Braneworld holography from $T\bar{T}$ flows

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Based on 2510.01099 and 2607.xxxx with Matteo Selle

Overview of talk

- Motivation
  - Holography with different boundary conditions
  - Holography with Neumann boundary conditions or ‘braneworld holo’: status and challenges

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  - Extension to 3D/2d braneworld holo with non-zero cc and island rule application (non-asymptotic!)

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- Comparison to literature: different modes
  - Carlip's ϕ (asymptotic Weyl mode)
  - Neuenfeld's $\tilde{\phi}$ (radial fluctuation of brane location)

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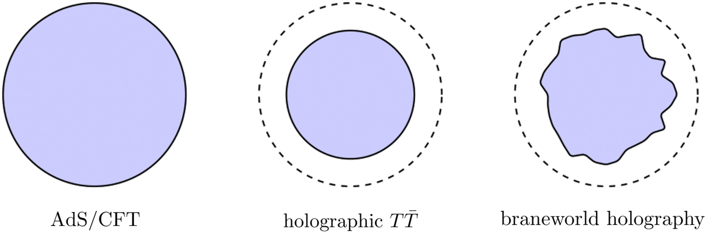
- Outlook

Holography with different boundary conditions



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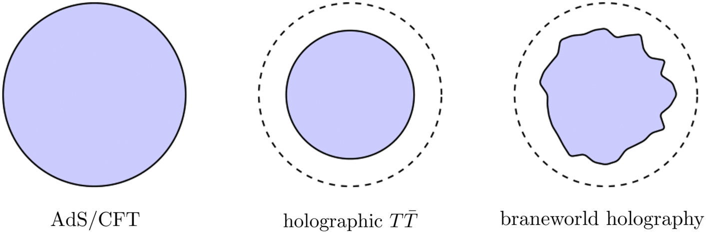

Asymptotic DBC
'97 Maldacena, GKPW

'Finite' DBC
'16 McGough Mezei Verlinde

NBC
'99 Gubser; Verlinde
'19 Almheiri et al 'islands'
$\rightsquigarrow$ revival e.g. 'Islands made easy papers' by Myers et al

$$Z_{DBC}(g_{(0)}) = Z_{CFT}(g_{(0)}), \quad Z_{DBC}(g) = Z_{T^2}(g), \quad Z_{NBC} = ?$$

Holography with NBC or ‘braneworld holo’: status

Historically, end of ’90s:
1) holography (Maldacena), 2) braneworld (Randall Sundrum)

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Gubser '99 "AdS/CFT and gravity"
AdS/CFT: AdS gravity with DBC $\quad \sim \quad$ CFT
bw holo: AdS gravity with NBC $\quad \sim \quad$ "cut-off CFT" + gravity

Scale in "cut-off" CFT: info where in the bulk brane is placed (inprecise concept)

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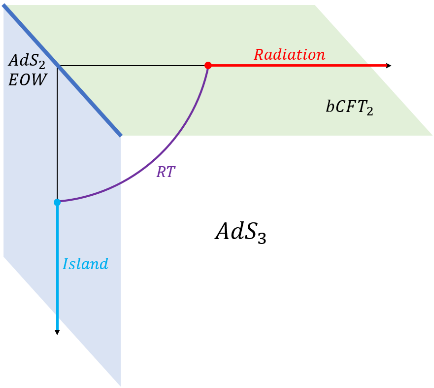
Systematic discussion of braneworld (NBC) holo?

Holographic set-up for island calculations



Bath region (green): CFT

$\sim$ AdS gravity with Dirichlet bc on bulk metric

Brane region (blue): (cut-off) CFT + (2D) gravity

$\sim$ AdS gravity with Neumann bc on bulk metric

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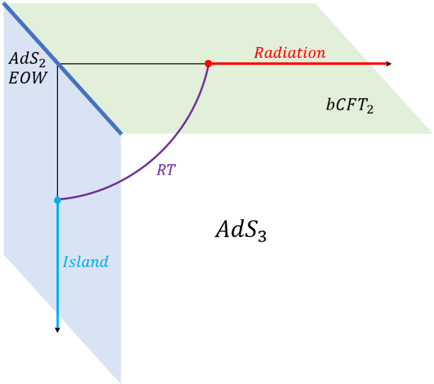

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Braneworld holo

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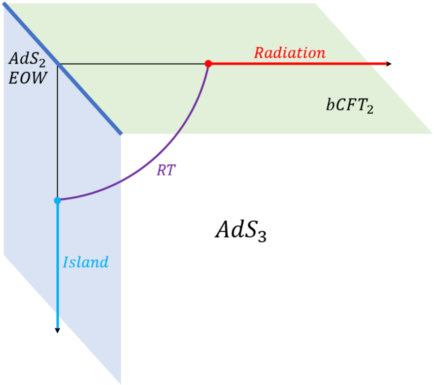

vN entropy for radiation region S_R can be calculated

- 2D: replica trick calculation
- 3D: holographic Ryu-Takayanagi prescription

giving rise to 'island rule' $S_R = \mathrm{Min}_I \, \mathrm{Ext}_I \left[\frac{\mathcal{A}(\partial I)}{4G_N} + S_{CFT}(R \cup I) \right]$

that reproduces desired Page curve (no black hole info paradox)

Holographic set-up for island calculations: challenges

Criticisms of derivation and interpretation of island rule formula

1. Ad hoc JT gravity on brane

   Work on holo derivation of gravity theory on brane [Geng et al '22] [Neuenfeld et al '24] but debated, e.g. [Wang et al '25] "AdS$_2$ brane cannot be described by JT gravity"

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2. Massive gravity on the brane and relevance for interpretation of Page curve as resolution to black hole info paradox

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Signals of underdeveloped AdS_3 braneworld holo
$\rightsquigarrow$ What is bulk-induced 2d set-up?

Holography with NBC or ‘braneworld holo’: our results

We improve AdS_3 braneworld holo [NC Selle ’25]

from

AdS_3 gravity with NBC $\sim$ “cut-off CFT” + gravity

to

AdS_3 gravity with NBC $\sim$ “$T\bar{T}$-deformed CFT”
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Precise, well-defined appearance of scale which breaks conformal invariance to $T\bar{T}$ theory

Precise, well-defined gravity theory on brane *derived from holo bulk*

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$\sim$ effective theory $W[\sigma, \hat{g}]$
ito Weyl mode σ

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Strategy

Dirichlet theory for $S_{grav} = S_{EH} + S_{GHY} + S_{ct}$

$$Z_{DBC}(g) \equiv Z_{grav}(g) = \int_{G|_\partial = g} DG\, e^{iS_{grav}[G]}$$

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Neumann theory for $S_{tot} = S_{EH} + S_{GHY} + S_T$

$$Z_{NBC} = \int_{NBC} DG\, e^{iS_{tot}[G]} = \int Dg\, Z_{DBC}(g) e^{i(S_T - S_{ct})}$$

'setting free the Dirichlet theory'

At saddle, δg imposes NBC from $\delta S_{tot} = \int (EOM)\delta G + \int (NBC)\delta g$
with $NBC \equiv K^{\mu\nu} - Kg^{\mu\nu} + Tg^{\mu\nu} = T^{\mu\nu}_{BY} - T^{\mu\nu}_{BY,ct\,contrib} + T^{\mu\nu}_{BY,T\,contrib}$.

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To obtain braneworld holo, use known holo for $Z_{DBC}(g)$ [Hartman et al '18] and definition $S_{bwgrav} \equiv S_T - S_{ct}$

$$Z_{DBC}(g) = Z_{T^2}(g) \quad \Rightarrow \quad Z_{NBC} = \int Dg\, Z_{T^2}(g) e^{iS_{bwgrav}}$$

Z_{NBC} is dual to matter T^2 theory coupled to gravity S_{bwgrav}.

Strategy for $D = 3$

In $D = d + 1$,

$$S_{bwgrav} = \frac{1}{\kappa_{d+1}} \int_{\partial\mathcal{M}} d^d x \sqrt{-g} \Bigg[(d-1) - T + \frac{1}{2(d-2)} R$$
$$+ \frac{1}{2(d-4)(d-2)^2} \left(R_{\mu\nu} R^{\mu\nu} - \frac{d}{4(d-1)} R^2 \right) + \cdots \Bigg].$$

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with $S_{bwgrav} \equiv S_T - S_{ct} \sim (T-1) \int d^2 x \sqrt{-g}$ just a volume term (no kinetic terms for g).

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$\rightsquigarrow$ Use known holo for $Z_{DBC}(g)$ and 'dissection of Z_{DBC}', i.e. rewriting of $Z_{DBC}(g)$ ito Weyl mode of 2D metric g. We specify to $T = 1$ case.

Known holo for $Z_{DBC}(g)$: canonical formulation

Hamiltonian description of AdS_3 gravity $S_{grav}(G)$: parametrize bulk metric G in terms of 2-metric $g_{\mu\nu}$ on radial slices

Canonically conjugate variables $\quad (g_{\mu\nu}, \pi^{\mu\nu})$

Momenta promoted to operators $\pi^{\mu\nu} = -i\frac{\delta}{\delta g_{\mu\nu}}$ acting on gravitational wavefunctional in 'coordinate representation' $Z_{grav}(g)$ (or $\Psi(g_{\mu\nu})$)

Hamiltonian constraint $\mathcal{H}(g_{\mu\nu}, \pi^{\mu\nu}) = 0$ or semi-classically, gravitational Hamilton-Jacobi equation $\hat{\mathcal{H}}(g_{\mu\nu}, \pi^{\mu\nu})Z_{grav} = 0$ given by

$$\left(\frac{(16\pi G_N)^2}{g}\left(\pi_{\mu\nu}\pi^{\mu\nu} - (\pi^\mu_\mu)^2\right) + \frac{32\pi G_N}{\sqrt{-g}}\pi^\mu_\mu - R(g_{\mu\nu})\right) Z_{grav} = 0$$

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Asymptotic (close to AdS boundary) gravitational HJ equation

$$\pi^\mu_\mu \, Z_{grav} = \frac{\sqrt{-g}}{32\pi G_N} \, R \, Z_{grav}$$

for semi-cl $Z_{grav}(g) = \int_{DBC\ G|_\partial = g} DG\, e^{iS_{grav}[G]}$

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the conformal Ward identity

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for c large $Z_{CFT}(g) = \int D\phi \, e^{iS_{CFT}[\phi]}$

[Freidel '08] [Heemskerk Polchinski '10]

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Cfr. evolution in AdS bulk direction $\sim$ change of scale in bdy theory

Particularly nice formulation of AdS/CFT, as the statement that the partition functions are the same, $Z_{grav} = Z_{CFT}$, because they obey the same differential equation, one interpreted as gravitational evolution equation, the other as conformal Ward identity

$$-i\, g_{\mu\nu} \frac{\delta}{\delta g_{\mu\nu}}\, Z_{CFT} = \frac{c}{48\pi} \sqrt{-g}\, R(g_{\mu\nu})\, Z_{CFT}$$

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the trace flow equation of a $T\bar{T}$-deformed CFT

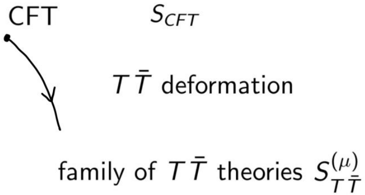
$$\pi^\mu_\mu \, Z_{T\bar{T}} = \frac{c}{48\pi} \sqrt{-g} \, R \, Z_{T\bar{T}} + \frac{\mu}{\sqrt{-g}} \left(\pi_{\mu\nu}\pi^{\mu\nu} - (\pi^\mu_\mu)^2\right) \, Z_{T\bar{T}}$$

for c large $Z_{T\bar{T}}(g) = \int D\phi \, e^{iS^{(\mu)}_{T\bar{T}}[\phi]}$ and deformation parameter $\mu = 4\pi G_N l$

[Freidel '08] [McGough Mezei Verlinde '16]

$T\bar{T}$-deformations of QFT

Discovery of whole class of deformations of 2d QFT that preserve integrability. Strange (irrelevant deformation so non-locality) but integrable: interesting!



$T\bar{T}$ flow equation $\frac{d}{d\mu}S^{(\mu)}_{T\bar{T}} = \int d^2x \underbrace{\left(T_{\mu\nu}T^{\mu\nu} - (T^{\mu}_{\mu})^2\right)}_{\mathcal{O}_{T\bar{T}}}$

[Smirnov Zamolodchikov '16, Cavaglia Negro Szécsényi Tateo '16]

- in context of investigating subspace of integrable QFT in space of all 2D QFT

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CFT $\qquad S_{CFT}$

$T\bar{T}$ deformation

family of $T\bar{T}$ theories $S^{(\mu)}_{T\bar{T}}$

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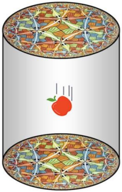

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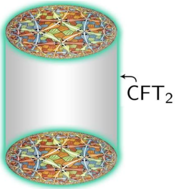
" AdS / CFT "



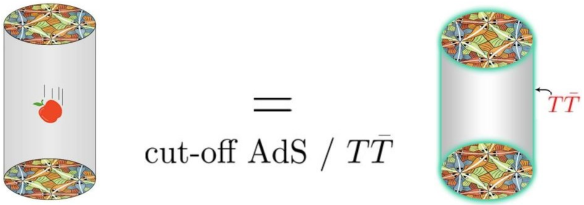
$$Z_{DBC} = Z_{CFT}$$

$$Z_{DBC}(g) = \int_{G|_\partial = g} DG\, e^{iS_{grav}[G]} \qquad\qquad Z_{CFT}(g) = \int D\phi\, e^{iS_{CFT}[\phi;g]}$$

$$\lim_{\text{near bdy}} \hat{\mathcal{H}}(g_{\mu\nu}, \pi^{\mu\nu}) Z_{DBC} = 0$$

The CFT path integral solves the *asymptotic* radial HJ equation

Known holo for $Z_{DBC}(g)$

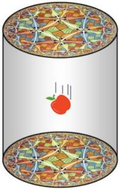


$$Z_{DBC} = Z_{T\bar{T}}$$

$$Z_{DBC}(g) = \int_{G|_{\partial}=g} DG\, e^{iS_{grav}[G]} \qquad\qquad Z_{T\bar{T}}(g) = \int D\phi\, e^{iS_{T\bar{T}}[\phi;g]}$$

$$\hat{\mathcal{H}}(g_{\mu\nu}, \pi^{\mu\nu}) Z_{grav} = 0$$

The $T\bar{T}$ path integral solves the radial HJ equation



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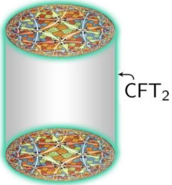
" AdS / CFT "



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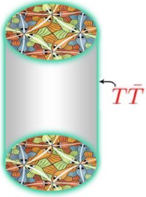
cut-off AdS / $T\bar{T}$



Gravity in smaller box, dual to deformed CFT

[McGough Mezei Verlinde '16]

Dissection of $Z_{DBC}(g)$

Writing the 2D metric as $g = e^{\sigma}\hat{g}$,

$$Z_{CFT}(g) = Z_{CFT}(\hat{g})e^{iS_L[\sigma]}$$

solves the conformal Ward identity (near-bdy gravitational HJ eq) if S_L is Liouville action $S_L = \frac{c}{48\pi}\int d^2x\sqrt{-\hat{g}}\left(\sigma\,\hat{R} + \frac{1}{2}\hat{g}^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma\right)$

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and similarly

$$Z_{T\bar{T}}(g) = Z_{T\bar{T}}(\hat{g})e^{iS_{\tilde{L}}[\sigma]}$$

solves the $T\bar{T}$ flow (gravitational HJ eq) if $S_{\tilde{L}}$ is a deformed Liouville theory solving the flow (with $\mathcal{O}^{\tilde{L}}_{T\bar{T}} \equiv t^{\mu\nu}_{\tilde{L}}t^{\alpha\beta}_{\tilde{L}}\hat{g}_{\alpha\mu}\hat{g}_{\beta\nu} - (t^{\mu\nu}_{\tilde{L}}\hat{g}_{\mu\nu})^2$)

$$\frac{d}{d\mu}S^{(\mu)}_{\tilde{L}} = \frac{1}{4\pi}\int d^2x\sqrt{-\hat{g}}\,e^{-\sigma}\mathcal{O}^{\tilde{L}}_{T\bar{T}}, \qquad S^{(0)}_{\tilde{L}} = S_L$$

Dissection of $Z_{DBC}(g)$: deformed Liouville

$$\frac{d}{d\mu} S_{\tilde{L}}^{(\mu)} = \frac{1}{4\pi} \int d^2x \sqrt{-\hat{g}}\, e^{-\sigma} \mathcal{O}_{T\bar{T}}^{\tilde{L}}, \qquad S_{\tilde{L}}^{(0)} = S_L$$

- Deformation that includes a Weyl factor, $\mu \to \mu e^{-\sigma}$, compared to a $T\bar{T}$ flow
- Straightforwardly solved order by order in μ to $S_{\tilde{L}} = S_L +$ higher-order derivative corrections

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$$S_{\tilde{L}} = \frac{c}{48\pi} \int d^2x \sqrt{-\hat{g}} \left\{ \sigma \hat{R} + \frac{1}{2} \hat{g}^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma \right.$$
$$\left. + \frac{\mu c}{16\pi} \frac{e^{-\sigma}}{12^2} \left(\frac{1}{2} (\partial\sigma)^4 - 2 \hat{\Box} \sigma (\partial\sigma)^2 \right) + \mathcal{O}(\mu^2 c^2) \right\}$$

Apply strategy, main result

Use known holo for $Z_{DBC}(g)$ and 'dissection of Z_{DBC}',
i.e. rewriting of $Z_{DBC}(g)$ ito Weyl mode of 2D metric g, to obtain braneworld holo interpretation of large c Neumann theory

$$Z_{NBC} = \int Dg\, Z_{DBC}(g)$$

'setting free the Dirichlet theory'

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$$Z_{NBC} = \int Dg\, Z_{T\bar{T}}(g) = \int D\sigma\, e^{iS_{\tilde{L}}[\sigma]}\, Z_{T\bar{T}}(\hat{g})$$

AdS$_3$ gravity with NBC $\quad \sim \quad$ '$T\bar{T}$ set free' $\quad$ or
"$T\bar{T}$-deformed CFT" + timelike deformed Liouville

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For asymptotic brane, reduces to 'CFT + timelike Liouville' lore
[Compere Marolf '08, ...], but with explicit bulk derivation

Generalization to non-zero cc on brane

3D/2d braneworld for any tension $T \neq 1 \rightsquigarrow$ any cc on brane

[Upcoming work with Selle]

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Dirichlet theory for $S_{tot} = S_{EH} + S_{GHY} + S_T$

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Radial gravitational HJ equation

$$\hat{\pi}^\mu_\mu \, Z_{grav} = \frac{\sqrt{-g}}{32\pi G_N} R\, Z_{grav} + \frac{8\pi G_N}{\sqrt{-g}} \left(\hat{\pi}_{\mu\nu}\hat{\pi}^{\mu\nu} - (\hat{\pi}^\mu_\mu)^2 \right) Z_{grav}$$

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Generalization to non-zero cc on brane

Our strategy: Solve radial HJ as diff eq for $S_{tot}(g)$ or $W(\sigma, \hat{g})$, then set free

$$Z_{bw} := \int D\sigma \int D\hat{g}\, e^{iW(\sigma,\hat{g})}$$
$$Z_{CBC} := \int D\sigma e^{iW(\sigma,\hat{g})}$$

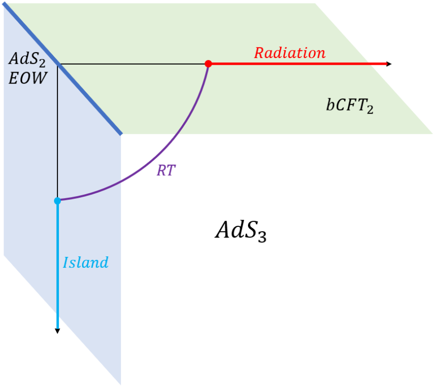
so $W(\sigma, \hat{g})$ is the sought after effective theory on brane/bdy.

$\leftrightarrow$ Typical bw strategy: Compute $S_{tot}(g)$ up to fluctuating radial brane location $\rho = \bar{\rho} + \tilde{\phi}$

* Our method straightforwardly applies to CBC

* $W(\sigma, \hat{g})$ obtained to first orders in derivative expansion but all orders in asymptotic expansion [NC Selle 2607.xxxxx]

Island rule for non-asymptotic brane



$$S_{R,island} = S_{RT} = \frac{c}{6}\log\left(\frac{2L}{\delta}\right) + \frac{c}{6}\mathrm{arccosh}\left(\frac{1}{\sqrt{1-T^2}}\right)$$

Asymptotic brane case: bulk derivation of $S_L(\phi)$

Lore: asymptotic brane theory is 'CFT + timelike Liouville'
[Compere Marolf '08, Takayanagi '18, Suzuki Takayanagi '22, ...]

- Surprisingly difficult to find explicit discussion/derivation in AdS/CFT literature
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Finite difference: Discussed general bulk derivation based on differential equation for the bulk on-shell gravitational action $\delta S_{grav}/\delta\sigma$ that is then integrated. Useful to work directly with finite difference $S_{grav}(e^{\sigma}g) - S_{grav}(g)$.

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New mode: Write 2D conformal boundary metric $g_{(0)} = e^{\phi}\hat{g}_{(0)}$ in terms of *asymptotic* Weyl mode ϕ, to be distinguished from 2D induced metric on EOW brane $g = e^{\sigma}\hat{g}$ in terms of non-asymptotic Weyl mode σ. Only for asymptotic brane, $\sigma \to \phi$.

Bulk derivation of $S_L(\phi)$

Weyl anomaly $\langle T^\mu_\mu \rangle = -\dfrac{c}{12} R_{(0)}$ in CFT

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for bulk solutions (in Fefferman-Graham gauge)

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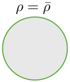
Different bulk sols $\hat{G}(X)$ and $G(X)$ with asymptotic DBC $\hat{g}_{(0)}$ and $g_{(0)} = e^\phi \hat{g}_{(0)}$ at $\rho = \bar{\rho} \to \infty$

Bulk derivation of $S_L(\phi)$

Bulk solutions $\widehat{FG}$ and FG



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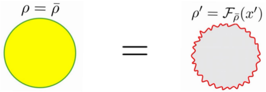

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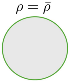

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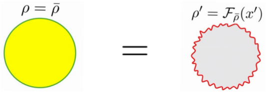

are connected by BrH diffs $X'(X)$ labeled by ϕ:

$$\begin{cases} \rho'(\rho, x) = \rho + \frac{1}{2}\phi(x) + e^{-2\rho} f_\rho(x) + \cdots \\ x'^\mu(\rho, x) = x^\mu + e^{-2\rho} f_x(x) + \cdots \end{cases}$$

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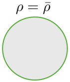

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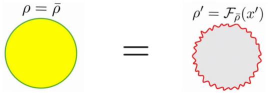

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that are known order by order in $e^{-2\rho}$, and in closed form for $\widehat{FG} \equiv$ Poincaré AdS_3 and $FG \equiv Banados(\phi)$.

Banados(ϕ)

Bulk solutions $\widehat{FG} \equiv$ Poincaré AdS_3 and $FG \equiv Banados(\phi)$

$$\widehat{FG}: \quad \hat{G}(X)dX^2 = d\rho^2 + e^{2\rho}\eta\, dx^2$$

$$FG: \quad G(X)dX^2 = d\rho^2 + e^{2\rho}e^{\phi}(-2dfd\bar{f}) - \frac{6}{c}t^L_{ff}df^2 - \frac{6}{c}t^L_{\bar{f}\bar{f}}d\bar{f}^2 - \frac{18}{c^2}\frac{e^{-2\rho}}{e^{\phi}}t^L_{ff}t^L_{\bar{f}\bar{f}}\,dfd\bar{f}$$

are connected by finite BrH diffs $X'(X)$ labeled by ϕ:

$$\rho'(\rho, x) = \rho + \frac{1}{2}\phi(x) + \log\left(1 + \frac{1}{16}e^{-2\rho}e^{-\phi(x)}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right)$$

$$x'^{\mu}(\rho, x) = x^\mu + \frac{e^{-2\rho}e^{-\phi(x)}\eta^{\mu\nu}\partial_\nu\phi}{4 + \frac{1}{4}e^{-2\rho}e^{-\phi(x)}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi}$$

with Liouville stress tensor $t^L_{ff} = \frac{c}{24}\left((\partial_f\phi)(\partial_f\phi) - 2\partial_f^2\phi\right)$.

Bulk derivation of $S_L(\phi)$

$$\lim_{\bar\rho\to\infty} \quad W_{CFT}(g_{(0)}) \;-\; W_{CFT}(\hat g_{(0)}) = S_L[\phi]$$

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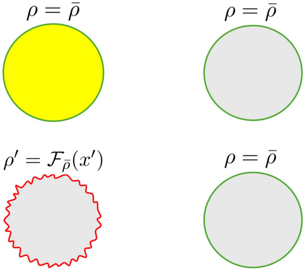

$\leftarrow$ Carlip '05 calculation

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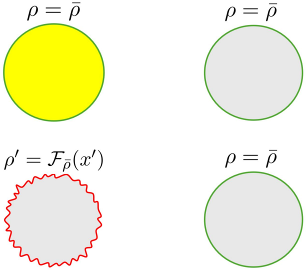

$\leftarrow$ Carlip '05 calculation

Conclusion: Carlip calculates holographically the integrated Weyl anomaly of CFT, *not* an effective Liouville theory with dynamical ϕ. When we set CFT free by integrating $\mathcal{D}g$ or $\mathcal{D}\phi$, we obtain asymptotic brane theory $\int \mathcal{D}\phi \, Z_{CFT}(\hat g_{(0)}) \exp iS_L[\phi]$.

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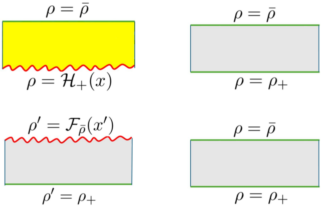
$$\lim_{\bar\rho\to\infty} \quad S_{grav}(FG) \;-\; S_{grav}(\widehat{FG})$$



$\leftarrow$ Poincaré horizon

Bulk derivation of $S_L(\phi)$

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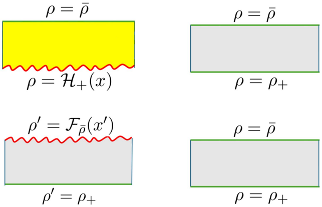


$\leftarrow$ Poincaré horizon

Conclusion: Integrated Weyl anomaly physics $S_L[\phi]$ also describes horizon physics of Banados geometry!

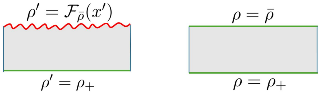
Bulk derivation of $S_{\tilde{L}}(\tilde{\phi})$

New mode: To compare to other strategies in literature for deriving braneworld holo theory [Neuenfeld et al '24, Geng et al '22], introduce third mode $\tilde{\phi}$ ($\neq \phi \neq \sigma$) as fluctuation in radial location of the brane

$$\rho = \bar{\rho} + \tilde{\phi}(x)$$

For finite $\bar{\rho}$,

$$\begin{array}{ccccc} W_{T\bar{T}}(g) & - & W_{T\bar{T}}(\hat{g}) & = & S_{\tilde{L}}[\tilde{\phi}] \\ S_{grav}(FG) & - & S_{grav}(\widehat{FG}) & & \end{array}$$



Bulk derivation of $S_{\tilde{L}}(\tilde{\phi})$

$$S_{\tilde{L}} = \frac{1}{2\kappa}\int d^2x\sqrt{-\hat{g}_{(0)}}\Bigg\{\tilde{\phi}\hat{R}_{(0)} + 2e^{2(\bar{\rho}+\tilde{\phi})} + 2(\partial\tilde{\phi})^2$$

$$- 2e^{2(\bar{\rho}+\tilde{\phi})}\sqrt{1+e^{-2(\bar{\rho}+\tilde{\phi})}(\partial\tilde{\phi})^2}$$

$$+ \frac{\partial^\mu\tilde{\phi}\left[-2e^{-2(\bar{\rho}+\tilde{\phi})}\partial_\mu\tilde{\phi}(\partial\tilde{\phi})^2 + e^{-2(\bar{\rho}+\tilde{\phi})}\partial_\mu(\partial\tilde{\phi})^2\right]}{1+e^{-2(\bar{\rho}+\tilde{\phi})}(\partial\tilde{\phi})^2}\Bigg\}$$

which can also be written as a perturbative in $e^{-2\bar{\rho}}$ result for $S_{\tilde{L}}[\phi]$.

Bulk derivation of $S_{\tilde{L}}(\tilde{\phi})$

$$
\begin{aligned}
S_{\tilde{L}} = \frac{1}{2\kappa}\int d^2x\sqrt{-\hat{g}_{(0)}}\Bigg\{&\tilde{\phi}\hat{R}_{(0)} + 2e^{2(\bar{\rho}+\tilde{\phi})} + 2(\partial\tilde{\phi})^2 \\
&- 2e^{2(\bar{\rho}+\tilde{\phi})}\sqrt{1+e^{-2(\bar{\rho}+\tilde{\phi})}(\partial\tilde{\phi})^2} \\
&+ \frac{\partial^\mu\tilde{\phi}\left[-2e^{-2(\bar{\rho}+\tilde{\phi})}\partial_\mu\tilde{\phi}(\partial\tilde{\phi})^2 + e^{-2(\bar{\rho}+\tilde{\phi})}\partial_\mu(\partial\tilde{\phi})^2\right]}{1+e^{-2(\bar{\rho}+\tilde{\phi})}(\partial\tilde{\phi})^2}\Bigg\}
\end{aligned}
$$

which can also be written as a perturbative in $e^{-2\bar{\rho}}$ result for $S_{\tilde{L}}[\phi]$.

Compare to [Neuenfeld et al '24 "Liouville gravity at the end of the world"]: same $S_{\tilde{L}}(\tilde{\phi})$ but Neuenfeld et al conclude that $S_{\tilde{L}}(\tilde{\phi})$ with dynamic $\tilde{\phi}$ *and* $\hat{g}_{(0)}$ is braneworld gravity theory $\leftrightarrow$ we argue to integrate over $\sigma \neq \tilde{\phi}$. Similar disagreements with [Geng et al '22 "Jackiw-Teitelboim Gravity from the Karch-Randall Braneworld"].

Literature comparison and conclusion

- We argue for dynamical mode $\sigma(\neq \tilde{\phi} \neq \phi)$
- For asymptotic branes $(\sigma \approx \tilde{\phi} \approx \phi)$ CFT+timelike Liouville
- Integrated Weyl anomaly physics $S_L[\phi]$ also describes horizon physics of Banados geometry
- For CBC, our $S_{\tilde{L}}[\sigma]$ and its defining flow equation also appear in [Allameh Shaghoulian '25]

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In summary,

$$\begin{aligned} \text{AdS}_3 \text{ gravity with NBC} \quad &\sim \quad \text{“}T\bar{T}\text{-deformed CFT”} \\ &\qquad\qquad + \text{timelike deformed Liouville} \\ &\sim \quad \text{effective theory } W[\sigma, \hat{g}] \end{aligned}$$

with deformed Liouville $S_{\tilde{L}}^{(\mu)}[\sigma]$ defined by flow equation

$$\frac{d}{d\mu} S_{\tilde{L}}^{(\mu)} = \frac{1}{4\pi} \int d^2x \sqrt{-\hat{g}}\, e^{-\sigma} \mathcal{O}_{T\bar{T}}^{\tilde{L}}, \qquad S_{\tilde{L}}^{(0)} = S_L$$

Thank you!