Multiscale Structure of Eigenstates Thermalization in Integrable Systems

Pavel Orlov

Based on
arXiv:2605.08079

In collaboration with:
Rustem Sharipov,
Enej Ilievski

UNIVERZA V LJUBLJANI
University of Ljubljana

My scientific gang

Rustem Sharipov

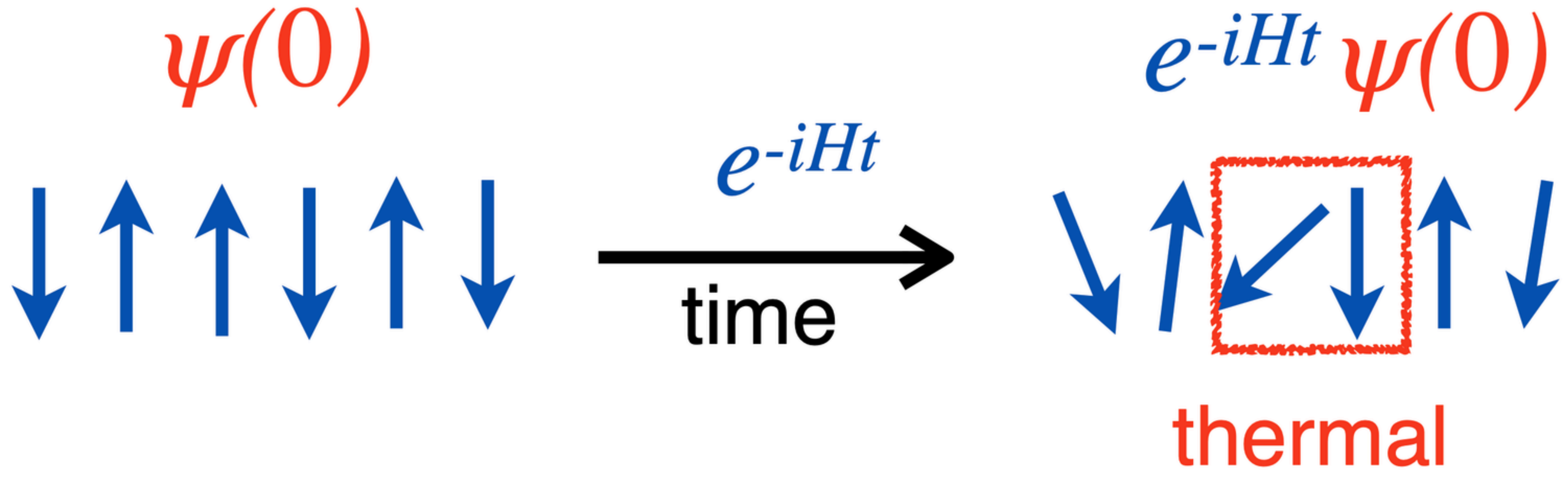
Enej Ilievski

Plan:

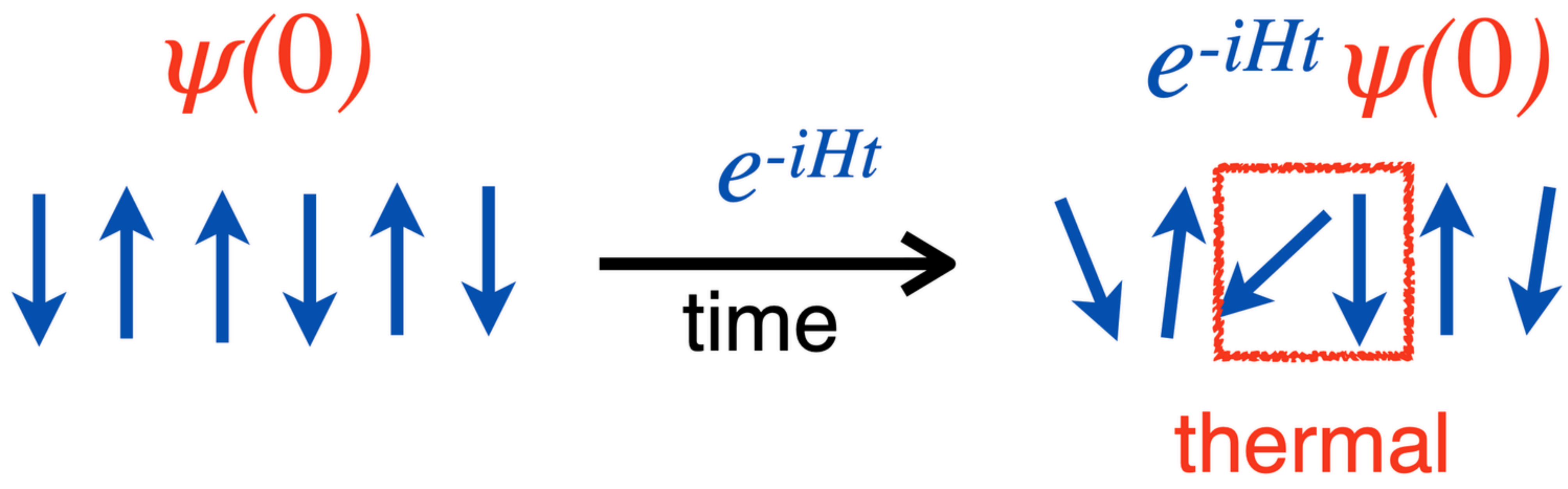
- Quantum thermalization and the ETH ansatz
- Integrable systems: can they help to study ETH?
- A minimal model for probing matrix elements:
  - what we want to study
  - where we study these questions
  - what are the results

ETH – Eigenstate Thermalization Hypothesis

Thermalization in quantum systems



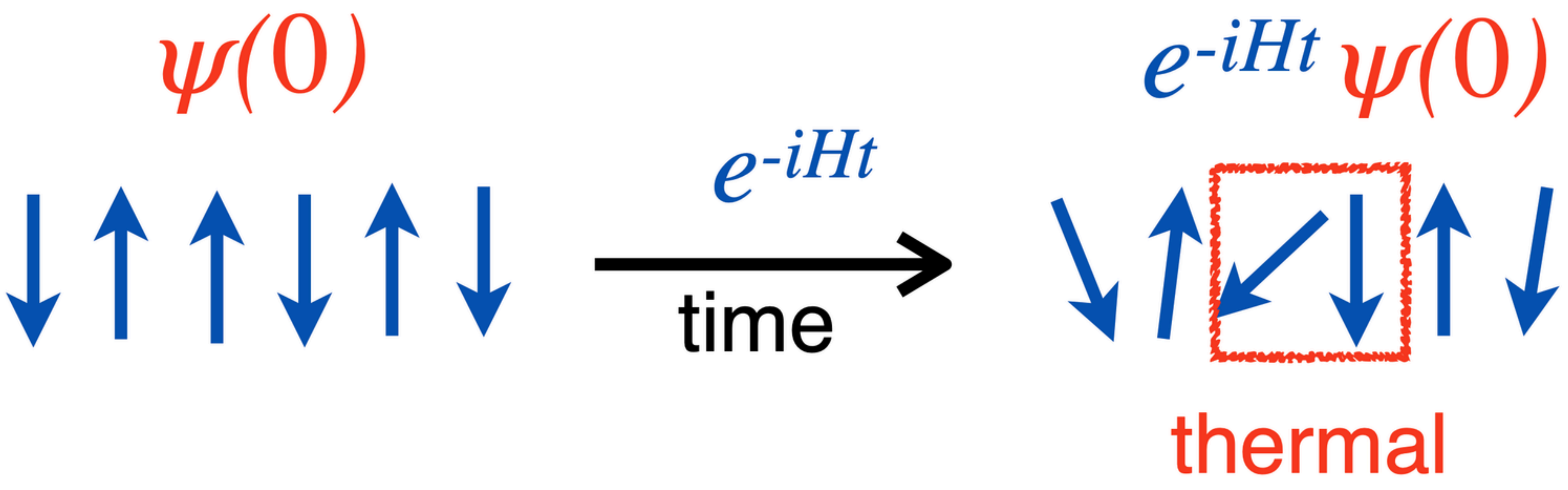
Thermalization in quantum systems



For any $|\psi(0)\rangle = \sum_n c_n |n\rangle$ **the dynamics is explicit:**

$$\langle\psi(t)|\hat{A}|\psi(t)\rangle = \sum_n |c_n|^2 A_{nn} + \sum_{n\neq m} e^{-i(E_m - E_n)t} c_m^* c_n A_{mn}$$

Thermalization in quantum systems

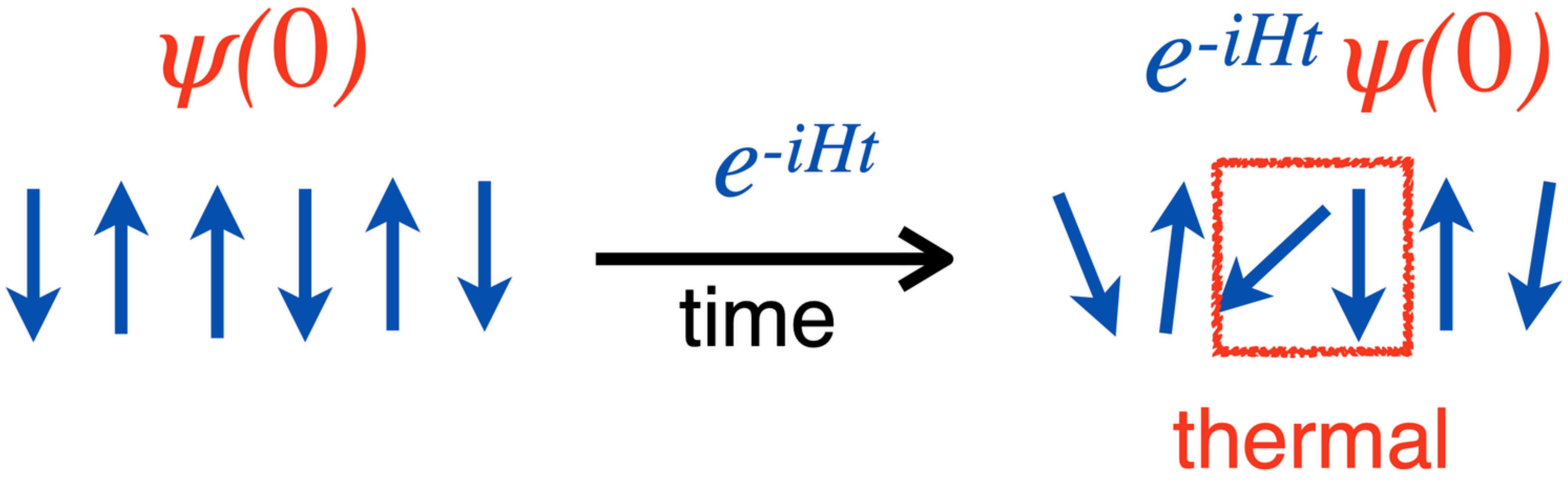


For any $|\psi(0)\rangle = \sum_n c_n |n\rangle$ **the dynamics is explicit:**

$$\langle\psi(t)|\hat{A}|\psi(t)\rangle = \sum_n |c_n|^2 A_{nn} + \sum_{n\neq m} e^{-i(E_m - E_n)t} c_m^* c_n A_{mn}$$

Diagonal ensemble for long-time average: $\overline{A}_\psi = \sum_n |c_n|^2 A_{nn}$

Thermalization in quantum systems



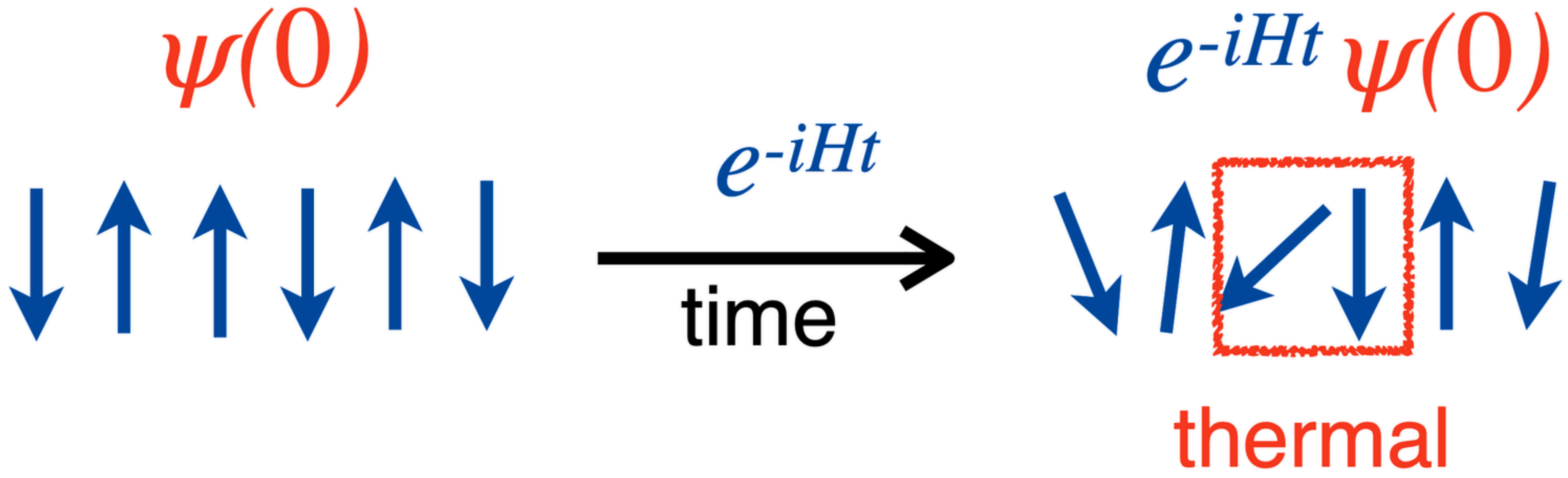
For any $|\psi(0)\rangle = \sum_n c_n |n\rangle$ **the dynamics is explicit:**

$$\langle\psi(t)|\hat{A}|\psi(t)\rangle = \sum_n |c_n|^2 A_{nn} + \sum_{n\neq m} e^{-i(E_m - E_n)t} c_m^* c_n A_{mn}$$

Diagonal ensemble for long-time average: $\overline{A}_\psi = \sum_n |c_n|^2 A_{nn}$

How system can forget about its initial condition?

Thermalization and the ETH ansatz



How system can forget about its initial condition?

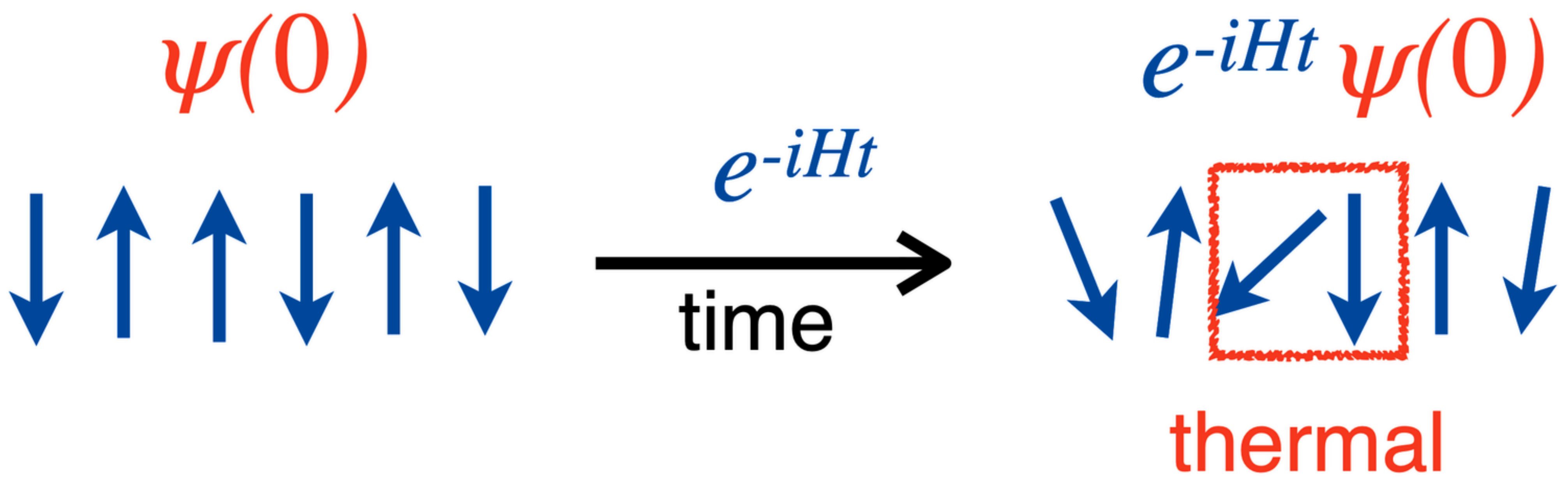
$$\overline{A}_\psi = \sum_n |c_n|^2 A_{nn}$$

Hint: Good clustering properteis of $|\psi(0)\rangle$ $\Rightarrow$

$\overline{E}_\psi = e_\psi L$ extensive mean

$\delta E_\psi \sim L^\gamma$ sub-extensive variance $(\gamma < 1)$

Thermalization and the ETH ansatz



How system can forget about its initial condition?

$$\overline{A}_\psi = \sum_n |c_n|^2 A_{nn}$$

Hint: Good clustering properteis of $|\psi(0)\rangle$ $\Rightarrow$

$\overline{E}_\psi = e_\psi L$ **extensive mean**

$\delta E_\psi \sim L^\gamma$ **sub-extensive variance** $(\gamma < 1)$

Diagonal ETH: $A_{nn} = \mathcal{A}(e) + o(1)$ for states sampled from the same macro-state $\Rightarrow$ $\overline{A}_\psi = \mathcal{A}(e_\psi) + o(1)$

M. Srednicki, Journal of Physics A **32**, 1163-1175 (1999)

The ETH ansatz

Diagonal ETH:

$$A_{nn} = \mathcal{A}(e) + o(1)$$

for states sampled from the same macro-state

- explains thermalization of long-time average

Usually normally distributed pseudo-random numbers

Off-diagonal ETH:

$$A_{mn} = e^{-S(e)/2} f_A(e, \omega) R_{mn}$$

for $\omega = E_n - E_m = O(1)$

- bounds temporal fluctuations
- encodes corr. function

M. Srednicki, Journal of Physics A **32**, 1163-1175 (1999)

The ETH ansatz = "RMT on steroids"

$$\overline{A} = \frac{1}{N}\mathrm{tr}A, \quad \sigma_A^2 = \frac{1}{N}\mathrm{tr}A^2 - \overline{A}^2$$

Matrix elements in RMT:

$$A_{mn} = \delta_{mn}\overline{A} + N^{-1/2}\sigma_A R_{mn} + o(N^{-1/2})$$

ETH ansatz:

$$A_{mn} = \delta_{mn}\mathcal{A}(e) + e^{-S(e)/2} f_A(e,\omega) R_{mn}$$

$$\mathcal{A}(e) = A_{\mathrm{mc}}$$

$$\langle A(t)A(0)\rangle_{\beta,c} = \int d\omega e^{i\omega t} e^{-\beta\omega/2} |f_A(e_\beta,\omega)|^2$$

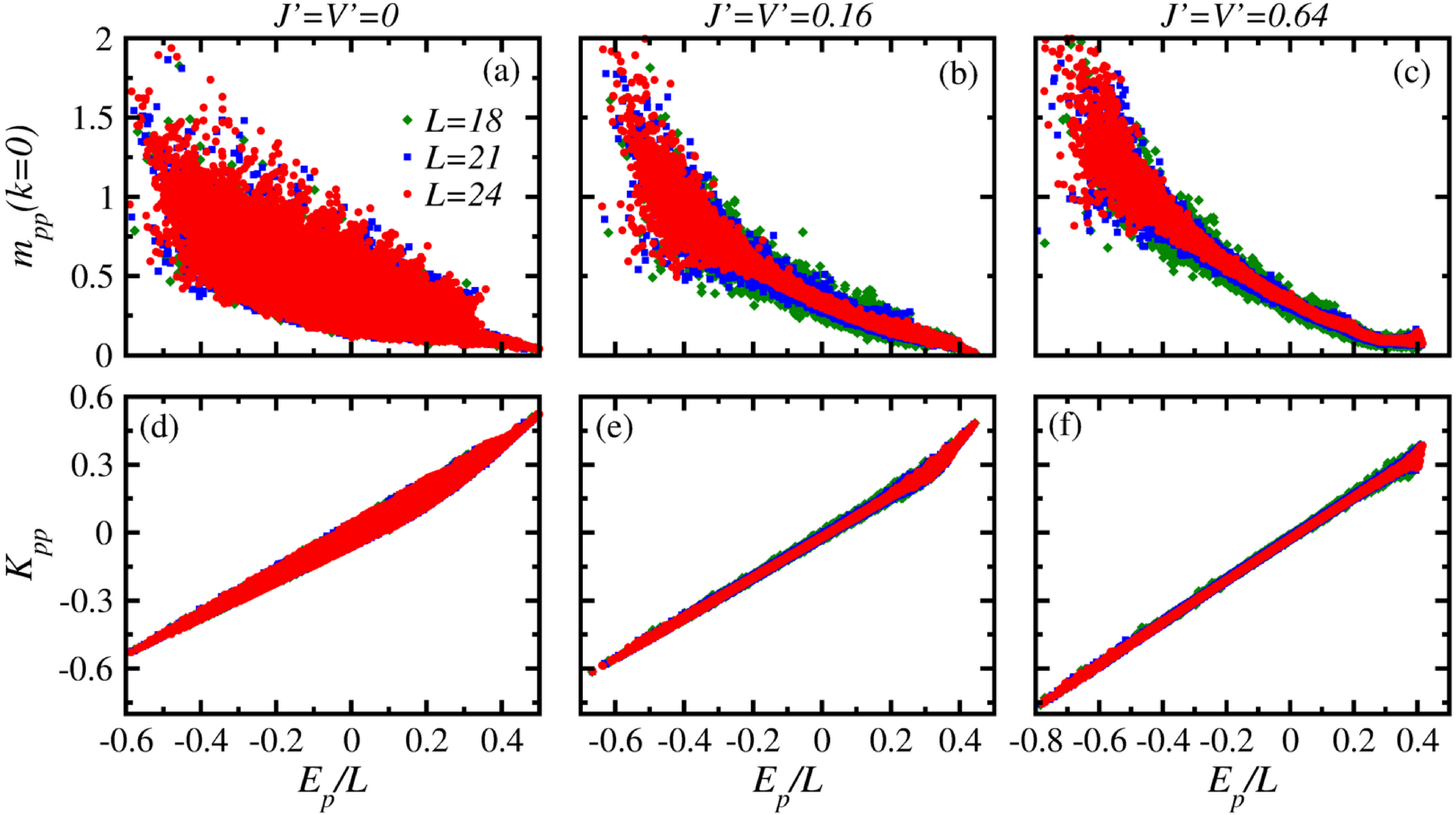
How to verify ETH?

Physical systems: **spin chains**, since Hilbert space is finite-dim

Main tool: **Exact Diagonalization** of the Hamiltonian

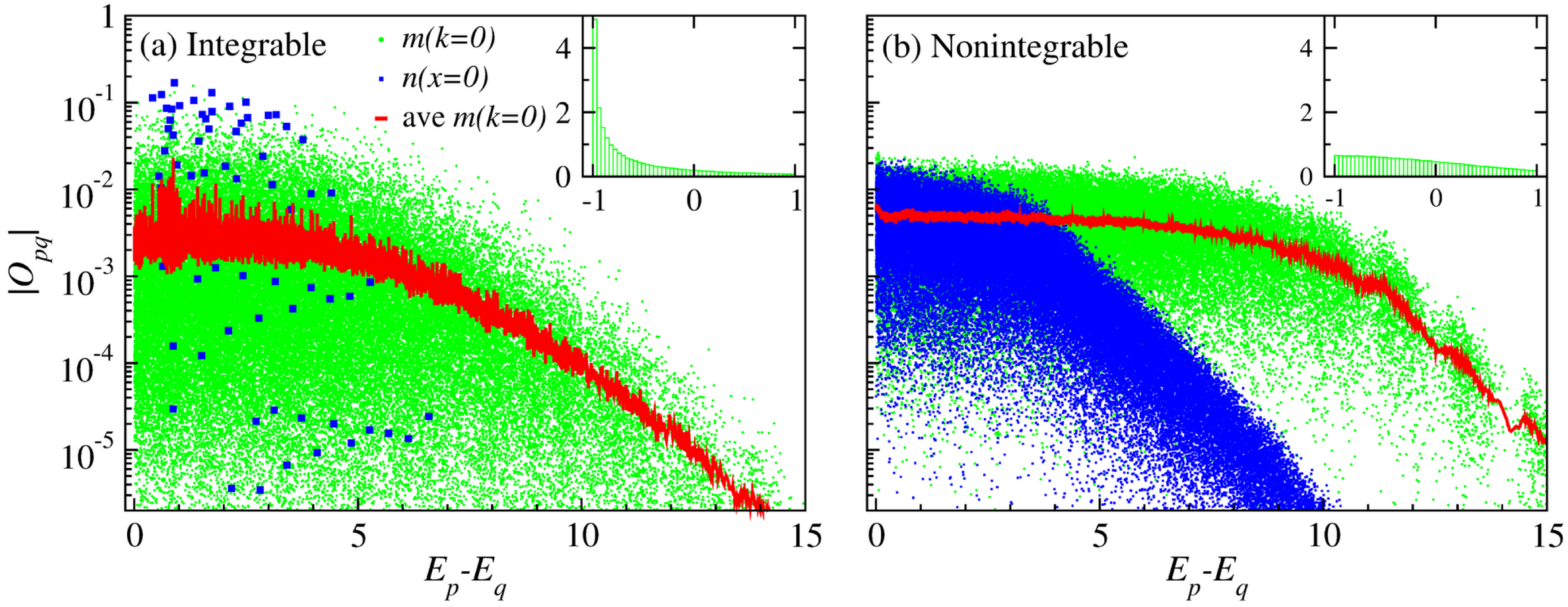
Bottlenecks:
1) ETH is about $L \to \infty$, ED is about $L \sim 20$
2) No analytical control

How to verify ETH? Diagonal



D'Alessio, Kafri, Polkovnikov, Rigol, review paper from 2016

How to verify ETH? Off-diagonal



D'Alessio, Kafri, Polkovnikov, Rigol, review paper from 2016

How to verify ETH?

What about such structures in integrable systems?

1. If similar => integrable systems as toy models to probe ETH

2. If different => a way to understand chaos/integrability interplay

Towards ETH in integrable systems

Do integrable systems thermalize (equilibrate)?

Well, maybe **not to Gibbs** $\hat{\rho}_{Gibbs} = \frac{1}{Z} e^{-\beta \hat{H}}$,

but **to GGE**: $\hat{\rho}_{GGE} = \frac{1}{Z} e^{-\sum_k \mu_k \hat{Q}^{(k)}}$.

And what about integrable systems?

Do integrable systems thermalize (equilibrate)?

Well, maybe **not to Gibbs** $\hat{\rho}_{Gibbs} = \frac{1}{Z} e^{-\beta \hat{H}}$,

but **to GGE**: $\hat{\rho}_{GGE} = \frac{1}{Z} e^{-\sum_k \mu_k \hat{Q}^{(k)}}$.

Can we explain this in a similar manner to the ETH?

Yes, but keep in mind **what the macrostate is**.

The ETH ansatz in integrable systems

Diagonal ETH:

$$A_{nn} = \mathcal{A}(\{q^{(k)}\}) + o(1)$$

for states sampled from the same macro-state

- explains equilibration to GGE of long-time average

Cassidy, Clark and Rigol, PRL **106** (2011)

Mestyán and Pozsgay, J. Stat. Mech. (2014) P09020

A. Dymarsky and K. Pavlenko, PRL **123** (2019)

The ETH ansatz in integrable systems

Diagonal ETH:

$$A_{nn} = \mathcal{A}(\{q^{(k)}\}) + o(1)$$

for states sampled from the same macro-state

- explains equilibration to GGE of long-time average

Cassidy, Clark and Rigol, PRL **106** (2011)

Mestyán and Pozsgay, J. Stat. Mech. (2014) P09020

A. Dymarsky and K. Pavlenko, PRL **123** (2019)

Off-diagonal ETH:

?

The off-diagonal ETH in integrable systems

In all existing studies so far **only energy difference** between eigenstates **was taken into account**…

The most recent "Bethe ansatz" studies:

- **Lieb-Liniger**, Essler and de Klerk, PRX **14** (2024)
- **XXX chain**, Rottoli and Alba, PRB **113** (2026)

Structure **incompatible with** conventional **ETH**:

$$A_{mn} \sim \exp\left(-c_A L \log L - L M_{mn}^{A}\right)$$

But what is the “distance” between eigenstates?

Just keep in mind **what the macrostate is** in your system!

	Macrostate defined by:	**Distance between states:**
Chaotic system	Energy	$\text{dist}(n,m) = \lvert E_n - E_m \rvert = \lvert \omega_{nm} \rvert$
Integrable system	Local charges	$\text{dist}(n,m) = \frac{1}{N_Q} \sum_{k=1}^{N_Q} \lvert Q_n^{(k)} - Q_m^{(k)} \rvert$

Ensembles with different fluctuation scales

An **ensemble of states** $\mathcal{E}_\gamma$ corresponds to a macrostate $\{q^{(k)}\}_{k=1}^{N_Q}$ and has a **fluctuation scale** $\gamma < 1$ if:

$$Q^{(k)} = \langle \hat{Q}^{(k)} \rangle_{\mathcal{E}_\gamma} = Lq^{(k)} + o(1)$$

$$\delta Q^{(k)} = \sqrt{\langle (\hat{Q}^{(k)})^2 \rangle_{\mathcal{E}_\gamma} - \langle \hat{Q}^{(k)} \rangle_{\mathcal{E}_\gamma}^2} \sim L^\gamma \qquad (\mathrm{dist}_{\mathcal{E}_\gamma} \sim L^\gamma)$$

Ensembles with different fluctuation scales

An **ensemble of states** $\mathcal{E}_\gamma$ corresponds to a macrostate $\{q^{(k)}\}_{k=1}^{N_Q}$ and has a **fluctuation scale** $\gamma < 1$ if:

$$Q^{(k)} = \langle \hat{Q}^{(k)} \rangle_{\mathcal{E}_\gamma} = Lq^{(k)} + o(1)$$

$$\delta Q^{(k)} = \sqrt{\langle (\hat{Q}^{(k)})^2 \rangle_{\mathcal{E}_\gamma} - \langle \hat{Q}^{(k)} \rangle_{\mathcal{E}_\gamma}^2} \sim L^\gamma \qquad (\mathrm{dist}_{\mathcal{E}_\gamma} \sim L^\gamma)$$

Example in chaotic case:

microcanonical $\rho_{mc} \sim \sum_{n:|E_n - eL| < d} |n\rangle\langle n|$

with width $d \sim L^\gamma$

Ensembles with different fluctuation scales

An **ensemble of states** $\mathcal{E}_\gamma$ corresponds to a macrostate $\{q^{(k)}\}_{k=1}^{N_Q}$ and has a **fluctuation scale** $\gamma < 1$ if:

$$Q^{(k)} = \langle \hat{Q}^{(k)} \rangle_{\mathcal{E}_\gamma} = L q^{(k)} + o(1)$$

$$\delta Q^{(k)} = \sqrt{\langle (\hat{Q}^{(k)})^2 \rangle_{\mathcal{E}_\gamma} - \langle \hat{Q}^{(k)} \rangle_{\mathcal{E}_\gamma}^2} \sim L^\gamma \qquad (\mathrm{dist}_{\mathcal{E}_\gamma} \sim L^\gamma)$$

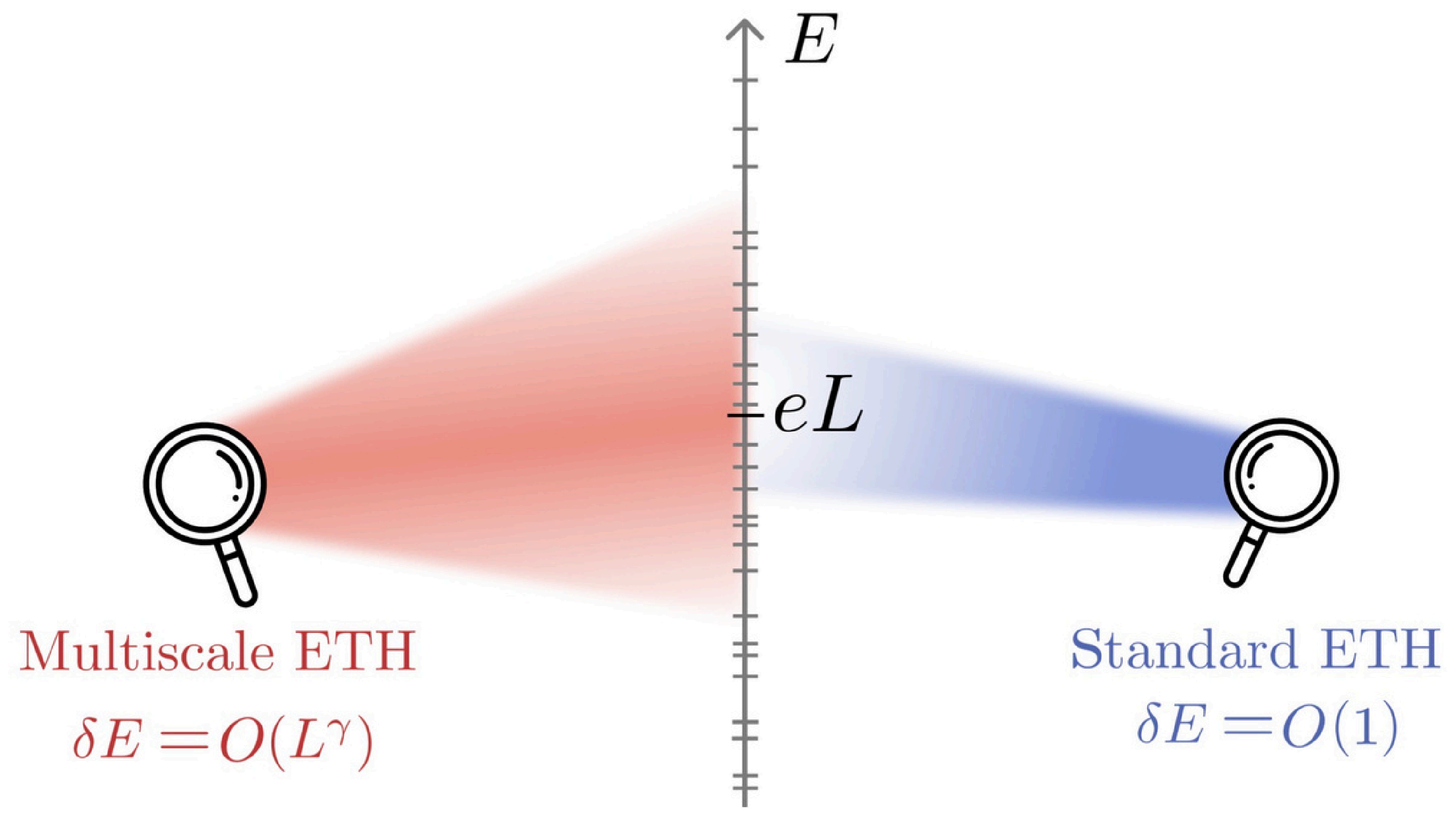
Example in chaotic case: microcanonical $\rho_{mc} \sim \sum_{n:|E_n - eL| < d} |n\rangle\langle n|$

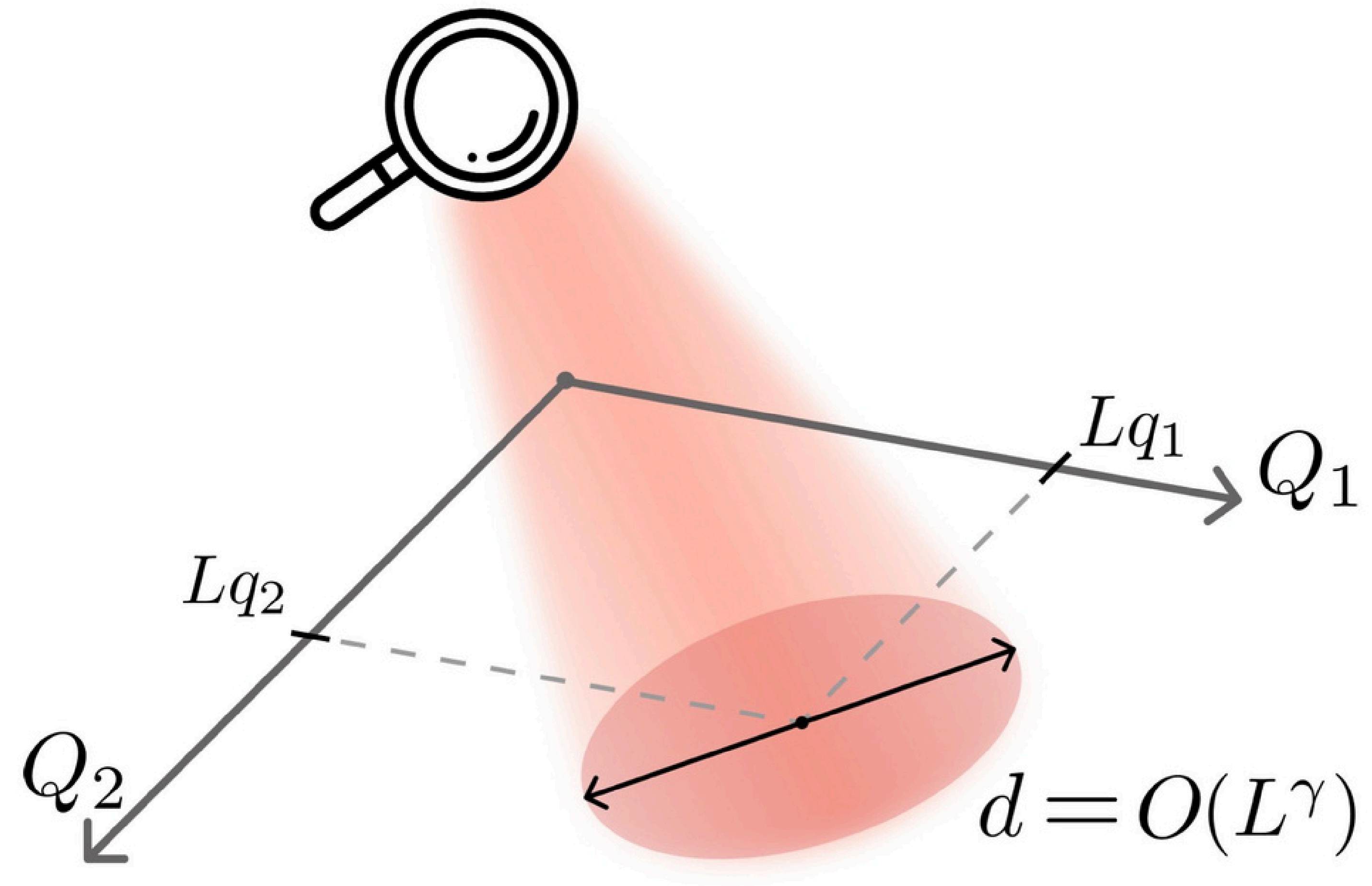
with width $d \sim L^\gamma$

Example in integrable case: generalized microcanonical with extra widths for all other charges

Note! All studies before dealt with microcan. ensemble, **even for integrable systems!**

Ensembles with different fluctuation scales





Matrix elements at different fluctuation scales

Having an ensemble of states $\mathcal{E}_\gamma$, we are interested in:

$$\kappa_{mn} = -\frac{1}{L}\log|A_{mn}|^2$$

(expected to be of order 1)

Matrix elements at different fluctuation scales

Having an ensemble of states $\mathcal{E}_\gamma$, we are interested in:

$$\kappa_{mn} = -\frac{1}{L}\log|A_{mn}|^2$$

(expected to be of order 1)

In particular, we define:

suppression rate $$\kappa \equiv \langle \kappa_{mn} \rangle_{\mathcal{E}_\gamma}$$

distribution width $$\delta\kappa \equiv \sqrt{\langle \kappa_{mn}^2 \rangle_{\mathcal{E}_\gamma} - \langle \kappa_{mn} \rangle_{\mathcal{E}_\gamma}^2}$$

Matrix elements at different fluctuation scales

Having an ensemble of states $\mathcal{E}_\gamma$, we are interested in:

$$\kappa_{mn} = -\frac{1}{L}\log|A_{mn}|^2$$ (expected to be of order of 1)

In particular, we define:

suppression rate $$\kappa \equiv \langle \kappa_{mn} \rangle_{\mathcal{E}_\gamma}$$

distribution width $$\delta\kappa \equiv \sqrt{\langle \kappa_{mn}^2 \rangle_{\mathcal{E}_\gamma} - \langle \kappa_{mn} \rangle_{\mathcal{E}_\gamma}^2}$$

and their **scaling exponents** $$p(\gamma) \equiv \frac{\partial \log(\kappa)}{\partial \log(L)}, \quad q(\gamma) \equiv \frac{\partial \log(\delta\kappa)}{\partial \log(L)}$$

About the Model

To study questions we ask requires:

- to be able **to define ensembles** $\mathcal{E}_\gamma$ from which states can be effectively sampled for any γ,

- to have **access to the matrix elements** of some local observables.

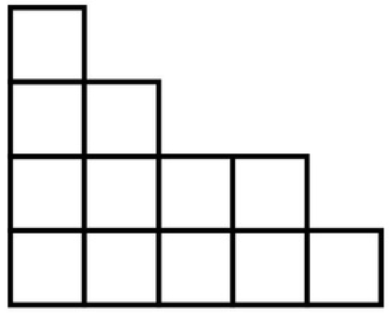
About the Model – Quantum Benjamin-Ono (qBO)

1. **qBO** - integrable hierarchy defined in CFTs

About the Model – Quantum Benjamin-Ono (qBO)

1. **qBO** - integrable hierarchy defined in CFTs

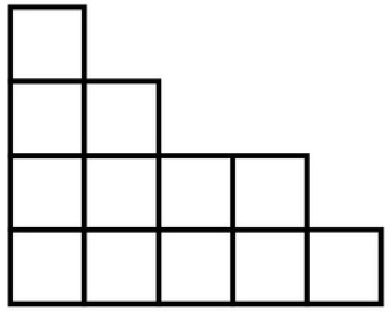
2. **qBO** - system where everything can be computed using only

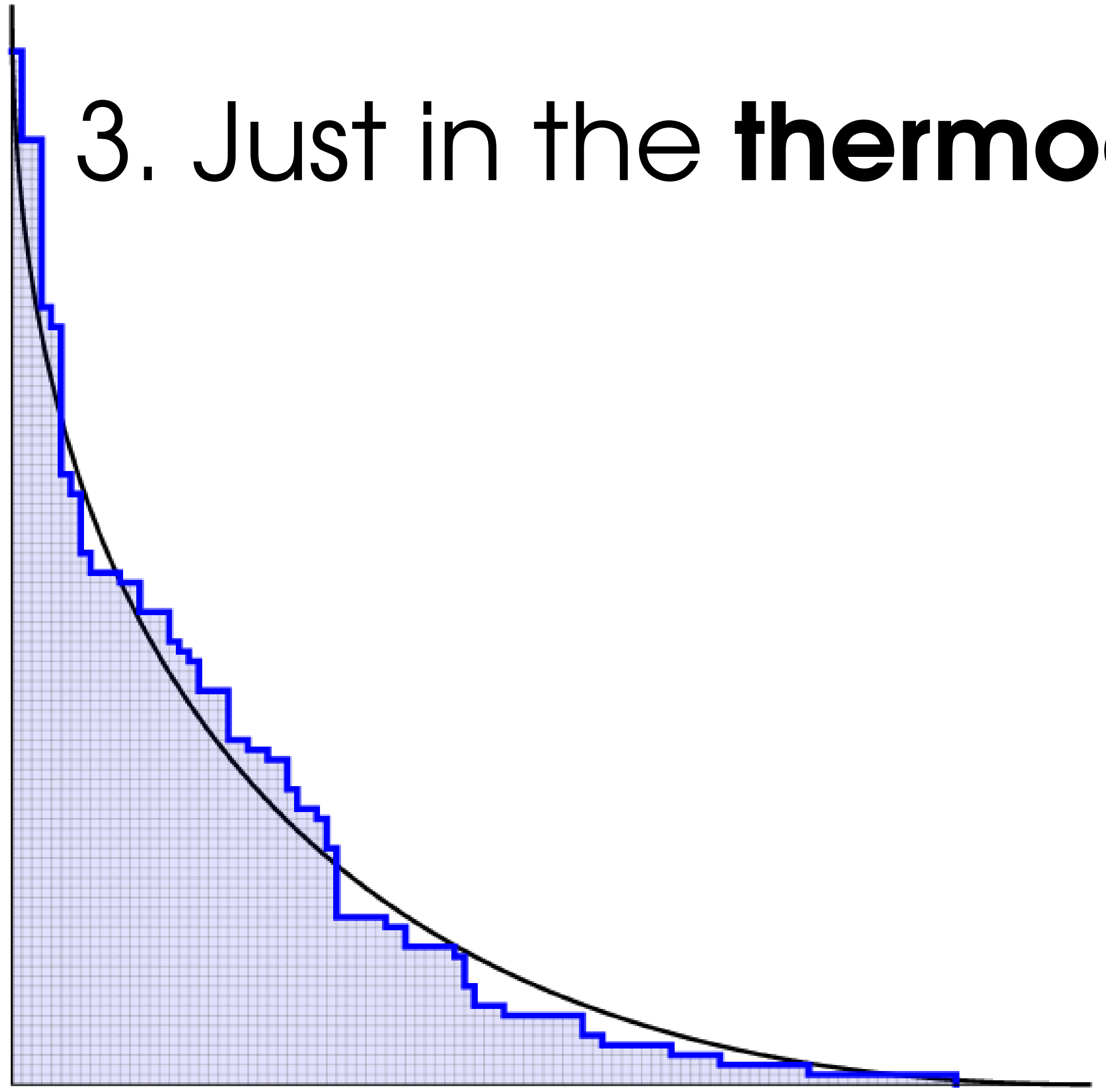
combinatorics of these small objects:  - **Young diagrams**!

About the Model – Quantum Benjamin-Ono (qBO)

1. **qBO** - integrable hierarchy defined in CFTs

2. **qBO** - system where everything can be computed using only

combinatorics of these small objects:  - **Young diagrams**!



3. Just in the **thermodynamic limit** they are **not as small** anymore,

but still tractable

both analytically and numerically!

Benjamin-Ono hierarchy

To define charges we need **a single bosonic field**

$$\hat{\varphi}(\mathrm{x}) = -i \sum_{n \neq 0} \frac{\hat{a}_n}{n} e^{-i(2\pi n/L)\mathrm{x}} \qquad \text{with} \qquad [\hat{a}_n, \hat{a}_m] = n\,\delta_{n,-m}$$

and associated **U(1) current**

$$\hat{J}(\mathrm{x}) = -\partial_{\mathrm{x}} \hat{\varphi}(\mathrm{x}) = \frac{2\pi}{L} \sum_{n \neq 0} \hat{a}_n e^{-i(2\pi n/L)\mathrm{x}}$$

Benjamin-Ono hierarchy

Using U(1) current one can build a family of charges $\hat{Q}^{(k)} = \int_0^L \mathrm{dx}\, \hat{q}^{(k)}(\mathrm{x})$

with **the first densities**

$$\hat{q}^{(1)} = \frac{1}{4\pi} : J^2 :, \quad \hat{q}^{(2)} = \frac{1}{4\pi}\left(\frac{1}{3} : J^3 : -\frac{1}{2}(\beta - \beta^{-1}) : J' \,\mathbb{H}[J] :\right)$$

with Hilbert transform $\mathbb{H}[f](\mathrm{x}) = \frac{1}{L}\mathrm{p.v.}\int_0^L \mathrm{dy}\; f(\mathrm{y})\; \mathrm{ctg}\left(\frac{\pi}{L}(\mathrm{x}-\mathrm{y})\right)$

Awata et. al, PRB **347** (1995); Nazarov and Sklyanin (2013)

Benjamin-Ono hierarchy

Using U(1) current one can build a family of charges $\hat{Q}^{(k)} = \int_0^L \mathrm{dx}\, \hat{q}^{(k)}(\mathrm{x})$

with **the first densities**

$$\hat{q}^{(1)} = \frac{1}{4\pi} : J^2 :, \qquad \hat{q}^{(2)} = \frac{1}{4\pi}\left(\frac{1}{3} : J^3 : -\frac{1}{2}(\beta - \beta^{-1}) : J' \,\mathbb{H}[J] :\right)$$

with Hilbert transform $\mathbb{H}[f](\mathrm{x}) = \frac{1}{L}\mathrm{p.v.}\int_0^L \mathrm{dy}\; f(\mathrm{y})\,\mathrm{ctg}\left(\frac{\pi}{L}(\mathrm{x}-\mathrm{y})\right)$

Or **in modes:**

$$\hat{Q}^{(1)} = \left(\frac{2\pi}{L}\right)\sum_{n>0}\hat{a}_{-n}\hat{a}_n$$

$$\hat{Q}^{(2)} = \left(\frac{2\pi}{L}\right)^2\left(\frac{1}{6}\sum_{i+j+k=0} : \hat{a}_i\hat{a}_j\hat{a}_k : +\frac{1}{4}(\beta-\beta^{-1})\sum_{n\neq 0}|n| : \hat{a}_{-n}\hat{a}_n :\right)$$

Awata et. al, PRB **347** (1995); Nazarov and Sklyanin (2013)

Why qBO? System with known spectrum

1) Eigenstates are labeled by **partitions** $\boldsymbol{\lambda} = (\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_l > 0)$
and constructed using **Jack polynomials**

Why qBO? System with known spectrum

1) Eigenstates are labeled by **partitions** $\boldsymbol{\lambda} = (\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_l > 0)$ and constructed using **Jack polynomials**

2) The eigenvalues are expressed simply using **combinatorics of Young diagrams**:

$$\hat{Q}^{(k)}|\boldsymbol{\lambda}\rangle = \left(\frac{2\pi}{L}\right)^k \sum_{\square\in\boldsymbol{\lambda}} u_\square^{k-1}|\boldsymbol{\lambda}\rangle$$

with $u_\square = \beta^{-1}(i_\square - 1) - \beta(j_\square - 1)$

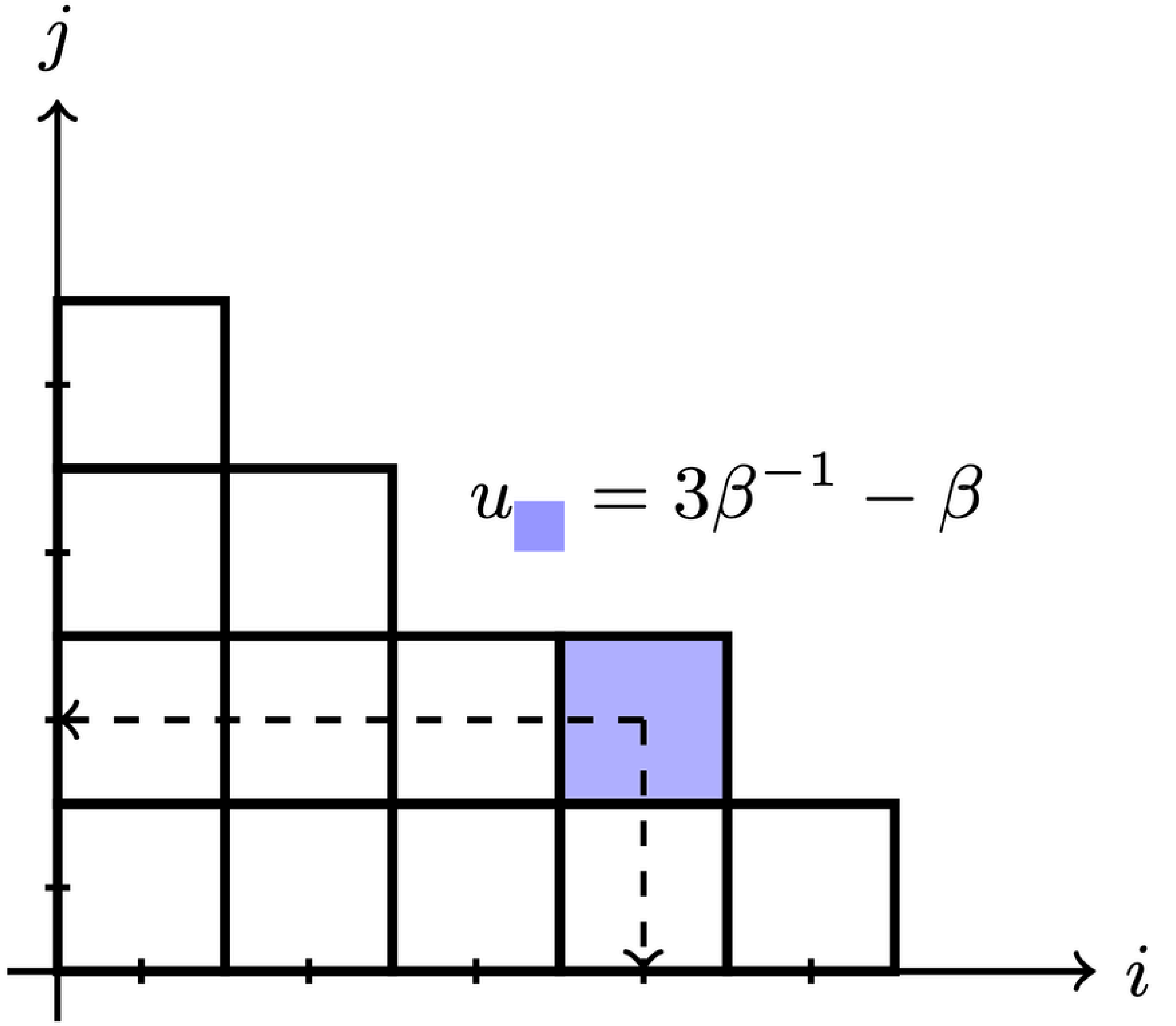


Fig. Young diagram for $\boldsymbol{\lambda} = (5,4,2,1)$

Why qBO? System with known spectrum

1) Eigenstates are labeled by **partitions** $\boldsymbol{\lambda} = (\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_l > 0)$ and constructed using **Jack polynomials**

2) The eigenvalues are expressed simply using **combinatorics of Young diagrams**:

$$\hat{Q}^{(k)}|\boldsymbol{\lambda}\rangle = \left(\frac{2\pi}{L}\right)^k \sum_{\square\in\boldsymbol{\lambda}} u_\square^{k-1}|\boldsymbol{\lambda}\rangle$$

with $u_\square = \beta^{-1}(i_\square - 1) - \beta(j_\square - 1)$

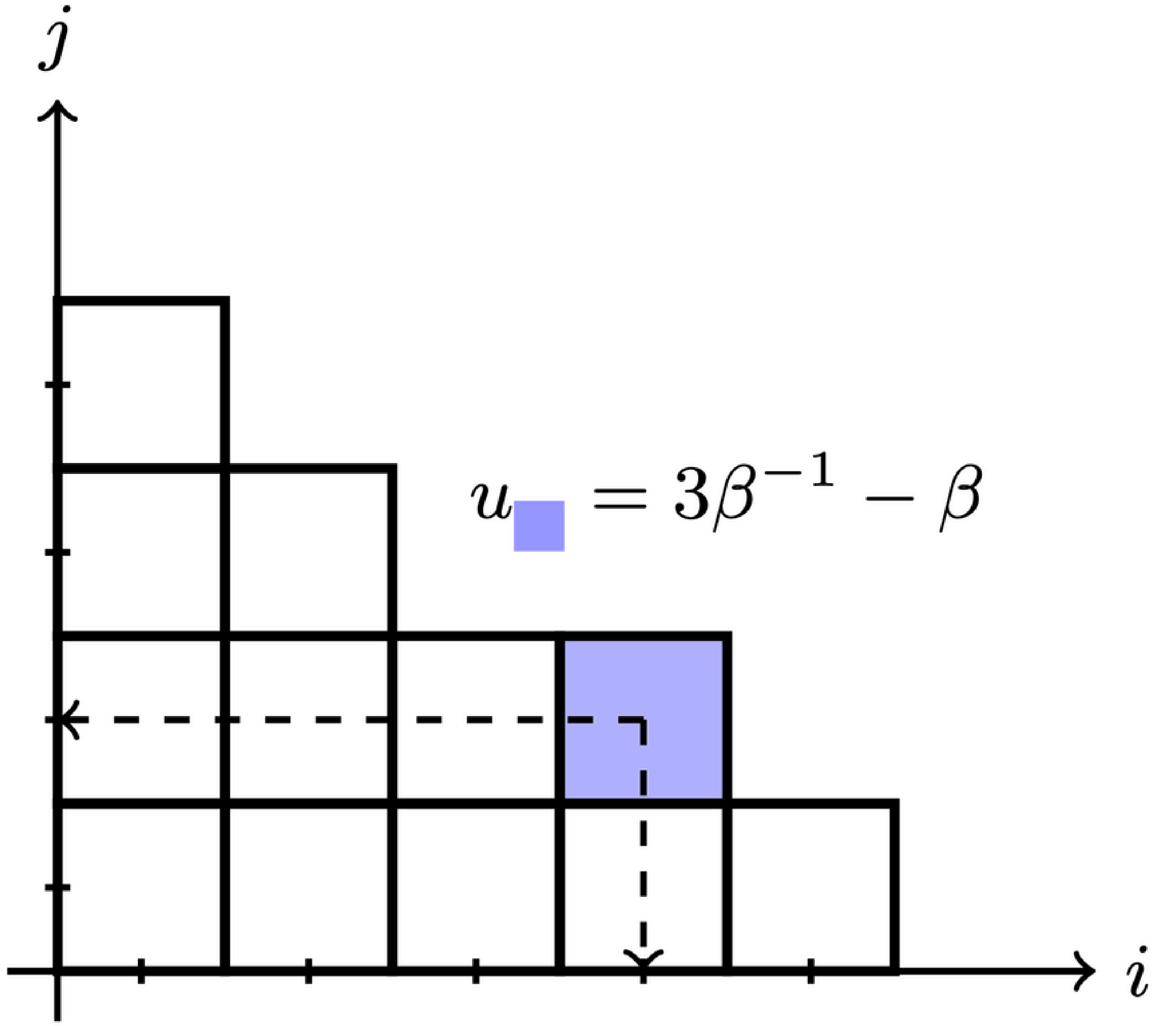


Fig. Young diagram for $\boldsymbol{\lambda} = (5,4,2,1)$

Why qBO? System with known matrix elements

Modified vertex operators: $$V_\alpha(\mathrm{x}) = e^{i(\alpha+\alpha_0)\varphi_-} e^{i\alpha\varphi_+}$$

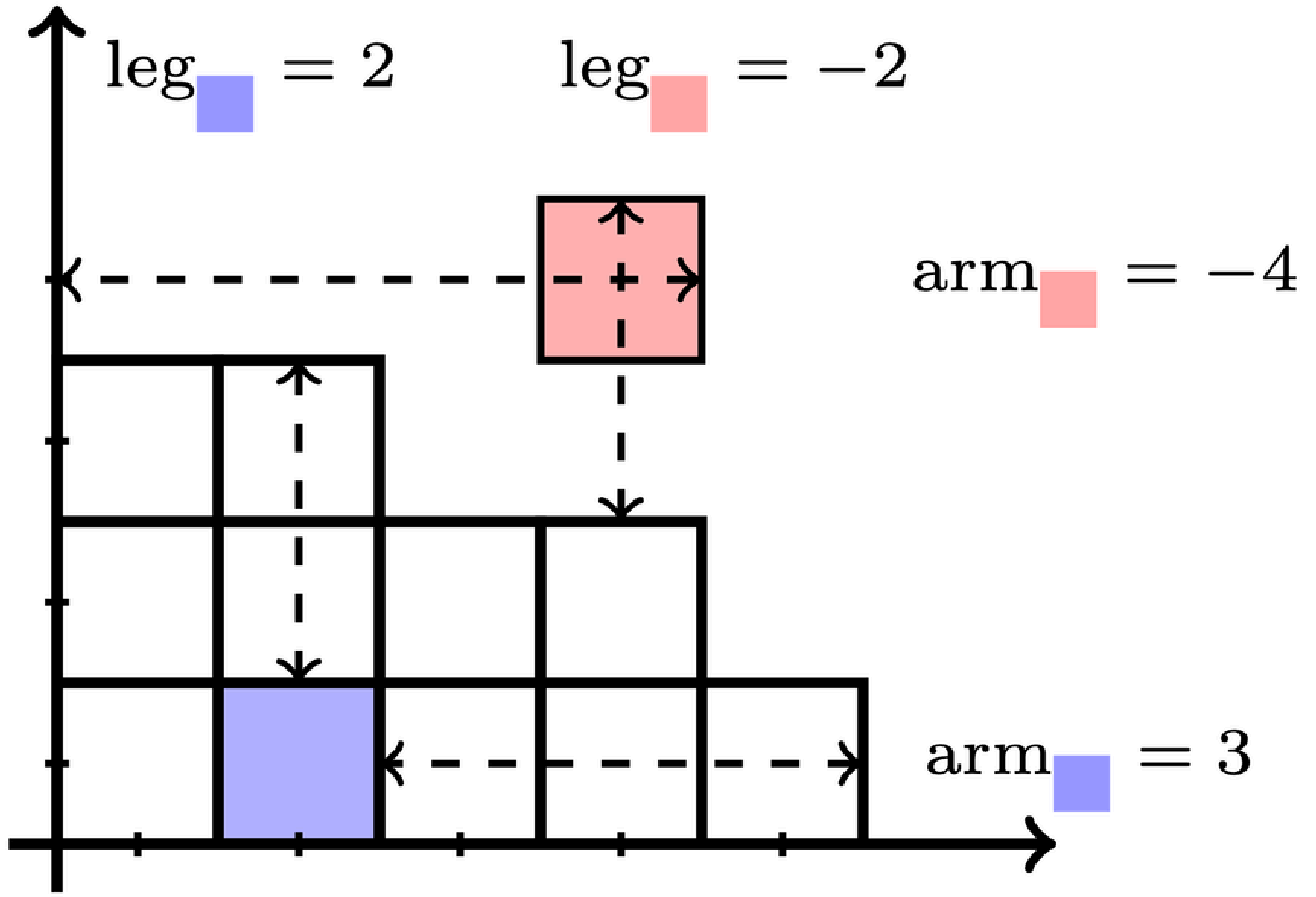
with $\varphi_- = -i \sum_{n<0} \frac{\hat{a}_n}{n} e^{-i(2\pi n/L)\mathrm{x}}, \quad \varphi_+ = -i \sum_{n>0} \frac{\hat{a}_n}{n} e^{-i(2\pi n/L)\mathrm{x}}$ and $\alpha_0 = \beta - \beta^{-1}.$

Why qBO? System with known matrix elements

Modified vertex operators: $$V_\alpha(\mathrm{x}) = e^{i(\alpha+\alpha_0)\varphi_-} e^{i\alpha\varphi_+}$$

with $\varphi_- = -i\sum_{n<0}\frac{\hat{a}_n}{n}e^{-i(2\pi n/L)\mathrm{x}}, \quad \varphi_+ = -i\sum_{n>0}\frac{\hat{a}_n}{n}e^{-i(2\pi n/L)\mathrm{x}}$ and $\alpha_0 = \beta - \beta^{-1}$.

Matrix elements of such operators, again, expressed using combinatorics of diagrams:
legs and arms



Why qBO? System with known matrix elements

Modified vertex operators: $$V_\alpha(\mathrm{x}) = e^{i(\alpha+\alpha_0)\varphi_-} e^{i\alpha\varphi_+}$$

Diagonal element: $$\langle \boldsymbol{\lambda} | V_\alpha(0) | \boldsymbol{\lambda} \rangle = \prod_{\square \in \boldsymbol{\lambda}} \left(1 - \frac{\alpha}{F_1^{\boldsymbol{\lambda}}(\square)}\right)\left(1 + \frac{\alpha}{F_2^{\boldsymbol{\lambda}}(\square)}\right)$$

with
$$F_1^{\boldsymbol{\lambda}} = \beta\, \mathrm{arm}_{\boldsymbol{\lambda}}(\square) + \beta^{-1}\left(\mathrm{leg}_{\boldsymbol{\lambda}}(\square) + 1\right)$$
$$F_2^{\boldsymbol{\lambda}} = \beta\left(\mathrm{arm}_{\boldsymbol{\lambda}}(\square) + 1\right) + \beta^{-1}\, \mathrm{leg}_{\boldsymbol{\lambda}}(\square)$$

V. Alba et. al, Lett Math Phys **98**, 33–64 (2011)

Why qBO? System with known matrix elements

Modified vertex operators: $$V_\alpha(\mathrm{x}) = e^{i(\alpha+\alpha_0)\varphi_-} e^{i\alpha\varphi_+}$$

Diagonal element: $$\langle \boldsymbol{\lambda} | V_\alpha(0) | \boldsymbol{\lambda} \rangle = \prod_{\square \in \boldsymbol{\lambda}} \left(1 - \frac{\alpha}{F_1^{\boldsymbol{\lambda}}(\square)}\right)\left(1 + \frac{\alpha}{F_2^{\boldsymbol{\lambda}}(\square)}\right)$$

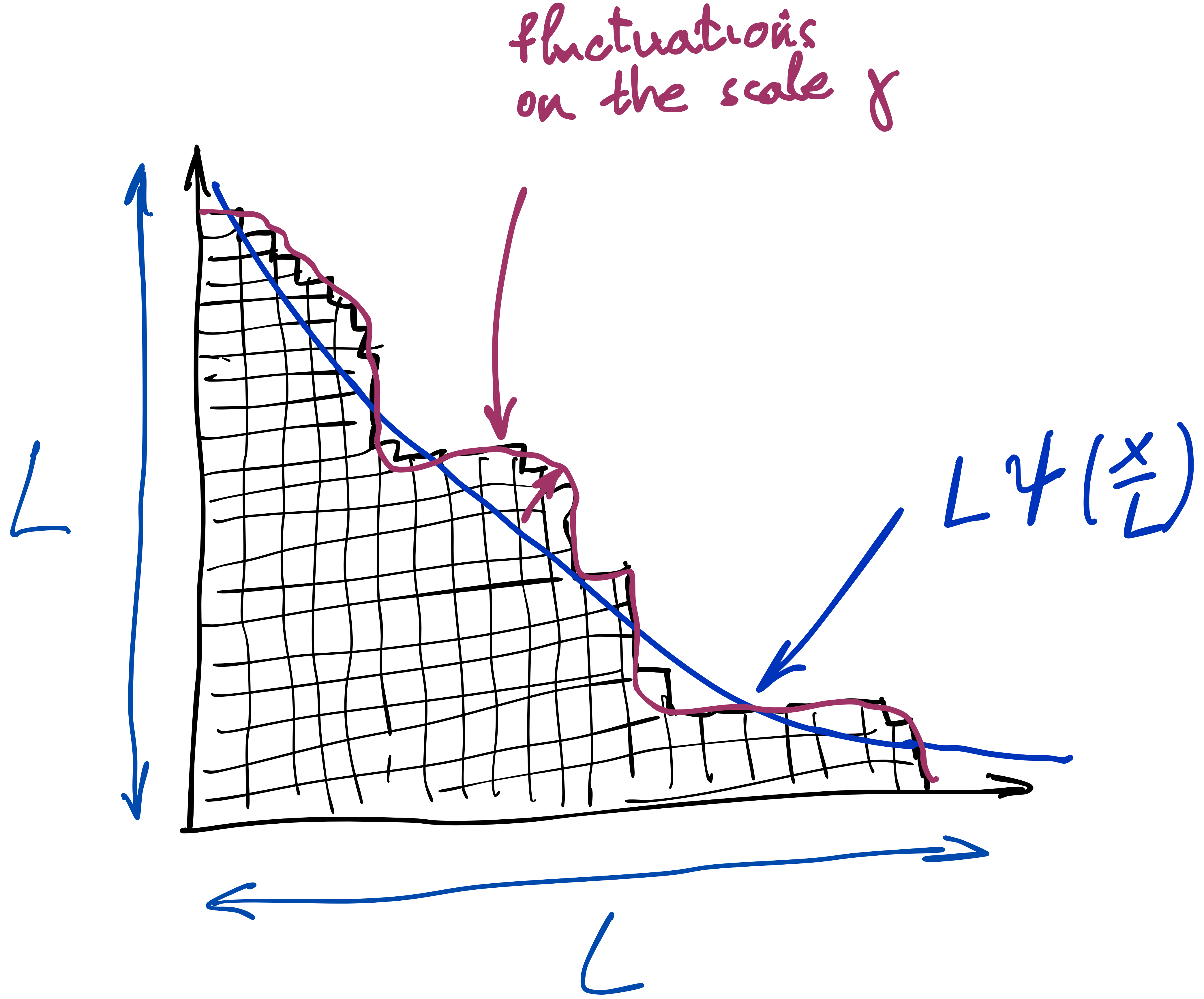
with
$$F_1^{\boldsymbol{\lambda}} = \beta\,\mathrm{arm}_{\boldsymbol{\lambda}}(\square) + \beta^{-1}\left(\mathrm{leg}_{\boldsymbol{\lambda}}(\square) + 1\right)$$
$$F_2^{\boldsymbol{\lambda}} = \beta\left(\mathrm{arm}_{\boldsymbol{\lambda}}(\square) + 1\right) + \beta^{-1}\,\mathrm{leg}_{\boldsymbol{\lambda}}(\square)$$

Off-diagonal element: $$|\langle \boldsymbol{\lambda} | V_\alpha(1) | \boldsymbol{\mu} \rangle|^2 = \prod_{\square \in \boldsymbol{\lambda}} \frac{(F_1^{\boldsymbol{\lambda\mu}} - \alpha)^2}{F_1^{\boldsymbol{\lambda}} F_2^{\boldsymbol{\lambda}}} \prod_{\square \in \boldsymbol{\mu}} \frac{(F_2^{\boldsymbol{\lambda\mu}} + \alpha)^2}{F_1^{\boldsymbol{\mu}} F_2^{\boldsymbol{\mu}}}$$

with
$$F_1^{\boldsymbol{\lambda\mu}}(\square) = \beta\,\mathrm{arm}_{\boldsymbol{\mu}}(\square) + \beta^{-1}(\mathrm{leg}_{\boldsymbol{\lambda}}(\square) + 1)$$
$$F_2^{\boldsymbol{\lambda\mu}}(\square) = \beta\left(\mathrm{arm}_{\boldsymbol{\lambda}}(\square) + 1\right) + \beta^{-1}\,\mathrm{leg}_{\boldsymbol{\mu}}(\square)$$

V. Alba et. al, Lett Math Phys **98**, 33–64 (2011)

Towards ensembles



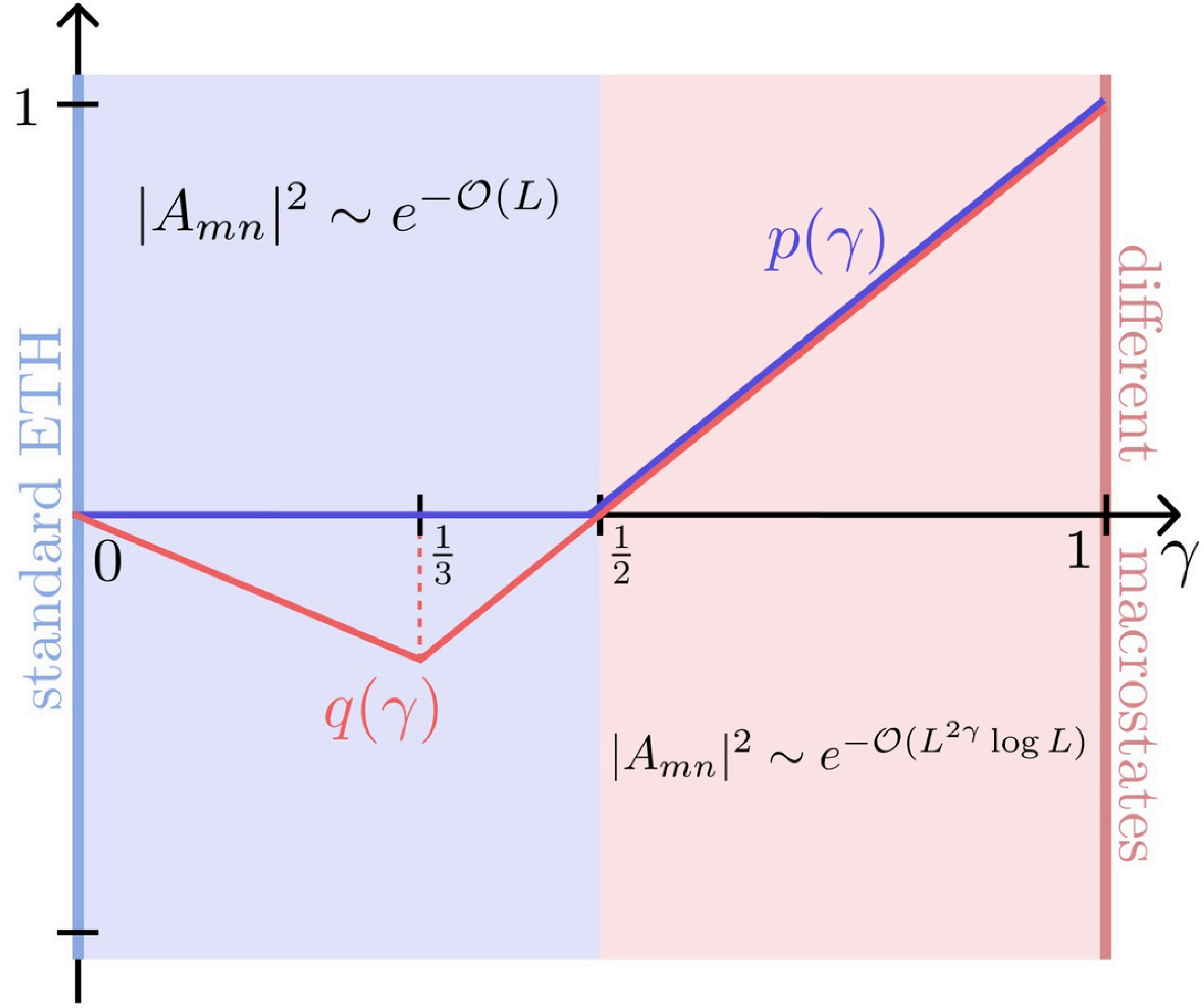
1. Macrostate = limit shape

2. Ensemble — fluctuations on top of the limit shape with a given scale

3. For simplest Gibbs state both the limit shape and the fluctuations are known analytically

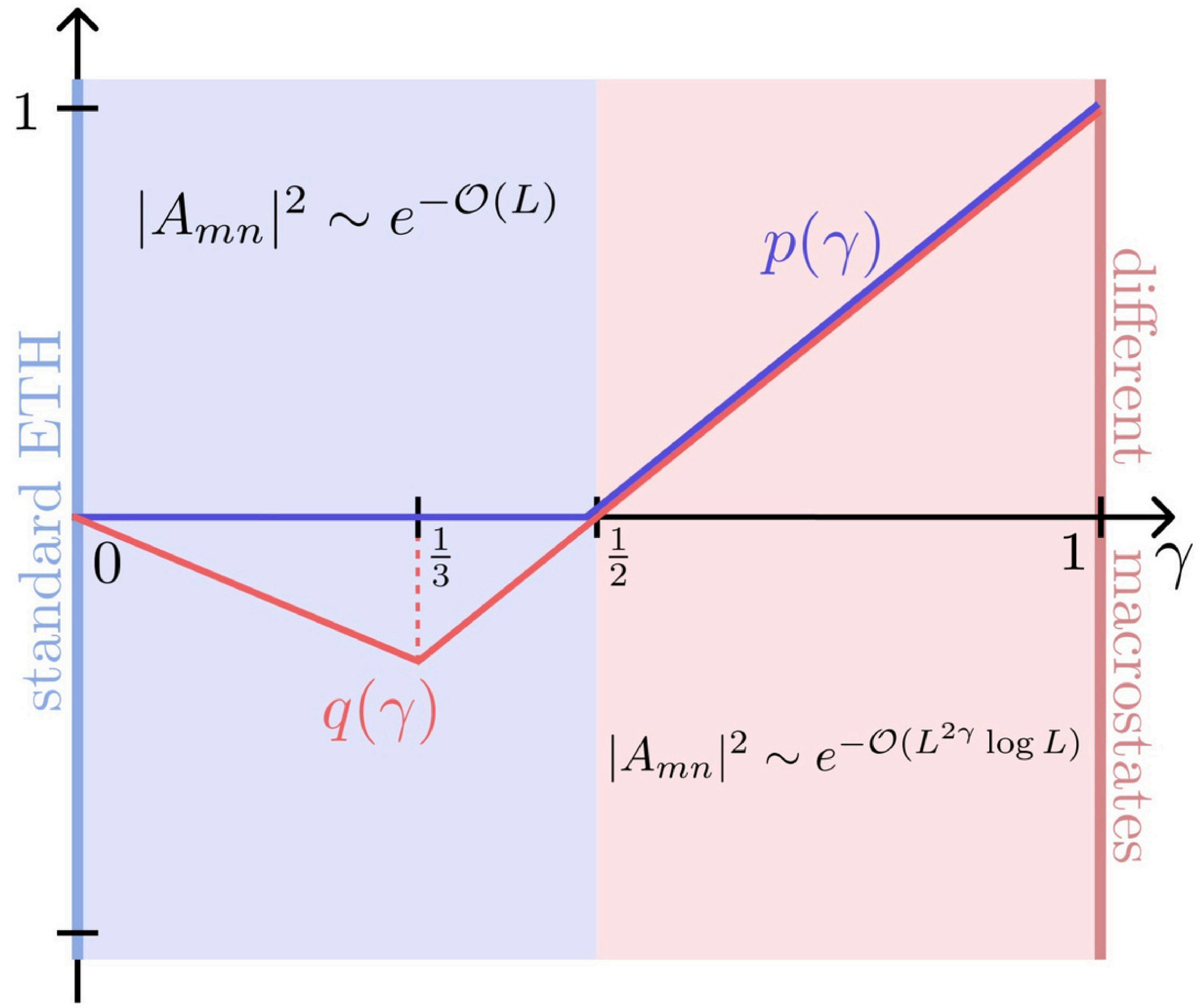
Summary of main results

$$p(\gamma) \equiv \frac{\partial \log(\kappa)}{\partial \log(L)}, \quad q(\gamma) \equiv \frac{\partial \log(\delta\kappa)}{\partial \log(L)}$$



Summary of main results

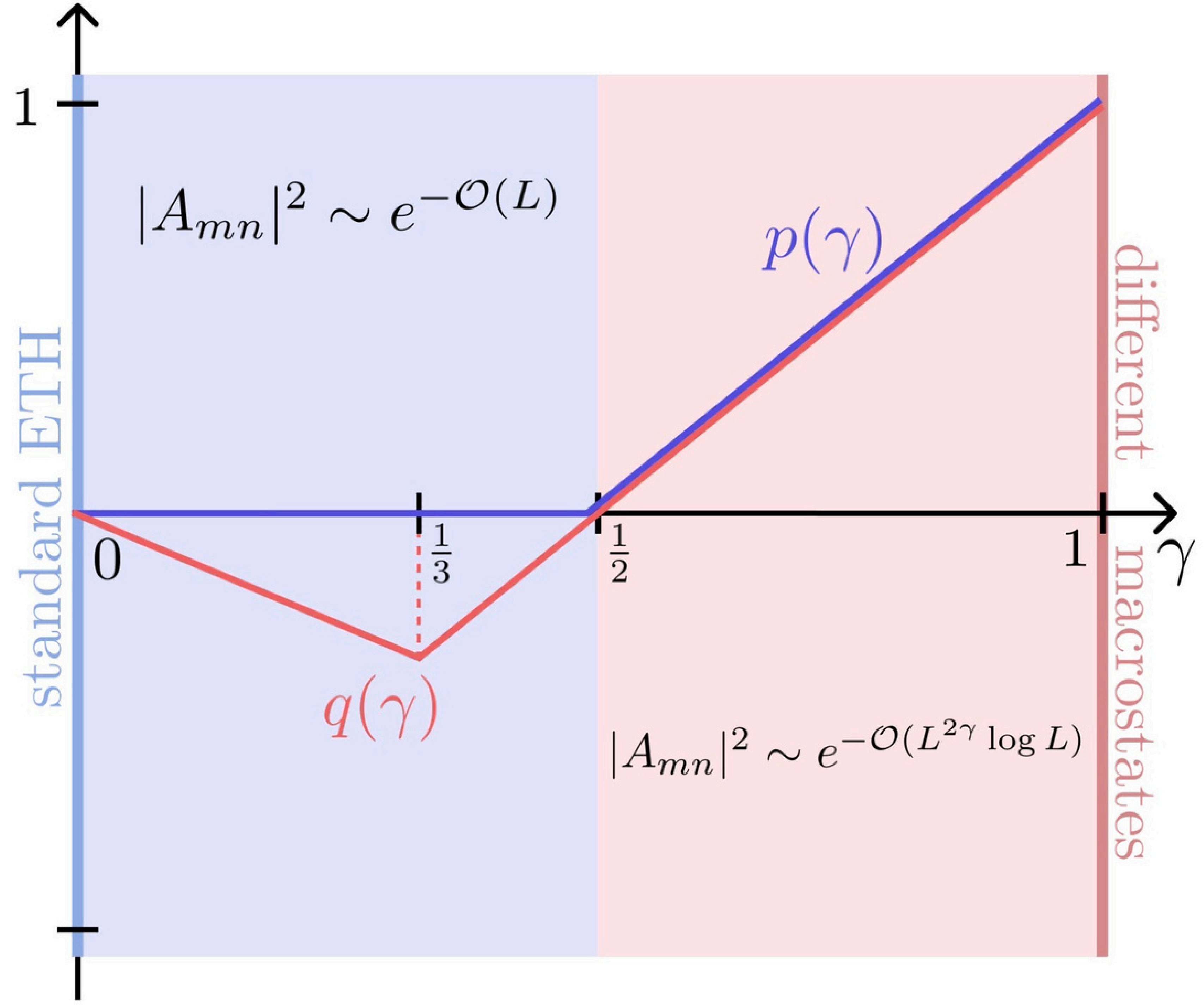
$$p(\gamma) \equiv \frac{\partial \log(\kappa)}{\partial \log(L)}, \quad q(\gamma) \equiv \frac{\partial \log(\delta\kappa)}{\partial \log(L)}$$



1. $p(\gamma)$ - analytics and numerics
   $q(\gamma)$ - numerics, **but** $L_{spins} \sim 10^6$

Summary of main results

$$p(\gamma) \equiv \frac{\partial \log(\kappa)}{\partial \log(L)}, \quad q(\gamma) \equiv \frac{\partial \log(\delta\kappa)}{\partial \log(L)}$$

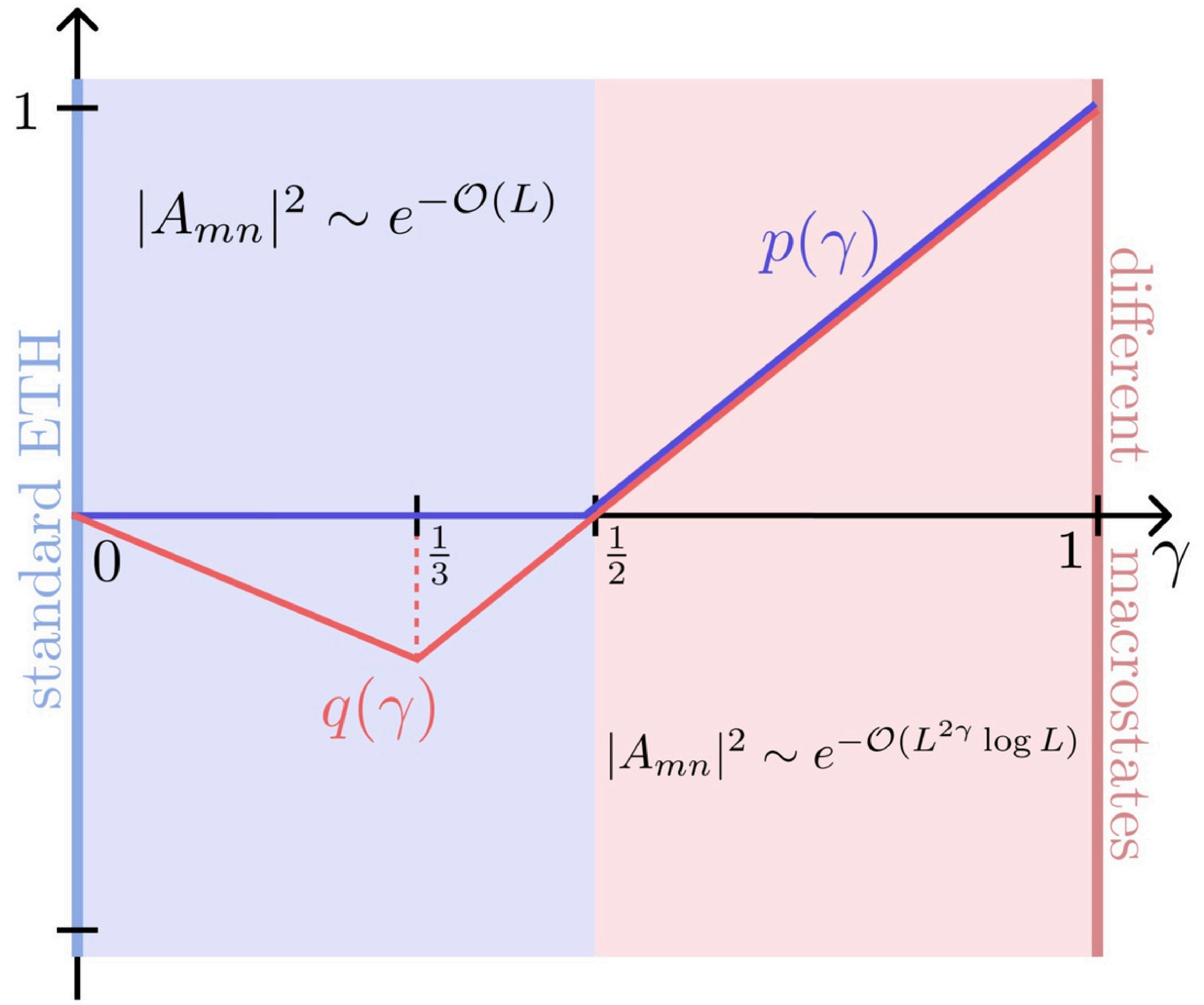


1. $p(\gamma)$ - analytics and numerics
   $q(\gamma)$ - numerics, **but** $L_{spins} \sim 10^5$

2. $\gamma < 1/2$ phase of **exp suppression**
   $\gamma \geq 1/2$ suppression is **more than exp**

Summary of main results

$$p(\gamma) \equiv \frac{\partial \log(\kappa)}{\partial \log(L)}, \qquad q(\gamma) \equiv \frac{\partial \log(\delta\kappa)}{\partial \log(L)}$$



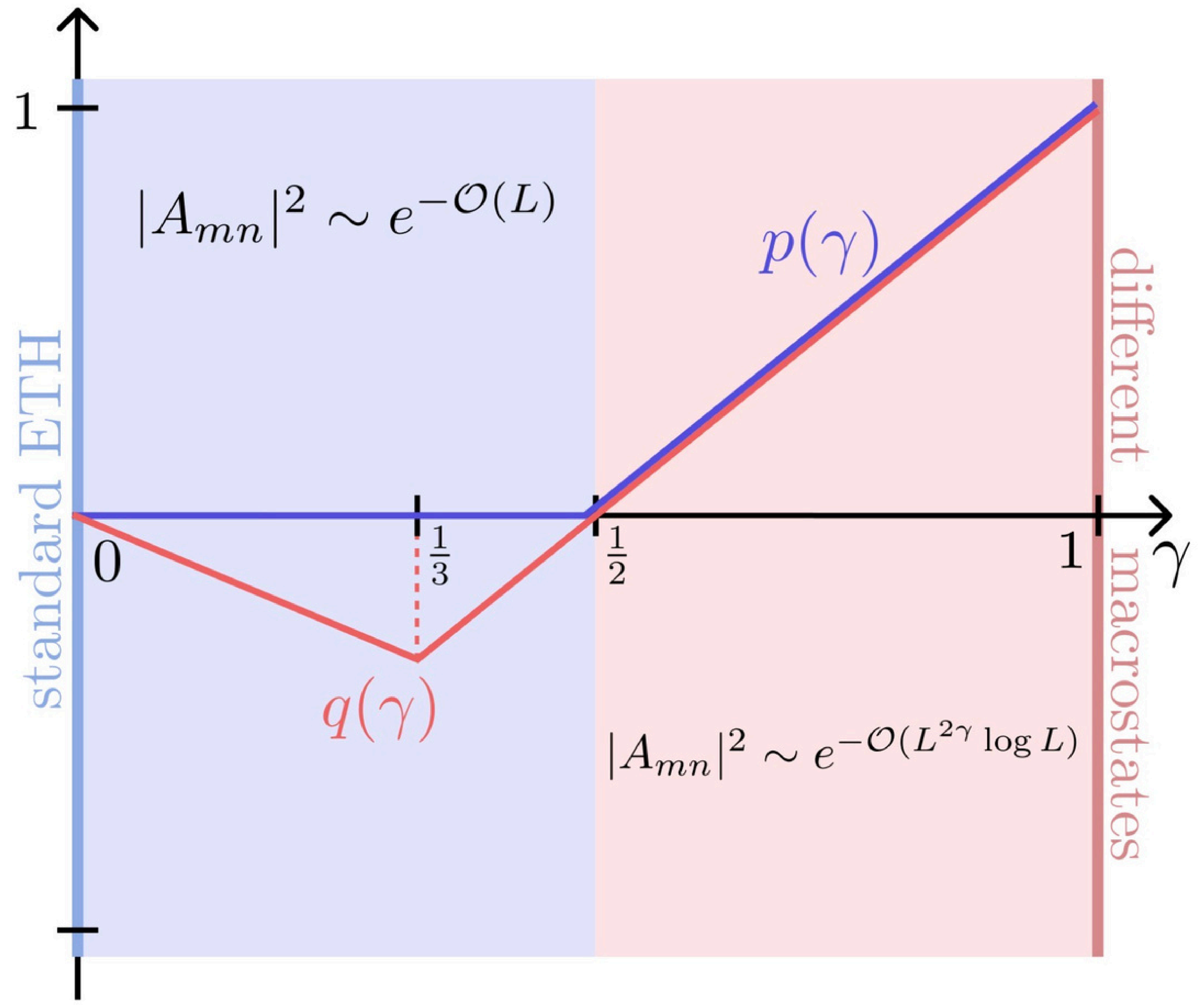
1. $p(\gamma)$ - analytics and numerics
   $q(\gamma)$ - numerics, **but** $L_{spins} \sim 10^5$

2. $\gamma < 1/2$ phase of **exp suppression**
   $\gamma \geq 1/2$ suppression is **more than exp**

3. Presence of **log corrections**:
   for $\gamma \geq 1/2$ $\quad \kappa = O(L^{2\gamma-1}\log L)$

Summary of main results

$$p(\gamma) \equiv \frac{\partial \log(\kappa)}{\partial \log(L)}, \qquad q(\gamma) \equiv \frac{\partial \log(\delta\kappa)}{\partial \log(L)}$$

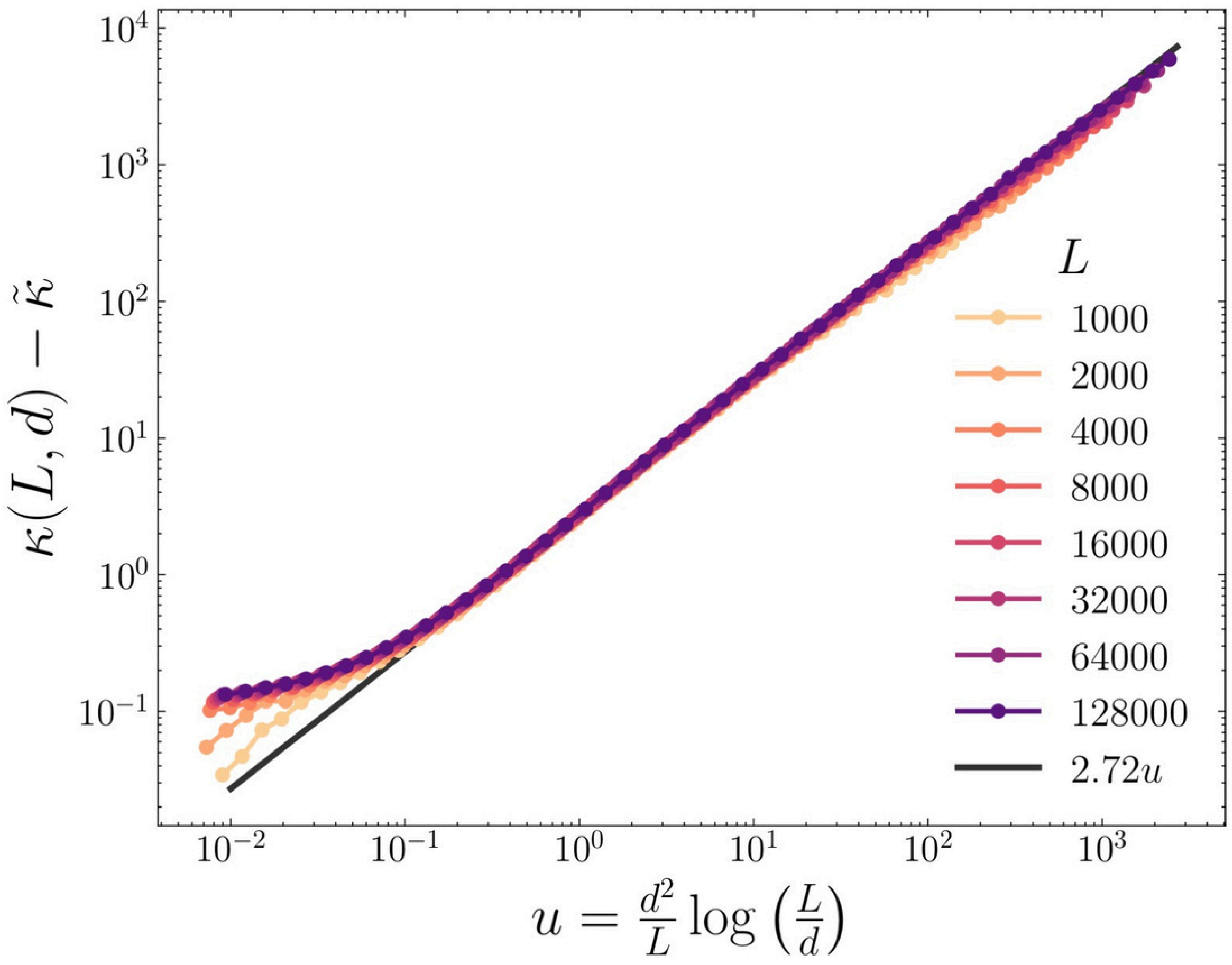


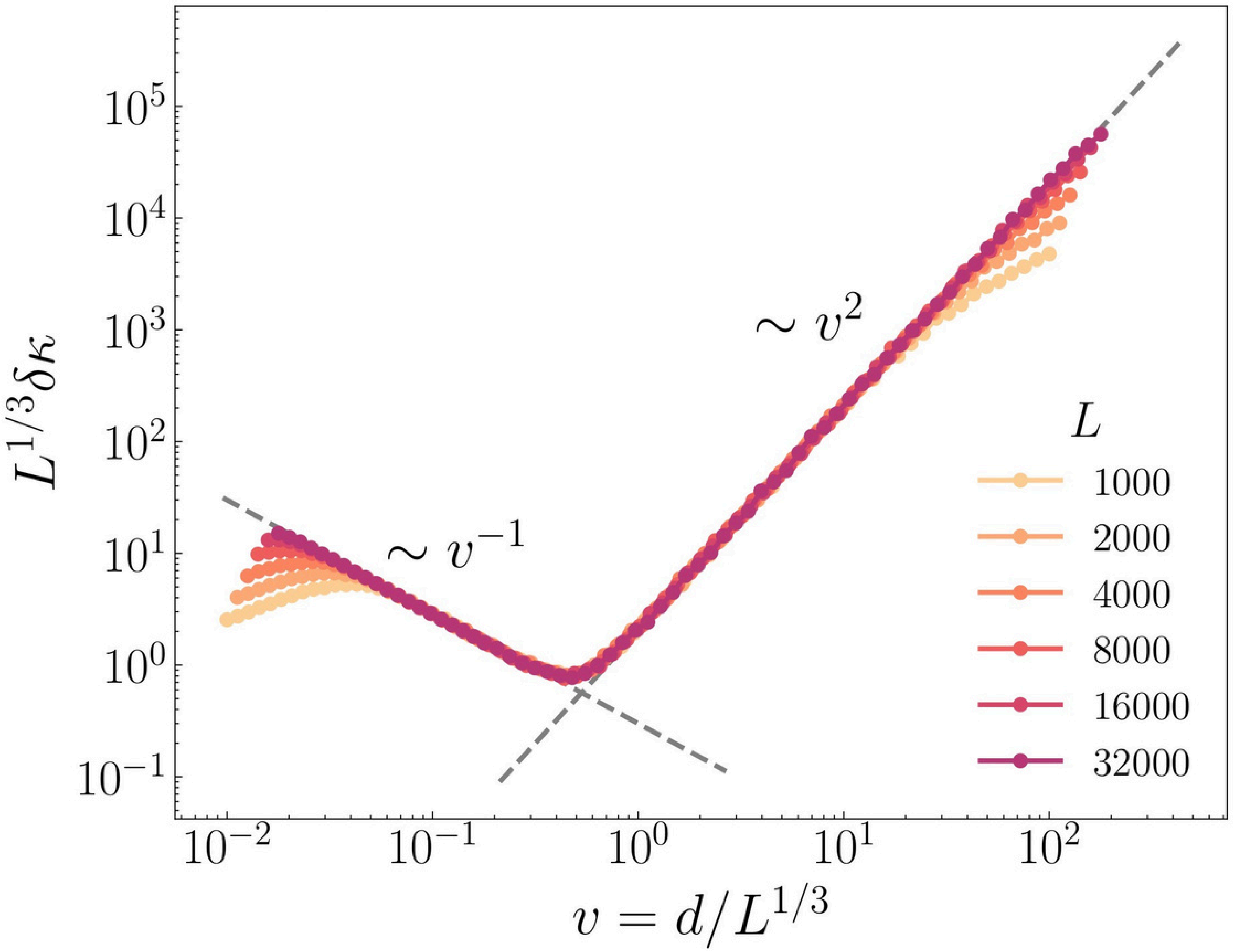
1. $p(\gamma)$ - analytics and numerics
   $q(\gamma)$ - numerics, **but** $L_{spins} \sim 10^5$
2. $\gamma < 1/2$ phase of **exp suppression**
   $\gamma \geq 1/2$ suppression is **more than exp**
3. Presence of **log corrections**:
   for $\gamma \geq 1/2$ $\quad \kappa = O(L^{2\gamma-1} \log L)$
4. Two scales $\gamma = 1/2,\ 1$ match with previous studies in XXX and LL models

From scaling functions to scaling exponents

If d is a fluctuation magnitude in the ensemble of states

$$\kappa = f_\kappa\left(d^2/L\log(L/d)\right) \qquad \delta\kappa = L^{-1/3}f_{\delta\kappa}(d/L^{1/3})$$





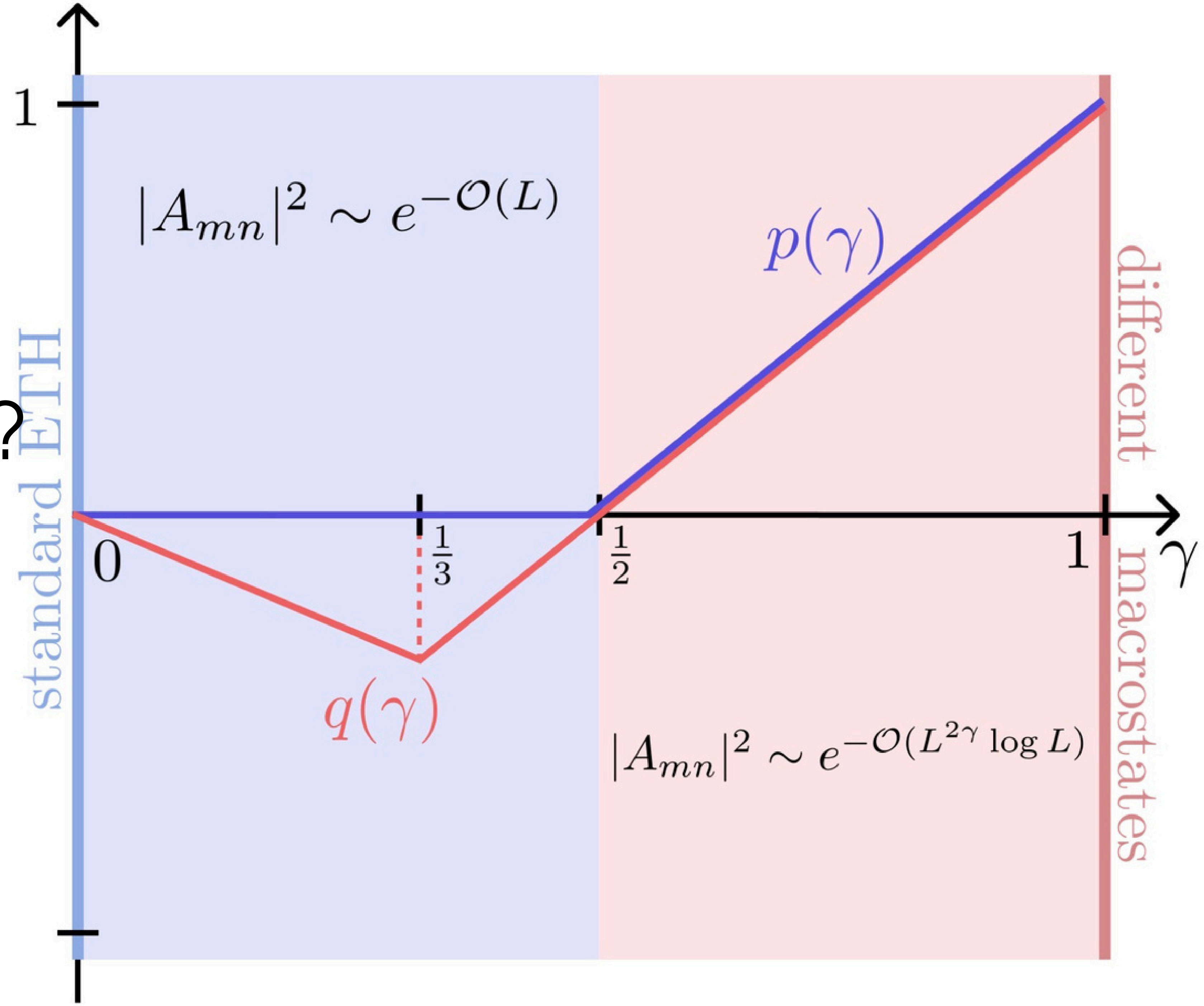
Summary

1. **ETH ansatz = “RMT on steroids”**.
A lot of numerical evidence that it holds in systems without conservation laws.

2. What about **ETH in integrable systems**?

3. An attempt to study this question incroporating **proper distance**.

4. The **fluctuation scale** influences the scaling of matrix elements.



Outlook

1. Universality on the level of m.el.?
2. Is it even useful for corr. functions?