; P^0 \geq 0$

Poincaré

Wigner bootstrap $\longrightarrow$

massless	*massive*
0	0
1/2	1/2
1	1
3/2	3/2
2	2
3	3
⋮	⋮

EFT Pillars

Principles + *Symmetries* + *light d.o.f.*

Wigner bootstrap

massless	massive
0	0
1/2	1/2
1	1
3/2	3/2
2	2
~~3~~	3
~~⋮~~	⋮

SUSY

Grisaru, Pendleton, van Nieuwenhuizen '77

GR

No HS

Porrati 0804.4672

Weinberg-Witten '80

Benincasa Cachazo 0705.4305

QM

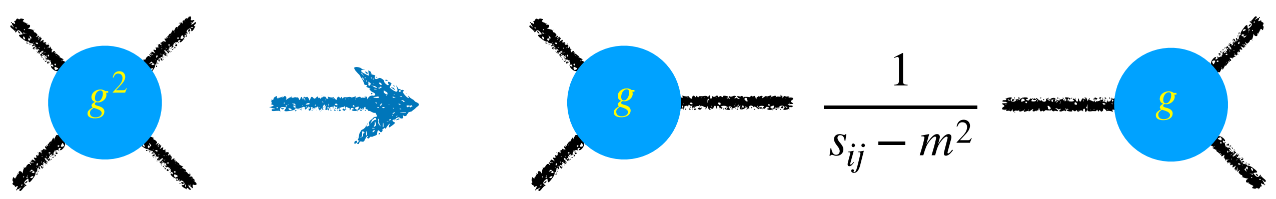
$P^2 \geq 0 \; P^0 \geq 0$

Poincaré

+

Locality

$$\mathcal{S} = \int dx \mathcal{L}(\mathcal{O}, \partial)$$



EFT Pillars

Principles + *Symmetries* + *light d.o.f.*

Wigner bootstrap

	massless	massive
Higgs	0	0
	$\cancel{1/2}$	1/2
YM, gauge	1	1
SUSY	$\boxed{3/2}$	3/2
GR	$\boxed{2}$	2
No HS	$\cancel{3}$	3
	⋮	⋮

Grisaru, Pendleton, van Nieuwenhuizen '77

Porrati 0804.4672

Weinberg-Witten '80

Benincasa Cachazo 0705.4305

QM

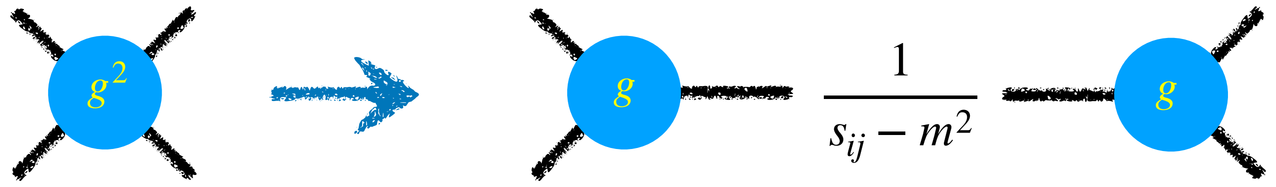
$P^2 \geq 0 \; P^0 \geq 0$

Poincaré

+

Locality

$\mathcal{S} = \int dx \mathcal{L}(\mathcal{O}, \partial)$



EFT Pillars

Principles + *Symmetries* + *light d.o.f.*

↓ ↓

QM

$P^2 \geq 0 \; P^0 \geq 0$

Poincaré

Wigner bootstrap

	massless	massive
Higgs	0	0
	1/2	1/2
YM, gauge	1	1
SUSY	3/2	3/2
GR	2	2
No HS	3 ⋮	3 ⋮

Grisaru, Pendleton, van Nieuwenhuizen '77

BB, Pomarol, Romano, Sciotti 2304.02550

BB, Isabella, Ricossa, Riva 2304.02550

Porrati 0804.4672

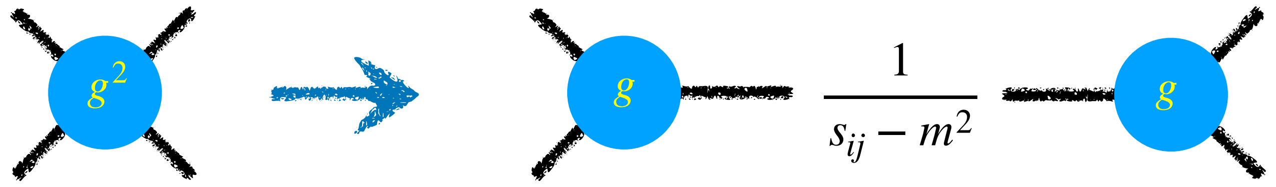
Weinberg-Witten '80

Benincasa Cachazo 0705.4305

+

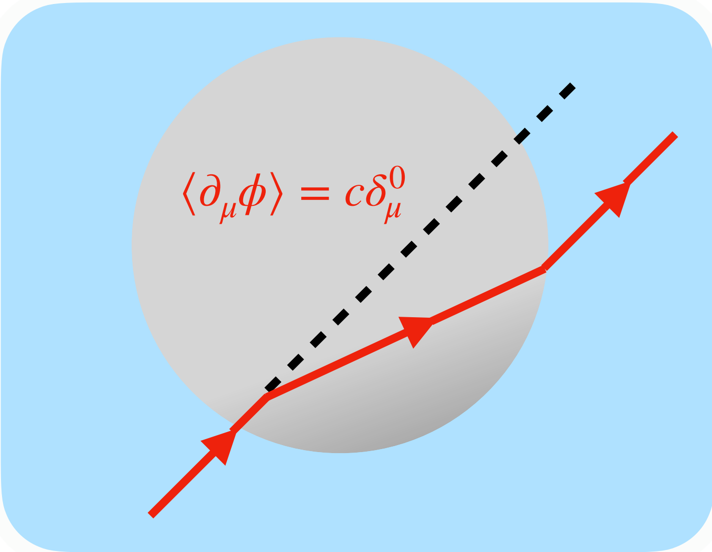
<u>Locality</u>

$\mathcal{S} = \int dx \mathcal{L}(\mathcal{O}, \partial)$



+

<u>Causality/Positivity</u>



Adams, Arkani-Hamed, Dubovsky, Nicolis Rattazzi '06

LEGO - Philosophy

Complex Universe from simple building block Principles

SM & GR

largely fixed by consistency

$$\mathscr{L}^{\text{EFT}}_{\text{leading}} = -R + F^2_{\mu\nu} + \psi^\dagger i\partial\psi + (\partial\phi)^2 + \psi\psi\phi - V(\phi)$$

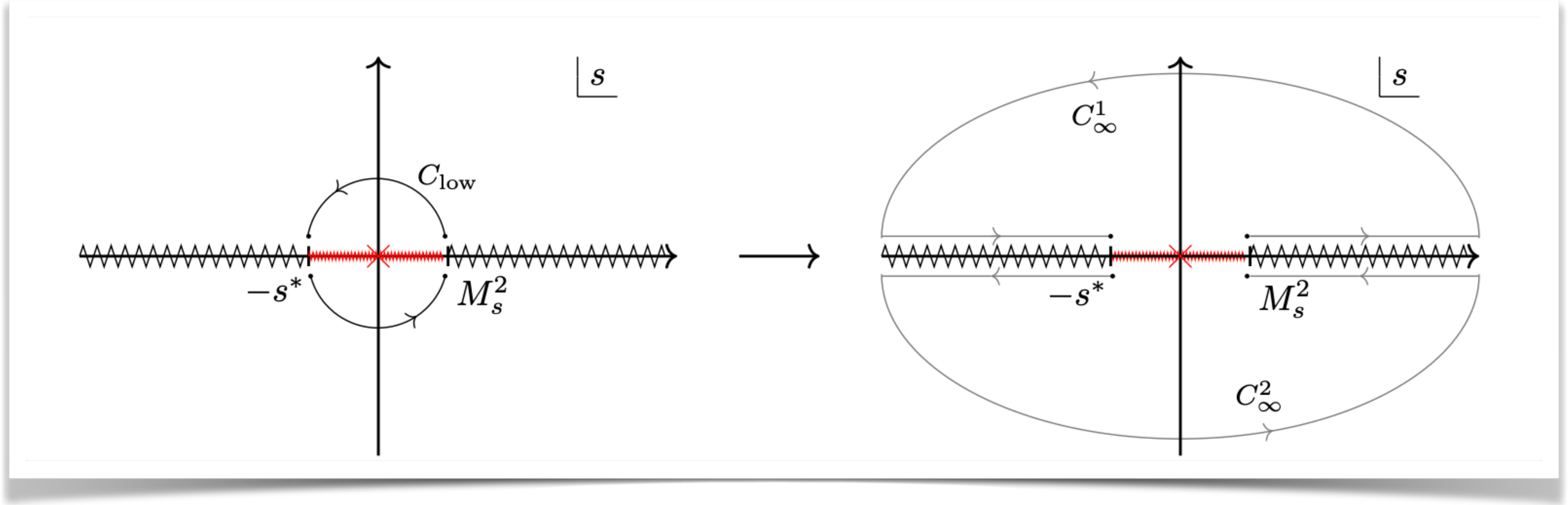
Q: Does consistency fix/constrain subleading terms?

Yes!

Examples

Example-1: SUSY-breaking space

$$\mathcal{M}(1^+2^+3^-4^-) = [12]\langle 14\rangle\left(c_{1,0}s + c_{2,0}s^2 + c_{2,1}tu + \ldots\right)$$

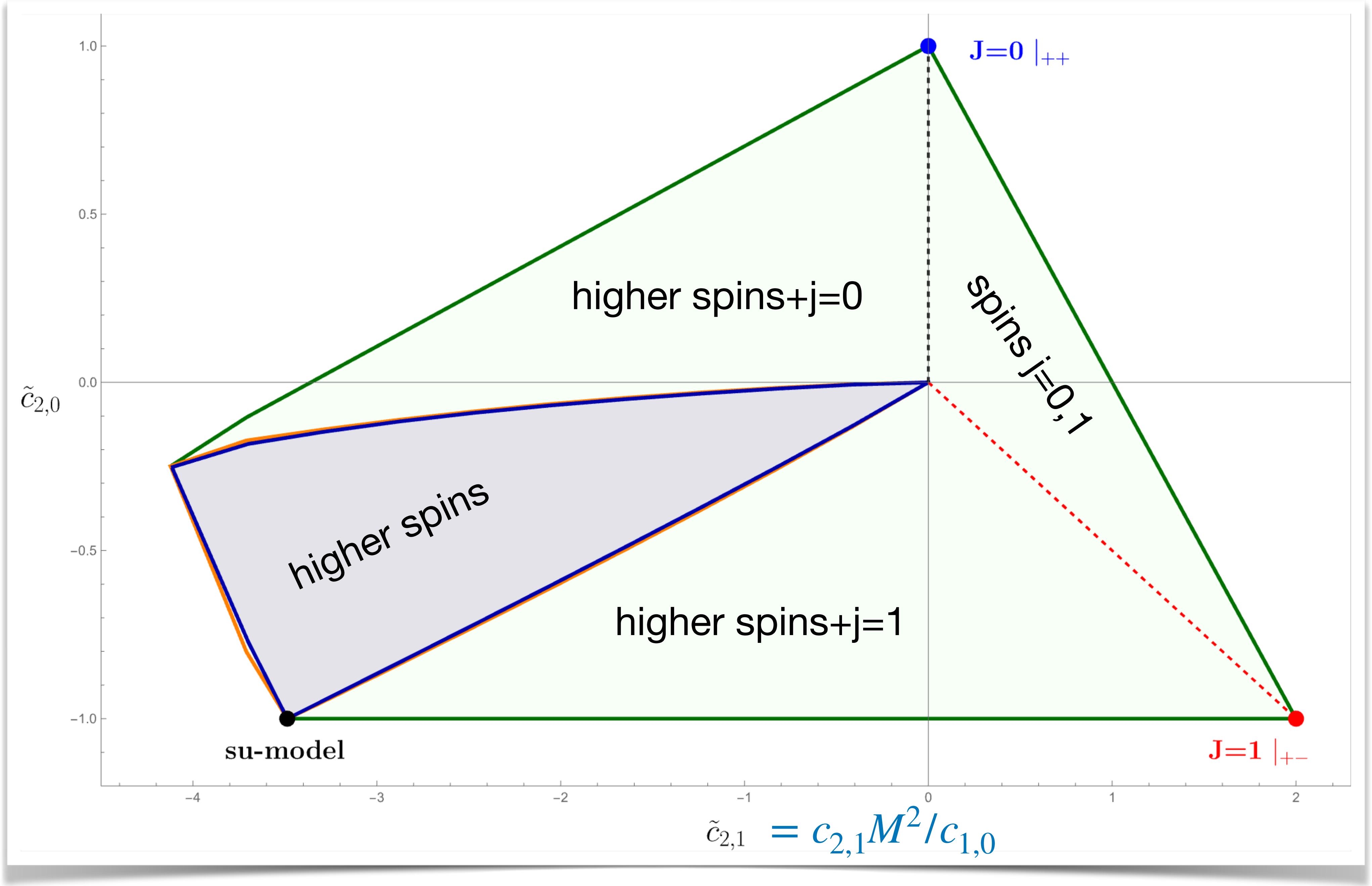


$$c_{1,0} = -\langle 1\rangle_{--} - \langle 1\rangle_{-+}$$

$$c_{2,0} = -\langle 1/s\rangle_{--} + \langle 1/u\rangle_{-+}$$

$$\vdots$$

$$\langle(\cdots)\rangle_{--} = \sum_\ell \int_{M^2}^\infty \frac{ds}{s^3}(\cdots)\mathrm{Im}\mathcal{M}_\ell^{--++}(s)$$

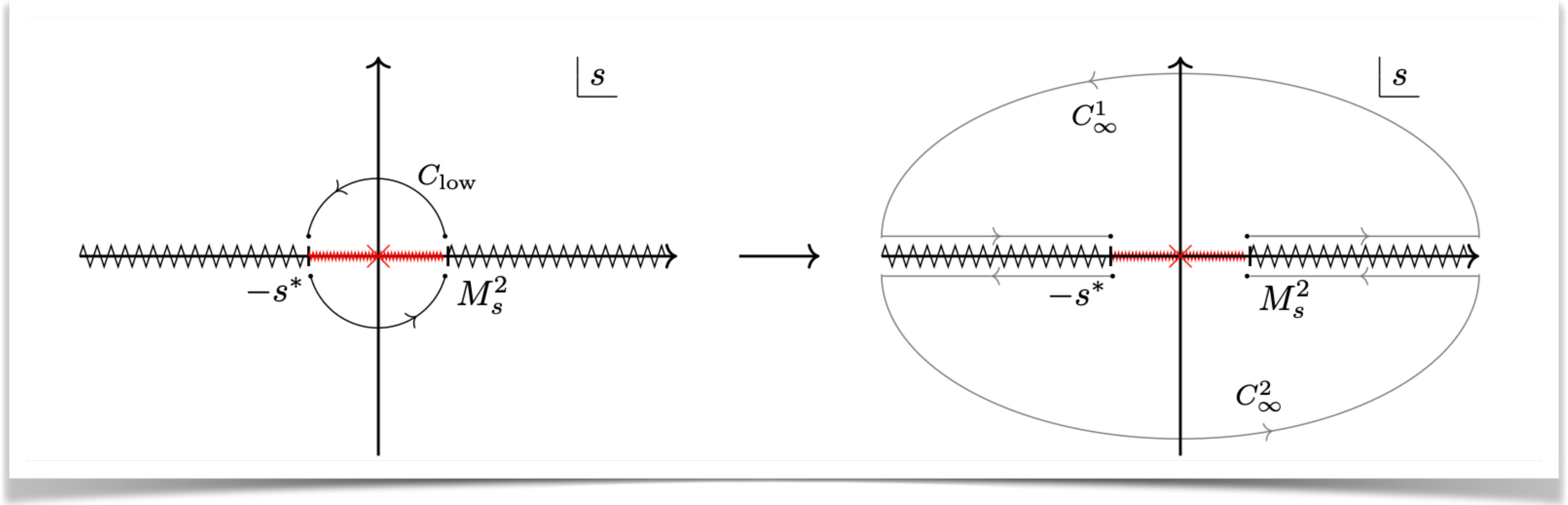


The Goldstino-hedron

BB, Pomarol, Romano, Sciotti 2304.02550

Example-1: SUSY-breaking space

$$\mathcal{M}(1^+2^+3^-4^-) = [12]\langle 14\rangle\left(c_{1,0}s + c_{2,0}s^2 + c_{2,1}tu + \ldots\right)$$



$$c_{1,0} = -\langle 1\rangle_{--} - \langle 1\rangle_{-+}$$

$$c_{2,0} = -\langle 1/s\rangle_{--} + \langle 1/u\rangle_{-+}$$

$$\vdots$$

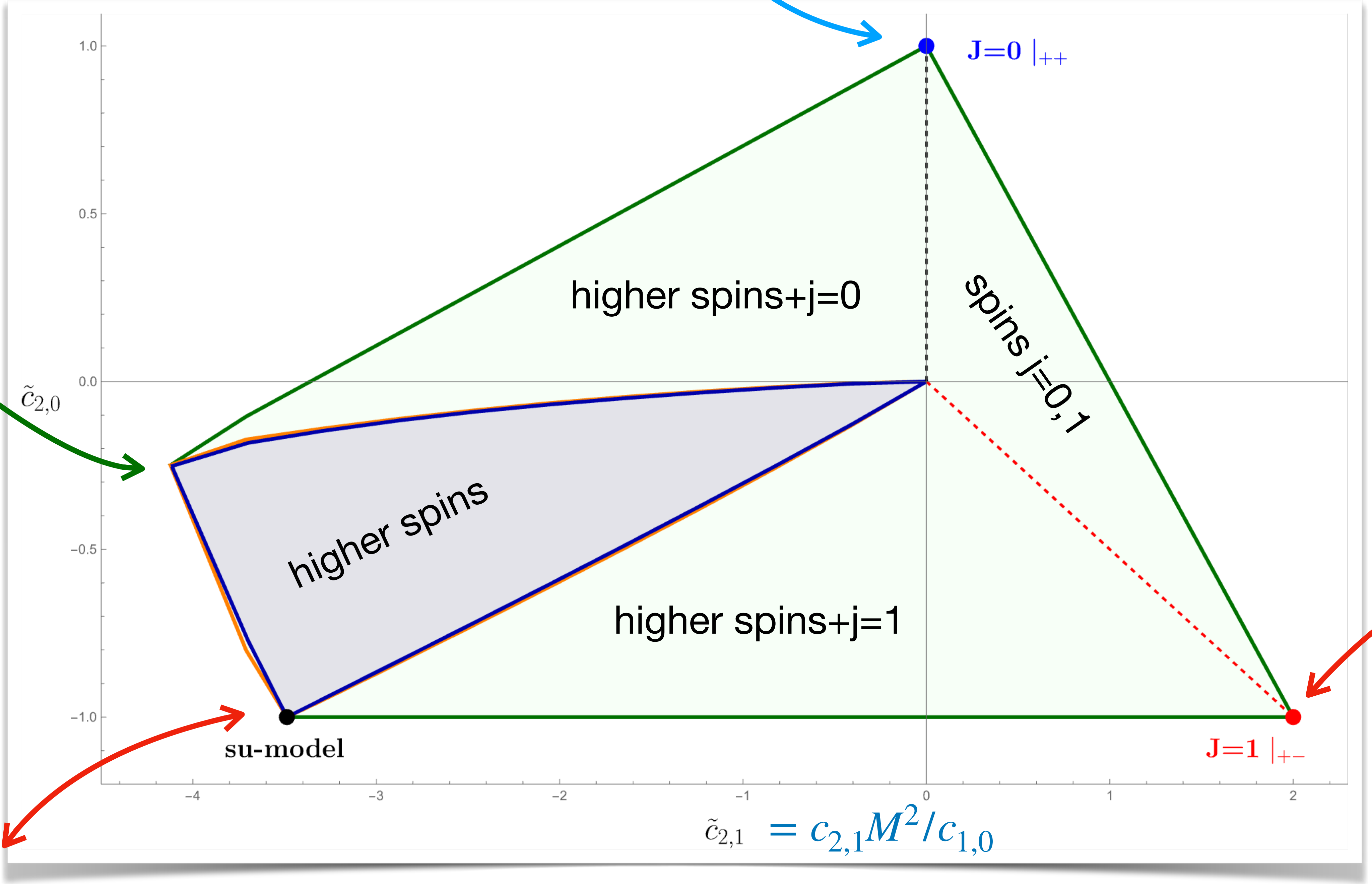
$$\langle(\cdots)\rangle_{--} = \sum_\ell \int_{M^2}^\infty \frac{ds}{s^3}(\cdots)\mathrm{Im}\mathcal{M}_\ell^{--++}(s)$$

single spin-0 exchange

$$\frac{1}{s-M^2} = \frac{\mathcal{M}(1^+2^+3^-4^-)}{[12]\langle 14\rangle}$$

infinitely many degenerate spins in +- & ++

$$\frac{M^2 s + \alpha s^2 + \beta ut + \gamma stu/M^2}{(s-M^2)(u-M^2)(t-M^2)}$$



single spin-1

$$\frac{1}{M^2}\left(\frac{u}{u-M^2} + \frac{t}{t-M^2}\right)$$

A new kind of Model Building

infinitely many degenerate spins in +-

$$\frac{\alpha s + ut/M^2}{(u-M^2)(t-M^2)}$$

The Goldstino-hedron

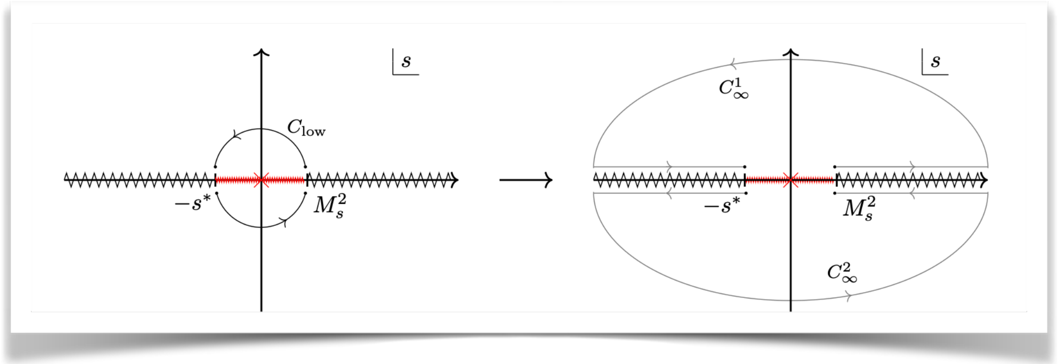
BB, Pomarol, Romano, Sciotti 2304.02550

Example-2: Goldstones

$$\mathscr{L} = \frac{1}{2}(\partial_\mu\phi)^2 + g_2(\partial_\mu\phi)^4 + g_3(\partial\phi)^2(\partial\partial\phi)^2 + g_4(\partial\partial\phi)^4 + \dots$$

$g_2 > 0$

Adams, Arkani-Hamed, Dubovsky, Nicolis Rattazzi '06

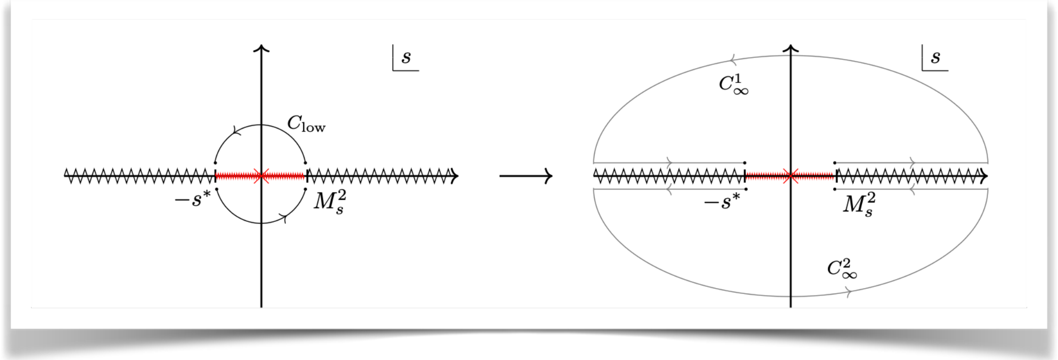


Example-2: *Goldstones*

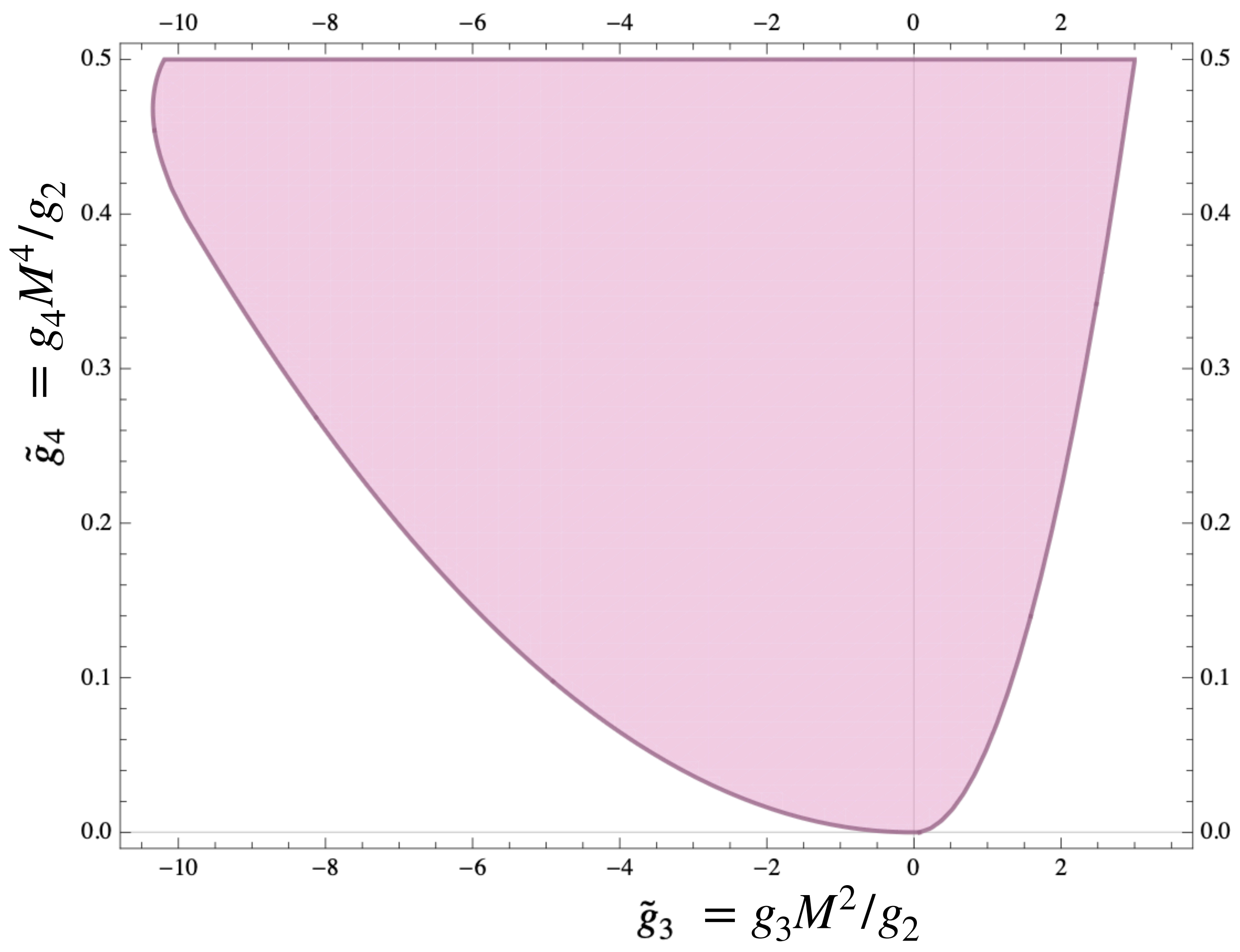
$$\mathscr{L} = \frac{1}{2}(\partial_\mu \phi)^2 + g_2(\partial_\mu \phi)^4 + g_3(\partial\phi)^2(\partial\partial\phi)^2 + g_4(\partial\partial\phi)^4 + \dots$$

$g_2 > 0$

Adams, Arkani-Hamed, Dubovsky, Nicolis Rattazzi '06

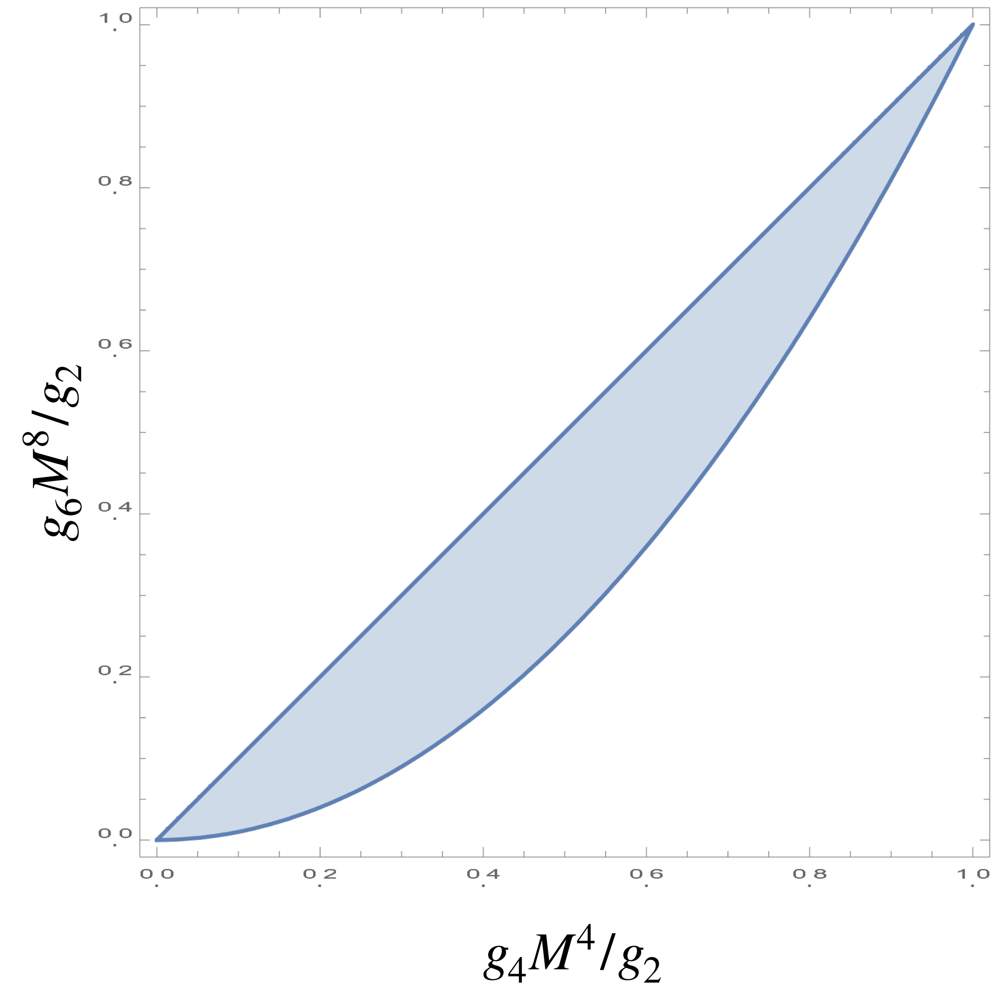


Clash: "Symmetries" vs "Causality"



Caron-huot, van Duong 2011.02957

Tolley, Wang, Zhou 2011.02400



BB, Miro, Rattazzi, Riembau, Riva 2011.00037

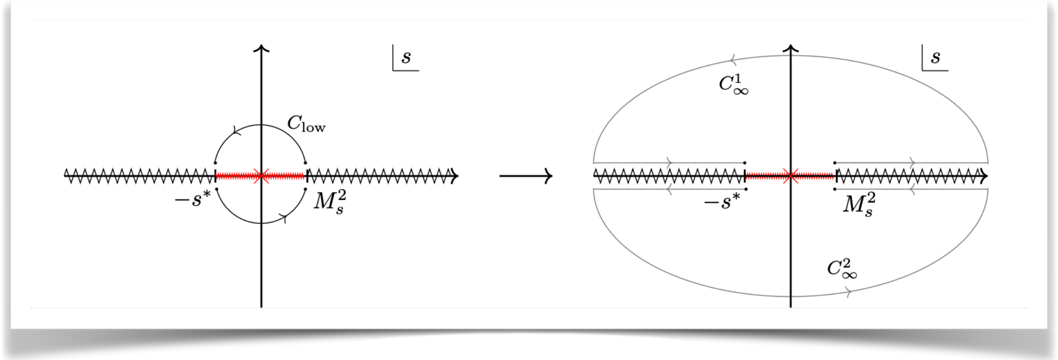
Arkani-Hamed, Huang, Huang, 2012.15849

Example-2: Goldstones

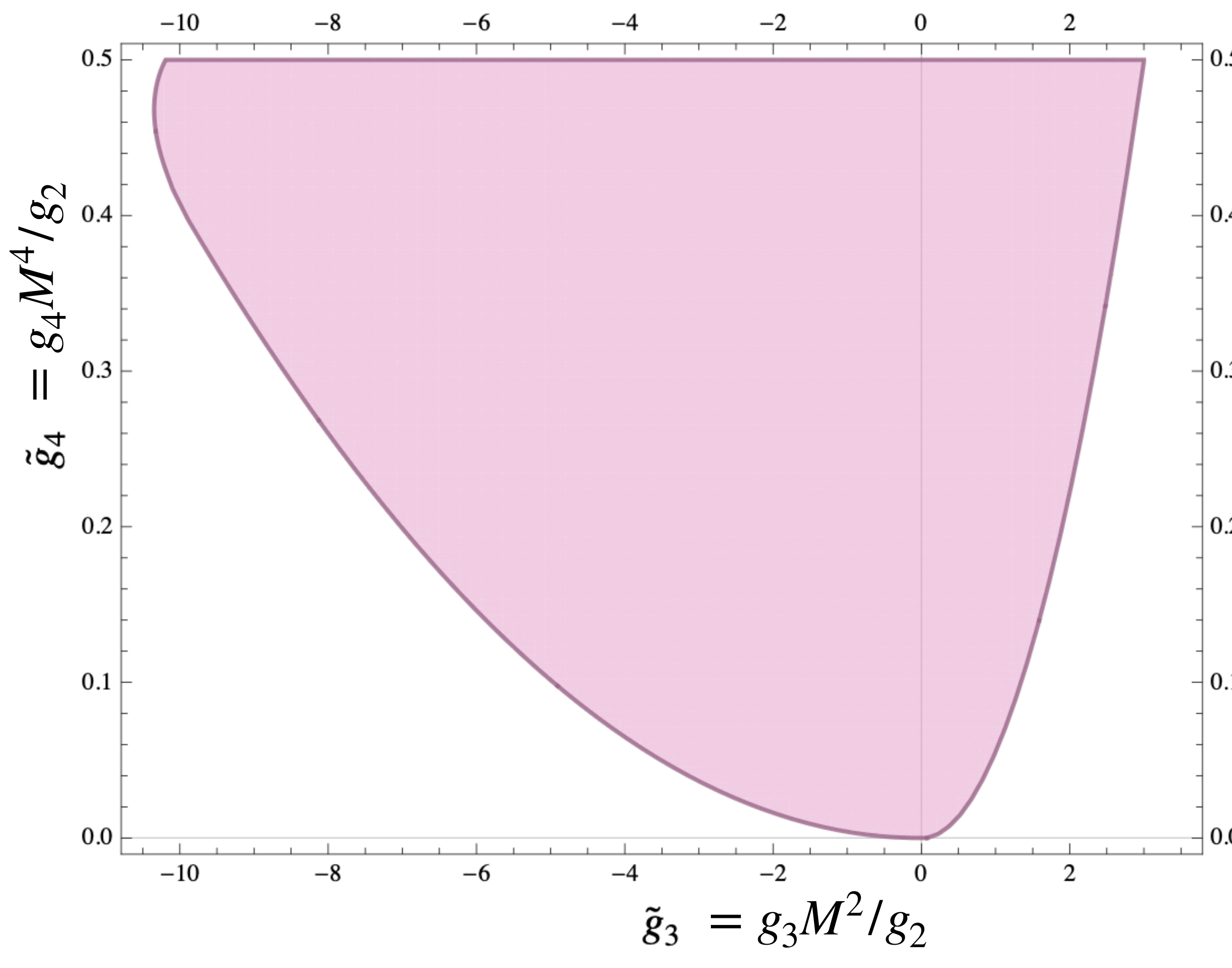
$$\mathscr{L} = \frac{1}{2}(\partial_\mu \phi)^2 + g_2(\partial_\mu \phi)^4 + g_3(\partial\phi)^2(\partial\partial\phi)^2 + g_4(\partial\partial\phi)^4 + \ldots$$

$g_2 > 0$

Adams, Arkani-Hamed, Dubovsky, Nicolis Rattazzi '06

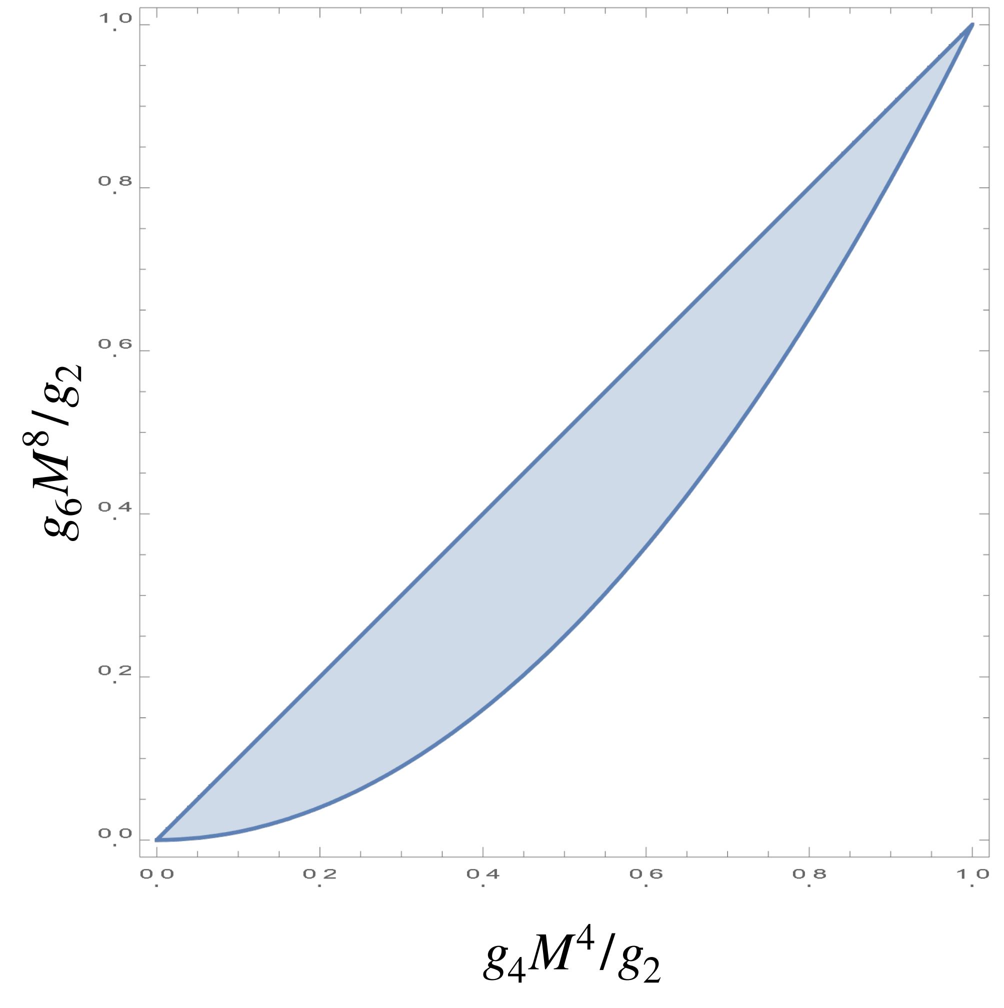


Clash: "Symmetries" vs "Causality"



Caron-huot, van Duong 2011.02957

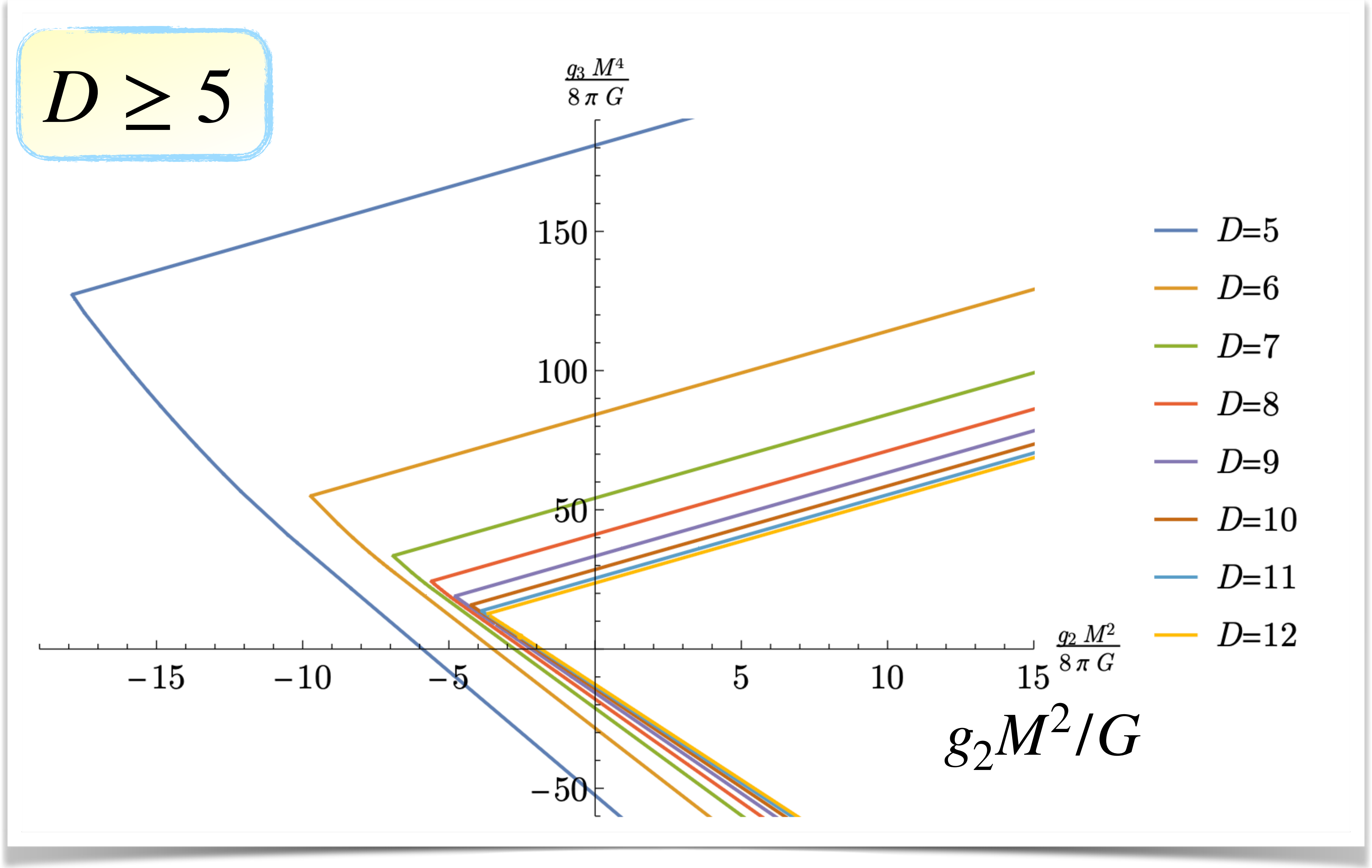
Tolley, Wang, Zhou 2011.02400



BB, Miro, Rattazzi, Riembau, Riva 2011.00037

Arkani-Hamed, Huang, Huang, 2012.15849

Adding gravity:



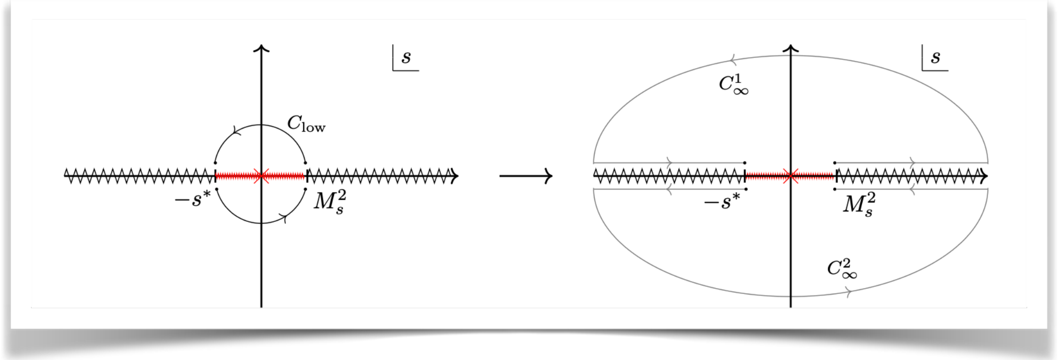
Caron-huot, Mazac, Rastelli, Simmons-Duffin 2102.08951

Example-2: Goldstones

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \phi)^2 + g_2(\partial_\mu \phi)^4 + g_3(\partial\phi)^2(\partial\partial\phi)^2 + g_4(\partial\partial\phi)^4 + \dots$$

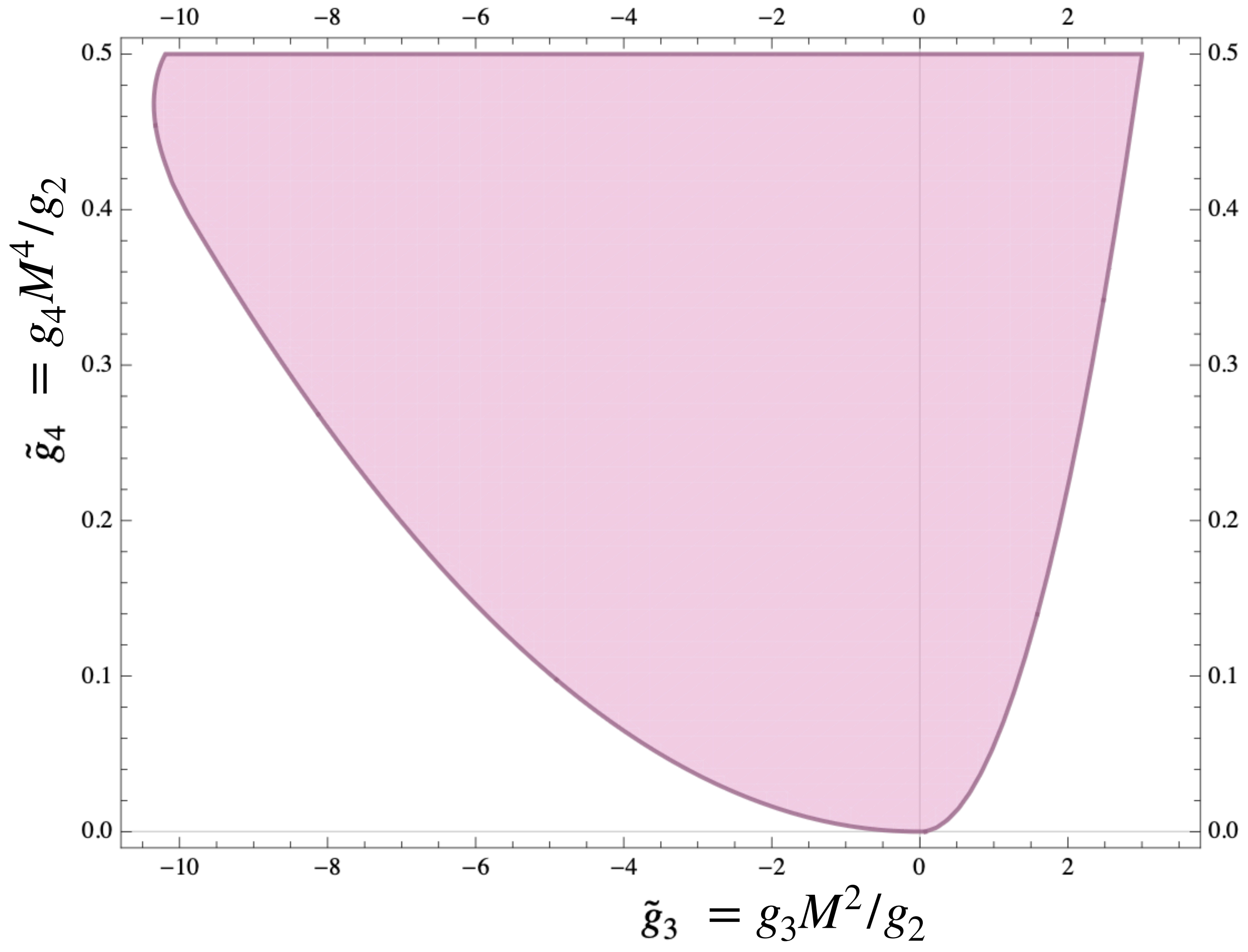
$g_2 > 0$

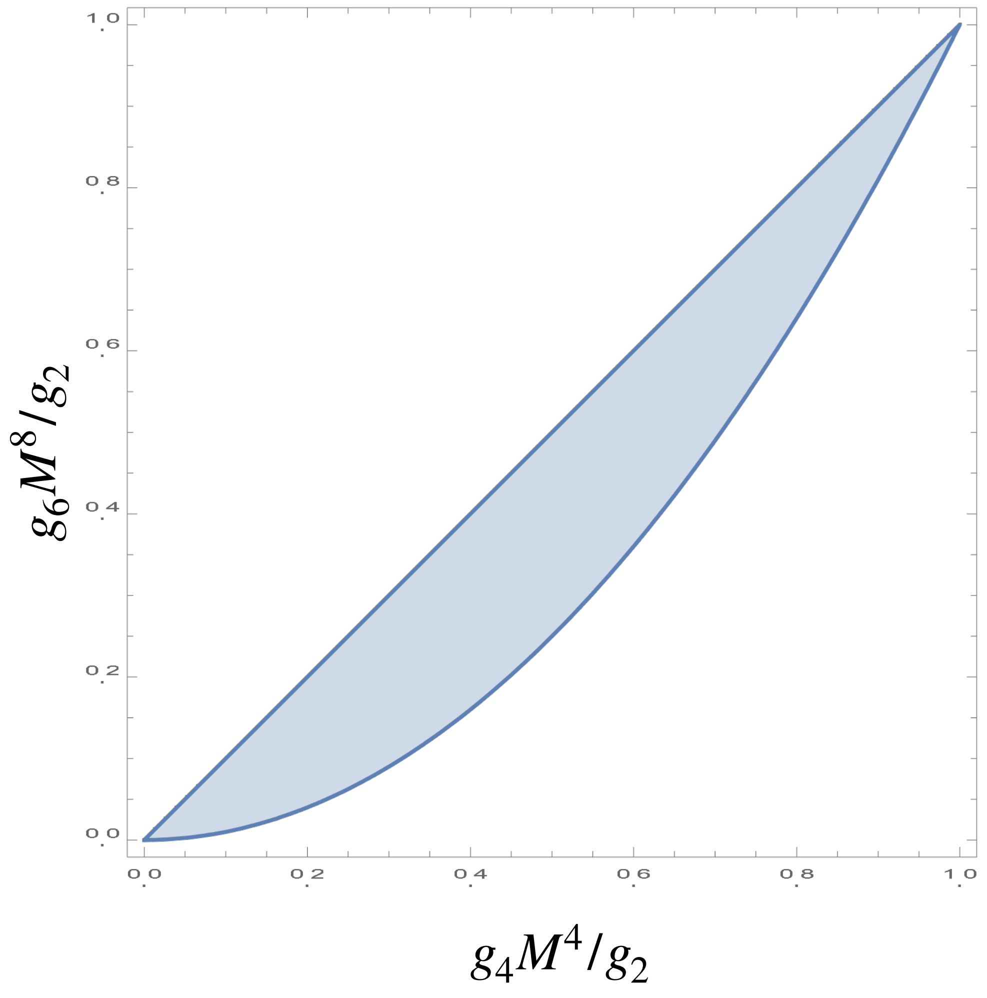
Adams, Arkani-Hamed, Dubovsky, Nicolis Rattazzi ‘06

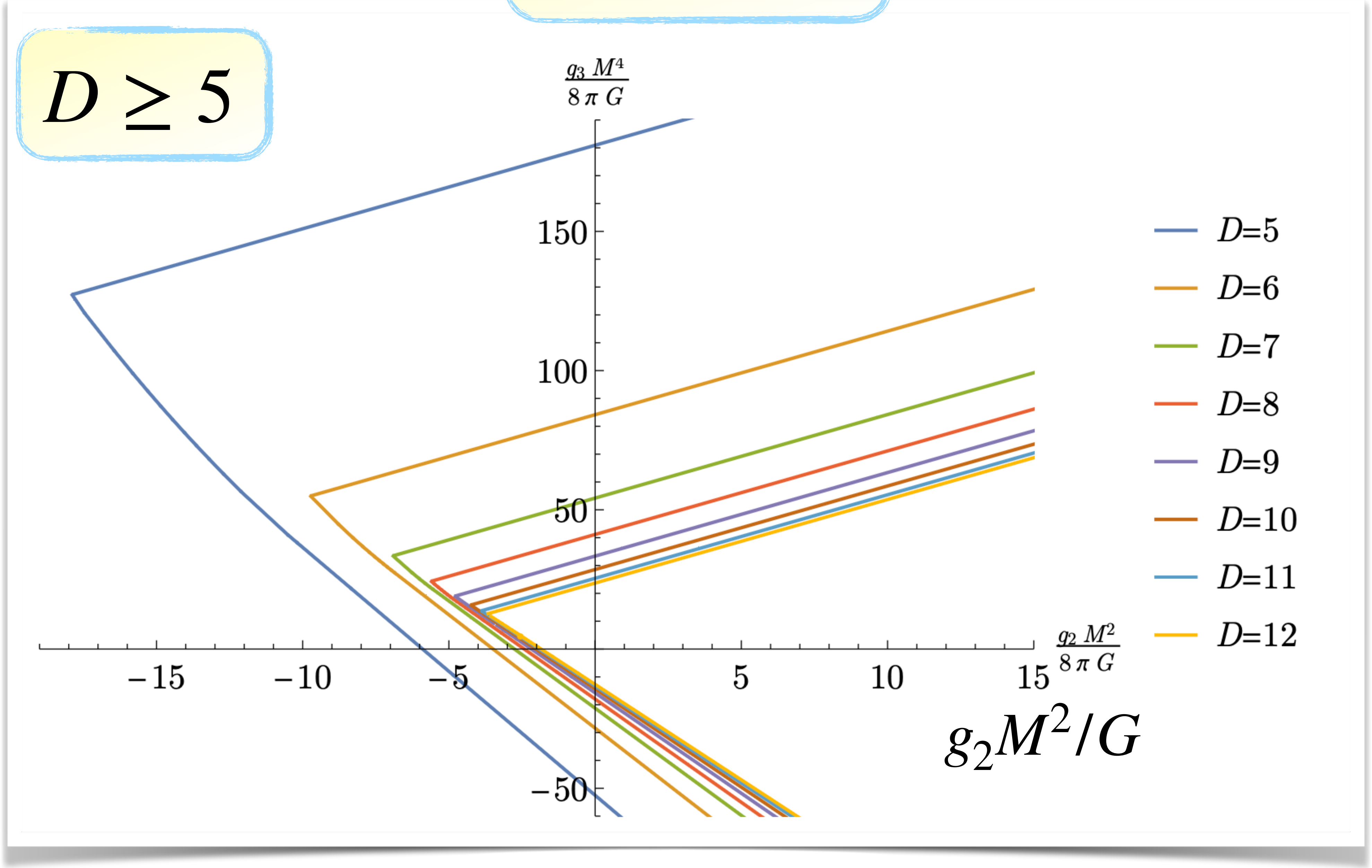


Clash: “Symmetries” vs “Causality”

Adding gravity: $D = 4???$







Caron-huot, van Duong 2011.02957

Tolley, Wang, Zhou 2011.02400

BB, Miro, Rattazzi, Riembau, Riva 2011.00037

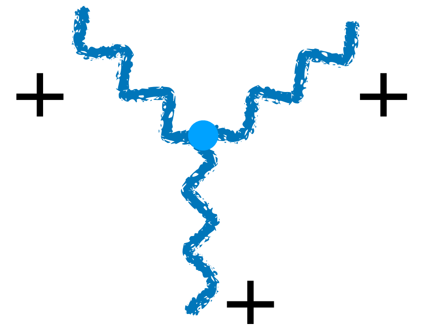
Arkani-Hamed, Huang, Huang, 2012.15849

Caron-huot, Mazac, Rastelli, Simmons-Duffin 2102.08951

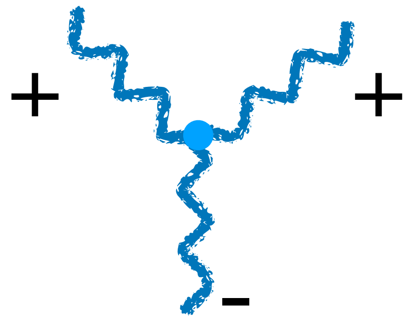
Example-3: EFT Gravity

$$\mathscr{L} = \frac{1}{G}\left(-R + \ell_2^2 Rie^2 + \ell_3^4 Rie^3 + \ell_4^6 Rie^4 + \ldots\right) \qquad \ell_i^n = \frac{\alpha_i}{M^n}$$

$|\alpha_i| \gg 1\,?$



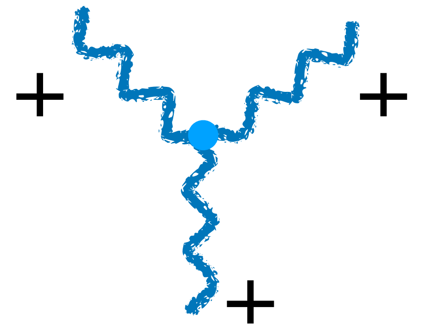
vs



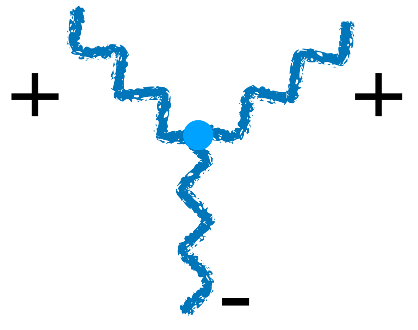
Example-3: EFT Gravity

$$\mathscr{L} = \frac{1}{G}\left(-R + \ell_2^2 Rie^2 + \ell_3^4 Rie^3 + \ell_4^6 Rie^4 + \ldots\right) \qquad \ell_i^n = \frac{\alpha_i}{M^n}$$

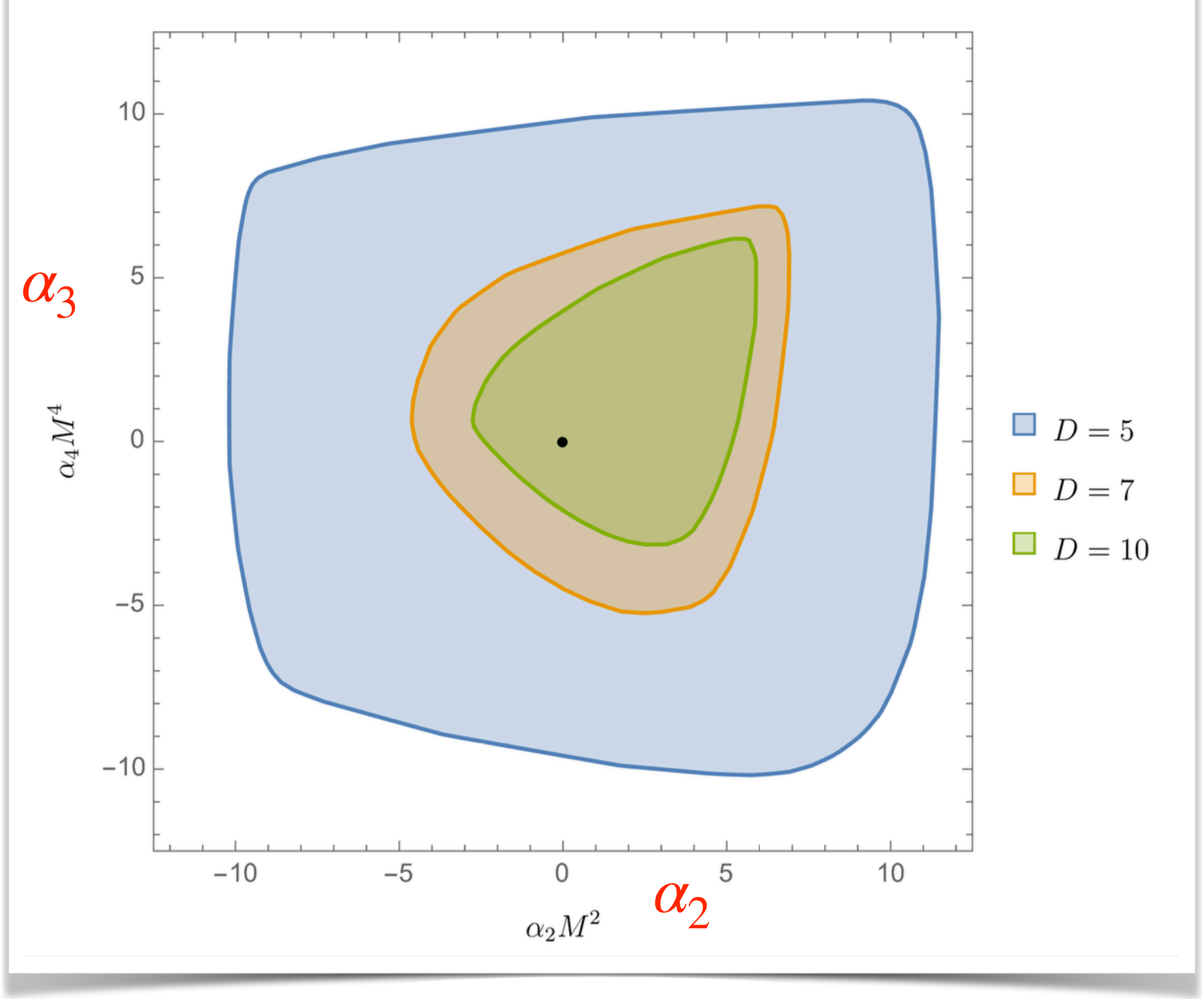
$$|\alpha_i| \gg 1\,?$$



vs



Caron-huot, Parra-Martinez, Li, Simmons-Duffin 2205.01495



$$D \geq 5$$

$$\Delta T \propto R_s \left(\frac{R_s}{b}\right)^{D-4} \left(\frac{1}{(\ell_2/b)^2 + \ldots} \quad \frac{(\ell_2/b)^2 + \ldots}{1} \right) \succ 0$$

Camanho, Edelstein, Maldacena, Zhiboedov 1407.5597

$$0 < \alpha_4, \alpha_4' < O(1) G M^2$$

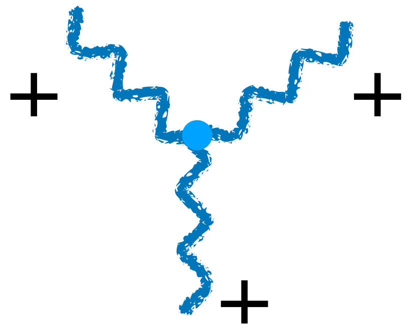
BB, Cheung, Remmen 1509.00851

Caron-huot, Parra-Martinez, Li, Simmons-Duffin 2201.06602

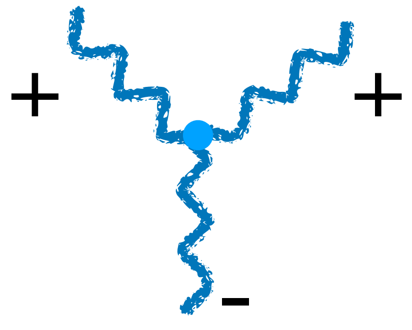
Example-3: *EFT Gravity*

$$\mathscr{L} = \frac{1}{G}\left(-R + \ell_2^2 Rie^2 + \ell_3^4 Rie^3 + \ell_4^6 Rie^4 + \ldots\right) \qquad \ell_i^n = \frac{\alpha_i}{M^n}$$

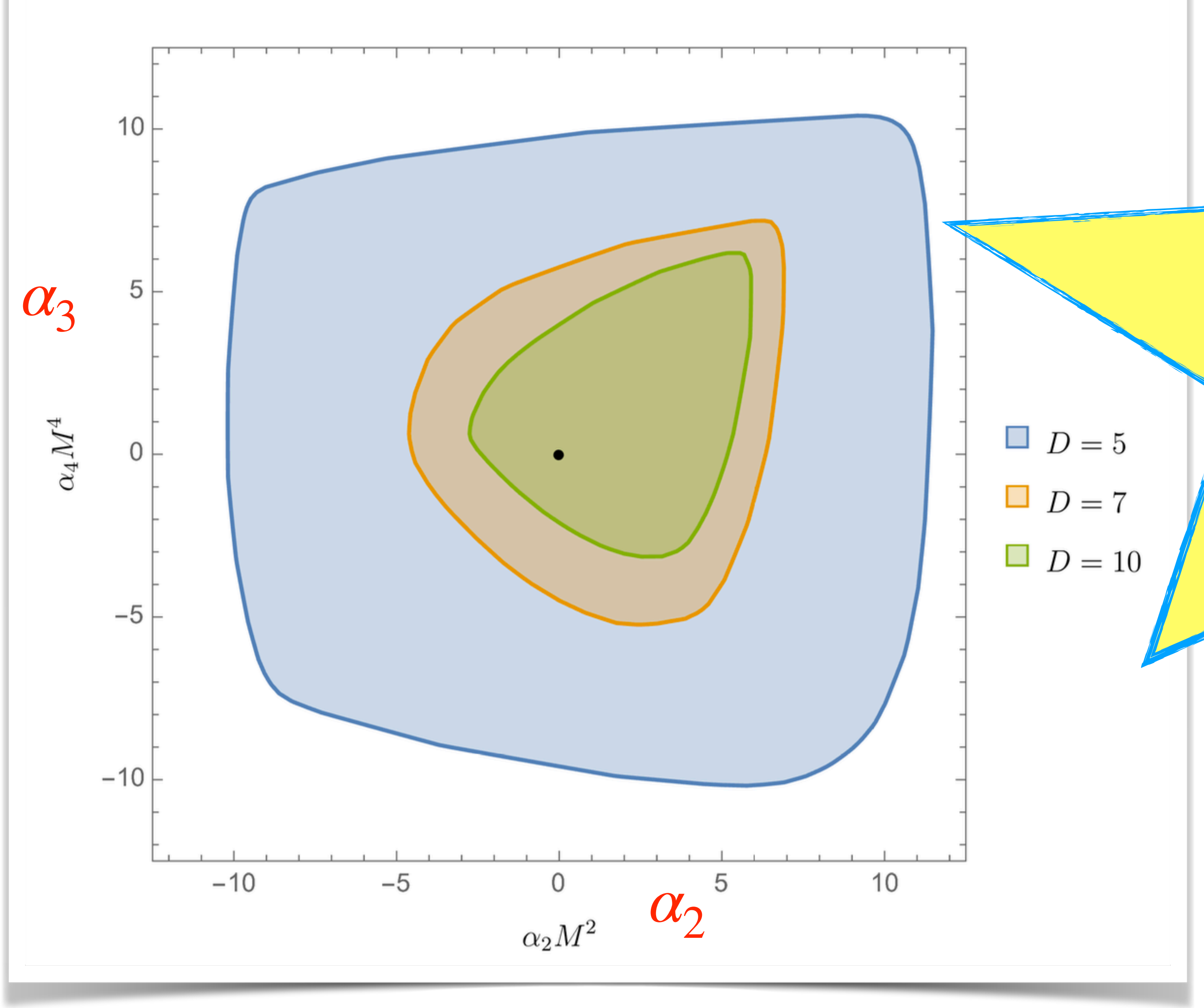
$$|\alpha_i| \gg 1\,?$$



vs



Caron-huot, Parra-Martinez, Li, Simmons-Duffin 2205.01495



$$D \geq 5$$

$$D = 4???$$

$$\propto R_s \left(\frac{R_s}{b}\right)^{D-4} \begin{pmatrix} \frac{1}{(\ell_2/b)^2 + \ldots} & \frac{(\ell_2/b)^2 + \ldots}{1} \end{pmatrix} \succ 0$$

manho, Edelstein, Maldacena, Zhiboedov 1407.5597

$$0 < \alpha_4, \alpha_4' < O(1) G M^2$$

BB, Cheung, Remmen 1509.00851

Caron-huot, Parra-Martinez, Li, Simmons-Duffin 2201.06602

Long-Range Interactions

Long-Range forces in D=4

E&M or Gravity

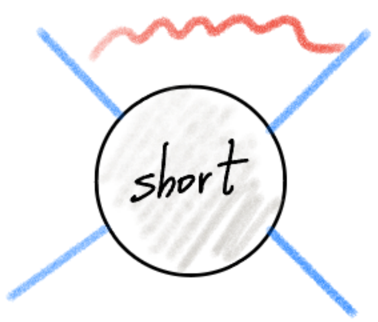
(Ordinary) Amplitudes are <u>NOT</u> good observables

Long-Range forces in D=4

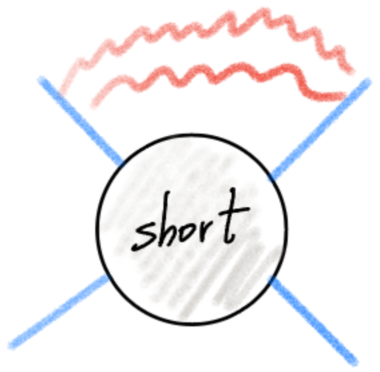
E&M or Gravity

(Ordinary) Amplitudes are <u>NOT</u> good observables

$+$



$+$



$+\dots$

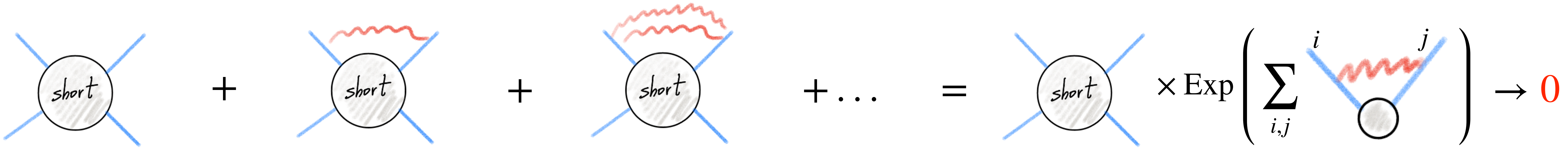
$$\frac{GE^2}{\epsilon} \qquad \left(\frac{GE^2}{\epsilon}\right)^2$$

$$\frac{\alpha}{\epsilon} \qquad \left(\frac{\alpha}{\epsilon}\right)^2$$

Long-Range forces in D=4

E&M or Gravity

(Ordinary) Amplitudes are NOT good observables



$$\frac{GE^2}{\epsilon} \qquad \left(\frac{GE^2}{\epsilon}\right)^2$$

$$\frac{\alpha}{\epsilon} \qquad \left(\frac{\alpha}{\epsilon}\right)^2$$

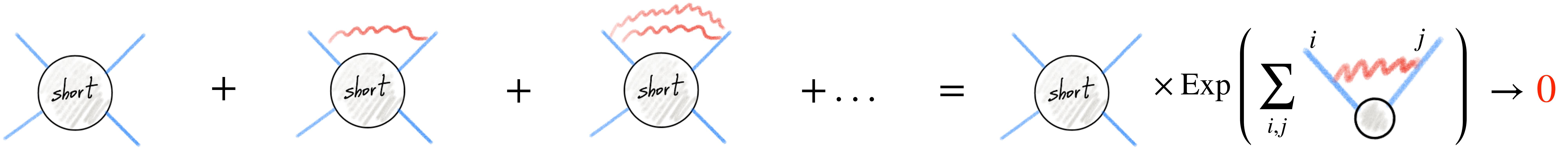
$$\boxed{\mathscr{W}^{\epsilon} \sim e^{\frac{-GE^2}{\epsilon}} \to e^{-\infty}}$$

Weinberg Soft Exp

Long-Range forces in D=4

E&M or Gravity

(Ordinary) Amplitudes are NOT good observables



$\frac{GE^2}{\epsilon}$ $\quad \left(\frac{GE^2}{\epsilon}\right)^2$

$\frac{\alpha}{\epsilon}$ $\quad \left(\frac{\alpha}{\epsilon}\right)^2$

$$\boxed{\mathscr{W}^{\epsilon} \sim e^{\frac{-GE^2}{\epsilon}} \to e^{-\infty}}$$

Weinberg Soft Exp

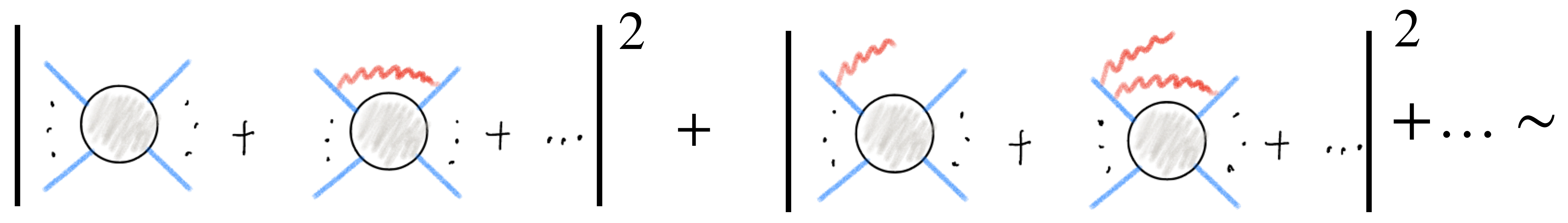
(paradoxical) Lessons

1. NOT a small perturbation, no matter how small G or α
2. No robust bootstrap program
3. UV/IR mixing?? Good place where to look!

Long-Range forces in D=4

E&M or Gravity

Inclusive X-sections? Good observables but …

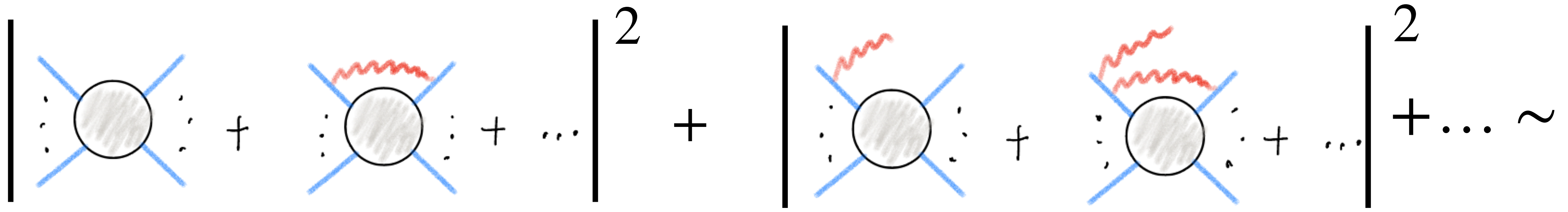


Long-Range forces in D=4

E&M or Gravity

Inclusive X-sections? Good observables but …

…NOT analytic



$$\sim \boxed{\sum_{E_{\gamma,h}<\Delta E}^{\infty} \mathcal{M}^\dagger \mathcal{M}} \quad \propto \left(\frac{\Delta E}{E}\right)^{\alpha, GE^2}$$

Universal log-behavior
on detector resolution

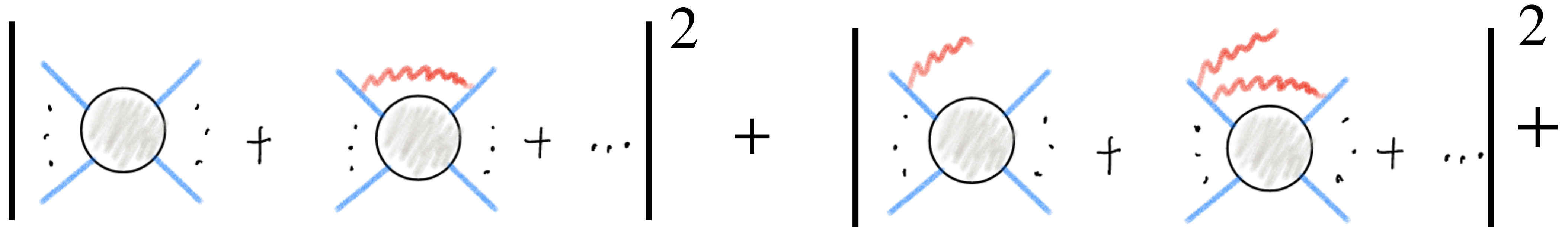
$$d\sigma_{inclusive} \to 0 \text{ if } \Delta E \to 0$$

Long-Range forces in D=4

E&M or Gravity

Inclusive X-sections? Good observables but …

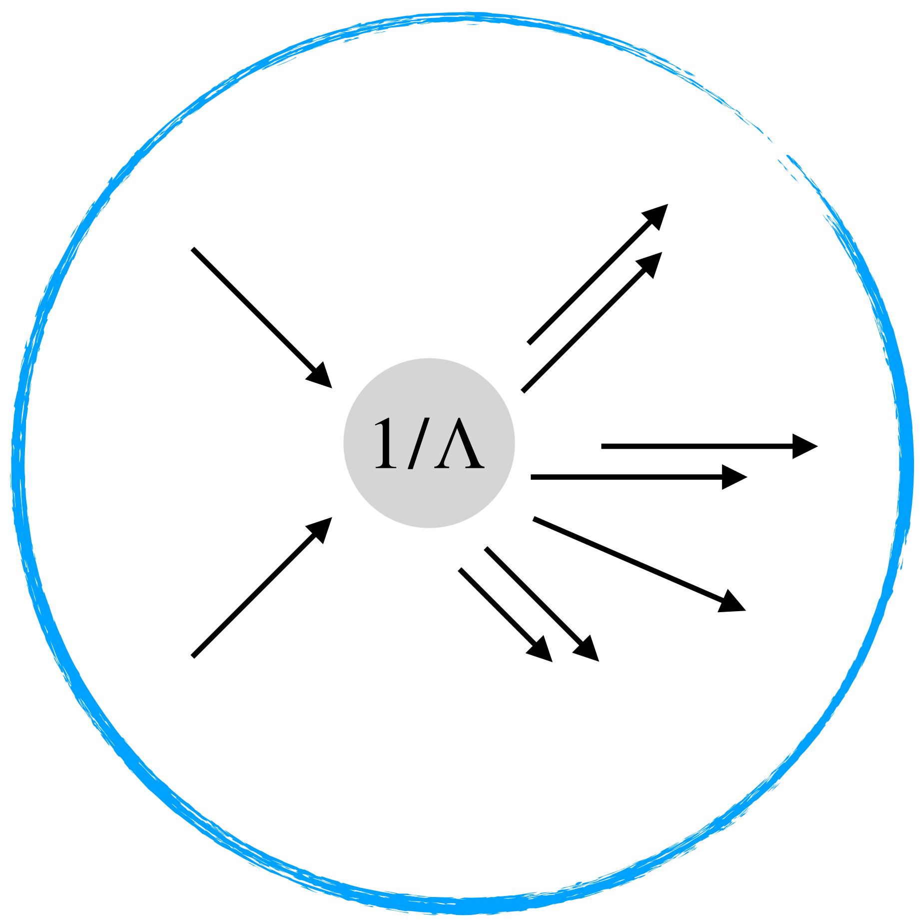
…NOT analytic



$$+\ldots \sim \boxed{\sum_{E_{\gamma,h}<\Delta E}^{\infty} \mathcal{M}^\dagger \mathcal{M}} \quad \propto \left(\frac{\Delta E}{E}\right)^{\alpha, GE^2}$$

Universal log-behavior on detector resolution

$$d\sigma_{inclusive} \to 0 \text{ if } \Delta E \to 0$$



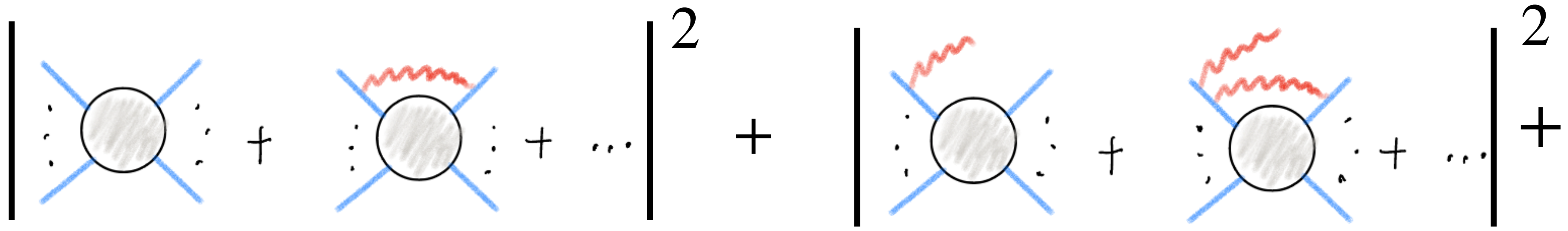
$L = 1/\mathscr{E} \simeq \Delta E_{opt}$

Long-Range forces in D=4

E&M or Gravity

Inclusive X-sections? Good observables but …

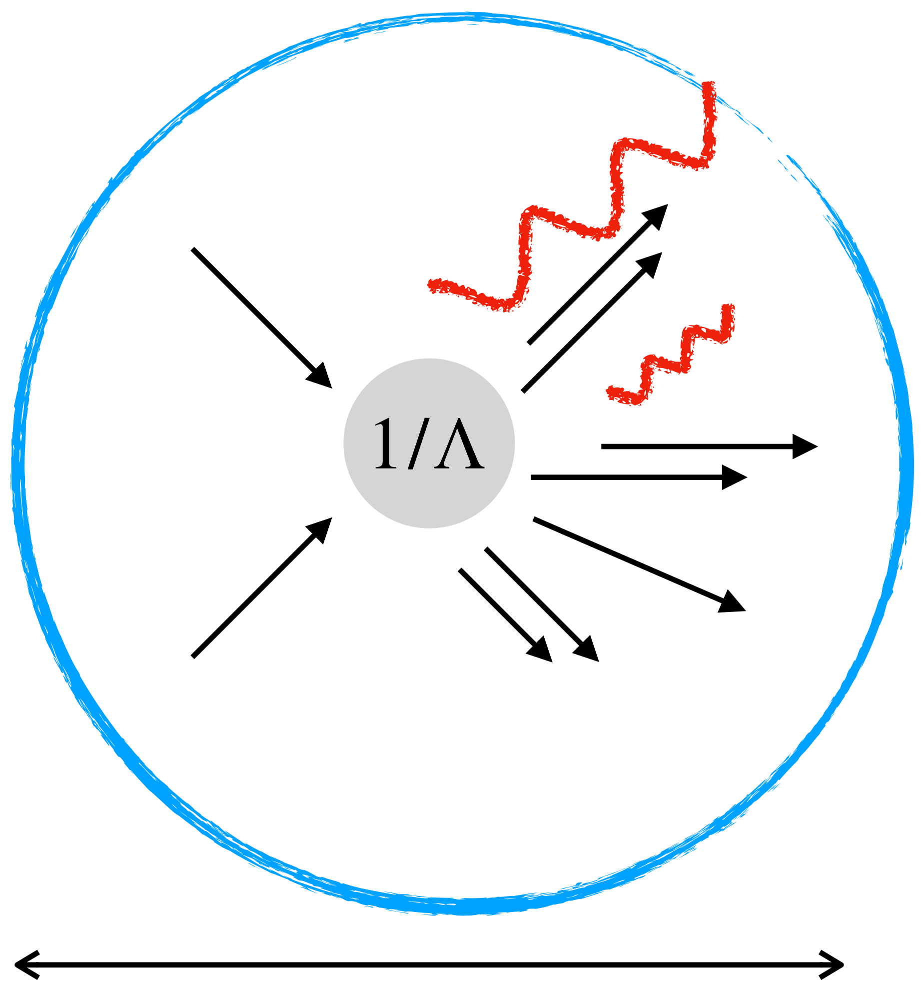
…NOT analytic



$$+\ldots \sim \boxed{\sum_{E_{\gamma,h}<\Delta E}^{\infty} \mathcal{M}^\dagger \mathcal{M}} \propto \left(\frac{\Delta E}{E}\right)^{\alpha, GE^2}$$

Universal log-behavior
on detector resolution

$d\sigma_{inclusive} \to 0$ if $\Delta E \to 0$



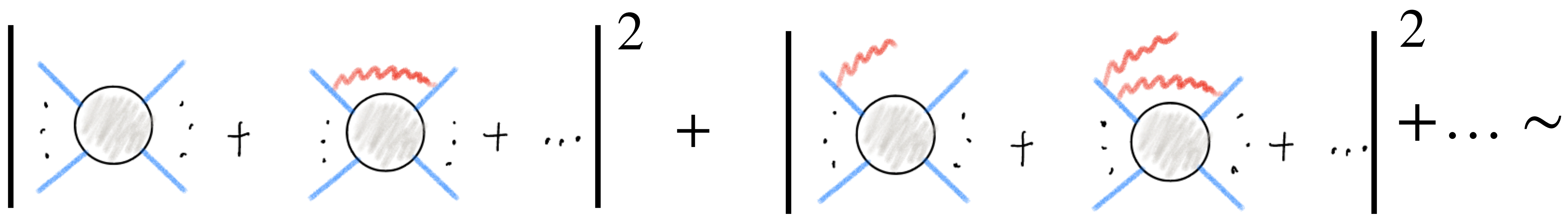
$L = 1/\mathscr{E} \simeq \Delta E_{opt}$

Long-Range forces in D=4

E&M or Gravity

Inclusive X-sections? Good observables but …

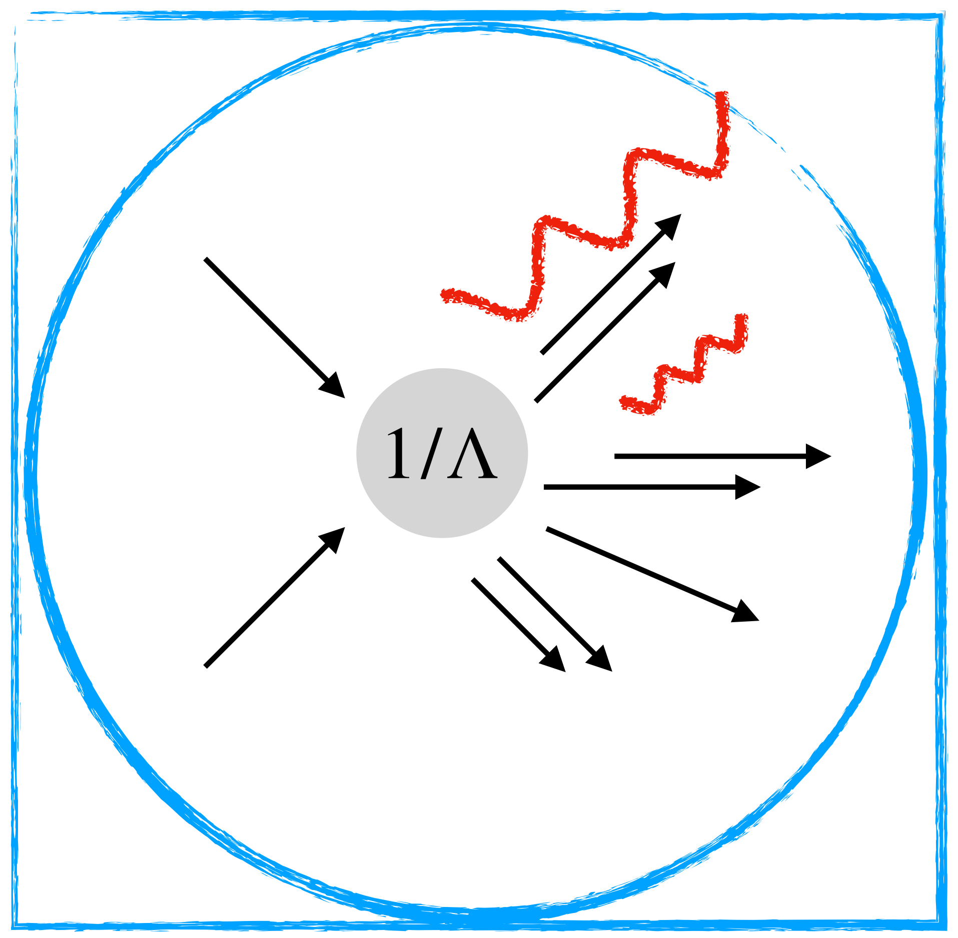
…NOT analytic



$$\sim \sum_{E_{\gamma,h}<\Delta E}^{\infty} \mathscr{M}^{\dagger}\mathscr{M} \propto \left(\frac{\Delta E}{E}\right)^{\alpha, GE^2}$$

Universal log-behavior
on detector resolution

$$d\sigma_{inclusive} \to 0 \text{ if } \Delta E \to 0$$



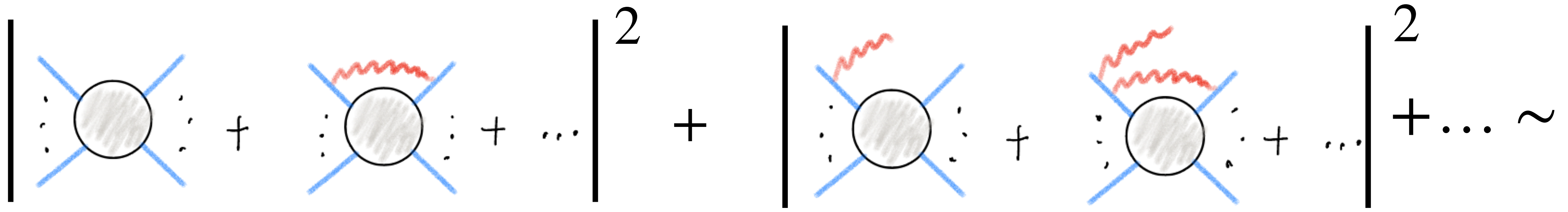
$$L = 1/\mathscr{E} \simeq \Delta E_{opt}$$

Long-Range forces in D=4

E&M or Gravity

Inclusive X-sections? Good observables but …

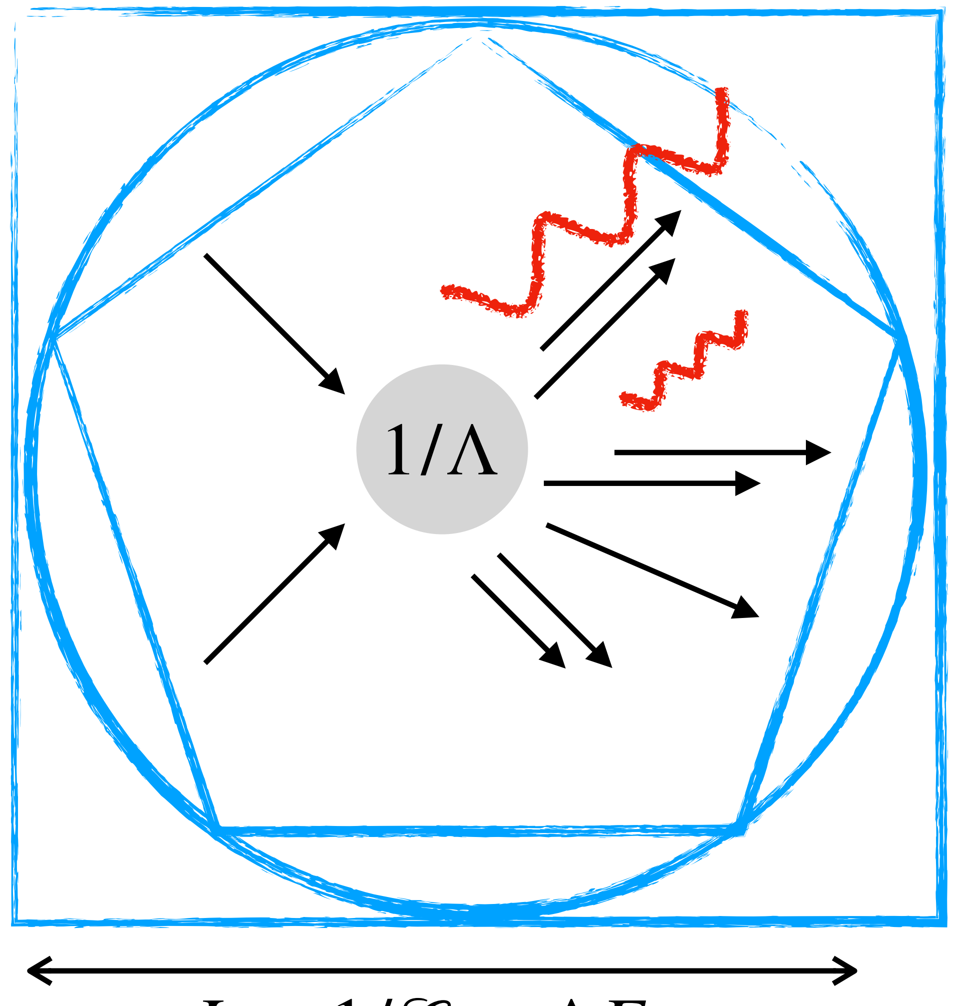
…NOT analytic



$$\sim \sum_{E_{\gamma,h}<\Delta E}^{\infty} \mathscr{M}^\dagger \mathscr{M} \propto \left(\frac{\Delta E}{E}\right)^{\alpha, GE^2}$$

Universal log-behavior
on detector resolution

$$d\sigma_{inclusive} \to 0 \text{ if } \Delta E \to 0$$

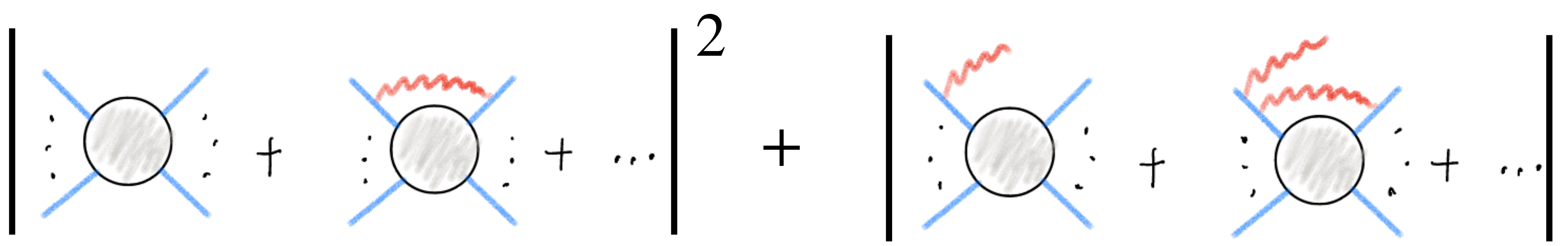


$L = 1/\mathscr{E} \simeq \Delta E_{opt}$

Long-Range forces in D=4

E&M or Gravity

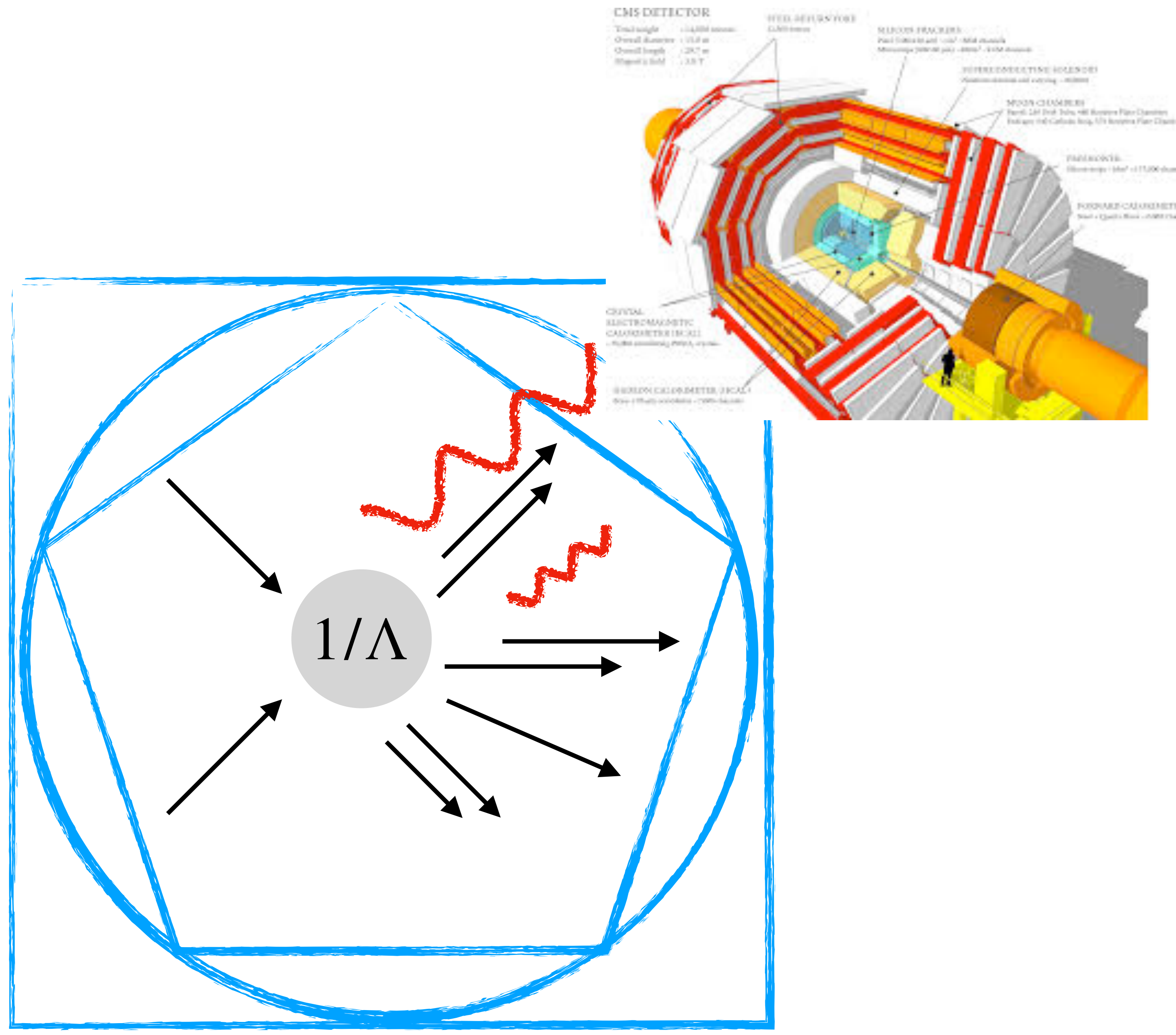
Inclusive X-sections? Good observables but … …NOT analytic



$$+\ldots \sim \boxed{\sum_{E_{\gamma,h}<\Delta E}^{\infty} \mathscr{M}^\dagger \mathscr{M}} \quad \boxed{\propto \left(\frac{\Delta E}{E}\right)^{\alpha, GE^2}}$$

Universal log-behavior on detector resolution

$$d\sigma_{inclusive} \to 0 \text{ if } \Delta E \to 0$$



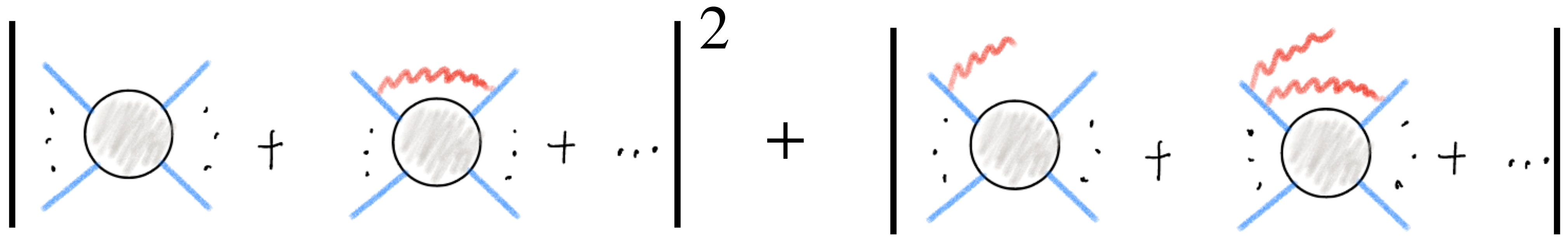
$L = 1/\mathscr{E} \simeq \Delta E_{opt}$

Long-Range forces in D=4

E&M or Gravity

Inclusive X-sections? Good observables but …

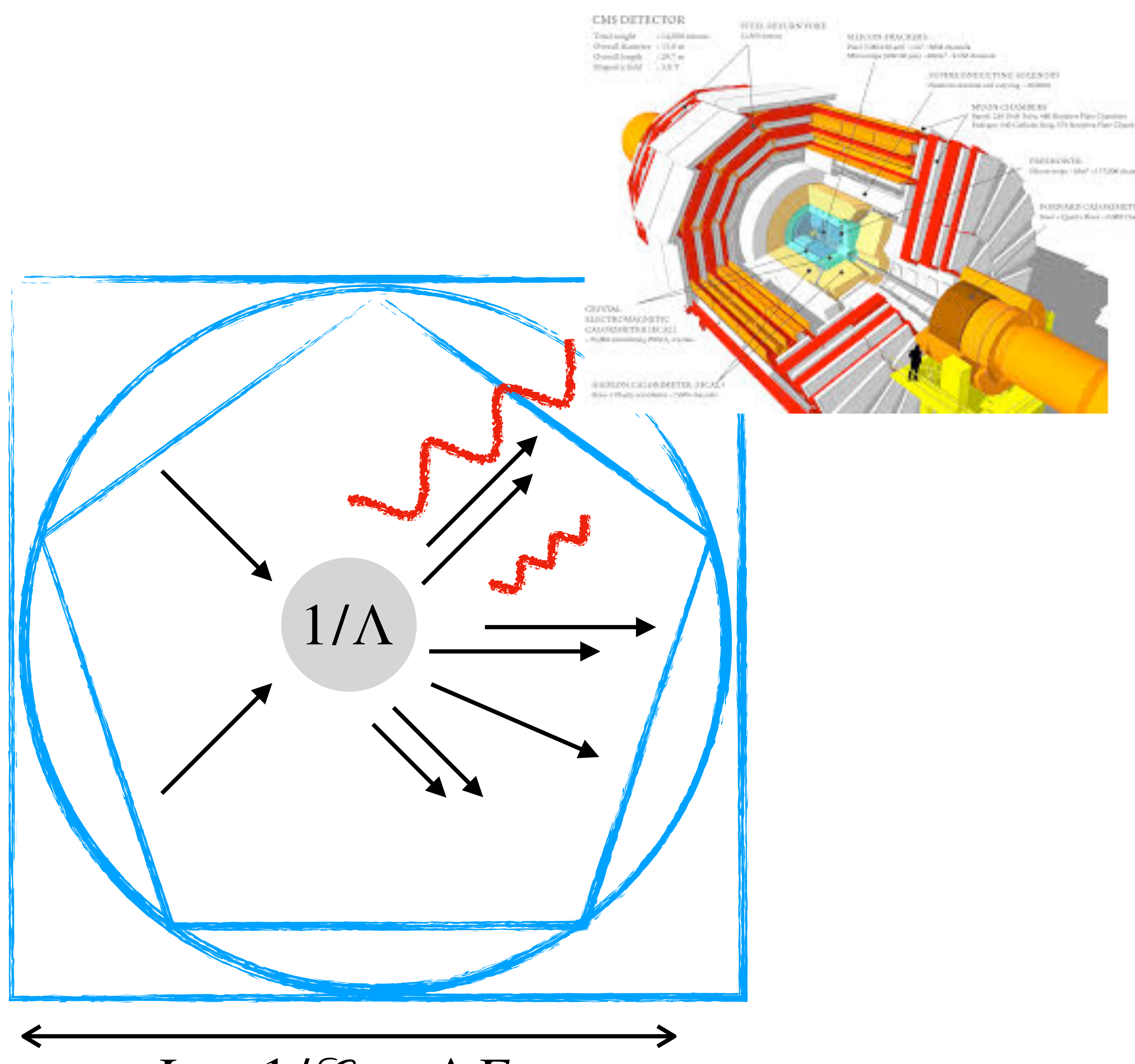
…NOT analytic



$$+\ldots \sim \sum_{E_{\gamma,h}<\Delta E}^{\infty} \mathscr{M}^\dagger \mathscr{M} \propto \left(\frac{\Delta E}{E}\right)^{\alpha, GE^2}$$

Universal log-behavior on detector resolution

$$d\sigma_{inclusive} \to 0 \text{ if } \Delta E \to 0$$



$L = 1/\mathscr{E} \simeq \Delta E_{opt}$

Main lesson:

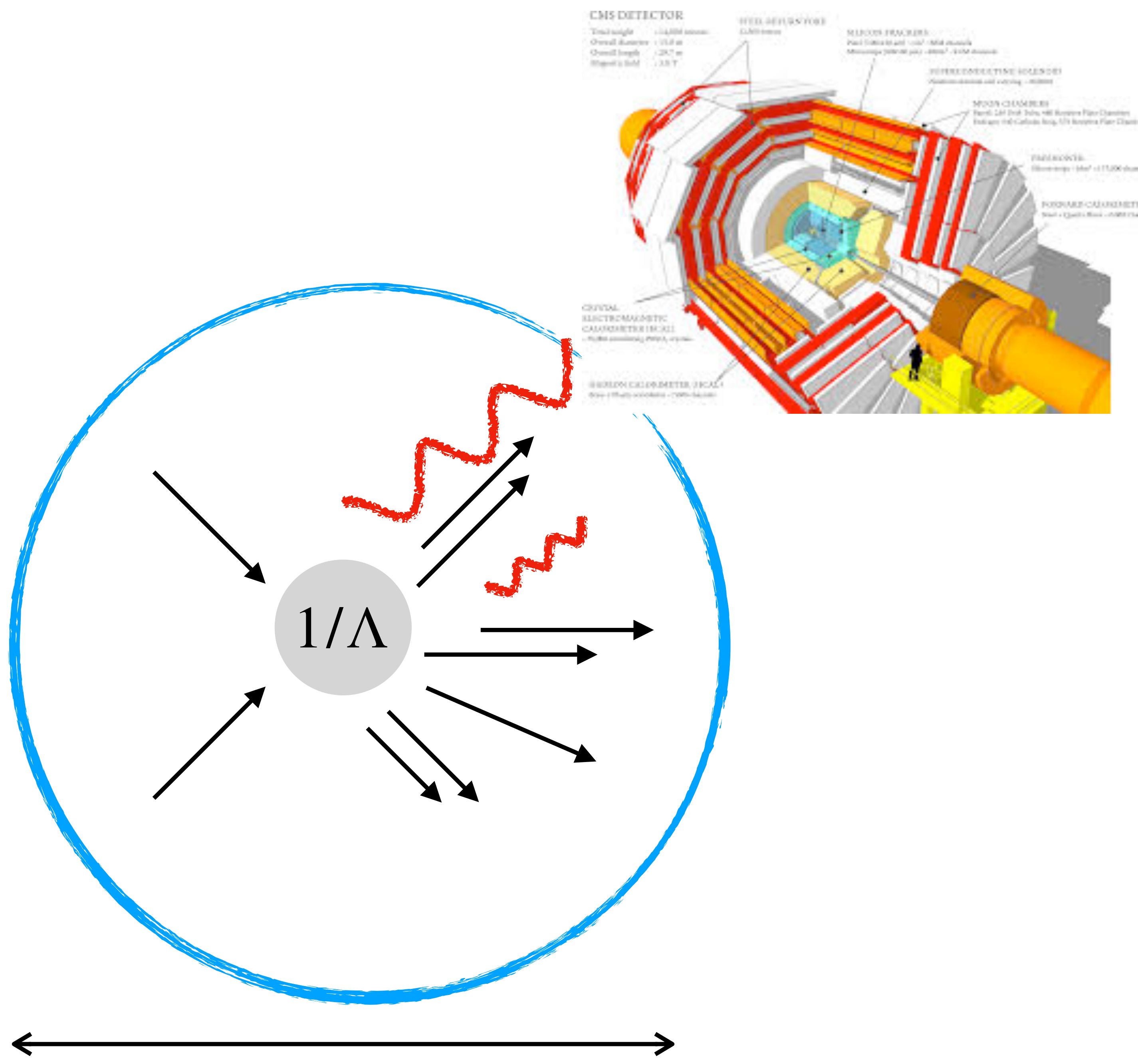
Cannot decouple the detector!

Yet: most of its details are irrelevant

What are good observables?

E&M or Gravity

1. IR finite, 2. not too much IR sensitive, 3. Know about causality??!

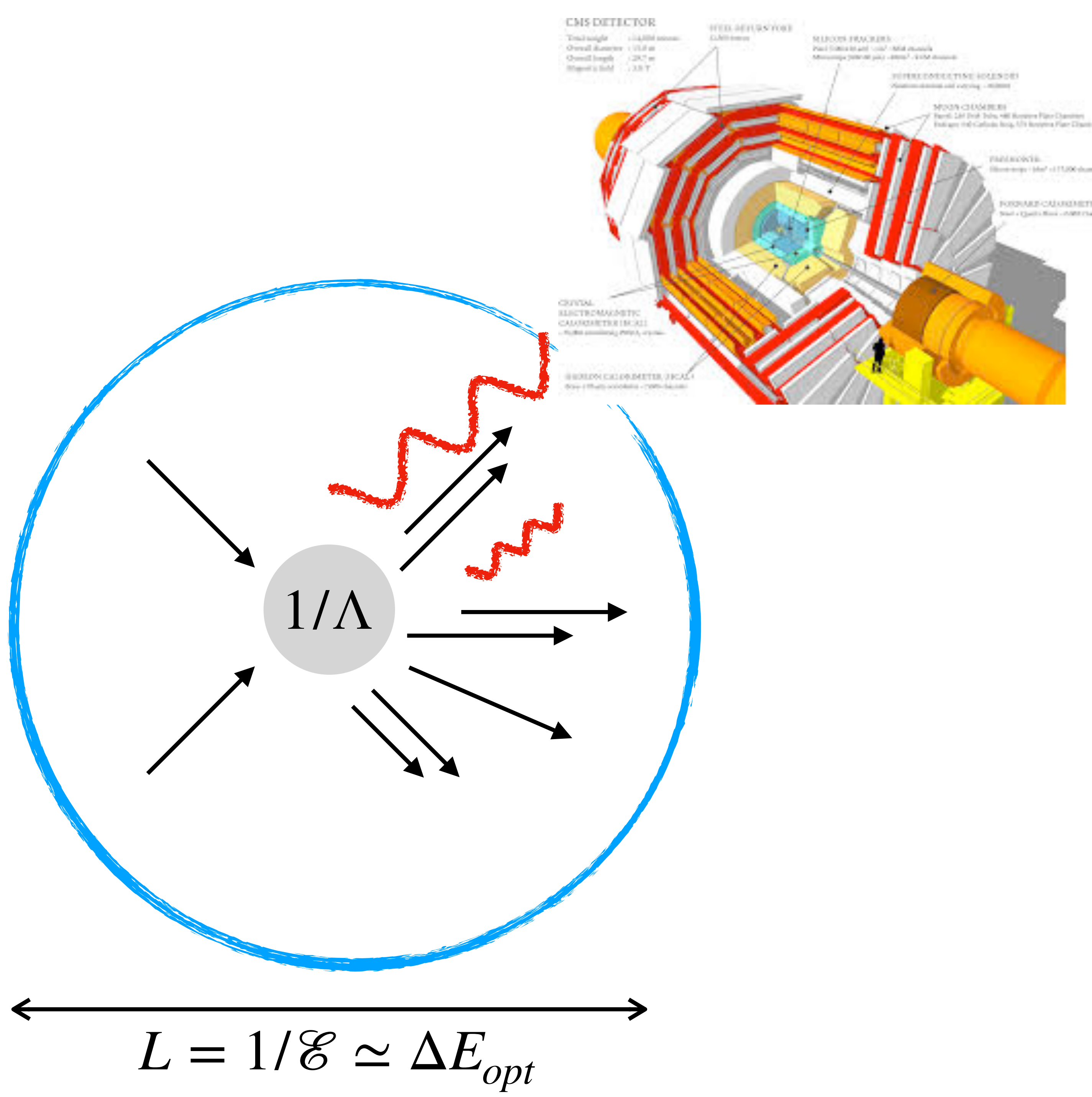


$L = 1/\mathscr{E} \simeq \Delta E_{opt}$

What are good observables?

E&M or Gravity

1. IR finite, 2. not too much IR sensitive, 3. Know about causality??!

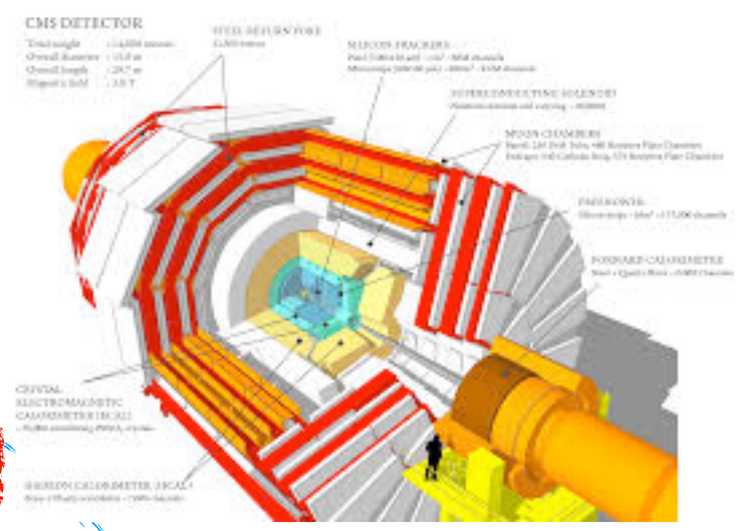


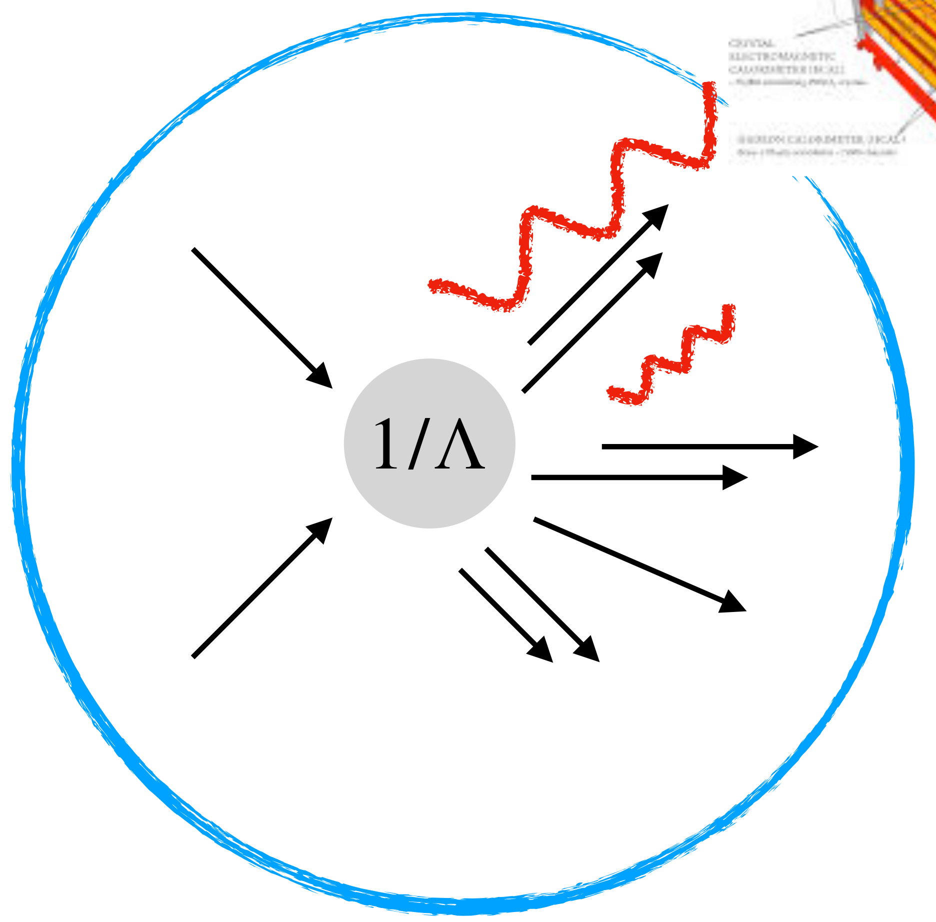
Obs checklist	*IR finite*	*Gauge inv.*	*Causal/ Analytic*	*Un-Ambiguous*
Amplitudes	✗	✓	✓	✓
Inclusive X-sections	✓	✓	✗	✓
Energy-Energy Corr.	✓	✓	✗ ?	✓
Faddeev-Kulish Amp	✓	✓	?	✗
Off-shell Correlators	✓	✓ ✗	?	?
?	✓	✓	✓	✓

What are good observables?

E&M or Gravity

1. IR finite, 2. not too much IR sensitive, 3. Know about causality??!





$L = 1/\mathscr{E} \simeq \Delta E_{opt}$

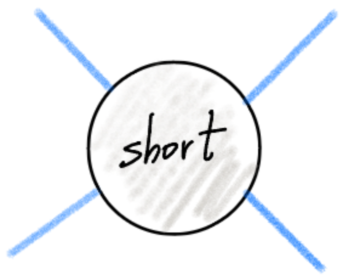
Obs checklist	IR finite	Gauge inv.	Causal/ Analytic	Un-Ambiguous
Amplitudes	✗	✓	✓	✓
Inclusive X-sections	✓	✓	✗	✓
Energy-Energy Corr.	✓	✓	✗ ?	✓
Faddeev-Kulish Amp	✓	✓	?	✗
Off-shell Correlators	✓	✓ ✗	?	?
?	✓	✓	✓	✓

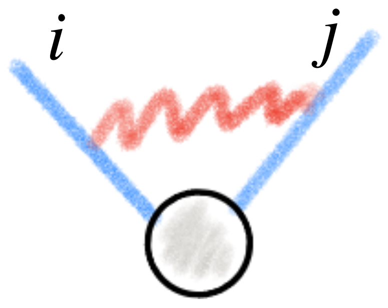
This talk:

Stripped/Detector Amplitudes

Stripped, Detector Amplitudes

Stripped, Detector Amplitudes



$\times \, \mathrm{Exp} \left(\sum_{i,j} \right.$  $\left. \right)$

Weinberg Soft Exp

E&M:

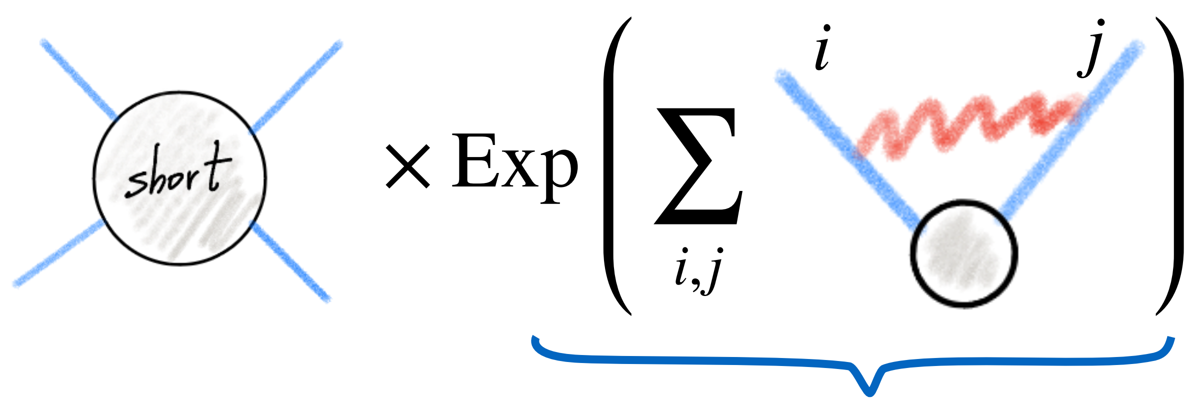
$$\mathscr{W}^{\epsilon}_{\mathscr{E}} = \exp\left\{ \frac{\alpha}{8\pi}\frac{1}{\epsilon}\left[1+\epsilon\log\frac{\mathcal{E}^2}{\mu^2}\right]\left(\sum_{i,j}\eta_i\eta_j q_i q_j \frac{1}{\beta_{ij}}\log\frac{1+\beta_{ij}}{1-\beta_{ij}} - i\sum_{\substack{i\neq j\\ \eta_i\eta_j=1}} q_i q_j \frac{2\pi}{\beta_{ij}}\right)\right\}$$

(Analogous GR))

$\eta_i = \pm = in/out$

$$\beta_{ij}^2 = \frac{\left(s_{ij}-(m_i-m_j)^2\right)\left(s_{ij}-(m_i+m_j)^2\right)}{\left(s_{ij}-m_i^2-m_j^2\right)^2}$$

Stripped, Detector Amplitudes



Weinberg Soft Exp

E&M:

$$\mathscr{W}^{\epsilon}_{\mathscr{E}} = \exp\left\{ \frac{\alpha}{8\pi}\frac{1}{\epsilon}\left[1+\epsilon\log\frac{\mathcal{E}^2}{\mu^2}\right]\left(\sum_{i,j}\eta_i\eta_j q_i q_j \frac{1}{\beta_{ij}}\log\frac{1+\beta_{ij}}{1-\beta_{ij}} - i\sum_{\substack{i\neq j\\ \eta_i\eta_j=1}} q_i q_j \frac{2\pi}{\beta_{ij}}\right)\right\}$$

(Analogous GR))

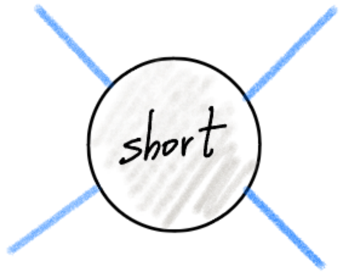
$\eta_i = \pm = in/out$

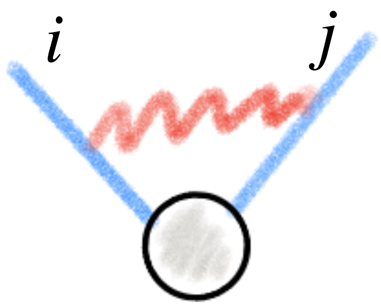
$$\beta_{ij}^2 = \frac{\left(s_{ij}-(m_i-m_j)^2\right)\left(s_{ij}-(m_i+m_j)^2\right)}{\left(s_{ij}-m_i^2-m_j^2\right)^2}$$

$$\cancel{\frac{\text{Amplitude}}{\text{Weinberg Soft Exp}}} = \text{“Stripped Amplitude”} = \lim_{\epsilon\to 0}\frac{\mathscr{M}^{\epsilon}}{\mathscr{W}^{\epsilon}_{\mathscr{E}}} = \mathscr{M}_{\mathscr{E}}$$

1-parameter $\mathscr{E}$
family of amplitudes

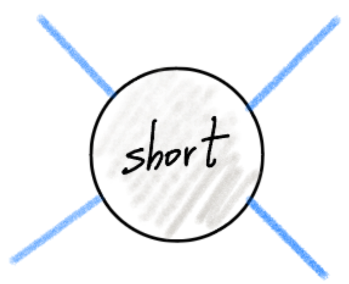
Stripped, Detector Amplitudes

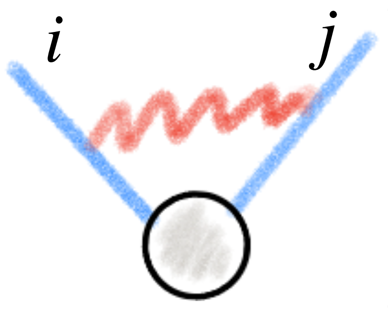


$\times \, \mathrm{Exp}\Bigg(\sum_{i,j}$  $\Bigg)$

$$\frac{\text{Amplitude}}{\text{Weinberg Soft Exp}} = \text{“Stripped Amplitude”} = \lim_{\epsilon \to 0} \frac{\mathscr{M}^{\epsilon}}{\mathscr{W}^{\epsilon}_{\mathscr{E}}} = \mathscr{M}_{\mathscr{E}}$$

Stripped, Detector Amplitudes



$\times\,\mathrm{Exp}\Bigg(\sum_{i,j}$ 

$\Bigg)$

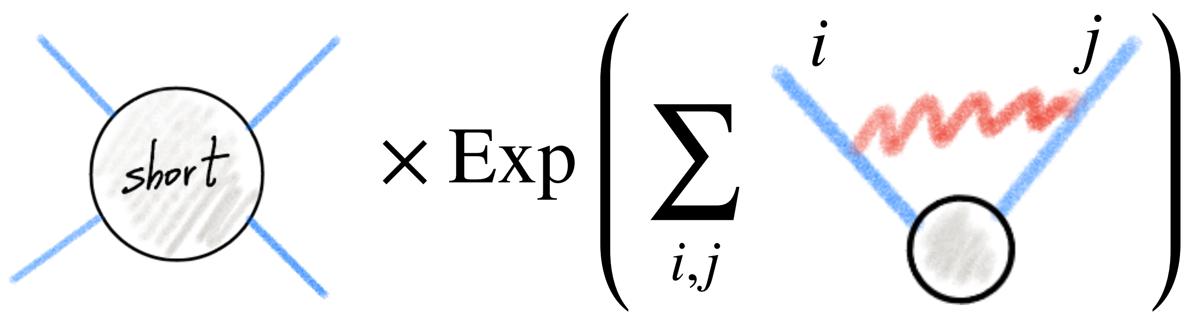
$$\frac{\text{Amplitude}}{\text{Weinberg Soft Exp}} = \text{“Stripped Amplitude”} = \lim_{\epsilon\to 0}\frac{\mathcal{M}^{\epsilon}}{\mathcal{W}^{\epsilon}_{\mathcal{E}}} = \mathcal{M}_{\mathcal{E}}$$

$\mathcal{M}_{\mathcal{E}}$ is

$$\left.\begin{array}{l} \text{1. IR finite} \\ \text{2. Lorentz invariant} \\ \text{3. Analytic \& X-ssing} \\ \text{4. Regge/Froissart Bound} \end{array}\right\} \text{Exactly}$$

BB, Berman, Isabella, Riva, Romano, Sciotti 2512.13780

Stripped, Detector Amplitudes



$$\frac{\text{Amplitude}}{\text{Weinberg Soft Exp}} = \text{“Stripped Amplitude”} = \lim_{\epsilon \to 0} \frac{\mathscr{M}^{\epsilon}}{\mathscr{W}^{\epsilon}_{\mathscr{E}}} = \mathscr{M}_{\mathscr{E}}$$

$\mathscr{M}_{\mathscr{E}}$ is

1. IR finite
2. Lorentz invariant
3. Analytic & X-ssing
4. Regge/Froissart Bound

} Exactly

5. Unitary
6. Universal, $\mathscr{E} \sim \Delta E$

} In a limit

BB, Berman, Isabella, Riva, Romano, Sciotti 2512.13780

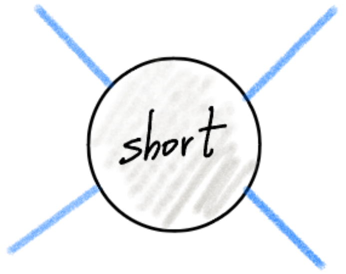
“Detector Amplitude”

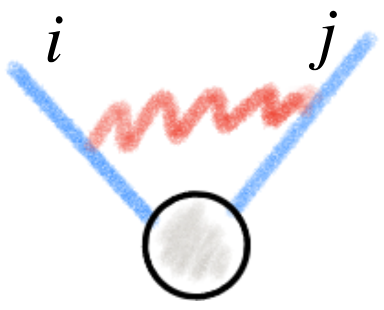
$$\mathscr{M}_{\mathscr{E}} = \mathscr{M}(\alpha_{\mathscr{E}}, G_{\mathscr{E}}, g) + O(\alpha, G, \mathscr{E})$$

$\alpha \to 0, \quad \alpha \log \mathscr{E} = \alpha_{\mathscr{E}}$ - fixed (all order)

$G \to 0, \quad G \log \mathscr{E} = G_{\mathscr{E}}$ -fixed (all order)

Stripped, Detector Amplitudes



$\times\,\mathrm{Exp}\Big(\sum_{i,j}$ 

$\Big)$

$$\frac{\text{Amplitude}}{\text{Weinberg Soft Exp}} = \text{“Stripped Amplitude”} = \lim_{\epsilon\to 0}\frac{\mathscr{M}^{\epsilon}}{\mathscr{W}^{\epsilon}_{\mathscr{E}}} = \mathscr{M}_{\mathscr{E}}$$

$\mathscr{M}_{\mathscr{E}}$ is

1. IR finite
2. Lorentz invariant
3. Analytic & X-ssing
4. Regge/Froissart Bound
5. Unitary
6. Universal, $\mathscr{E} \sim \Delta E$

Items 1–4: Exactly

Items 5–6: In a limit

BB, Berman, Isabella, Riva, Romano, Sciotti 2512.13780

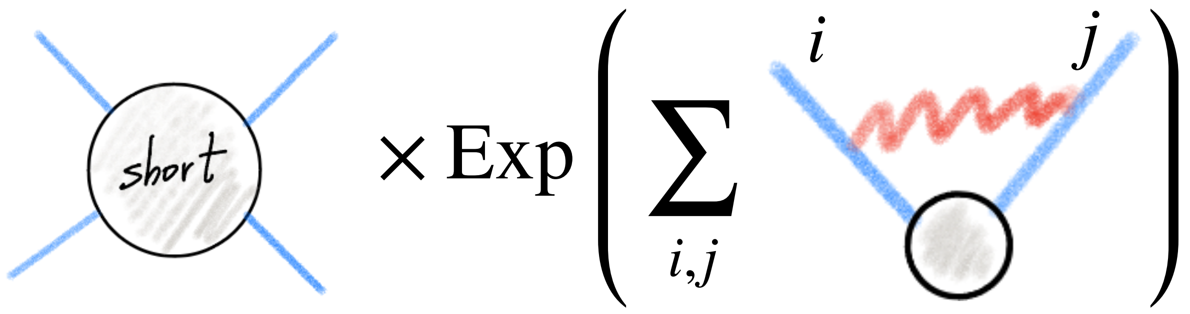
“Detector Amplitude”

$$\mathscr{M}_{\mathscr{E}} = \mathscr{M}(\alpha_{\mathscr{E}}, G_{\mathscr{E}}, g) + O(\alpha, G, \mathscr{E})$$

$\alpha \to 0, \quad \alpha \log \mathscr{E} = \alpha_{\mathscr{E}}$ - fixed (all order)

$G \to 0, \quad G \log \mathscr{E} = G_{\mathscr{E}}$ -fixed (all order)

Stripped, Detector Amplitudes



$$\frac{\text{Amplitude}}{\text{Weinberg Soft Exp}} = \text{“Stripped Amplitude”} = \lim_{\epsilon\to 0}\frac{\mathscr{M}^{\epsilon}}{\mathscr{W}^{\epsilon}_{\mathscr{E}}} = \mathscr{M}_{\mathscr{E}}$$

$\mathscr{M}_{\mathscr{E}}$ is

1. IR finite ✓
2. Lorentz invariant ✓
3. Analytic & X-ssing
4. Regge/Froissart Bound

} Exactly

5. Unitary
6. Universal, $\mathscr{E} \sim \Delta E$

} In a limit

BB, Berman, Isabella, Riva, Romano, Sciotti 2512.13780

“Detector Amplitude”

$$\mathscr{M}_{\mathscr{E}} = \mathscr{M}(\alpha_{\mathscr{E}}, G_{\mathscr{E}}, g) + O(\alpha, G, \mathscr{E})$$

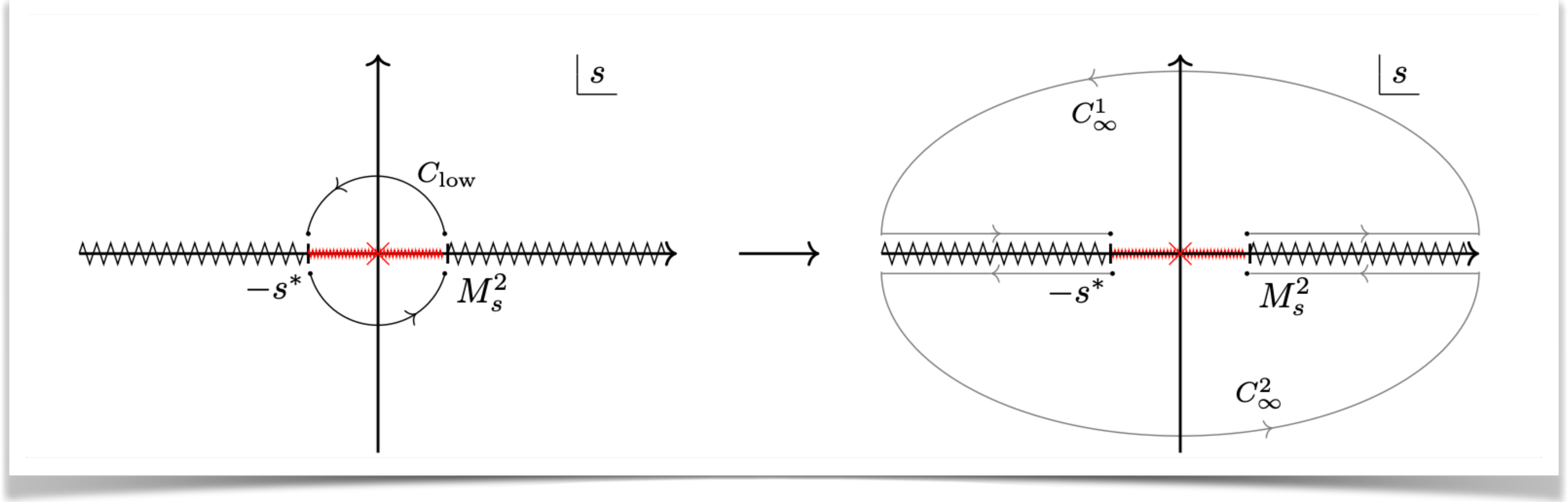
$\alpha \to 0, \quad \alpha \log \mathscr{E} = \alpha_{\mathscr{E}}$ - fixed (all order)

$G \to 0, \quad G \log \mathscr{E} = G_{\mathscr{E}}$ -fixed (all order)

Stripped-Amplitudes:

~~good observables~~ & inputs bootstrap/positivity

Tool!



Analyticity & Crossing

Analyticity & Crossing

$$\frac{\text{Amplitude}}{\text{Weinberg Soft Exp}}$$

Analytic

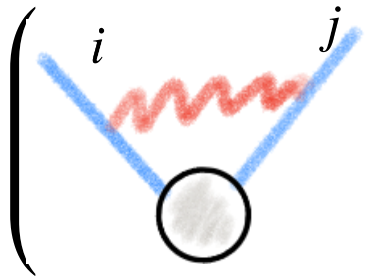
NOT (manifestly) analytic
and crossing symmetric

Analyticity & Crossing

1. defined on real kinematics

2. distinguish ingoing/outgoing

$$\frac{\text{Amplitude}}{\text{Weinberg Soft Exp}} \qquad \prod_{ij}\mathscr{W}_{ij} = \prod_{ij}\mathrm{Exp}\left(\right)$$



$$\overset{\text{GR}}{=} \exp\left\{-\frac{G}{8\pi}\frac{1}{\epsilon}\left[1+\epsilon\log\frac{\mathcal{E}^2}{\mu^2}\right]\left(\sum_{i,j}\eta_i\eta_j m_i m_j\frac{1+\beta_{ij}^2}{\beta_{ij}\sqrt{1-\beta_{ij}^2}}\log\frac{1+\beta_{ij}}{1-\beta_{ij}} - i\sum_{\substack{i\neq j\\ \eta_i\eta_j=1}} m_i m_j\frac{2\pi(1+\beta_{ij}^2)}{\beta_{ij}\sqrt{1-\beta_{ij}^2}}\right)\right\}$$

Weinberg '65

Analytic

NOT (manifestly) analytic and crossing symmetric

$$\beta_{ij}^2 = \frac{\left(s_{ij}-(m_i-m_j)^2\right)\left(s_{ij}-(m_i+m_j)^2\right)}{\left(s_{ij}-m_i^2-m_j^2\right)^2}$$

Analyticity & Crossing

1. defined on real kinematics

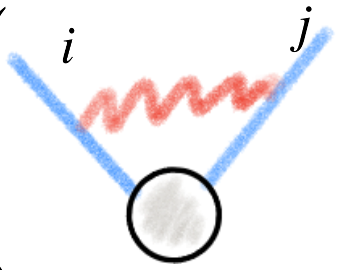
2. distinguish ingoing/outgoing

$$\frac{\text{Amplitude}}{\text{Weinberg Soft Exp}}$$

Analytic

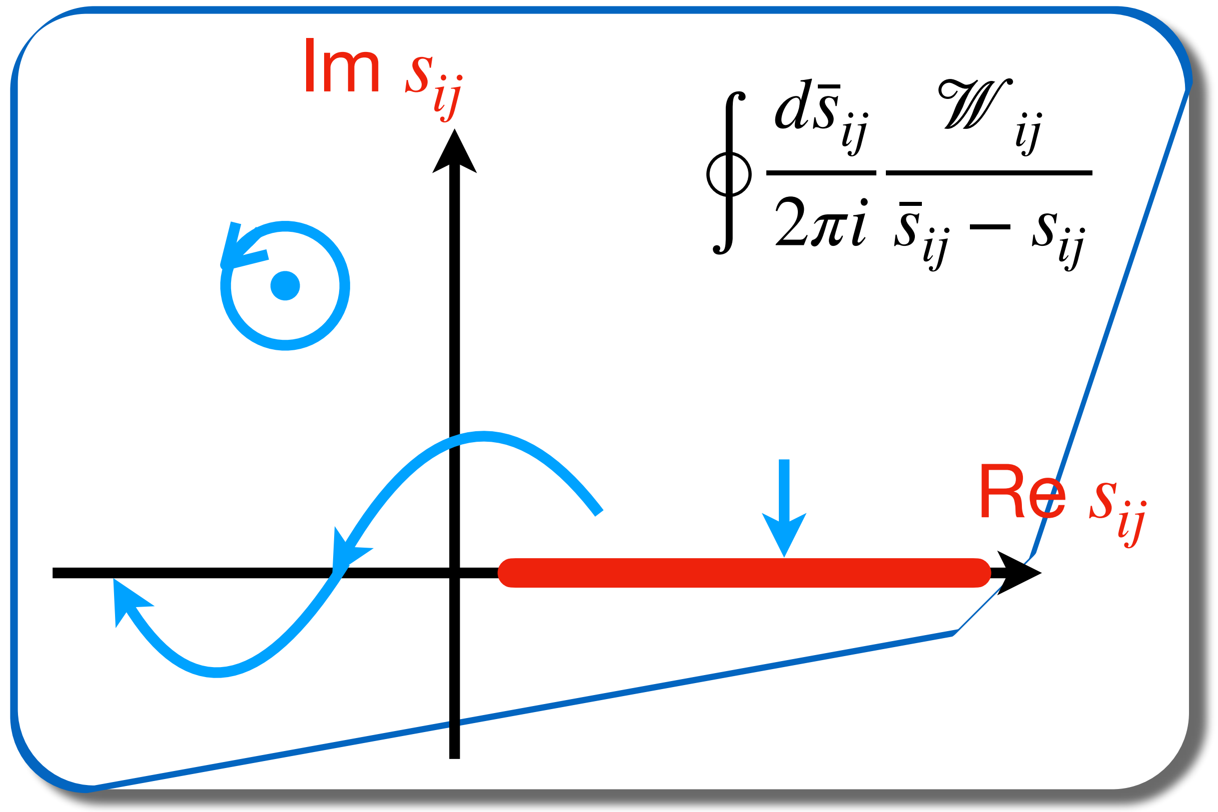
NOT (manifestly) analytic
and crossing symmetric

$$\prod_{ij}\mathscr{W}_{ij} = \prod_{ij}\mathrm{Exp}\left(\quad \right) \overset{\text{GR}}{=} \exp\left\{-\frac{G}{8\pi}\frac{1}{\epsilon}\left[1+\epsilon\log\frac{\mathcal{E}^2}{\mu^2}\right]\left(\underbrace{\sum_{i,j}\eta_i\eta_j m_i m_j \frac{1+\beta_{ij}^2}{\beta_{ij}\sqrt{1-\beta_{ij}^2}}\log\frac{1+\beta_{ij}}{1-\beta_{ij}}}_{\text{Real part}} - i\underbrace{\sum_{\substack{i\neq j\\ \eta_i\eta_j=1}} m_i m_j\frac{2\pi(1+\beta_{ij}^2)}{\beta_{ij}\sqrt{1-\beta_{ij}^2}}}_{\text{Imaginary part}}\right)\right\}$$



Weinberg '65

$$\beta_{ij}^2 = \frac{\left(s_{ij}-(m_i-m_j)^2\right)\left(s_{ij}-(m_i+m_j)^2\right)}{\left(s_{ij}-m_i^2-m_j^2\right)^2}$$



Analyticity & Crossing

$$\frac{\text{Amplitude}}{\text{Weinberg Soft Exp}}$$

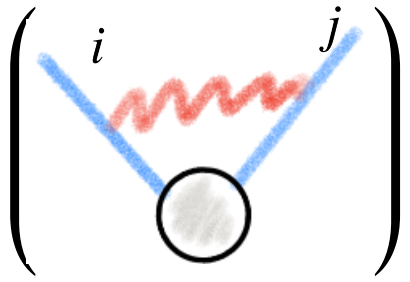
Analytic

NOT (manifestly) analytic
and crossing symmetric

1. defined on real kinematics

2. distinguish ingoing/outgoing

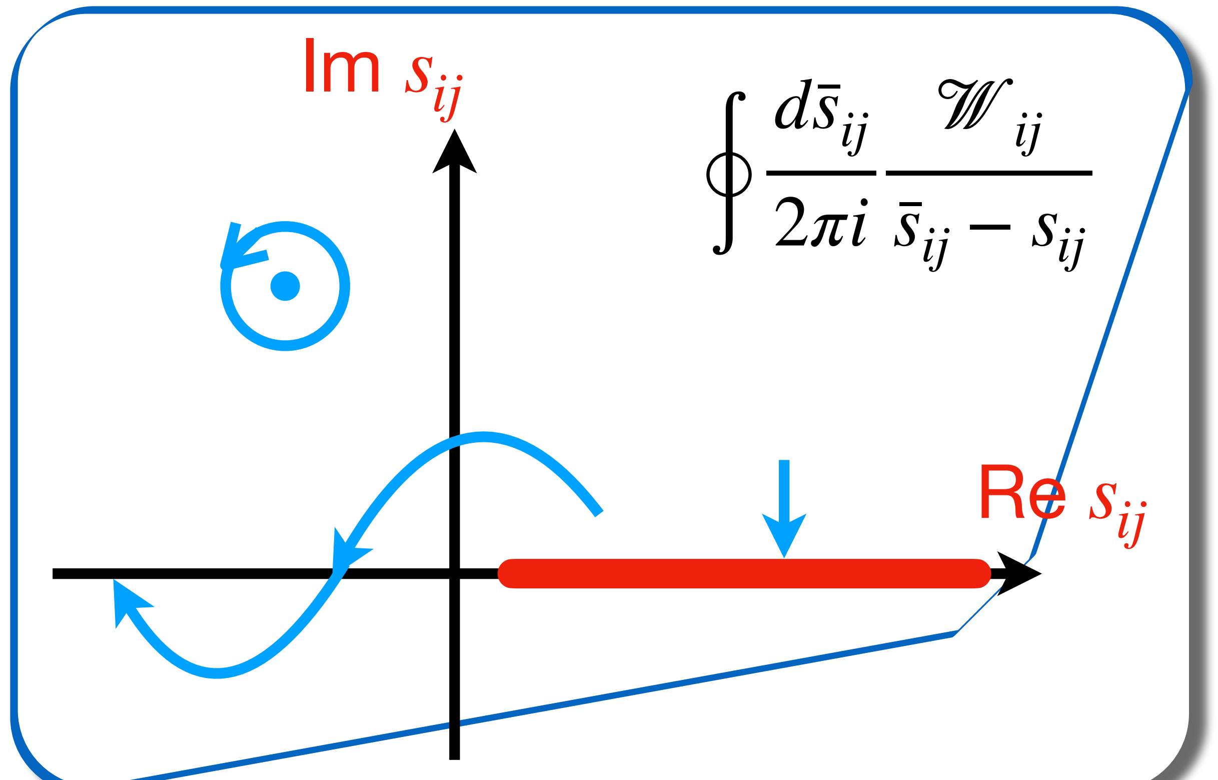
$$\prod_{ij} \mathscr{W}_{ij} = \prod_{ij} \mathrm{Exp}\left(\cdots\right) \overset{\text{GR}}{=} \exp\left\{-\frac{G}{8\pi}\frac{1}{\epsilon}\left[1+\epsilon\log\frac{\mathcal{E}^2}{\mu^2}\right]\left(\underbrace{\sum_{i,j}\eta_i\eta_j m_i m_j \frac{1+\beta_{ij}^2}{\beta_{ij}\sqrt{1-\beta_{ij}^2}}\log\frac{1+\beta_{ij}}{1-\beta_{ij}}}_{\text{Real part}} - i\underbrace{\sum_{\substack{i\neq j\\ \eta_i\eta_j=1}} m_i m_j \frac{2\pi(1+\beta_{ij}^2)}{\beta_{ij}\sqrt{1-\beta_{ij}^2}}}_{\text{Imaginary part}}\right)\right\}$$



Weinberg '65

$$\beta_{ij}^2 = \frac{\left(s_{ij}-(m_i-m_j)^2\right)\left(s_{ij}-(m_i+m_j)^2\right)}{\left(s_{ij}-m_i^2-m_j^2\right)^2}$$

$$\oint \frac{d\bar{s}_{ij}}{2\pi i}\frac{\mathscr{W}_{ij}}{\bar{s}_{ij}-s_{ij}}$$



"The Analytic Soft Exp"

BB, Berman, Isabella, Riva, Romano, Sciotti '25

$$\mathcal{W}_{ij} \overset{\text{GR}}{\equiv} \begin{cases} \exp\left[\frac{G}{2\pi}\frac{1}{\epsilon}\left(1+\epsilon\log\frac{\mathcal{E}^2}{\mu^2}\right)\frac{(s_{ij}-m_i^2-m_j^2)^2-2m_i^2m_j^2}{\sqrt{(s_{ij}-(m_i-m_j)^2)(-s+(m_i+m_j)^2)}}\arctan\frac{\sqrt{s_{ij}-(m_i-m_j)^2}}{\sqrt{-s_{ij}+(m_i+m_j)^2}}\right] & i\neq j \\ \exp\left[-\frac{Gm_i^2}{4\pi}\frac{1}{\epsilon}\left(1+\epsilon\log\frac{\mathcal{E}^2}{\mu^2}\right)\right] & i=j \end{cases}$$

$m_{i,j} \to 0$

$$\mathcal{W}_{ij} = \exp\left[-\frac{1}{4\pi\epsilon}\left(1+\epsilon\log\frac{\mathcal{E}^2}{\mu^2}\right)G s_{ij}\log\left(\frac{-s_{ij}}{\mu^2}\right)\right]$$

Analyticity & Crossing ✅

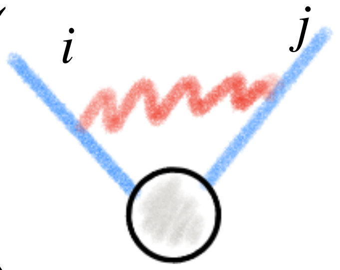
$\frac{\text{Amplitude}}{\text{Weinberg Soft Exp}}$

Analytic

NOT (manifestly) analytic and crossing symmetric

1. defined on real kinematics

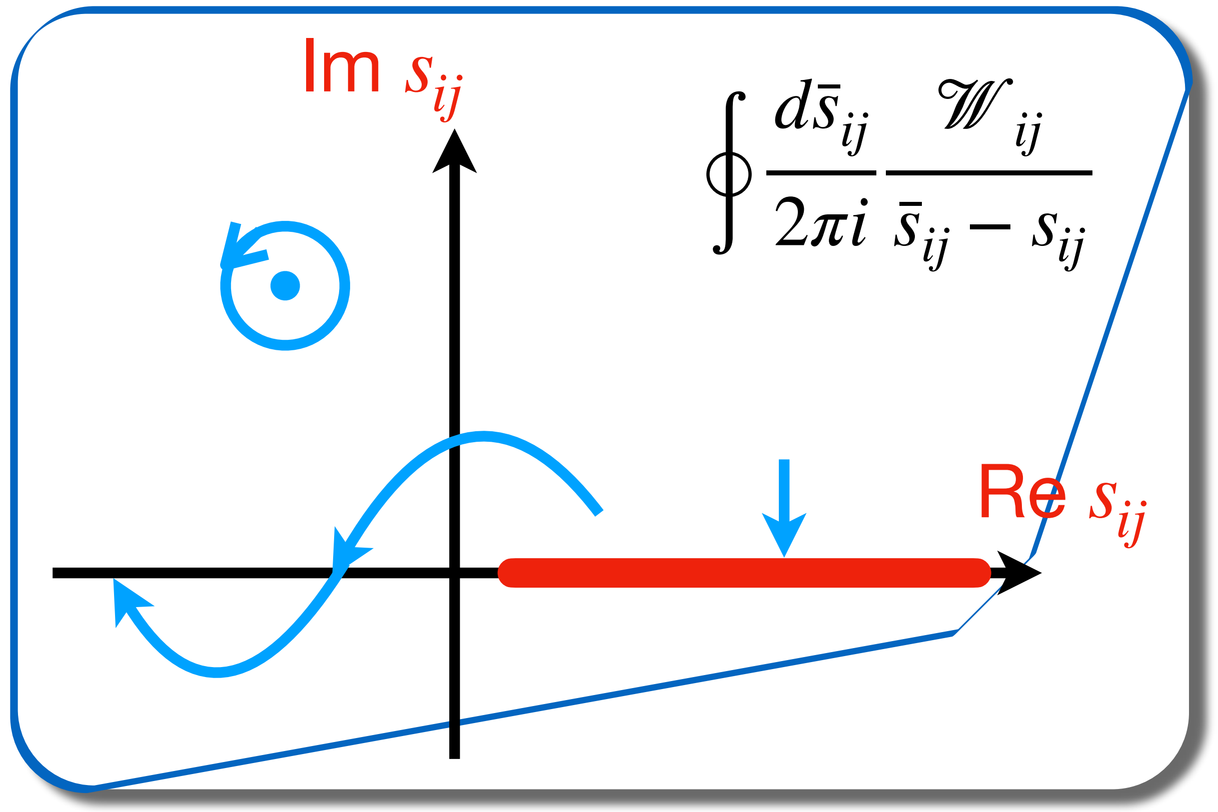
2. distinguish ingoing/outgoing

$\prod_{ij} \mathscr{W}_{ij} = \prod_{ij} \mathrm{Exp}$ ()

$$\overset{\text{GR}}{=} \exp\left\{ -\frac{G}{8\pi}\frac{1}{\epsilon}\left[1+\epsilon\log\frac{\mathcal{E}^2}{\mu^2}\right]\left(\underbrace{\sum_{i,j} \eta_i\eta_j m_i m_j \frac{1+\beta_{ij}^2}{\beta_{ij}\sqrt{1-\beta_{ij}^2}}\log\frac{1+\beta_{ij}}{1-\beta_{ij}}}_{\text{Real part}} - i \underbrace{\sum_{\substack{i\neq j\\ \eta_i\eta_j=1}} m_i m_j \frac{2\pi(1+\beta_{ij}^2)}{\beta_{ij}\sqrt{1-\beta_{ij}^2}}}_{\text{Imaginary part}} \right)\right\}$$

Weinberg ‘65

$$\beta_{ij}^2 = \frac{\left(s_{ij}-(m_i-m_j)^2\right)\left(s_{ij}-(m_i+m_j)^2\right)}{\left(s_{ij}-m_i^2-m_j^2\right)^2}$$



$$\oint \frac{d\bar{s}_{ij}}{2\pi i}\frac{\mathscr{W}_{ij}}{\bar{s}_{ij}-s_{ij}}$$

“The Analytic Soft Exp”

BB, Berman, Isabella, Riva, Romano, Sciotti ‘25

$$\mathcal{W}_{ij} \overset{\text{GR}}{\equiv} \begin{cases} \exp\left[\frac{G}{2\pi}\frac{1}{\epsilon}\left(1+\epsilon\log\frac{\mathcal{E}^2}{\mu^2}\right)\frac{(s_{ij}-m_i^2-m_j^2)^2-2m_i^2m_j^2}{\sqrt{(s_{ij}-(m_i-m_j)^2)(-s+(m_i+m_j)^2)}}\arctan\frac{\sqrt{s_{ij}-(m_i-m_j)^2}}{\sqrt{-s_{ij}+(m_i+m_j)^2}}\right] & i\neq j \\ \exp\left[-\frac{Gm_i^2}{4\pi}\frac{1}{\epsilon}\left(1+\epsilon\log\frac{\mathcal{E}^2}{\mu^2}\right)\right] & i=j \end{cases}$$

$m_{i,j} \to 0$

$$\mathcal{W}_{ij} = \exp\left[-\frac{1}{4\pi\epsilon}\left(1+\epsilon\log\frac{\mathcal{E}^2}{\mu^2}\right)G s_{ij}\log\left(\frac{-s_{ij}}{\mu^2}\right)\right]$$

Regge-scaling

Regge-scaling

- Gapped theory

  Froissart 61, Martin '63

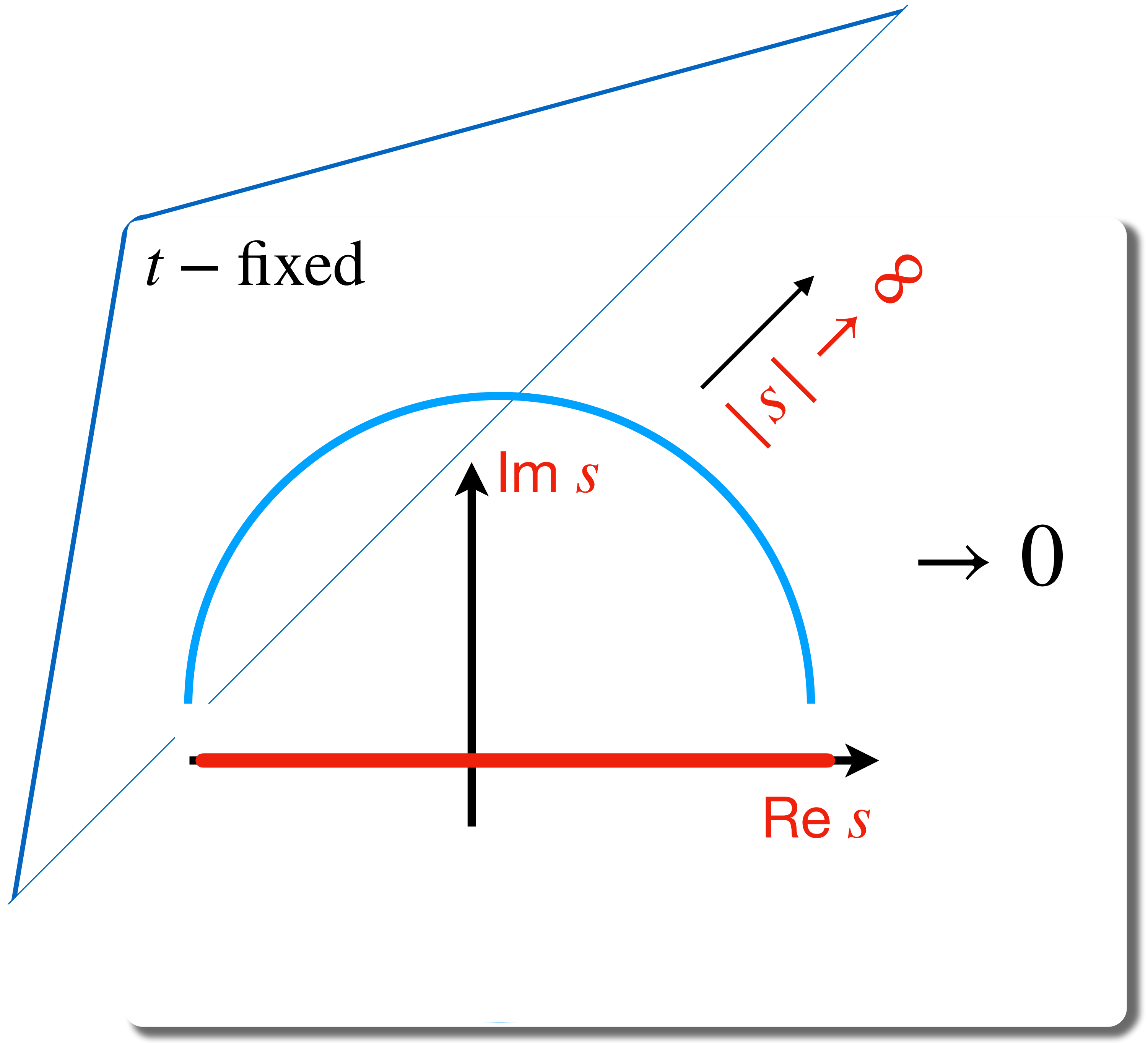
$$\frac{\mathcal{M}(s,t)}{s^2} \to 0$$

- Gravity in D>4

  Haring Zhiboedov '22

  Caron-hoot, Parra-Martinez, Li, Simmons-Duffin '22

$$\int dq \psi(q) \frac{\mathcal{M}(s,-q^2)}{s^2} \to 0$$



Regge-scaling

- Gapped theory

  Froissart 61, Martin '63

$$\frac{\mathcal{M}(s,t)}{s^2} \to 0$$

- Gravity in D>4

  Haring Zhiboedov '22

  Caron-hoot, Parra-Martinez, Li, Simmons-Duffin '22

$$\int dq \psi(q) \frac{\mathcal{M}(s,-q^2)}{s^2} \to 0$$

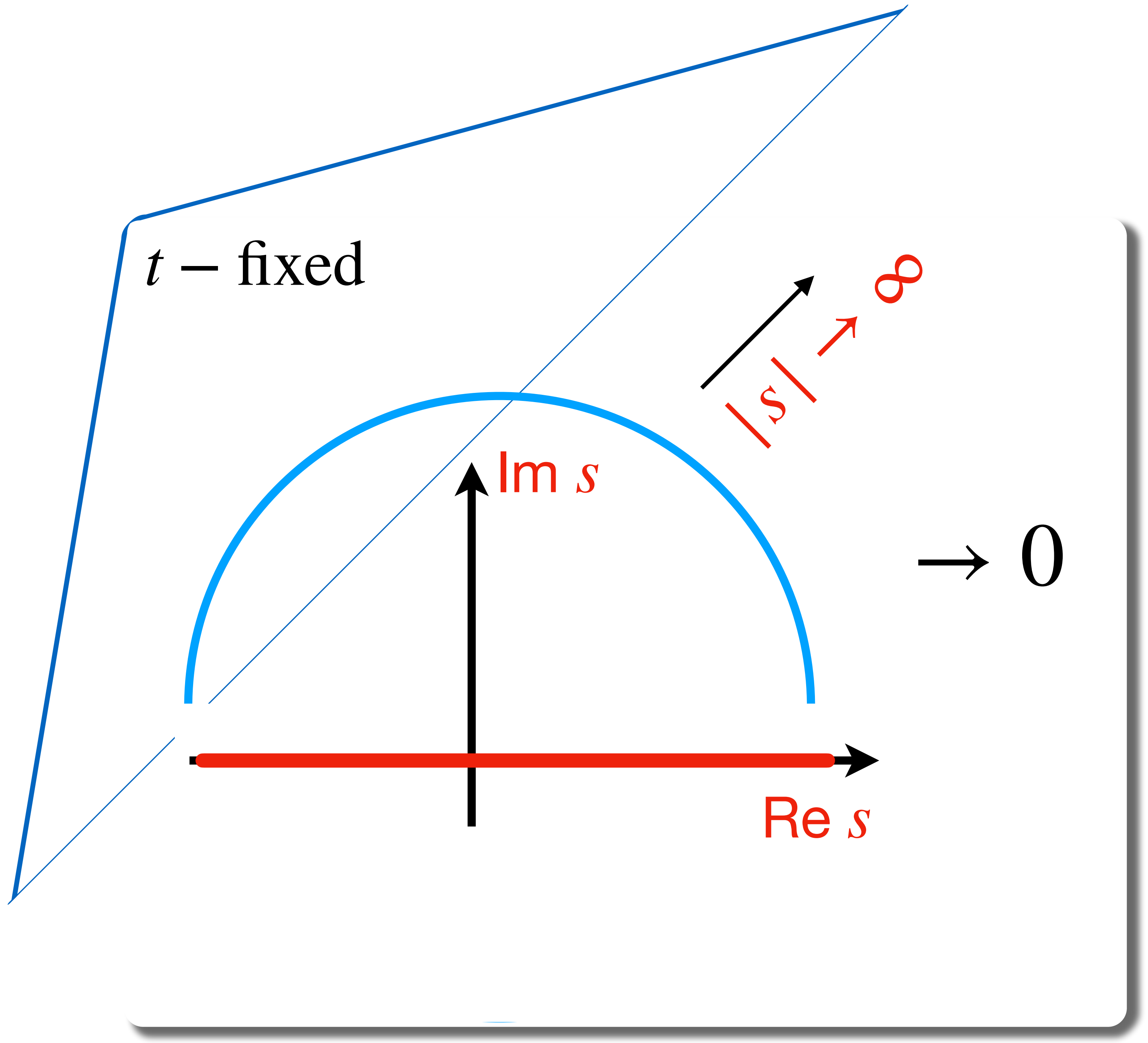
- Gravity in D=4

  BB, Berman, Isabella, Riva, Romano, Sciotti '25

$$\oint_{|s|\to\infty} \frac{ds}{s} \frac{\mathcal{M}_{\mathcal{E}}(s,t)}{s^2} \to 0$$

"Average Regge"

Good enough for dispersion



Regge-scaling

- Gapped theory

  Froissart 61, Martin '63

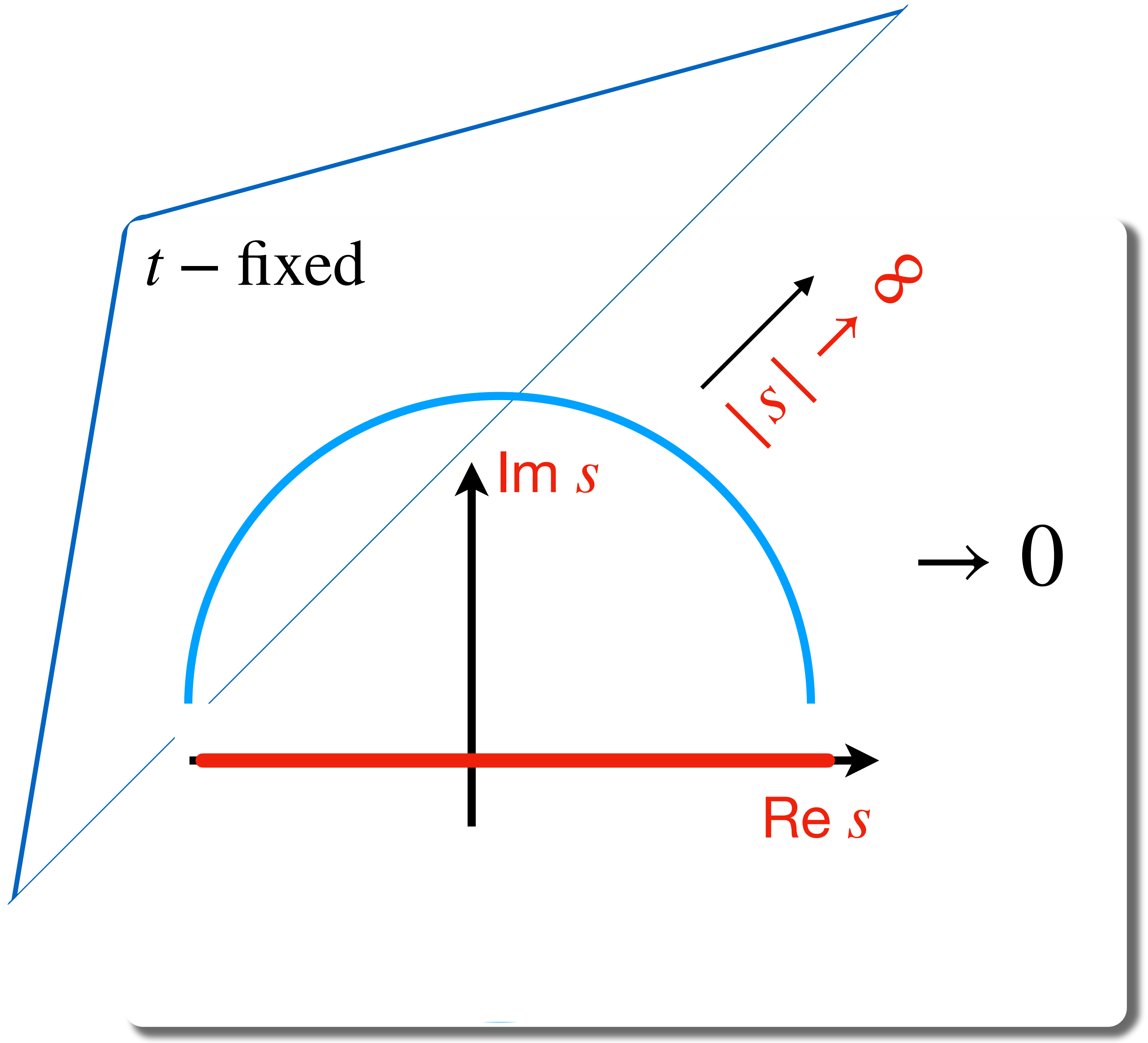
$$\frac{\mathcal{M}(s,t)}{s^2} \to 0$$

- Gravity in D>4

  Haring Zhiboedov '22

  Caron-hoot, Parra-Martinez, Li, Simmons-Duffin '22

$$\int dq \psi(q) \frac{\mathcal{M}(s,-q^2)}{s^2} \to 0$$



- Gravity in D=4

  BB, Berman, Isabella, Riva, Romano, Sciotti '25

$$\oint_{|s|\to\infty} \frac{ds}{s} \frac{\mathcal{M}_{\mathscr{E}}(s,t)}{s^2} \to 0$$

"Average Regge"

Good enough for dispersion

Eikonal regime

$$\propto \oint \frac{ds}{s} \frac{\mathrm{Exp}\left[\frac{Gs}{\pi\epsilon}\big(\log(s)-\log(-s)\big)\big(1+\epsilon\log(-t/\mu^2)\big)\right]}{\mathrm{Exp}\left[\frac{Gs}{\pi\epsilon}\big(\log(s)-\log(-s)\big)\big(1+\epsilon\log \mathscr{E}^2/\mu^2\big)\right]} = \oint \frac{ds}{s}\mathrm{Exp}\left[\frac{Gs}{\pi}\big(\log(s)-\log(-s)\big)\log(-t/\mathscr{E}^2)\right] \propto \pi - 2\mathrm{Si}(G|s|\log(-t/\mathscr{E}^2)) = O(1/|s|)$$

Regge-scaling

- Gapped theory

  Froissart 61, Martin '63

$$\frac{\mathcal{M}(s,t)}{s^2} \to 0$$

- Gravity in D>4

  Haring Zhiboedov '22

  Caron-hoot, Parra-Martinez, Li, Simmons-Duffin '22

$$\int dq\,\psi(q)\frac{\mathcal{M}(s,-q^2)}{s^2} \to 0$$

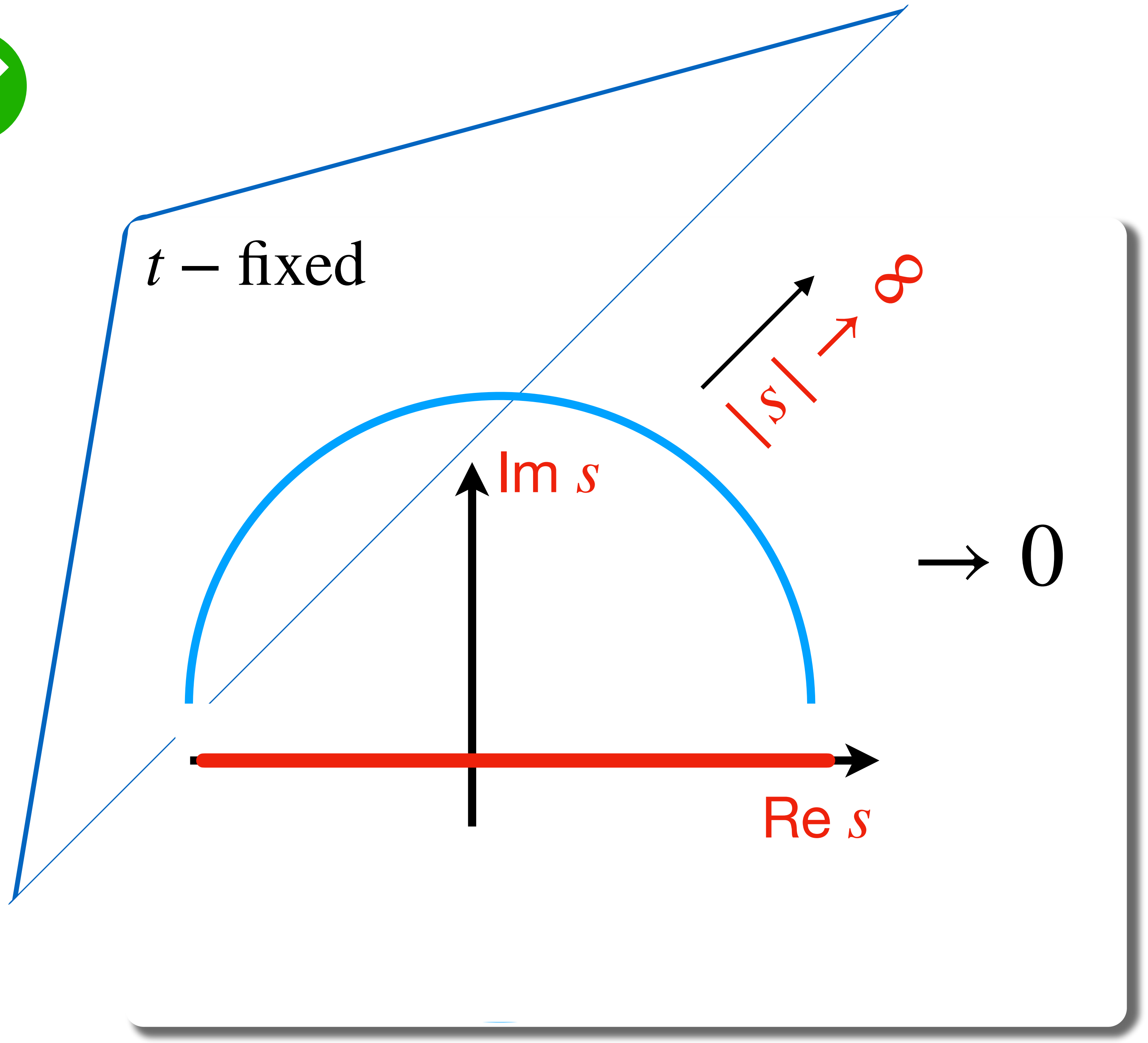
- Gravity in D=4

  BB, Berman, Isabella, Riva, Romano, Sciotti '25

$$\oint_{|s|\to\infty} \frac{ds}{s}\,\frac{\mathcal{M}_{\mathcal{E}}(s,t)}{s^2} \to 0$$

"Average Regge"

Good enough for dispersion

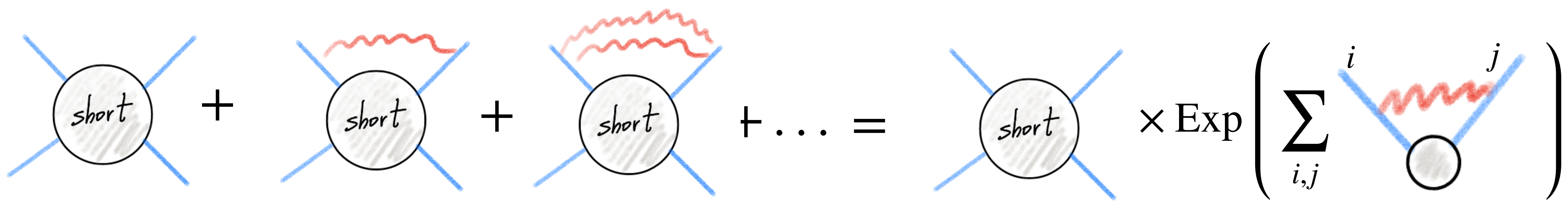


Eikonal regime

$$\propto \oint \frac{ds}{s}\,\frac{\mathrm{Exp}\left[\frac{Gs}{\pi\epsilon}\big(\log(s)-\log(-s)\big)\big(1+\epsilon\log(-t/\mu^2)\big)\right]}{\mathrm{Exp}\left[\frac{Gs}{\pi\epsilon}\big(\log(s)-\log(-s)\big)\big(1+\epsilon\log \mathcal{E}^2/\mu^2\big)\right]} = \oint \frac{ds}{s}\mathrm{Exp}\left[\frac{Gs}{\pi}\big(\log(s)-\log(-s)\big)\log(-t/\mathcal{E}^2)\right] \propto \boxed{\pi - 2\mathrm{Si}(G|s|\log(-t/\mathcal{E}^2)) = O(1/|s|)}$$

Unitarity

Unitary Detector Amplitudes



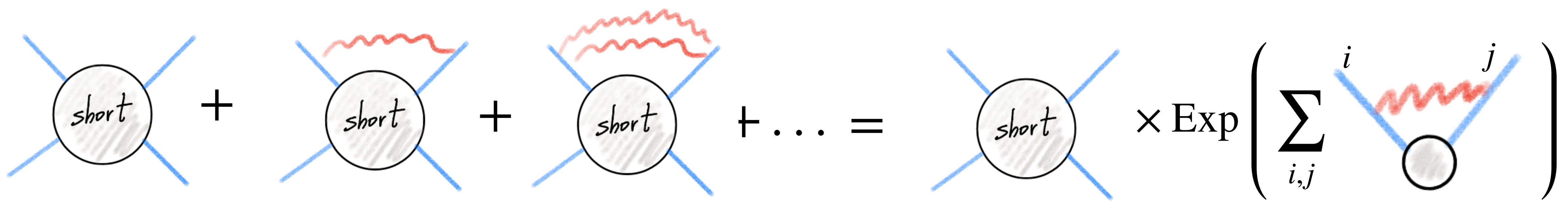
$$\longrightarrow \mathscr{W}^{\epsilon}_{\mathscr{E}} = \exp\left\{\frac{\alpha}{8\pi}\frac{1}{\epsilon}\left[1+\epsilon\log\frac{\mathcal{E}^2}{\mu^2}\right]\left(\sum_{i,j}\eta_i\eta_j q_i q_j \frac{1}{\beta_{ij}}\log\frac{1+\beta_{ij}}{1-\beta_{ij}} - i\sum_{\substack{i\neq j\\ \eta_i\eta_j=1}} q_i q_j \frac{2\pi}{\beta_{ij}}\right)\right\}$$

E&M

Why Unitary?

$$\frac{\text{Amplitude}}{\text{Weinberg Soft Exp}}$$

Unitary Detector Amplitudes



$$\longrightarrow \quad \mathscr{W}^{\epsilon}_{\mathscr{E}} = \exp\left\{\frac{\alpha}{8\pi}\frac{1}{\epsilon}\left[1+\epsilon\log\frac{\mathcal{E}^2}{\mu^2}\right]\left(\sum_{i,j}\eta_i\eta_j q_i q_j\frac{1}{\beta_{ij}}\log\frac{1+\beta_{ij}}{1-\beta_{ij}} - i\sum_{\substack{i\neq j\\ \eta_i\eta_j=1}} q_i q_j\frac{2\pi}{\beta_{ij}}\right)\right\}$$

E&M

Why Unitary?

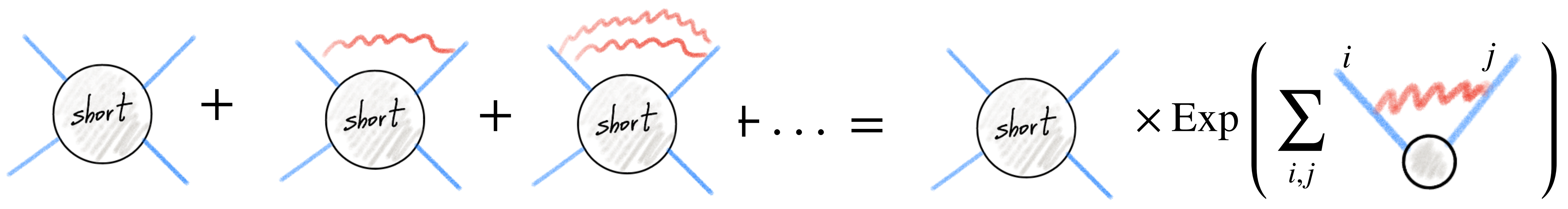
$$\frac{\text{Amplitude}}{\text{Weinberg Soft Exp}} = \boxed{\mathscr{M}^{\text{hard}}_{\mathscr{E}}} + \delta_{\text{sub.soft}}$$

Loops: $\underbrace{\int_0^{\mathscr{E}} d^{4+\epsilon}k}_{\text{Soft}} + \underbrace{\int_{\mathscr{E}}^{\infty} d^{4+\epsilon}k}_{\text{Hard}}$

Leading Soft

Sub-leading Soft

Unitary Detector Amplitudes



$$\mathscr{W}^{\epsilon}_{\mathscr{E}} = \exp\left\{\frac{\alpha}{8\pi}\frac{1}{\epsilon}\left[1+\epsilon\log\frac{\mathcal{E}^2}{\mu^2}\right]\left(\sum_{i,j}\eta_i\eta_j q_i q_j \frac{1}{\beta_{ij}}\log\frac{1+\beta_{ij}}{1-\beta_{ij}} - i\sum_{\substack{i\neq j\\ \eta_i\eta_j=1}} q_i q_j \frac{2\pi}{\beta_{ij}}\right)\right\}$$

E&M

Why Unitary?

$$\frac{\text{Amplitude}}{\text{Weinberg Soft Exp}} = \boxed{\mathscr{M}^{\text{hard}}_{\mathscr{E}}} + \cancel{\delta_{\text{sub.soft}}}$$

$\mathscr{M}^{\text{detector}}$

$\alpha \to 0$

$\alpha \log \mathscr{E} = \alpha_{\mathscr{E}}$

$G \to 0$

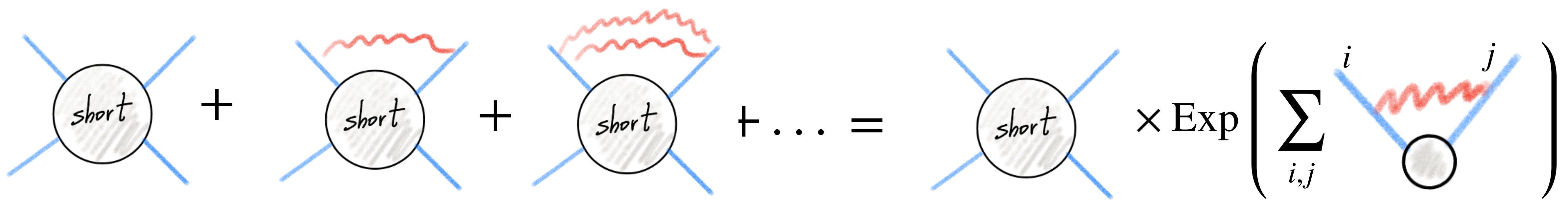
$G \log \mathscr{E} = G_{\mathscr{E}}$

Loops: $\underbrace{\int_0^{\mathscr{E}} d^{4+\epsilon}k}_{\text{Soft}} + \underbrace{\int_{\mathscr{E}}^{\infty} d^{4+\epsilon}k}_{\text{Hard}}$

Leading Soft

Sub-leading Soft

Unitary Detector Amplitudes



$$\longrightarrow \quad \mathscr{W}^{\epsilon}_{\mathscr{E}} = \exp\left\{\frac{\alpha}{8\pi}\frac{1}{\epsilon}\left[1+\epsilon\log\frac{\mathcal{E}^2}{\mu^2}\right]\left(\sum_{i,j}\eta_i\eta_j q_i q_j\frac{1}{\beta_{ij}}\log\frac{1+\beta_{ij}}{1-\beta_{ij}} - i\sum_{\substack{i\neq j\\ \eta_i\eta_j=1}} q_i q_j\frac{2\pi}{\beta_{ij}}\right)\right\}$$

E&M

Why Unitary?

$$\frac{\text{Amplitude}}{\text{Weinberg Soft Exp}} = \boxed{\mathscr{M}^{\text{hard}}_{\mathscr{E}}} + \cancel{\delta_{\text{sub.soft}}}$$

$\mathscr{M}^{\text{detector}}$

$\alpha \to 0$

$\alpha \log \mathscr{E} = \alpha_{\mathscr{E}}$

$G \to 0$

$G \log \mathscr{E} = G_{\mathscr{E}}$

Loops: $\underbrace{\int_0^{\mathscr{E}} d^{4+\epsilon}k}_{\text{Soft}} + \underbrace{\int_{\mathscr{E}}^{\infty} d^{4+\epsilon}k}_{\text{Hard}}$

Leading Soft

Sub-leading Soft

Manifestly Unitary in the Hilbert space of Hard photons/gravitons

$$H^{\text{hard}}_{\mathcal{E}} = \sum_{h=\pm}\int_{\boxed{|\boldsymbol{k}|>\mathcal{E}}}\frac{d^3\boldsymbol{k}}{(2\pi)^3}\left\{\omega(\boldsymbol{k})a_h^\dagger(\boldsymbol{k})a_h(\boldsymbol{k}) - e\left[a_h(\boldsymbol{k})\boldsymbol{\varepsilon}^h(\boldsymbol{k})\cdot\boldsymbol{J} + \text{h.c.}\right] + V_{\text{Coul}}\right\} + c.t. + \dots$$

Unitarity

3 couplings: 2 long-distance couplings + EFT $\mathcal{M}_{\mathscr{E}} = \mathcal{M}(\alpha_{\mathscr{E}}, g_{EFT}) + O(\alpha, \mathscr{E})$ $\mathscr{E} \ll p_{\text{hard}}$

$\alpha, \quad \alpha \log \mathscr{E} = \alpha_{\mathscr{E}}$

$G, \quad G \log \mathscr{E} = G_{\mathscr{E}}$

Unitarity

3 couplings: 2 long-distance couplings + EFT $\quad \mathcal{M}_{\mathscr{E}} = \mathcal{M}(\alpha_{\mathscr{E}}, g_{EFT}) + O(\alpha, \mathscr{E})$ $\quad \mathscr{E} \ll p_{\text{hard}}$

$\alpha, \quad \alpha \log \mathscr{E} = \alpha_{\mathscr{E}}$

$G, \quad G \log \mathscr{E} = G_{\mathscr{E}}$

~~Stripped Amplitude~~ $\longrightarrow$ Detector Amplitude $\longleftarrow$ Hard Amplitude

~~Analytic, X-ssing, Regge~~

~~$\alpha, G \to 0$~~

~~$\alpha \log \mathscr{E} = \alpha_{\mathscr{E}}$~~

~~$G \log \mathscr{E} = G_{\mathscr{E}}$~~

$\mathcal{M}^{\text{detector}}(\alpha_{\mathscr{E}}, G_{\mathscr{E}}, g_{EFT})$

$\alpha, G \to 0$

$\alpha \log \mathscr{E} = \alpha_{\mathscr{E}}$

$G \log \mathscr{E} = G_{\mathscr{E}}$

Unitary

Unitarity

3 couplings: 2 long-distance couplings + EFT $\quad \mathcal{M}_{\mathscr{E}} = \mathcal{M}(\alpha_{\mathscr{E}}, g_{EFT}) + O(\alpha, \mathscr{E})$ $\qquad \mathscr{E} \ll p_{\text{hard}}$

$\alpha, \quad \alpha \log \mathscr{E} = \alpha_{\mathscr{E}}$

$G, \quad G \log \mathscr{E} = G_{\mathscr{E}}$

Stripped Amplitude $\xrightarrow[\substack{\alpha, G \to 0 \\ \alpha \log \mathscr{E} = \alpha_{\mathscr{E}} \\ G \log \mathscr{E} = G_{\mathscr{E}}}]{}$ *Detector Amplitude* $\xleftarrow[\substack{\alpha, G \to 0 \\ \alpha \log \mathscr{E} = \alpha_{\mathscr{E}} \\ G \log \mathscr{E} = G_{\mathscr{E}}}]{}$ *Hard Amplitude*

Analytic, X-ssing, Regge

$\mathcal{M}^{\text{detector}}(\alpha_{\mathscr{E}}, G_{\mathscr{E}}, g_{EFT})$

Unitary

$$\text{Disc}\,\mathcal{M}_{\mathscr{E}} \xrightarrow[\substack{\alpha, G \to 0 \\ \alpha \log \mathscr{E} = \alpha_{\mathscr{E}} \\ G \log \mathscr{E} = G_{\mathscr{E}}}]{} \text{Disc}\,\mathcal{M}^{\text{Detector}} \succ 0$$

Unitarity ✓

3 couplings: 2 long-distance couplings + EFT $\quad \mathcal{M}_{\mathcal{E}} = \mathcal{M}(\alpha_{\mathcal{E}}, g_{EFT}) + O(\alpha, \mathcal{E})$ $\quad \mathcal{E} \ll p_{\text{hard}}$

$\alpha, \quad \alpha \log \mathcal{E} = \alpha_{\mathcal{E}}$

$G, \quad G \log \mathcal{E} = G_{\mathcal{E}}$

~~Stripped Amplitude~~ $\xrightarrow[\substack{\alpha, G \to 0 \\ \alpha \log \mathcal{E} = \alpha_{\mathcal{E}} \\ G \log \mathcal{E} = G_{\mathcal{E}}}]{}$ Detector Amplitude $\xleftarrow[\substack{\alpha, G \to 0 \\ \alpha \log \mathcal{E} = \alpha_{\mathcal{E}} \\ G \log \mathcal{E} = G_{\mathcal{E}}}]{}$ Hard Amplitude

~~Analytic, X-ssing, Regge~~

$\mathcal{M}^{\text{detector}}(\alpha_{\mathcal{E}}, G_{\mathcal{E}}, g_{EFT})$

Unitary

$$\mathrm{Disc}\,\mathcal{M}_{\mathcal{E}} \xrightarrow[\substack{\alpha, G \to 0 \\ \alpha \log \mathcal{E} = \alpha_{\mathcal{E}} \\ G \log \mathcal{E} = G_{\mathcal{E}}}]{} \mathrm{Disc}\,\mathcal{M}^{\text{Detector}} \succ 0$$

Universality

Inclusive Detector Amplitudes

$$\frac{\text{Amplitude}}{\text{Weinberg Soft Exp}} = \mathscr{M}_{\mathscr{E}} \longleftarrow \; ?$$

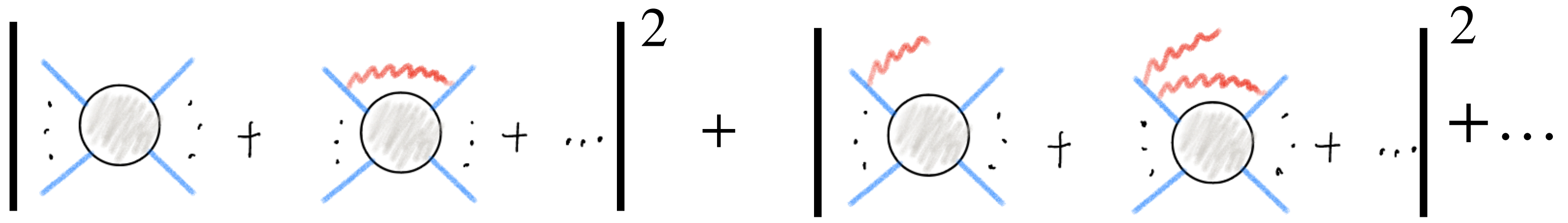
Inclusive Detector Amplitudes

$$\frac{\text{Amplitude}}{\text{Weinberg Soft Exp}} = \mathscr{M}_{\mathscr{E}} \longleftarrow \quad ?$$

Exclusive |Detector-Amplitudes|^2

$$\left|\mathscr{M}^{\text{hard}}_{\mathscr{E}}]\right|^2$$

$$\frac{\left|\mathscr{M}_{\mathscr{E}}\right|^2}{\sum_{E_{\gamma,h}<\Delta E}^{\infty}\left|\mathscr{M}\right|^2} = \frac{\left|\mathscr{M}_{\mathscr{E}}\right|^2}{\left|\quad + \quad + \cdots\right|^2 + \left|\quad + \quad + \cdots\right|^2 + \ldots}$$



$$\left|\mathscr{M}^{\text{hard}}_{\Delta E_{\gamma,h}}\right|^2$$

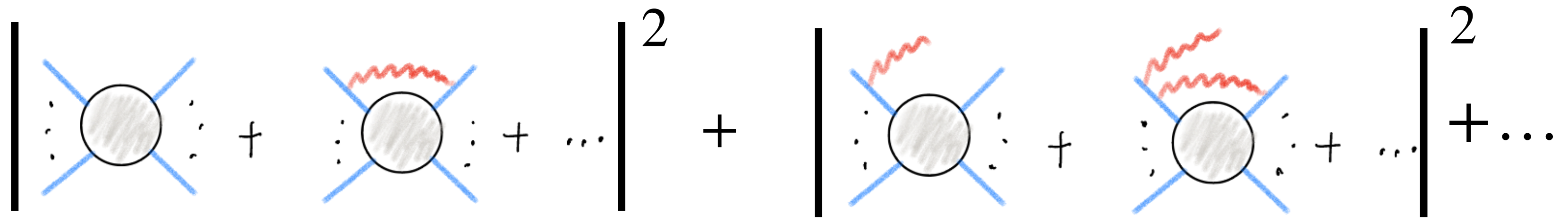
Inclusive X-sections

Inclusive Detector Amplitudes

$$\frac{\text{Amplitude}}{\text{Weinberg Soft Exp}} = \mathscr{M}_{\mathscr{E}} \longleftarrow \; ?$$

Exclusive |Detector-Amplitudes|^2

$$\frac{\left|\mathscr{M}_{\mathscr{E}}\right|^2}{\sum^{\infty}_{E_{\gamma,h}<\Delta E}\left|\mathscr{M}\right|^2} = \frac{\left|\mathscr{M}_{\mathscr{E}}\right|^2}{\cdots}$$



$$\left|\mathscr{M}^{\text{hard}}_{\mathscr{E}}]\right|^2$$

$$\left|\mathscr{M}^{\text{hard}}_{\Delta E_{\gamma,h}}\right|^2$$

$$\longrightarrow \left(\frac{\Delta E}{\mathscr{E}}\right)^{-\frac{\alpha}{8\pi^2}\left(\sum_{i,j}\eta_i\eta_j q_i q_j \frac{1}{\beta_{ij}}\log\frac{1+\beta_{ij}}{1-\beta_{ij}}\right)} = 1$$

$$\Delta E_{\gamma,h} = \mathscr{E}$$

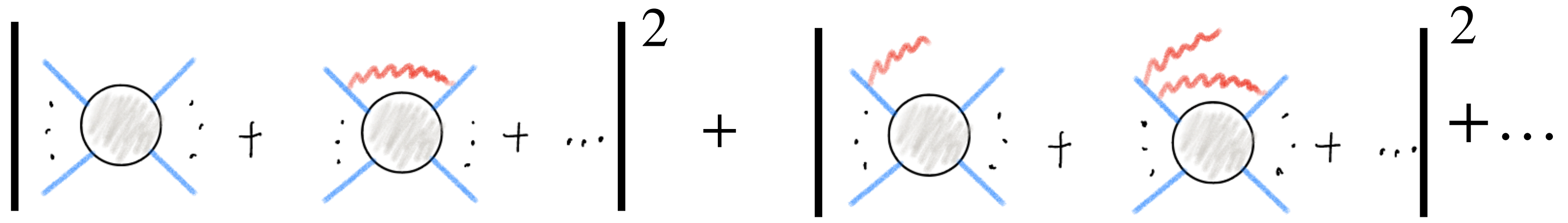
Inclusive X-sections

Inclusive Detector Amplitudes

$$\frac{\text{Amplitude}}{\text{Weinberg Soft Exp}} = \mathscr{M}_{\mathscr{E}} \leftarrow \;?$$

Exclusive |Detector-Amplitudes|^2

$$\frac{\left|\mathscr{M}_{\mathscr{E}}\right|^2}{\sum_{E_{\gamma,h}<\Delta E}^{\infty}\left|\mathscr{M}\right|^2} = \frac{\left|\mathscr{M}_{\mathscr{E}}\right|^2}{\cdots}$$



$$\left|\mathscr{M}^{\text{hard}}_{\mathscr{E}}]\right|^2 \qquad \left|\mathscr{M}^{\text{hard}}_{\Delta E_{\gamma,h}}\right|^2$$

$$\longrightarrow \left(\frac{\Delta E}{\mathscr{E}}\right)^{-\frac{\alpha}{8\pi^2}\left(\sum_{i,j}\eta_i\eta_j q_i q_j \frac{1}{\beta_{ij}}\log\frac{1+\beta_{ij}}{1-\beta_{ij}}\right)} = 1$$

$$\Delta E_{\gamma,h} = \mathscr{E}$$

Inclusive X-sections

Lessons:

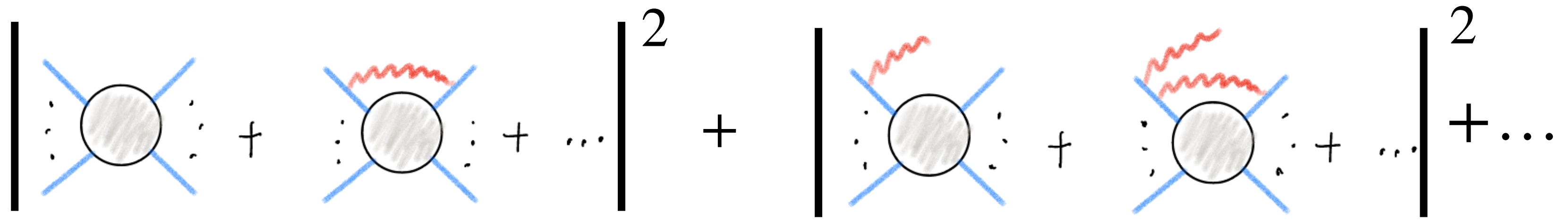
1. $\mathscr{E}$ ~ Detector Energy Resolution; optimally 1/“size”
2. IR sensitive, but not too much
3. Universal leading-log of any IR-finite amplitude

Inclusive Detector Amplitudes

$$\frac{\text{Amplitude}}{\text{Weinberg Soft Exp}} = \mathscr{M}_{\mathscr{E}} \longleftarrow \ ?$$

Exclusive |Detector-Amplitudes|^2

$$\frac{\left|\mathscr{M}_{\mathscr{E}}\right|^2}{\sum_{E_{\gamma,h}<\Delta E}^{\infty}\left|\mathscr{M}\right|^2} = \frac{\left|\mathscr{M}_{\mathscr{E}}\right|^2}{\text{(diagram series)}} \longrightarrow \left(\frac{\Delta E}{\mathscr{E}}\right)^{-\frac{\alpha}{8\pi^2}\left(\sum_{i,j}\eta_i\eta_j q_i q_j \frac{1}{\beta_{ij}}\log\frac{1+\beta_{ij}}{1-\beta_{ij}}\right)} = 1$$



$$\left|\mathscr{M}^{\text{hard}}_{\mathscr{E}}]\right|^2$$

$$\left|\mathscr{M}^{\text{hard}}_{\Delta E_{\gamma,h}}\right|^2$$

$$\Delta E_{\gamma,h} = \mathscr{E}$$

Inclusive X-sections

Lessons:

1. $\mathscr{E}$ ~ Detector Energy Resolution; optimally 1/“size”
2. IR sensitive, but not too much
3. Universal leading-log of any IR-finite amplitude

$\mathscr{M}_{\mathscr{E}}$ is

1. IR finite
2. Lorentz invariant
3. Analytic & X-ssing
4. Regge/Froissart Bound
5. Unitary
6. Universality, $\mathscr{E} \sim \Delta E$

Exactly

In a limit

BB, Berman, Isabella, Riva, Romano, Sciotti 2512.13780

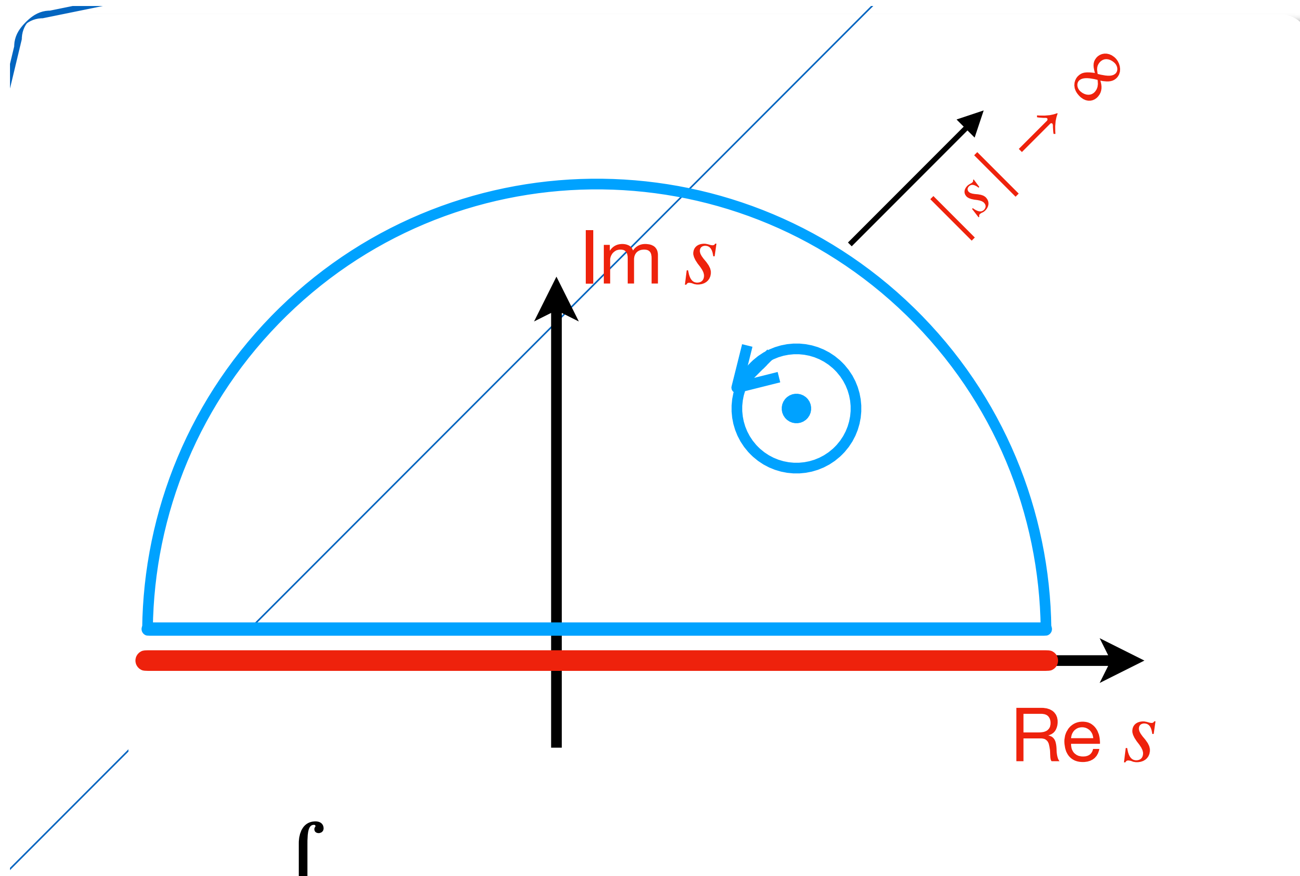
Positivity

Dispersion Relations

$\mathcal{M}_{\mathcal{E}}$ is

1. IR finite
2. Lorentz invariant
3. Analytic
4. Average Regge
5. Unitary
6. Inclusive, $\mathcal{E} \sim \Delta E$

Exactly



$$\oint \mathcal{M}_{\mathcal{E}} \mathcal{K}(s) = \text{“Arc } \mathcal{A}\text{”}$$

$$g_{EFT} = \langle (\cdots) \rangle = \sum_{\ell} \int_{M^2}^{\infty} \frac{ds}{s^3} (\cdots) \mathrm{Im} \mathcal{M}_{\mathcal{E}}^{\ell}(s)$$

EFT Dispersive Sum Rule

Dispersion Relations

$\mathscr{M}_{\mathscr{E}}$ is

1. IR finite
2. Lorentz invariant
3. Analytic
4. Average Regge

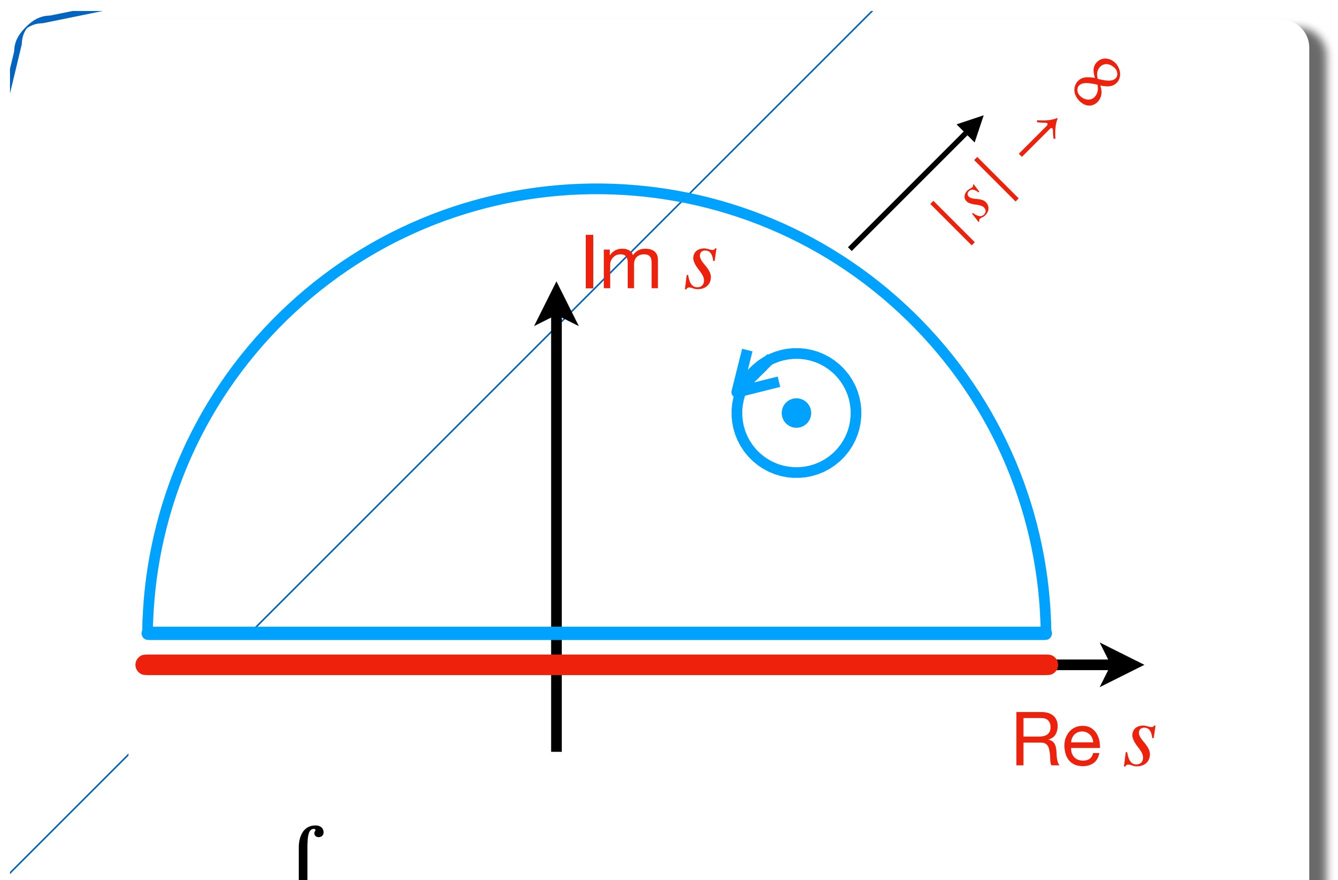
} Exactly

5. Unitary
6. Inclusive, $\mathscr{E} \sim \Delta E$

} In the limit $\alpha \to 0, \quad \alpha \log \mathscr{E} = \alpha_{\mathscr{E}}$ - fixed

$G \to 0, \quad G \log \mathscr{E} = G_{\mathscr{E}}$ -fixed

(or tree-level)



$\oint \mathscr{M}_{\mathscr{E}} \mathscr{K}(s)$ =“Arc $\mathscr{A}$”

$$g_{EFT} = \langle (\cdots) \rangle = \sum_{\ell} \int_{M^2}^{\infty} \frac{ds}{s^3} (\cdots) \mathrm{Im} \mathscr{M}^{\ell}_{\mathscr{E}}(s)$$

EFT Dispersive Sum Rule

EFT Positivity bounds

$$\mathscr{A}(g_{EFT}, \alpha_{\mathscr{E}}, G_{\mathscr{E}}) + O(\alpha, G) \geq 0$$

Dispersion Relations

$\mathscr{M}_{\mathscr{E}}$ is

1. IR finite
2. Lorentz invariant
3. Analytic
4. Average Regge

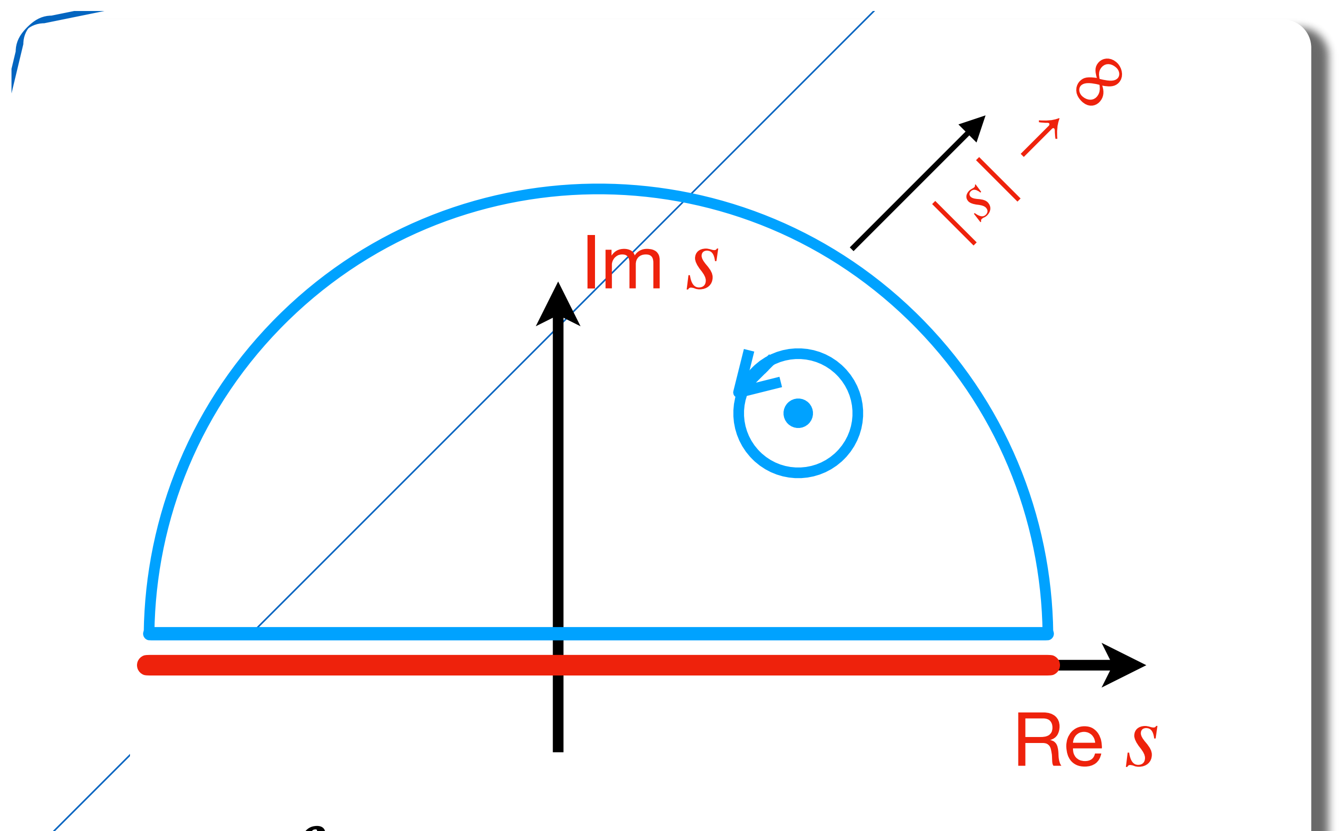
} Exactly

5. Unitary
6. Inclusive, $\mathscr{E} \sim \Delta E$

} In the limit $\alpha \to 0, \quad \alpha \log \mathscr{E} = \alpha_{\mathscr{E}}$ - fixed

$G \to 0, \quad G \log \mathscr{E} = G_{\mathscr{E}}$ -fixed

(or tree-level)



$$\oint \mathscr{M}_{\mathscr{E}} \mathscr{K}(s) = \text{“Arc } \mathscr{A}\text{”}$$

$$g_{EFT} = \langle (\cdots) \rangle = \sum_{\ell} \int_{M^2}^{\infty} \frac{ds}{s^3} (\cdots) \mathrm{Im} \mathscr{M}^{\ell}_{\mathscr{E}}(s)$$

EFT Dispersive Sum Rule

$$\mathscr{M}_{\mathscr{E}} = \mathscr{M}^{\text{detector}}(\alpha_{\mathscr{E}}, G_{\mathscr{E}}, g_{EFT}) \left[1 + O\left(\frac{G}{G_{\mathscr{E}}}, \frac{\alpha}{\alpha_{\mathscr{E}}}, \mathscr{E}/E \right) \right]$$

Beyond Tree:

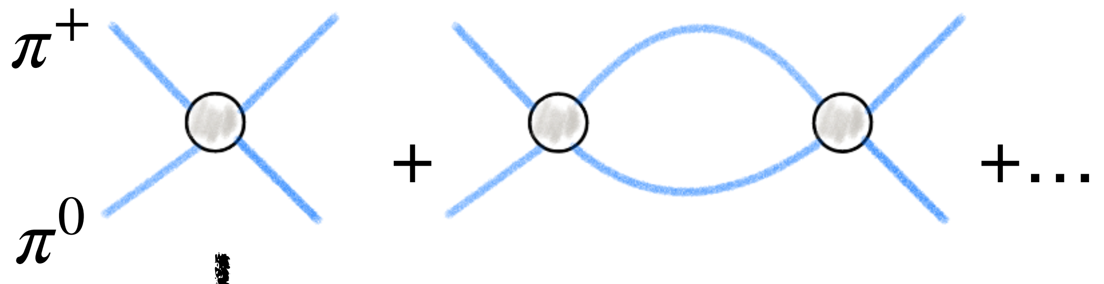
Effective bounds on g_{EFT} as long as: $\alpha_{\mathscr{E}}/g_{EFT} \ll 1$, $G_{\mathscr{E}} E^2/g_{EFT} \ll 1$ while $1/\log \mathscr{E}/E \ll 1$

EFT Positivity bounds

$$\mathscr{A}(g_{EFT}, \alpha_{\mathscr{E}}, G_{\mathscr{E}}) + O(\alpha, G) \geq 0$$

Examples

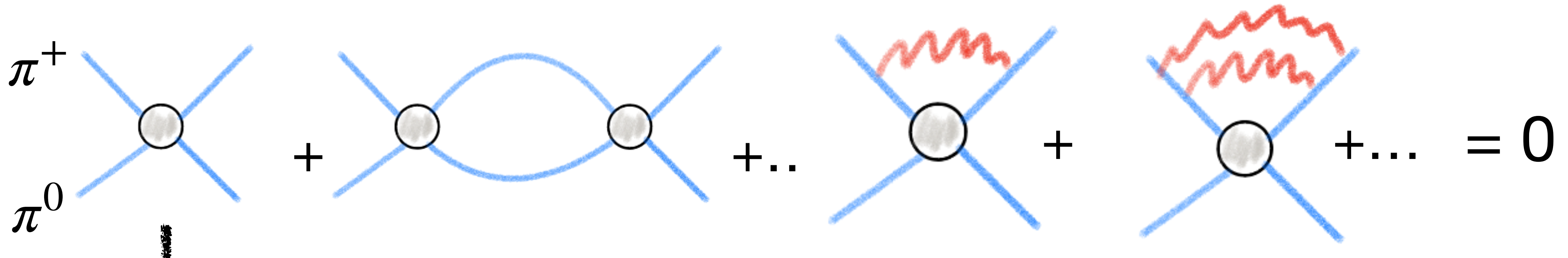
$\pi^0\pi^+ + \mathcal{E}\&\mathcal{M}$



$$c_{2,0} \geq 0 \qquad \frac{c_{3,1}M^2}{c_{2,0}} \geq -\frac{3}{2}$$

$$\mathcal{M}^{0+} = \left\{ c_{0,0} + c_{1,1}t + c_{2,0}(s - m_\pm^2 + t/2)^2 + c_{2,2}t^2 + c_{3,1}t(s - m_\pm^2 + t/2)^2 + \ldots \right\}$$

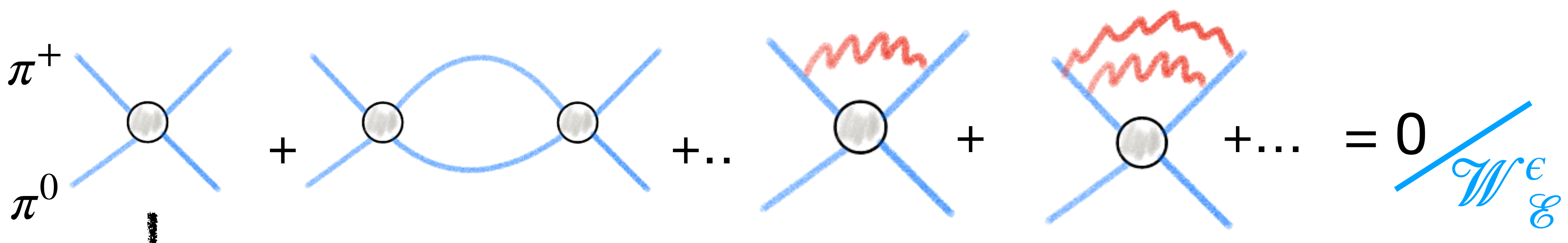
$\pi^0\pi^+ + \mathcal{E}\&\mathcal{M}$



$$\boxed{c_{2,0} \geq 0 \qquad \frac{c_{3,1}M^2}{c_{2,0}} \geq -\frac{3}{2}} \longrightarrow \quad c_{2,0} \geq ?? \qquad \frac{c_{3,1}M^2}{c_{2,0}} \geq ??$$

$$\mathscr{M}^{0+} = \left\{ c_{0,0} + c_{1,1}t + \boxed{c_{2,0}}(s - m_\pm^2 + t/2)^2 + c_{2,2}t^2 + \boxed{c_{3,1}}t(s - m_\pm^2 + t/2)^2 + \ldots \right\}$$

$\pi^0\pi^+ + \mathcal{E}\&\mathcal{M}$

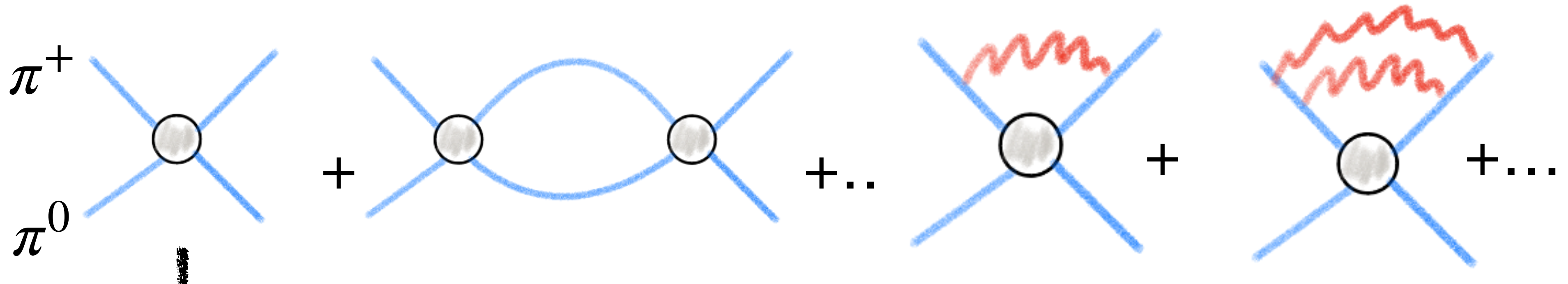


$\longrightarrow \mathscr{M}^{0+}_{\mathscr{E}}$

$$\boxed{c_{2,0} \geq 0 \qquad \frac{c_{3,1}M^2}{c_{2,0}} \geq -\frac{3}{2}} \longrightarrow \quad c_{2,0} \geq ?? \qquad \frac{c_{3,1}M^2}{c_{2,0}} \geq ??$$

$$\mathscr{M}^{0+}_{\mathscr{E}} = \left\{c_{0,0} + c_{1,1}t + \boxed{c_{2,0}}(s - m_\pm^2 + t/2)^2 + c_{2,2}t^2 + \boxed{c_{3,1}}t(s - m_\pm^2 + t/2)^2 + \ldots\right\} \times \left[1 + \frac{\alpha}{2\pi}\log\left(\frac{m_\pm^2(4m_\pm^2 - t)}{\mathscr{E}^4}\right)\frac{t - 2m_\pm^2}{\sqrt{t(4m_\pm^2 - t)}}\arctan\left(\frac{\sqrt{t}}{\sqrt{4m_\pm^2 - t}}\right) + \frac{\alpha}{2\pi}\log\frac{4\pi\mu^2}{\mathscr{E}^2}\right] + O(\alpha_{\mathscr{E}}^2)$$

$\pi^0\pi^+ + \mathcal{E}\&\mathcal{M}$



$= 0 \, / \, \mathscr{W}^{\epsilon}_{\mathscr{E}} \longrightarrow \mathscr{M}^{0+}_{\mathscr{E}}$

$$\boxed{c_{2,0} \geq 0 \qquad \frac{c_{3,1}M^2}{c_{2,0}} \geq -\frac{3}{2}} \longrightarrow \quad c_{2,0} \geq ?? \qquad \frac{c_{3,1}M^2}{c_{2,0}} \geq ??$$

$$\mathscr{M}^{0+}_{\mathscr{E}} = \left\{c_{0,0} + c_{1,1}t + c_{2,0}(s - m_\pm^2 + t/2)^2 + c_{2,2}t^2 + c_{3,1}t(s - m_\pm^2 + t/2)^2 + \ldots\right\} \times \left[1 + \frac{\alpha}{2\pi}\log\left(\frac{m_\pm^2(4m_\pm^2 - t)}{\mathscr{E}^4}\right)\frac{t - 2m_\pm^2}{\sqrt{t(4m_\pm^2 - t)}}\arctan\left(\frac{\sqrt{t}}{\sqrt{4m_\pm^2 - t}}\right) + \frac{\alpha}{2\pi}\log\frac{4\pi\mu^2}{\mathscr{E}^2}\right] + O(\alpha^2_{\mathscr{E}})$$

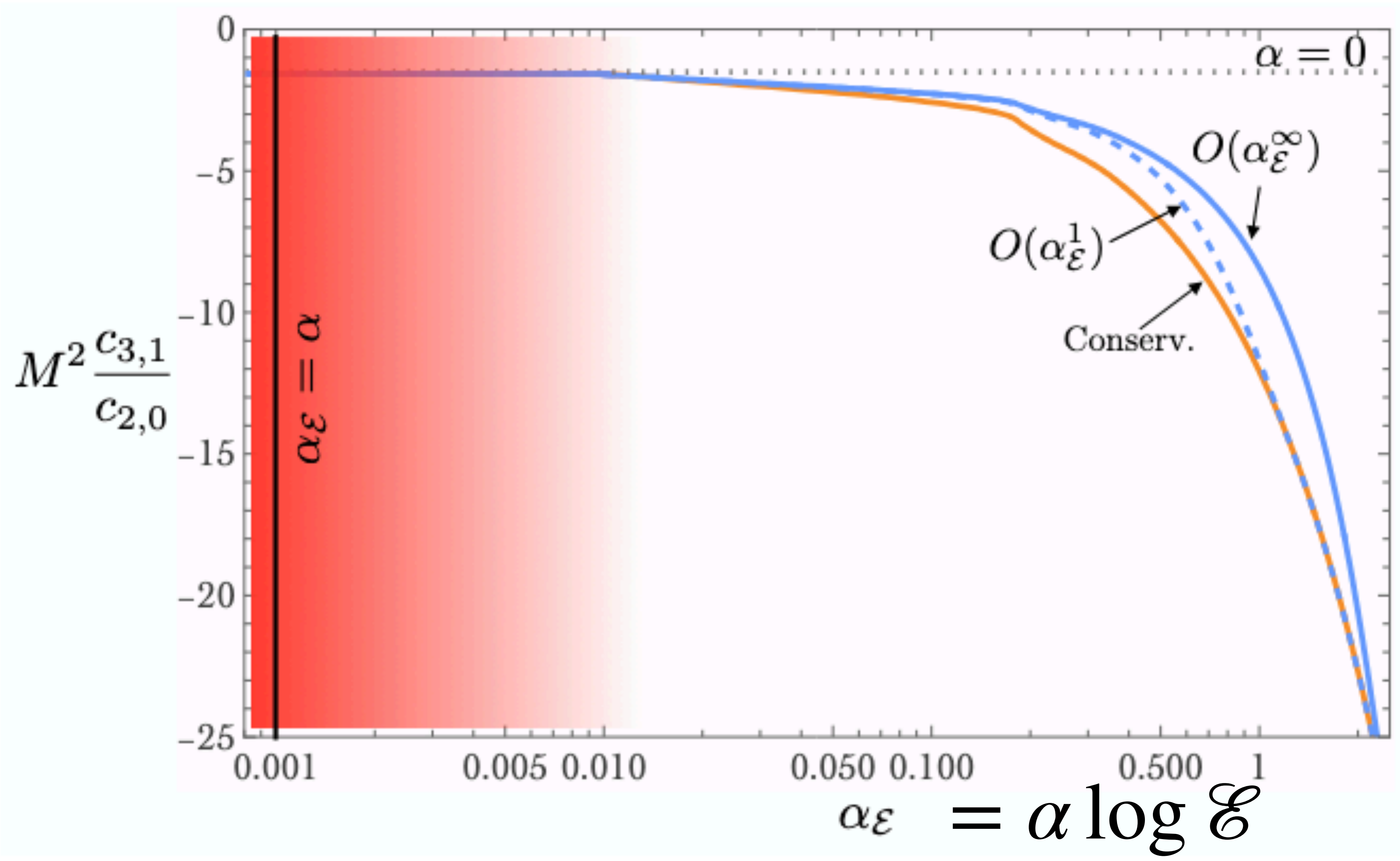
Smoothly connected to $\alpha = 0$

$$c_{2,0} \geq 0$$

$$\frac{c_{3,1}M^2}{c_{2,0}} > -\frac{3}{2} + \frac{M^2}{m_\pm^2}\frac{\alpha \log \mathscr{E}/m_\pm}{3\pi} + O(\alpha, \alpha^2_{\mathscr{E}}, c^2_{n,k})$$

BB, Berman, Isabella, Riva, Romano, Sciotti '25

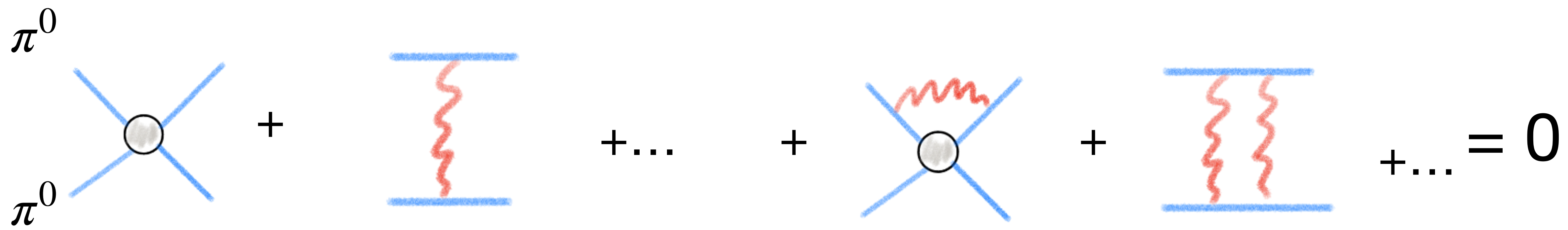
Optimal bounds: $1 \ll \log m_\pm/\mathscr{E} < \infty$



$\longleftarrow$ -3/2

$= \alpha \log \mathscr{E}$

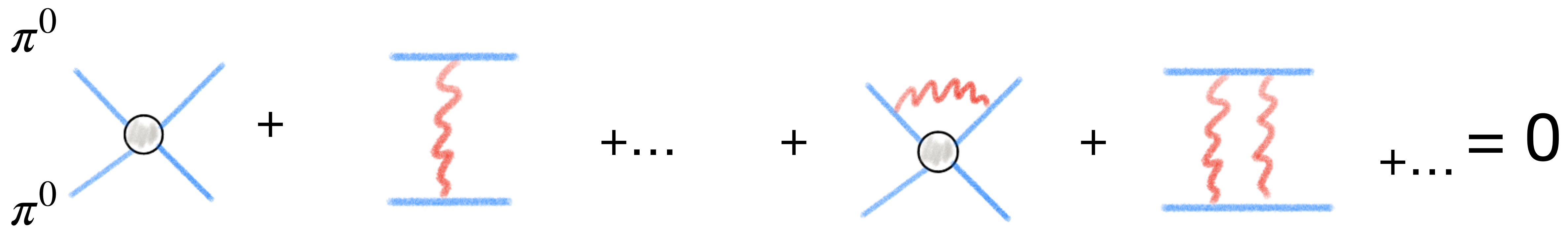
$\pi^0\pi^0$+gravity



$$\mathcal{M}^{\text{tree,GR}} = 8\pi G\left(\frac{tu}{s}+\frac{us}{t}+\frac{st}{u}\right) + c_{2,0}\left(\frac{s^2+t^2+u^2}{2}\right) + c_{3,1}stu + \dots$$

$$\boxed{c_{2,0} \geq 0 \qquad \frac{c_{3,1}M^2}{c_{2,0}} \leq \frac{3}{2}} \longrightarrow \quad c_{2,0} \geq ?? \qquad \frac{c_{3,1}M^2}{c_{2,0}} \leq ??$$

$\pi^0\pi^0$+gravity



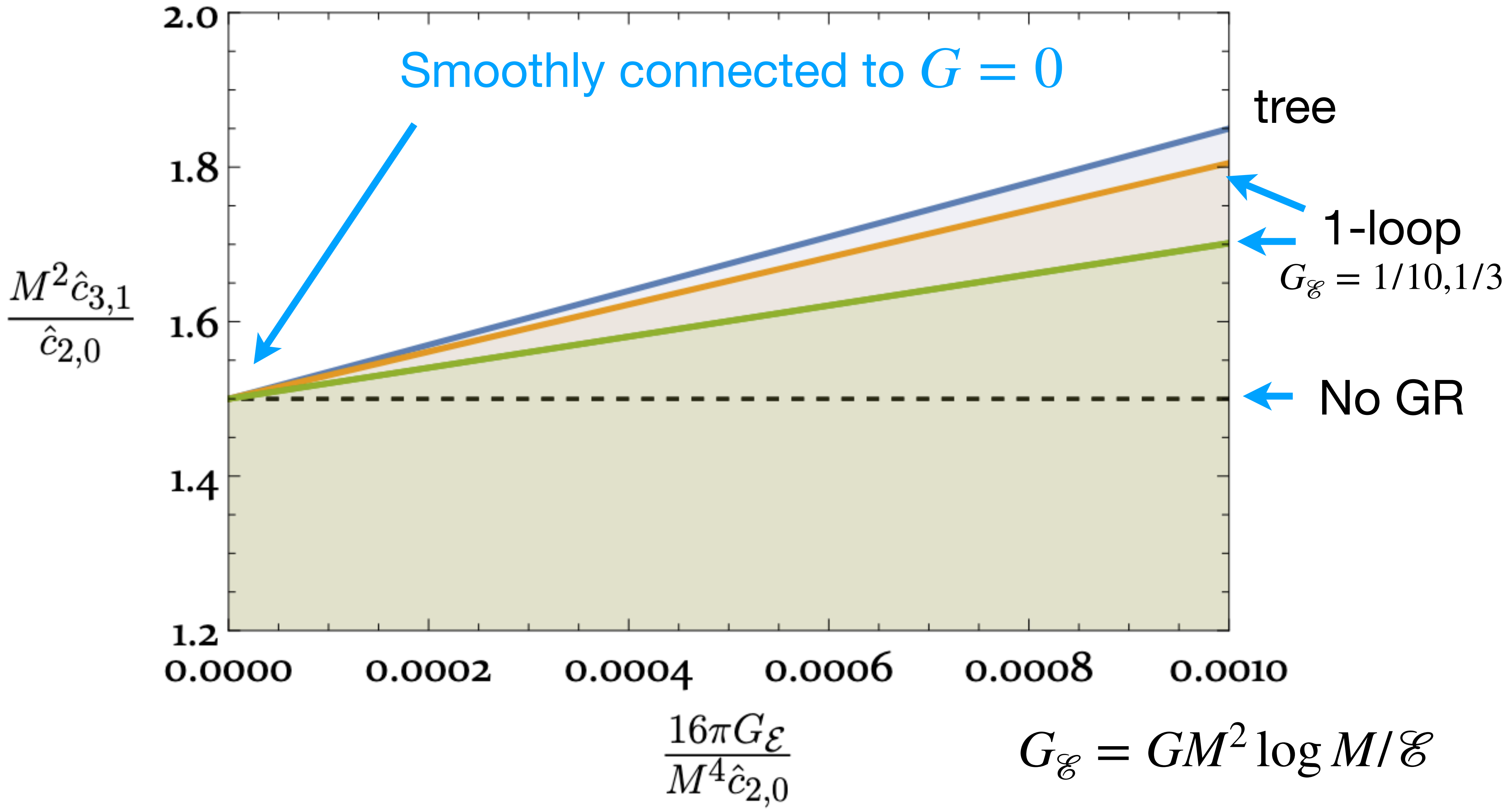
$$\mathcal{M}^{\text{tree,GR}} = 8\pi G\left(\frac{tu}{s}+\frac{us}{t}+\frac{st}{u}\right) + \boxed{c_{2,0}}\left(\frac{s^2+t^2+u^2}{2}\right) + \boxed{c_{3,1}}stu + \ldots$$

$$\boxed{c_{2,0} \geq 0 \qquad \frac{c_{3,1}M^2}{c_{2,0}} \leq \frac{3}{2}}$$

$$\longrightarrow \quad c_{2,0} \geq \,?? \qquad \frac{c_{3,1}M^2}{c_{2,0}} \leq \,??$$

$$\boxed{\begin{gathered} c_{2,0}M^4 \geq -526\pi G_{\mathscr{E}} + O(G) \\ 0 \leq \frac{c_{2,0}M^4}{5600\pi G_{\mathscr{E}}}\left(\frac{3}{2} - \frac{c_{3,1}M^2}{c_{2,0}}\right) + 1 + \frac{4}{\pi}G_{\mathscr{E}} + \ldots + O\left(\frac{1}{\log M/\mathscr{E}}\right) \end{gathered}}$$

BB, Berman, Isabella, Riva, Romano, Sciotti '25

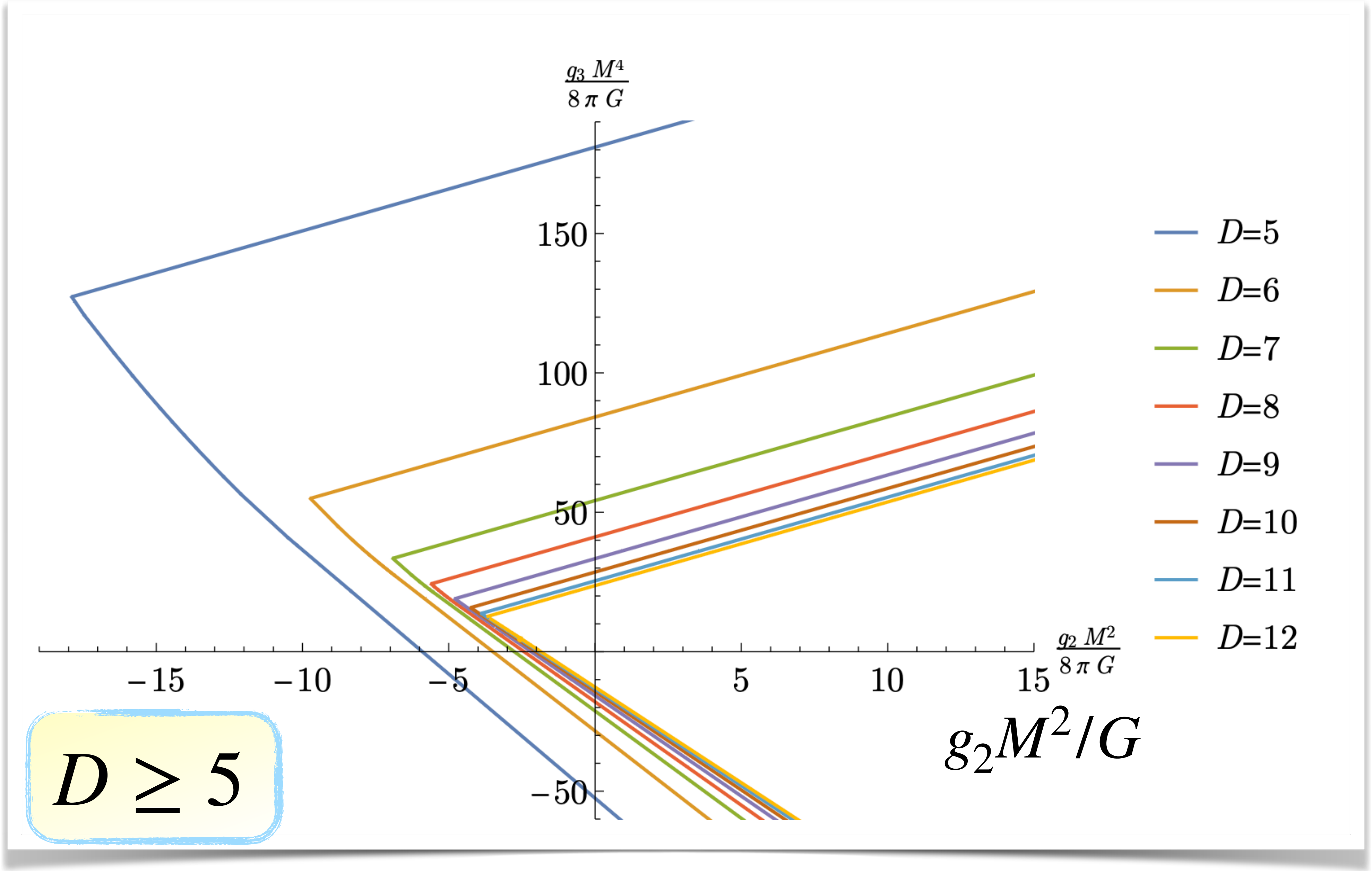


$G_{\mathscr{E}} = GM^2 \log M/\mathscr{E}$

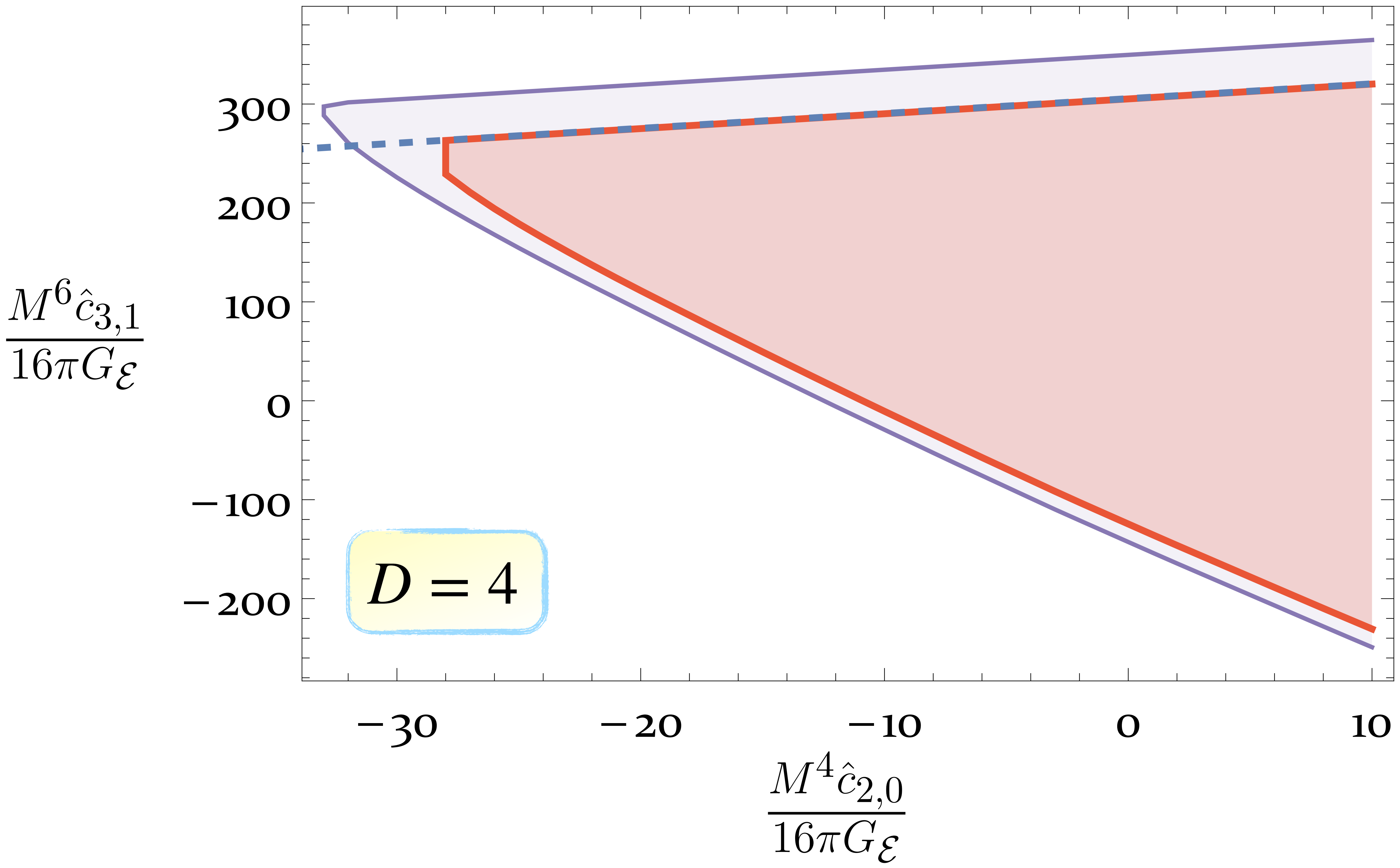
Optimal bounds: $1 \ll \log M/\mathscr{E} < \infty$

$\pi^0\pi^0$+gravity

Caron-huot, Mazac, Rastelli, Simmons-Duffin 2102.08951



BB, Berman, Isabella, Riva, Romano, Sciotti 2512.13780

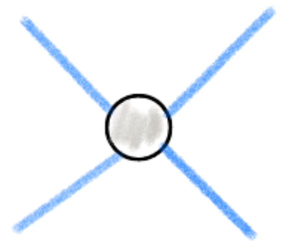


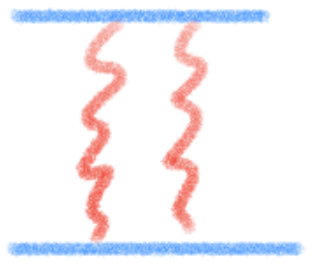
Earlier, IR-divergent Bounds?

without gravity $\qquad c_{2,0} \geq 0$

with gravity, IR-div
(Tree&Loops)

$$\left(1 + \dots \qquad \right) c_{2,0} M^4 + \left(\frac{GM^2}{\epsilon} + \left(\frac{GM^2}{\epsilon}\right)^2 + \dots \right) \geq 0$$





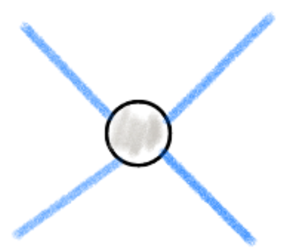
Earlier, IR-divergent Bounds?

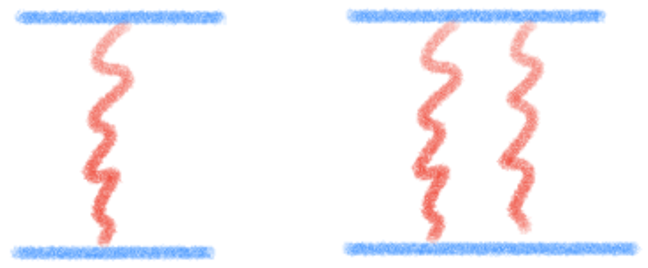
without gravity $\quad c_{2,0} \geq 0$

with gravity, IR-div (Tree&Loops) $\quad \overset{??}{c_{2,0} + 256 \geq 0}$

Chang Parra-Martinez '25

$$\left(1 + \dots \qquad\qquad \right) c_{2,0} M^4 + \overbrace{\left(\frac{GM^2}{\epsilon} + \left(\frac{GM^2}{\epsilon}\right)^2 + \dots \right)}^{F(GM^2/\epsilon)\,\to\,256} \geq 0$$





Earlier, IR-divergent Bounds?

without gravity $\quad c_{2,0} \geq 0$

with gravity, IR-div (Tree&Loops) $\quad \overset{??}{c_{2,0} + 256 \geq 0}$

With IR-div: Trivialize $\quad 256 \geq 0$

$$\overbrace{\left(1+\ldots\frac{GM^2}{\epsilon}+\ldots\right)}^{\to 0} c_{2,0}M^4 + \overbrace{\left(\frac{GM^2}{\epsilon}+\left(\frac{GM^2}{\epsilon}\right)^2+\ldots\right)}^{F(GM^2/\epsilon)\,\to\,256} \geq 0$$

Chang Parra-Martinez '25

Earlier, IR-divergent Bounds?

without gravity $c_{2,0} \geq 0$

??

with gravity, IR-div (Tree&Loops) $c_{2,0} + 256 \geq 0$

$$\overbrace{\left(1+\ldots\frac{GM^2}{\epsilon}+\ldots\right)}^{\to 0} c_{2,0}M^4 + \overbrace{\left(\frac{GM^2}{\epsilon}+\left(\frac{GM^2}{\epsilon}\right)^2+\ldots\right)}^{F(GM^2/\epsilon)\,\to\, 256} \geq 0$$

Chang Parra-Martinez '25

With IR-div: Trivialize $256 \geq 0$

Issue is not the t-pole

Remove the IR-divergences and all is good.

Conclusions

Conclusions & Outlook

Principles + *Symmetries* + *light d.o.f.* ⟶ Non-trivial IR-bootstrap problem

↓

Photons, gravitons in D=4

E&M or Gravity

Ordinary S-matrix not a well-defined observable

What are good observables/tools? 1. IR finite, 2. not (too) IR sensitive, 3. Causality

In detector limit —> stripped amplitudes are one such tool

Conclusions & Outlook

Some lessons:

1. Detector/IR data are crucial inputs: energy resolution $\mathscr{E}$ (or apparatus size)
2. Leading IR effects: universal, captured by Stripped/Detector Amplitudes $\mathscr{M}_{\mathscr{E}}$ theory tool
   (analytic, crossing-symmetric, Regge behaved, …)
3. Smooth EFT bounds exist: finite, calculable long-range corrections for QED and GR in D=4
4. Optimal resolution $\mathscr{E}$ is not necessarily optimal for bounding EFT coefficients
   (e.g. too large apparatus —> too large GR time delay)

Thank you!

Backup Slides

T-channel pole & Negativity

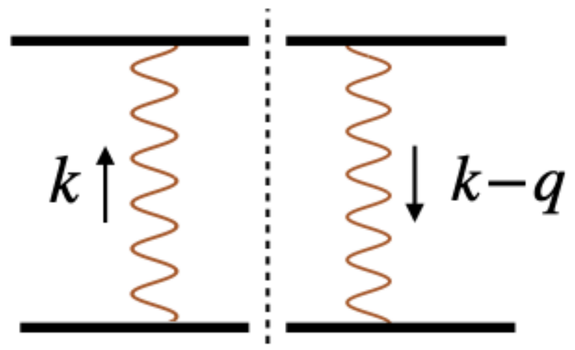
$$\int_{\mathscr{E}}^{M} dq\ \psi(q)\ \mathscr{M}_{\mathscr{E}}^{\text{hard}\,\dagger}\mathscr{M}_{\mathscr{E}}^{\text{hard}} \overset{?}{\succ} 0 \longrightarrow \underbrace{\int_{0}^{M} dq\ \psi(q)\ \mathscr{M}_{\mathscr{E}}^{\text{hard}\,\dagger}\mathscr{M}_{\mathscr{E}}^{\text{hard}}}_{\text{Exist } \psi(q) \text{ s.t. } >0} - \underbrace{\int_{0}^{\mathscr{E}} dq\ \psi(q)\ \mathscr{M}_{\mathscr{E}}^{\text{hard}\,\dagger}\mathscr{M}_{\mathscr{E}}^{\text{hard}}}_{\text{negativity} = O(G^2) \text{ sub-leading}}$$

forward region excluded

Exist $\psi(q)$ s.t. >0

negativity = $O(G^2)$ sub-leading

$$\int_{0}^{\mathscr{E}} dq\ \psi(q)\ \mathscr{M}_{\mathscr{E}}^{\text{hard}\,\dagger}\mathscr{M}_{\mathscr{E}}^{\text{hard}} \sim \int_0^{\mathcal{E}} q\, dq$$



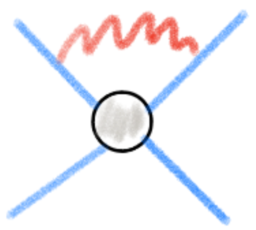
$$\sim G^2 \int_0^{\mathscr{E}} q\, dq \left[\int \frac{k\, dk}{2\pi} \frac{\theta(k-\mathscr{E})}{k^2} \frac{\theta(|k-q|-\mathscr{E})}{(k-q)^2} \right] \sim G^2$$

No graviton for $|q| < \mathscr{E}$ in $\mathscr{M}_{\mathscr{E}}^{\text{hard}}$

Contrast to previous papers with IR-divergence

$$\int_0^{\mathscr{E}} dq\ \psi(q)\ \mathscr{M}_{\epsilon}^{\dagger}\mathscr{M}_{\epsilon} = F\left(\frac{GM^2}{\epsilon}\right) \rightarrow G-\text{independent}$$

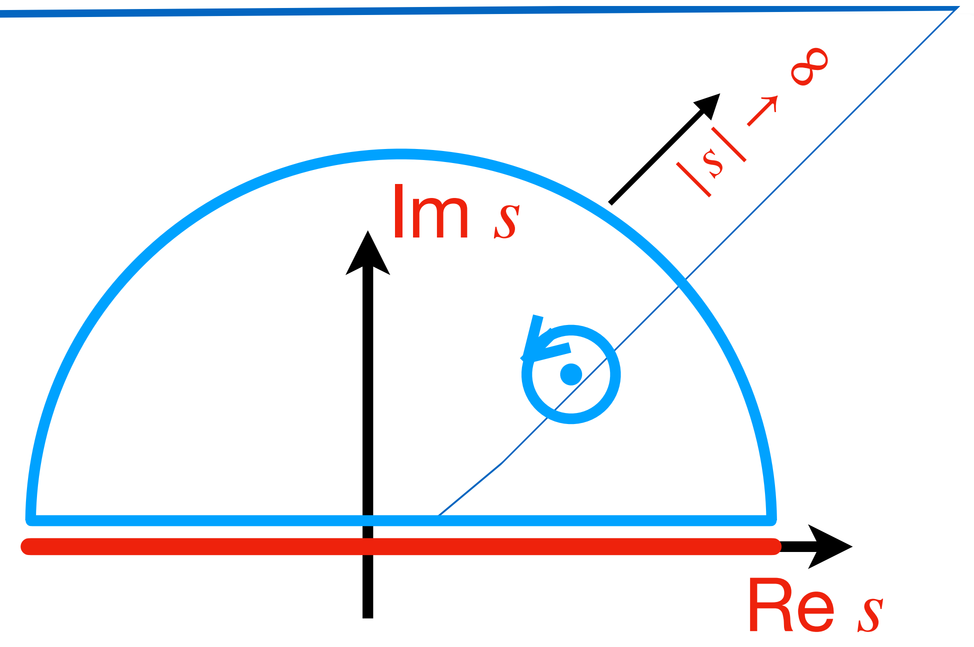
Chang Parra-Martinez '25 +trivialize the bound



$$\int_0^{\mathscr{E}} dq\ \psi(q)\ \mathscr{M}_{\lambda}^{\text{hard}\,\dagger}\mathscr{M}_{\lambda}^{\text{hard}} = F(GM^2 \log \mathscr{E}/\lambda)$$

Analogous in other regularizations

T-channel pole & Unitarity



$$\oint \mathscr{M}_{\mathscr{E}} \mathscr{K}(s) = \text{“Arc } \mathscr{A}\text{”}$$

1. IR finite
2. Lorentz invariant
3. Analytic
4. Average Regge

} Exactly $\quad g_{EFT} = \langle (\cdots) \rangle = \sum_{\ell} \int_{M^2}^{\infty} \frac{ds}{s^3} (\cdots) \mathrm{Im} \mathscr{M}_{\mathscr{E}}^{\ell}(s)$

5. Unitary
6. Inclusive, $\mathscr{E} \sim \Delta E$

} Match both sides in a limit

$$\int_{\mathscr{E}}^{M} dq \; \psi(q) \, \overline{\mathrm{Im}} \, \mathscr{M}_{\mathscr{E}}(s, -q^2) \to \int_{\mathscr{E}}^{M} dq \; \psi(q) \, \mathscr{M}_{\mathscr{E}}^{\mathrm{hard}\,\dagger} \mathscr{M}_{\mathscr{E}}^{\mathrm{hard}}$$

Unitary limit, examples:

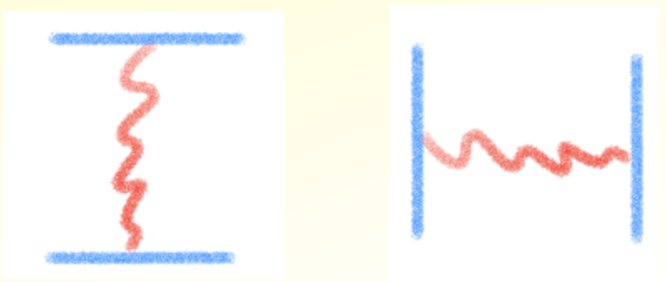
(a) Leading-Log (before smearing)

$\alpha \to 0, \quad \alpha \log \mathscr{E} = \alpha_{\mathscr{E}}$ - fixed

$G \to 0, \quad G \log \mathscr{E} = G_{\mathscr{E}}$ -fixed

(b) Tree-level

α, G —finite



(c) Leading T-channel sing.

α, G —finite

$\mathscr{E} \ll \sqrt{-t} \ll E$

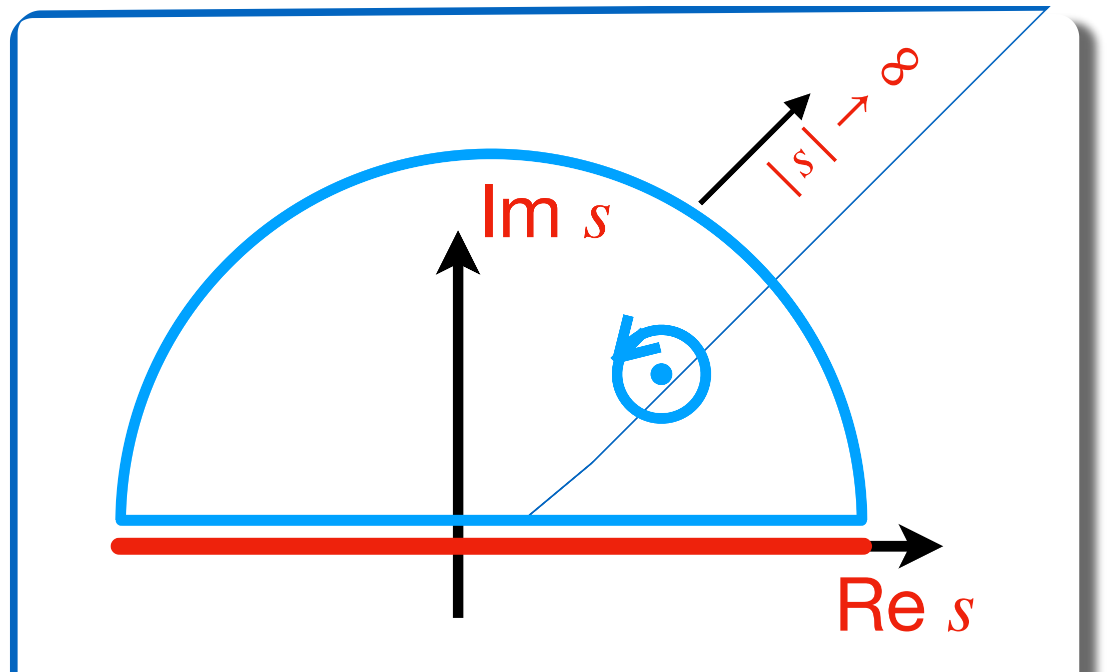
...

$$G s^2 / t \to G s^2 \log M / \mathscr{E}$$

T-channel pole & Unitarity

1. IR finite
2. Lorentz invariant
3. Analytic
4. Average Regge

Exactly $g_{EFT} = \langle(\cdots)\rangle = \sum_{\ell} \int_{M^2}^{\infty} \frac{ds}{s^3} (\cdots) \mathrm{Im}\mathscr{M}^{\ell}_{\mathscr{E}}(s)$



$\oint \mathscr{M}_{\mathscr{E}} \mathscr{K}(s) =$ "Arc $\mathscr{A}$"

5. Unitary
6. Inclusive, $\mathscr{E} \sim \Delta E$

Match both sides in a limit

$$\int_{\mathscr{E}}^{M} dq\, \psi(q)\, \overline{\mathrm{Im}}\, \mathscr{M}_{\mathscr{E}}(s, -q^2) \to \int_{\mathscr{E}}^{M} dq\, \psi(q)\, \mathscr{M}^{\mathrm{hard}\,\dagger}_{\mathscr{E}} \mathscr{M}^{\mathrm{hard}}_{\mathscr{E}}$$

Unitary limit, examples:

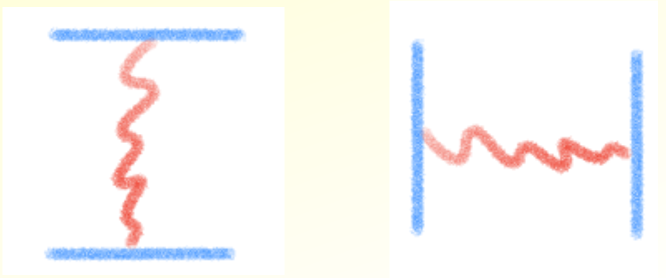
(a) Leading-Log (before smearing)

$\alpha \to 0, \quad \alpha \log \mathscr{E} = \alpha_{\mathscr{E}}$ - fixed

$G \to 0, \quad G \log \mathscr{E} = G_{\mathscr{E}}$ -fixed

(b) Tree-level

α, G —finite



(c) Leading T-channel sing.

α, G —finite

$\mathscr{E} \ll \sqrt{-t} \ll E$

...

$Gs^2/t \to Gs^2 \log M/\mathscr{E}$

t-pole matches