Fermionic Basis in CFT: the Free Fermion Point

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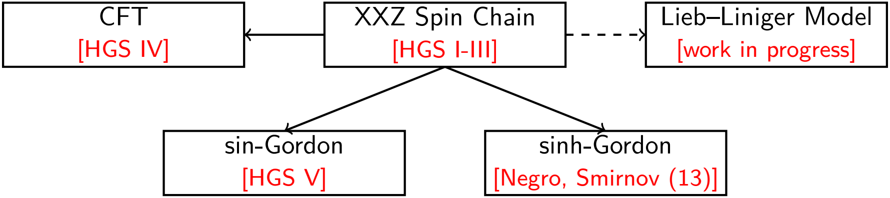
Overview and plan of the talk

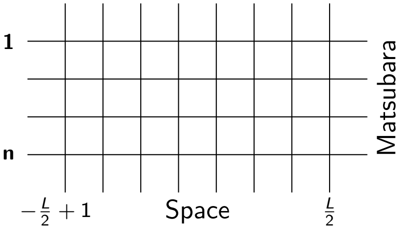
- Introduction and motivation to study fermionic basis
- A brief overview of the fermionic basis construction
- Applications to CFT: three-point correlation functions at the free fermion point
- Summary and overview

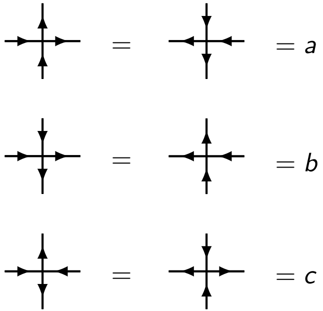
Introduction and motivation



- The fermionic basis provides a universal tool for working with $U_q(\widehat{\mathfrak{sl}}_2)$–symmetric theories
- Factorization of the algebraic and physical parts
  - The generators of the fermionic basis are constructed using the representation theory of $U_q(\widehat{\mathfrak{sl}}_2)$ and are independent of any physical data
  - All physics is absorbed by two functions ρ and ω
  - The Jimbo–Miwa–Smirnov theorem: all correlation functions of (quasi-)local operators can be expressed via those two functions only

The XXZ spin chain and the 6-vertex model





- Related spin-chain model:

$$H_L = \sum_{j=-L/2+1}^{L/2} \sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y + \Delta \sigma_j^z \sigma_{j+1}^z$$

- Integrable structure is generated by the L-operator

$$L_{j,\mathbf{m}}(\zeta) = r(\zeta) \begin{pmatrix} 1 & & & \\ & \beta(\zeta) & \gamma(\zeta) & \\ & \gamma(\zeta) & \beta(\zeta) & \\ & & & 1 \end{pmatrix}$$

$$\beta(\zeta) = \frac{\zeta - \zeta^{-1}}{q\zeta - q^{-1}\zeta^{-1}}, \quad \gamma(\zeta) = \frac{q - q^{-1}}{q\zeta - q^{-1}\zeta^{-1}}$$

$$\Delta = \frac{1}{2}(q + q^{-1}), \quad q = \mathrm{e}^{\mathrm{i}\pi\nu}$$

- The main problem is to compute the partition function $Z = \sum\limits_{\text{conf.}} \prod\limits_{\text{vert.}} W_{i,\mathbf{m}}$
- This can be done with the help of the monodromy matrices

$$\left.\begin{array}{l} T_{j,\mathbf{M}}(\zeta) = L_{j,\mathbf{n}}(\zeta/\tau_{\mathbf{n}})L_{j,\mathbf{n}-1}(\zeta/\tau_{\mathbf{n}-1})\dots L_{j,\mathbf{1}}(\zeta/\tau_{\mathbf{1}}) \\ T_{\mathrm{S},\mathbf{M}} = T_{-L/2+1,\mathbf{M}}(1)\,T_{-L/2+2,\mathbf{M}}(1)\dots T_{L/2,\mathbf{M}}(1) \end{array}\right\} \Longrightarrow Z = \operatorname*{tr}_{\mathrm{S}}\operatorname*{tr}_{\mathbf{M}} T_{\mathrm{S},\mathbf{M}}$$

- The twisted Matsubara transfer matrix

$$T_{\mathbf{M}}(\zeta,\kappa) = \operatorname*{tr}_{j}\left\{L_{j,\mathbf{n}}(\zeta/\tau_{\mathbf{n}})\dots L_{j,\mathbf{1}}(\zeta/\tau_{\mathbf{1}})q^{\kappa\sigma_j^z}\right\},$$

$$[T_{\mathbf{M}}(\zeta_1,\kappa),\, T_{\mathbf{M}}(\zeta_2,\kappa)] = 0$$

- The Baxter TQ-relation

$$T_{\mathbf{M}}(\zeta,\kappa)Q^{\pm}_{\mathbf{M}}(\zeta,\kappa) = d(\zeta)Q^{\pm}_{\mathbf{M}}(q\zeta,\kappa) + a(\zeta)Q^{\pm}_{\mathbf{M}}(q^{-1}\zeta,\kappa),$$

where

$$d(\zeta) = \prod_{\mathbf{m}=1}^{\mathbf{n}}\left(\frac{\zeta^2}{\tau_{\mathbf{m}}^2} - 1\right), \quad a(\zeta) = d(q\zeta), \quad Q(\zeta,\kappa) = \zeta^{-\kappa}\prod_{r=1}^{M}\left(1 - \frac{\zeta^2}{\lambda_r^2}\right)$$

- The Bethe ansatz equations:

$$\mathfrak{a}(\zeta,\kappa) = \frac{d(\zeta)Q(q\zeta,\kappa)}{a(\zeta)Q(q^{-1}\zeta,\kappa)}, \qquad \mathfrak{a}(\lambda_r,\kappa) = -1, \quad r = 1,\dots,M$$

The main object

- We consider quasi-local operators of the form

$$X = q^{2\alpha S(0)}\mathcal{O}, \qquad S(k) = \frac{1}{2}\sum_{j=-L/2+1}^{k}\sigma_j^z$$

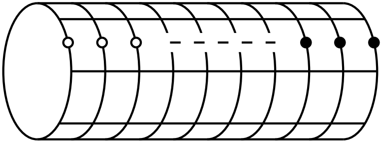
where $\mathcal{O}$ acts non-trivially only on a finite segment of the lattice in the Space direction

- Support is the minimal interval $[k,l]$ such that

$$X = q^{2\alpha S(k-1)}X_{[k,l]}$$

- The main object of our study:

$$Z_{\mathbf{n}}^{\kappa}\left\{q^{2\alpha S(0)}\mathcal{O}\right\} = \frac{\mathrm{tr}_{\mathrm{S}}\,\mathrm{tr}_{\mathbf{M}}\left(T_{\mathrm{S},\mathbf{M}}q^{2\kappa S+2\alpha S(0)}\mathcal{O}\right)}{\mathrm{tr}_{\mathrm{S}}\,\mathrm{tr}_{\mathbf{M}}\left(T_{\mathrm{S},\mathbf{M}}q^{2\kappa S+2\alpha S(0)}\right)}$$



$= q^{(\alpha+\kappa)\sigma^z}$ (open dot) $\qquad = q^{\kappa\sigma^z}$ (filled dot)

Fermionic basis construction

- Consider the space of quasi-local operators of tail α:

$$\mathcal{W}^{(\alpha)} = \bigoplus_{s=-\infty}^{+\infty} \mathcal{W}_{\alpha-s,s}$$

with $\mathcal{W}_{\alpha,s}$ being a subspace of quasi-local operators of spin s

- The fermionic basis consists of operators $\mathbf{t}^*(\zeta)$, $\mathbf{b}^*(\zeta)$, $\mathbf{c}^*(\zeta)$, $\mathbf{b}(\zeta)$, $\mathbf{c}(\zeta)$, acting on $\mathcal{W}_{\alpha-s,s}$ as follows:

$$\begin{aligned}
&\mathbf{t}^*(\zeta,\alpha) &&: \quad \mathcal{W}_{\alpha-s,s} \to \mathcal{W}_{\alpha-s,s},\\
&\mathbf{b}^*(\zeta,\alpha),\ \mathbf{c}(\zeta,\alpha) &&: \quad \mathcal{W}_{\alpha-s+1,s-1} \to \mathcal{W}_{\alpha-s,s},\\
&\mathbf{c}^*(\zeta,\alpha),\ \mathbf{b}(\zeta,\alpha) &&: \quad \mathcal{W}_{\alpha-s-1,s+1} \to \mathcal{W}_{\alpha-s,s}.
\end{aligned}$$

- Operator $\mathbf{t}^*$ is bosonic and generates commuting integrals of motion. The only non-trivial anti-commutation relations are

$$[\mathbf{c}(\xi),\mathbf{c}^*(\zeta)]_+ = \psi(\xi/\zeta,\alpha), \quad [\mathbf{b}(\xi),\mathbf{b}^*(\zeta)]_+ = -\psi(\zeta/\xi,\alpha), \quad \psi(\zeta) = \frac{\zeta^\alpha}{2}\frac{\zeta^2+1}{\zeta^2-1}$$

- In the homogeneous case:

$$\mathbf{t}^*(\zeta) = \sum_{p=1}^{\infty} \left(\zeta^2 - 1\right)^{p-1} \mathbf{t}^*_p,$$

$$\mathbf{b}^*(\zeta) = \zeta^{2+\alpha+\mathbb{S}} \sum_{p=1}^{\infty} \left(\zeta^2 - 1\right)^{p-1} \mathbf{b}^*_p, \quad \mathbf{c}^*(\zeta) = \zeta^{2-\alpha-\mathbb{S}} \sum_{p=1}^{\infty} \left(\zeta^2 - 1\right)^{p-1} \mathbf{c}^*_p,$$

$$\mathbf{b}(\zeta) = \zeta^{-\alpha-\mathbb{S}} \sum_{p=0}^{\infty} \left(\zeta^2 - 1\right)^{-p} \mathbf{b}_p, \quad \mathbf{c}(\zeta) = \zeta^{\alpha+\mathbb{S}} \sum_{p=0}^{\infty} \left(\zeta^2 - 1\right)^{-p} \mathbf{c}_p$$

- The annihilation operators $\mathbf{b}_p$, $\mathbf{c}_p$ do not increase the support of quasi-local operators
- The creation operators $\mathbf{t}^*_p$, $\mathbf{b}^*_p$, $\mathbf{c}^*_p$ increase the support by at most p
- Operators $\mathbf{b}$ and $\mathbf{c}$ annihilate the "primary field" $q^{2\alpha S(0)}$

$$\mathbf{b}(\zeta)(q^{2\alpha S(0)}) = 0, \quad \mathbf{c}(\zeta)(q^{2\alpha S(0)}) = 0,$$

and the entire space of states is spanned by the vectors of the form

$$\mathbf{t}^*(\zeta_1^0)\dots\mathbf{t}^*(\zeta_p^0)\mathbf{b}^*(\zeta_1^+)\dots\mathbf{b}^*(\zeta_r^+)\mathbf{c}^*(\zeta_r^-)\dots\mathbf{c}^*(\zeta_1^-)(q^{2\alpha S(0)})$$

Jimbo–Miwa–Smirnov theorem

- Correlation functions of quasi-local operators are described by two transcendental functions $\rho(\zeta)$ and $\omega(\zeta,\xi)$. The former is related to the one-point function, and the latter is related to the nearest neighbour correlators

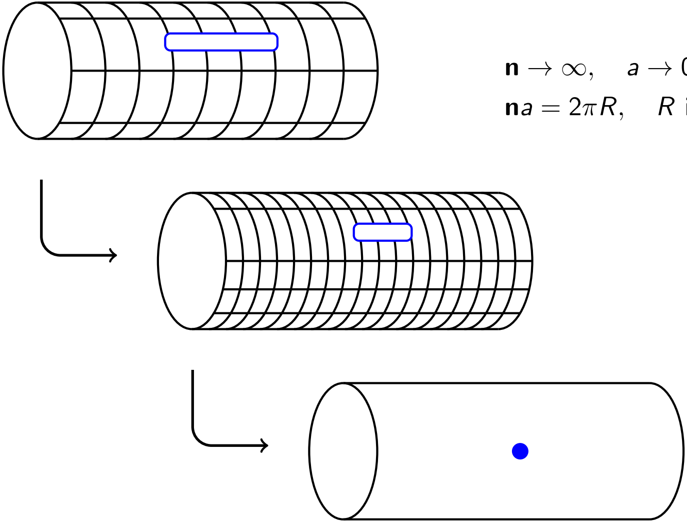
$$\omega(\zeta,\xi) = Z^{\kappa}_{\mathbf{n}}\left\{\mathbf{b}^*(\zeta)\mathbf{c}^*(\xi)q^{2\alpha S(0)}\right\}.$$

- The JMS theorem allows us to explicitly calculate

$$Z^{\kappa}_{\mathbf{n}}\{\mathbf{t}^*(\zeta^0_1)\dots\mathbf{t}^*(\zeta^0_p)\mathbf{b}^*(\zeta^+_1)\dots\mathbf{b}^*(\zeta^+_r)\mathbf{c}^*(\zeta^-_r)\dots\mathbf{c}^*(\zeta^-_1)(q^{2\alpha S(0)})\}$$
$$= \prod_{i=1}^{p} 2\rho(\zeta^0_i) \times \det(\omega(\zeta^+_i,\zeta^-_j))_{i,j=1,\dots,r}.$$

- By definition, $\rho(\zeta;\kappa,\alpha) = \frac{T(\zeta,\kappa+\alpha)}{T(\zeta,\kappa)}$, but the definition of $\omega(\zeta,\xi)$ is more involved.

Applications to CFT: continuum limit



$$\mathbf{n} \to \infty, \quad a \to 0,$$
$$\mathbf{n}a = 2\pi R, \quad R \text{ is fixed.}$$

The Verma modules

- If CFT is perturbed by some local field, the conformal invariance of such QFT is violated. In [Zamolodchikov (87)], local integrals of motion that survive perturbations of CFT were introduced.
- Local integrals of motion are commuting operators acting on the fields as

$$(\mathbf{i}_{2n-1}\phi_{|h\rangle})(x,\overline{x}) = \oint\limits_{y} \frac{\mathrm{d}y}{2\pi\mathrm{i}} h_{2n}(y)\phi_{|h\rangle}(x,\overline{x}), \qquad n \geq 1$$

- The densities $h_{2n}(y)$ are certain descendants of the identity operator:

$$h_2(y) = (\mathbf{l}_{-2}\,\mathrm{id})\,(y) = T(y), \qquad h_4(y) = (\mathbf{l}_{-2}\,T)\,(y)$$

- The three-point function of a $\mathbf{i}_{2n-1}$-descendant of a field reduces to that of the field itself:

$$\langle\Delta_-|(\mathbf{i}_{2n-1}\phi_{|\Delta\rangle})(y,\overline{y})|\Delta_+\rangle = \left(I^+_{2n-1} - I^-_{2n-1}\right)\langle\Delta_-|\phi_{|\Delta\rangle}(y,\overline{y})|\Delta_+\rangle$$

- The entire Verma module $\mathcal{V}_\Delta$ is spanned by the elements

$$\left(\mathbf{i}_{2k_1-1}\ldots\mathbf{i}_{2k_p-1}\mathbf{l}_{-2m_1}\ldots\mathbf{l}_{-2m_q}\right)\phi_{|\Delta\rangle}(0,0)$$

- We obtain a CFT with $c = 1 - \frac{6\nu^2}{1-\nu}$, $q = \mathrm{e}^{\mathrm{i}\pi\nu}$
- The eigenvector $T_{\mathbf{M}}(\zeta,\kappa)|\kappa\rangle = T_0(\zeta,\kappa)|\kappa\rangle$ becomes the primary state of the CFT with

$$\Delta_{\kappa+1} = \frac{\nu^2}{4(1-\nu)}(\kappa^2 - 1)$$

- Introduce the "tuning" constant $\bar{a} = C(\nu)a$
- In the $s = 0$ sector we define the scaling versions

$$T^{(\mathrm{sc})}(\zeta,\kappa) = \lim_{\text{scaling}} T(\zeta\bar{a}^{\nu},\kappa,0), \qquad Q^{(\mathrm{sc})}(\zeta,\kappa) = \lim_{\text{scaling}} \bar{a}^{\nu\kappa} Q(\zeta\bar{a}^{\nu},\kappa,0)$$

- In the spin s sector we define the scaling versions

$$T^{(\mathrm{sc})}(\zeta,\kappa+\alpha-s,s) = T^{(\mathrm{sc})}(\zeta,\kappa'), \qquad Q^{(\mathrm{sc})}(\zeta,\kappa+\alpha-s,s) = Q^{(\mathrm{sc})}(\zeta,\kappa')$$

- Thus, we can define

$$\rho^{(\mathrm{sc})}(\zeta;\kappa,\kappa') = \lim_{\text{scaling}} \rho(\zeta\bar{a}^{\nu};\kappa,\alpha,s),$$
$$4\omega^{(\mathrm{sc})}(\zeta,\xi;\kappa,\kappa',\alpha) = \lim_{\text{scaling}} \omega(\zeta\bar{a}^{\nu},\xi\bar{a}^{\nu};\kappa,\alpha,s),$$

where $\kappa' = \kappa + \alpha + 2\frac{1-\nu}{\nu}s$ can be considered an independent parameter

Scaling limit in the Space direction

- Formally, we define for finite $ja = x$

$$2\boldsymbol{\tau}^*(\zeta) = \lim_{a\to 0} \mathbf{t}^*(\zeta\overline{a}^{\nu}), \quad 2\beta^*(\zeta) = \lim_{a\to 0} \mathbf{b}^*(\zeta\overline{a}^{\nu}), \quad 2\gamma^*(\zeta) = \lim_{a\to 0} \mathbf{c}^*(\zeta\overline{a}^{\nu})$$

- The asymptotic expansions at $\zeta \to \infty$ look like

$$\log \boldsymbol{\tau}^*(\zeta) \simeq \sum_{j=1}^{\infty} \boldsymbol{\tau}^*_{2j-1}\zeta^{-\frac{2j-1}{\nu}},$$

$$\frac{1}{\sqrt{\boldsymbol{\tau}^*(\zeta)}}\boldsymbol{\beta}^*(\zeta) \simeq \sum_{j=1}^{\infty} \boldsymbol{\beta}^*_{2j-1}\zeta^{-\frac{2j-1}{\nu}}, \quad \frac{1}{\sqrt{\boldsymbol{\tau}^*(\zeta)}}\boldsymbol{\gamma}^*(\zeta) \simeq \sum_{j=1}^{\infty} \boldsymbol{\gamma}^*_{2j-1}\zeta^{-\frac{2j-1}{\nu}}$$

- It can be shown that the lattice primary field scales to CFT primary field

$$\frac{\phi_\alpha(0)}{\phi_\alpha(-\infty)} = \lim_{a\to 0} q^{2\alpha S(0)}, \qquad \Delta_\alpha = \frac{\nu^2\alpha(\alpha-2)}{4(1-\nu)}$$

- The right way of understanding the scaling limit in the Space direction is via the JMS theorem.

The scaling limit of the JMS theorem

- The JMS theorem becomes

$$Z^{\kappa,\kappa'}\left\{\boldsymbol{\tau}^*(\zeta_1^0)\dots\boldsymbol{\tau}^*(\zeta_p^0)\boldsymbol{\beta}^*(\zeta_1^+)\dots\boldsymbol{\beta}^*(\zeta_r^+)\boldsymbol{\gamma}^*(\zeta_r^-)\dots\boldsymbol{\gamma}^*(\zeta_1^-)\phi_{|\alpha\rangle}(0)\right\}$$
$$=\prod_{i=1}^{p}\rho^{(\mathrm{sc})}(\zeta_i^0;\kappa,\kappa')\times\det(\omega^{(\mathrm{sc})}(\zeta_i^+,\zeta_j^-;\kappa,\kappa',\alpha))_{i,j=1,\dots,r}.$$

- The asymptotic expansions for $\zeta,\xi\to\infty$ are

$$\log\rho^{(\mathrm{sc})}(\zeta;\kappa,\kappa')\simeq\sum_{j=1}^{\infty}\zeta^{-\frac{2j-1}{\nu}}C_j(I_{2j-1}^+-I_{2j-1}^-),$$

$$\omega^{(\mathrm{sc})}(\zeta,\xi)\simeq\sqrt{\rho^{(\mathrm{sc})}(\zeta)\rho^{(\mathrm{sc})}(\xi)}\sum_{i,j=1}^{\infty}\zeta^{-\frac{2i-1}{\nu}}\xi^{-\frac{2j-1}{\nu}}\omega_{i,j}(\kappa,\kappa',\alpha).$$

- As a result, the partition functions of the six-vertex model in the scaling limit become the three-point correlation functions of CFT:

$$\lim_{\text{scaling}}Z_{\mathbf{n}}^{\kappa}\left\{q^{2\alpha S(0)}\mathcal{O}\right\}=\frac{\langle\Delta_{-\kappa'+1}|P_\alpha(\{\mathbf{I}_{-k}\})\phi_\alpha(0)|\Delta_{\kappa+1}\rangle}{\langle\Delta_{-\kappa'+1}|\phi_\alpha(0)|\Delta_{\kappa+1}\rangle}$$

Definitions of $\omega(\zeta,\xi)$

- The original definition via deformed Abelian integrals [HGS III] explicitly states the analytic properties of $\omega(\zeta,\xi)$, but is not suitable for taking the thermodynamic limit $\mathbf{n}\to\infty$

- A method to calculate the function $\omega(\zeta,\xi)$ was developed in the case when $\kappa=\kappa', \rho=\rho^{(\mathrm{sc})}=1$, i.e. modulo integrals of motion [Boos, Jimbo, Miwa, Smirnov (10)]. It is based on a linear integral equation for the function $\omega(\zeta,\xi)$ and employs the standard Wiener-Hopf method.

- To extend the result for the case when $\kappa\neq\kappa'$, a new method was proposed in [Boos, Göhmann, (10, 12)]. It utilizes the so-called Master function $\Phi(\zeta,\xi;\kappa,\kappa',\alpha)$. The main result, interesting for us, is the relation

$$\omega^{(\mathrm{sc})}(\zeta,\xi;\kappa,\kappa',\alpha)=\delta^-_\zeta\delta^-_\xi\Phi(\zeta,\xi;\kappa,\kappa',\alpha),$$

where

$$\delta^-_\zeta f(\zeta)=f(q\zeta)-\rho(\zeta)f(\zeta)$$

The master function approach

- The master function satisfies the equation

$$\underset{\zeta=\lambda_r}{\mathrm{res}}\left(\frac{\Delta_\zeta\widetilde{\Phi}(\zeta,\xi;\kappa,\kappa',\alpha)+\psi(\zeta/\xi,\alpha)}{\rho(\zeta;\kappa,\kappa')(1+\mathfrak{a}(\zeta,\kappa))}+\widetilde{\Phi}(\zeta,\xi;\kappa,\kappa',\alpha)\right)=0,$$

$$\Phi(\zeta,\xi;\kappa,\kappa',\alpha)=\widetilde{\Phi}(\zeta,\xi;\kappa,\kappa',\alpha)+\Delta_\zeta^{-1}\psi(\zeta/\xi,\alpha),$$

$$\Delta_\zeta f(\zeta)=f(q\zeta)-f(q^{-1}\zeta),$$

which is a consequence of the TQ-relation.

- This equation can be solved explicitly at the free fermion point $\nu=1/2$ with the help of the ODE/IM correspondence:

$$\left(-\frac{d^2}{dz^2}+z^2+\left(\kappa^2-\frac{1}{4}\right)\frac{1}{z^2}\right)\psi_n(z)=E_n\psi_n(z)$$

$$\psi_n(z)=z^{\kappa+\frac{1}{2}}\,\mathrm{e}^{-\frac{z^2}{2}}\,L_n^{(\kappa)}(z^2),\quad \psi_n^\dagger(z)=2\frac{n!}{\Gamma(n+\kappa+1)}z^{\kappa+\frac{1}{2}}\,\mathrm{e}^{-\frac{z^2}{2}}\,L_n^{(\kappa)}(z^2)$$

- By construction, the master function has poles at the Bethe roots

$$\pi\lambda_n^2 = \frac{1+\kappa}{2} + n, \quad n = 0, \pm 1, \pm 2, \dots$$

- Using the ansatz

$$\widetilde{\Phi}(\zeta,\xi;\kappa,\kappa',\alpha) = \zeta^\alpha \xi^{2-\alpha} \sum_{m=0}^{\infty} \frac{R_m}{\pi\zeta^2 - \left(m + \frac{1+\kappa}{2}\right)},$$

we rewrite the equation for the master function as system of linear equations

$$R_n + \frac{1}{\rho_n} \frac{\sin\left(\frac{\pi\alpha}{2}\right)}{\pi} \sum_{m=0}^{\infty} \frac{R_m}{m+n+\kappa+1} = \frac{1}{\rho_n} M_n$$

- Define

$$\widetilde{R}(z) = \sum_{m=0}^{\infty} \frac{\Gamma(m+1)}{\Gamma(m+\frac{\kappa-\kappa'}{2}+1)} R_m \psi_m(z), \quad \widetilde{M}(z) = \sum_{m=0}^{\infty} \frac{\Gamma(m+\frac{\kappa+\kappa'}{2}+1)}{\Gamma(m+\kappa+1)} M_m \psi_m(z)$$

- The master function equation can be rewritten in the integral form

$$\widetilde{R}(z) + \frac{\sin(\frac{\pi\alpha}{2})}{\pi} \int_0^\infty \widetilde{G}(z,w)\widetilde{R}(w)\mathrm{d}w = \widetilde{M}(z)$$

- Explicitly,

$$\widetilde{G}(z,w) = \sum_{m,n=0}^{\infty} \frac{\Gamma(n+\frac{\kappa+\kappa'}{2}+1)}{\Gamma(n+\kappa+1)}\psi_n(z)\frac{1}{m+n+\kappa+1}\frac{\Gamma(m+\frac{\kappa-\kappa'}{2}+1)}{\Gamma(m+1)}\psi^\dagger_m(w)$$
$$= 2\left(\frac{z}{w}\right)^{-\kappa'}(zw)^{\frac{1}{2}}\frac{\mathrm{e}^{-\frac{1}{2}(z^2+w^2)}}{z^2+w^2} = \left(\frac{z}{w}\right)^{-\kappa'} G(z,w).$$

- The integral operator with the kernel $G(z,w)$ is diagonalizable:

$$\int_0^\infty \mathrm{d}w\, G(z,w) f(w,k) = \frac{\pi}{\cosh(\pi k)} f(z,k), \qquad f(z,k) = z^{-\frac{1}{2}} K_{\mathrm{i}k}\left(\frac{z^2}{2}\right),$$

where $K_{\mathrm{i}k}(x)$ is the modified Bessel function of the second kind.

- We can compute the inverse of the following integral operator

$$\left(1+\frac{\sin\frac{\pi\alpha}{2}}{\pi}G\right)^{-1}(z,w) = \frac{4}{\pi^2}\int_0^\infty \mathrm{d}k\,\frac{k\sinh\pi k\cosh\pi k}{\cosh\pi k+\sin\frac{\pi\alpha}{2}}\frac{1}{(zw)^{\frac{1}{2}}}K_{\mathrm{i}k}(\frac{1}{2}z^2)K_{\mathrm{i}k}(\frac{1}{2}w^2)$$

After further simplifications, the free fermion master function explicitly reads

$$\widetilde{\Phi}(\zeta,\xi;\kappa,\kappa',\alpha)=\frac{\mathrm{i}}{\pi}\zeta^{\alpha}\xi^{2-\alpha}\Gamma(-\pi\zeta^2+\frac{1+\kappa}{2})\Gamma(-\pi\xi^2+\frac{1+\kappa}{2})$$
$$\times\int_0^\infty \mathrm{d}m_\alpha(k)\frac{F(\zeta,k;\kappa,\kappa')F(\xi,k;\kappa,\kappa')}{\Gamma(\frac{1+\kappa-\kappa'}{2}+\mathrm{i}k)\Gamma(\frac{1+\kappa-\kappa'}{2}-\mathrm{i}k)\Gamma(\frac{1+\kappa+\kappa'}{2}+\mathrm{i}k)\Gamma(\frac{1+\kappa+\kappa'}{2}-\mathrm{i}k)}$$

Here

$$F(\zeta,k;\kappa,\kappa')=\int_{\mathrm{i}\mathbb{R}-0}\frac{\mathrm{d}s}{2\pi\mathrm{i}}\frac{\Gamma(-s)\Gamma(-s+\frac{\kappa+\kappa'}{2})\Gamma(-s+\frac{\kappa-\kappa'}{2})\Gamma(s+\mathrm{i}k+\frac{1}{2})\Gamma(s-\mathrm{i}k+\frac{1}{2})}{\Gamma(-s-\pi\zeta^2+\frac{1+\kappa}{2})},$$

$$\mathrm{d}m_\alpha(k)=\mathrm{d}k\frac{k\sinh\pi k\cosh\pi k}{\cosh\pi k+\sin\frac{\pi\alpha}{2}}$$

- The master function has poles at the Bethe roots!

Relating two bases

- Calculating the asymptotic expansion of $\delta^-_\zeta \delta^-_\xi \Phi(\zeta, \xi; \kappa, \kappa', \alpha)$, we obtain the modes $\omega_{i,j}(\kappa, \kappa', \alpha)$.
- This allows us to find the relation between the generators of the fermionic basis and the Virasoro algebra:

$$
\begin{aligned}
\boldsymbol{\beta}_1^*\boldsymbol{\gamma}_1^* &\simeq \mathbf{l}_{-2} + \frac{2}{\alpha(\alpha-2)}\mathbf{i}_1^2, \\
\boldsymbol{\beta}_1^*\boldsymbol{\gamma}_3^* &\simeq \mathbf{l}_{-2}^2 + \left(\frac{2c-32}{9} + \frac{2}{3}d_\alpha\right)\mathbf{l}_{-4} + \frac{16\left(\alpha^3+7\alpha^2-17\alpha-6\right)}{3(\alpha-6)(\alpha-4)(\alpha-2)\alpha(\alpha+2)}\mathbf{i}_1\mathbf{i}_3 \\
&\quad + \frac{4(\alpha+18)}{(\alpha-6)(\alpha-4)(\alpha+2)}\mathbf{i}_1^2\mathbf{l}_{-2} + \frac{4\left(\alpha^2-4\alpha+6\right)}{3(\alpha-6)(\alpha-4)(\alpha-2)\alpha}\mathbf{i}_1^4
\end{aligned}
$$

- In order to proceed with higher levels, the particle-hole excitations have to be considered as well.

Summary and outlook

- Using the fermionic basis approach, we were able to calculate all the correlation functions of the CFT at the free fermion point. The results incorporate the action of the integrals of motion as well.
- The most natural next step is to consider the three-point correlation functions with excited asymptotic states. We plan to apply the same techniques using BLZ monstrous potentials [Bazhanov, Lukyanov, Zamolodchikov (03)].
- It would be interesting to understand more systematically how the fermionic basis and the ODE/IM correspondence are related saway from the free fermion point. Is it possible to write the master function explicitly via the solutions of the ODE?

Thank you for your attention!