The Black Hole at the End of the Cone

localizing the anomaly polynomial on toric geometries

Davide Cassani (INFN Padova)

INSTITUT D'ÉTÉ DE L'ENS ~ 26 June 2026

Motivation

- A very robust partition function: **refined Witten index**

$$\mathcal{I}(\omega,\varphi) = \mathrm{Tr}\,(-1)^{F}\, \mathrm{e}^{-\beta\{\mathcal{Q},\overline{\mathcal{Q}}\}+\omega J+\varphi Q}$$

only susy states $\{\mathcal{Q},\overline{\mathcal{Q}}\}=0$ contribute, no dependence on β

independent of continuous deformations, RG flow invariant

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- Can be used to explore **quantum gravity** :
  - successful BH microstate counting reduce to an index computation
  - holography: $Z_{\mathbf{QG}} = Z_{\mathbf{CFT}}$

- Our aim: **define and study index *directly in supergravity***,

using the **gravitational path integral**

Motivation

$$Z = \int \underset{\text{metric}}{Dg_{\mu\nu}} \, \underset{\text{other fields}}{D\psi} \, \mathrm{e}^{-\mathbf{Action}[g_{\mu\nu},\,\psi]}$$

metric

other fields

Gibbons,
Hawking '77

with suitable boundary conditions

Motivation

$$Z = \int Dg_{\mu\nu} D\psi \, e^{-\text{Action}[g_{\mu\nu}, \psi]}$$

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- Hard ➜ **semiclassical approximation**

$$Z \simeq e^{-I_1} + e^{-I_2} + \ldots$$

  - saddles : classical solutions with Euclidean action $I_1, I_2, \ldots$

our main goal today

- We will work in 5d supergravity

SUPERGRAVITIES IN 5 DIMENSIONS

E. CREMMER

Laboratoire de Physique Théorique de l'Ecole Normale Supérieure+

Invited Paper at the Nuffield Gravity Workshop, June 22 – July 12, 1980, Cambridge (England)

LPTENS 80/17
August 1980

+Laboratoire Propre du CNRS, associé à l'Ecole Normale Supérieure et à l'Université de Paris-Sud.
Postal address : 24 rue Lhomond, 75231 PARIS cedex 05 (France)

Outline

- 5d (gauged) supergravity and some of its solutions
- the **gravitational index**, and its **saddles**
- **susy on-shell action, including higher-derivatives,** using **equivariant localization**
- **Black hole** and **black ring** examples

Based on:

2606.16986 with Enrico Turetta

2403.02410, 2409.01332, 2507.12650 with Alejandro Ruipérez & Enrico Turetta

5D (gauged) supergravity

$$\mathcal{S} = \int \left[(R - 2\mathcal{V}) \star 1 - a_{IJ}\, \mathrm{d}X^I \wedge \star \mathrm{d}X^J - a_{IJ} F^I \wedge \star F^J - \tfrac{1}{6} C_{IJK} A^I \wedge F^J \wedge F^K \right]$$

+ *fermions*

scalars

gauge fields $F^I = \mathrm{d}A^I$

- $I = 0, \ldots, n$

  vector multiplets

- C_{IJK} symmetric tensor, determines a_{IJ} and **Chern-Simons term**

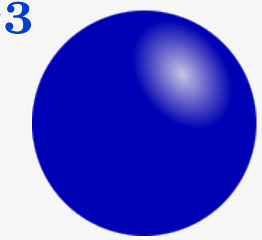
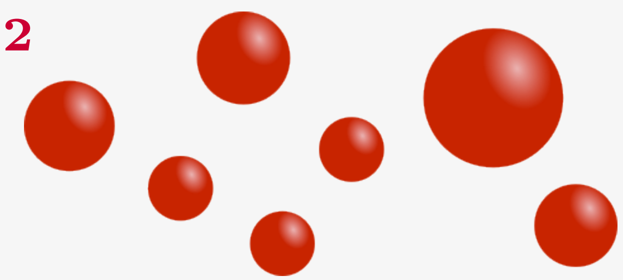
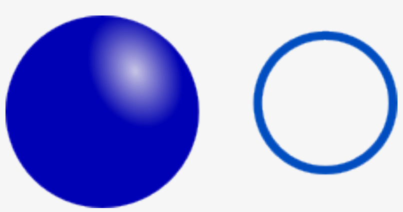
- $\mathcal{V}$ scalar potential, determined by constants g_I ➜ **Fayet-Iliopoulos** supergravity

  $g_I \neq 0$ ➜ AdS$_5$ vacuum

  $g_I = 0$ ➜ $\mathcal{V} \equiv 0$ ➜ Minkowski vacuum

- Many known embeddings in string or M-theory

Susy & extremal solutions with $R_t \times U(1)^2$ symmetry

Horizon topology	asymptotically flat	asymptotically AdS$_5$
S^3  black hole	Strominger, Vafa; BMPV '96	Gutowski, Reall '04; Chong, Cvetic, Lu, Pope '05; Kunduri, Lucietti, Reall '06
S^2  horizonless topological solitons	infinitely many Giusto, Mathur, Saxena '04; Bena, Warner '05; Berglund, Gimon, Levi '05;	<u>very</u> few examples Cvetic, Gibbons, Lu, Pope '05; Chong, Cvetic, Lu, Pope '05; Durgut, Kunduri '23
$S^2 \times S^1$  black ring	Elvang, Emparan, Mateos, Reall '04	? ? ? near-horizon Kunduri,Lucietti '07
$S^3/\mathbb{Z}_p$ black lens	Kunduri, Lucietti '14	? ? ?
combinations of the above	classified in: Breunhölder, Lucietti '17	? ? ?

Competing solutions

Microcanonical ensemble: fix charges, compare entropy.

In 5d, fixing the asymptotic charges does not determine the solution uniquely.

E.g. BMPV black hole and black ring can have equal charges.

Black hole - black ring transition   Alexandrov, Klemm, Pioline '26

Partition functions playing a role in BH microstate counting

often are *not* microcanonical: some of the variables are fugacities, not charges.

Study and compare solutions with *grand-canonical* boundary conditions

Which solutions dominate?   **New phase transitions?**

Boundary conditions

- Asymptotically AdS space with compactified Euclidean time.

$$\mathbf{d}s^2 \to \frac{\mathbf{d}r^2}{r^2} + r^2\left[\frac{\beta^2}{4\pi^2}\mathbf{d}\phi_0^2 + \mathbf{d}\theta^2 + \sin^2\theta\left(\mathbf{d}\phi_1 + \frac{\beta\Omega_1}{2\pi\mathrm{i}}\mathbf{d}\phi_0\right)^2 + \cos^2\theta\left(\mathbf{d}\phi_2 + \frac{\beta\Omega_2}{2\pi\mathrm{i}}\mathbf{d}\phi_0\right)^2\right]$$

$$\beta\Phi^I = -\mathrm{i}\int_{S^1_\infty} A^I$$

$\phi_0,\ \phi_1,\ \phi_2\ :\ 2\pi$ -periodic angles

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$$\beta\Phi^I = -\mathrm{i}\int_{S^1_\infty} A^I$$

$\phi_0,\ \phi_1,\ \phi_2\ :\ 2\pi$ -periodic angles

- Asymptotically:

energy, angular momenta, electric charges

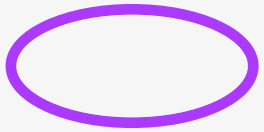
conserved **charges** $\qquad E\,,\ J_1\,,\ J_2\,,\ Q_I$

$\Updownarrow$

conjugate **potentials** $\qquad \beta\,,\ \Omega_1\,,\ \Omega_2\,,\ \Phi^I$

inverse temperature, angular velocities, electrostatic potentials

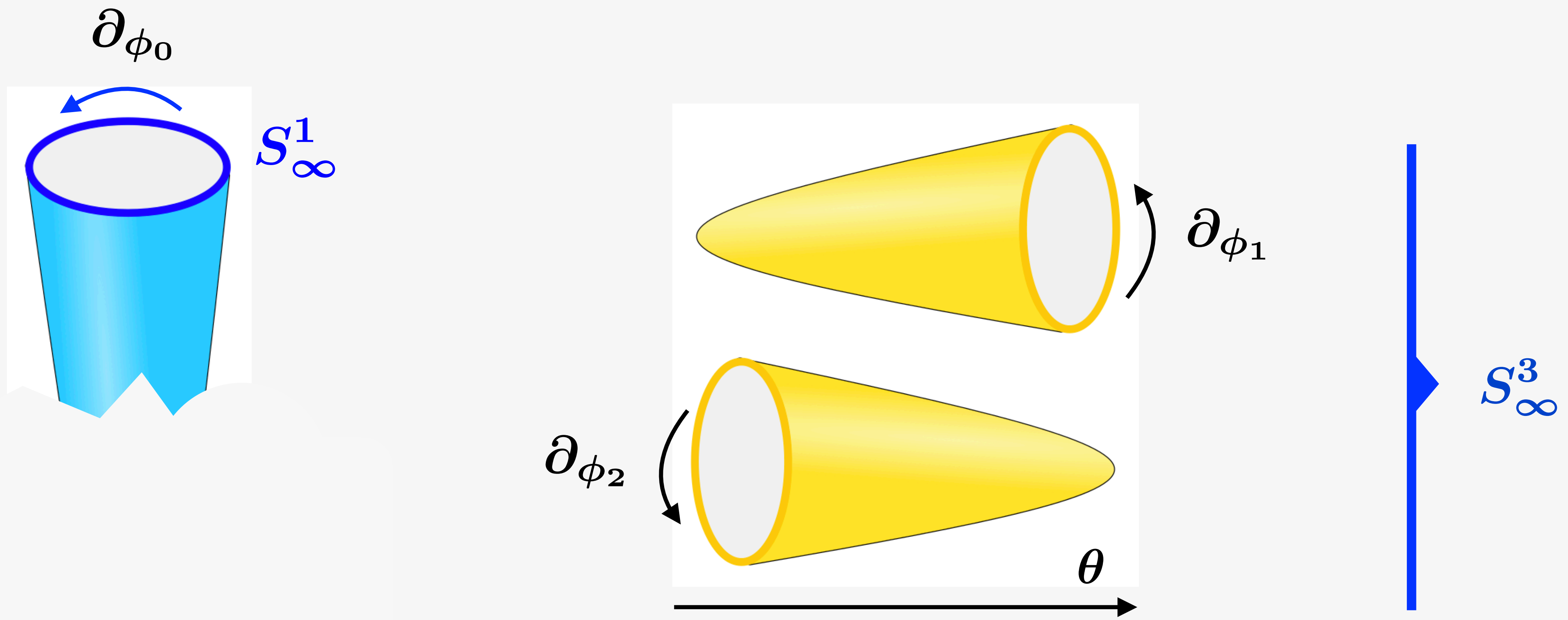
S^3_∞



S^1_∞

Boundary conditions

- toric $\mathbf{U(1)^3}$ action



Gravitational index

$$Z \;=\; \int D g_{\mu\nu} D\psi \; \mathrm{e}^{-\mathrm{Action}[g_{\mu\nu},\,\psi]}$$

Dirichlet bdry conditions ➜ grand-canonical partition function:

$$Z(\beta,\Omega,\Phi) \;=\; \mathrm{Tr}\; \mathrm{e}^{-\beta(E-\Omega_i J_i-\Phi^I Q_I)}$$

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$$Z(\beta,\Omega,\Phi) \;=\; \mathrm{Tr}\; \mathrm{e}^{-\beta(E-\Omega_i J_i-\Phi^I Q_I)}$$

$$\{\mathcal{Q},\overline{\mathcal{Q}}\} = E - \Omega_i^* J_i - \Phi^{*I} Q_I$$

supercharge

const

const

$$\omega_i = \beta(\Omega_i - \Omega_i^*)\;, \qquad \varphi^I = \beta(\Phi^I - \Phi^{*I})$$

$$➜\;\; Z(\beta,\omega,\varphi) \;=\; \mathrm{Tr}\; \mathrm{e}^{-\beta\{\mathcal{Q},\overline{\mathcal{Q}}\}+\omega_i J_i+\varphi^I Q_I}$$

Gravitational index

$$Z(\beta,\omega,\varphi) = \mathrm{Tr}\ \mathrm{e}^{-\beta\{\mathcal{Q},\overline{\mathcal{Q}}\}+\omega_i J_i+\varphi^I Q_I}$$

contributions **only from susy configurations ?**

Gravitational index

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contributions **only from susy configurations ?**

fix: $$\boxed{\omega_1+\omega_2-2g_I\varphi^I = 2\pi i}$$ supersymmetric boundary condition

can solve e.g. for ω_1 and use: $e^{2\pi i J_1} = (-1)^F$

➜ $$Z(\omega,\varphi) = \mathrm{Tr}\,(-1)^F\, e^{-\beta\{\mathcal{Q},\overline{\mathcal{Q}}\}+\omega_2(J_2-J_1)+\varphi^I Q_I+2g_I\varphi^I J_1}$$

➜ **gravitational index** it has the nice properties of a Witten index

Cabo-Bizet, Murthy, DC, Martelli '18;

Iliesiu, Kologlu, Turiaci '21 . . .

Microstate counting

- Laplace transform gives degeneracies at fixed charges ➜ **entropy**

➜ microstate counting, to be compared with microscopic approaches using string theory or holography

- **Asymptotically flat**: compare with actual microscopic computation, e.g. from brane constructions

  Strominger, Vafa '96
  Maldacena, Strominger, Witten '97, ...

- **Asymptotically AdS**: comparison with dual SCFT index and its large-N saddles

  last decade

➜ **microscopic derivation of susy black hole entropy**

Saddles?

- **What are the saddles of the gravitational index?**

$\Downarrow$

supersymmetric Euclidean solutions with finite β ➜ non-extremal

Lorentzian sol not well-defined.

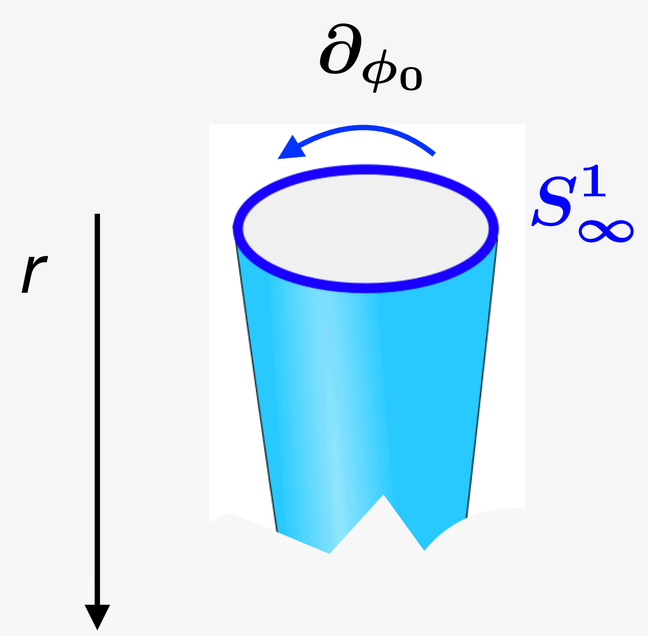
Euclidean sol ok if allow for **complexification**

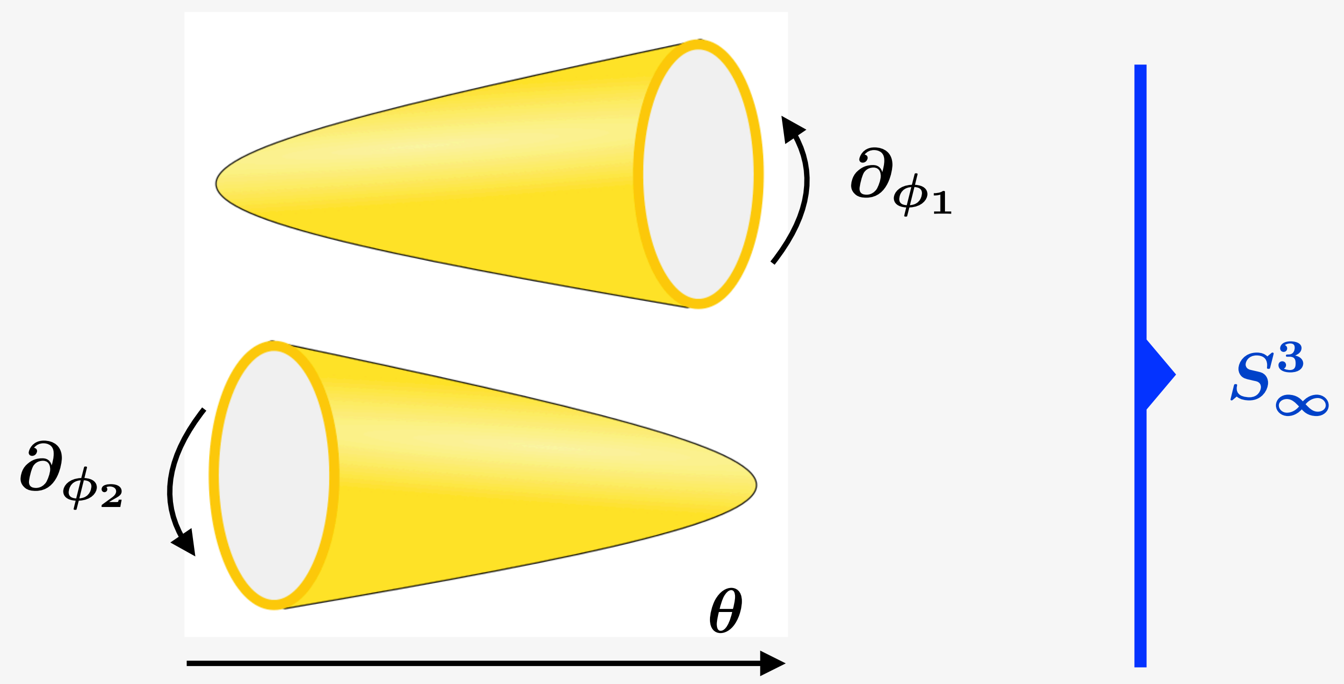
Cabo-Bizet, Murthy, DC, Martelli '18

Eventually send $\beta \to \infty$ and recover the physical, extremal solution

Topology of saddles

- toric $\mathrm{U}(1)^3$ action at boundary. We assume it extends to the bulk



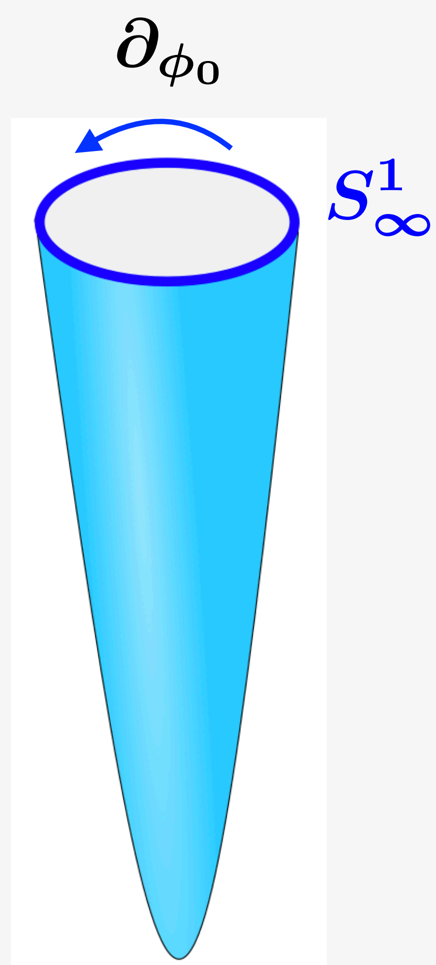


Different U(1) Killing vectors can degenerate in the bulk:

$$V = n\,\partial_{\phi_0} + r\,\partial_{\phi_1} + p\,\partial_{\phi_2}\,, \qquad (n,\, r,\, p) \in \mathbb{Z}^3$$

Collection of all V's: **fan** of the geometry. Specifying it fixes the topology.

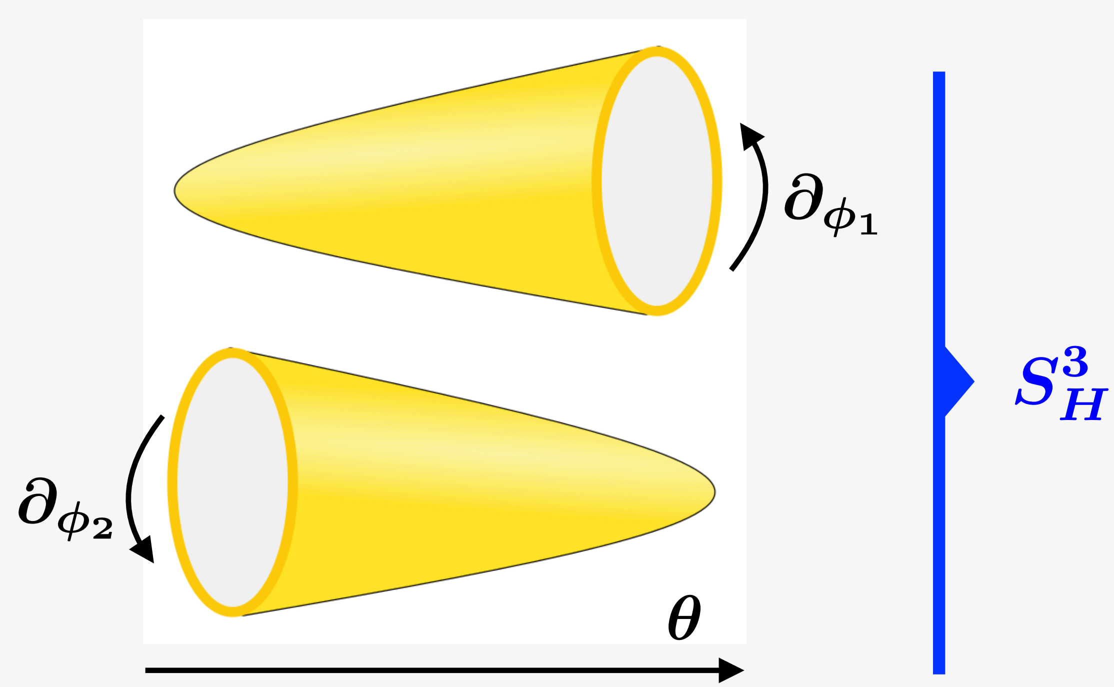
Black hole saddle (spherical horizon)



$$V^0 = (0,\ 1,\ 0)$$

$$V^1 = (1,\ 0,\ 0)$$

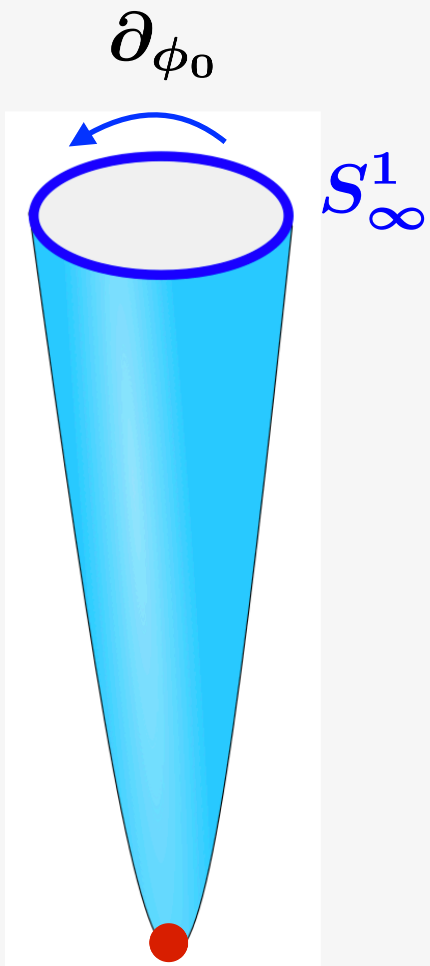
$$V^2 = (0,\ 0,\ 1)$$

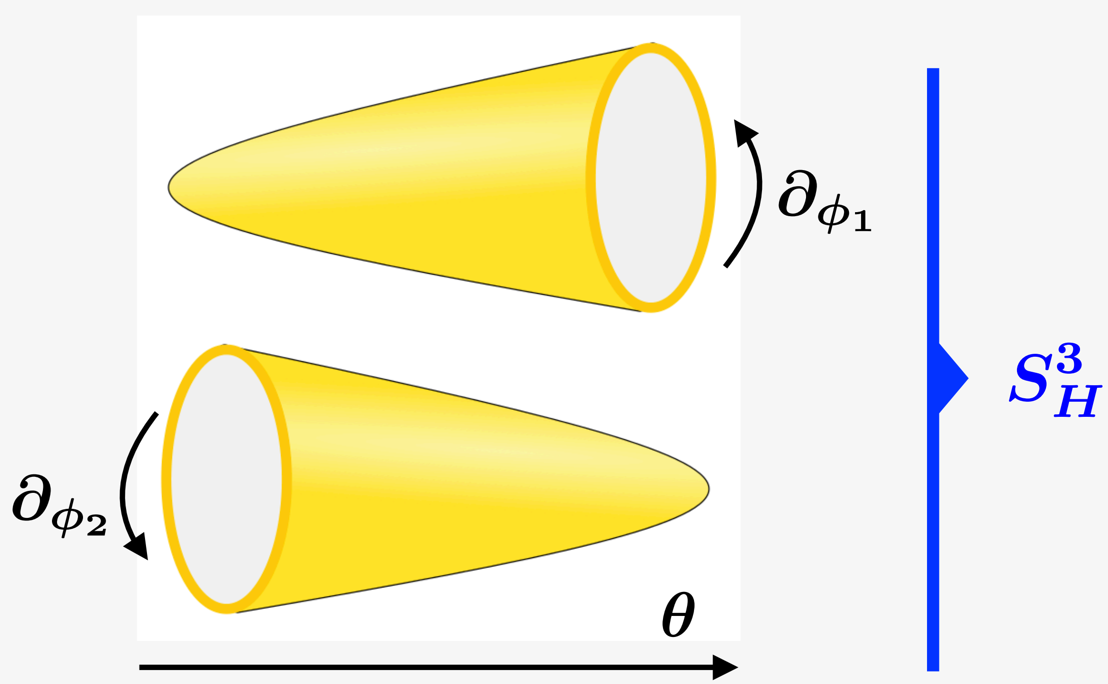


Black hole saddle (spherical horizon)

- $\partial_{\phi_0} = 0$ at horizon

  fixed locus : the whole S^3_H





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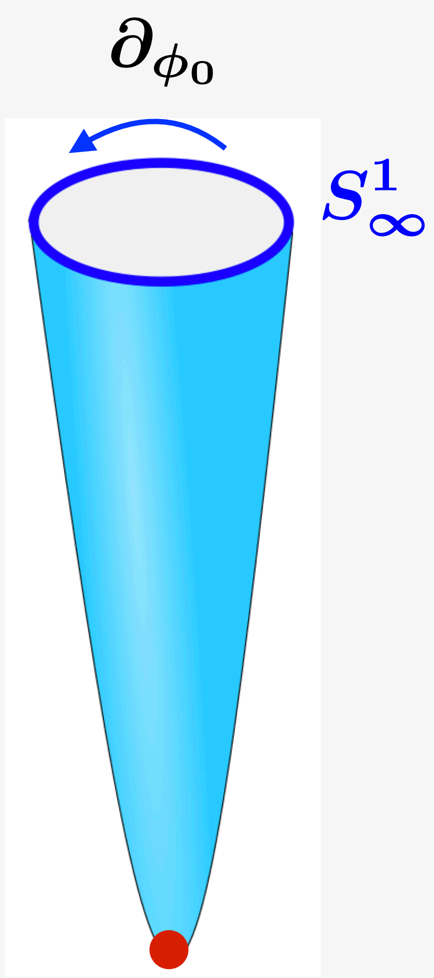
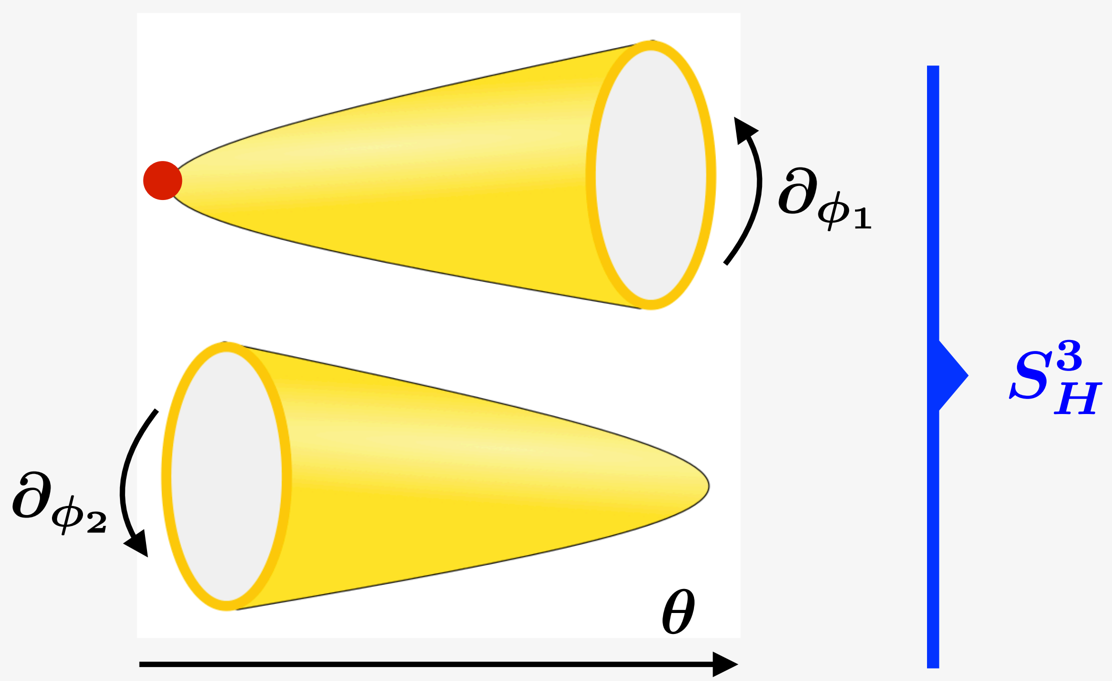
- $\partial_{\phi_0} + \partial_{\phi_1} = 0$ at horizon & $\theta = 0$

  fixed locus : orbits of ∂_{ϕ_2}





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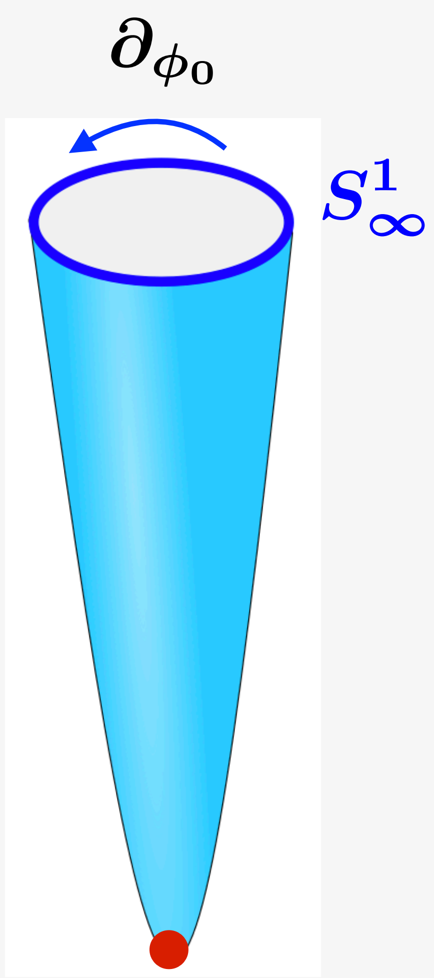

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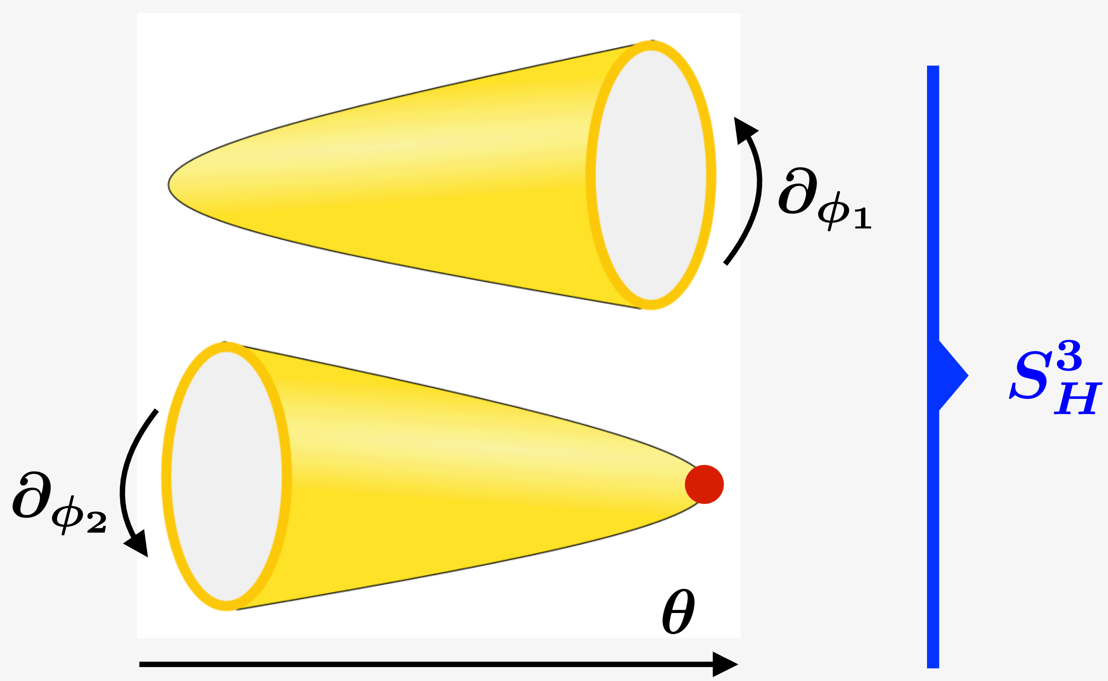
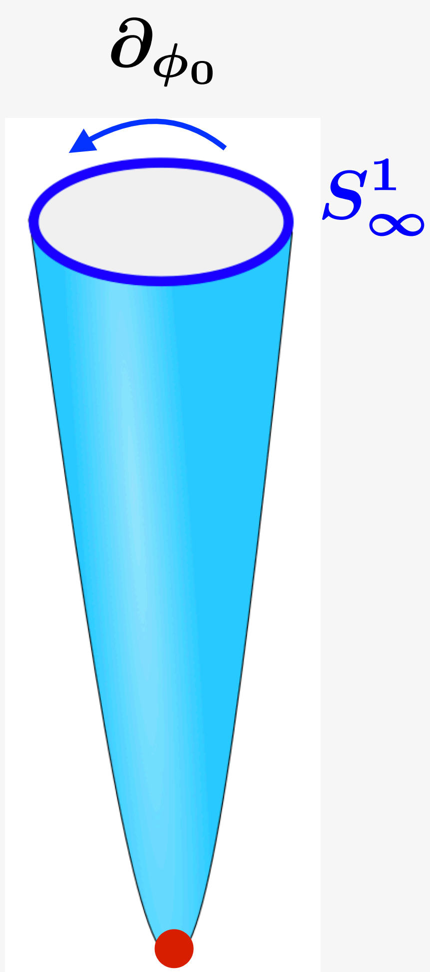
- $\partial_{\phi_0} + \partial_{\phi_2}$ $= 0$ at horizon & $\theta = \dfrac{\pi}{2}$

  fixed locus : orbits of ∂_{ϕ_1}



Black ring saddle

keep same asymptotics, but different bulk topology

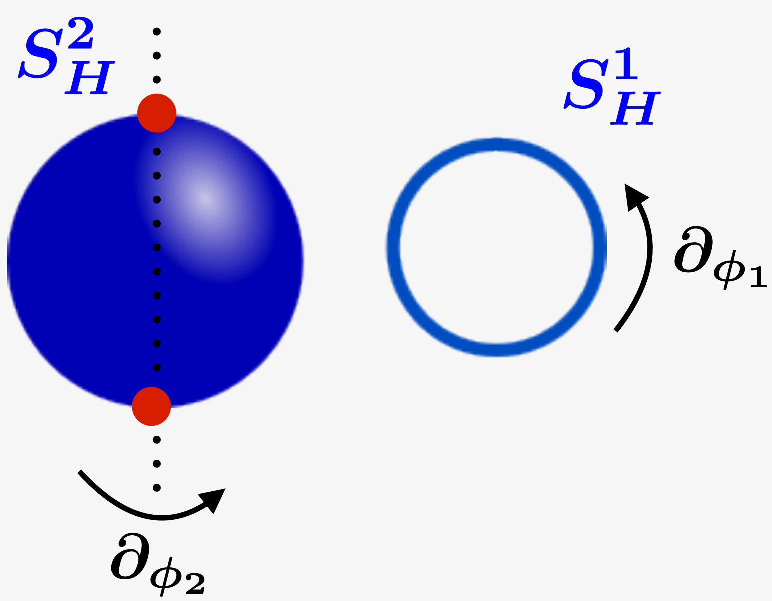


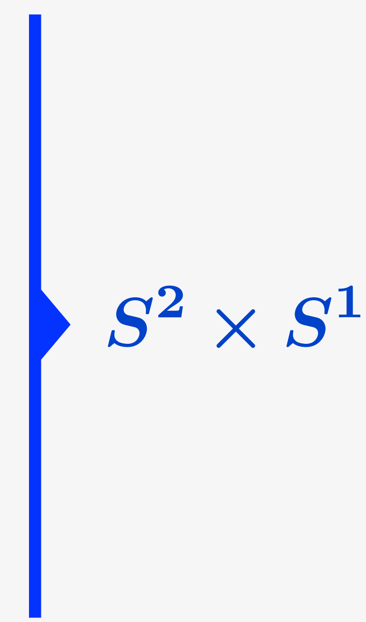
- ∂_{ϕ_0} vanishes at horizon

  fixed locus : the whole $S^2 \times S^1$

- $\partial_{\phi_0} + \partial_{\phi_2}$ vanishes at horizon & **both** poles of S^2

  fixed loci : orbits of ∂_{ϕ_1}





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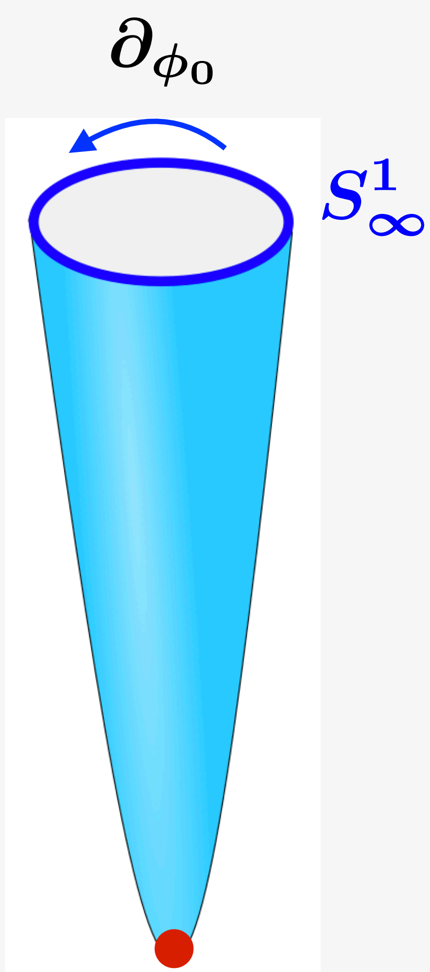

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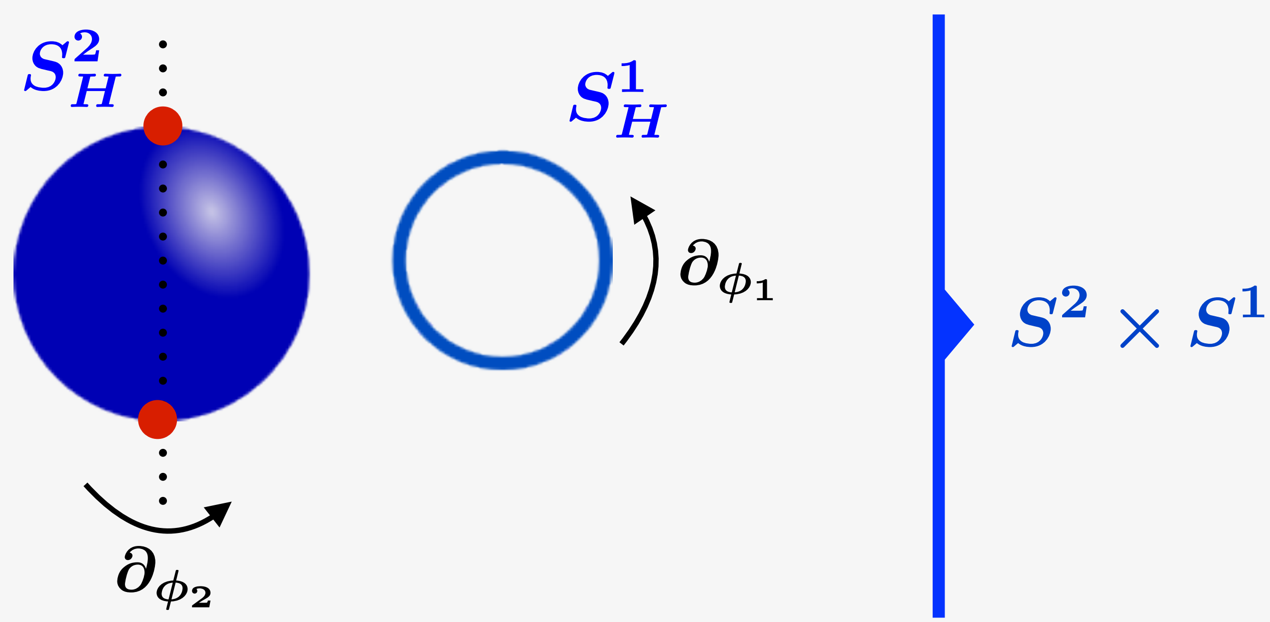

Equivariant localization

- Recently: **equivariant localization** allows to evaluate the action of supersymmetric solutions, even if not explicitly known

Benetti Genolini, Gauntlett, Sparks '23

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Berline-Vergne-Atiyah-Bott (BVAB) theorem

Integral of equivariantly closed forms localize to fixed loci of Killing vector

ξ Killing vector, Ψ polyform such that $(\mathrm{d} - \iota_\xi)\Psi = 0 = \mathcal{L}_\xi \Psi$

$$\int_{\mathcal{M}} \Psi = \int_{\mathcal{M}_0} \frac{\Psi}{e_\xi(\mathcal{N})}$$

equivariant Euler form of normal bundle $\mathcal{N}$ to $\mathcal{M}_0$

fixed locus of ξ

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- different approaches for 5d supergravity:

stay in 5d — DC, Ruiperez, Turetta '24

reduce to 4d — Colombo, Dimitrov, Martelli, Zaffaroni '25; Benetti Genolini, Gauntlett, Jiao, Park, Sparks '25

The on-shell action

$$\mathcal{S} = \int \left[(R - 2\mathcal{V}) \star 1 - a_{IJ}\, \mathrm{d}X^I \wedge \star \mathrm{d}X^J - a_{IJ} F^I \wedge \star F^J - \tfrac{1}{6} C_{IJK} A^I \wedge F^J \wedge F^K\right]$$

There is an interesting story about boundary terms. Eventually they cancel out.

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- Background-subtracted action of supersymmetric solutions:

$$I = \int_{\mathcal{M}_5} \tfrac{1}{6} C_{IJK}\, \eta^I \wedge \mathrm{d}\eta^J \wedge \mathrm{d}\eta^K$$

Colombo, Dimitrov, Martelli, Zaffaroni '25

$$\eta^I = -A^I - \mathrm{i} X^I e^0 \,, \qquad \mathrm{d}s^2 = (e^0)^2 + \mathrm{d}s^2(\mathbb{B}) \longrightarrow \text{Kähler base}$$

Usual subtleties of Chern-Simons terms: in general defined patchwise

The on-shell action

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Kähler base

Usual subtleties of Chern-Simons terms: in general defined patchwise

- **"ascent" to 6d**

$$I = \int_{\mathcal{M}_6} \tfrac{1}{6}\, C_{IJK}\, \mathrm{d}\eta^I \wedge \mathrm{d}\eta^J \wedge \mathrm{d}\eta^K \qquad \mathcal{M}_5 = \partial \mathcal{M}_6$$

Higher-derivative corrections

- Higher-derivative action contains only two Chern-Simons terms:

$$\mathcal{S} = \int \left(-\frac{1}{6} k_{IJK} A^I \wedge \mathrm{d}A^J \wedge \mathrm{d}A^K + \frac{1}{48} k_I A^I \wedge R_{\alpha\beta} \wedge R^{\alpha\beta} \right) + \ldots$$

many other terms

$$k_{IJK} = C_{IJK} + \ldots$$

possible corrections

- We conjecture: the full action of gravitational index saddles is captured by:

$$I = \int_{\mathcal{M}_6} \left(\frac{1}{6} k_{IJK} \mathrm{d}\eta^I \wedge \mathrm{d}\eta^J \wedge \mathrm{d}\eta^K - \frac{1}{48} k_I \mathrm{d}\eta^I \wedge R_{\alpha\beta} \wedge R^{\alpha\beta} \right)$$

Same form as **integral of anomaly polynomial** on $\mathcal{M}_6$

Evaluating the action

- We are ideally placed for localizing the integral. For two reasons:
  - can localize to **isolated fixed points**
  - can use **supersymmetric Killing vector** (does not vanish on $\mathcal{M}_5$)

    Defined through Killing spinor bilinear

$$\xi = \frac{1}{\beta}\left(2\pi\partial_{\phi_0} + \mathrm{i}\omega_1\partial_{\phi_1} + \mathrm{i}\omega_2\partial_{\phi_2}\right)$$

- Idea:

  decompose the fan into triplets of vectors generating cones in 6d

Fan decomposition

- Each cone $\mathcal{C}_k$ is generated by three independent vectors

$$\{ V^{k_1},\ V^{k_2},\ V^{k_3} \}$$

of the form $\quad V^{k_1} = n_{k_1}\,\partial_{\phi_0} + r_{k_1}\,\partial_{\phi_1} + p_{k_1}\,\partial_{\phi_2}\,, \qquad n_{k_1}, r_{k_1}, p_{k_1} \in \mathbb{Z}$

Susy Killing vector can be expresses as

$$\xi = \epsilon_{k_1} V^{k_1} + \epsilon_{k_2} V^{k_2} + \epsilon_{k_3} V^{k_3}$$

$\vec{\epsilon}_k = (\epsilon_{k_1}, \epsilon_{k_2}, \epsilon_{k_3})$ equivariant parameters

depend on ω_1, ω_2 and on the n, r, p integers

- In each cone, ξ has precisely one fixed point at the tip

$\rightarrow$ can use localization

General formula for on-shell action

$$I = \sum_{\mathcal{C}_k} \left(\frac{1}{6} k_{IJL} I^{IJL}(\mathcal{C}_k) - \frac{1}{24} k_J I^J(\mathcal{C}_k) \right)$$

sum over cones

$$I^{IJL}(\mathcal{C}_k) = \frac{\vec{\epsilon}_k \cdot (\vec{n}_k \varphi^I - 2\pi \mathrm{i} \vec{q}_k^{\,I}) \;\; \vec{\epsilon}_k \cdot (\vec{n}_k \varphi^J - 2\pi \mathrm{i} \vec{q}_k^{\,J}) \;\; \vec{\epsilon}_k \cdot (\vec{n}_k \varphi^L - 2\pi \mathrm{i} \vec{q}_k^{\,L})}{(2\pi)^2 \, \epsilon_{k_1} \epsilon_{k_2} \epsilon_{k_3}}$$

$$I^J(\mathcal{C}_k) = -\frac{\vec{\epsilon}_k \cdot (\vec{n}_k \varphi^J - 2\pi \mathrm{i} \vec{q}_k^{\,J}) \;\; (\vec{\epsilon}_k \cdot \vec{\epsilon}_k)}{\epsilon_{k_1} \epsilon_{k_2} \epsilon_{k_3}}$$

$$\vec{n}_k = (n_{k_1}, n_{k_2}, n_{k_3}) \qquad \vec{q}_k = (q_{k_1}, q_{k_2}, q_{k_3})$$

gauge field data

- Depends just on: boundary conditions $\omega_1, \omega_2, \varphi^I$

  discrete bulk data n, r, p, q

Black hole saddle (spherical horizon)

Just one cone

$$\boxed{I = \frac{k_{IJK}}{6}\frac{\varphi^I\varphi^J\varphi^K}{\omega_1\omega_2} - \frac{k_I}{24}\frac{\varphi^I\left(\omega_1^2+\omega_2^2-4\pi^2\right)}{\omega_1\omega_2}}$$

Valid both for asymptotically flat and AdS_5

DC, Ruiperez, Turetta '24

only distinguished by the constraint $\omega_1+\omega_2-2g_I\varphi^I = 2\pi i$

vanish in asympt flat

- asymptotically AdS_5

we match Cardy-like asymptotics of dual SCFT index precisely

Legendre transform gives entropy of Gutowski-Reall and CCLP black holes, with corrections.

DC, Ruiperez, Turetta '24; Gaar, Gauntlett, Park, Sparks '26

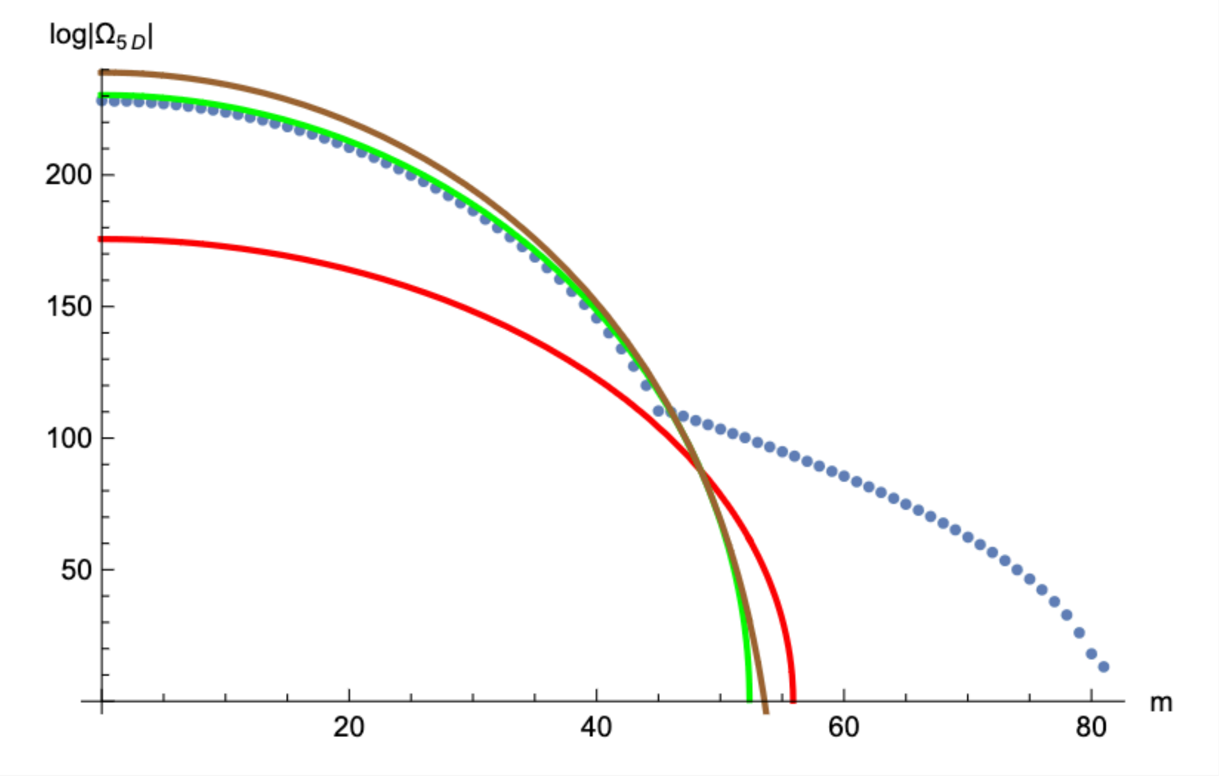
Black hole saddle (spherical horizon)

- asymptotically flat

Legendre transform gives corrected entropy for BMPV black hole

$$\mathcal{S} = 4\sqrt{\pi}\sqrt{C^{IJL}Q_IQ_J\left(Q_L + 18\pi\, k_L\right) - \frac{\pi}{16}(J_1 - J_2)^2\left(1 + 24\pi\frac{C^{IJL}Q_IQ_Jk_K}{C^{IJL}Q_IQ_JQ_L}\right)}$$

Very nice agreement with microscopics



picture from

Alexandrov, Klemm, Pioline '26

Black ring saddle

Two cones.

$$\boxed{I = \mp k_{IJK} q^I \frac{3\varphi^J\varphi^K \pm 3\,\omega_2\, q^J \varphi^K + \omega_2^2\, q^J q^K}{6\omega_1} \pm k_I q^I \frac{\omega_1^2 + \omega_2^2 - 4\pi^2}{24\omega_1}}$$

- upper sign matches asymptotically flat black ring, including HD corrections

$$\mathcal{S} = 2\pi\sqrt{\frac{(kqqq) - 2(kq)}{6}\left[J_1 + \frac{(kqqq)}{24} + \frac{1}{2}(Q\tilde{k}^{-1}Q)\right]}$$

$$\tilde{k}_{IJ} = k_{IJL} q^L$$

$$J_2 = \frac{(Qq)}{2} - \frac{(kqqq) - (kq)}{12}$$

new?

- lower sign compatible with near-horizon solution in gauged supergravity

see also Colombo, Dimitrov, Martelli, Zaffaroni '25

Outlook

- assigned chemical potentials and action to large classes of susy solutions: black holes and rings. Also: black lenses, topological solitons

- proposed a method for evaluating the action via equivariant localization of anomaly polynomial

- Equivariant localization most useful when solution not known explicitly
  - asymptotically AdS_5 beyond isolated spherical horizon
  - higher-derivative corrections

- **Longstanding open question: is there an AdS_5 black ring?**
  If so, does it contribute to the gravitational index?
  If so, can we match the on-shell action with a SCFT calculation?

thank you !