Multiparty entanglement and holography

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Overview

Entanglement is central to the emergence of spacetime in holography.

Is multiparty entanglement important?

Outline:

- Review entanglement, define multiparty entanglement.
- New signals: reflected entropy, multi-entropy
- Multiparty entanglement in holography.
- Holographic inequalities.
- Time dependence.

Entanglement

Consider pure states on two parties:

$|\psi\rangle_{AB} \in \mathcal{H}_A \otimes \mathcal{H}_B$ is entangled if not seperable: $|\psi\rangle_{AB} \neq |\phi\rangle_A \times |\chi\rangle_B$.

LU-invariant information in reduced density matrix $\rho_A = \mathrm{tr}_B(|\psi\rangle\langle\psi|)$ - diagonalizable.

von Neumann entropy $S_A = -\mathrm{tr}_A \rho_A \ln \rho_A$ is an entanglement measure:

- $S_A = 0$ iff $|\psi\rangle_{AB}$ is not entangled.
- $0 \leq S_A \leq \ln \min(\dim \mathcal{H}_A, \dim \mathcal{H}_B)$.
- Monotonic under LOCC.
- Example: $|\psi_{AB}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$. $S_A = \ln 2$.

In holography, given by RT formula,

$$S_A = \min_\gamma \frac{A_\gamma}{4G}.$$

What is multiparty entanglement?

A state $|\psi\rangle_{ABC} \in \mathcal{H}_A \otimes \mathcal{H}_B \otimes H_C$ is separable if it's separable with respect to any (non-trivial) partition, eg. $|\psi\rangle_{ABC} = |\phi_1\rangle_{AB} \times |\phi_2\rangle_C$.

Non-seperable states could just have two-party entanglement: e.g.

$$|\psi\rangle_{ABC} = |\chi_1\rangle_{A_1 B_1} \times |\chi_2\rangle_{A_2 C_1} \times |\chi_3\rangle_{B_2 C_2}$$

Define a state to have *n*-party entanglement if it *can't* be written as a product of $(n-1)$-party entangled states.

Examples:

- GHZ state: $|\psi\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$
- W state: $|\psi\rangle = \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle)$.

Multiparty entanglement is complicated

- n-party entanglement is LU invariant, but *not* monotonic under LOCC: given a state like

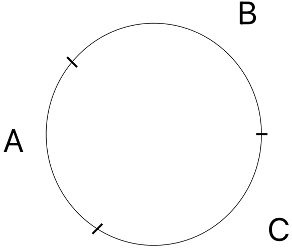
$$|\psi\rangle_{ABC} = |\chi_1\rangle_{A_1B_1} \times |\chi_2\rangle_{A_2C_1} \times |\chi_3\rangle_{B_2C_2},$$

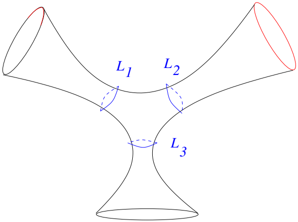
  can use quantum teleportation to create a state with three-party entanglement.
- Expect multiple kinds of multiparty entanglement:
  - GHZ, W states inequivalent.
  - Many possibilities for larger $\mathcal{H}$.
- LU invariant information in reduced density matrix ρ_{AB}; not diagonalisable.

Is multiparty entanglement important in holography?

Potentially relevant for:

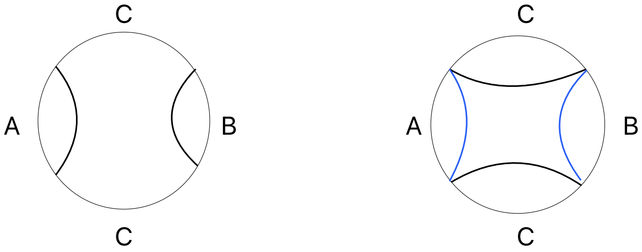
- Dividing the vacuum into more than two regions.
- Multiboundary wormholes: generalisation of ER bridge. Explicit solutions in $2 + 1$ pure gravity.





Is multiparty entanglement important in holography?

Example:



- When AB is less than half the boundary, entanglement is purely bipartite between A, C, B, C.
- When AB is more than half the boundary, there is multiparty entanglement.

How to detect multiparty entanglement?

Consider reduced density matrix ρ_{AB}. Mutual information:

$$I(A:B) = S_A + S_B - S_{AB}$$

Measures correlation between A, B.
Zero for bipartite states $|\chi\rangle_{AC}$, $|\chi\rangle_{BC}$, but not for $|\chi\rangle_{AB}$.

$\star$ Doesn't distinguish multiparty, bipartite entanglement.

$\Rightarrow$ Need new tools to characterise multiparty entanglement.

Reflected entropy

Canonical purification:

- Given a mixed state $\rho = \sum_i p_i |i\rangle\langle i|$, define the canonical purification $|\sqrt{\rho}\rangle = \sum_i \sqrt{p_i} |i\rangle \otimes \langle i| \in \mathcal{H} \otimes \mathcal{H}^*$.
- Trace over one factor gives back ρ.

Reflected entropy: Dutta & Faulkner

- For $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$, define $\rho_{AA^*} = \mathrm{tr}_{BB^*}(|\sqrt{\rho}\rangle\langle\sqrt{\rho}|)$
- Reflected entropy $S_R(A:B) = S(\rho_{AA^*})$.

Properties:

- $S_R(A:B) \geq I(A:B)$.
- Violates monotonicity: $S_R(A:BC) \ngeq S_R(A:B)$. Hayden Lemm Sorce

Reflected entropy

Akers
Rath

On bipartite states:

- For $|\chi\rangle_{AC}$, $|\chi\rangle_{BC}$, $\rho_{AB} = \rho_A \times \rho_B$, so $|\sqrt{\rho}\rangle_{ABA^*B^*} = |\sqrt{\rho_A}\rangle_{AA^*} \times |\sqrt{\rho_B}\rangle_{BB^*}$, and $S_R(A:B) = 0$.
  Recall these cases also had $I(A:B) = 0$.
- For $|\chi\rangle_{AB}$, $\rho_{AB} = |\chi\rangle\langle\chi|$, so $S_R(A:B) = 2S(A)$
  Also has $I(A:B) = S(A) + S(B) - S(AB) = 2S(A)$.
- $\Rightarrow$ for bipartite states, $S_R - I = 0$.

Residual information $R_3(A:B) = S_R(A:B) - I(A:B)$ is a *signal* of multipartite entanglement.

Residual information

If $R_3 \neq 0$, state has 3-party entanglement. But $R_3 = 0$ does not imply the state is *not* multiparty entangled; in particular $R_3 = 0$ for the GHZ state.

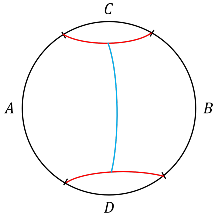
Definition is not permutation symmetric in A, B, C. Three signals: $R_3(A : B)$, $R_3(A : C)$, $R_3(B : C)$.

Bounds: $R_3(A : B) \geq 0$, $S_B + S_C \geq S_A + R_3(A : B)$, and permutations.

Reflected entropy in holography

In a time-independent holographic state $|\psi\rangle$,

- The reduced density matrix ρ is dual to the entanglement wedge.
- The canonical purification is dual to a "doubled" spacetime, formed from two copies of the entanglement wedge. Engelhardt Wall Takayangai Umemoto
- Applying RT to this spacetime, S_R is the area of the "entanglement wedge cross-section". Dutta Faulkner



Holographic properties

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Multiparty entanglement significant in holographic states:

- Dividing the vacuum into three connected regions, $R_3 = \frac{1}{4G} \ln 4$.
- If $EW(AB) = EW(A) \cup EW(B)$, $R_3 = 0$.
  (If $EW(AB)$ is disconnected, $S_R = I = 0$.)
- Can change discontinuously. Signals phase transitions in entanglement structure.
- In the three-boundary wormhole, we can saturate bounds, signalling purely 3-party entanglement.

See also Mori Yoshida on absence of distillable entanglement.

More parties

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For a four-party state $|\psi\rangle_{ABCD}$, consider $\rho_{ABC} = \mathrm{tr}_D(|\psi\rangle\langle\psi|)$.

- Triple information $I_3 = S_A + S_B + S_C - (S_{AB} + S_{BC} + S_{AC}) + S_{ABC}$ is a four-party entanglement signal.
- This is permutation symmetric.
- In holography, $-I_3 \geq 0$.
- Can construct another signal using canonical purification:
  - Take $\bar{\rho}_{ABA^*B^*} = \mathrm{tr}_{CC^*}(|\sqrt{\rho}\rangle\langle\sqrt{\rho}|)$, canonical purification $|\sqrt{\bar{\rho}}\rangle_{ABA^*B^*A_*B_*A_*^*B_*^*}$.
  - Define $Q_4(A:B) = S(AA^*A_*A_*^*) - 2S_R(A:B)$.
  - In holography, $Q_4 \geq 0$.

There are analogues of R_3 for all odd numbers and analogues of I_3 for all even numbers of parties. Multiparty entanglement signals, but not sign-definite, even in holography.

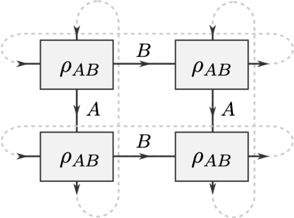
Multi-entropy

Gadde
Krishna
Sharma

Penington
Walter
Witteveen

Iizuka
Nishida

Define $Z_n^{(3)}$ by contracting n^2 copies of ρ_{AB} in a toroidal lattice:

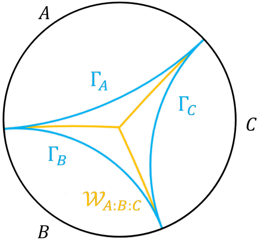


- Renyi multi-entropy is $S_n^{(3)} = \frac{1}{1-n} \ln[Z_n^{(3)}/(Z_1^{(3)})^{n^2}]$.
- Multi-entropy is $S^{(3)} = \lim_{n\to 1} S_n^{(3)}$.
- Genuine multi-entropy $GM^{(3)} = S^{(3)} - \frac{1}{2}(S_A + S_B + S_C)$ is also a three-party entanglement signal.
- For the GHZ state, $GM^{(3)} = \ln\sqrt{2}$.

Multi-entropy

Gadde
Krishna
Sharma

Conjectured holographic dual to $S^{(3)}$ is minimal area brane web $\mathcal{W}_{A:B:C}$:



Gives

$$GM^{(3)} = \frac{1}{4G}[A(\mathcal{W}) - \frac{1}{2}(A(\Gamma_A) + A(\Gamma_B) + A(\Gamma_C)].$$

Relating multiparty signals

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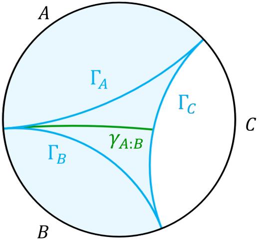
Compare multi-entropy to R_3:



$$R_3 = \frac{1}{4G}[2A(\gamma_{A:B}) - (A(\Gamma_A) + A(\Gamma_B) - A(\Gamma_C))].$$

But

$$A(\gamma_{A:B}) + A(\Gamma_C) \geq A(\mathcal{W}),$$

so

$$\frac{1}{2}R_3 \geq GM^{(3)}.$$

Relating multiparty signals

$$\frac{1}{2} R_3 \geq GM^{(3)}.$$

- First holographic constraint on pure three-party states.
- Extension of the holographic entropy cone.
- Since GHZ has $R_3 = 0$ and $GM^{(3)} > 0$, implies holographic states can't have just GHZ-like multiparty entanglement.

Relating multiparty signals

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Chan
Kang
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Four parties:

- Multi-entropy $S^{(4)}$. Three four-party signals: Gadde

$$M_{AB|CD} = S^{(4)}(A, B, C, D) - S^{(3)}(AB, C, D) - S^{(3)}(A, B, CD),$$

 and similarly $M_{AC|BD}$, $M_{AD|BC}$.
- In holography, $-I_3 \geq M_{AB|CD} \geq 0$: multi-entropy signals are weaker than R_3, I_3.

Holographic 4-party cone including multi-entropies analysed by Ju Zhao : additional inequalities, show these bounds are tight.

Holographic time dependence

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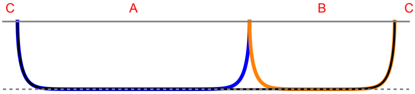
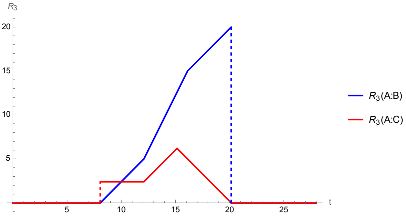
Consider the late time evolution of a thermalizing system: entanglement described by an effective membrane picture. Mezei Jiang Mezei Virrueta

For example, A, B two finite strips in the boundary:



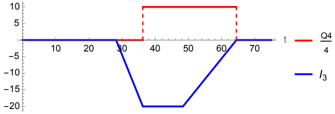
Time evolution of R_3:



Holographic time dependence

Balasubramanian
Bernamonti
Copland
Craps
Galli

Time evolution of I_3, Q_4:



- Different signals behave differently
- Evolution is non-monotonic: multiparty entanglement at intermediate stage of evolution.
- Discontinuities indicate phase transitions in entanglement structure.
- Bounds saturated, e.g. in plateau in I_3

Discussion

- Multiparty entanglement complicated, important in holography.
- New signals of multiparty entanglement provide some handle on it.
- Many signals; inequalities relating them and entropies form a cone, extending holographic entropy cone. More data on the structure of holographic states. Compatible with random tensor network picture.
- Time dependence shows dynamical phase transitions in the entanglement structure.

Discussion

Future directions:

- Characterising states which saturate bounds on multiparty signals.
- Further signals?
- Fuller treatment of time dependent case.
- Algebraic perspective?
- Multi-entropy in random tensor network states. Hu Lin Nechita