

Detector Amplitudes

how Positivity Meets the Real World



Brando Bellazzini

arXiv:2512.13780

with J. Berman, G. Isabella, F. Riva, M. Romano, F. Sciotti



EFT Pillars

Principles + *Symmetries* + *light d.o.f.*



EFT Pillars

Principles + *Symmetries* + *light d.o.f.*



QM

$$P^2 \geq 0 \quad P^0 \geq 0$$



Poincaré



EFT Pillars

Principles + *Symmetries* + *light d.o.f.*



QM
 $P^2 \geq 0 \quad P^0 \geq 0$



Poincaré

Wigner bootstrap



<i>massless</i>	<i>massive</i>
0	0
1/2	1/2
1	1
3/2	3/2
2	2
3	3
$\vdots$	$\vdots$



EFT Pillars

Principles + *Symmetries* + *light d.o.f.*



QM
 $P^2 \geq 0 \quad P^0 \geq 0$

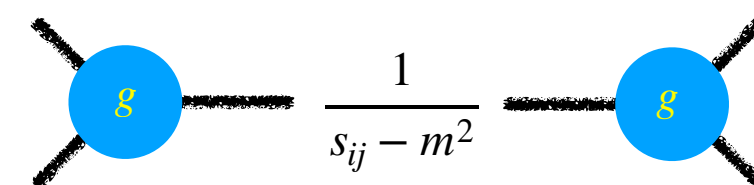
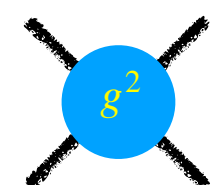
+

Locality



Poincaré

$$\mathcal{S} = \int dx \mathcal{L}(\mathcal{O}, \partial)$$



Wigner bootstrap

SUSY
Grisaru, Pendleton,
van Nieuwenhuizen '77

GR

No
HS

<i>massless</i>	<i>massive</i>
0	0
1/2	1/2
1	1
3/2	3/2
2	2
3 ⋮	3 ⋮

Porrati 0804.4672

Weinberg-Witten '80

Benincasa Cachazo 0705.4305



EFT Pillars

Principles + *Symmetries* + *light d.o.f.*



QM

$$P^2 \geq 0 \quad P^0 \geq 0$$

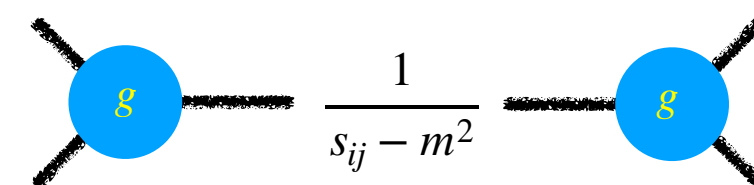
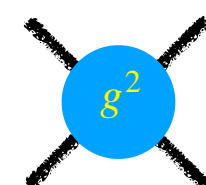
+

Locality



Poincaré

$$\mathcal{S} = \int dx \mathcal{L}(\mathcal{O}, \partial)$$



Wigner bootstrap



Higgs

YM, gauge

SUSY

Grisaru, Pendleton,
van Nieuwenhuizen '77

GR

No
HS

massless | *massive*

0

0

1/2

1/2

1

1

3/2

3/2

2

2

3

3

⋮

⋮

Porrati 0804.4672

Weinberg-Witten '80

Benincasa Cachazo 0705.4305



EFT Pillars

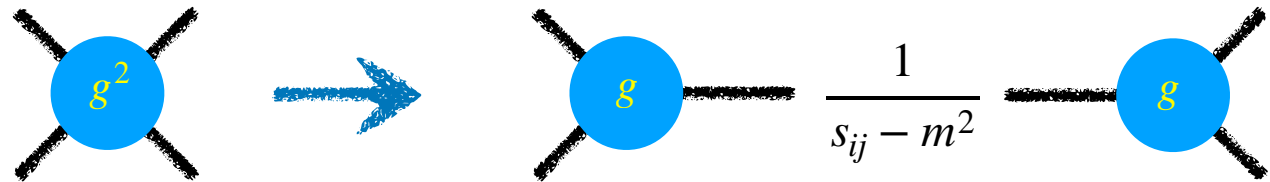
Principles + Symmetries + light d.o.f.

QM
 $P^2 \geq 0 \quad P^0 \geq 0$

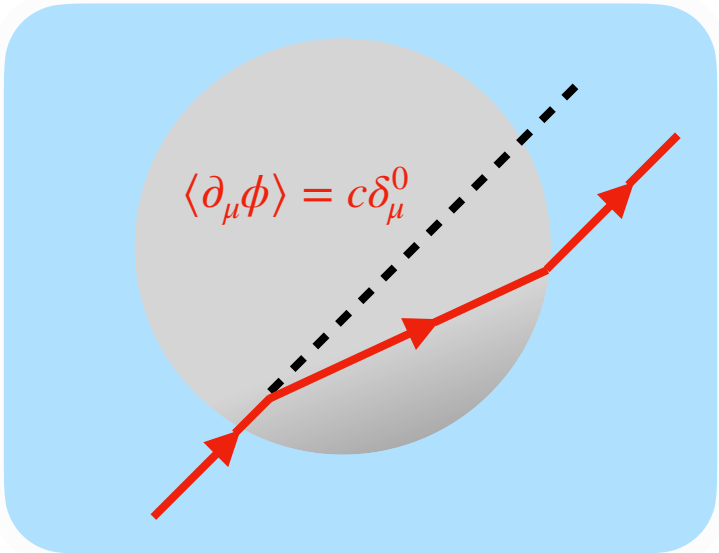
Poincaré

Locality

$$\mathcal{S} = \int dx \mathcal{L}(\mathcal{O}, \partial)$$



Causality/Positivity



Adams, Arkani-Hamed, Dubovsky, Nicolis Rattazzi '06

Wigner bootstrap

- Higgs
- YM, gauge
- SUSY
- GR
- No HS

Grisaru, Pendleton, van Nieuwenhuizen '77

	massless	massive
Higgs	0	0
YM, gauge	1/2 1	1/2 1
SUSY	3/2	3/2
GR	2	2
No HS	3 ⋮	3 ⋮

BB, Pomarol, Romano, Sciotti 2304.02550

BB, Isabella, Ricossa, Riva 2304.02550

Porrati 0804.4672

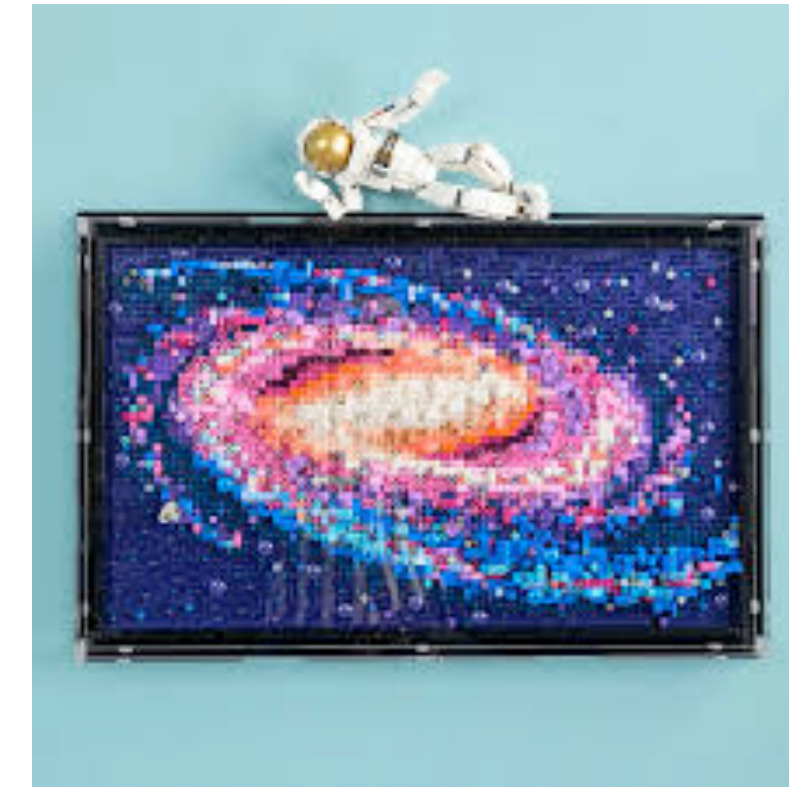
Weinberg-Witten '80

Benincasa Cachazo 0705.4305



- Philosophy

Complex Universe from simple building block Principles



SM & GR
largely fixed by consistency

$$\mathcal{L}_{\text{leading}}^{\text{EFT}} = -R + F_{\mu\nu}^2 + \psi^\dagger i \partial \psi + (\partial \phi)^2 + \psi \psi \phi - V(\phi)$$

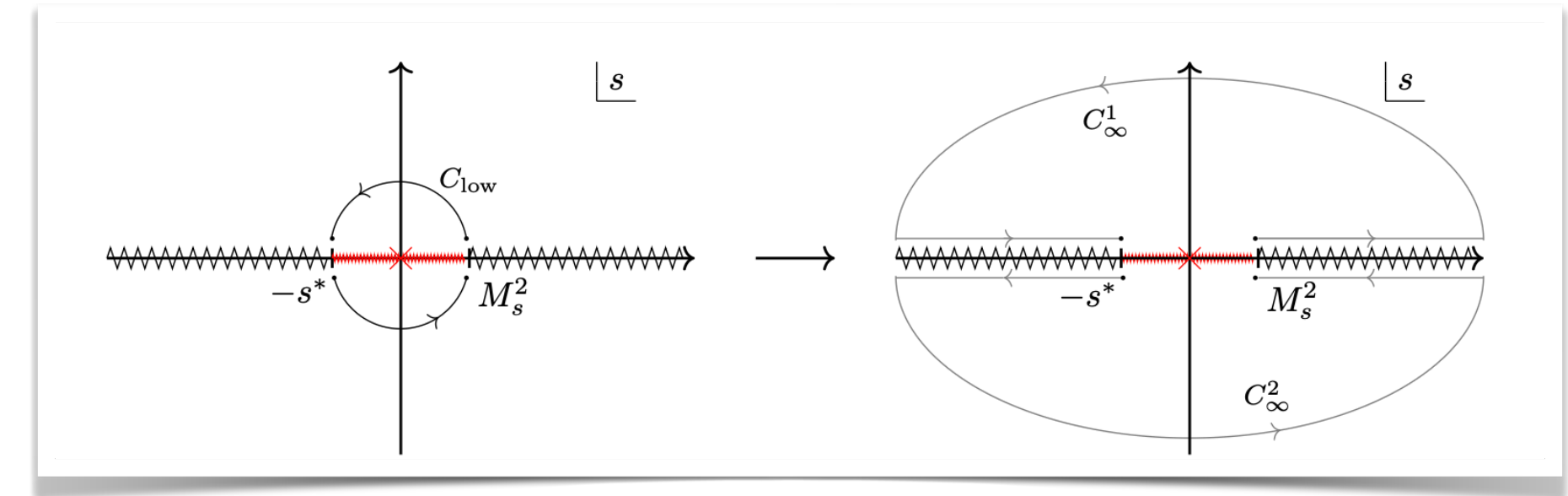
Q: Does consistency fix/constrain subleading terms?

Yes!

Examples

Example-1: SUSY-breaking space

$$\mathcal{M}(1^+2^+3^-4^-) = [12]\langle 14 \rangle (c_{1,0}s + c_{2,0}s^2 + c_{2,1}tu + \dots)$$

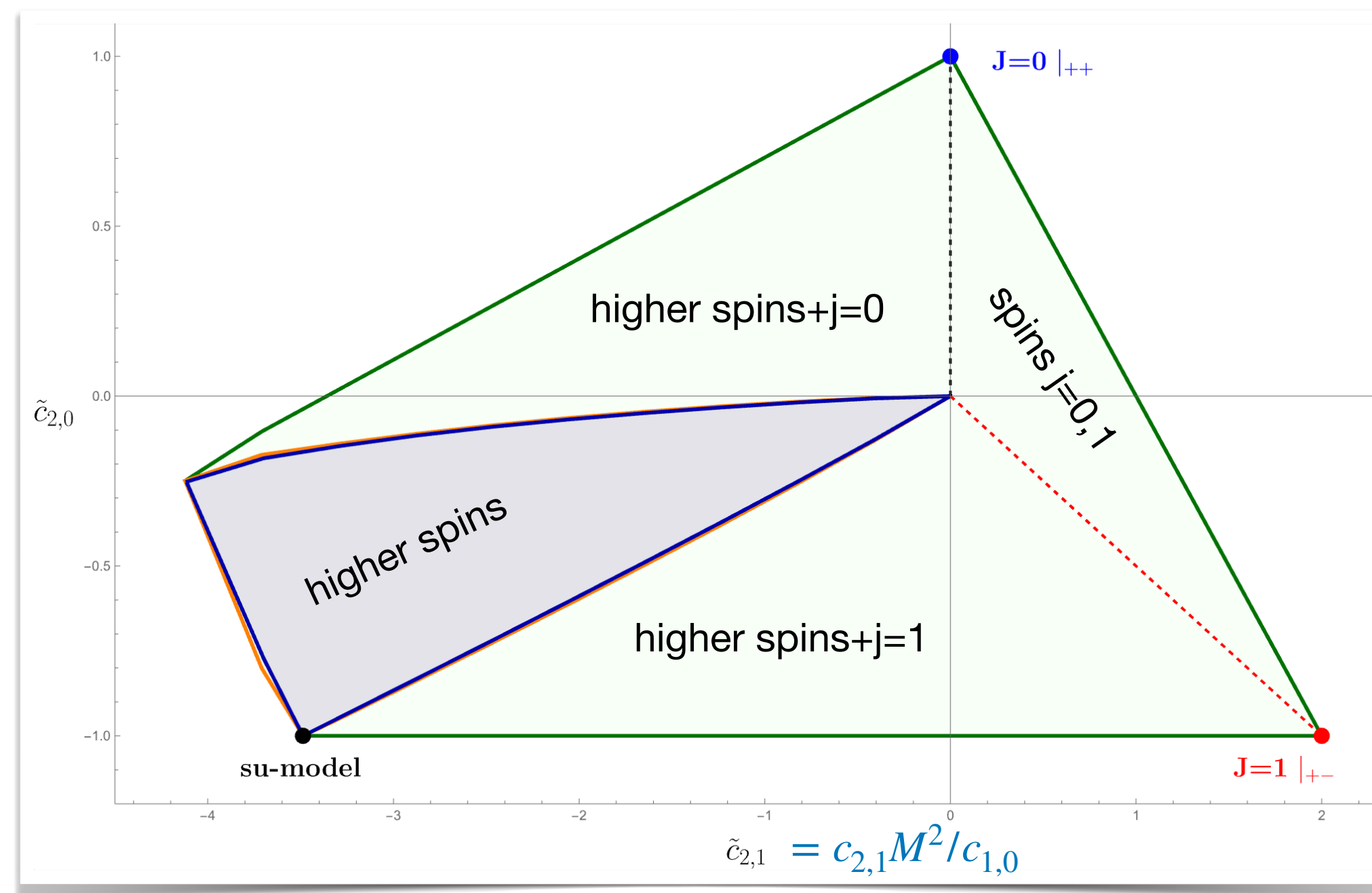


$$c_{1,0} = -\langle 1 \rangle_{--} - \langle 1 \rangle_{-+}$$

$$c_{2,0} = -\langle 1/s \rangle_{--} + \langle 1/u \rangle_{-+}$$

$$\vdots$$

$$\langle (\dots) \rangle_{--} = \sum_{\ell} \int_{M^2}^{\infty} \frac{ds}{s^3} (\dots) \text{Im} \mathcal{M}_{\ell}^{--++}(s)$$

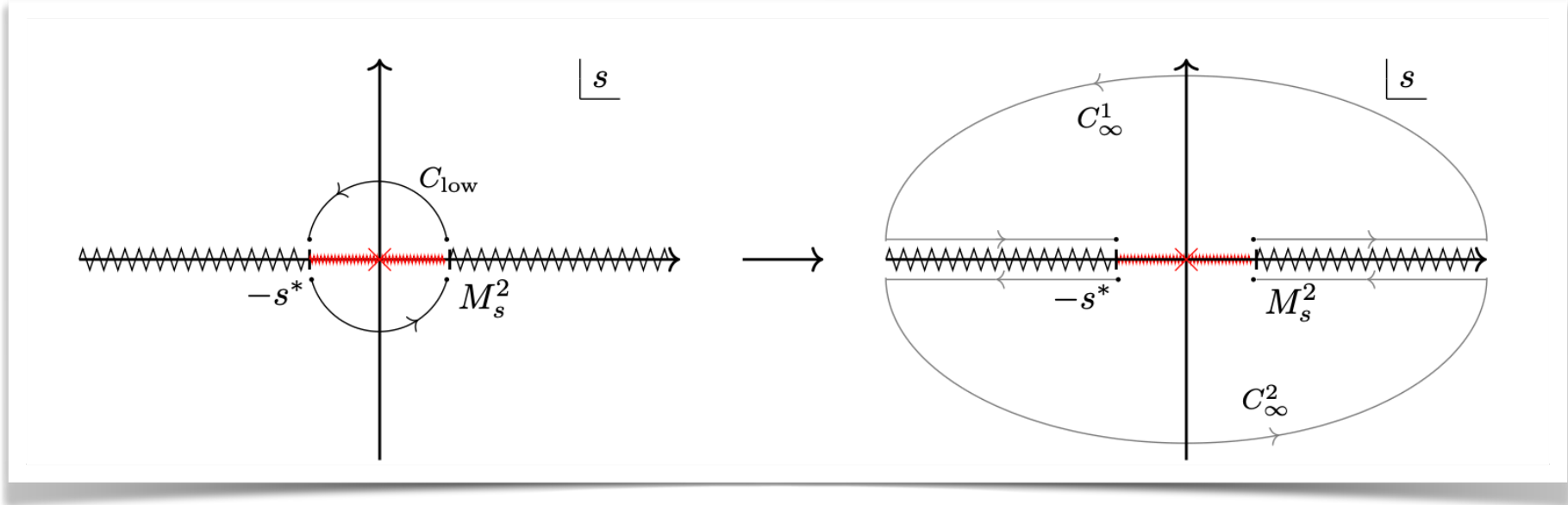


The Goldstino-hedron

BB, Pomarol, Romano, Sciotti 2304.02550

Example-1: SUSY-breaking space

$$\mathcal{M}(1^+2^+3^-4^-) = [12]\langle 14 \rangle (c_{1,0}s + c_{2,0}s^2 + c_{2,1}tu + \dots)$$

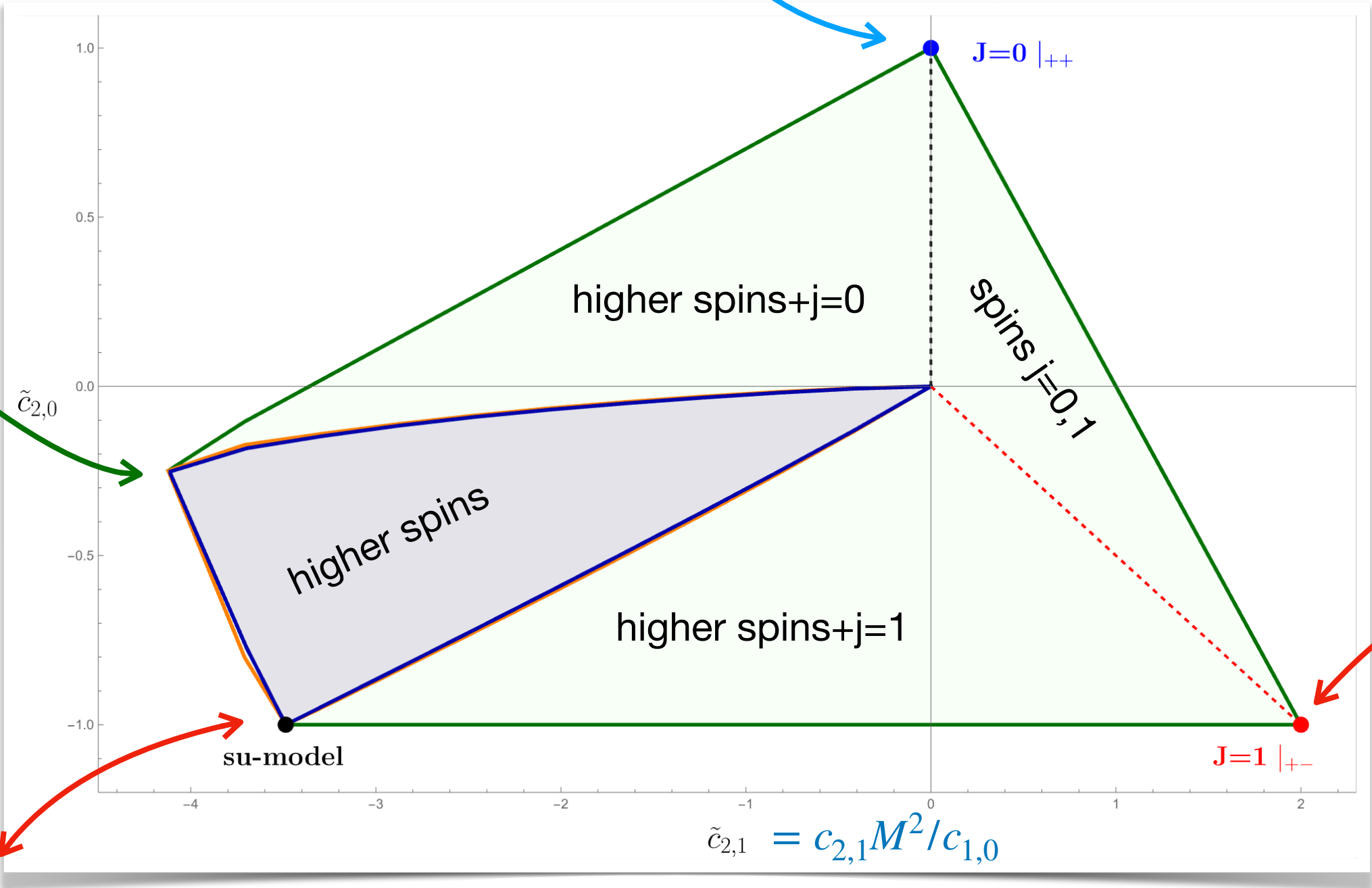


single spin-0 exchange

$$\frac{1}{s - M^2} = \frac{\mathcal{M}(1^+2^+3^-4^-)}{[12]\langle 14 \rangle}$$

infinitely many
degenerate spins in
+- & ++

$$\frac{M^2s + \alpha s^2 + \beta ut + \gamma stu/M^2}{(s - M^2)(u - M^2)(t - M^2)}$$



$$c_{1,0} = -\langle 1 \rangle_{--} - \langle 1 \rangle_{-+}$$

$$c_{2,0} = -\langle 1/s \rangle_{--} + \langle 1/u \rangle_{-+}$$

⋮

$$\langle (\dots) \rangle_{--} = \sum_{\ell} \int_{M^2}^{\infty} \frac{ds}{s^3} (\dots) \text{Im} \mathcal{M}_{\ell}^{--++}(s)$$

single spin-1

$$\frac{1}{M^2} \left(\frac{u}{u - M^2} + \frac{t}{t - M^2} \right)$$

A new kind
of Model Building

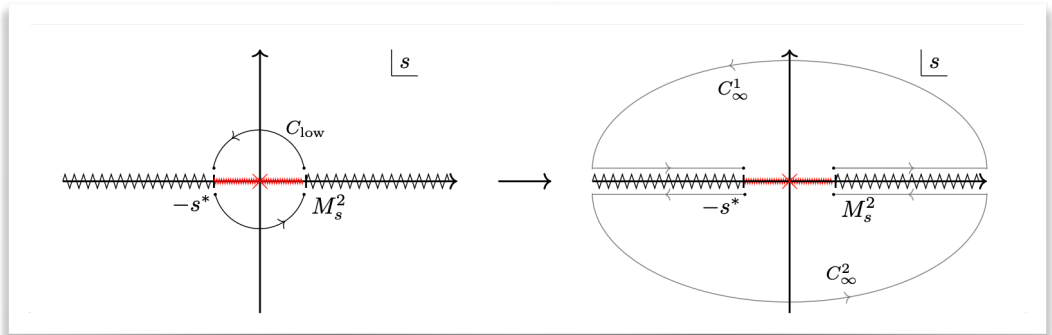
infinitely many
degenerate spins in +-
 $\frac{\alpha s + ut/M^2}{(u - M^2)(t - M^2)}$

Example-2: Goldstones

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + g_2(\partial_\mu \phi)^4 + g_3(\partial \phi)^2(\partial \partial \phi)^2 + g_4(\partial \partial \phi)^4 + \dots$$

$g_2 > 0$

Adams, Arkani-Hamed, Dubovsky,
Nicolis Rattazzi '06

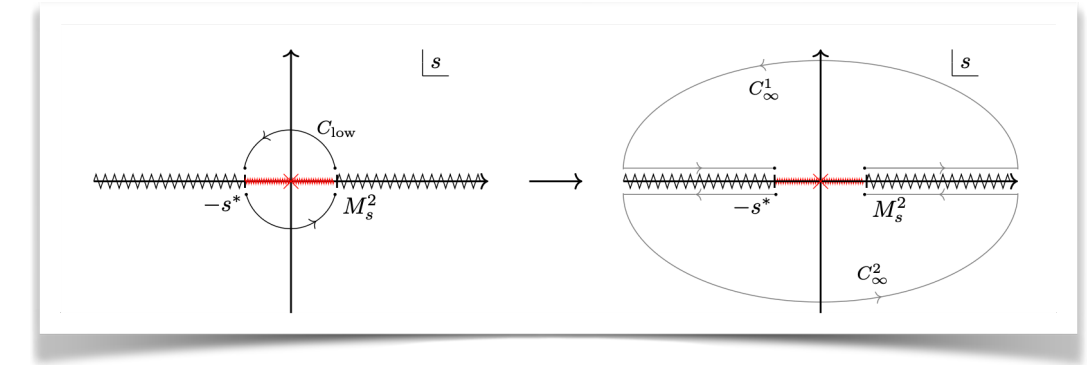


Example-2: Goldstones

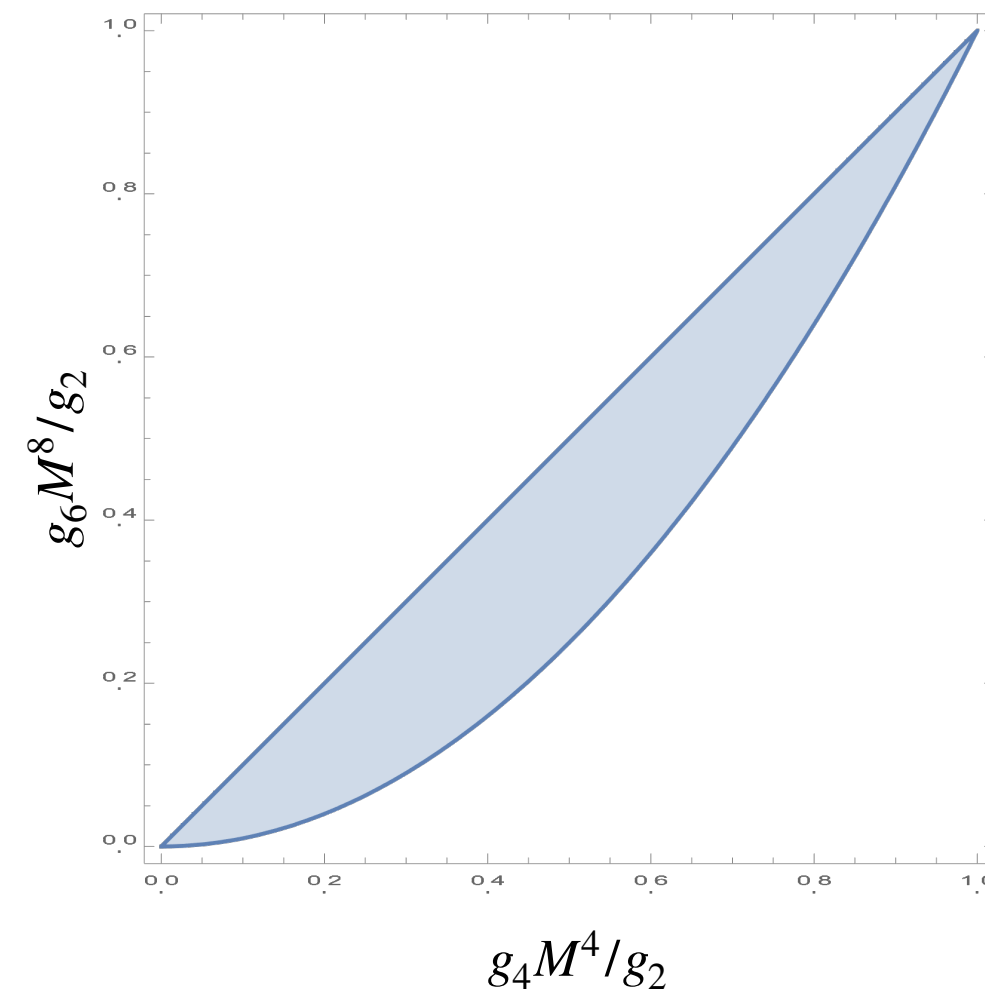
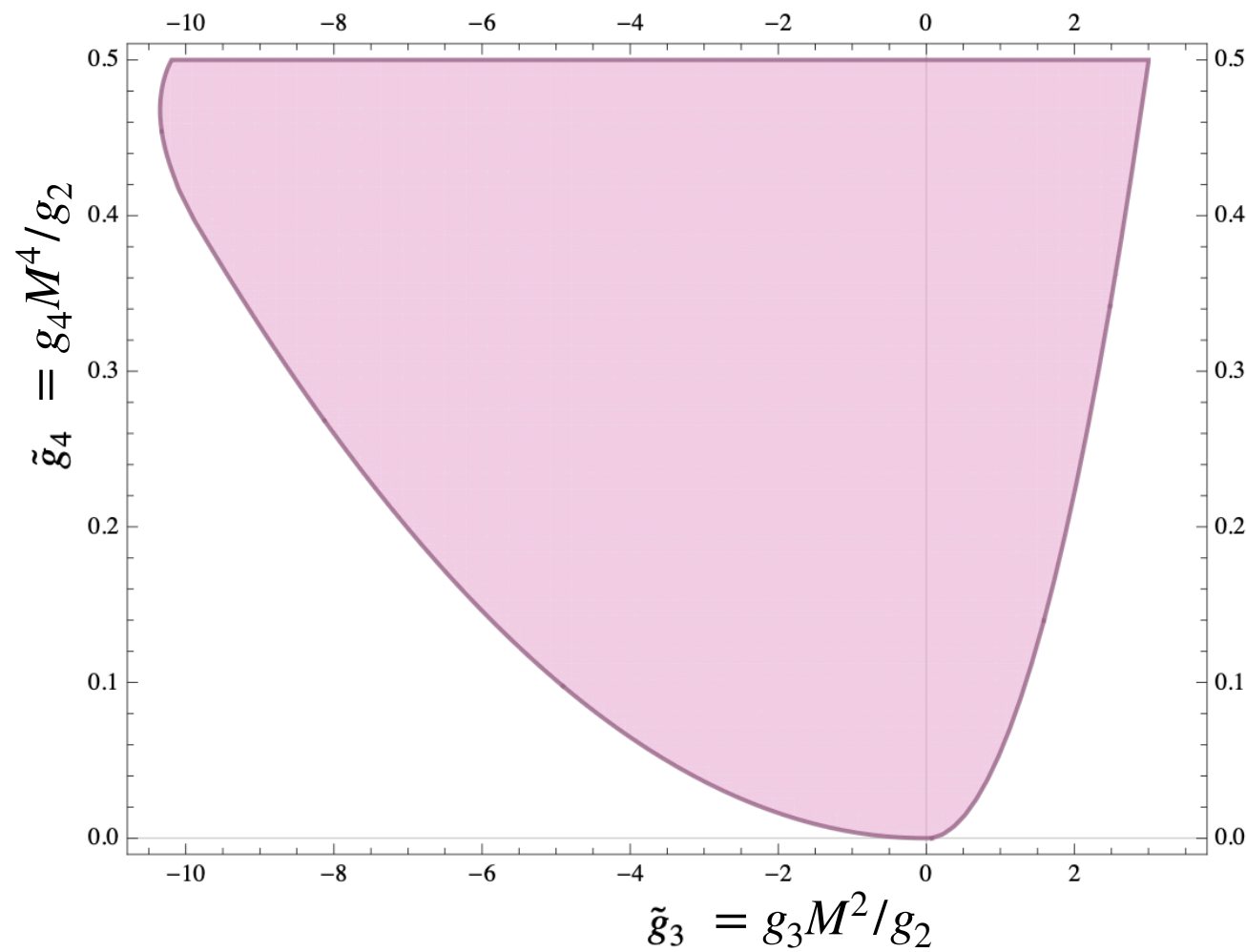
$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + g_2(\partial_\mu \phi)^4 + g_3(\partial \phi)^2(\partial \partial \phi)^2 + g_4(\partial \partial \phi)^4 + \dots$$

$$g_2 > 0$$

Adams, Arkani-Hamed, Dubovsky,
Nicolis Rattazzi '06



Clash: “Symmetries” vs “Causality”



Caron-huot, van Duong 2011.02957

Tolley, Wang, Zhou 2011.02400

BB, Miro, Rattazzi, Riembau, Riva 2011.00037

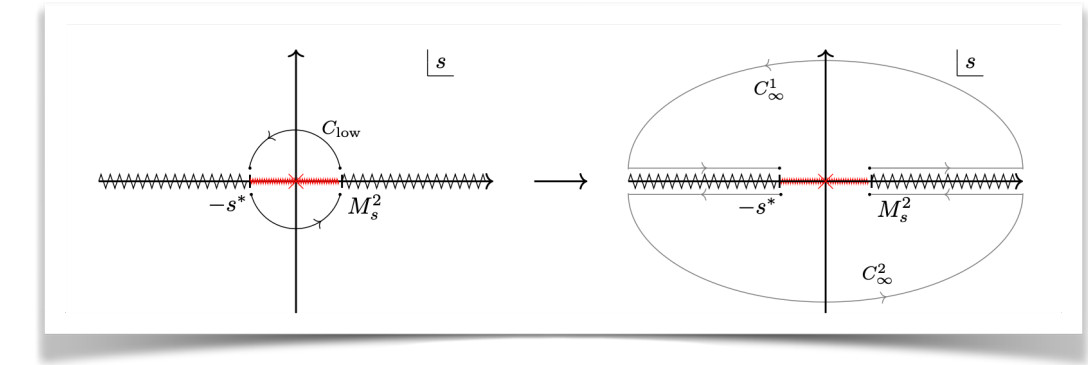
Arkani-Hamed, Huang, Huang, 2012.15849

Example-2: Goldstones

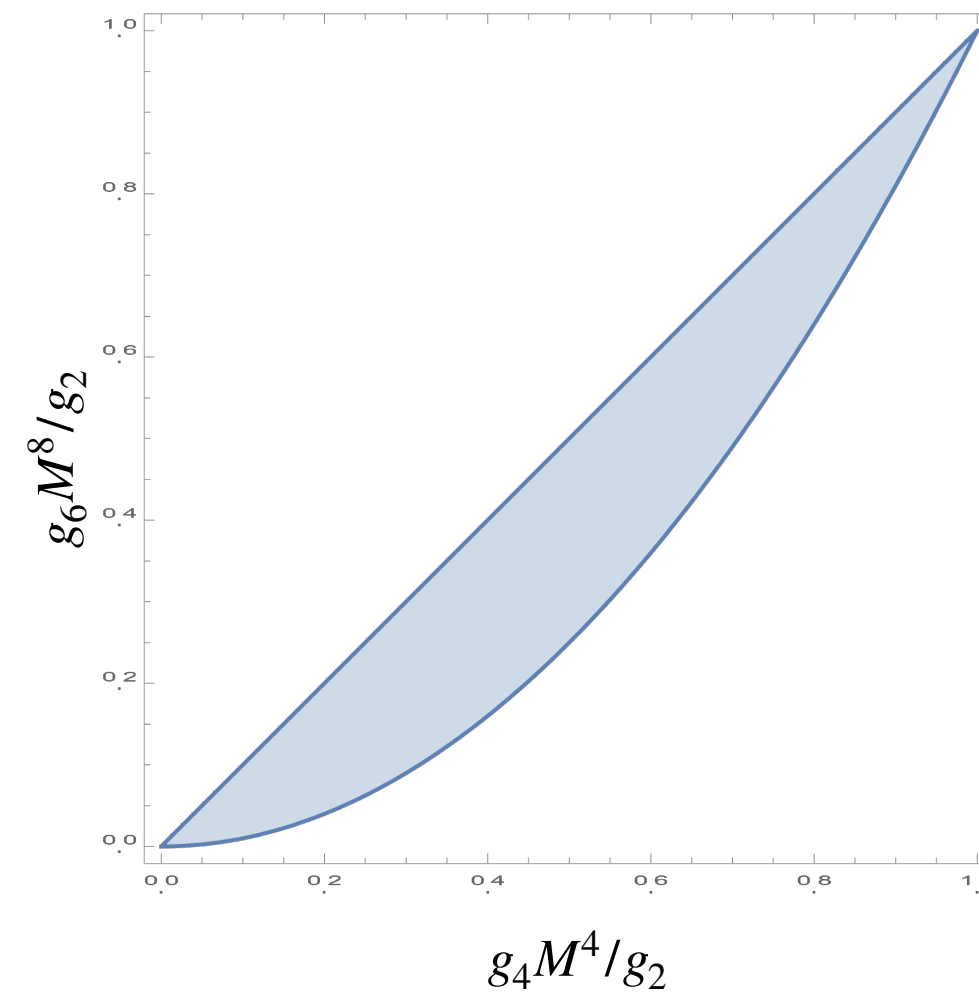
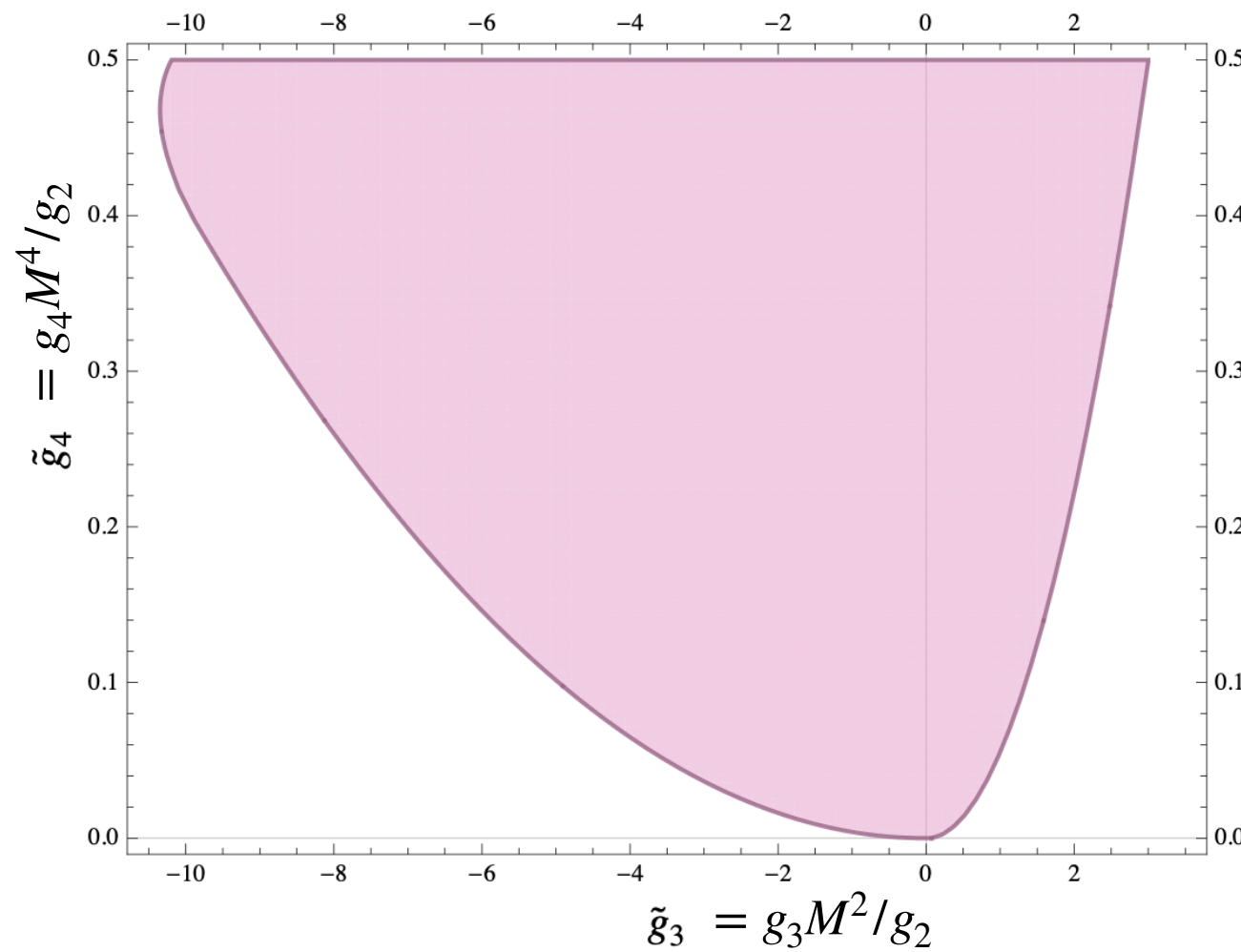
$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + g_2(\partial_\mu \phi)^4 + g_3(\partial \phi)^2(\partial \partial \phi)^2 + g_4(\partial \partial \phi)^4 + \dots$$

$$g_2 > 0$$

Adams, Arkani-Hamed, Dubovsky,
Nicolis Rattazzi '06



Clash: “Symmetries” vs “Causality”



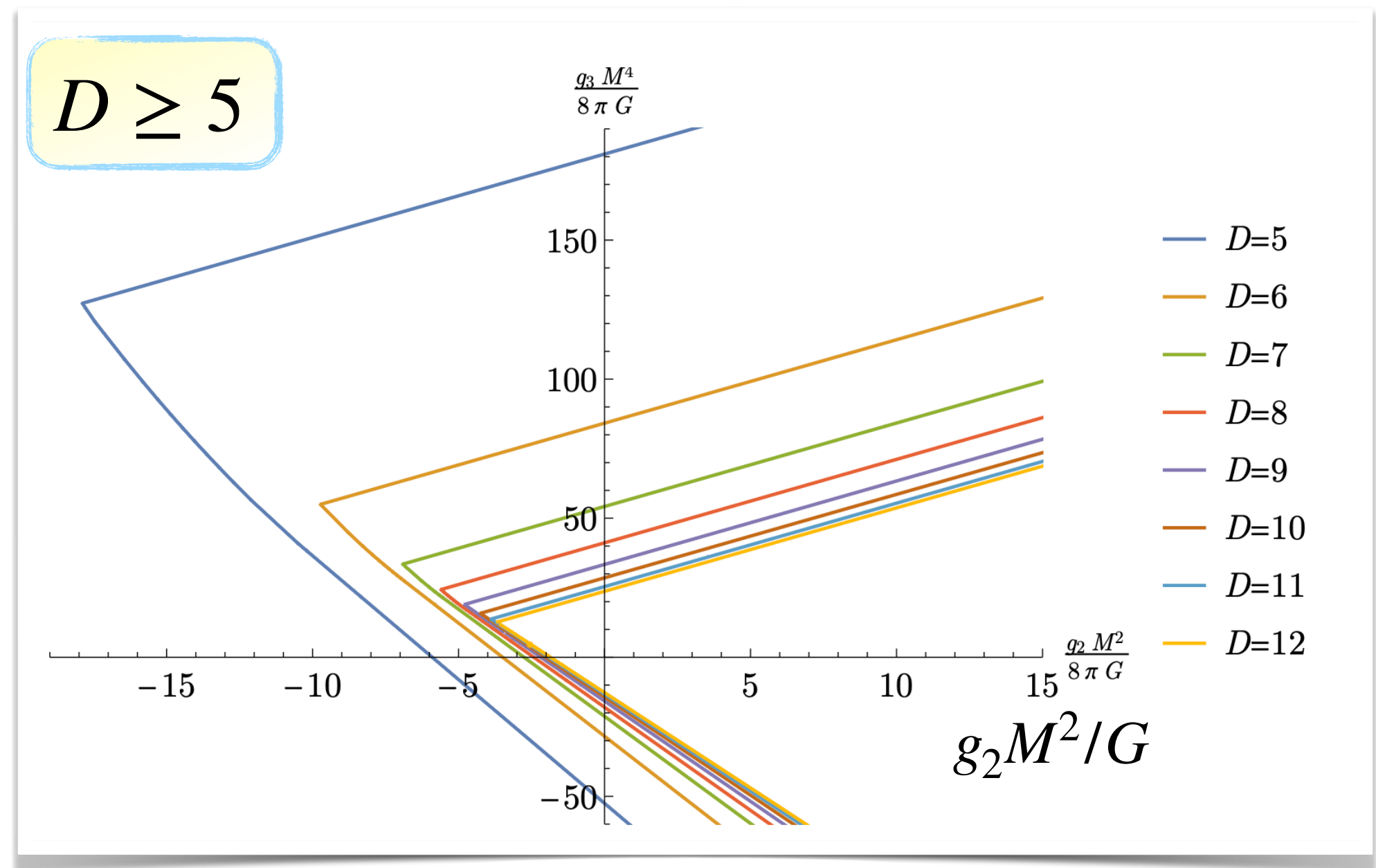
Caron-huot, van Duong 2011.02957

Tolley, Wang, Zhou 2011.02400

BB, Miro, Rattazzi, Riembau, Riva 2011.00037

Arkani-Hamed, Huang, Huang, 2012.15849

Adding gravity:



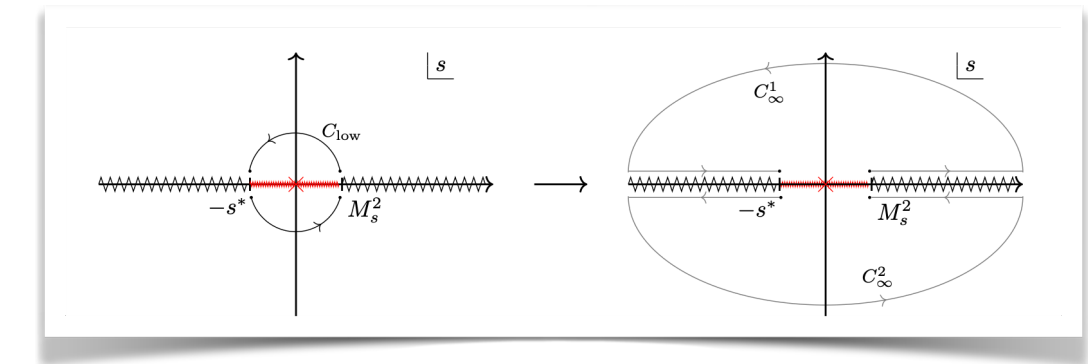
Caron-huot, Mazac, Rastelli, Simmons-Duffin 2102.08951

Example-2: Goldstones

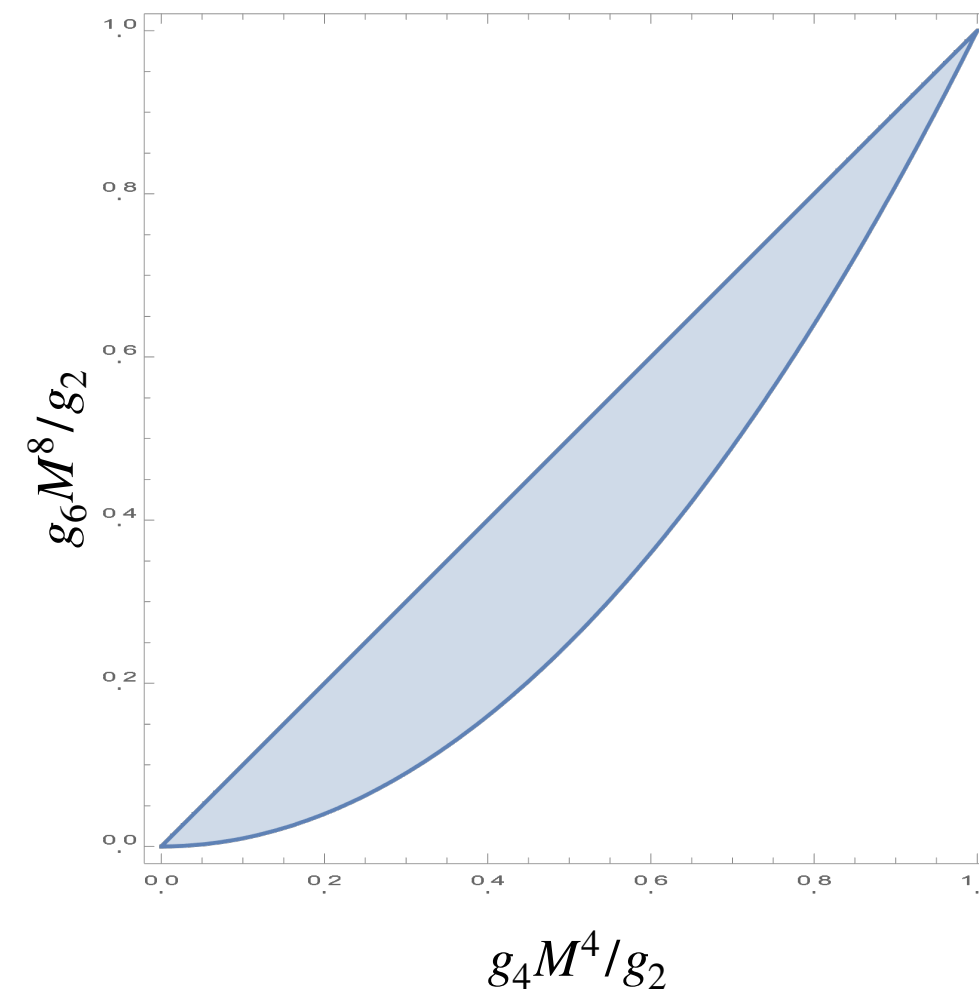
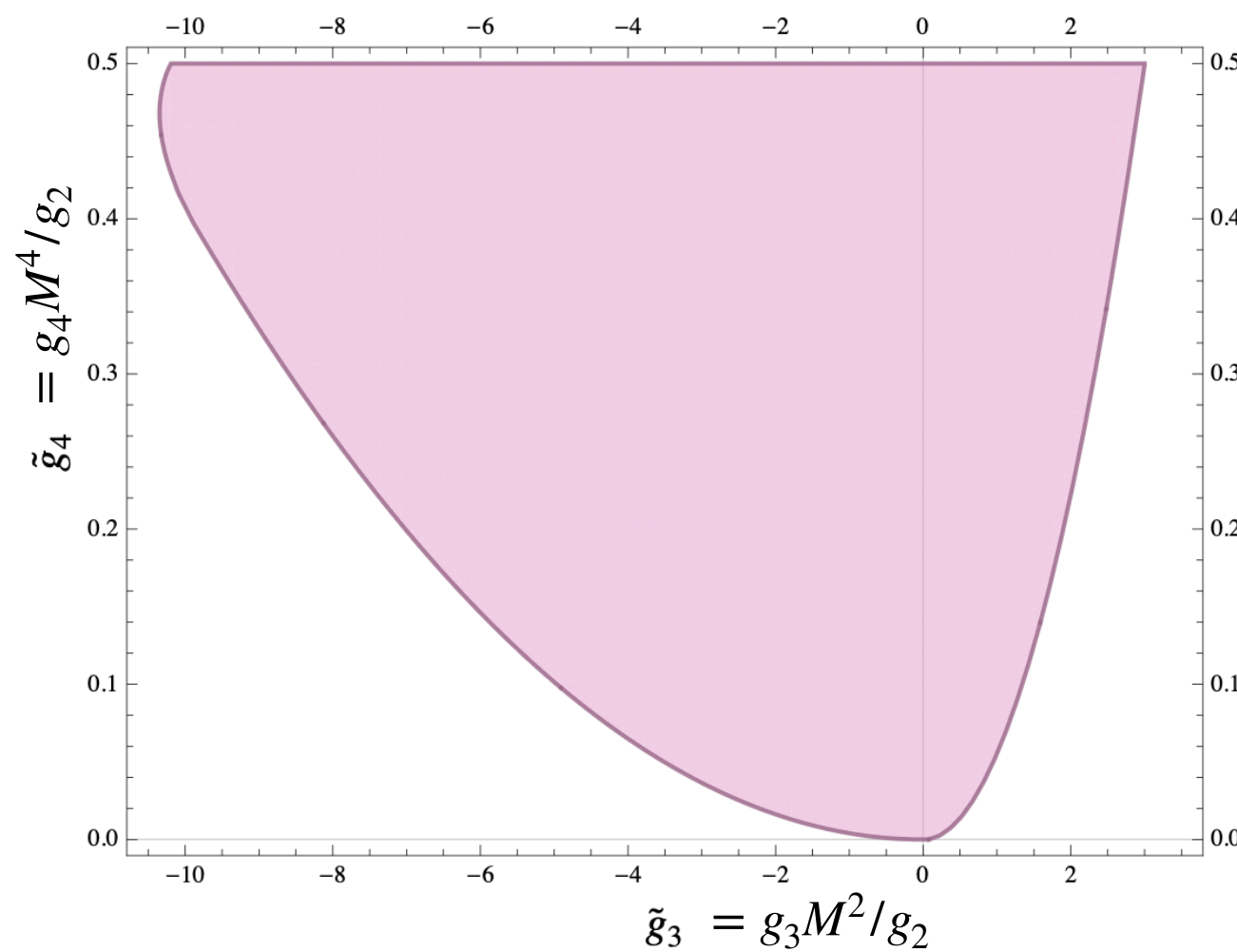
$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + g_2(\partial_\mu \phi)^4 + g_3(\partial \phi)^2(\partial \partial \phi)^2 + g_4(\partial \partial \phi)^4 + \dots$$

$$g_2 > 0$$

Adams, Arkani-Hamed, Dubovsky,
Nicolis Rattazzi '06



Clash: “Symmetries” vs “Causality”



Caron-huot, van Duong 2011.02957

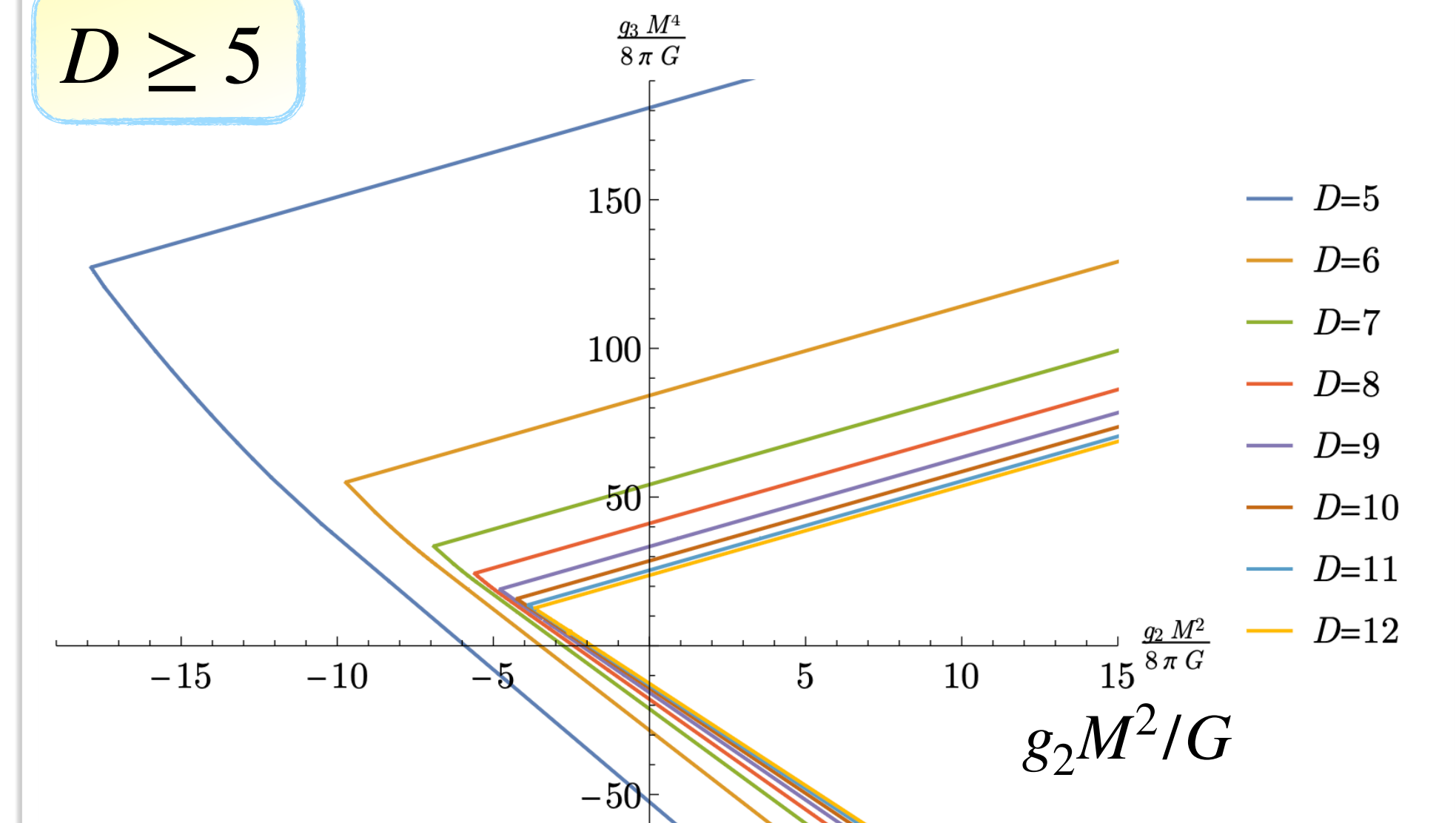
Tolley, Wang, Zhou 2011.02400

BB, Miro, Rattazzi, Riembau, Riva 2011.00037

Arkani-Hamed, Huang, Huang, 2012.15849

Adding gravity: $D = 4???$

$D \geq 5$

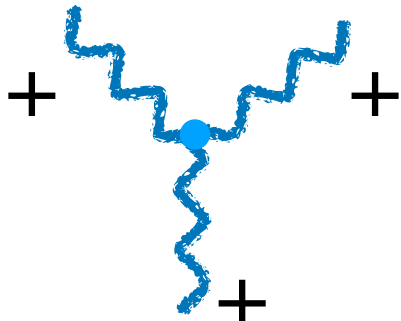


Caron-huot, Mazac, Rastelli, Simmons-Duffin 2102.08951

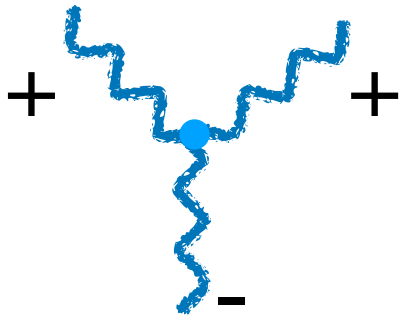
Example-3: EFT Gravity

$$\mathcal{L} = \frac{1}{G} \left(-R + \ell_2^2 R^2 + \ell_3^4 R^3 + \ell_4^6 R^4 + \dots \right) \qquad \ell_i^n = \frac{\alpha_i}{M^n}$$

$|\alpha_i| \gg 1?$

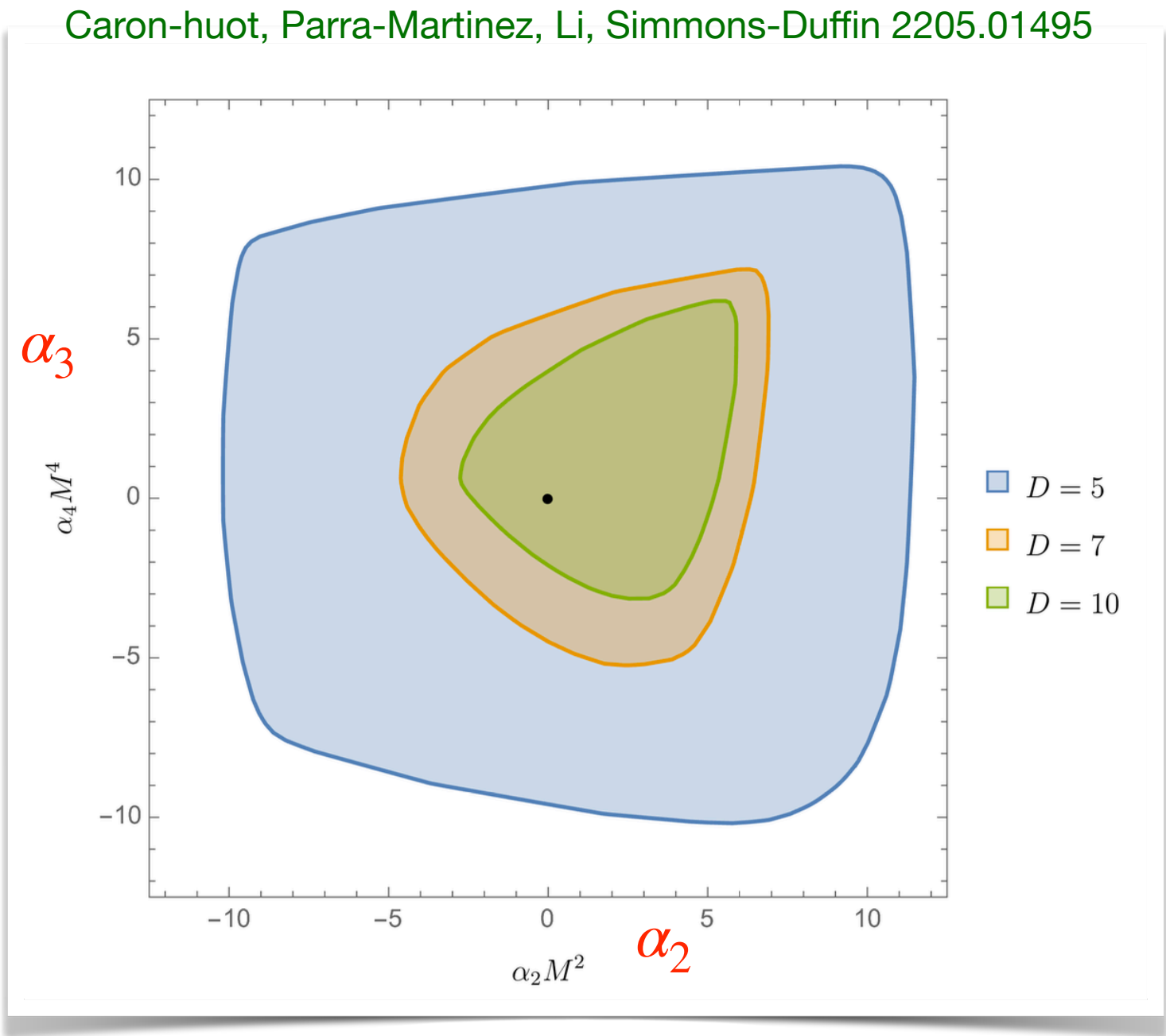
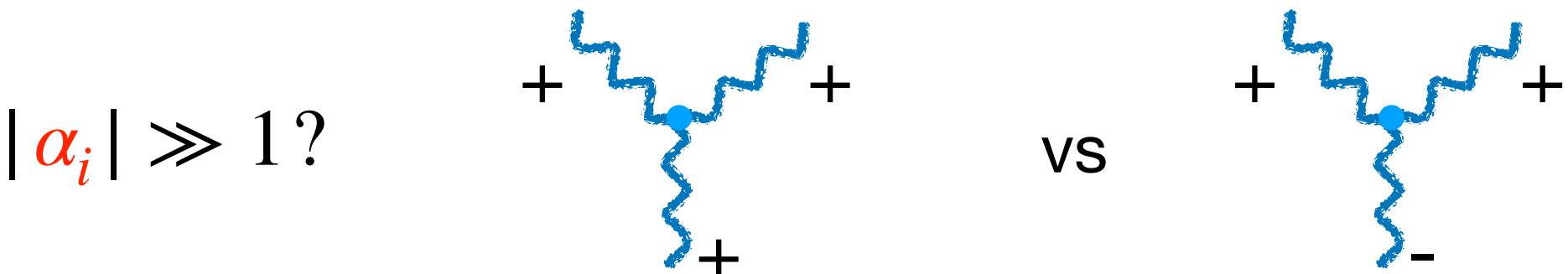


vs



Example-3: EFT Gravity

$$\mathcal{L} = \frac{1}{G} \left(-R + \ell_2^2 Rie^2 + \ell_3^4 Rie^3 + \ell_4^6 Rie^4 + \dots \right) \qquad \ell_i^n = \frac{\alpha_i}{M^n}$$



$D \geq 5$

$$\Delta T \propto R_s \left(\frac{R_s}{b} \right)^{D-4} \left(\begin{matrix} 1 & (\ell_2/b)^2 + \dots \\ (\ell_2/b)^2 + \dots & 1 \end{matrix} \right) \succ 0$$

Camanho, Edelstein, Maldacena, Zhiboedov 1407.5597

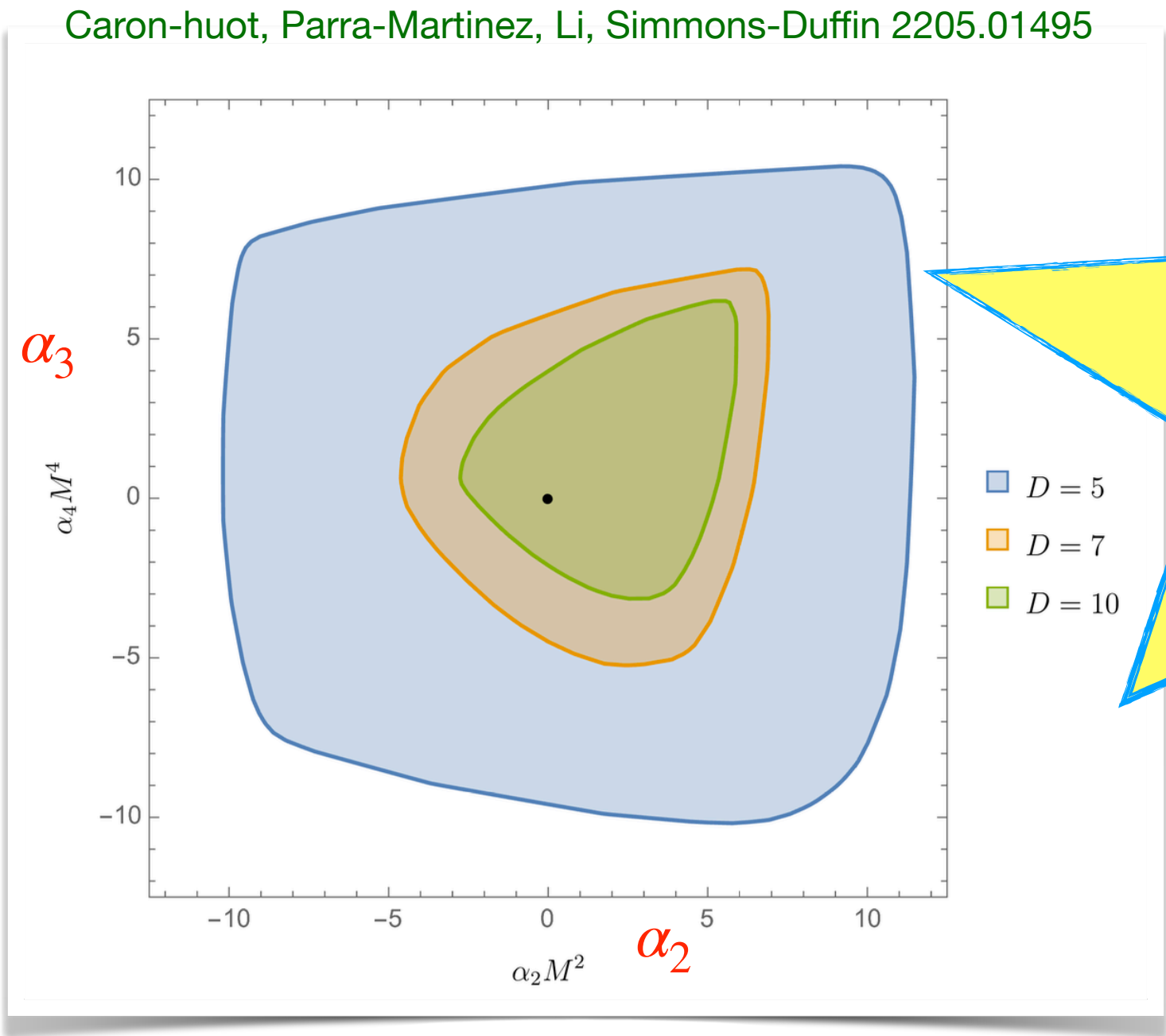
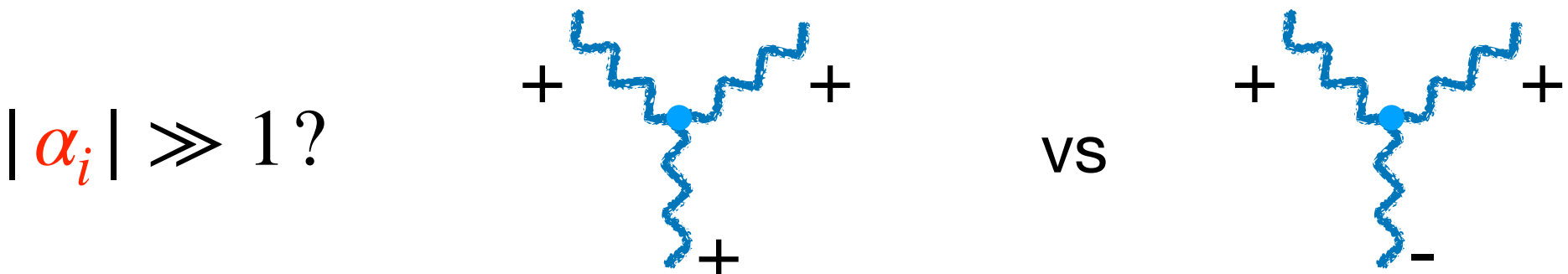
$0 < \alpha_4, \alpha_4' < O(1)GM^2$

BB, Cheung, Remmen 1509.00851

Caron-huot, Parra-Martinez, Li, Simmons-Duffin 2201.06602

Example-3: EFT Gravity

$$\mathcal{L} = \frac{1}{G} \left(-R + \ell_2^2 Rie^2 + \ell_3^4 Rie^3 + \ell_4^6 Rie^4 + \dots \right) \quad \ell_i^n = \frac{\alpha_i}{M^n}$$



$$D \geq 5$$

$D = 4???$

$$\propto R_s \left(\frac{R_s}{b} \right)^{D-4} \left(\frac{1}{(\ell_2/b)^2 + \dots} \quad \frac{(\ell_2/b)^2 + \dots}{1} \right) \succ 0$$

manho, Edelstein, Maldacena, Zhiboedov 1407.5597

$$0 < \alpha_4, \alpha_4' < O(1)GM^2$$

BB, Cheung, Remmen 1509.00851
Caron-huot, Parra-Martinez, Li, Simmons-Duffin 2201.06602

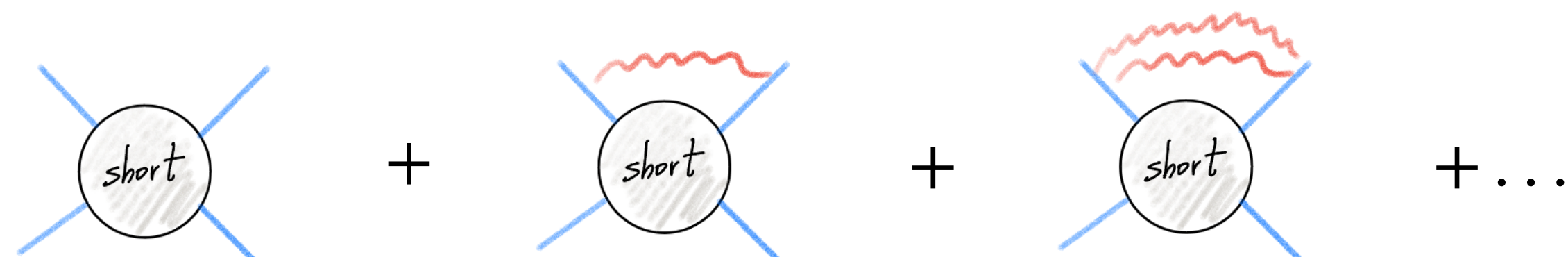
Long-Range Interactions

Long-Range forces in $D=4$ E&M or Gravity

(Ordinary) Amplitudes are NOT good observables

Long-Range forces in $D=4$ E&M or Gravity

(Ordinary) Amplitudes are NOT good observables



The diagram shows a series of Feynman diagrams representing a scattering process. Each diagram consists of a central shaded circle labeled "short" with two blue lines entering from the left and two exiting to the right. The first diagram is a simple circle. The second diagram has a red wavy line connecting the top of the circle to itself. The third diagram has a red wavy line connecting the top of the circle to a second, identical circle, which then has two blue lines exiting to the right. This represents a series expansion of a scattering amplitude.

$$+ \frac{GE^2}{\epsilon} + \left(\frac{GE^2}{\epsilon} \right)^2 + \dots$$
$$+ \frac{\alpha}{\epsilon} + \left(\frac{\alpha}{\epsilon} \right)^2 + \dots$$

Long-Range forces in $D=4$ E&M or Gravity

(Ordinary) Amplitudes are NOT good observables

$$\begin{array}{c}
 \text{[Diagram: circle labeled 'short' with two blue lines crossing it]} + \text{[Diagram: circle labeled 'short' with two blue lines and a red wavy line]} + \text{[Diagram: circle labeled 'short' with two blue lines and two red wavy lines]} + \dots = \text{[Diagram: circle labeled 'short' with two blue lines]} \times \text{Exp} \left(\sum_{i,j} \text{[Diagram: vertex with two blue lines labeled } i, j \text{ and a red wavy line]} \right) \rightarrow 0
 \end{array}$$

$\frac{GE^2}{\epsilon}$
 $\frac{\alpha}{\epsilon}$

$\left(\frac{GE^2}{\epsilon} \right)^2$
 $\left(\frac{\alpha}{\epsilon} \right)^2$

$\mathcal{W}^\epsilon \sim e^{\frac{-GE^2}{\epsilon}} \rightarrow e^{-\infty}$

Weinberg Soft Exp

Long-Range forces in $D=4$ E&M or Gravity

(Ordinary) Amplitudes are NOT good observables

$$\begin{array}{c}
 \text{short} \\
 \text{---} \bigcirc \text{---} \\
 \text{---} \text{---}
 \end{array}
 +
 \begin{array}{c}
 \text{short} \\
 \text{---} \bigcirc \text{---} \\
 \text{---} \text{---}
 \end{array}
 \frac{GE^2}{\epsilon}
 +
 \begin{array}{c}
 \text{short} \\
 \text{---} \bigcirc \text{---} \\
 \text{---} \text{---}
 \end{array}
 \left(\frac{GE^2}{\epsilon} \right)^2
 + \dots
 =
 \begin{array}{c}
 \text{short} \\
 \text{---} \bigcirc \text{---} \\
 \text{---} \text{---}
 \end{array}
 \times \text{Exp} \left(\sum_{i,j} \text{---} \bigcirc \text{---} \right) \rightarrow 0$$

$\frac{\alpha}{\epsilon}$
 $\left(\frac{\alpha}{\epsilon} \right)^2$

$\mathcal{W}^\epsilon \sim e^{\frac{-GE^2}{\epsilon}} \rightarrow e^{-\infty}$
 Weinberg Soft Exp

(paradoxical)
Lessons

1. NOT a small perturbation, no matter how small G or α
2. No robust bootstrap program
3. UV/IR mixing?? Good place where to look!

Long-Range forces in $D=4$

E&M or Gravity

Inclusive X-sections? Good observables but ...

$$\left| \begin{array}{c} \text{diagram 1} \\ \vdots \\ \text{diagram 2} \\ \vdots \\ \text{diagram } n \end{array} \right|^2 + \left| \begin{array}{c} \text{diagram 1} \\ \vdots \\ \text{diagram 2} \\ \vdots \\ \text{diagram } n \end{array} \right|^2 + \dots \sim$$

The diagram shows a mathematical expression for inclusive cross-sections. It consists of two large terms, each being the square of a sum of diagrams, followed by an ellipsis and a tilde symbol. Each sum is enclosed in large vertical bars. The first sum contains diagrams with blue external lines and a shaded circular interaction region. The second sum contains diagrams with blue external lines, a shaded circular interaction region, and a red wavy line representing a long-range force. The diagrams are separated by plus signs and an ellipsis, indicating a series of higher-order terms.

Long-Range forces in $D=4$

E&M or Gravity

Inclusive X-sections? Good observables but ...

...NOT analytic

$$\left| \begin{array}{c} \text{diagram 1} \\ \vdots \\ \text{diagram } n \end{array} \right|^2 + \left| \begin{array}{c} \text{diagram 1} \\ \vdots \\ \text{diagram } n \end{array} \right|^2 + \dots \sim$$

$$\sum_{E_{\gamma,h} < \Delta E}^{\infty} \mathcal{M}^{\dagger} \mathcal{M}$$

$$\propto \left(\frac{\Delta E}{E} \right)^{\alpha, GE^2}$$

Universal log-behavior
on detector resolution

$$d\sigma_{\text{inclusive}} \rightarrow 0 \text{ if } \Delta E \rightarrow 0$$

Long-Range forces in $D=4$

E&M or Gravity

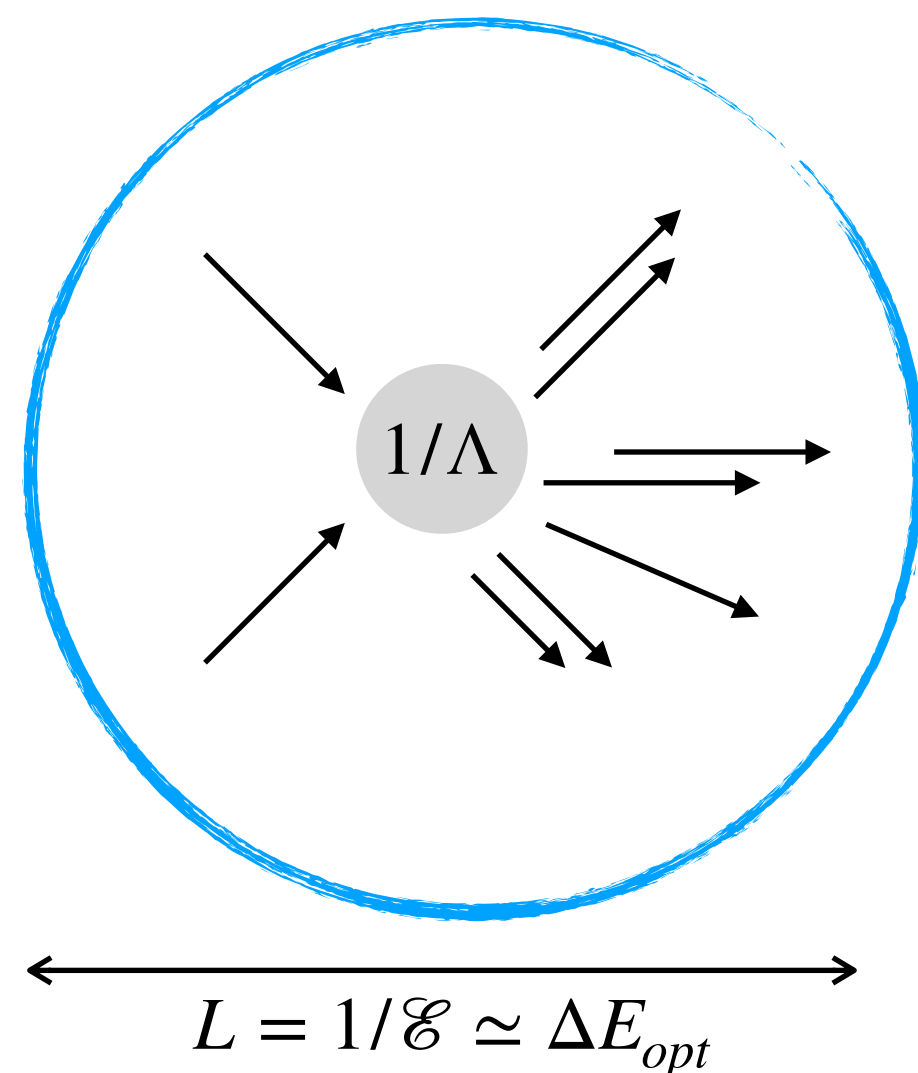
Inclusive X-sections? Good observables but ...

...NOT analytic

$$\left| \begin{array}{c} \text{diagram 1} \\ \vdots \\ \text{diagram } n \end{array} \right|^2 + \left| \begin{array}{c} \text{diagram } n+1 \\ \vdots \\ \text{diagram } m \end{array} \right|^2 + \dots \sim \sum_{E_{\gamma,h} < \Delta E}^{\infty} \mathcal{M}^\dagger \mathcal{M} \propto \left(\frac{\Delta E}{E} \right)^{\alpha, GE^2}$$

Universal log-behavior
on detector resolution

$$d\sigma_{\text{inclusive}} \rightarrow 0 \text{ if } \Delta E \rightarrow 0$$



Long-Range forces in $D=4$

E&M or Gravity

Inclusive X-sections? Good observables but ...

...NOT analytic

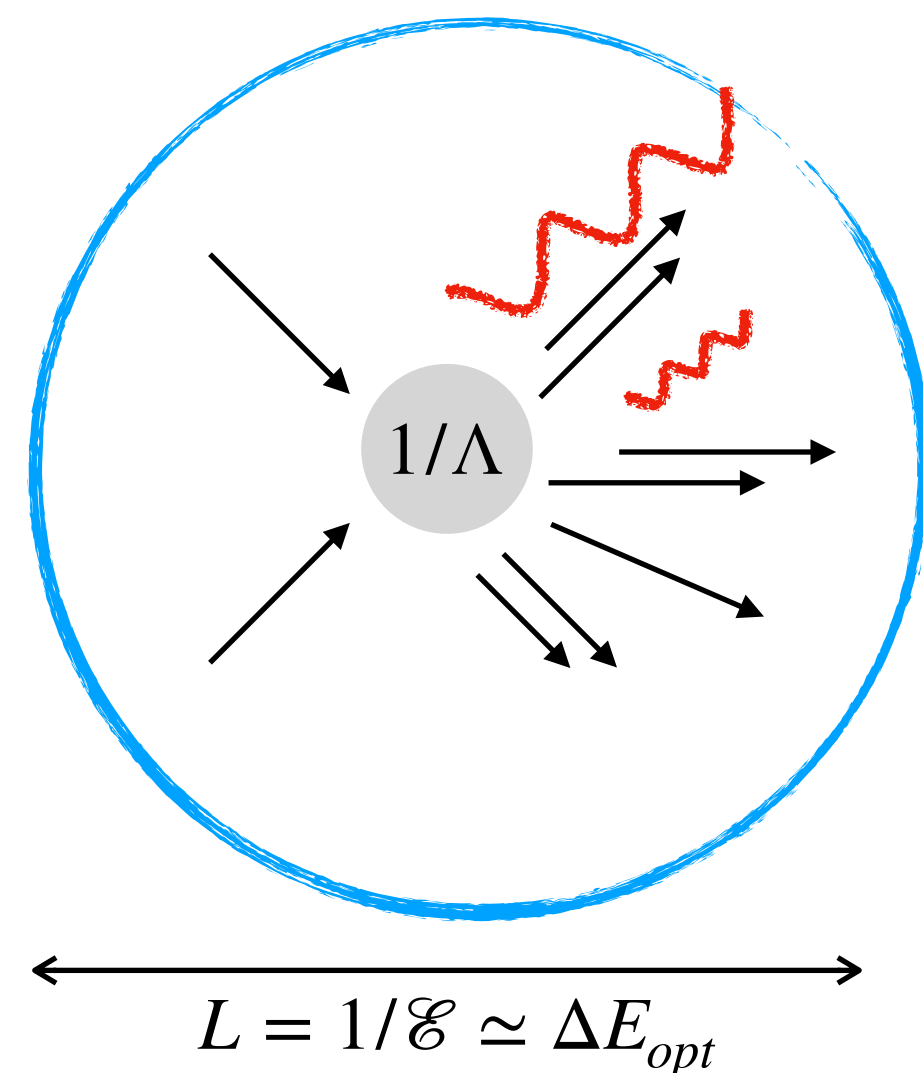
$$\left| \begin{array}{c} \text{diagram 1} \\ \vdots \\ \text{diagram } n \end{array} \right|^2 + \left| \begin{array}{c} \text{diagram } n+1 \\ \vdots \\ \text{diagram } m \end{array} \right|^2 + \dots \sim$$

$$\sum_{E_{\gamma,h} < \Delta E}^{\infty} \mathcal{M}^{\dagger} \mathcal{M}$$

$$\propto \left(\frac{\Delta E}{E} \right)^{\alpha, GE^2}$$

Universal log-behavior
on detector resolution

$$d\sigma_{inclusive} \rightarrow 0 \text{ if } \Delta E \rightarrow 0$$



Long-Range forces in $D=4$

E&M or Gravity

Inclusive X-sections? Good observables but ...

...NOT analytic

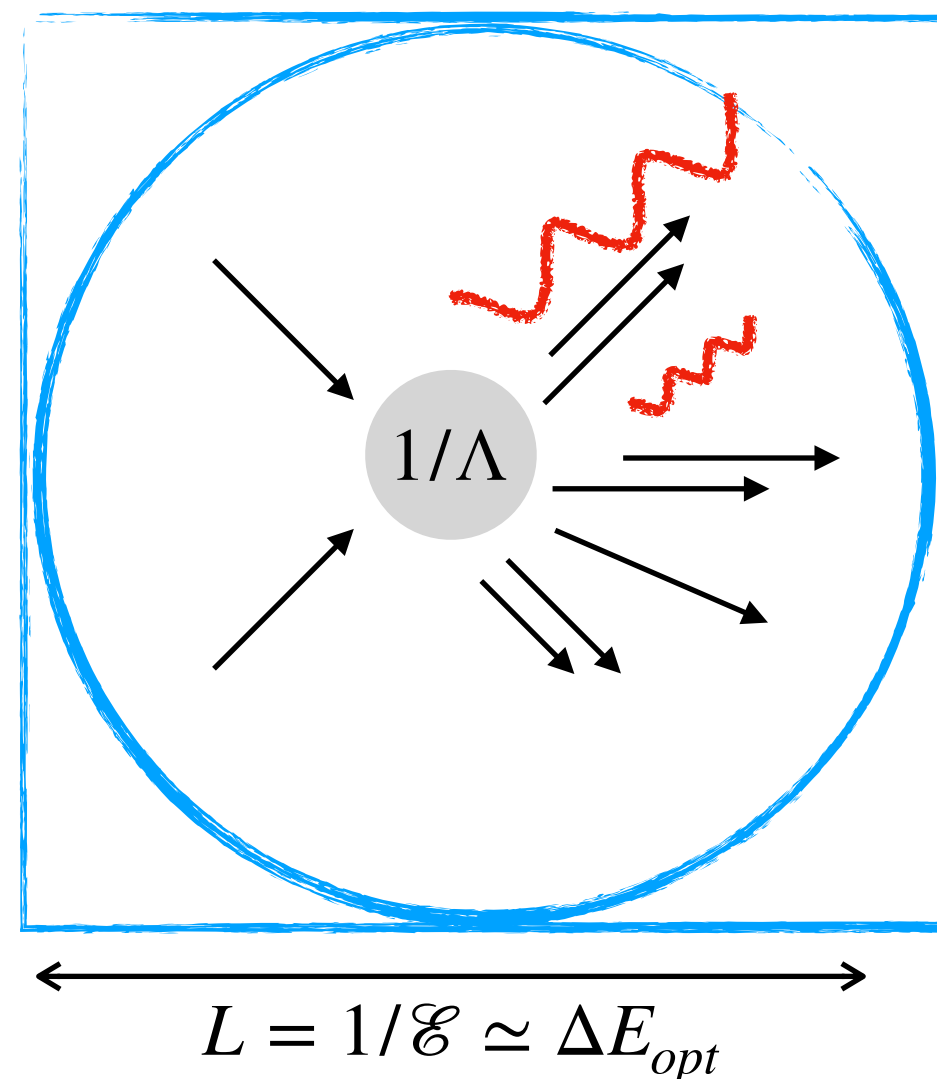
$$\left| \begin{array}{c} \text{diagram 1} \\ \vdots \\ \text{diagram } n \end{array} \right|^2 + \left| \begin{array}{c} \text{diagram } n+1 \\ \vdots \\ \text{diagram } m \end{array} \right|^2 + \dots \sim$$

$$\sum_{E_{\gamma,h} < \Delta E}^{\infty} \mathcal{M}^{\dagger} \mathcal{M}$$

$$\propto \left(\frac{\Delta E}{E} \right)^{\alpha, GE^2}$$

Universal log-behavior
on detector resolution

$$d\sigma_{inclusive} \rightarrow 0 \text{ if } \Delta E \rightarrow 0$$



Long-Range forces in $D=4$

E&M or Gravity

Inclusive X-sections? Good observables but ...

...NOT analytic

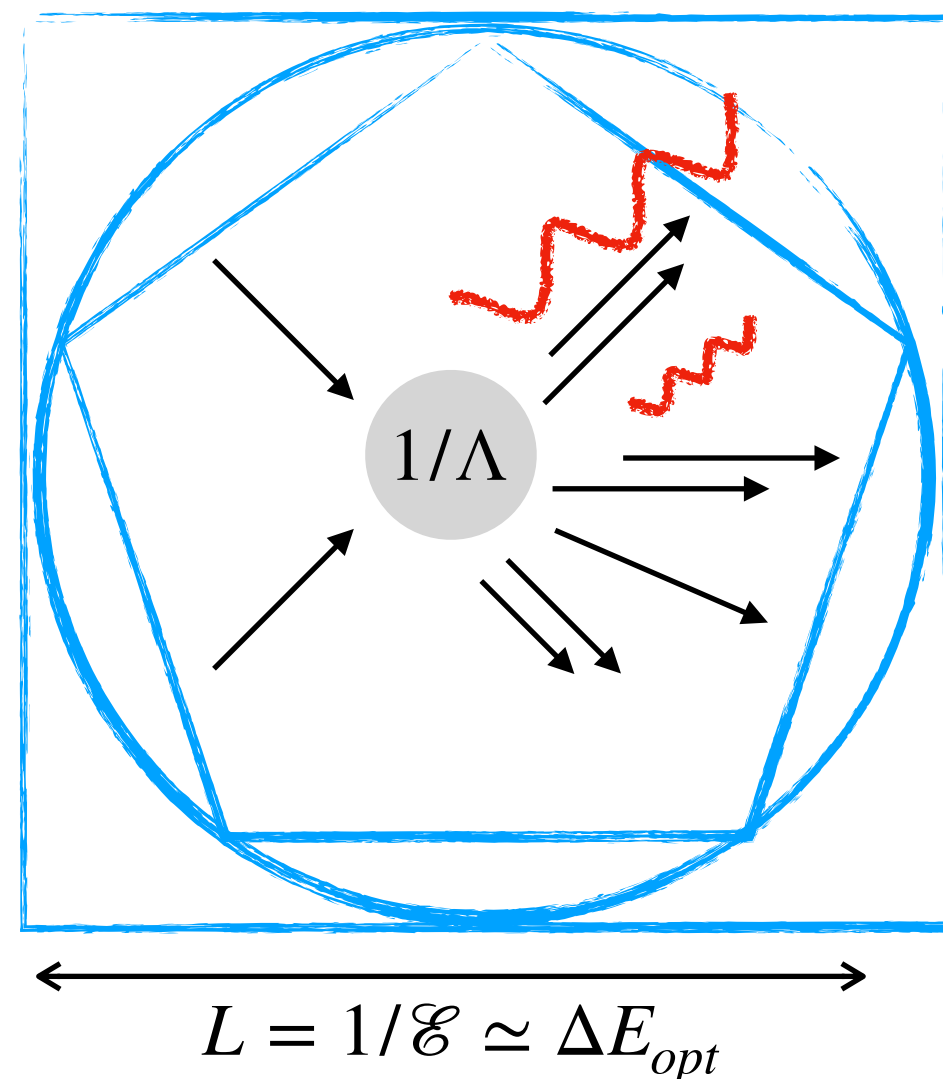
$$\left| \begin{array}{c} \text{diagram 1} \\ \vdots \\ \text{diagram } n \end{array} \right|^2 + \left| \begin{array}{c} \text{diagram } n+1 \\ \vdots \\ \text{diagram } m \end{array} \right|^2 + \dots \sim$$

$$\sum_{E_{\gamma,h} < \Delta E}^{\infty} \mathcal{M}^{\dagger} \mathcal{M}$$

$$\propto \left(\frac{\Delta E}{E} \right)^{\alpha, GE^2}$$

Universal log-behavior
on detector resolution

$$d\sigma_{inclusive} \rightarrow 0 \text{ if } \Delta E \rightarrow 0$$



Long-Range forces in $D=4$

E&M or Gravity

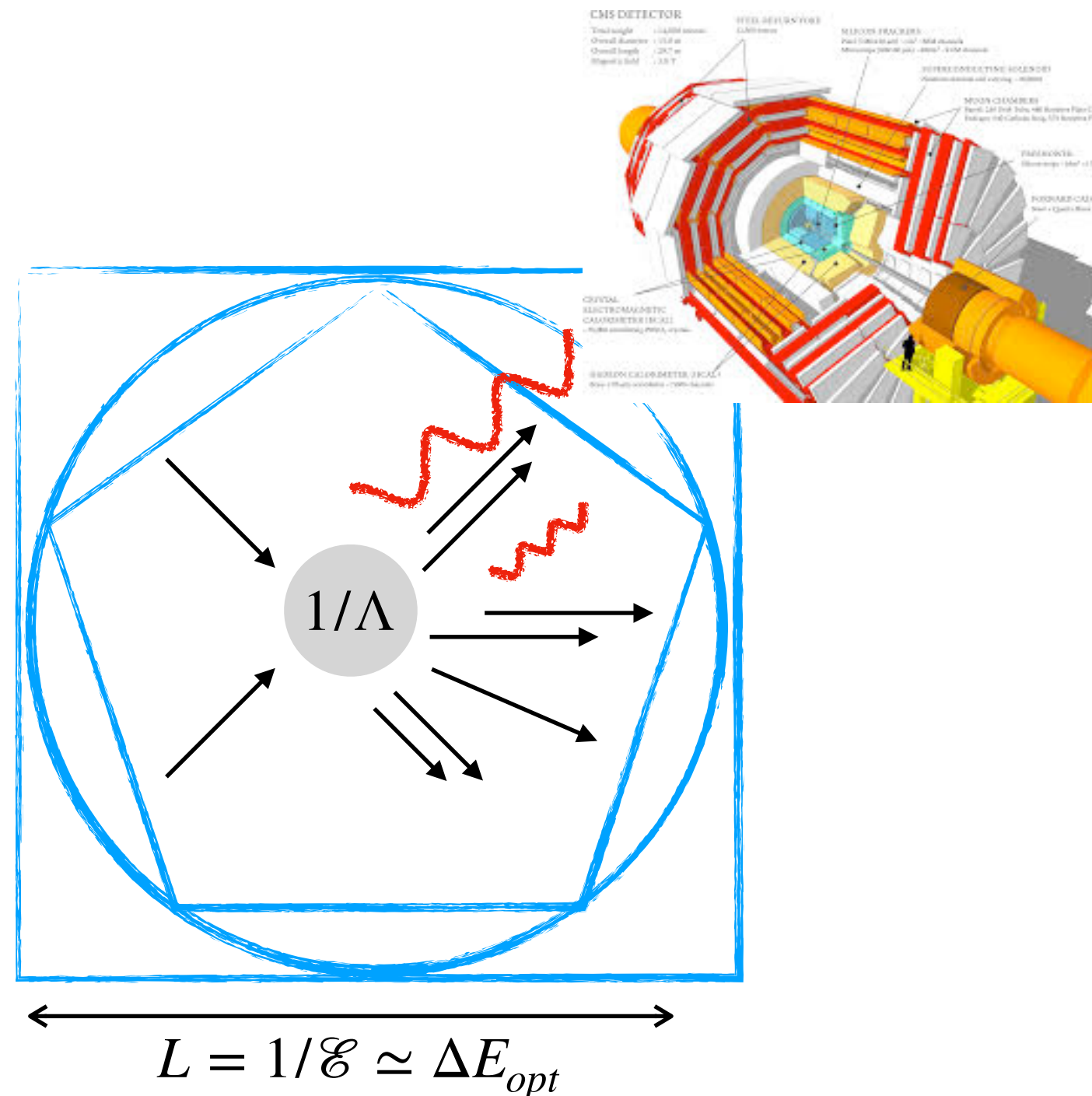
Inclusive X-sections? Good observables but ...

...NOT analytic

$$\left| \begin{array}{c} \text{diagram 1} \\ \vdots \\ \text{diagram } n \end{array} \right|^2 + \left| \begin{array}{c} \text{diagram } n+1 \\ \vdots \\ \text{diagram } m \end{array} \right|^2 + \dots \sim$$

$$\sum_{E_{\gamma,h} < \Delta E} \mathcal{M}^\dagger \mathcal{M}$$

$$\propto \left(\frac{\Delta E}{E} \right)^{\alpha, GE^2}$$



Universal log-behavior
on detector resolution

$$d\sigma_{inclusive} \rightarrow 0 \text{ if } \Delta E \rightarrow 0$$

Long-Range forces in $D=4$

E&M or Gravity

Inclusive X-sections? Good observables but ...

...NOT analytic

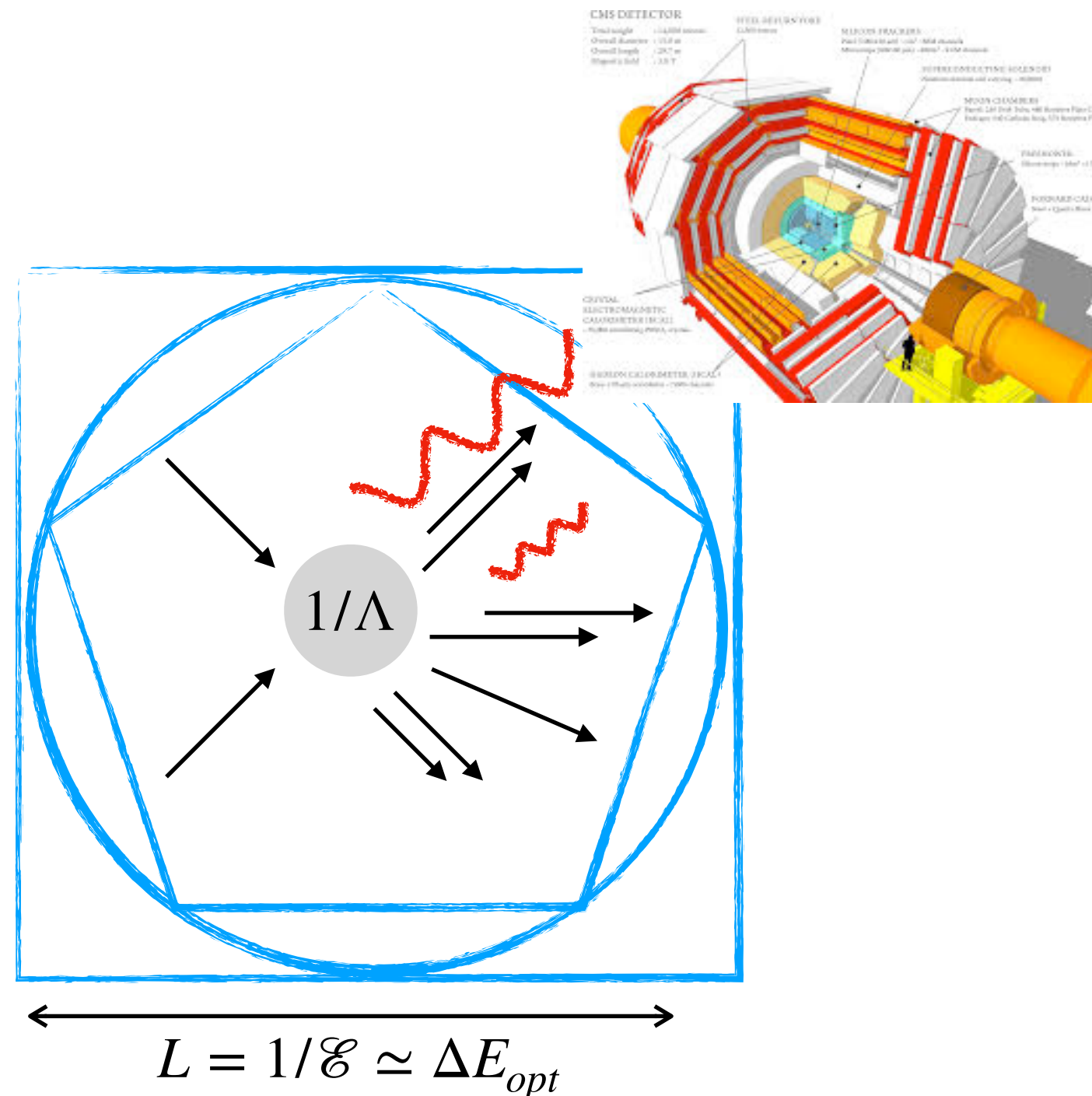
$$\left| \begin{array}{c} \text{diagram 1} \\ \vdots \\ \text{diagram } n \end{array} \right|^2 + \left| \begin{array}{c} \text{diagram } n+1 \\ \vdots \\ \text{diagram } m \end{array} \right|^2 + \dots \sim$$

$$\sum_{E_{\gamma,h} < \Delta E} \mathcal{M}^\dagger \mathcal{M}$$

$$\propto \left(\frac{\Delta E}{E} \right)^{\alpha, GE^2}$$

Universal log-behavior
on detector resolution

$$d\sigma_{inclusive} \rightarrow 0 \text{ if } \Delta E \rightarrow 0$$



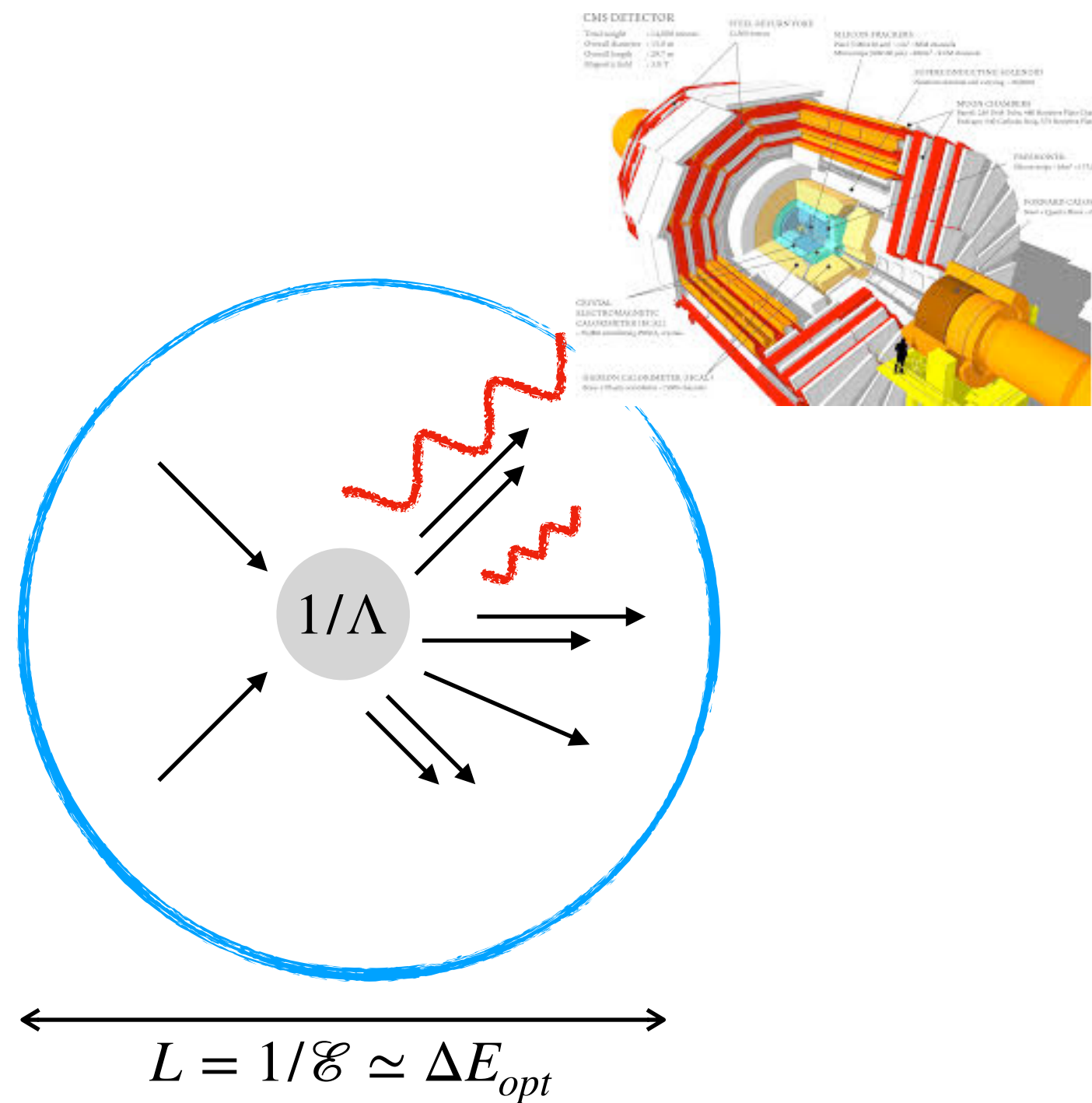
Main lesson:

Cannot decouple the detector!

Yet: most of its details are irrelevant

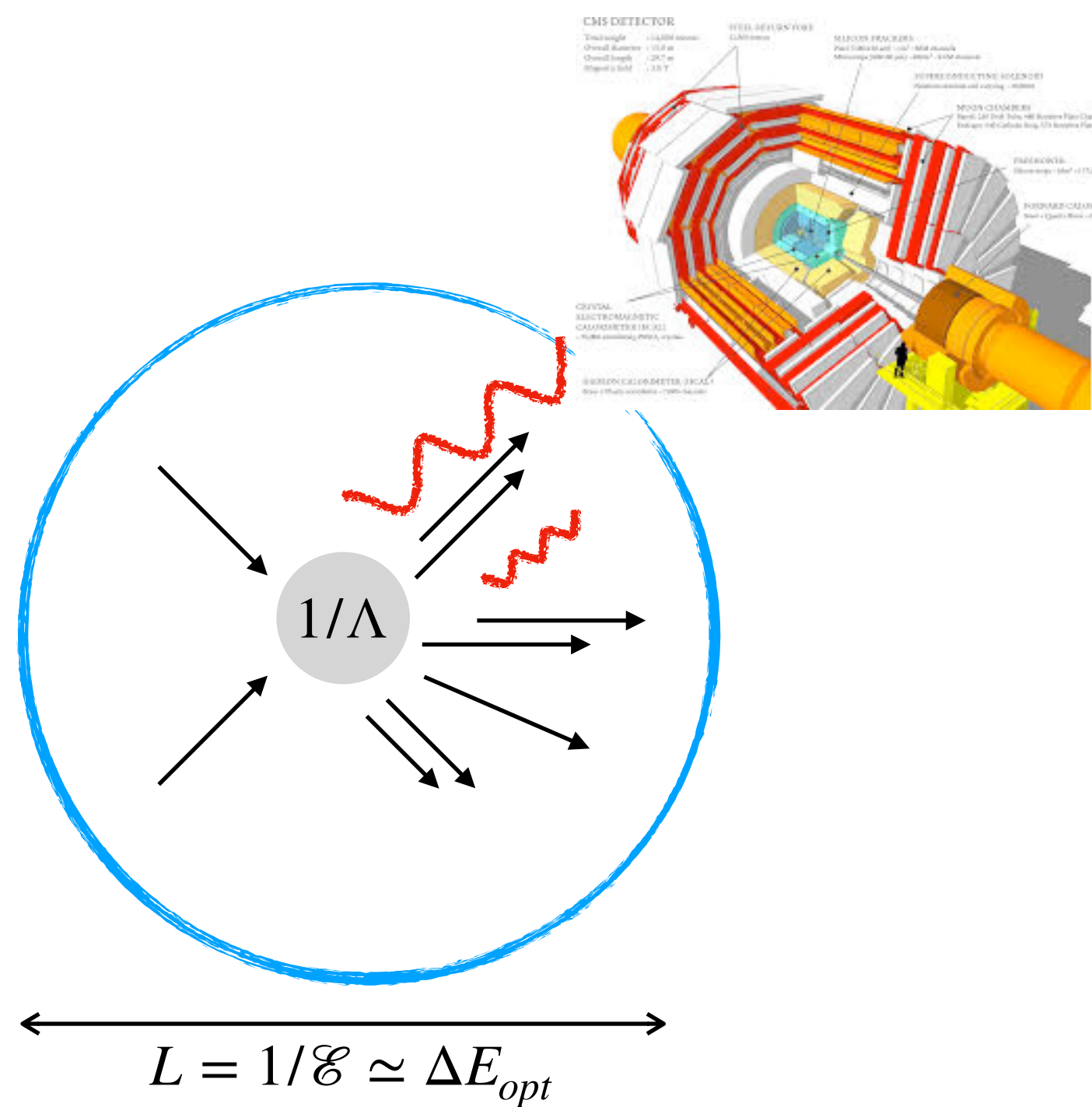
What are good observables? E&M or Gravity

1. IR finite,
2. not too much IR sensitive,
3. Know about causality??!



What are good observables? E&M or Gravity

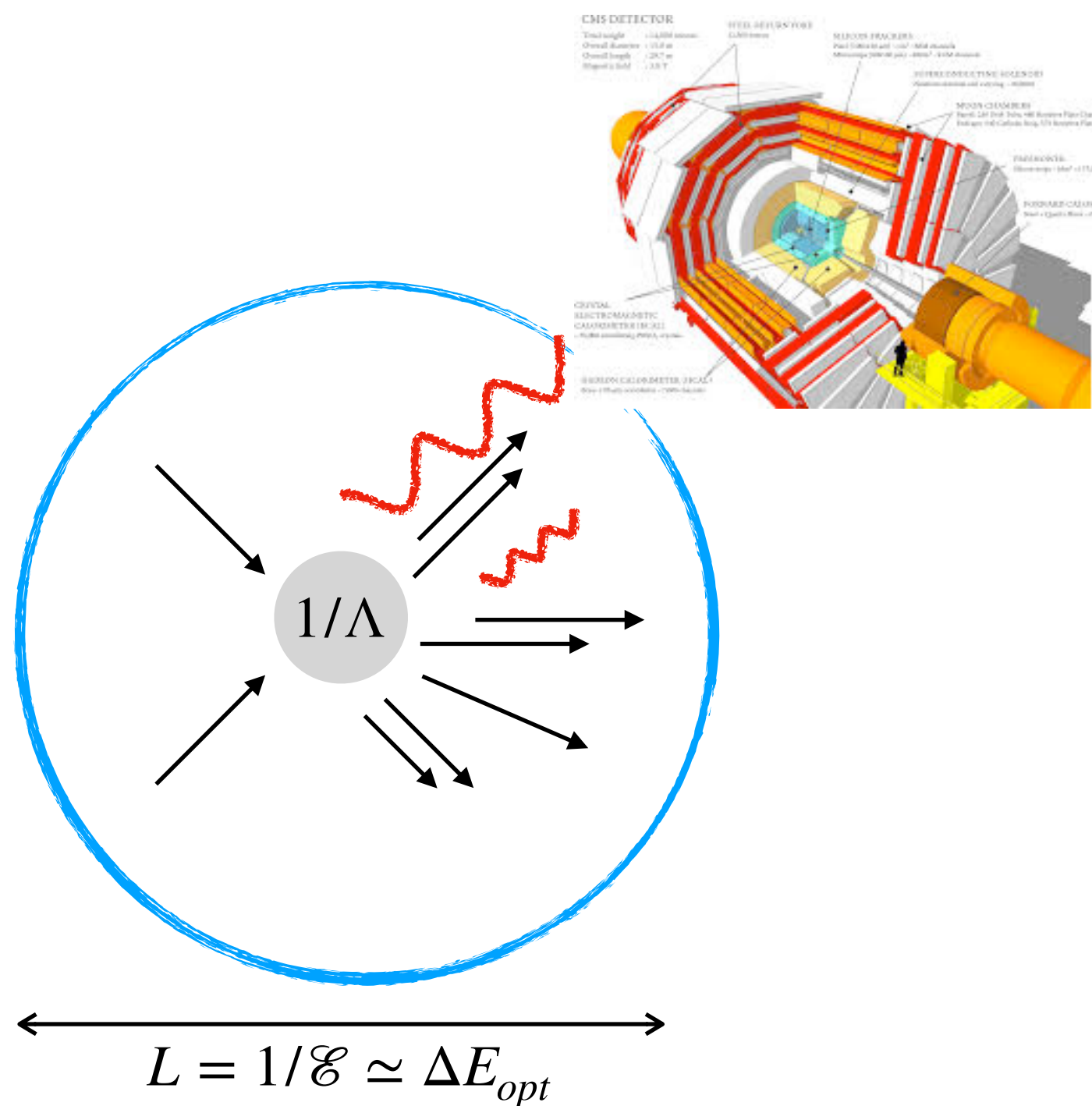
1. IR finite, 2. not too much IR sensitive, 3. Know about causality??!



Obs checklist	<i>IR finite</i>	<i>Gauge inv.</i>	<i>Causal/ Analytic</i>	<i>Un- Ambiguous</i>
Amplitudes	✗	✓	✓	✓
Inclusive X-sections	✓	✓	✗	✓
Energy-Energy Corr.	✓	✓	✗ ?	✓
Faddeev-Kulish Amp	✓	✓	?	✗
Off-shell Correlators	✓	✓ ✗	?	?
?	✓	✓	✓	✓

What are good observables? E&M or Gravity

1. IR finite, 2. not too much IR sensitive, 3. Know about causality??!



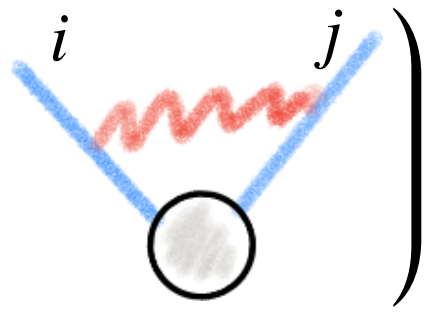
Obs checklist	<i>IR finite</i>	<i>Gauge inv.</i>	<i>Causal/ Analytic</i>	<i>Un- Ambiguous</i>
Amplitudes	✗	✓	✓	✓
Inclusive X-sections	✓	✓	✗	✓
Energy-Energy Corr.	✓	✓	✗ ?	✓
Faddeev-Kulish Amp	✓	✓	?	✗
Off-shell Correlators	✓	✓ ✗	?	?
?	✓	✓	✓	✓

This talk:

Stripped/Detector Amplitudes

Stripped, Detector Amplitudes

Stripped, Detector Amplitudes

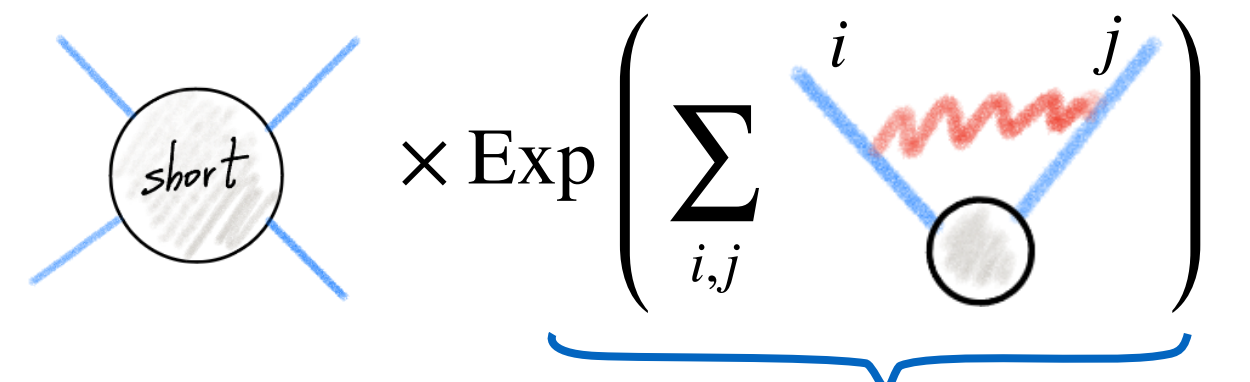
$$\text{short} \times \underbrace{\text{Exp} \left(\sum_{i,j} \text{diagram}_{ij} \right)}_{\text{Weinberg Soft Exp}}$$


$$\mathcal{W}_{\mathcal{E}}^{\epsilon} \stackrel{\text{E\&M:}}{=} \exp \left\{ \frac{\alpha}{8\pi} \frac{1}{\epsilon} \left[1 + \epsilon \log \frac{\mathcal{E}^2}{\mu^2} \right] \left(\sum_{i,j} \eta_i \eta_j q_i q_j \frac{1}{\beta_{ij}} \log \frac{1 + \beta_{ij}}{1 - \beta_{ij}} - i \sum_{\substack{i \neq j \\ \eta_i \eta_j = 1}} q_i q_j \frac{2\pi}{\beta_{ij}} \right) \right\}$$

(Analogous GR)

$\eta_i = \pm = in/out$
 $\beta_{ij}^2 = \frac{(s_{ij} - (m_i - m_j)^2)(s_{ij} - (m_i + m_j)^2)}{(s_{ij} - m_i^2 - m_j^2)^2}$

Stripped, Detector Amplitudes



$$\times \text{Exp} \left(\sum_{i,j} \text{diagram}(i,j) \right)$$

Weinberg Soft Exp

↓

$$\mathcal{W}_{\mathcal{E}}^{\epsilon} \stackrel{\text{E\&M:}}{=} \exp \left\{ \frac{\alpha}{8\pi} \frac{1}{\epsilon} \left[1 + \epsilon \log \frac{\mathcal{E}^2}{\mu^2} \right] \left(\sum_{i,j} \eta_i \eta_j q_i q_j \frac{1}{\beta_{ij}} \log \frac{1 + \beta_{ij}}{1 - \beta_{ij}} - i \sum_{\substack{i \neq j \\ \eta_i \eta_j = 1}} q_i q_j \frac{2\pi}{\beta_{ij}} \right) \right\}$$

(Analogous GR))

$\eta_i = \pm = in/out$

$$\beta_{ij}^2 = \frac{(s_{ij} - (m_i - m_j)^2)(s_{ij} - (m_i + m_j)^2)}{(s_{ij} - m_i^2 - m_j^2)^2}$$

Amplitude

Weinberg Soft Exp

= “Stripped Amplitude” =

$\lim_{\epsilon \rightarrow 0} \frac{\mathcal{M}^{\epsilon}}{\mathcal{W}_{\mathcal{E}}^{\epsilon}} = \mathcal{M}_{\mathcal{E}}$

1-parameter $\mathcal{E}$
family of amplitudes

Stripped, Detector Amplitudes

$$\text{Diagram: a circle with a diagonal line through it, labeled "short"} \times \text{Exp} \left(\sum_{i,j} \text{Diagram: a vertex with two blue lines labeled } i \text{ and } j \text{ and a red wavy line} \right)$$

$$\frac{\text{Amplitude}}{\text{Weinberg Soft Exp}} = \text{"Stripped Amplitude"} = \lim_{\epsilon \rightarrow 0} \frac{\mathcal{M}^\epsilon}{\mathcal{W}_\epsilon^\epsilon} = \mathcal{M}_\epsilon$$

Stripped, Detector Amplitudes

$$\text{short} \times \text{Exp} \left(\sum_{i,j} \text{diagram}(i,j) \right) \frac{\text{Amplitude}}{\text{Weinberg Soft Exp}} = \text{"Stripped Amplitude"} = \lim_{\epsilon \rightarrow 0} \frac{\mathcal{M}^\epsilon}{\mathcal{W}_\epsilon^\epsilon} = \mathcal{M}_\epsilon$$

$\mathcal{M}_\epsilon$ is

- | | | |
|---|---|---------|
| <ol style="list-style-type: none"> 1. IR finite 2. Lorentz invariant 3. Analytic & X-ssing 4. Regge/Froissart Bound | } | Exactly |
|---|---|---------|

Stripped, Detector Amplitudes

$$\text{short} \times \text{Exp} \left(\sum_{i,j} \text{diagram}(i,j) \right) \frac{\text{Amplitude}}{\text{Weinberg Soft Exp}} = \text{"Stripped Amplitude"} = \lim_{\epsilon \rightarrow 0} \frac{\mathcal{M}^\epsilon}{\mathcal{W}_\epsilon^\epsilon} = \mathcal{M}_\mathcal{E}$$

$\mathcal{M}_\mathcal{E}$ is

- | | | |
|---|---|------------|
| 1. IR finite | } | Exactly |
| 2. Lorentz invariant | | |
| 3. Analytic & X-ssing | | |
| 4. Regge/Froissart Bound | | |
| 5. Unitary | } | In a limit |
| 6. Universal, $\mathcal{E} \sim \Delta E$ | | |

"Detector Amplitude"

$$\begin{aligned}
 \mathcal{M}_\mathcal{E} &= \mathcal{M}(\alpha_\mathcal{E}, G_\mathcal{E}, g) + O(\alpha, G, \mathcal{E}) \\
 \alpha \rightarrow 0, \quad \alpha \log \mathcal{E} &= \alpha_\mathcal{E} \text{ - fixed (all order)} \\
 G \rightarrow 0, \quad G \log \mathcal{E} &= G_\mathcal{E} \text{ - fixed (all order)}
 \end{aligned}$$

Stripped, Detector Amplitudes

$$\text{short} \times \text{Exp} \left(\sum_{i,j} \text{diagram}(i,j) \right) \frac{\text{Amplitude}}{\text{Weinberg Soft Exp}} = \text{"Stripped Amplitude"} = \lim_{\epsilon \rightarrow 0} \frac{\mathcal{M}^\epsilon}{\mathcal{W}_\epsilon^\epsilon} = \mathcal{M}_\mathcal{E}$$

$\mathcal{M}_\mathcal{E}$ is

- | | | |
|---|---|------------|
| 1. IR finite | ✓ | Exactly |
| 2. Lorentz invariant | ✓ | |
| 3. Analytic & X-ssing | | |
| 4. Regge/Froissart Bound | | |
| 5. Unitary | | In a limit |
| 6. Universal, $\mathcal{E} \sim \Delta E$ | | |

"Detector Amplitude"

$$\mathcal{M}_\mathcal{E} = \mathcal{M}(\alpha_\mathcal{E}, G_\mathcal{E}, g) + O(\alpha, G, \mathcal{E})$$

$$\alpha \rightarrow 0, \quad \alpha \log \mathcal{E} = \alpha_\mathcal{E} \text{ - fixed (all order)}$$

$$G \rightarrow 0, \quad G \log \mathcal{E} = G_\mathcal{E} \text{ - fixed (all order)}$$

Stripped, Detector Amplitudes

$$\text{short} \times \text{Exp} \left(\sum_{i,j} \text{diagram} \right) \frac{\text{Amplitude}}{\text{Weinberg Soft Exp}} = \text{"Stripped Amplitude"} = \lim_{\epsilon \rightarrow 0} \frac{\mathcal{M}^\epsilon}{\mathcal{W}_\epsilon^\epsilon} = \mathcal{M}_\mathcal{E}$$

$\mathcal{M}_\mathcal{E}$ is

- | | | |
|---|---|--------------|
| 1. IR finite | ✓ | } Exactly |
| 2. Lorentz invariant | ✓ | |
| 3. Analytic & X-ssing | | |
| 4. Regge/Froissart Bound | | } In a limit |
| 5. Unitary | | |
| 6. Universal, $\mathcal{E} \sim \Delta E$ | | |

"Detector Amplitude"

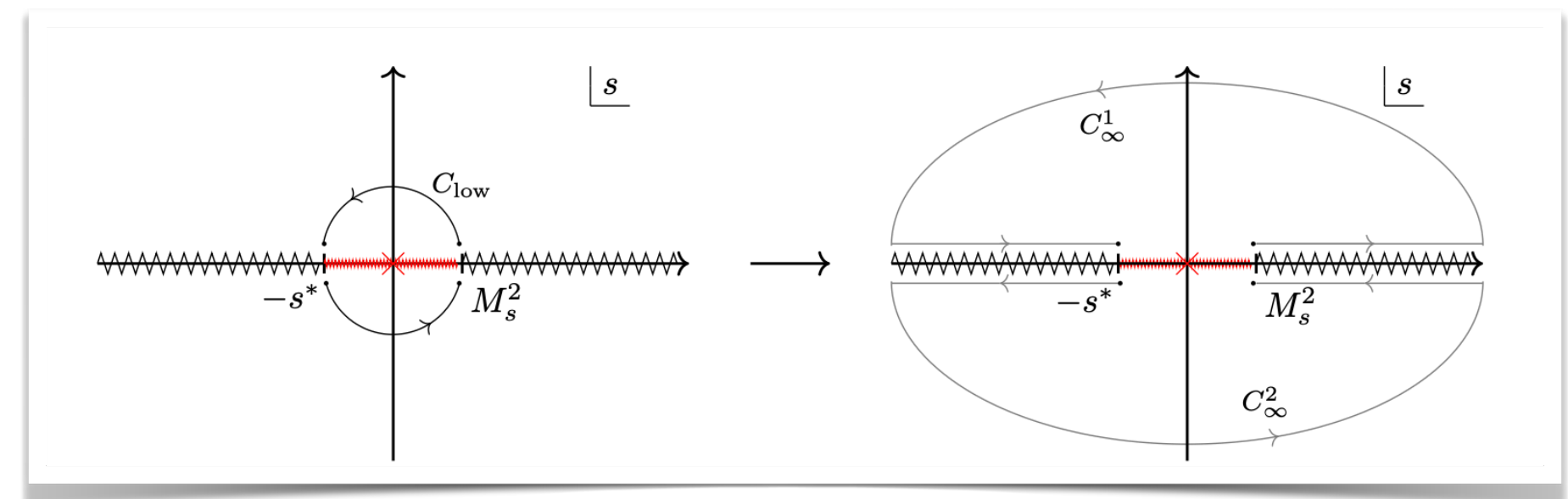
$$\mathcal{M}_\mathcal{E} = \mathcal{M}(\alpha_\mathcal{E}, G_\mathcal{E}, g) + O(\alpha, G, \mathcal{E})$$

$$\alpha \rightarrow 0, \quad \alpha \log \mathcal{E} = \alpha_\mathcal{E} \text{ - fixed (all order)}$$

$$G \rightarrow 0, \quad G \log \mathcal{E} = G_\mathcal{E} \text{ - fixed (all order)}$$

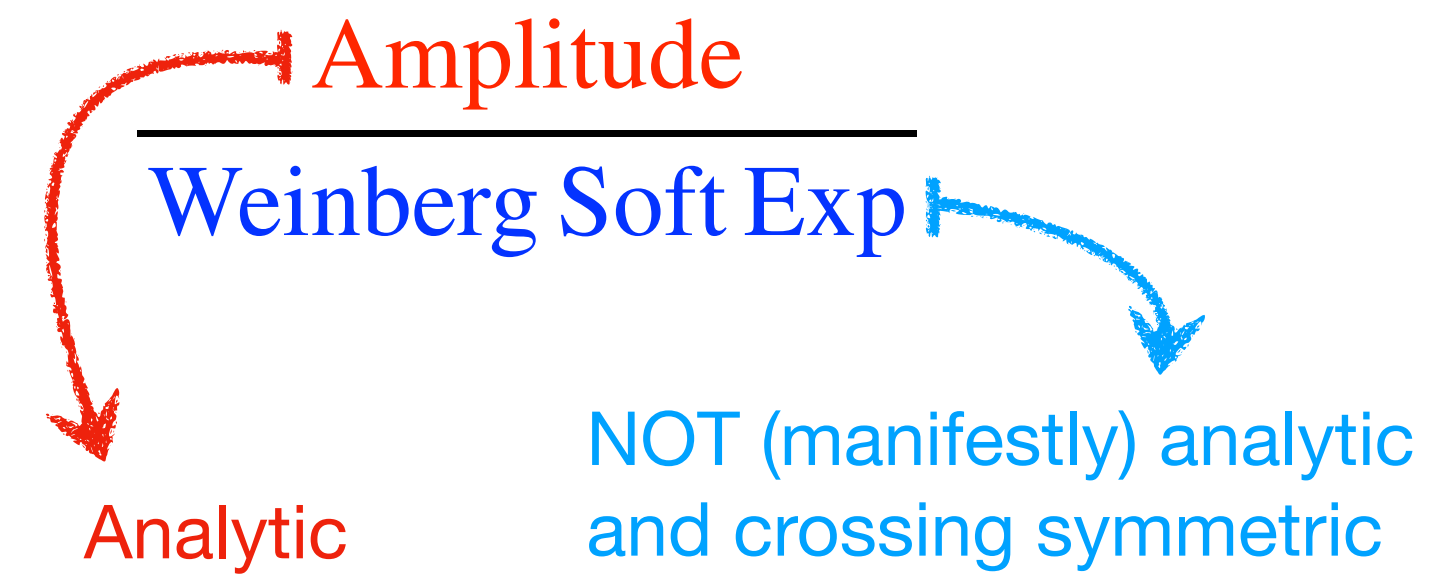
BB, Berman, Isabella, Riva, Romano, Sciotti 2512.13780

Stripped-Amplitudes:
~~good observables~~ & inputs bootstrap/positivity
 Tool!



Analyticity & Crossing

Analyticity & Crossing



Analyticity & Crossing

1. defined on real kinematics

2. distinguish ingoing/outgoing

Amplitude

Weinberg Soft Exp

Analytic

NOT (manifestly) analytic and crossing symmetric

$$\prod_{ij} \mathcal{W}_{ij} = \prod_{ij} \text{Exp} \left(\text{diagram} \right) \stackrel{\text{GR}}{=} \exp \left\{ -\frac{G}{8\pi} \frac{1}{\epsilon} \left[1 + \epsilon \log \frac{\mathcal{E}^2}{\mu^2} \right] \left(\sum_{i,j} \eta_i \eta_j m_i m_j \frac{1 + \beta_{ij}^2}{\beta_{ij} \sqrt{1 - \beta_{ij}^2}} \log \frac{1 + \beta_{ij}}{1 - \beta_{ij}} - i \sum_{\substack{i \neq j \\ \eta_i \eta_j = 1}} m_i m_j \frac{2\pi(1 + \beta_{ij}^2)}{\beta_{ij} \sqrt{1 - \beta_{ij}^2}} \right) \right\}$$

Weinberg '65

$$\beta_{ij}^2 = \frac{(s_{ij} - (m_i - m_j)^2)(s_{ij} - (m_i + m_j)^2)}{(s_{ij} - m_i^2 - m_j^2)^2}$$

Analyticity & Crossing

1. defined on real kinematics

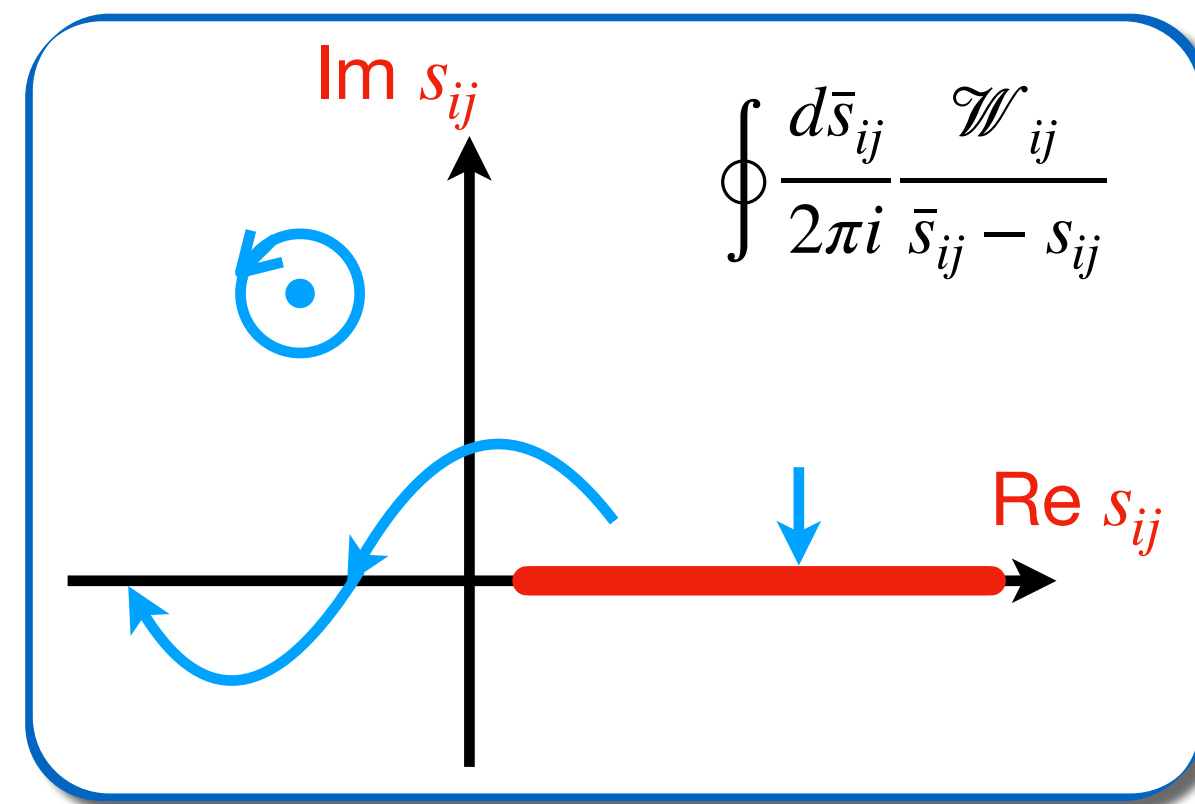
2. distinguish ingoing/outgoing

Amplitude
Weinberg Soft Exp

NOT (manifestly) analytic
and crossing symmetric

Analytic

$$\prod_{ij} \mathcal{W}_{ij} = \prod_{ij} \text{Exp} \left(\text{diagram} \right) \stackrel{\text{GR}}{=} \exp \left\{ \underbrace{-\frac{G}{8\pi} \frac{1}{\epsilon} \left[1 + \epsilon \log \frac{\mathcal{E}^2}{\mu^2} \right]}_{\text{Weinberg '65}} \left(\underbrace{\sum_{i,j} \eta_i \eta_j m_i m_j \frac{1 + \beta_{ij}^2}{\beta_{ij} \sqrt{1 - \beta_{ij}^2}} \log \frac{1 + \beta_{ij}}{1 - \beta_{ij}}}_{\text{Real part}} - \underbrace{i \sum_{\substack{i \neq j \\ \eta_i \eta_j = 1}} m_i m_j \frac{2\pi(1 + \beta_{ij}^2)}{\beta_{ij} \sqrt{1 - \beta_{ij}^2}}}_{\text{Imaginary part}} \right) \right\}$$



$$\beta_{ij}^2 = \frac{(s_{ij} - (m_i - m_j)^2)(s_{ij} - (m_i + m_j)^2)}{(s_{ij} - m_i^2 - m_j^2)^2}$$

Analyticity & Crossing

1. defined on real kinematics

2. distinguish ingoing/outgoing

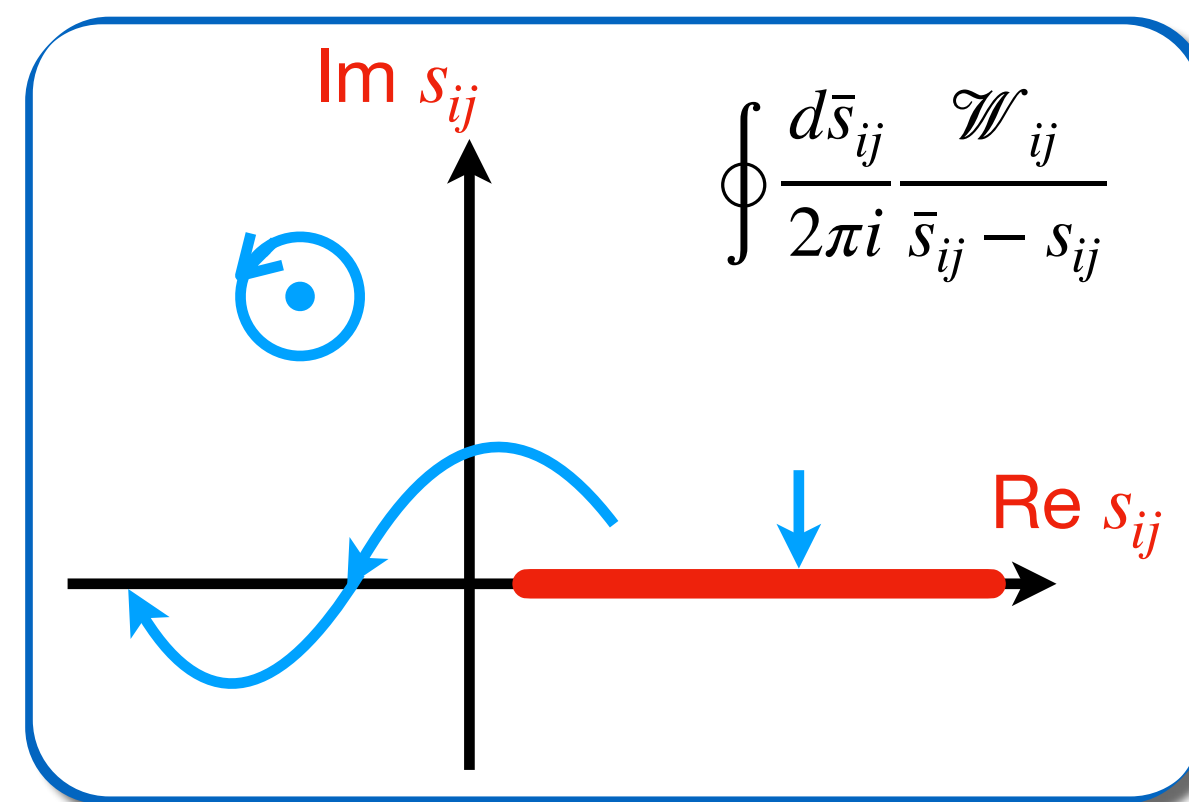
Amplitude
Weinberg Soft Exp

NOT (manifestly) analytic
and crossing symmetric

Analytic

$$\prod_{ij} \mathcal{W}_{ij} = \prod_{ij} \text{Exp} \left(\text{diagram} \right) \stackrel{\text{GR}}{=} \exp \left\{ -\frac{G}{8\pi} \frac{1}{\epsilon} \left[1 + \epsilon \log \frac{\mathcal{E}^2}{\mu^2} \right] \left(\underbrace{\sum_{i,j} \eta_i \eta_j m_i m_j \frac{1 + \beta_{ij}^2}{\beta_{ij} \sqrt{1 - \beta_{ij}^2}} \log \frac{1 + \beta_{ij}}{1 - \beta_{ij}}}_{\text{Real part}} - \underbrace{i \sum_{\substack{i \neq j \\ \eta_i \eta_j = 1}} m_i m_j \frac{2\pi(1 + \beta_{ij}^2)}{\beta_{ij} \sqrt{1 - \beta_{ij}^2}}}_{\text{Imaginary part}} \right) \right\}$$

Weinberg '65



$$\beta_{ij}^2 = \frac{(s_{ij} - (m_i - m_j)^2)(s_{ij} - (m_i + m_j)^2)}{(s_{ij} - m_i^2 - m_j^2)^2}$$

“The Analytic Soft Exp”

BB, Berman, Isabella, Riva, Romano, Sciotti '25

$$\mathcal{W}_{ij}^{\text{GR}} \equiv \begin{cases} \exp \left[\frac{G}{2\pi} \frac{1}{\epsilon} \left(1 + \epsilon \log \frac{\mathcal{E}^2}{\mu^2} \right) \frac{(s_{ij} - m_i^2 - m_j^2)^2 - 2m_i^2 m_j^2}{\sqrt{(s_{ij} - (m_i - m_j)^2)(-s_{ij} + (m_i + m_j)^2)}} \arctan \frac{\sqrt{s_{ij} - (m_i - m_j)^2}}{\sqrt{-s_{ij} + (m_i + m_j)^2}} \right] & i \neq j \\ \exp \left[-\frac{G m_i^2}{4\pi} \frac{1}{\epsilon} \left(1 + \epsilon \log \frac{\mathcal{E}^2}{\mu^2} \right) \right] & i = j \end{cases}$$

$m_{i,j} \rightarrow 0$

$$\mathcal{W}_{ij} = \exp \left[-\frac{1}{4\pi\epsilon} \left(1 + \epsilon \log \frac{\mathcal{E}^2}{\mu^2} \right) G s_{ij} \log \left(\frac{-s_{ij}}{\mu^2} \right) \right]$$

Analyticity & Crossing

1. defined on real kinematics

2. distinguish ingoing/outgoing

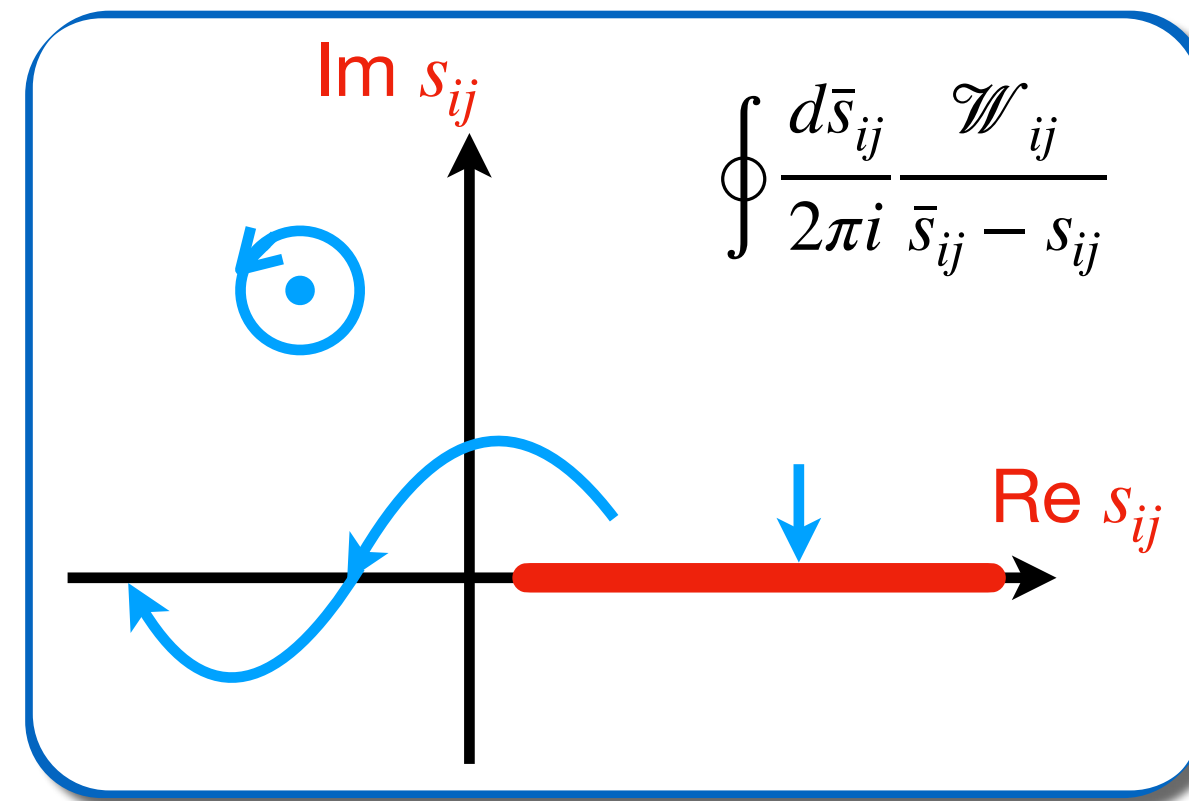
Amplitude
Weinberg Soft Exp

NOT (manifestly) analytic
and crossing symmetric

Analytic

$$\prod_{ij} \mathcal{W}_{ij} = \prod_{ij} \text{Exp} \left(\text{diagram} \right) \stackrel{\text{GR}}{=} \exp \left\{ -\frac{G}{8\pi} \frac{1}{\epsilon} \left[1 + \epsilon \log \frac{\mathcal{E}^2}{\mu^2} \right] \left(\underbrace{\sum_{i,j} \eta_i \eta_j m_i m_j \frac{1 + \beta_{ij}^2}{\beta_{ij} \sqrt{1 - \beta_{ij}^2}} \log \frac{1 + \beta_{ij}}{1 - \beta_{ij}}}_{\text{Real part}} - \underbrace{i \sum_{\substack{i \neq j \\ \eta_i \eta_j = 1}} m_i m_j \frac{2\pi(1 + \beta_{ij}^2)}{\beta_{ij} \sqrt{1 - \beta_{ij}^2}}}_{\text{Imaginary part}} \right) \right\}$$

Weinberg '65



$$\beta_{ij}^2 = \frac{(s_{ij} - (m_i - m_j)^2)(s_{ij} - (m_i + m_j)^2)}{(s_{ij} - m_i^2 - m_j^2)^2}$$

“The Analytic Soft Exp”

BB, Berman, Isabella, Riva, Romano, Sciotti '25

$$\mathcal{W}_{ij} \stackrel{\text{GR}}{=} \begin{cases} \exp \left[\frac{G}{2\pi} \frac{1}{\epsilon} \left(1 + \epsilon \log \frac{\mathcal{E}^2}{\mu^2} \right) \frac{(s_{ij} - m_i^2 - m_j^2)^2 - 2m_i^2 m_j^2}{\sqrt{(s_{ij} - (m_i - m_j)^2)(-s_{ij} + (m_i + m_j)^2)}} \arctan \frac{\sqrt{s_{ij} - (m_i - m_j)^2}}{\sqrt{-s_{ij} + (m_i + m_j)^2}} \right] & i \neq j \\ \exp \left[-\frac{G m_i^2}{4\pi} \frac{1}{\epsilon} \left(1 + \epsilon \log \frac{\mathcal{E}^2}{\mu^2} \right) \right] & i = j \end{cases}$$

$m_{i,j} \rightarrow 0$

$$\mathcal{W}_{ij} = \exp \left[-\frac{1}{4\pi\epsilon} \left(1 + \epsilon \log \frac{\mathcal{E}^2}{\mu^2} \right) G s_{ij} \log \left(\frac{-s_{ij}}{\mu^2} \right) \right]$$

Regge-scaling

Regge-scaling

- Gapped theory

Froissart 61, Martin '63

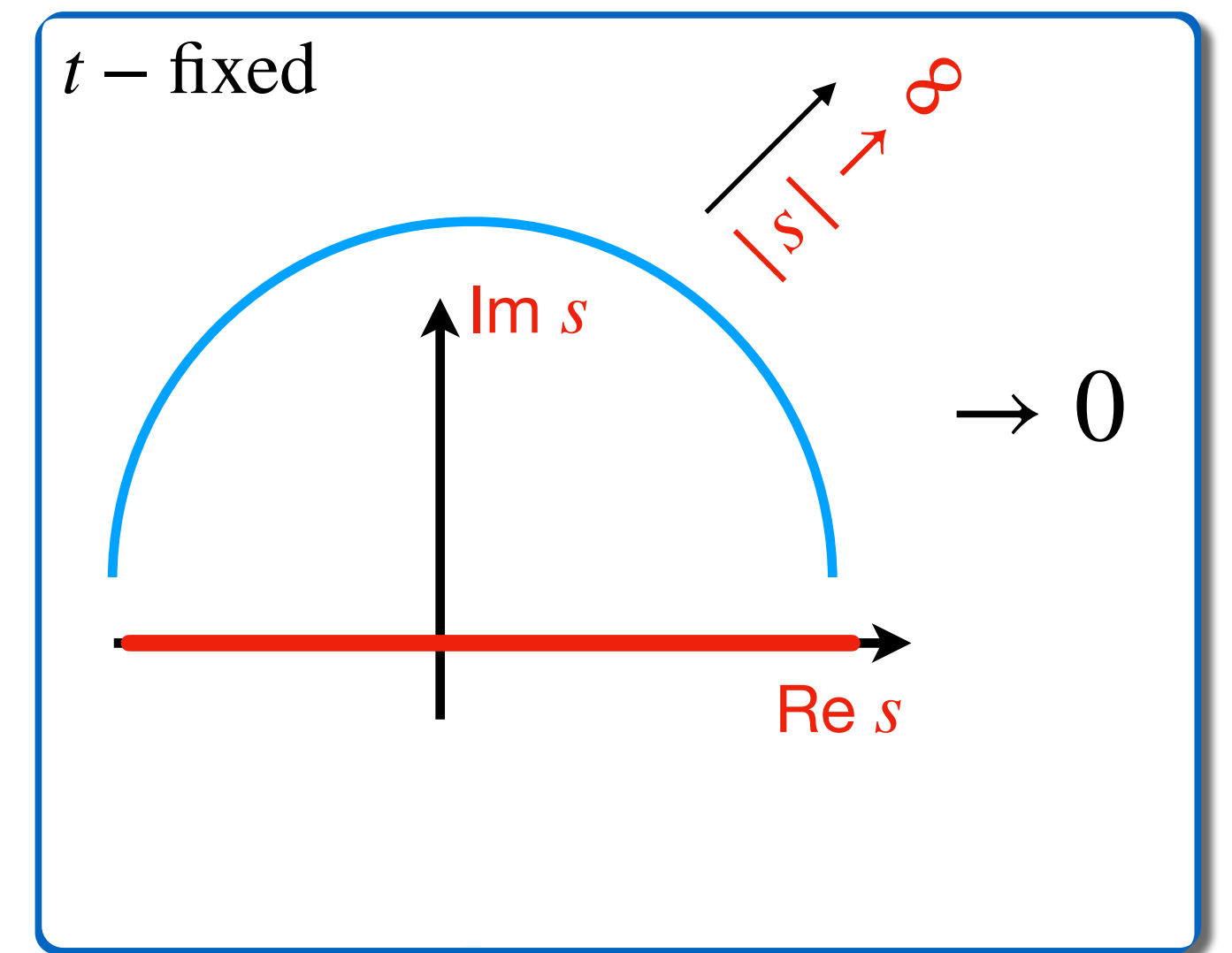
- Gravity in $D > 4$

Haring Zhiboedov '22

Caron-hoot, Parra-Martinez, Li, Simmons-Duffin '22

$$\frac{\mathcal{M}(s, t)}{s^2} \rightarrow 0$$

$$\int dq \psi(q) \frac{\mathcal{M}(s, -q^2)}{s^2} \rightarrow 0$$



Regge-scaling

- Gapped theory

Froissart 61, Martin '63

- Gravity in $D > 4$

Haring Zhiboedov '22

Caron-hoot, Parra-Martinez, Li, Simmons-Duffin '22

- Gravity in $D = 4$

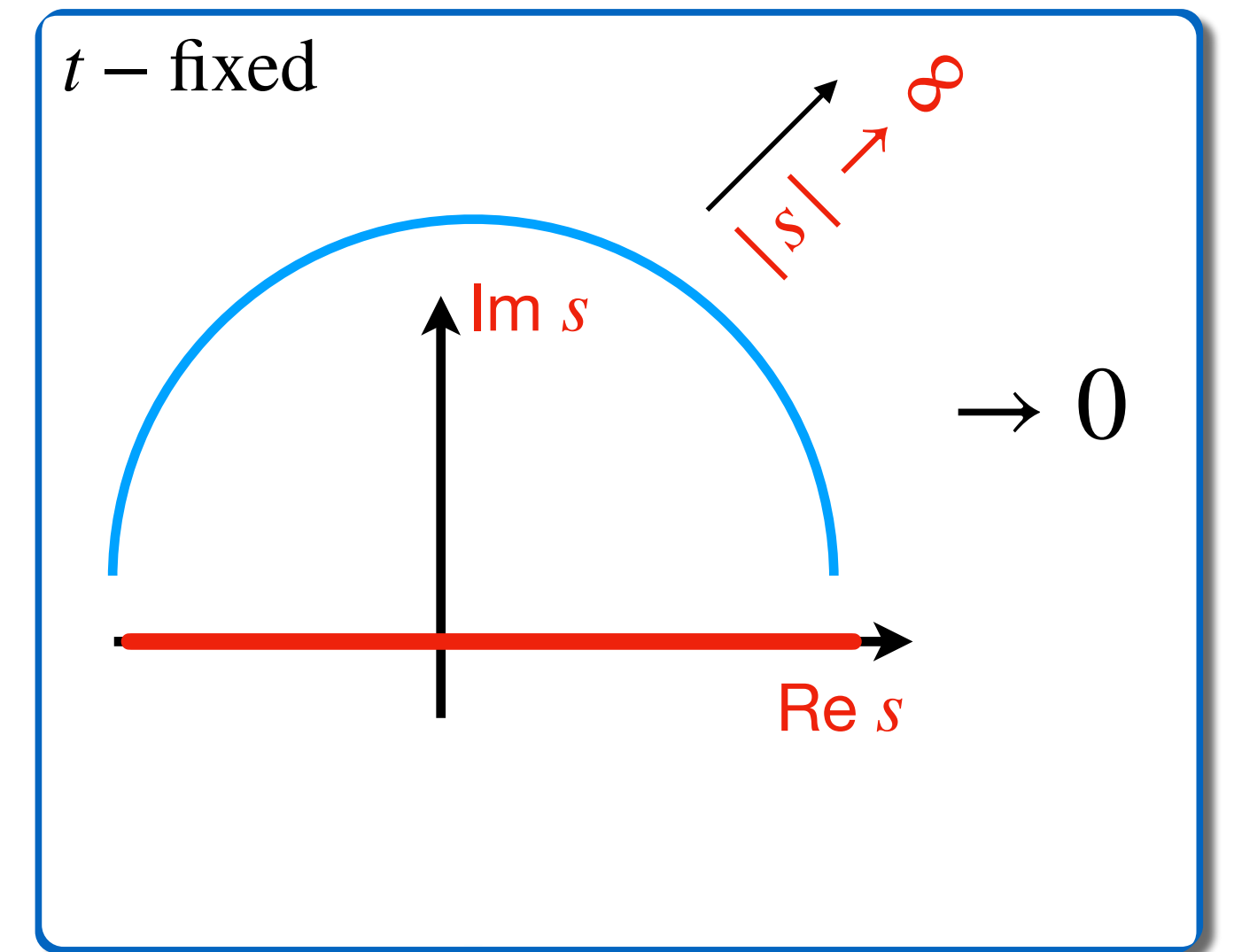
BB, Berman, Isabella, Riva, Romano, Sciotti '25

$$\frac{\mathcal{M}(s, t)}{s^2} \rightarrow 0$$

$$\int dq \psi(q) \frac{\mathcal{M}(s, -q^2)}{s^2} \rightarrow 0$$

$$\oint_{|s| \rightarrow \infty} \frac{ds}{s} \frac{\mathcal{M}_{\mathcal{G}}(s, t)}{s^2} \rightarrow 0 \quad \text{“Average Regge”}$$

Good enough for dispersion



Regge-scaling

- Gapped theory

Froissart 61, Martin '63

$$\frac{\mathcal{M}(s, t)}{s^2} \rightarrow 0$$

- Gravity in $D > 4$

Haring Zhiboedov '22

Caron-hoot, Parra-Martinez, Li, Simmons-Duffin '22

$$\int dq \psi(q) \frac{\mathcal{M}(s, -q^2)}{s^2} \rightarrow 0$$

- Gravity in $D=4$

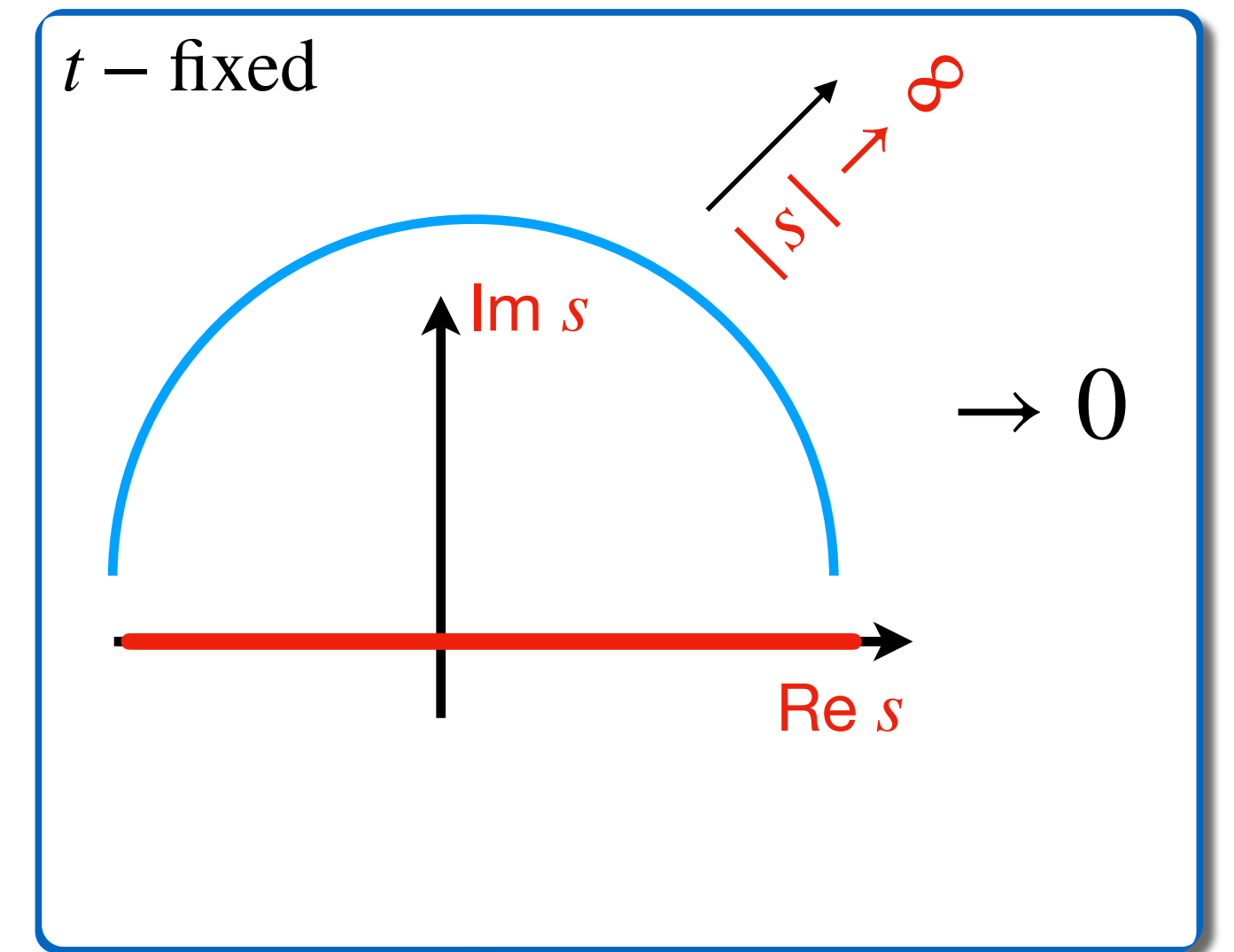
BB, Berman, Isabella, Riva, Romano, Sciotti '25

$$\oint_{|s| \rightarrow \infty} \frac{ds}{s} \frac{\mathcal{M}_{\mathcal{E}}(s, t)}{s^2} \rightarrow 0 \quad \text{"Average Regge"}$$

Good enough for dispersion

Eikonal regime

$$\propto \oint \frac{ds}{s} \frac{\text{Exp} \left[\frac{Gs}{\pi \epsilon} (\log(s) - \log(-s)) (1 + \epsilon \log(-t/\mu^2)) \right]}{\text{Exp} \left[\frac{Gs}{\pi \epsilon} (\log(s) - \log(-s)) (1 + \epsilon \log \mathcal{E}^2/\mu^2) \right]} = \oint \frac{ds}{s} \text{Exp} \left[\frac{Gs}{\pi} (\log(s) - \log(-s)) \log(-t/\mathcal{E}^2) \right] \propto \pi - 2\text{Si}(G|s| \log(-t/\mathcal{E}^2)) = O(1/|s|)$$



Regge-scaling

- Gapped theory

Froissart 61, Martin '63

$$\frac{\mathcal{M}(s, t)}{s^2} \rightarrow 0$$

- Gravity in D>4

Haring Zhiboedov '22

Caron-hoot, Parra-Martinez, Li, Simmons-Duffin '22

$$\int dq \psi(q) \frac{\mathcal{M}(s, -q^2)}{s^2} \rightarrow 0$$

- Gravity in D=4

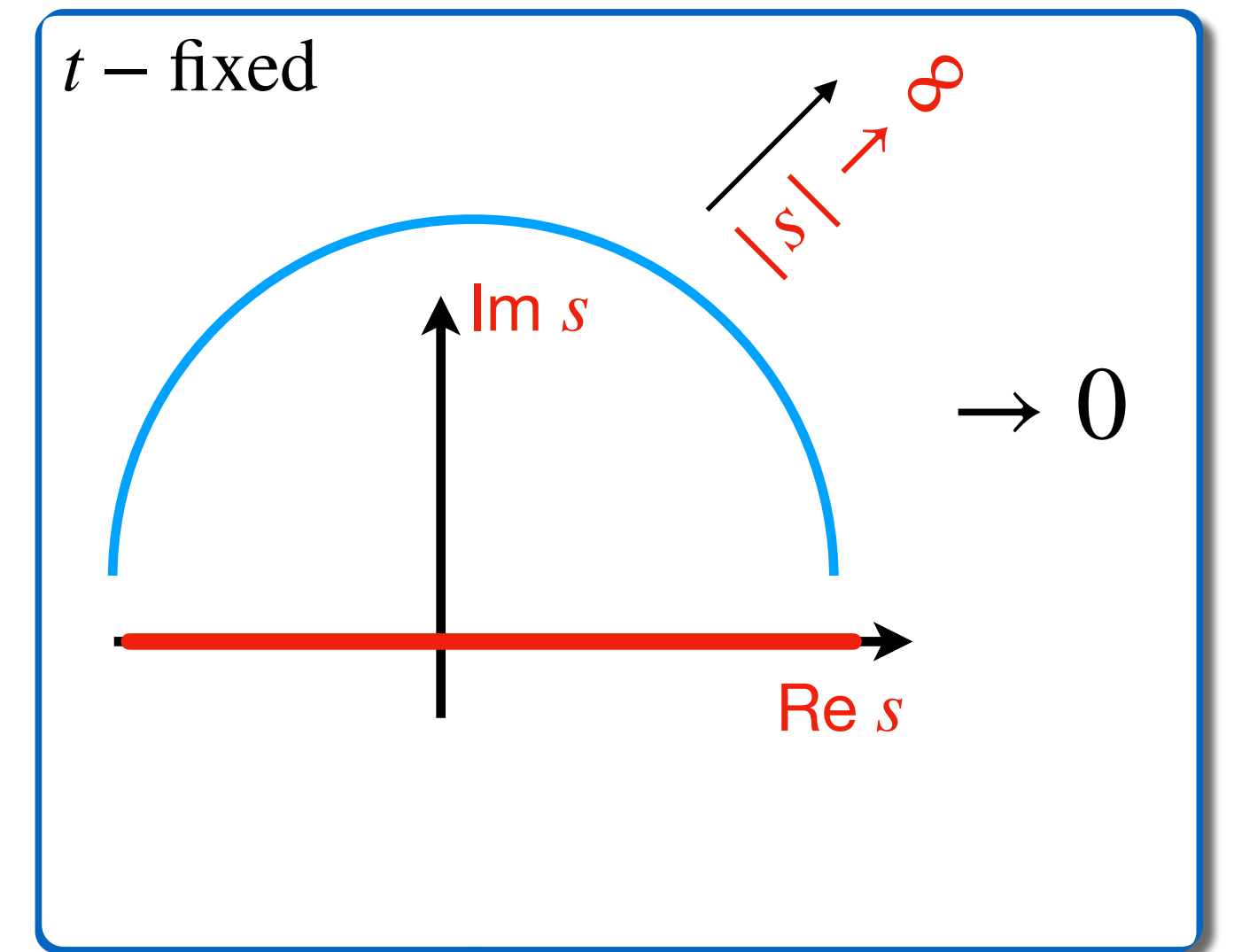
BB, Berman, Isabella, Riva, Romano, Sciotti '25

$$\oint_{|s| \rightarrow \infty} \frac{ds}{s} \frac{\mathcal{M}_{\mathcal{E}}(s, t)}{s^2} \rightarrow 0 \quad \text{“Average Regge”}$$

Good enough for dispersion

Eikonal regime

$$\propto \oint \frac{ds}{s} \frac{\text{Exp} \left[\frac{Gs}{\pi \epsilon} (\log(s) - \log(-s)) (1 + \epsilon \log(-t/\mu^2)) \right]}{\text{Exp} \left[\frac{Gs}{\pi \epsilon} (\log(s) - \log(-s)) (1 + \epsilon \log \mathcal{E}^2/\mu^2) \right]} = \oint \frac{ds}{s} \text{Exp} \left[\frac{Gs}{\pi} (\log(s) - \log(-s)) \log(-t/\mathcal{E}^2) \right] \propto \pi - 2\text{Si}(G|s| \log(-t/\mathcal{E}^2)) = O(1/|s|)$$



Unitarity

Unitary Detector Amplitudes

$$\begin{array}{c} \diagup \\ \text{short} \\ \diagdown \end{array} + \begin{array}{c} \text{wavy} \\ \diagup \\ \text{short} \\ \diagdown \end{array} + \begin{array}{c} \text{wavy} \\ \text{wavy} \\ \diagup \\ \text{short} \\ \diagdown \end{array} + \dots = \begin{array}{c} \diagup \\ \text{short} \\ \diagdown \end{array} \times \text{Exp} \left(\sum_{i,j} \begin{array}{c} i \\ \text{wavy} \\ j \\ \bigcirc \end{array} \right) \rightarrow \mathcal{W}_{\text{E\&M}}^\epsilon = \exp \left\{ \frac{\alpha}{8\pi} \frac{1}{\epsilon} \left[1 + \epsilon \log \frac{\mathcal{E}^2}{\mu^2} \right] \left(\sum_{i,j} \eta_i \eta_j q_i q_j \frac{1}{\beta_{ij}} \log \frac{1 + \beta_{ij}}{1 - \beta_{ij}} - i \sum_{\substack{i \neq j \\ \eta_i \eta_j = 1}} q_i q_j \frac{2\pi}{\beta_{ij}} \right) \right\}$$

Why Unitary?

Amplitude

Weinberg Soft Exp

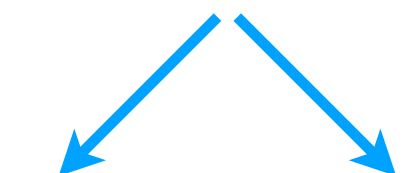
Unitary Detector Amplitudes

$$\begin{array}{c} \text{short} \end{array} + \begin{array}{c} \text{short} \end{array} + \begin{array}{c} \text{short} \end{array} + \dots = \begin{array}{c} \text{short} \end{array} \times \text{Exp} \left(\sum_{i,j} \begin{array}{c} i \quad j \\ \text{ } \end{array} \right) \rightarrow \mathcal{W}_{\text{E\&M}}^{\epsilon} = \exp \left\{ \frac{\alpha}{8\pi} \frac{1}{\epsilon} \left[1 + \epsilon \log \frac{\mathcal{E}^2}{\mu^2} \right] \left(\sum_{i,j} \eta_i \eta_j q_i q_j \frac{1}{\beta_{ij}} \log \frac{1 + \beta_{ij}}{1 - \beta_{ij}} - i \sum_{\substack{i \neq j \\ \eta_i \eta_j = 1}} q_i q_j \frac{2\pi}{\beta_{ij}} \right) \right\}$$

Why Unitary?

$$\frac{\text{Amplitude}}{\text{Weinberg Soft Exp}} = \mathcal{M}_{\mathcal{E}}^{\text{hard}} + \delta_{\text{sub.soft}}$$

Loops: $\underbrace{\int_0^{\mathcal{E}} d^{4+\epsilon} k}_{\text{Soft}} + \underbrace{\int_{\mathcal{E}}^{\infty} d^{4+\epsilon} k}_{\text{Hard}}$



Leading Soft Sub-leading Soft

Unitary Detector Amplitudes

$$\begin{array}{c} \text{short} \end{array} + \begin{array}{c} \text{short} \end{array} + \begin{array}{c} \text{short} \end{array} + \dots = \begin{array}{c} \text{short} \end{array} \times \text{Exp} \left(\sum_{i,j} \begin{array}{c} i \quad j \\ \text{ } \end{array} \right) \rightarrow \mathcal{W}_{\mathcal{E}}^{\epsilon} = \exp \left\{ \frac{\alpha}{8\pi} \frac{1}{\epsilon} \left[1 + \epsilon \log \frac{\mathcal{E}^2}{\mu^2} \right] \left(\sum_{i,j} \eta_i \eta_j q_i q_j \frac{1}{\beta_{ij}} \log \frac{1 + \beta_{ij}}{1 - \beta_{ij}} - i \sum_{\substack{i \neq j \\ \eta_i \eta_j = 1}} q_i q_j \frac{2\pi}{\beta_{ij}} \right) \right\}$$

Why Unitary?

$$\frac{\text{Amplitude}}{\text{Weinberg Soft Exp}} = \mathcal{M}_{\mathcal{E}}^{\text{hard}} + \cancel{\delta_{\text{sub.soft}}}$$

$\mathcal{M}^{\text{detector}} \rightarrow \mathcal{M}_{\mathcal{E}}^{\text{hard}}$

$\alpha \rightarrow 0$
 $\alpha \log \mathcal{E} = \alpha_{\mathcal{E}}$

$G \rightarrow 0$
 $G \log \mathcal{E} = G_{\mathcal{E}}$

Loops:

$$\underbrace{\int_0^{\mathcal{E}} d^{4+\epsilon} k}_{\text{Soft}} + \underbrace{\int_{\mathcal{E}}^{\infty} d^{4+\epsilon} k}_{\text{Hard}}$$

Leading Soft Sub-leading Soft

Unitary Detector Amplitudes

$$\begin{array}{c} \text{short} \end{array} + \begin{array}{c} \text{short} \end{array} + \begin{array}{c} \text{short} \end{array} + \dots = \begin{array}{c} \text{short} \end{array} \times \text{Exp} \left(\sum_{i,j} \begin{array}{c} i \quad j \\ \text{ } \end{array} \right) \rightarrow \mathcal{W}_{\text{E\&M}}^\epsilon = \exp \left\{ \frac{\alpha}{8\pi} \frac{1}{\epsilon} \left[1 + \epsilon \log \frac{\mathcal{E}^2}{\mu^2} \right] \left(\sum_{i,j} \eta_i \eta_j q_i q_j \frac{1}{\beta_{ij}} \log \frac{1 + \beta_{ij}}{1 - \beta_{ij}} - i \sum_{\substack{i \neq j \\ \eta_i \eta_j = 1}} q_i q_j \frac{2\pi}{\beta_{ij}} \right) \right\}$$

Why Unitary?

$$\frac{\text{Amplitude}}{\text{Weinberg Soft Exp}} = \mathcal{M}_{\mathcal{E}}^{\text{hard}} + \cancel{\delta_{\text{sub.soft}}}$$

$\mathcal{M}^{\text{detector}} \rightarrow \mathcal{M}_{\mathcal{E}}^{\text{hard}}$

$\alpha \rightarrow 0$
 $\alpha \log \mathcal{E} = \alpha_{\mathcal{E}}$

$G \rightarrow 0$
 $G \log \mathcal{E} = G_{\mathcal{E}}$

Loops:

$$\underbrace{\int_0^{\mathcal{E}} d^{4+\epsilon} k}_{\text{Soft}} + \underbrace{\int_{\mathcal{E}}^{\infty} d^{4+\epsilon} k}_{\text{Hard}}$$

Leading Soft Sub-leading Soft

Manifestly Unitary in the Hilbert space of Hard photons/gravitons

$$H_{\mathcal{E}}^{\text{hard}} = \sum_{h=\pm} \int_{|\mathbf{k}| > \mathcal{E}} \frac{d^3 \mathbf{k}}{(2\pi)^3} \left\{ \omega(\mathbf{k}) a_h^\dagger(\mathbf{k}) a_h(\mathbf{k}) - e \left[a_h(\mathbf{k}) \boldsymbol{\epsilon}^h(\mathbf{k}) \cdot \mathbf{J} + \text{h.c.} \right] + V_{\text{Coul}} \right\} + c.t. + \dots$$

Unitarity

3 couplings: 2 long-distance couplings + EFT

$$\mathcal{M}_{\mathcal{E}} = \mathcal{M}(\overset{\downarrow}{\alpha}_{\mathcal{E}}, g_{EFT}) + O(\overset{\downarrow}{\alpha}, \mathcal{E}) \quad \mathcal{E} \ll p_{\text{hard}}$$

$\alpha, \quad \alpha \log \mathcal{E} = \alpha_{\mathcal{E}}$
 $G, \quad G \log \mathcal{E} = G_{\mathcal{E}}$

Unitarity

3 couplings: 2 long-distance couplings + EFT $\mathcal{M}_{\mathcal{E}} = \mathcal{M}(\alpha_{\mathcal{E}}, g_{EFT}) + O(\alpha, \mathcal{E})$ $\mathcal{E} \ll p_{\text{hard}}$

$\alpha,$ $\alpha \log \mathcal{E} = \alpha_{\mathcal{E}}$
 $G,$ $G \log \mathcal{E} = G_{\mathcal{E}}$

Stripped Amplitude

Analytic, X-ssing,
Regge



$$\begin{aligned} \alpha, G &\rightarrow 0 \\ \alpha \log \mathcal{E} &= \alpha_{\mathcal{E}} \\ G \log \mathcal{E} &= G_{\mathcal{E}} \end{aligned}$$

Detector Amplitude

$$\mathcal{M}^{\text{detector}}(\alpha_{\mathcal{E}}, G_{\mathcal{E}}, g_{EFT})$$



Hard Amplitude

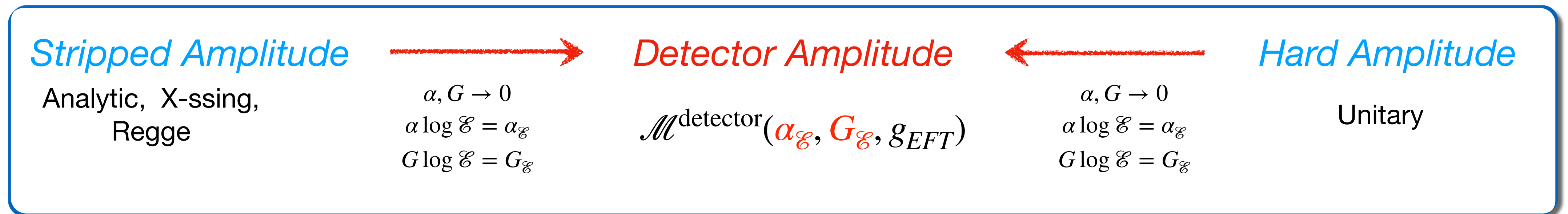
$$\begin{aligned} \alpha, G &\rightarrow 0 \\ \alpha \log \mathcal{E} &= \alpha_{\mathcal{E}} \\ G \log \mathcal{E} &= G_{\mathcal{E}} \end{aligned}$$

Unitary

Unitarity

3 couplings: 2 long-distance couplings + EFT $\mathcal{M}_{\mathcal{E}} = \mathcal{M}(\alpha_{\mathcal{E}}, g_{EFT}) + O(\alpha, \mathcal{E})$ $\mathcal{E} \ll p_{\text{hard}}$

$\alpha, \quad \alpha \log \mathcal{E} = \alpha_{\mathcal{E}}$
 $G, \quad G \log \mathcal{E} = G_{\mathcal{E}}$



$\text{Disc } \mathcal{M}_{\mathcal{E}} \longrightarrow \text{Disc } \mathcal{M}^{\text{Detector}} > 0$

$\alpha, G \rightarrow 0$
 $\alpha \log \mathcal{E} = \alpha_{\mathcal{E}}$
 $G \log \mathcal{E} = G_{\mathcal{E}}$

Unitarity

3 couplings: 2 long-distance couplings + EFT $\mathcal{M}_{\mathcal{E}} = \mathcal{M}(\alpha_{\mathcal{E}}, g_{EFT}) + O(\alpha, \mathcal{E})$ $\mathcal{E} \ll p_{\text{hard}}$

$\alpha, \quad \alpha \log \mathcal{E} = \alpha_{\mathcal{E}}$
 $G, \quad G \log \mathcal{E} = G_{\mathcal{E}}$

Stripped Amplitude

Analytic, X-sing,
Regge



$$\begin{aligned} \alpha, G &\rightarrow 0 \\ \alpha \log \mathcal{E} &= \alpha_{\mathcal{E}} \\ G \log \mathcal{E} &= G_{\mathcal{E}} \end{aligned}$$

Detector Amplitude

$$\mathcal{M}^{\text{detector}}(\alpha_{\mathcal{E}}, G_{\mathcal{E}}, g_{EFT})$$



$$\begin{aligned} \alpha, G &\rightarrow 0 \\ \alpha \log \mathcal{E} &= \alpha_{\mathcal{E}} \\ G \log \mathcal{E} &= G_{\mathcal{E}} \end{aligned}$$

Hard Amplitude

Unitary

$$\text{Disc } \mathcal{M}_{\mathcal{E}} \longrightarrow \text{Disc } \mathcal{M}^{\text{Detector}} > 0$$

$$\begin{aligned} \alpha, G &\rightarrow 0 \\ \alpha \log \mathcal{E} &= \alpha_{\mathcal{E}} \\ G \log \mathcal{E} &= G_{\mathcal{E}} \end{aligned}$$

Universality

Inclusive Detector Amplitudes

$$\frac{\text{Amplitude}}{\text{Weinberg Soft Exp}} = \mathcal{M}_{\mathcal{E}} \leftarrow ?$$

Inclusive Detector Amplitudes

$$\frac{\text{Amplitude}}{\text{Weinberg Soft Exp}} = \mathcal{M}_{\mathcal{E}} \leftarrow ?$$

Exclusive |Detector-Amplitudes|^2

$$\frac{|\mathcal{M}_{\mathcal{E}}|^2}{\sum_{E_{\gamma,h} < \Delta E} |\mathcal{M}|^2} = \frac{|\mathcal{M}_{\mathcal{E}}|^2}{\left| \begin{array}{c} \text{diagram 1} + \text{diagram 2} + \dots \\ \text{diagram 3} + \text{diagram 4} + \dots \end{array} \right|^2 + \dots}$$

The diagram shows a sum of squared amplitudes for various processes. The first term is $|\mathcal{M}_{\mathcal{E}}|^2$, which is identified as the Exclusive |Detector-Amplitudes|^2. The second term is a sum of squared amplitudes for processes involving soft photons, represented by wavy red lines. The third term is a sum of squared amplitudes for processes involving hard photons, represented by wavy red lines. The final term is $|\mathcal{M}_{\Delta E_{\gamma,h}}^{\text{hard}}|^2$, which is identified as the Inclusive X-sections.

Inclusive X-sections

Inclusive Detector Amplitudes

$$\frac{\text{Amplitude}}{\text{Weinberg Soft Exp}} = \mathcal{M}_{\mathcal{E}} \leftarrow ?$$

Exclusive |Detector-Amplitudes|^2

$$\frac{|\mathcal{M}_{\mathcal{E}}|^2}{\sum_{E_{\gamma,h} < \Delta E} |\mathcal{M}|^2} = \frac{|\mathcal{M}_{\mathcal{E}}|^2}{\left| \begin{array}{c} \text{diagram 1} + \text{diagram 2} + \dots \\ \text{diagram 3} + \text{diagram 4} + \dots \end{array} \right|^2} + \dots$$

Inclusive X-sections

$$|\mathcal{M}_{\Delta E_{\gamma,h}}^{\text{hard}}|^2$$

$$\left(\frac{\Delta E}{\mathcal{E}} \right)^{-\frac{\alpha}{8\pi^2} \left(\sum_{i,j} \eta_i \eta_j q_i q_j \frac{1}{\beta_{ij}} \log \frac{1+\beta_{ij}}{1-\beta_{ij}} \right)} = 1$$

$$\Delta E_{\gamma,h} = \mathcal{E}$$

Inclusive Detector Amplitudes

$$\frac{\text{Amplitude}}{\text{Weinberg Soft Exp}} = \mathcal{M}_{\mathcal{E}} \leftarrow ?$$

Exclusive |Detector-Amplitudes|^2

$$\Delta E_{\gamma,h} = \mathcal{E}$$

$$\frac{|\mathcal{M}_{\mathcal{E}}|^2}{\sum_{E_{\gamma,h} < \Delta E} |\mathcal{M}|^2} = \frac{|\mathcal{M}_{\mathcal{E}}|^2}{\left| \begin{array}{c} \text{diagrams with soft photons} \\ + \text{diagrams with hard photons} \end{array} \right|^2} \xrightarrow{\text{red arrow}} \left(\frac{\Delta E}{\mathcal{E}} \right)^{-\frac{\alpha}{8\pi^2} \left(\sum_{i,j} \eta_i \eta_j q_i q_j \frac{1}{\beta_{ij}} \log \frac{1+\beta_{ij}}{1-\beta_{ij}} \right)} = 1$$

$\xrightarrow{\text{blue arrow}} |\mathcal{M}_{\mathcal{E}}^{\text{hard}}|^2$
 $\xrightarrow{\text{blue arrow}} |\mathcal{M}_{\Delta E_{\gamma,h}}^{\text{hard}}|^2$

Inclusive X-sections

Lessons:

1. $\mathcal{E} \sim$ Detector Energy Resolution; optimally 1/“size”
2. IR sensitive, but not too much
3. Universal leading-log of any IR-finite amplitude

Inclusive Detector Amplitudes



$$\frac{\text{Amplitude}}{\text{Weinberg Soft Exp}} = \mathcal{M}_{\mathcal{E}} \leftarrow ?$$

Exclusive |Detector-Amplitudes|^2

$$\Delta E_{\gamma,h} = \mathcal{E}$$

$$\frac{|\mathcal{M}_{\mathcal{E}}|^2}{\sum_{E_{\gamma,h} < \Delta E} |\mathcal{M}|^2} = \frac{|\mathcal{M}_{\mathcal{E}}|^2}{\left| \begin{array}{c} \text{diagrams with soft photons} \\ + \text{diagrams with hard photons} \end{array} \right|^2} \xrightarrow{\text{red arrow}} \left(\frac{\Delta E}{\mathcal{E}} \right)^{-\frac{\alpha}{8\pi^2} \left(\sum_{i,j} \eta_i \eta_j q_i q_j \frac{1}{\beta_{ij}} \log \frac{1+\beta_{ij}}{1-\beta_{ij}} \right)} = 1$$

$\xrightarrow{\text{blue arrow}} |\mathcal{M}_{\Delta E_{\gamma,h}}^{\text{hard}}|^2$

Inclusive X-sections

Lessons:

1. $\mathcal{E} \sim$ Detector Energy Resolution; optimally 1/“size”
2. IR sensitive, but not too much
3. Universal leading-log of any IR-finite amplitude

$\mathcal{M}_{\mathcal{E}}$ is

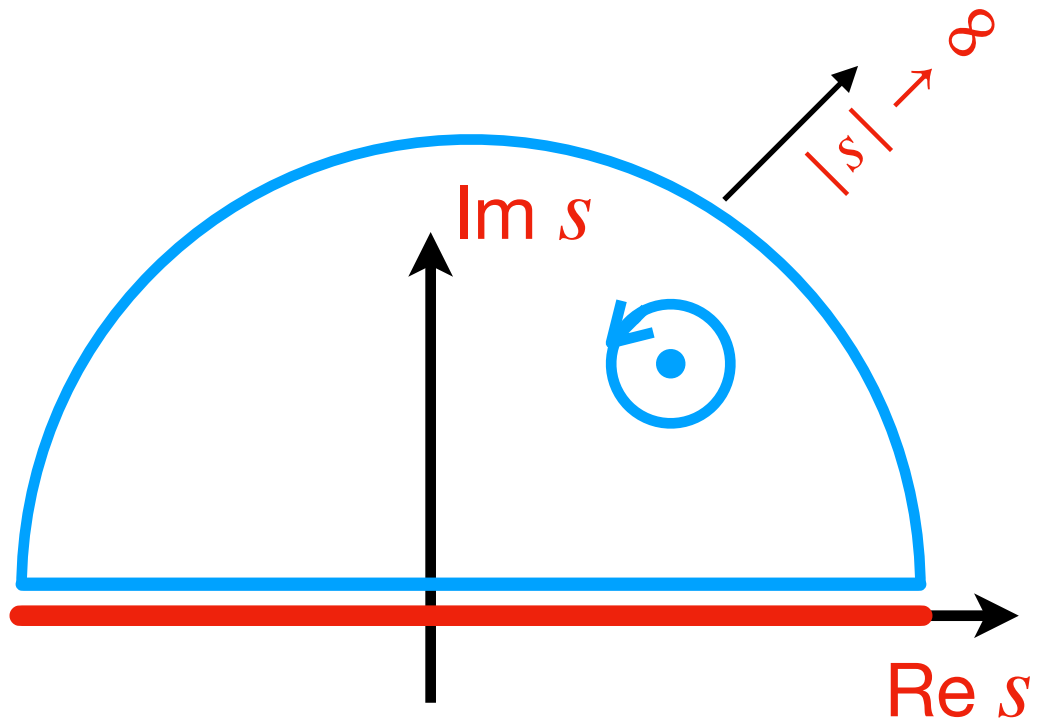
- | | | | |
|--|---|---|------------|
| 1. IR finite | ✓ | } | Exactly |
| 2. Lorentz invariant | ✓ | | |
| 3. Analytic & X-ssing | ✓ | | |
| 4. Regge/Froissart Bound | ✓ | | |
| 5. Unitary | ✓ | } | In a limit |
| 6. Universality, $\mathcal{E} \sim \Delta E$ | ✓ | | |

Positivity

Dispersion Relations

$\mathcal{M}_{\mathcal{E}}$ is

- | | |
|---|-----------|
| 1. IR finite | } Exactly |
| 2. Lorentz invariant | |
| 3. Analytic | |
| 4. Average Regge | |
| 5. Unitary | |
| 6. Inclusive, $\mathcal{E} \sim \Delta E$ | |



$\oint \mathcal{M}_{\mathcal{E}} \mathcal{K}(s) = \text{"Arc } \mathcal{A}"$

$$g_{EFT} = \langle (\dots) \rangle = \sum_{\ell} \int_{M^2}^{\infty} \frac{ds}{s^3} (\dots) \text{Im} \mathcal{M}_{\mathcal{E}}^{\ell}(s)$$

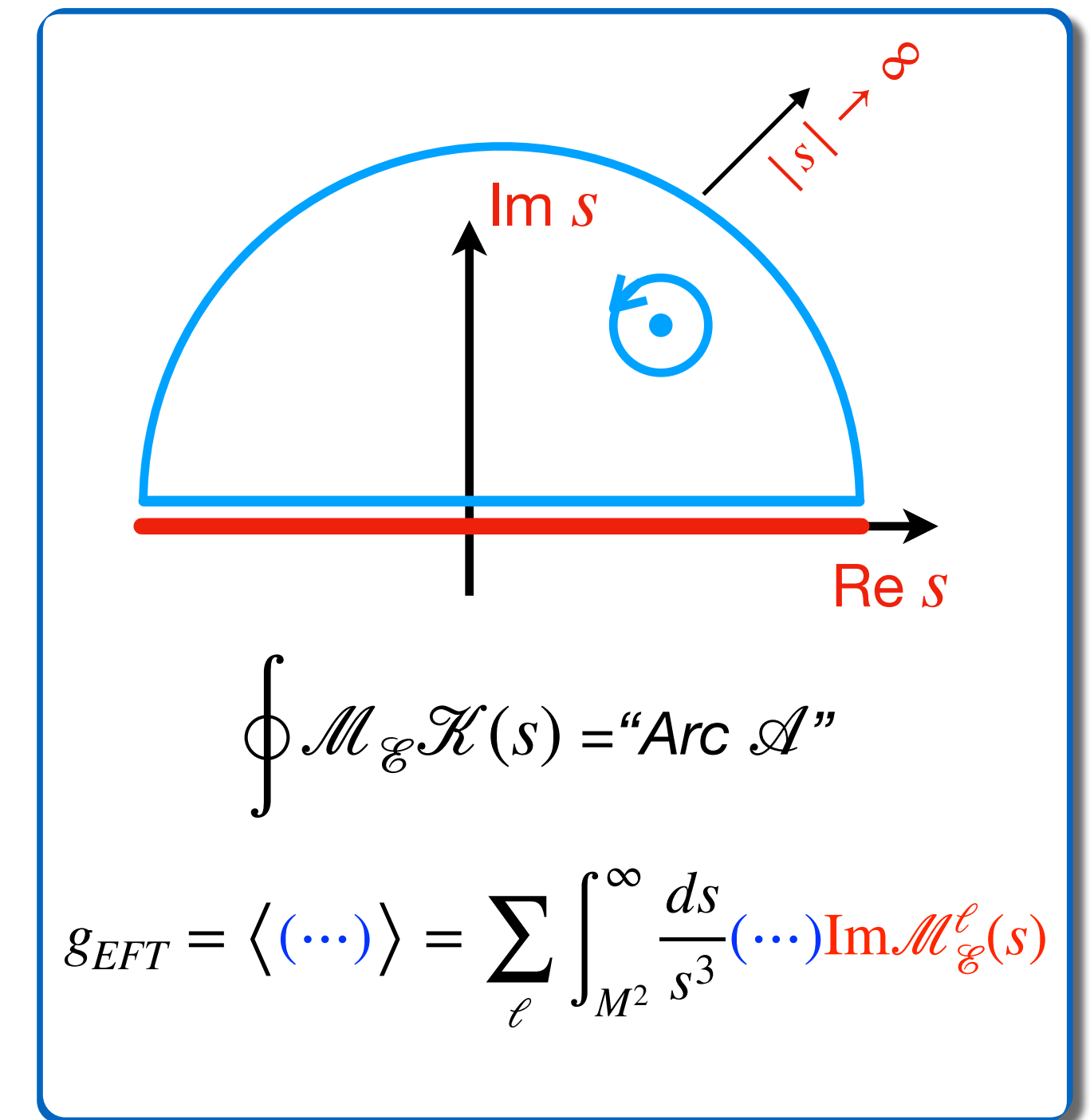
EFT Dispersive Sum Rule

Dispersion Relations

$\mathcal{M}_{\mathcal{E}}$ is

1. IR finite
 2. Lorentz invariant
 3. Analytic
 4. Average Regge
- } Exactly

5. Unitary
 6. Inclusive, $\mathcal{E} \sim \Delta E$
- } In the limit
- $\alpha \rightarrow 0, \quad \alpha \log \mathcal{E} = \alpha_{\mathcal{E}} \text{ - fixed}$
 $G \rightarrow 0, \quad G \log \mathcal{E} = G_{\mathcal{E}} \text{ -fixed}$
 (or tree-level)



EFT Dispersive Sum Rule

EFT Positivity bounds

$$\mathcal{A}(g_{EFT}, \alpha_{\mathcal{E}}, G_{\mathcal{E}}) + O(\alpha, G) \geq 0$$

Dispersion Relations

$\mathcal{M}_{\mathcal{E}}$ is

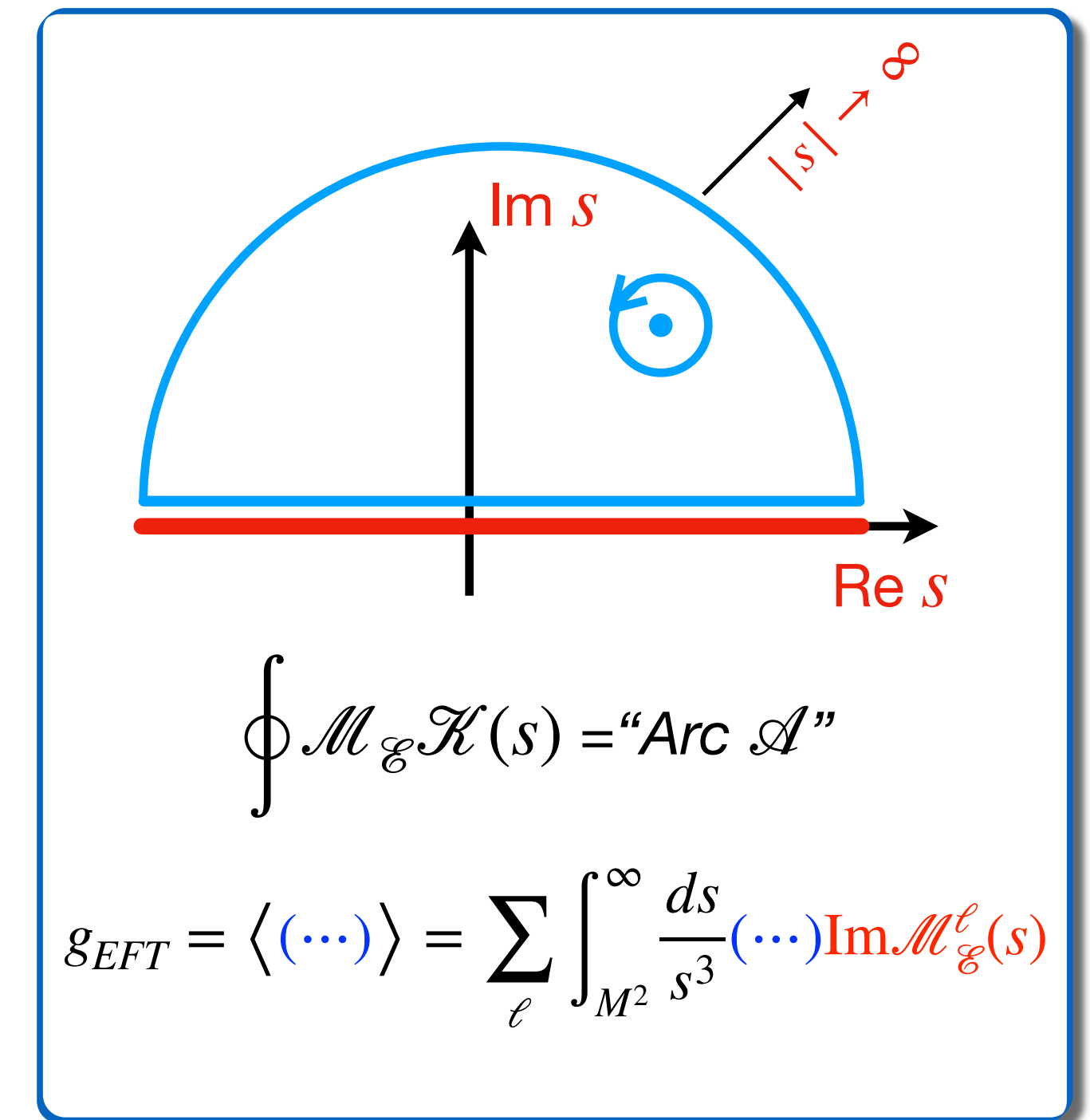
1. IR finite
2. Lorentz invariant
3. Analytic
4. Average Regge
5. Unitary
6. Inclusive, $\mathcal{E} \sim \Delta E$

Exactly

In the limit

$$\begin{aligned} \alpha \rightarrow 0, \quad \alpha \log \mathcal{E} = \alpha_{\mathcal{E}} \text{ - fixed} \\ G \rightarrow 0, \quad G \log \mathcal{E} = G_{\mathcal{E}} \text{ - fixed} \end{aligned}$$

(or tree-level)



EFT Dispersive Sum Rule

$$\mathcal{M}_{\mathcal{E}} = \mathcal{M}^{\text{detector}}(\alpha_{\mathcal{E}}, G_{\mathcal{E}}, g_{EFT}) \left[1 + O\left(\frac{G}{G_{\mathcal{E}}}, \frac{\alpha}{\alpha_{\mathcal{E}}}, \mathcal{E}/E\right) \right]$$

Beyond Tree:

Effective bounds on g_{EFT} as long as:

$$\begin{aligned} \alpha_{\mathcal{E}}/g_{EFT} &\ll 1 \\ G_{\mathcal{E}}E^2/g_{EFT} &\ll 1 \end{aligned}$$

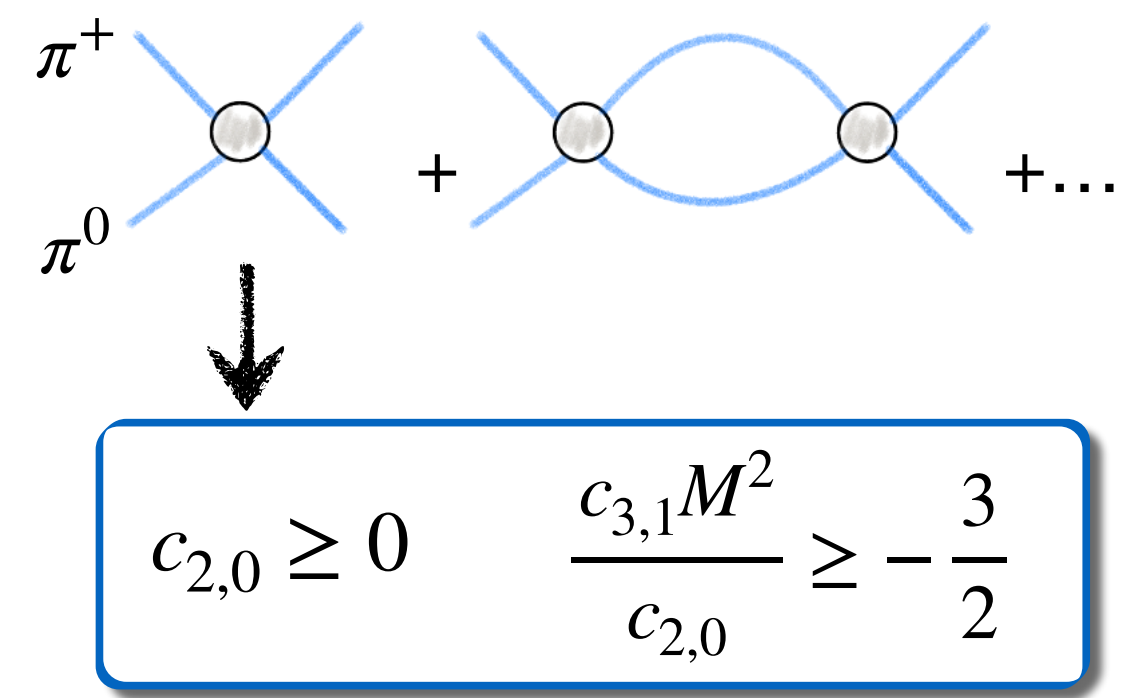
while $1/\log \mathcal{E}/E \ll 1$

EFT Positivity bounds

$$\mathcal{A}(g_{EFT}, \alpha_{\mathcal{E}}, G_{\mathcal{E}}) + O(\alpha, G) \geq 0$$

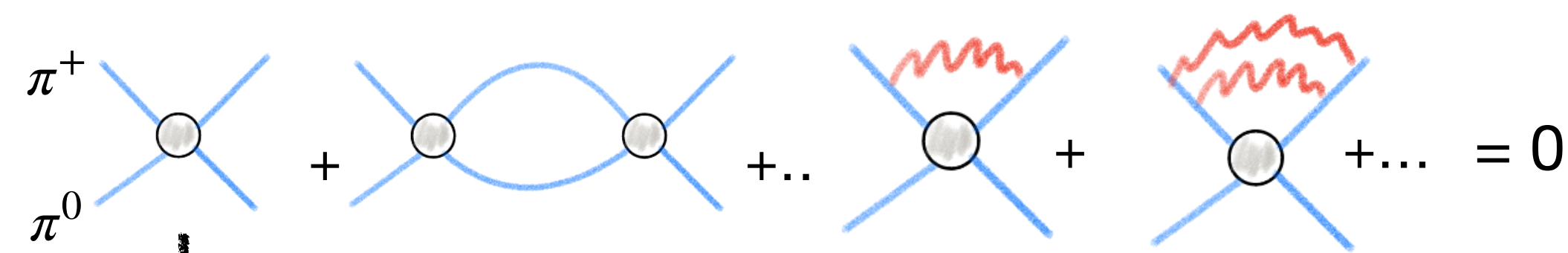
Examples

$$\pi^0\pi^+ + \mathcal{E} \& \mathcal{M}$$



$$\mathcal{M}^{0+} = \left\{ c_{0,0} + c_{1,1}t + c_{2,0}(s - m_{\pm}^2 + t/2)^2 + c_{2,2}t^2 + c_{3,1}t(s - m_{\pm}^2 + t/2)^2 + \dots \right\}$$

$$\pi^0 \pi^+ + \mathcal{E} \& \mathcal{M}$$



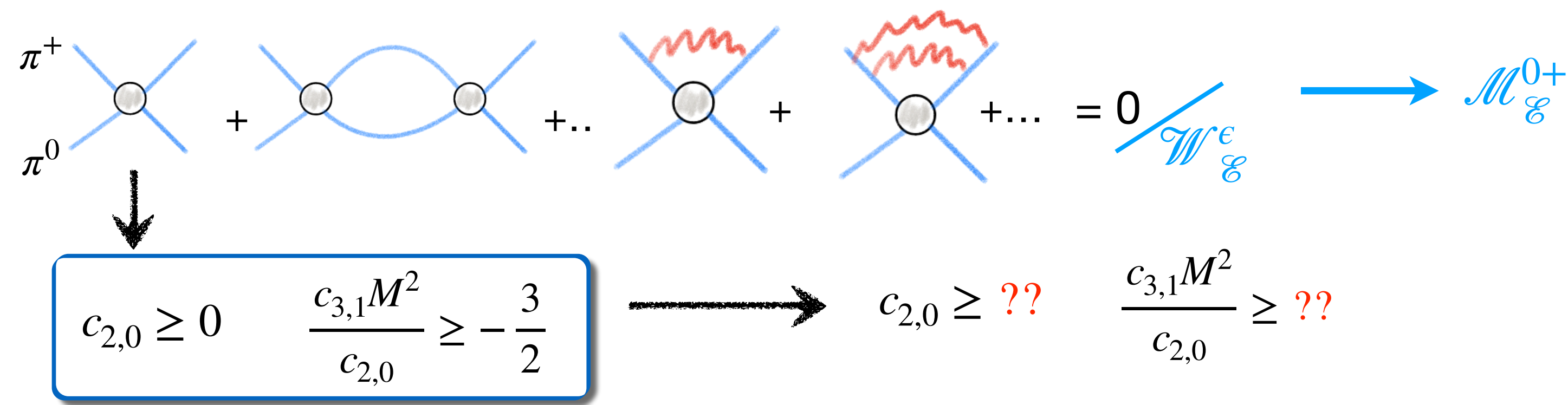
$c_{2,0} \geq 0$
 $\frac{c_{3,1} M^2}{c_{2,0}} \geq -\frac{3}{2}$

$\longrightarrow$

$c_{2,0} \geq ??$
 $\frac{c_{3,1} M^2}{c_{2,0}} \geq ??$

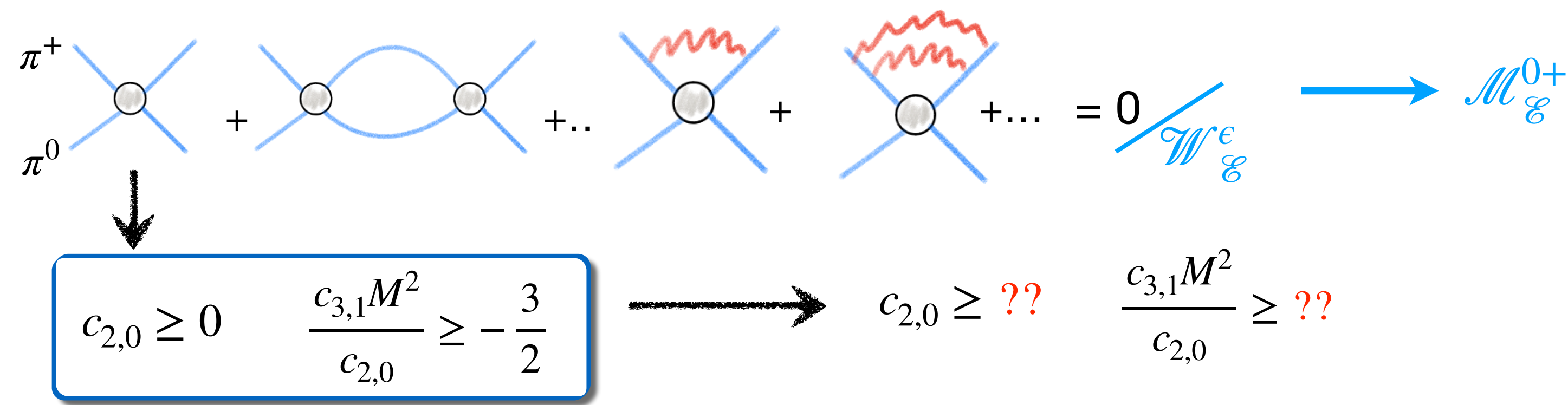
$$\mathcal{M}^{0+} = \left\{ c_{0,0} + c_{1,1}t + c_{2,0}(s - m_{\pm}^2 + t/2)^2 + c_{2,2}t^2 + c_{3,1}t(s - m_{\pm}^2 + t/2)^2 + \dots \right\}$$

$$\pi^0 \pi^+ + \mathcal{E} \& \mathcal{M}$$



$$\mathcal{M}_{\mathcal{E}}^{0+} = \left\{ c_{0,0} + c_{1,1}t + c_{2,0}(s - m_{\pm}^2 + t/2)^2 + c_{2,2}t^2 + c_{3,1}t(s - m_{\pm}^2 + t/2)^2 + \dots \right\} \times \left[1 + \frac{\alpha}{2\pi} \log \left(\frac{m_{\pm}^2(4m_{\pm}^2 - t)}{\mathcal{E}^4} \right) \frac{t - 2m_{\pm}^2}{\sqrt{t(4m_{\pm}^2 - t)}} \arctan \left(\frac{\sqrt{t}}{\sqrt{4m_{\pm}^2 - t}} \right) + \frac{\alpha}{2\pi} \log \frac{4\pi\mu^2}{\mathcal{E}^2} \right] + O(\alpha_{\mathcal{E}}^2)$$

$\pi^0 \pi^+ + \mathcal{E} \& \mathcal{M}$



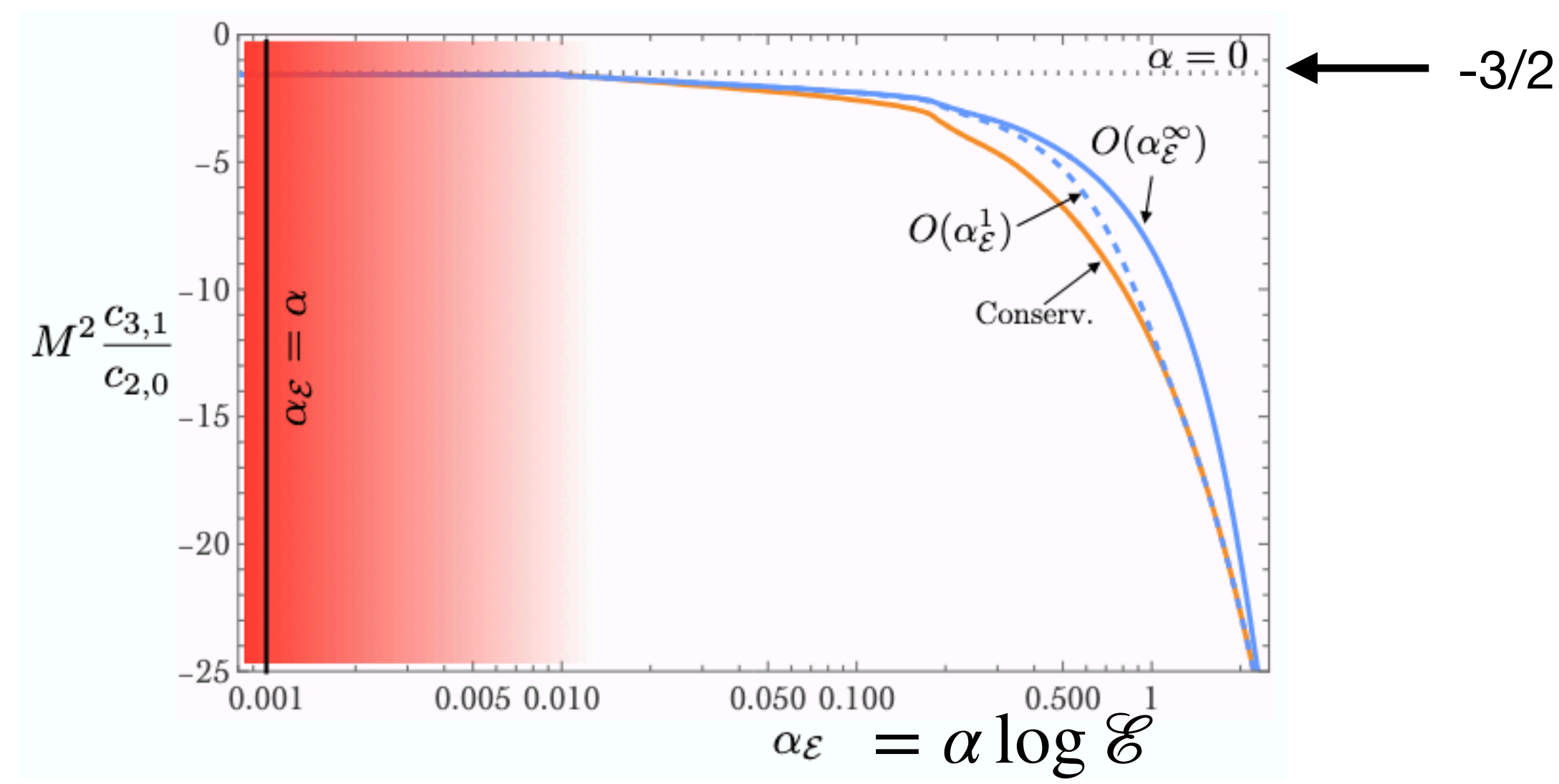
$$\mathcal{M}_\mathcal{E}^{0+} = \left\{ c_{0,0} + c_{1,1}t + c_{2,0}(s - m_\pm^2 + t/2)^2 + c_{2,2}t^2 + c_{3,1}t(s - m_\pm^2 + t/2)^2 + \dots \right\} \times \left[1 + \frac{\alpha}{2\pi} \log \left(\frac{m_\pm^2(4m_\pm^2 - t)}{\mathcal{E}^4} \right) \frac{t - 2m_\pm^2}{\sqrt{t(4m_\pm^2 - t)}} \arctan \left(\frac{\sqrt{t}}{\sqrt{4m_\pm^2 - t}} \right) + \frac{\alpha}{2\pi} \log \frac{4\pi\mu^2}{\mathcal{E}^2} \right] + O(\alpha_\mathcal{E}^2)$$

Smoothly connected to $\alpha = 0$

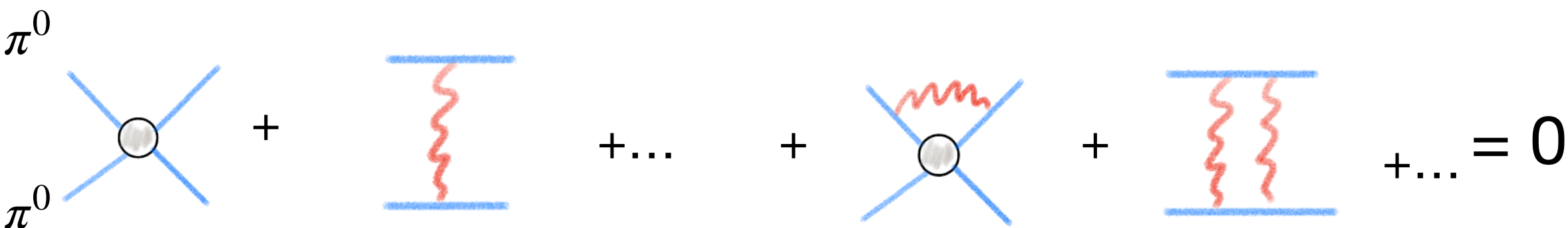
$$c_{2,0} \geq 0$$
$$\frac{c_{3,1}M^2}{c_{2,0}} > -\frac{3}{2} + \frac{M^2}{m_\pm^2} \frac{\alpha \log \mathcal{E}/m_\pm}{3\pi} + O(\alpha, \alpha_\mathcal{E}^2, c_{n,k}^2)$$

BB, Berman, Isabella, Riva, Romano, Sciotti '25

Optimal bounds: $1 \ll \log m_\pm/\mathcal{E} < \infty$



$\pi^0\pi^0+gravity$



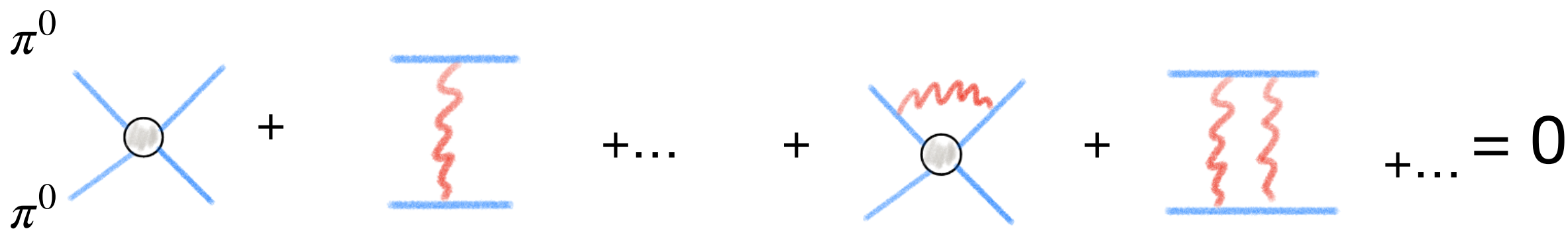
$$\mathcal{M}^{\text{tree,GR}} = 8\pi G \left(\frac{tu}{s} + \frac{us}{t} + \frac{st}{u} \right) + c_{2,0} \left(\frac{s^2 + t^2 + u^2}{2} \right) + c_{3,1} stu + \dots$$

$c_{2,0} \geq 0$ $\frac{c_{3,1}M^2}{c_{2,0}} \leq \frac{3}{2}$

$\longrightarrow$

$c_{2,0} \geq ??$ $\frac{c_{3,1}M^2}{c_{2,0}} \leq ??$

$\pi^0\pi^0+gravity$



$$\mathcal{M}^{\text{tree,GR}} = 8\pi G \left(\frac{tu}{s} + \frac{us}{t} + \frac{st}{u} \right) + c_{2,0} \left(\frac{s^2 + t^2 + u^2}{2} \right) + c_{3,1} stu + \dots$$

$c_{2,0} \geq 0 \qquad \frac{c_{3,1}M^2}{c_{2,0}} \leq \frac{3}{2}$

→

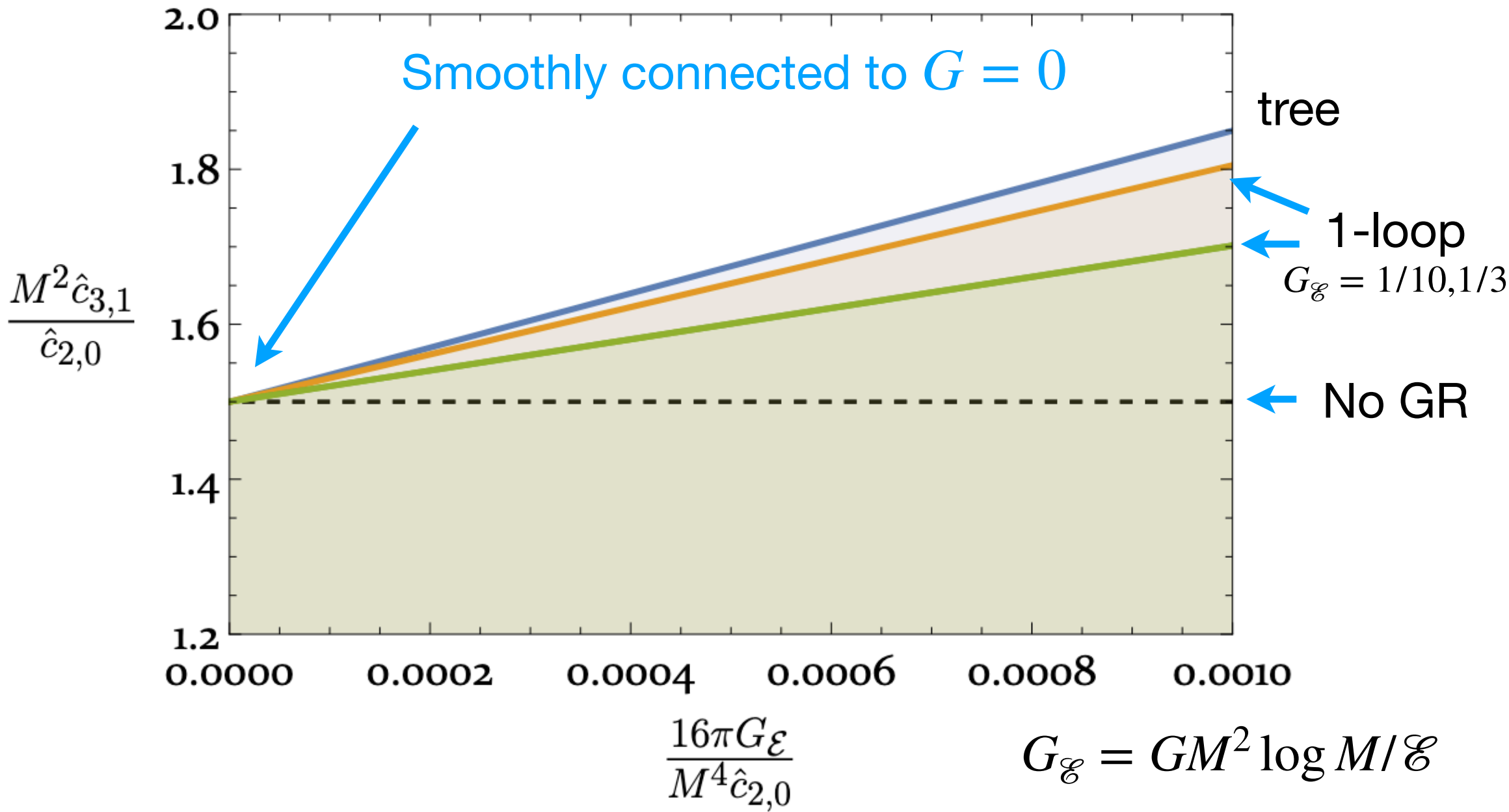
$c_{2,0} \geq ?? \qquad \frac{c_{3,1}M^2}{c_{2,0}} \leq ??$

$$c_{2,0}M^4 \geq -526\pi G_{\mathcal{E}} + O(G)$$

$$0 \leq \frac{c_{2,0}M^4}{5600\pi G_{\mathcal{E}}} \left(\frac{3}{2} - \frac{c_{3,1}M^2}{c_{2,0}} \right) + 1 + \frac{4}{\pi} G_{\mathcal{E}} + \dots + O\left(\frac{1}{\log M/\mathcal{E}}\right)$$

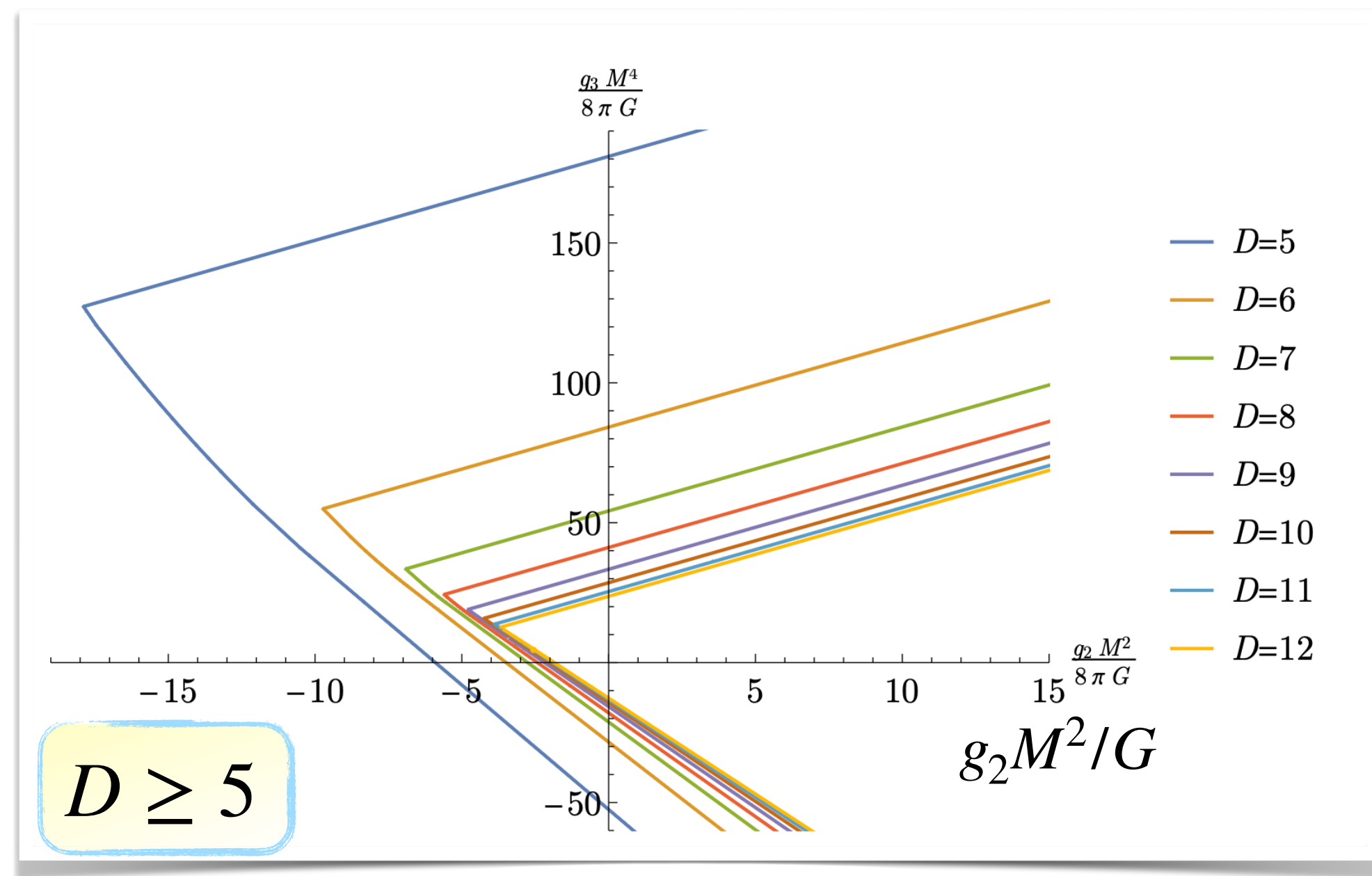
BB, Berman, Isabella, Riva, Romano, Sciotti '25

Optimal bounds: $1 \ll \log M/\mathcal{E} < \infty$

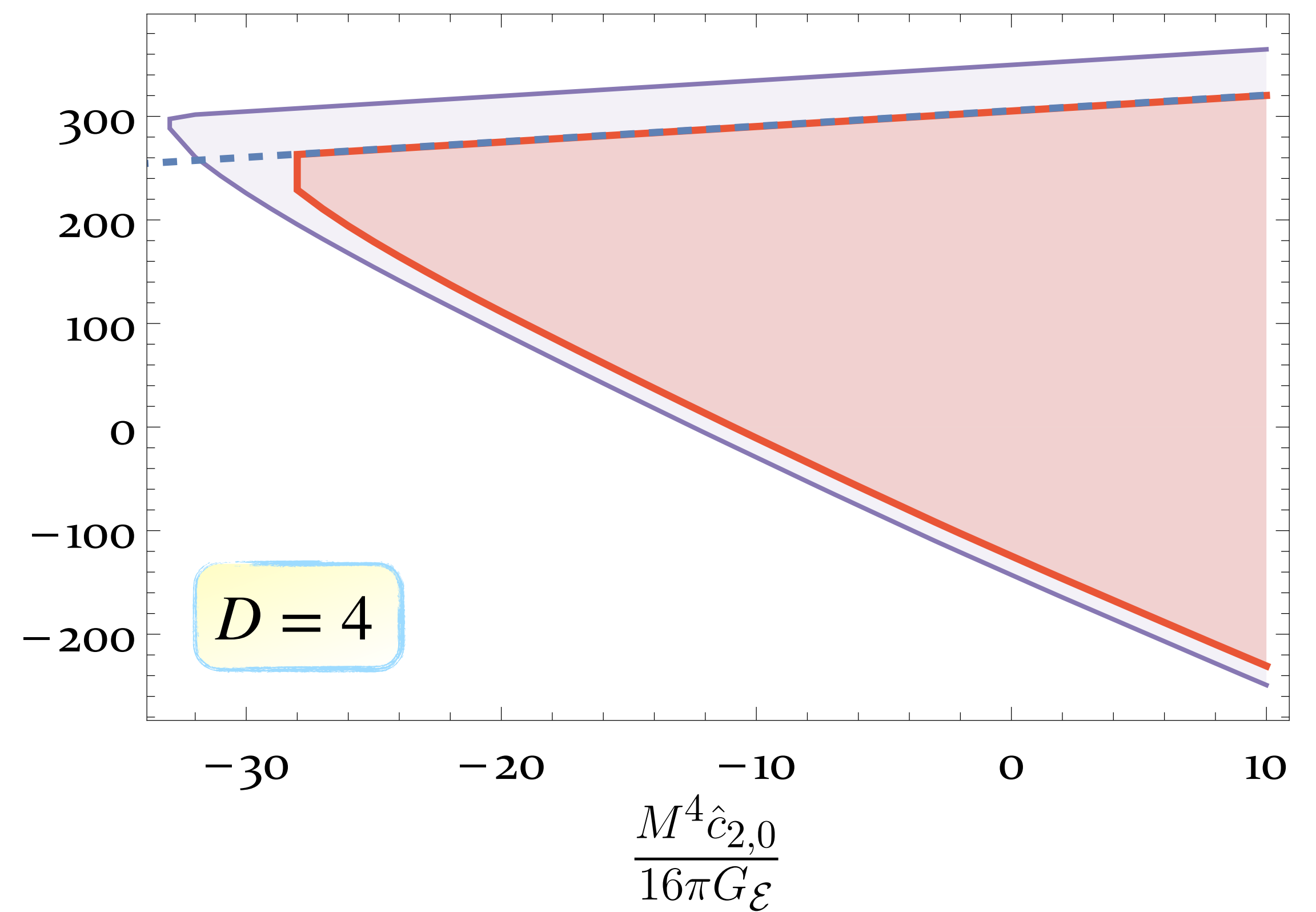


$\pi^0\pi^0$ + gravity

Caron-huot, Mazac, Rastelli, Simmons-Duffin 2102.08951



BB, Berman, Isabella, Riva, Romano, Sciotti 2512.13780

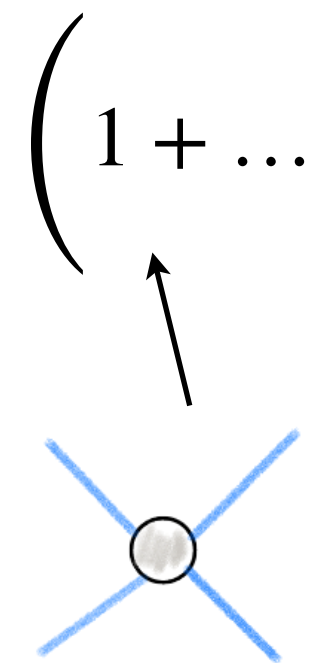


Earlier, IR-divergent Bounds?

without gravity

$$c_{2,0} \geq 0$$

with gravity, IR-div
(Tree&Loops)



$$\left(1 + \dots\right) c_{2,0} M^4 + \left(\frac{GM^2}{\epsilon} + \left(\frac{GM^2}{\epsilon} \right)^2 + \dots \right) \geq 0$$

The equation shows the infrared-divergent terms in the bound. The first term, $\frac{GM^2}{\epsilon}$, is associated with a one-loop diagram (a red wavy line forming a loop between two blue horizontal lines). The second term, $\left(\frac{GM^2}{\epsilon}\right)^2$, is associated with a two-loop diagram (two red wavy lines forming a loop between two blue horizontal lines). Arrows point from these diagrams to their respective terms in the equation.

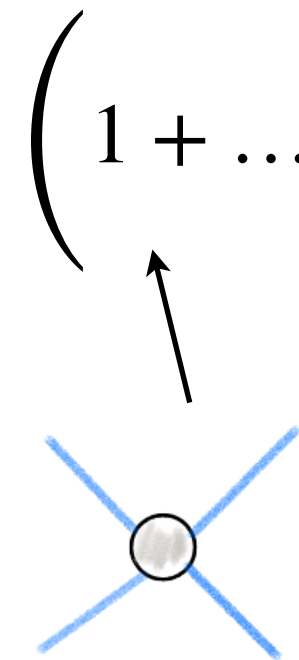
Earlier, IR-divergent Bounds?

without gravity

$$c_{2,0} \geq 0$$

with gravity, IR-div
(Tree&Loops)

$$c_{2,0} + 256 \geq 0$$



$$\left(1 + \dots\right) c_{2,0} M^4 + \overbrace{\left(\frac{GM^2}{\epsilon} + \left(\frac{GM^2}{\epsilon}\right)^2 + \dots\right)}^{F(GM^2/\epsilon) \rightarrow 256} \geq 0$$

Below the equation, two Feynman diagrams are shown. The first diagram, corresponding to the $\frac{GM^2}{\epsilon}$ term, shows two horizontal blue lines connected by a single red wavy line. The second diagram, corresponding to the $\left(\frac{GM^2}{\epsilon}\right)^2$ term, shows two horizontal blue lines connected by two red wavy lines. Arrows point from these diagrams to their respective terms in the equation.

Chang Parra-Martinez '25

Earlier, IR-divergent Bounds?

without gravity

$$c_{2,0} \geq 0$$

with gravity, IR-div
(Tree&Loops)

$$c_{2,0} + 256 \geq 0$$

With IR-div: Trivialize

$$256 \geq 0$$

$$\overbrace{\left(1 + \dots \frac{GM^2}{\epsilon} + \dots\right)}^{\rightarrow 0} c_{2,0} M^4 + \overbrace{\left(\frac{GM^2}{\epsilon} + \left(\frac{GM^2}{\epsilon}\right)^2 + \dots\right)}^{F(GM^2/\epsilon) \rightarrow 256} \geq 0$$

Chang Parra-Martinez '25

Earlier, IR-divergent Bounds?

without gravity

$$c_{2,0} \geq 0$$

with gravity, IR-div
(Tree&Loops)

$$c_{2,0} + 256 \geq 0$$

With IR-div: Trivialize

$$256 \geq 0$$

$$\overbrace{\left(1 + \dots \frac{GM^2}{\epsilon} + \dots\right)}^{\rightarrow 0} c_{2,0} M^4 + \overbrace{\left(\frac{GM^2}{\epsilon} + \left(\frac{GM^2}{\epsilon}\right)^2 + \dots\right)}^{F(GM^2/\epsilon) \rightarrow 256} \geq 0$$

Chang Parra-Martinez '25

Issue is not the t-pole

Remove the IR-divergences and all is good.

Conclusions

Conclusions & Outlook



Principles + *Symmetries* + *light d.o.f.*



Non-trivial IR-bootstrap problem



Photons, gravitons in D=4
E&M or Gravity

Ordinary S-matrix not a well-defined observable

What are good observables/tools?

1. IR finite, 2. not (too) IR sensitive, 3. Causality

In detector limit $\rightarrow$ stripped amplitudes are one such tool

Conclusions & Outlook

Some lessons:

1. Detector/IR data are crucial inputs: energy resolution $\mathcal{E}$ (or apparatus size)
2. Leading IR effects: universal, captured by Stripped/Detector Amplitudes $\mathcal{M}_{\mathcal{E}}$ theory tool (analytic, crossing-symmetric, Regge behaved, ...)
3. Smooth EFT bounds exist: finite, calculable long-range corrections for QED and GR in D=4
4. Optimal resolution $\mathcal{E}$ is not necessarily optimal for bounding EFT coefficients (e.g. too large apparatus $\rightarrow$ too large GR time delay)

Thank you!

Backup Slides

T-channel pole & Negativity

$$\int_{\mathcal{E}}^M dq \, \psi(q) \, \mathcal{M}_{\mathcal{E}}^{\text{hard} \dagger} \mathcal{M}_{\mathcal{E}}^{\text{hard}} \stackrel{?}{>} 0$$

forward region excluded

$$\underbrace{\int_0^M dq \, \psi(q) \mathcal{M}_{\mathcal{E}}^{\text{hard} \dagger} \mathcal{M}_{\mathcal{E}}^{\text{hard}}} - \underbrace{\int_0^{\mathcal{E}} dq \, \psi(q) \mathcal{M}_{\mathcal{E}}^{\text{hard} \dagger} \mathcal{M}_{\mathcal{E}}^{\text{hard}}}$$

Exist $\psi(q)$ s.t. >0

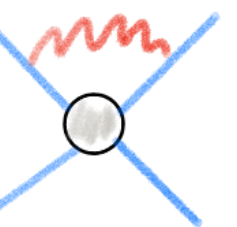
negativity = $O(G^2)$ sub-leading

$$\int_0^{\mathcal{E}} dq \, \psi(q) \mathcal{M}_{\mathcal{E}}^{\text{hard} \dagger} \mathcal{M}_{\mathcal{E}}^{\text{hard}} \sim \int_0^{\mathcal{E}} q \, dq \, \begin{array}{c} k \uparrow \\ \text{---} \end{array} \begin{array}{c} \text{---} \\ k - q \downarrow \end{array} \sim G^2 \int_0^{\mathcal{E}} q \, dq \left[\int \frac{k \, dk}{2\pi} \frac{\theta(k - \mathcal{E})}{k^2} \frac{\theta(|k - q| - \mathcal{E})}{(k - q)^2} \right] \sim G^2$$

No graviton for
 $|q| < \mathcal{E}$ in $\mathcal{M}_{\mathcal{E}}^{\text{hard}}$

Contrast to previous papers with IR-divergence

$$\left\{ \begin{array}{l} \int_0^{\mathcal{E}} dq \, \psi(q) \mathcal{M}_\epsilon^\dagger \mathcal{M}_\epsilon = F\left(\frac{GM^2}{\epsilon}\right) \rightarrow G - \text{independent} \quad \text{Chang Parra-Martinez '25} \quad +\text{trivialize the bound} \\ \int_0^{\mathcal{E}} dq \, \psi(q) \mathcal{M}_\lambda^{\text{hard} \dagger} \mathcal{M}_\lambda^{\text{hard}} = F(GM^2 \log \mathcal{E}/\lambda) \quad \text{Analogous in other regularizations} \end{array} \right.$$



T -channel pole & Unitarity

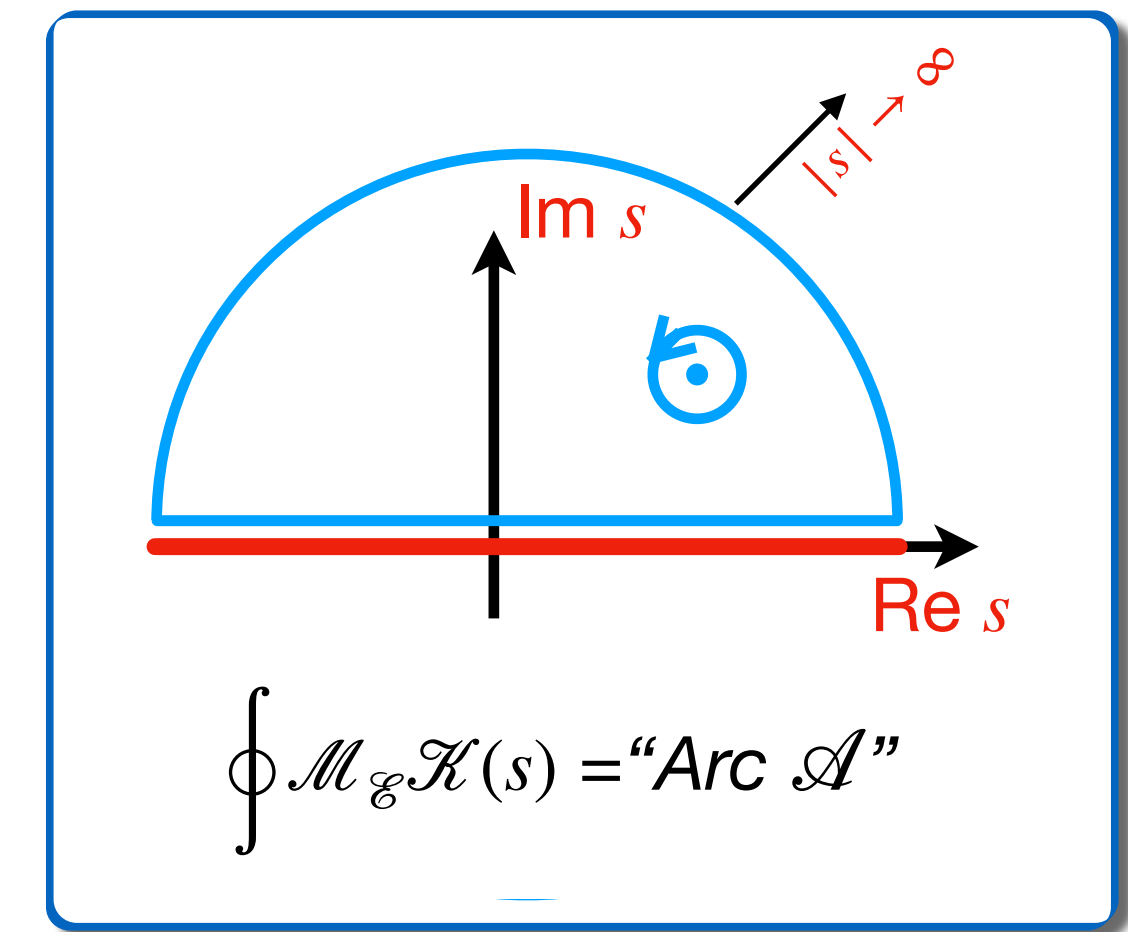
1. IR finite
2. Lorentz invariant
3. Analytic
4. Average Regge
5. Unitary
6. Inclusive, $\mathcal{E} \sim \Delta E$

Exactly

$$g_{EFT} = \langle (\dots) \rangle = \sum_{\ell} \int_{M^2}^{\infty} \frac{ds}{s^3} (\dots) \text{Im} \mathcal{M}_{\mathcal{E}}^{\ell}(s)$$

Match
both sides in a limit

$$\int_{\mathcal{E}}^M dq \psi(q) \overline{\text{Im}} \mathcal{M}_{\mathcal{E}}(s, -q^2) \rightarrow \int_{\mathcal{E}}^M dq \psi(q) \mathcal{M}_{\mathcal{E}}^{\text{hard} \dagger} \mathcal{M}_{\mathcal{E}}^{\text{hard}}$$



Unitary limit, examples:

(a) Leading-Log (before smearing)

$$\alpha \rightarrow 0, \quad \alpha \log \mathcal{E} = \alpha_{\mathcal{E}} \text{ - fixed}$$

$$G \rightarrow 0, \quad G \log \mathcal{E} = G_{\mathcal{E}} \text{ - fixed}$$

(b) Tree-level

α, G — finite



(c) Leading T-channel sing.

α, G — finite

$$\mathcal{E} \ll \sqrt{-t} \ll E$$



...

$$Gs^2/t \rightarrow Gs^2 \log M/\mathcal{E}$$

T -channel pole & Unitarity

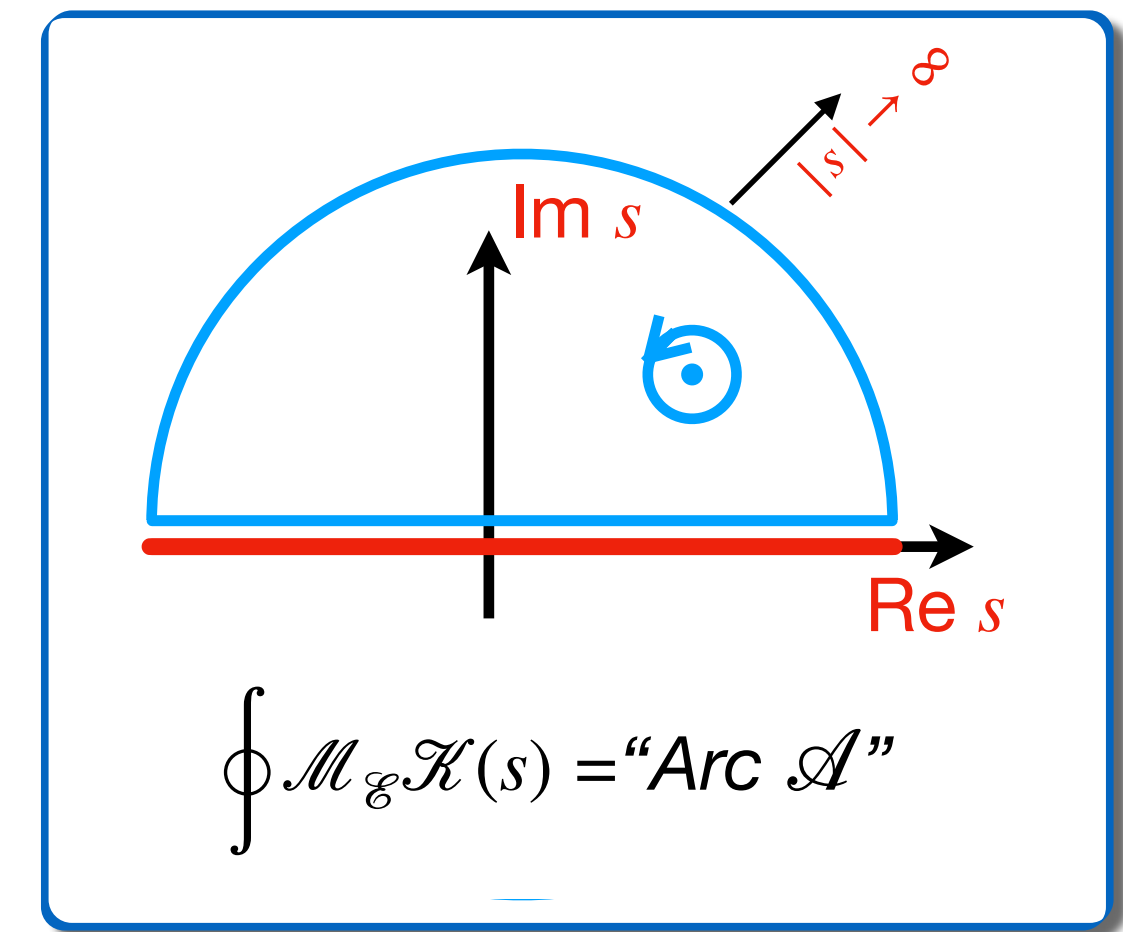
1. IR finite
2. Lorentz invariant
3. Analytic
4. Average Regge
5. Unitary
6. Inclusive, $\mathcal{E} \sim \Delta E$

Exactly

$$g_{EFT} = \langle (\dots) \rangle = \sum_{\ell} \int_{M^2}^{\infty} \frac{ds}{s^3} (\dots) \text{Im} \mathcal{M}_{\mathcal{E}}^{\ell}(s)$$

Match
both sides in a limit

$$\int_{\mathcal{E}}^M dq \psi(q) \overline{\text{Im}} \mathcal{M}_{\mathcal{E}}(s, -q^2) \rightarrow \int_{\mathcal{E}}^M dq \psi(q) \mathcal{M}_{\mathcal{E}}^{\text{hard} \dagger} \mathcal{M}_{\mathcal{E}}^{\text{hard}}$$



Unitary limit, examples:

(a) Leading-Log (before smearing)

$$\alpha \rightarrow 0, \quad \alpha \log \mathcal{E} = \alpha_{\mathcal{E}} \text{ - fixed}$$

$$G \rightarrow 0, \quad G \log \mathcal{E} = G_{\mathcal{E}} \text{ - fixed}$$

(b) Tree-level

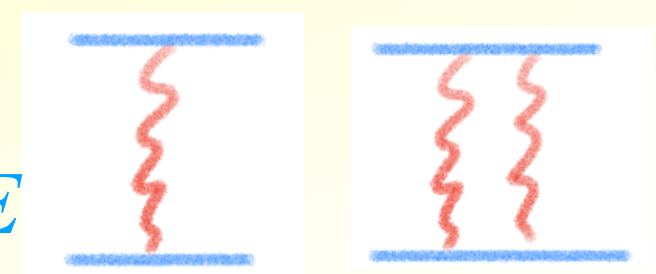
α, G — finite



(c) Leading T-channel sing.

α, G — finite

$$\mathcal{E} \ll \sqrt{-t} \ll E$$



...

$$Gs^2/t \rightarrow Gs^2 \log M/\mathcal{E}$$

t-pole matches