Exact Strong Zero Modes in Integrable Systems

Sascha Gehrmann

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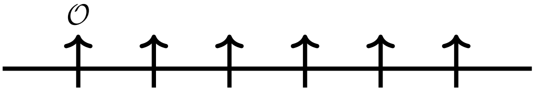
Engineering and Physical Sciences Research Council

UNIVERSITY OF OXFORD

The generic fate of correlators

Consider an infinite-temperature autocorrelation of a local boundary observable $\mathcal{O}_1$ in a quantum system, e.g., a spin chain of system size N with open BCs:

$$d^{-N}\,\mathrm{Tr}\big(\mathcal{O}_1(t)\mathcal{O}_1(0)\big) \qquad\qquad \mathcal{O}_1(t) = \mathbb{U}_t\big[\mathcal{O}_1\big] = \mathrm{e}^{\mathrm{i}\mathbb{H}t}\mathcal{O}_1\mathrm{e}^{-\mathrm{i}\mathbb{H}t}$$



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$$\lim_{T\to\infty}\frac{1}{T}\int_0^T \mathrm{d}t\;\mathbb{U}_t[\mathcal{O}_1] = \mathrm{Proj}_{\mathbb{C}}(\mathcal{O}_1)$$

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Interesting possible breaking cases: Many-body localisation, Hilbert-space fragmentation, ...

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In this talk: Exact Strong Zero Modes conserved normalisable operators with non-vanishing overlap for local operators at the boundaries as $N\to\infty$.

Exact Strong zero modes in the XYZ model

$$\mathbb{H} = \sum_{j=1}^{N-1} \left(J_x \, \sigma_j^x \sigma_{j+1}^x + J_y \, \sigma_j^y \sigma_{j+1}^y + J_z \, \sigma_j^z \sigma_{j+1}^z \right)$$

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Start at the edges and apply the brute force as in Ising [Fendley'15]:

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$$\Psi = \sigma_1^z + \frac{J_x J_y}{J_z^2} \sigma_2^z + \frac{J_x^2 J_y^2}{J_z^4} \sigma_3^z - \frac{J_y}{J_z} \left(1 - \frac{J_x^2}{J_z^2} \right) \sigma_1^x \sigma_2^x \sigma_3^z - \frac{J_x}{J_z} \left(1 - \frac{J_y^2}{J_z^2} \right) \sigma_1^y \sigma_2^y \sigma_3^z + \dots$$

$$\Psi \equiv \sum_{j=1}^{R} \Psi_j , \qquad [\mathbb{H}, \Psi] \sim \mathcal{O}\left(J_z^{-2R} \right) \qquad ||\Psi_j||^2 = \frac{1}{2^N} \mathrm{tr} \left(\Psi_j^\dagger \Psi_j \right) \sim \mathrm{e}^{-\alpha j}$$

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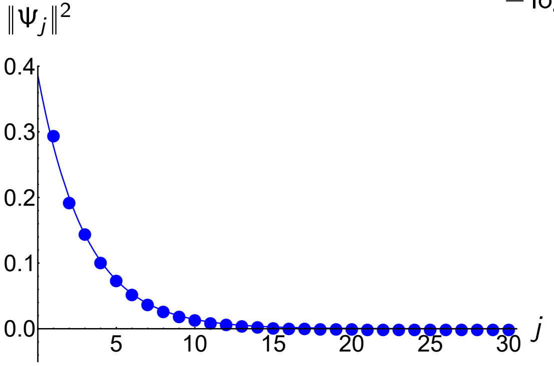
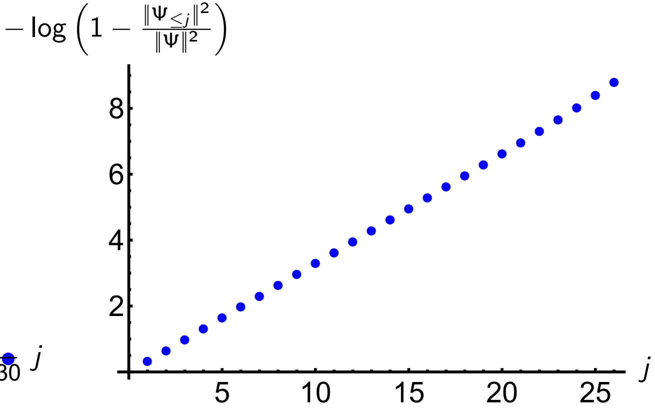
Also can be made exactly commuting, most general compatible BCs, defects and reinterpretation as MPO [Fendley, SG, Vernier, Verstraete'25, SG, Essler'25]





$$||\Psi_j||^2 \sim e^{-\alpha j} \quad \Longrightarrow \quad \|\Psi\|^2 = \sum_j ||\Psi_j||^2 = \sum_j e^{-\alpha j} < \infty$$

Infinite Coherence Time

Back to the infinite temperature correlation functions of a boundary-supported $\mathcal{O}$

$$C_0^{\mathcal{O}}(t) = \frac{1}{2^N} \mathrm{Tr}\left[\mathcal{O}(t)\mathcal{O}(0)\right]$$

Recall this generically decays in time to asymptotic values that go to zero as the system size increases.

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An exact SZM operator Ψ is changing this! We decompose

$$\mathcal{O}(0) = c_1^{\mathcal{O}}\Psi + c_2^{\mathcal{O}}\mathcal{O}' \ , \qquad \mathrm{Tr}\left(\Psi^\dagger\mathcal{O}'\right) = 0 \ , \qquad c_1^{\mathcal{O}} = \frac{1}{2^N}\mathrm{Tr}(\Psi^\dagger\mathcal{O}) \ ,$$

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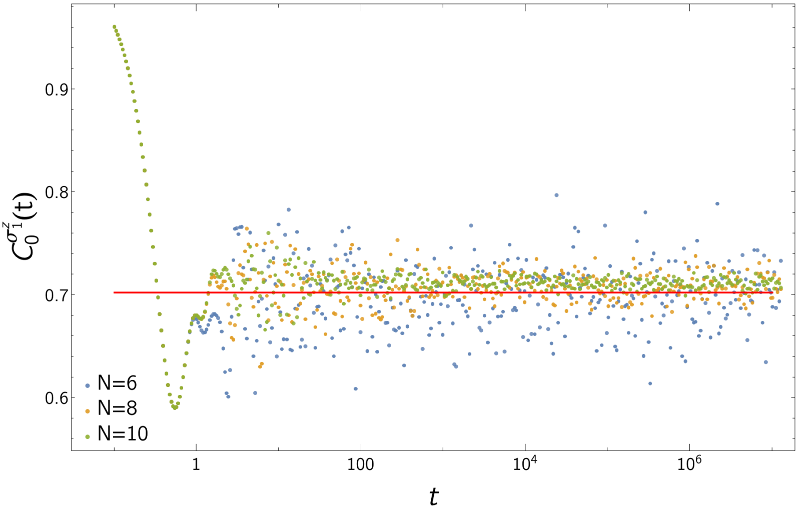
where $c_1^{\mathcal{O}} = O(N^0)$ for large system sizes and $\mathcal{O}'$ is by construction localized around the boundary. Using $(2)^{-N}\mathrm{Tr}\left[\Psi^\dagger\Psi\right] = 1$ gives

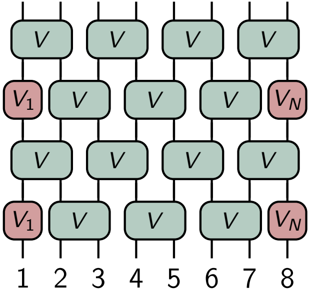
$$C_0^{\mathcal{O}}(t) = |c_1^{\mathcal{O}}|^2 + |c_2^{\mathcal{O}}|^2 C_0^{\mathcal{O}'}(t) \,.$$

The autocorrelator $C^{\mathcal{O}'}(t)$ is expected to decay in time to a value that vanishes as $N \to \infty$, such that $C^{\mathcal{O}}(t)$ decays to a finite value $|c_1^{\mathcal{O}}|^2$ set by the overlap of $\mathcal{O}(0)$ with Ψ.



$\Delta = 2.5$ and $h_1 = (1, 0.1, 0)$, $h_N = (0.25, 0.5, 1)$ and $\mathbb{H}^{\mathrm{BC}} = h_1 \cdot \boldsymbol{\sigma}_1 + h_N \cdot \boldsymbol{\sigma}_N$.
The red line is $|c_1^{\sigma_1^z}|^2$ in the thermodynamic limit.

Application: Quantum circuits in the lab



PBC XXZ circuit realized [Morvan et al '22].

An initial state $|\psi_0\rangle$ evolves as

$$|\psi_t\rangle = U^t|\psi_0\rangle,$$

under a brick-wall circuit

$$\begin{aligned} U &= U_{\mathrm{odd}}\, U_{\mathrm{even}}, \\ U_{\mathrm{even}} &= V_1\, V_{2,3}\, \ldots\, V_{N-2,N-1}\, V_N, \\ U_{\mathrm{odd}} &= V_{1,2}\, V_{3,4}\, \ldots\, V_{N-1,N}. \end{aligned}$$

The gates are

$$V_{1,2} = \mathrm{e}^{-\frac{\mathrm{i}\tau}{4}(\sigma_1^x\sigma_2^x+\sigma_1^y\sigma_2^y+\tilde{\Delta}(\sigma_1^z\sigma_2^z-1))}\,,\; V_1 = \mathrm{e}^{\mathrm{i}\boldsymbol{\ell}\cdot\boldsymbol{\sigma}_1}\,,\; V_N = \mathrm{e}^{\mathrm{i}\boldsymbol{r}\cdot\boldsymbol{\sigma}_N}$$

Existence of ESZM and robust under perturbation [Vernier, Yeh, Piroli, Mitra'24]

So far

Strong zero modes are remarkably robust:

- They survive in the presence of interactions.
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- They persist under perturbations.

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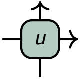
$\Longrightarrow$ **Generic in integrable fundamental spin systems with sufficiently large anisotropy**

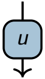
Basics of integrability

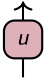
- Integrable quantum system: An extensive number $\#N$ of conserved local quantities $\mathbb{Q}^k$
- Originating from a generating functional called the transfer matrix $\mathbb{T}(u)$

$$\left[\mathbb{T}(u), \mathbb{T}(v)\right] = 0\,, \qquad \log(\mathbb{T}(u)) = c\,1 + u\,\mathbb{Q}^1 + u^2\,\mathbb{Q}^2 + \dots\,, \qquad \left[\mathbb{Q}^k, \mathbb{Q}^\ell\right] = 0 \tag{1}$$

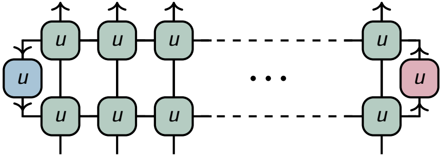
- Tensor notation:

$R(u) =$ 

$K^+(u) =$ 

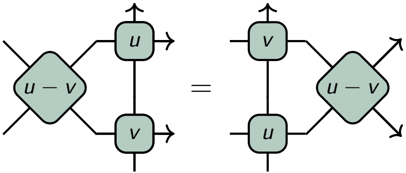
$K^-(u) =$ 

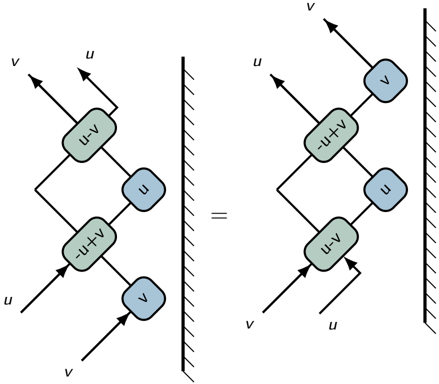
. (2)

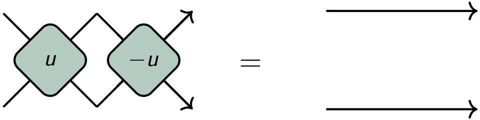
$\mathbb{T}(u) =$ 

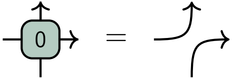
. (3)

Properties of boxes

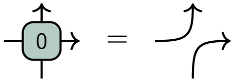




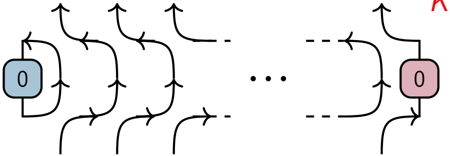


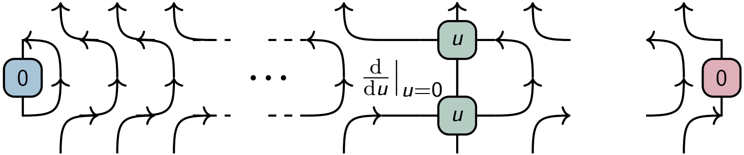


Properties of boxes: Via Regularity to $\mathbb{H}$



$K^{-}(0) \propto 1$

$\mathbb{T}(0) =$  $\propto 1$

$\mathbb{T}'(0) = \sum_{j=2}^{N}$ 

$+$ BCs terms $\qquad j-1 \qquad j$

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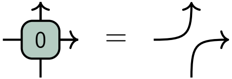

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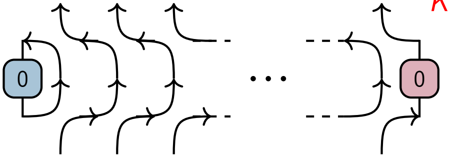

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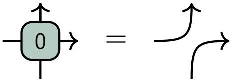

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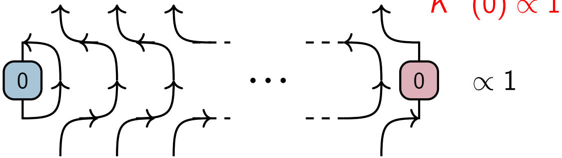

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$\mathbb{Q}^k = \sum_{j=1}^{N-k} q_{j,\dots,j+k} + q_{1,\dots k} + q_{N,\dots,N-k+1}$ sum over local densities!

How to generate conserved charge with other locality properties as strong zero modes?

$$\mathbb{\Psi} = \sum_{j=1}^{N} \Psi_j \qquad\qquad \Psi_j \quad \text{acting from 1 to } j \text{ and } \|\Psi_j\|^2 \sim \mathrm{e}^{-\alpha j}$$

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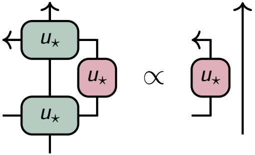
- Conserved $\implies$ use some functional of transfer matrix
- Different locality properties $\implies$ different expansion point

Pull though relations

Suppose there exists an $u_\star$ such that (XXZ quantum circuit [Vernier, Yeh, Piroli, Mitra'24])



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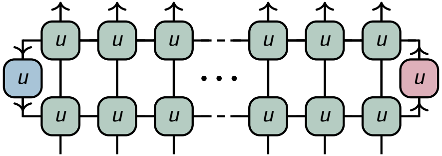

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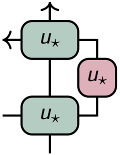
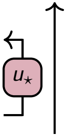


$\propto$



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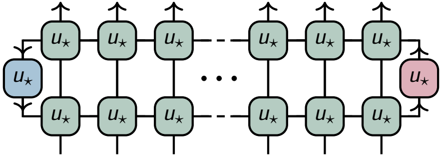

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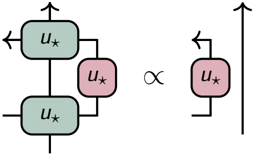

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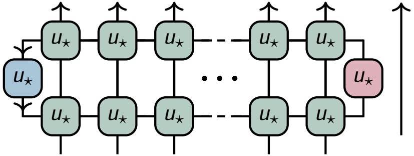

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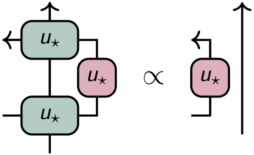
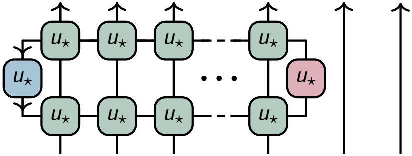


$\mathbb{T}(u_\star) \propto$ 

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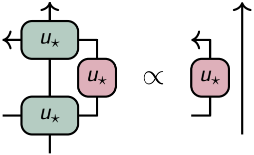

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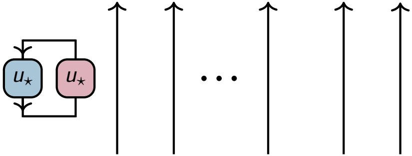

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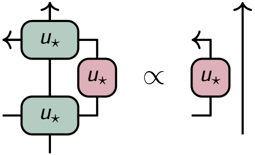

$$\mathbb{T}(u_\star) \propto 1$$

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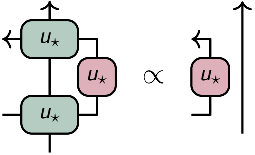
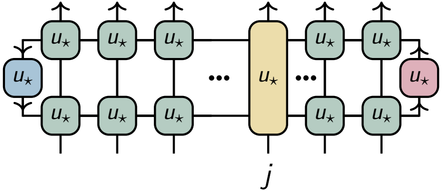


$\mathbb{T}'(u_\star) = \sum_j$ 

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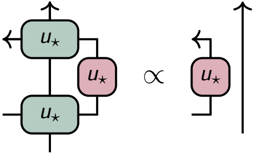

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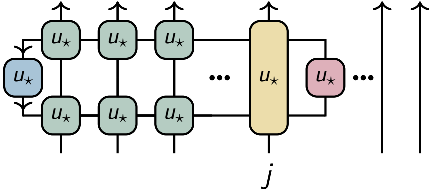

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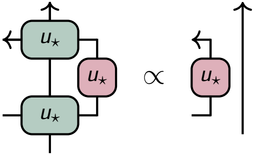

$\mathbb{T}'(u_\star) = \sum_j \Psi_j$

MPO formalism allows for efficient computation of $\|\Psi_j\|^2$

What is $u_\star$?

Consider the R-matrices of the broad class obtained by [Bazhanov'87, Jimbo'86] based on affine Lie-algebra series $A_n^{(1)}$, $A_{2n}^{(2)}$, $A_{2n+1}^{(2)}$, $B_n^{(1)}$, $C_n^{(1)}$, $D_n^{(1)}$ and use the associated K-matrices found by [Lima-Santos, Malara'06].

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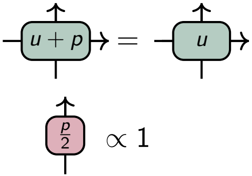

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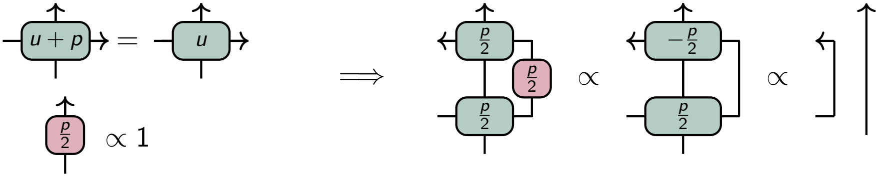


Localisation at the boundary

Back to

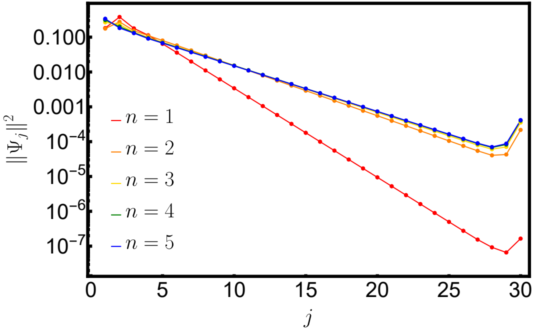
$$\mathbb{H}^{\mathrm{XXZ}} = \sum_{j=1}^{N-1} \left(\sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y + \Delta \sigma_j^z \sigma_{j+1}^z \right) + \vec{h}_N \vec{\sigma}_N + \vec{h}_1 \vec{\sigma}_1 .$$

For localisation:

$$\boxed{\Delta > 1 \,, \qquad h_1^z = 0}$$

For generalised models we need to pick

$$\boxed{|\Delta| = \frac{1}{2} \left| q + q^{-1} \right| > 1^* \,, \qquad \mathrm{Tr}(K^+(\tfrac{p}{2})) = 0^* .}$$



*Terms and Conditions apply

Conclusion and recent progress

- Strong Zero modes allow for infinite edge coherence times
- Strong Zero modes are a generic feature of integrable fundamental spin models for anisotropy $\Delta > 1^*$ [SG'26]
  - Originate from the pull-through relation
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- Strong Zero modes are a generic feature of integrable fundamental spin models for anisotropy $\Delta > 1^*$ [SG'26]
  - Originate from the pull-through relation
  - Pull-through relation can be obtained from periodicity and unitarity
- Very recently edge modes also for the **isotropic case** $\Delta = 1$ due to a boundary defect i.e. XXX [Prosen'26]
- Very recently also **exact** strong zero modes for **non integrable systems** [Moudgalya, Motrunich'26]
- More than one ESZM by fusion via different auxiliary spaces [Essler, Fendley, Vernier'26]
- Mapping to ASEP fails [SG, Essler'25]

Thank you!