ENS Summer Institute

Interfaces in Symmetric Orbifolds and Holography

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Based on work with

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HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

CLUSTER OF EXCELLENCE QUANTUM UNIVERSE

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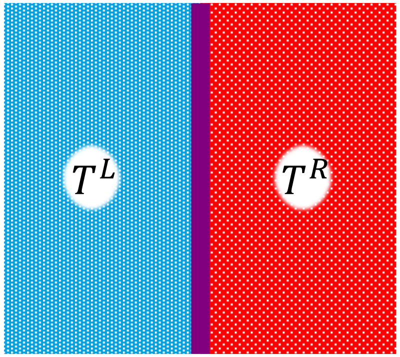
DESY.

Introduction: Conformal Interfaces

Exploring the space of CFTs

Conformal interfaces provide a powerful probe to explore CFTs & relations between them

Space of CFTs



Transmissivity $$\mathcal{T} := \frac{\langle T^L T^R \rangle}{\sqrt{\langle T^L T^L \rangle \langle T^R T^R \rangle}}$$ [Quella,Runkel, Watts]

Topological interfaces have $\mathcal{T}=1$; e.g. duality defects, non-invertible symmetries

[Petkova, Zuber][Fuchs, Froehlich, Runkel, Schweigert]…

<u>Permeable interfaces</u> $0 < \mathcal{T} < 1$ can exist between CFTs with same central charge

e.g. radius changing interface [Bachas et al.][QRW]…[Bachas,Brunner,Douglas,Rastelli]

and between CFTs with different central charge (= number of degrees of freedom)

First construction in [Quella,Schomerus 2002] e.g. RG interfaces of [Gaiotto]

Introduction: Gauge Theory Interfaces

From Nahm-poles to Janus – a well explored map

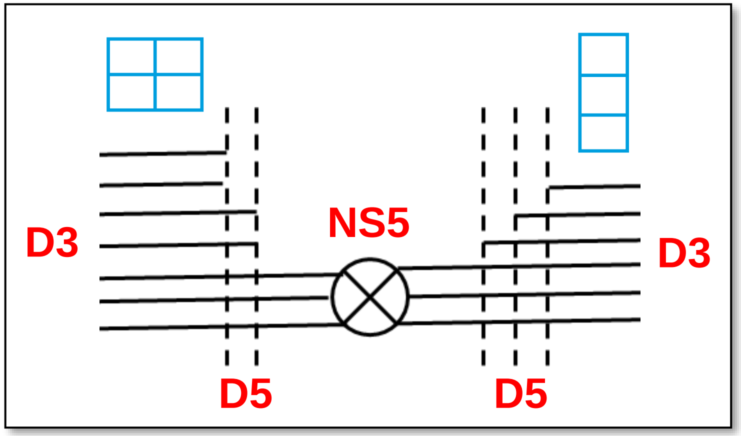
Rank-changing 1/2-BPS $SU(N_-)|SU(N_+)$ interfaces of N=4 SYM in [Gaiotto,Witten]

rank jump is encoded by Nahm-pole data

$$\frac{d\phi_i}{dy} = \epsilon_{ijk}[\phi_j, \phi_k] \qquad \phi_i \sim \frac{t_i}{y}$$

t_i **form** $(N_\pm - p)$ **−dim rep of** $su(2)$

Young diagram



Holographic description through Karch-Randall branes localized along $AdS_4 \times S^2$ with Nahm-pole data describing fluxes supporting S^2.

Green-Schwarz superstring with Karch-Randall $AdS_4 \times S^2$ boundary conditions was shown to be <u>classically integrable</u> [Dekel,Oz]

<u>Quantum integrability</u> of Nahm-pole interfaces in planar N = 4 SYM theory was also explored at weak coupling by [Kristjansen,Zarembo et. al.] **mostly 1-pt fcts**

Introduction: Symmetric Product Orbifolds

The 2D cousins of 4D Gauge Theories

SPOs $Sym^N(\mathcal{X}) = \mathcal{X}^N/S_N$ are permutation orbifold CFTs $\mathcal{M}/S_N$,

[Klemm,Schmidt]

- **Correlation functions possess a diagrammatic expansion**

  [Pakman,Rastelli, Razamat] based on [Arutyunov,][Lunin,Mathur]

- **Can possess a holographic/string theoretic description**

  [Dijkgraaf,Moore,Verlinde^2] ….[Eberhardt,Gaberdiel,Gopakumar]

I. Review of SPOs

III. SPO holography

Questions: **[Motivated by comparison with N=4 SYM theory]**

- **Are there analogues of Nahm-pole interfaces in SPOs ?**
- **If so, do they possess string theoretic dual (w. branes) ?**
- **And if so, is the dual open string theory also integrable ?**

II. SPO Interfaces

III. SPO holography

IV. Outlook & Conclusions

How does interface in CFT2 become boundary of string worldsheet ?

Symmetric Product Orbifolds

Introduction: Symmetric Product Orbifolds

A family of permutation orbifolds

To any ``seed CFT“ $\mathcal{X}$ one assigns a family of permutation orbifold CFTs through

$$Sym^N(\mathcal{X}) = \mathcal{M}/S_N \qquad \mathcal{M} = \mathcal{X}^N \qquad c_N\left(Sym^N(\mathcal{X})\right) = N\, c_{\mathcal{X}}$$

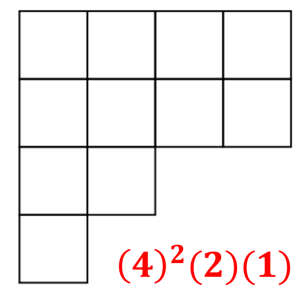
Partition fct:

$$Z_N(t,\bar{t}) = \frac{1}{|S_N|}\sum_{g\in S_N}\sum_{h\in C_g} h \underset{g}{\square} = \frac{1}{N!}\sum_{g\in S_N}\sum_{h\in C_g} tr_{\mathcal{H}_g}\left(h\,\eta^{L_0-\frac{c_N}{24}}\,\bar{\eta}^{\overline{L_0}-\frac{c_N}{24}}\right)$$

Conjugacy classes: $[g] = \{kgk^{-1}\}_{k\in S_N} = (1)^{m_1}(2)^{m_2}\cdots(N)^{m_N}$

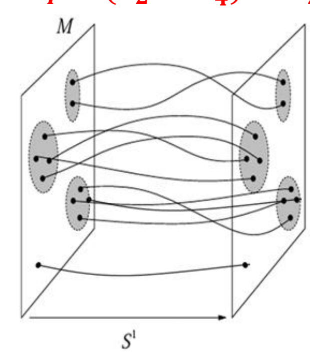
$$\sum_w w\, m_w = N$$

Centralizer $C_g \cong C_{[g]} \cong \{h|hg = gh\} \cong \prod_w S_{m_w} \ltimes \mathbb{Z}_w$



$(4)^2(2)(1)$

$C_Y = (S_2 \ltimes \mathbb{Z}_4^2) \times \mathbb{Z}_2$



Twist fields $\sigma^{(w)}$ associated to single cycle permutations of of length $w \in \mathbb{N}$ have weight

$$h_w = \frac{c_{\mathcal{X}}}{24}\left(w - \frac{1}{w}\right) = \bar{h}_w$$

DMVV Grand Canonical Ensemble

A free Bose gas of long strings

Dijkgraaf, Moore, Verlinde, Verlinde gave an elegant expression for partition function of ``grand canonical ensemble''

Hecke operator **PF of seed $\mathcal{X}$**

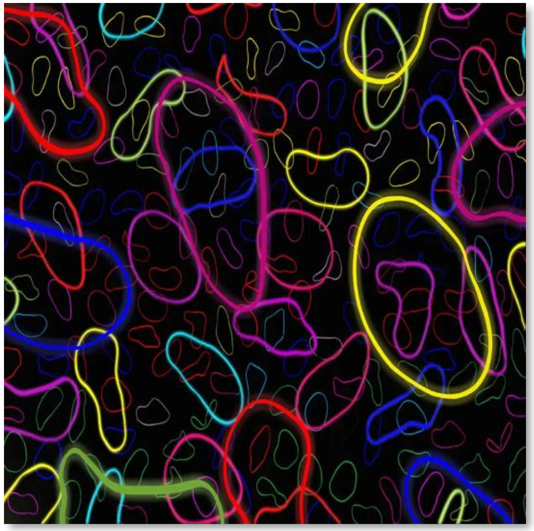
$$\boxed{\mathcal{Z}(\kappa;t,\bar{t}) := \sum_{N=0}^{\infty} \kappa^N Z_N(t,\bar{t}) = \exp\left[\sum_{k=1}^{\infty} \kappa^k T_k Z(t,\bar{t})\right]}$$

where

$$T_k Z_{\mathcal{X}}(t,\bar{t}) = \frac{1}{k}\sum_{w|k}\ \sum_{j=0}^{w-1} Z\left(\frac{kt}{w^2}+\frac{j}{w}, \frac{k\bar{t}}{w^2}+\frac{j}{w}\right) = Z^{conn}_{\mathcal{X}/\mathbb{Z}_k}(t,\bar{t})$$

contains a subset of connected contributions to the partition function of the orbifold theory $\mathcal{X}/\mathbb{Z}_k$.

DMVV function describes non-interacting Bose gas of long strings of any winding number k.



color = $(k, \psi \in \mathcal{H}_{\mathcal{X}})$

Correlators of Symmetric Orbifold

From correlators of the seed theory

Correlators of SPO on surface X computable from correlators of seed theory on covering surfaces Σ

$$\left\langle \sigma^{(w_1)}(x_1) \cdots \sigma^{(w_n)}(x_n) \right\rangle_X \sim \sum_{\Gamma} \alpha(\Gamma)\; Z_X(\Sigma)$$

Sum over ramified N-sheeted coverings $\Gamma : \Sigma \to X$ with ramifications points z_μ of order w_μ, i.e.

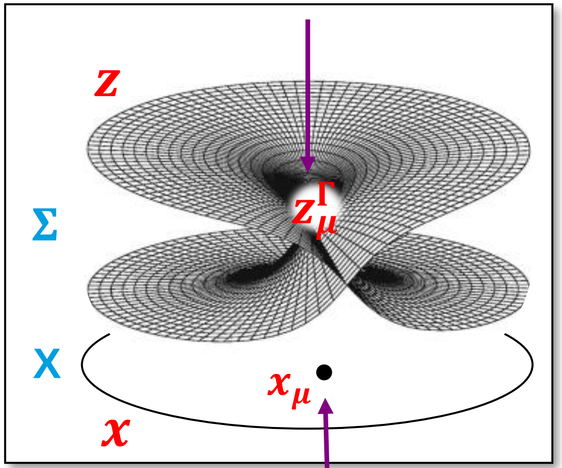
$$\Gamma(z) = x_\mu + a_\mu^\Gamma \left(z - z_\mu^\Gamma\right)^{w_\mu} + \cdots$$

Note: Given branch points x_μ, branching orders w_μ there are only finitely many R-sheeted coverings Γ

Genus g_Σ of Σ given by Riemann-Hurwitz formula

$$g_\Sigma = N \cdot (g_X - 1) + 1 + \frac{1}{2} \sum_\mu (w_\mu - 1)$$

ramification point



branch point

[Dixon et al.]

[Arutyunov,Frolov]

[Lunin. Mathur]

… [Dei, Eberhardt]

[Hikida, Liu]

A Feynman-like Diagrammatic Expansion

Pakman-Rastelli-Razamat diagrammatics

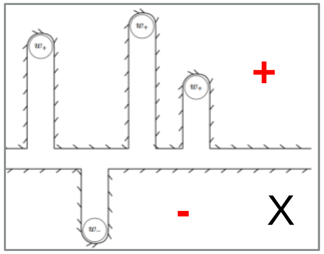
Covering maps Γ may be encoded in Feynman-like diagrams [Pakman,Rastelli,Razamat]

Build diagrams from the propagators and vertices shown on the left using rules:

(1) The number + and – loops must coincide (=N)

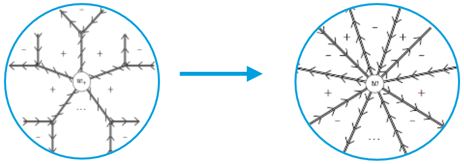
(2) in each loop: twist fields appear at most once.

(3) In each loop: order of twist fields is preserved.

Derivation: Lift 

from X to Σ

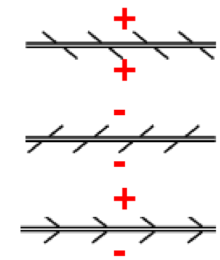
Equivalence with PRR diagrams: zippers can be removed



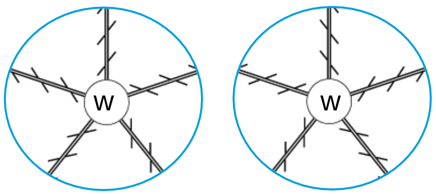
1 propagator

1 vertex

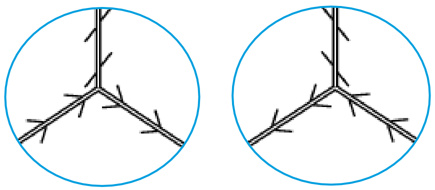
3 propagators



4 vertices



twist fld



zipper

Interfaces in SPOs

Permeable Interfaces in SPOs

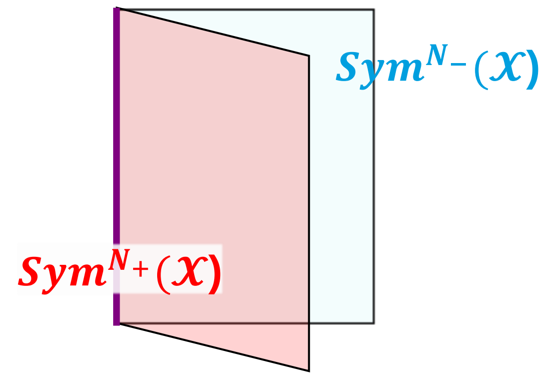
Hikida, Harris, Tsuda, Schomerus

Boundaries in SPOs [Belin,Biswas, Sully]

Topological dfcts in SPOs [Gutperle et al.]

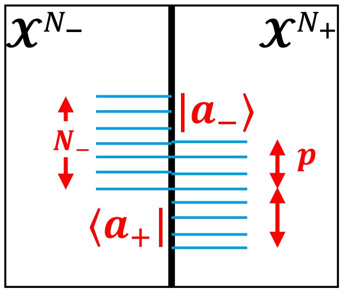
Use folding trick & construct boundary state in

$$Sym^{N_-}(\mathcal{X}) \times Sym^{N_+}(\mathcal{X}) = \mathcal{X}^{N_-+N_+} / S_{N_-} \times S_{N_+}$$



Choose a boundary state of numerator theory

apply orbifold projection; resolve fixed points.

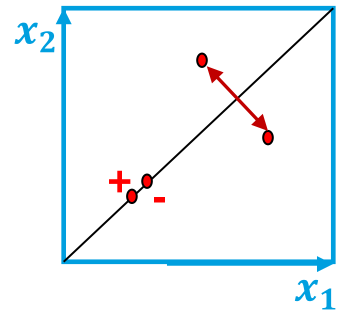


$|a_\pm\rangle_{\mathcal{X}}$ boundary states in seed $\mathcal{X}$

Orbifold projection with $S_{N_-} x\, S_{N_+}$

fixed by $S_{N_\pm,p} \coloneqq S_{N_--p}\; x\; S_p\; x\; S_{N_+-p}$

pick character χ of $S_{N_\pm,p}$ to resolve



Interface states $\left|\mathcal{I}^{p,\chi}_{a_\pm}\right\rangle_{N_\pm}$ **define permeable interface between** $Sym^{N_-}(\mathcal{X})$ **&** $Sym^{N_+}(\mathcal{X})$

transmissivity $\mathcal{T} = \dfrac{2p}{N_+ + N_-}$

[Harris,Hikida,VS,Tsuda]

Interface Grand Canonical Ensemble

A free Bose gas of long open strings

The generating function $\mathcal{Z}_{a_\pm^{L,R}}(\kappa_\pm, \rho_{L,R}; t)$ for overlaps

$$\langle \mathcal{J}^{p^L}_{a^L_\pm} | \eta^{L_0 - \frac{c_N}{24}} \eta^{L_0 - \frac{c_N}{24}} | \mathcal{J}^{p^R}_{a^R_\pm} \rangle$$

fugacities
for $N_\pm$ and $p^{L,R}$

between interfaces exponentiates, at least for $\chi = 1$.

$$\mathcal{Z}_{a_\pm^{L,R}}(\kappa, t) = e^{\mathcal{Z}_C(\kappa,t) + \mathcal{Z}_O(\kappa,t)}$$

where

$$\mathcal{Z}_C(\kappa, t) = \sum_{k=1}^{\infty} \kappa^{2k} H_k Z_\chi(t)$$

as in DMVV
but with $t = \bar{t}$

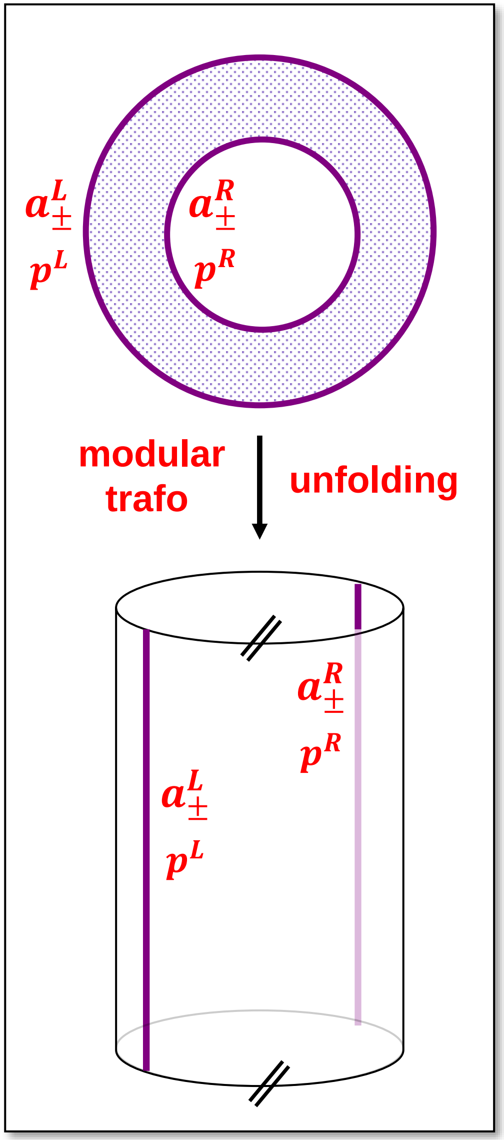
$$\boxed{Z_O(\kappa, t) = \sum_{A,B=L,R} \sum_{k=1}^{\infty} \kappa^{2k-1+\delta_A^B} S_{2k-1-\delta_A^B} Z_O^{AB}(t)}$$

$$S_\ell Z(t) = \sum_{w|\ell} \frac{1}{w} Z\left(\frac{\ell t}{w^2}\right)$$

$$Z_O^{LL} = Z_{a_+^L a_-^L} \quad Z_O^{RR} = Z_{a_-^R a_+^R}$$

$$Z_O^{RL} = Z_{a_+^R a_+^L} \quad Z_O^{LR} = Z_{a_-^L a_-^R}$$

There are four types of long open strings: LL,RR,LR,RL



Interface correlators

From boundary correlators of the seed theory

Place interface $\mathcal{I}^{p}_{a_\pm}$ along curve $\gamma \subset X; X \setminus \gamma = X_- \cup X_+$

$$\left\langle \ldots \sigma^{(w_\mu)}(x_\mu) \cdots \sigma^{(A_j B_j;\, w_j)}(y_j) \ldots \right\rangle_X \sim \sum_\Gamma \alpha(\Gamma)\; Z_X^{a_\pm}(\Sigma)$$

bulk twist fld **interface twist fld** $\partial\Sigma \neq \emptyset$

Sum over ramified ``interface coverings" $\Gamma: \Sigma \rightarrow X$

$\Gamma: \Sigma_\pm \rightarrow X_\pm$ are $N_\pm$ -sheeted ramified coverings of $X_\pm$

Degree p of transmissivity at y_* is $p = |\Gamma^{-1}(y_*) \cap \Sigma^o|$

$\partial\Sigma$ has at most $(N_- + N_+ - 2p)$ components

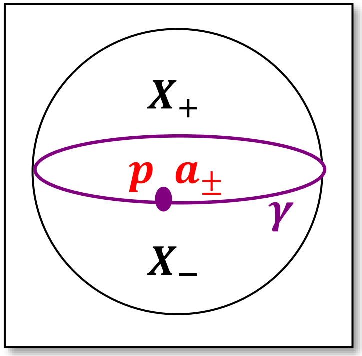
Ramification points z_i of order w_j on boundary of Σ

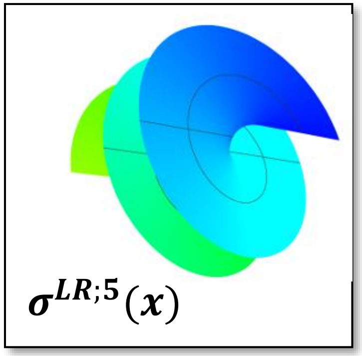
$\sigma^{LL;w}$ and $\sigma^{RR;w}$ shift $p \rightarrow p \pm 1$; induce jump $a_+ \leftrightarrow a_-$

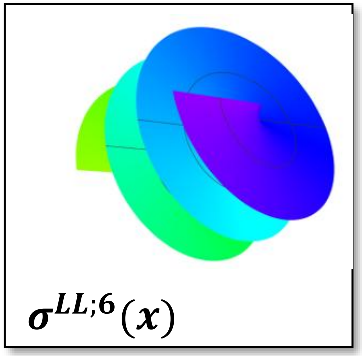
Euler character $\chi(\Sigma)$ from interface Riemann-Hurwitz

$$\chi(\Sigma) = N_-\chi(X_-) + N_+\chi(X_+) - \sum_\mu (w_\mu - 1) - \frac{1}{2}\sum_{A,B,j}(w_j^{AB} - 1)$$

$$\chi(\Sigma) = 2 - 2g_\Sigma + b_\Sigma \qquad \chi(X_\pm) = 1 - 2g_X$$







Diagrams for Interface Correlators

Boundary PRR diagrams [Harris,VS,Tsuda]

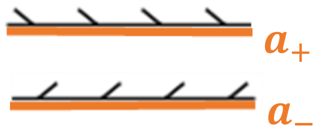
Build diagrams from the propagators and vertices shown on the left (and previous) using the rules

(1) The number + and – loops must is given by $N_\pm$

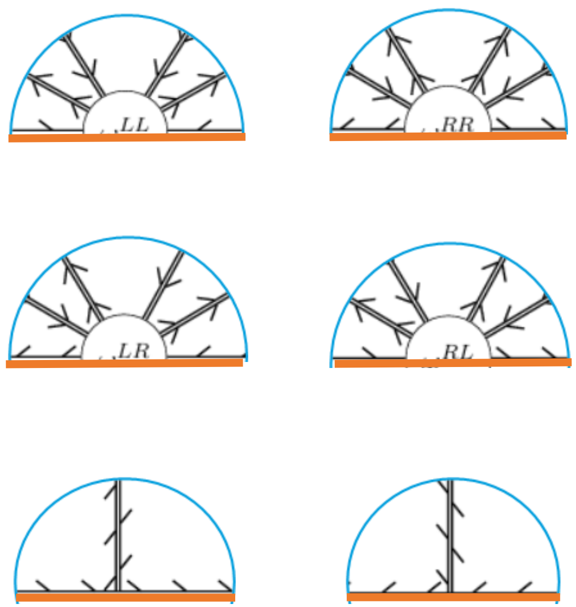
(2) in each loop: twist fields appear at most once.

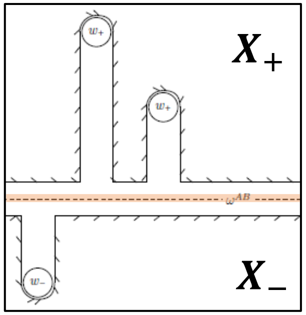
(3) In each loop: order of twist fields is preserved.

2 additional propagators



6 additional vertices



Derivation: Lift  **from X to Σ**

One can also read off the index p of transmissivity & the symmetry factors $\alpha(\Gamma)$ from these diagrams

p depends on position on interface

Holography

The Holographic Dual of SPOs

$Sym^N(\mathbb{T}^{4[1]})$ and tensionless strings

<u>Key example</u>: For $\mathcal{X}$ = $\mathbb{T}^{4[1]}$ symmetric product orbifold CFT $Sym^N(\mathbb{T}^{4[1]})$ is dual to IIB superstring theory in $AdS_3^{(k=1)} \times S^3 \times \mathbb{T}^4$ **[Eberhardt, Gaberdiel, Gopakumar]**

k = 1 unit of NSNS flux

Partition function of string theory was computed from exact worldsheet model & shown to coincide with logarithm of DMVV partition functions for $Sym^N(\mathbb{T}^{4[1]})$.

single-string states **[Eberhardt]**

Correlation functions of twist operators in $Sym^N(\mathbb{T}^{4[1]})$ on the sphere where also matched to tree-level string amplitudes of type IIB theory on $AdS_3^{(k=1)} \times S^3 \times \mathbb{T}^4$

[Eberhardt, Gaberdiel,Gopakumar] …. [Eberhardt,Dei]

Matching to all loop order was shown in [Hikida,VS]

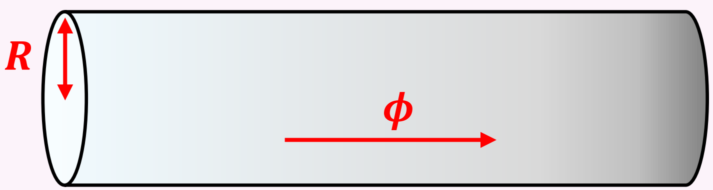
Now that there is a proof do we understand the mechanism ?

The Liouville – AdS3 Correspondence

Dual descriptions of sigma models

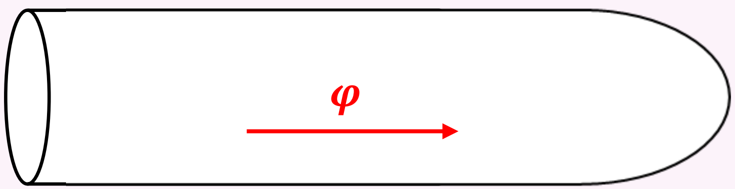
FZZ duality between sine-Liouville theory and Euclidean Black Hole sigma model

$$V(\phi, \widetilde{X}) \sim e^{\frac{1}{b}\phi} \cos(R\widetilde{X})$$



$$Q_\phi = b = \left(R^2 - 2\right)^{-1/2}$$

$$ds^2 \sim \frac{e^{-2\varphi}}{1 + e^{-2\varphi}} \left(d\varphi^2 + R^{-2} dX\right)$$



$$e^{-2\Phi} = e^{-2\Phi_0}\left(1 + e^{-2\varphi}\right)$$

Conjectured by [Fateev,Zamolodchikov^2] proven on arbitrary surfaces by [Hikida, VS]

Note: Radial coordinate φ of sigma model energes from Liouville direction ϕ by non-constant shift of zero mode and rotation in field space. **``bulk reconstruction''**

Proof based on closely related Liouville – eAdS3 correspondence [Ribault, Teschner], and its path integral derivation using Kac-Wakimoto realization of AdS3 in [Hikida,VS]

But where is the Liouville field in $Sym^N(\mathbb{T}^{4[1]})$?

Engineering Holographic Duals

Unraveling the Liouville field

There is an interesting extension of the $Sym^N(\mathbb{T}^{4[1]})$ duality NSNS flux $k \neq 1$

$$Sym^N\left(\mathbb{R}^{[1]}_{Q_b} \times \mathcal{X}\right)_{int} \xleftrightarrow{b^{-2}=k} \widetilde{AdS}{}_3^{(k)} \times \mathcal{X}$$

[Eberhardt]
[Sriprachyakul]

Interacting SPO, $Q = b - {}^1/_b$ *Critical IIB superstring*

Perturbative expansion of SPO correlators can be rewritten as string theory amplitudes without computing a single correlation function! [Hikida,VS]

all orders in $1/N$

Special case $\mathcal{X} = SU(2)^{[1]}_{k-2} \times \mathbb{T}^{4[1]}$ the dual is superstring in $AdS_3 \times S^3 \times \mathbb{T}^4$

→ seed contains *N = 4* supersymmetric linear dilaton CFT $\mathbb{R}^{[1]}_Q \times SU(2)^{[1]}_{k-2}$

$$c\left(\mathbb{R}^{[1]}_Q \times SU(2)^{[1]}_{k-2}\right) = 3 + 6\left(\mathrm{b} - \frac{1}{\mathrm{b}}\right)^2 + \frac{3(\mathrm{k}-2)}{\mathrm{k}} = 6\,(\mathrm{k}-1) = 0 \quad \text{for k = 1}$$

N=4 susy linear dilaton decouples and we are left with dual of $Sym^N(\mathbb{T}^{4[1]})$

The Holographic Dual of SPO Interfaces

AdS2 branes in tensionless string theory

Maximally symmetric D-branes geometries in SL(2,ℝ) were identified in [Bachas,Petropoulos] H_2, AdS_2, dS_2

Dual of spherical branes eH_2 in Euclidean AdS3 was identified in [Gaberdiel,Knighton,Vosmera]

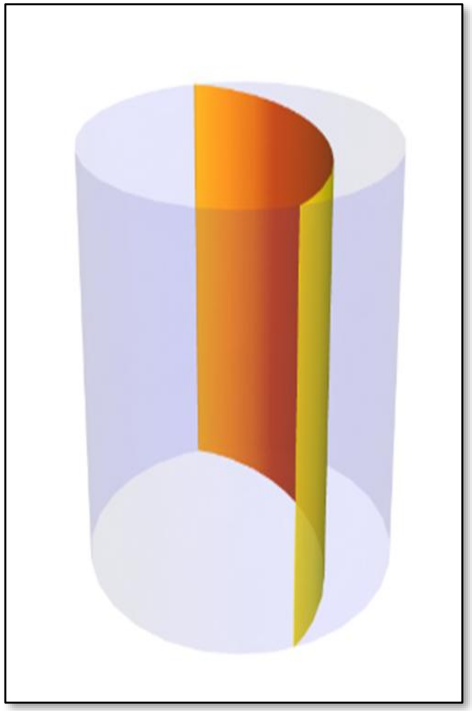
Boundary state in $Sym(\mathbb{T}^4)$

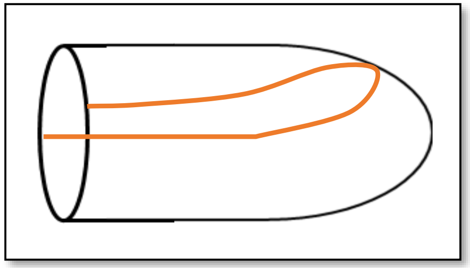
Worldsheet theory for AdS2 branes in AdS3 was constructed in [Lee,Ooguri,Park][Ponsot,VS,Teschner]

See also [Gaberdiel et al.] for tensionless case

Partition function of tensionless open string theory was computed and shown to coincide with logarithm of interface PF for $Sym(\mathbb{T}^{4[1]})$. [Harris et al.]

Proof of FZZ duality was extended to boundary theory w. D1 branes in Euclidean BH [Creutzig,Hikida,Rønne]





Outlook and Conclusion

Exploration of permeable interfaces in 2D CFT

Here we only explored only some observables for the simplest permeable interfaces in symmetric product orbifolds (↔ AdS2 branes in tensionless $AdS_3 \times S^3 \times \mathbb{T}^4$)

Interfaces with $\chi \neq 1$? Non-invertible transmissive factor ? Marginal deformations ?

Observables measuring various entropies ? **Boundary see [Belin,Biswas,Sully]**

[Karch,Ooguri et al.]

It would also be interesting to study the role of SPO interfaces in holographic pairs

Topological version ? Tensionless $AdS_3 \times S^3 \times S^3 \times S^1$? With less supersymmetry ?

Troost, Benizri **[Eberhardt,Gaberdiel]** **[Belin,Castro, Keller et. al.]**

Turn on RR-flux → geometric regime: worldsheet sigma model for $AdS_3 \times S^3 \times \mathbb{T}$ with mixed fluxes is classically integrable [Cagnazzo,Zarembo]. Open version with AdS_2-brane boundary conditions remains integrable [Bonomi,VS;WIP] **see also [Gil,Driezen]**

filling gap in [Dekel,Oz]

Quantum integrability of deformed interface CFTs ? Bootstrability ? ….

Backup

Perturbative Holography

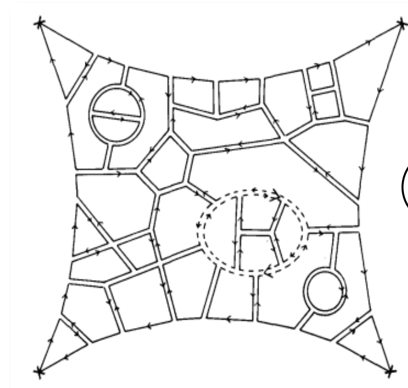
Ofer Aharony

2. What is the classical string theory dual to the large N limit of (3 + 1 dimensional) QCD?

Hint: For asymptotically free gauge theories, a good starting point could be understanding the string dual of the free gauge theory.

The Future of String Theory: 100 Open Questions
Strings 2024, CERN June 3-7

Particles/Fields **Strings**



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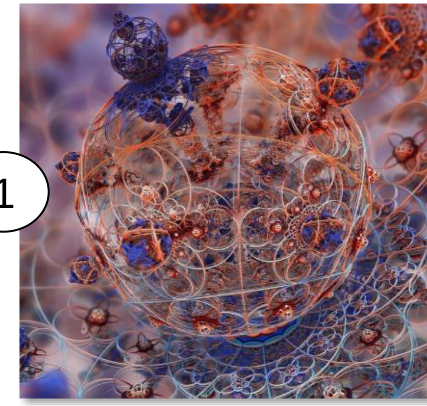

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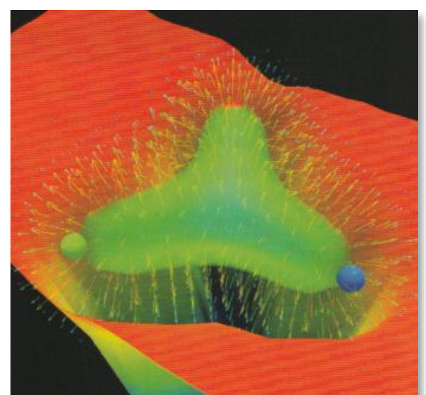
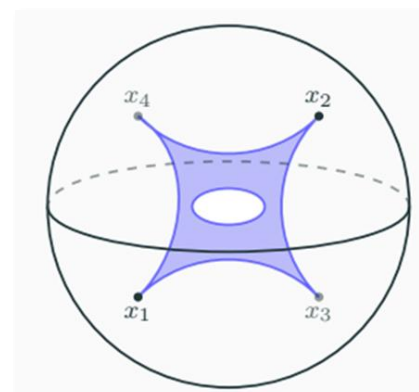




Gauge-Gravity Duality

=

Perturbative Holography 1

+

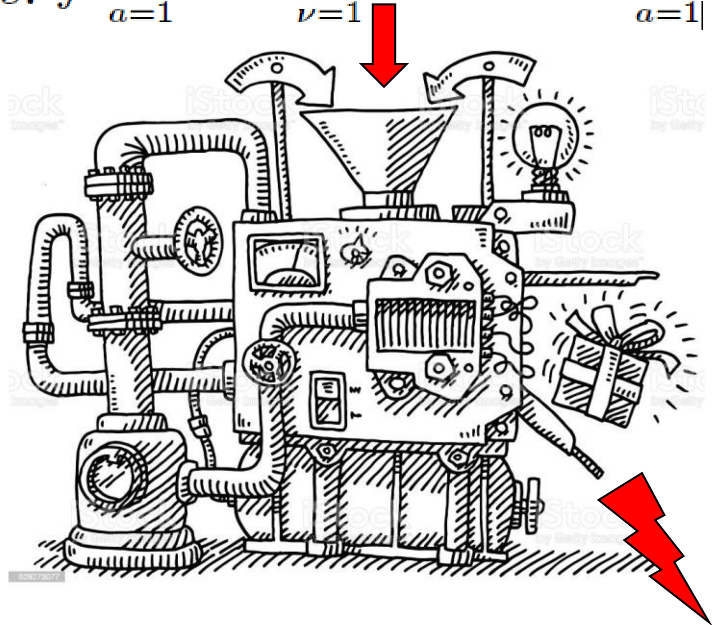
Emergent Geometry 2

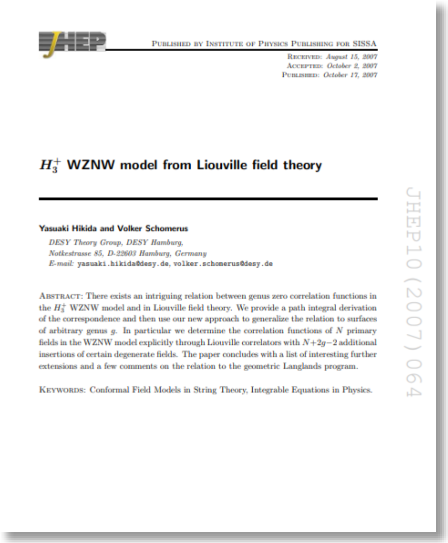
From Orbifold to SL(2)$_k$ KW Theory

An exact auxiliary formula

$$\frac{\rho^s}{s!}\int \prod_{a=1}^{s} d^2y_a \langle \prod_{\nu=1}^{n} V_{\alpha_\nu}^{w_\nu}(\psi_\nu;x_\nu) \prod_{a=1}^{s} V_{1/2b}^{2}(y_a)\rangle^{S^2}$$

s even





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H_3^+ WZNW model from Liouville field theory

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Abstract: There exists an intriguing relation between genus zero correlation functions in the H_3^+ WZNW model and in Liouville field theory. We provide a path integral derivation of the correspondence and then use our new approach to generalize the relation to surfaces of arbitrary genus g. In particular we determine the correlation functions of N primary fields in the WZNW model explicitly through Liouville correlators with $N+2g-2$ additional insertions of certain degenerate fields. The paper concludes with a list of interesting further extensions and a few comments on the relation to the geometric Langlands program.

Keywords: Conformal Field Models in String Theory, Integrable Equations in Physics.

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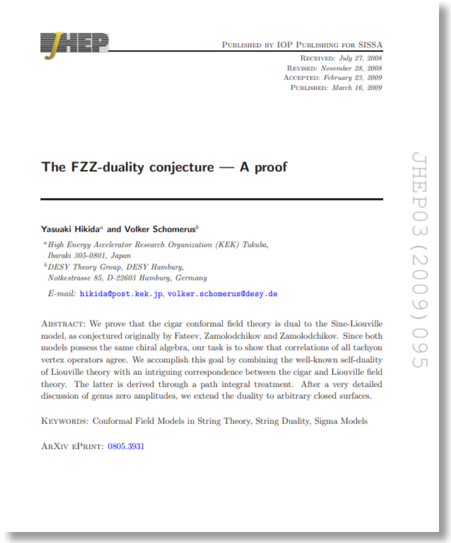

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The FZZ-duality conjecture — A proof

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Abstract: We prove that the cigar conformal field theory is dual to the Sine-Liouville model, as conjectured originally by Fateev, Zamolodchikov and Zamolodchikov. Since both models possess the same chiral algebra, our task is to show that correlations of all tachyon vertex operators agree. We accomplish this goal by combining the well-known self-duality of Liouville theory with an intriguing correspondence between the cigar and Liouville field theory. The latter is derived through a path integral treatment. After a very detailed discussion of genus zero amplitudes, we extend the duality to arbitrary closed surfaces.

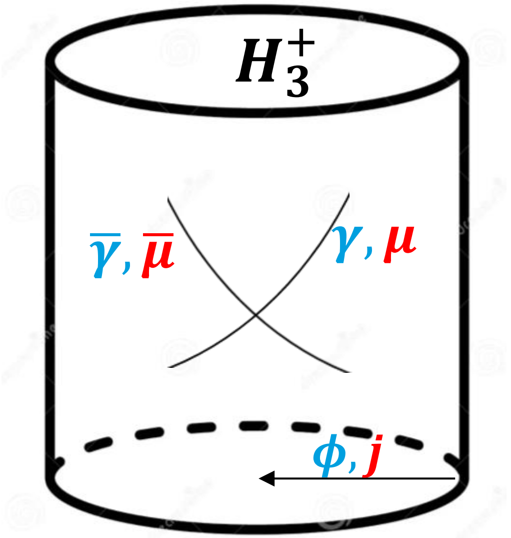
Keywords: Conformal Field Models in String Theory, String Duality, Sigma Models

ArXiv ePrint: 0805.3931

JHEP03(2009)095

$$\sim \rho^s \int \prod_{a=1}^{s} d^2u_a \quad \langle \prod_{\nu=1}^{n} \Phi^{j_\nu(w_\nu)}(\psi_\nu;z_\nu) \prod_{a=1}^{s} \Phi_{-k,-k}^{1/b^2(2)}(u_a) \prod_{j=1}^{R} \Phi_{k/2,k/2}^{1/2b^2(-1)}(v_j)\rangle^{\Sigma_g}$$

How can the machine work ?

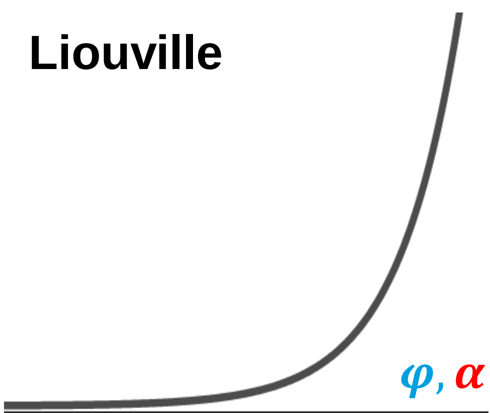


$$S(\phi,\beta,\gamma) \sim \int d^2z\,(\beta\,\bar{\partial}\,\gamma + \bar{\beta}\,\partial\,\gamma$$

$$-\,\partial\phi\,\bar{\partial}\phi + b^2\beta\bar{\beta}e^{2b\Phi} - \frac{Q_\phi}{4}\sqrt{g}\,R\,\phi)$$

$$\Phi_j(\mu|z) = |\mu|^{2j}e^{\mu\gamma(z)-\bar{\mu}\bar{\gamma}(\bar{z})}e^{2bj\phi(z,\bar{z})}$$

Liouville

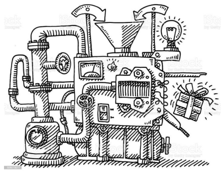


$$S(\phi,\beta,\gamma) \sim \int d^2z\,(\partial\varphi\,\bar{\partial}\varphi + \rho\,e^{2b\varphi} - \frac{Q_\varphi}{4}\sqrt{g}\,R\,\varphi)$$

$$V_\alpha(z) = e^{2\alpha\varphi(z,\bar{z})}$$

s. [Ribault,Teschner] …..

Explaining the Machine [Hikida,VS]



A lightening review

Represent vertex operators as $$\Phi^j_{m,\bar m}(z) = \int \frac{d\mu^2}{|\mu|^2}\mu^m \bar\mu^{\bar m}\Phi_j(\mu|z)$$

$$\Phi_j(\mu|z) = |\mu|^{2j} e^{\mu\gamma(z)-\bar\mu\bar\gamma(\bar z)} e^{2bj\phi(z,\bar z)}$$

$$S(\phi,\beta,\gamma) \sim \int d^2z\,(\partial\phi\,\bar\partial\phi + \frac{Q_\phi}{4}\sqrt{g}\,R\,\phi\; - b^2\beta\bar\beta e^{2b\Phi} - \beta\,\bar\partial\,\gamma - \bar\beta\,\partial\,\gamma)$$

1. Path integral over γ gives contraint for β

$$\bar\partial\,\beta(z) = \sum_\nu \mu_\nu\,\delta(z-z_\nu)$$

$$\beta(z) = \sum_\nu \frac{\mu_\nu}{z-z_\nu} \sim \frac{\prod(z-u_i)}{\prod(z-z_\nu)}$$

$\mu_\nu \to u_i$ resembles SOV

2. Shift linear dilaton ϕ $\quad \varphi :\sim \phi - \frac{1}{2b}\log\beta\bar\beta$

$$Q_\phi \to Q_\varphi = Q_\phi - \frac{1}{b}$$

$$\partial\bar\partial\,\varphi = \partial\,\bar\partial\,\phi\; - \frac{\pi}{b}\left(\sum_i \delta(z-u_i) - \sum_\nu \delta(z-z_\nu)\right)$$

Vertex operators shifted

Extension to $w \neq 0$ in [Hikida,VS;0805.3931]

The String Theory Dual

Worldsheet theory and duality statement

We were able to rewrite the perturbative expansion of $\mathrm{Sym}_\rho^N(\mathbb{R}_Q \times X)$ in terms of bosonic string theory **without computing a single correlation function!**

$$S_\rho(\phi,\beta,\gamma) \sim S_0(\phi,\beta,\gamma) + \int d^2z\, \Phi^-(z,\bar{z}) + \mu \int d^2u\, \Phi^+(u,\bar{u})$$

c=26 free CFT

with affine sl(2)$_k$ symmetry

branch cut creating screening charge $\leftrightarrow$ deformation $\rho \sim \mu^2$

Sheet creating screening charge

[Giveon, Itzhaki]

$$\boxed{\langle \prod_{\nu=1}^{n} V_{\alpha_\nu}^{w_\nu}(\psi_\nu; x_\nu) \rangle_{\rho}^{S^2} = \int d\tau \langle \prod_{\nu=1}^{n} \int d^2 z_\nu \Phi^{j_\nu,(w_\nu)}(\psi_\nu, x_\nu; z_\nu) \rangle_\mu^{\Sigma_\tau}}$$